Using Fermat Number Transform to Accelerate Convolutional Neural Network

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Outline

1. Introduction on CNNs
2. Motivation and Related Work
3. Proposed FNT Acceleration
4. Analysis and Implementation Results
5. Conclusion

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Convolutional Neural Networks

- Deep neural networks with convolution
- Processing 1D data (time series), 2D data (music) and 3D data (image)
- Strong capacities to extract features

Convolutional Neural Networks

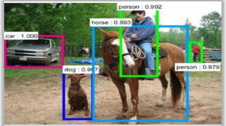
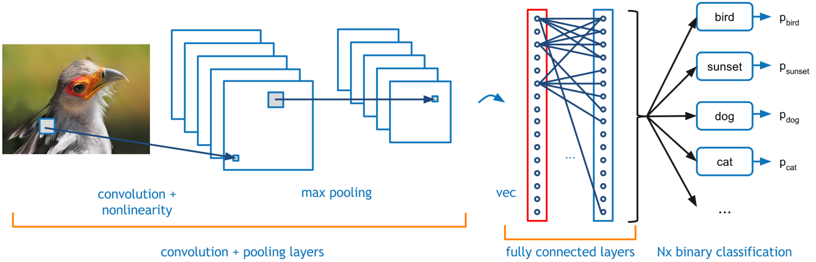


Image Segmentation

Game

....

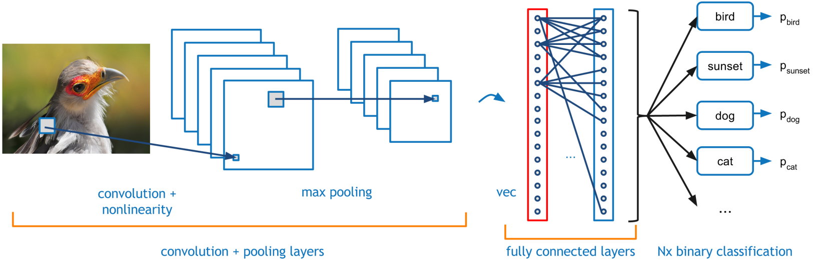
Convolutional Neural Networks



- **Convolution**, **Pooling**, **Non-linear** and **Fully-connected** layers stacked in stages

[1] https://bigsnarf.wordpress.com

Convolutional Neural Networks

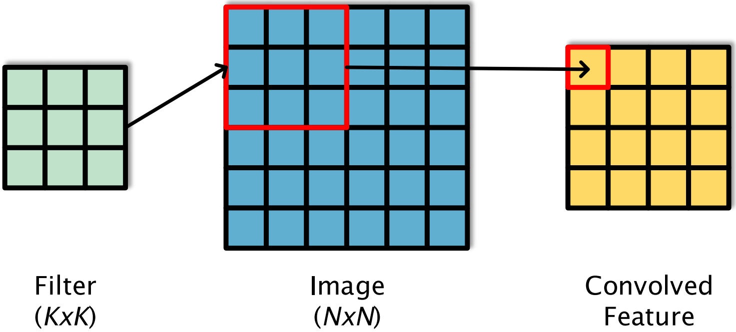


- **Convolution**: feature extraction by convolving various filters over input image
- **Fully-connected**: linear transform over input features
- **Pooling and Non-linear**: perform down sampling and non-linear function

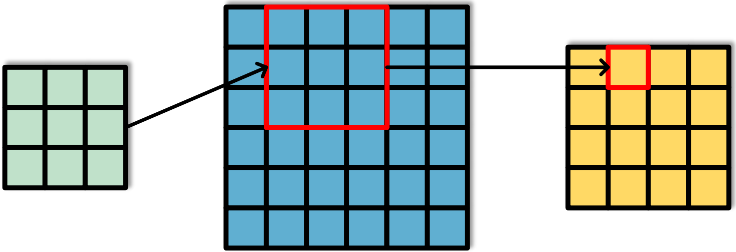
Major Challenges

- **Computation-intensive**: convolution takes up over 95% of ovarall complexity
- **Memory-intensive**: FC layers contribute 90% parameters

Convolution in Single Channel



Convolution in Single Channel



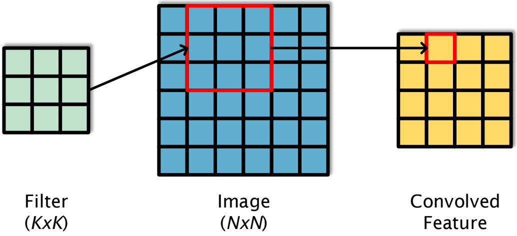
Filter
(KxK)

Image
(NxN)

Convolved
Feature

Outline

Prohibitive Complexity



- Computation Complexity: $\mathcal{O}(N^2K^2)$
- Multiplier is expensive!

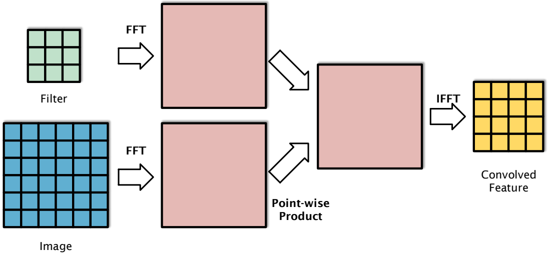
Related Work

1. N. Vasilache et al., "Fast Convolutional Nets With fbfft: A GPU Performance Evaluation," in *ICLR*, 2015
2. C. Zhang and V. Prasanna, "Frequency domain acceleration of convolutional neural networks on CPU-FPGA shared memory system," in *ACM/SIGDA ISFPGA*, 2017
3. T. Abtahi et al., "Accelerating convolutional neural network with fft on tiny cores," in *IEEE ISCAS*, 2017

Convolution based on FFT

Convolution property:

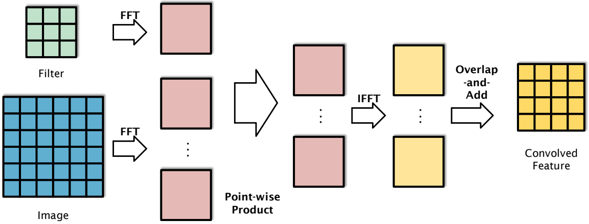
$$y(n) = x * h = \mathcal{F}^{-1}\left\{\mathcal{F}(x(n)). * \mathcal{F}(h(n))\right\}$$



- Computation Complexity: $\mathcal{O}(N^2 \log N)$
- Effective only when $\log N \geq K^2$
- Intensive intermediate memory requirement

Convolution based on Overlap-and-Add FFT

$$y(n) = \sum_k \mathcal{F}^{-1}\left\{\mathcal{F}(x(n-kL)).*\mathcal{F}(h(n))\right\}$$



- Computation Complexity: $\mathcal{O}(N^2 \log K)$
- Effective for small filters
- Negligible intermediate memory build up
- Additional modules for overlap and add

OaA-FFT is the best?

Problems to Solve

- Complex number operation of FFT
- Additional registers for twiddle factors
- Introducing roundoff noise during FFT/IFFT under low-bit fixed point
- Frequency information is not what we need

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Number Theoretic Transform

- Number Theoretic Transform:

$$X(k) = \mathcal{F}(x(n)) = \sum_{n=0}^{N-1} \left\langle x(n)\alpha^{nk} \right\rangle_m$$

$\langle a \rangle_b$: modular arithmetic

- To Dicrete Fourier Transform:

$$X(k) = \mathcal{F}(x(n)) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

Replace α with twiddle factor W_N to obtain DFT

Fermat Number Transform

- Replace α with 2:

$$X(k) = \mathcal{F}(x(n)) = \sum_{n=0}^{N-1} \left\langle x(n) 2^{nk} \right\rangle_{F_t}$$

F_t: the t-th Fermat number

- Constraints:

$$\begin{cases} F_t = 2^b + 1 \\ b = 2^t, \text{input bits} \\ N = 2^{t+1}, \text{sequence length.} \end{cases}$$

- Easy to satisfy with $F_4 = 2^{16} + 1, N = 32, b = 16$.

Overlap-and-Add FNT

- Similar to OaA FFT:

$$y(n) = \sum_k \mathcal{F}^{-1} \left\{ \mathcal{F}(x(n-kL)). * \mathcal{F}(h(n)) \right\},$$

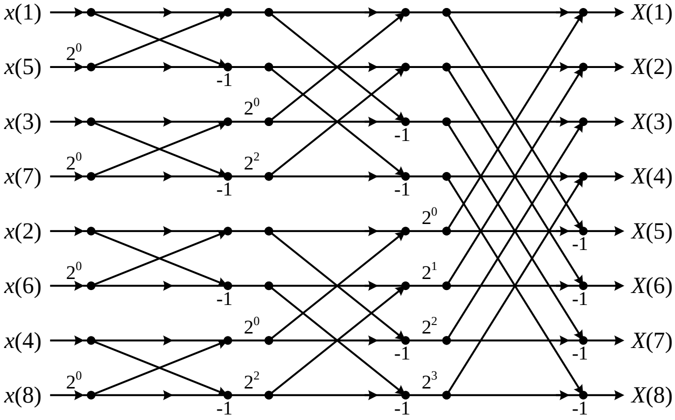
where L is an arbitrary segment length and $L \leq N - K + 1$.

- For maximum efficiency, select $L = N - K + 1$

- Run-time configurable for various input sizes by adjusting L and k

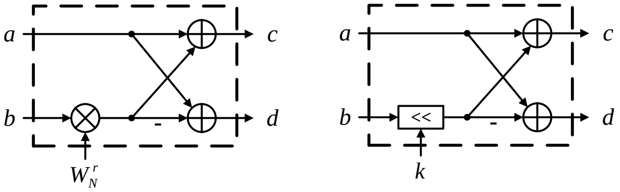
Factor Graph

- $t = 2$, $N = 2^{t+1} = 8$, $b = 2^t = 4$, $F_t = 2^b + 1 = 17$



Hardware Architecture

- No need for extra registers
- Area and power efficiency



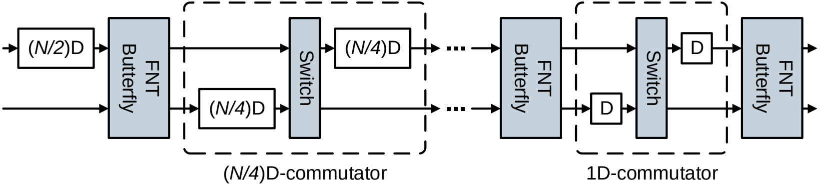
Butterfly modules of FFT and FNT

Complex multiplier $\longrightarrow$ Shift register

Complex adder $\longrightarrow$ Real adder and modular

Pipelined Architecture

- Increased efficiency of basic units
- Reduced area and IO bandwidth

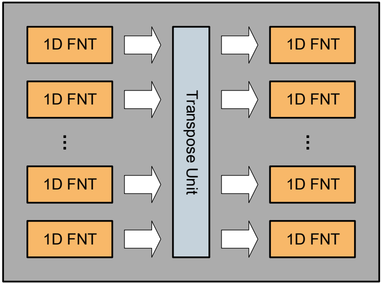


$$\text{Efficiency: } \frac{1}{\log_2 N} \longrightarrow {\color{red}50\%}$$

$$\text{Butterfly : } \frac{N}{2}\log_2 N \longrightarrow {\color{red}\log_2 N}$$

2D FNT based on 1D FNT

- First, perform 1D FNT on each row of the input image
- Then, perform 1D FNT on each column to the intermediate results



2D FNT Kernel

Data Processing for CNNs

- Range of FNT filtering data: $[0, 2^b]$
- Image and weights of CNNs are basically real numbers
- Transform input data with range $[-2^{b-1}, 2^{b-1}]$ into $[0, 2^b]$:

$$x^{'} = \begin{cases} x, & \text{if } 0 \le x \le 2^{b-1}, \\ x + F_t, & \text{if } -2^{b-1} \le x < 0. \end{cases}$$

- **Example:** $F_2 = 2^4 + 1 = 17$ and $2^{b-1} = 8$

$$x = [1, -2, 3, -4] \longrightarrow x^{'} = [1, 15, 3, 13]$$

Data Processing for CNNs

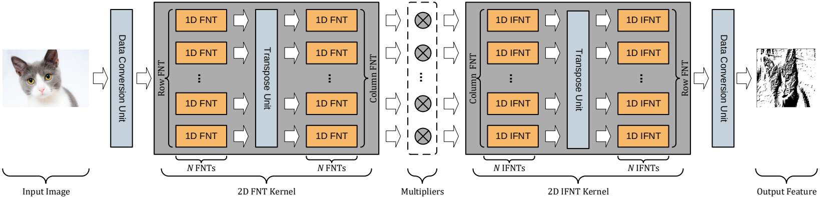
- Transform output data back to real domain:

$$y = \begin{cases} y^{'}, & \text{if } 0 \le y^{'} \le 2^{b-1}, \\ y^{'} - F_t, & \text{if } 2^{b-1} \le y^{'} \le 2^{b}. \end{cases}$$

- **Example:** $h = [1, 1, 1, 1]$, $F_2 = 2^4 + 1 = 17$ and $2^{b-1} = 8$

$$\begin{aligned} y^{'} &= x * h = [1, 16, 2, 15, 14, 16, 13, 0] \\ \longrightarrow y &= [1, -1, 2, -2, -3, -1, -4, 0] \end{aligned}$$

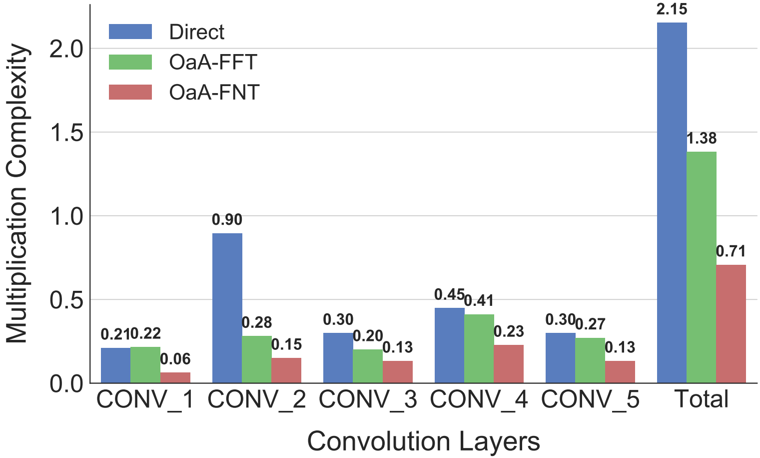
Overall Architecture



- Advantages
  - High data parallelism
  - Further reduction on arithmetic complexity
  - Suitable for various filter and image sizes
  - Round-off noise is zero

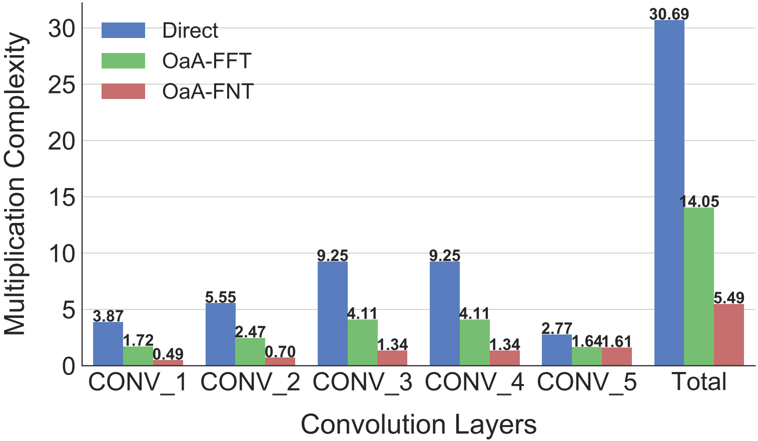
Outline

Complexity Analysis on AlexNet



[1][Zhang and Prasanna, *ISFPGA*, 2017]

Complexity Analysis on VGGNet



[1] [Zhang and Prasanna, *ISFPGA*, 2017]

Experimental Setup

- Xilinx Virtex-7 XC7VX485t FPGA Platform
- Intel i7-7700k CPU
- Quantize pre-trained VGGNet16 from Pytorch into 16 fixed point
- Pre-calculate FNT of weights and load to FPGA
- FNT parameters: $F_4 = 2^{16} + 1, N = 32, b = 16$
- We implement the accelerator for convolutional layer

Implementation Results

Design	Zhang et al. 2015	Qiu et al. 2016	This work
Platform	Virtex-7 VX485t	Zynq XC7Z045	Virtex-7 VX485t
Clock(MHz)	100	150	150
CNN Model	AlexNet	VGG16-SVD	VGG16
Quantization	32-bit float	16-bit fixed	16-bit fixed
LUT	186251	182616	188928
FF	205704	127653	142592
DSP	2240	780	**256**
BRAM	1024	486	512
CONV Throughput (GOP/s)	61.62	187.80	**264.60**
Resource Efficiency (GOP/s/DSP)	0.028	0.241	**1.033**

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Conclusion

- Efficient OaA-FNT-based 2D convolver to accelerate CNNs
- Data processing of CNNs to fit into FNT
- $\mathbf{1.41}\times$ convolution throughput with $\mathbf{3.05}\times$ less DSPs compared to state-of-the-art design on VGGNet.
- Future Work
  - Use data with lower-bit quantization
  - Build reconfigurable design for variable length

Thanks for Your Attention!

Q & A