Polar Decoding on Sparse Graphs with Deep Learning

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Outline

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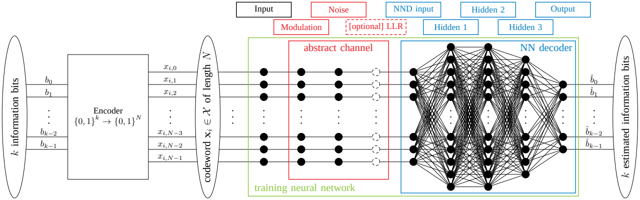
1. Related Work
2. Deep Learning for Polar Codes on Sparse Graphs
3. Results and Analysis
4. Conclusion

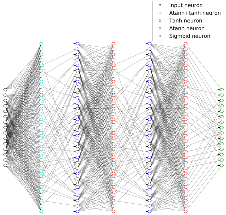
Neural Network Decoder for Polar Codes



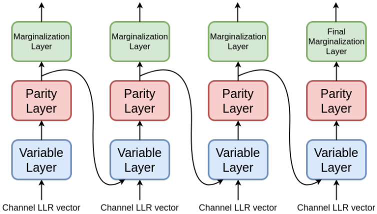
- Construct polar decoder based on fully-connected neural networks.
- **Pros & cons**:
  - ∗ Near-optimal performance for very short codes.
  - ∗ Hard to be extended to long codes.
  - ∗ Prohibitive complexity of NN inference.

[1] [Gruber, Cammerer, Hoydis, *et al.*, *CISS* 2017]

Neural Network Decoder for Linear Codes



Feed-forward network

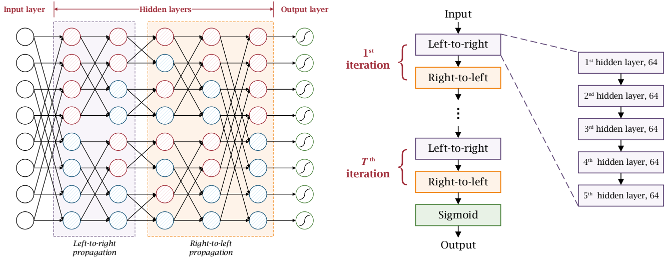


Recurrent network

- Tanner graph is unfolded into feed-forward or recurrent neural networks.
- **Pros & cons**:
  - ∗ Improving performance through training.
  - ∗ Easy to co-operate with other decoding methods, such as permutation.
  - ∗ **Complexity**: Near-BP. **Latency**: $2I$

[2][Nachmani, Marciano, Lugosch, *et al.*, *JSTSP* 2018]

Neural Network Decoder for Polar Codes



- Construct polar decoder based on factor graph.
- **Pros & cons**:
  - ∗ Near-BP performance.
  - ∗ Easy to extend.
  - ∗ **Complexity**: $\mathcal{O}(IN\log_2 N)$ with min-sum. **Latency**: $2I\log_2 N$

[3] [Xu, Wu, Ueng, *et al.*, *SiPS* 2017]

Some Questions

- Substantial weights are unfriendly for implementation.
- Is it necessary to parameterize every edge?
- Constructing neural network decoder of polar codes on Tanner graph?

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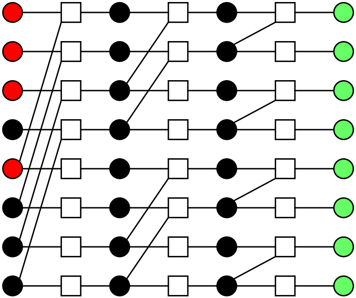
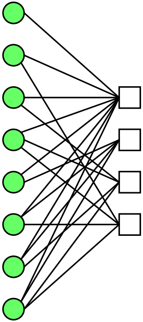
Two Types of Factor Graphs

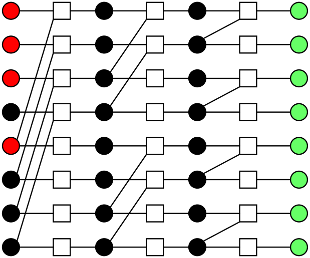


G-matirx-based



LDPC-like

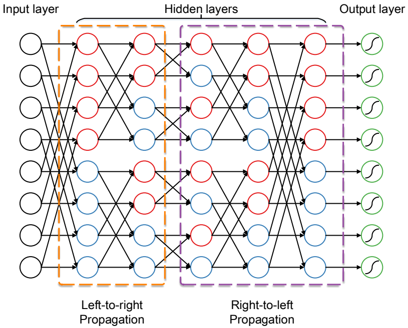
BP Decoding for Polar Codes



- **Original BP Decoding**:

$$\begin{cases} L_{i,j}^{(t)} = g(L_{i+1,j}^{(t-1)}, L_{i+1,j+N/2^i}^{(t-1)} + R_{i,j+N/2^i}^{(t)}), \\ L_{i,j+N/2^i}^{(t)} = g(L_{i+1,j}^{(t-1)}, R_{i,j}^{(t)}) + L_{i+1,j+N/2^i}^{(t-1)}, \\ R_{i+1,j}^{(t)} = g(R_{i,j}^{(t)}, L_{i+1,j+N/2^i}^{(t-1)} + R_{i,j+N/2^i}^{(t)}), \\ R_{i+1,j+N/2^i}^{(t)} = g(R_{i,j}^{(t)}, L_{i+1,j}^{(t-1)}) + R_{i,j+N/2^i}^{(t)}, \end{cases}$$

Optimization on G-matrix Factor Graph

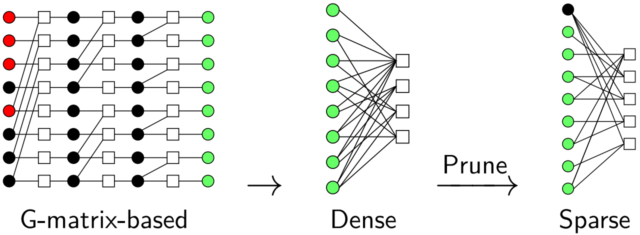


- Each node is approximated and parameterized:

$$g(x, y) = \ln \frac{1 + e^{x+y}}{e^x + e^y} \approx \alpha_{i,j} \times \mathsf{sign}(x)\mathsf{sign}(y) \times \min(|x|, |y|).$$

[3][Xu, Wu, Ueng, *et al.*, *SiPS* 2017]

BP Decoding for Polar Codes



▷ **BP Decoding on Tanner Graph**:

$$L_{t,e=(v,c)} = L_v + \sum_{e'=(c',v),c'\neq c} L_{t-1,e'},$$

$$L_{t,e=(c,v)} = \left(\prod_{e'=(v',c),v'\neq v} \mathrm{sign}\left(L_{t,e'}\right) \right) \times 2\tanh^{-1}\left(\prod_{e'=(v',c),v'\neq v} \tanh\left(\frac{\left|L_{t,e'}\right|}{2} \right) \right).$$

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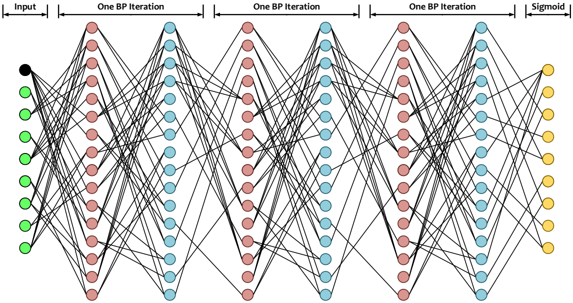
[4] [Cammerer, Ebada, Elkelesh, *et al.*, *ISIT* 2018]

Optimization on Tanner Graph

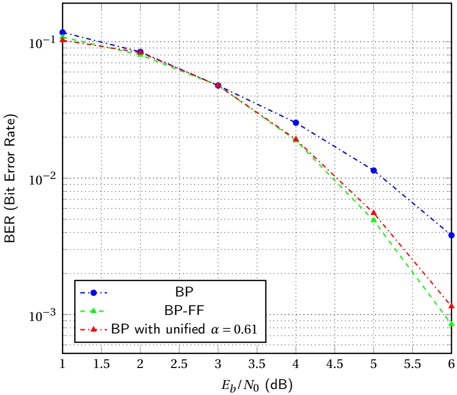


▷ Assign weights $\alpha_{i,e,e'}$ to check-to-variable edges:

$$x_{i,e=(c,v)} = \alpha_{i,e,e'} \times \prod_{e'} \mathsf{sign}(x_{i-1,e'}) \times \min_{e'}(|x_{i-1,e'}|).$$

An Example on BCH $(63, 36)$

- ▷ Using one unified weight achieves comparative performance.
- ▷ Assigning weights to every edge is redundant.



[2] [Nachmani, Marciano, Lugosch, *et al.*, *JSTSP* 2018]

Weights Reduction

- Assigning weights to every edge is redundant.
- Restrict training weights to one α parameter:

$$x_{i,e=(c,v)} = \alpha \times \prod_{e'} \mathsf{sign}(x_{i-1,e'}) \times \min_{e'}(|x_{i-1,e'}|).$$

Training Methods

- The output LLRs are squashed to $(0,1)$ probability by sigmoid:

$$o_i = \sigma(L_i) = \frac{1}{1 + e^{-L_i}}.$$

- **Binary cross entropy (BCE)** is adopted as loss function:

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{o}) = -\frac{1}{N} \sum x_i \log(o_i) + (1 - x_i) \log(1 - o_i).$$

- **Optimization target**: Search optimal parameters or their combination resulting in minimum BCE loss.

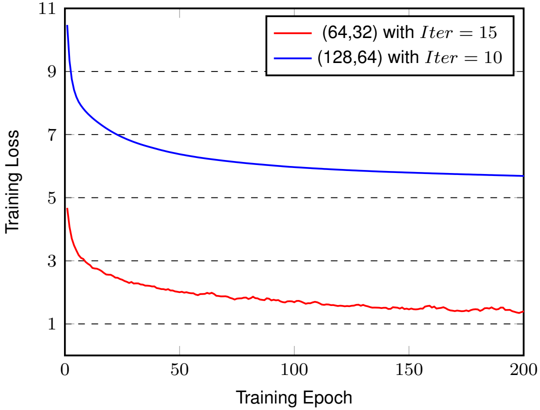
$$\alpha^* = \arg\min_{\alpha} \mathcal{L}(\boldsymbol{x}, \boldsymbol{o}).$$

Outline

Experiment on Tanner Graph

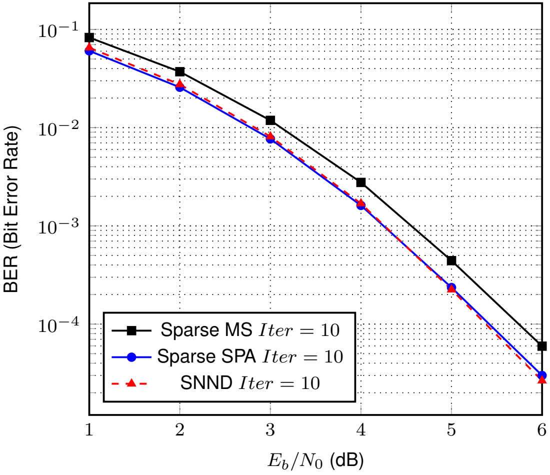
Parameters	Value
Code Length	64, 128, 256
Channel	AWGN with BPSK
SNR Range	1, 2, 3, 4, 5, 6
Optimizer	Mini-batch SGD with Adam
Learning Rate	Lr=0.001
Weights Initialization	$\mathcal{N} \sim (\mu = 1, \sigma = 0.1)$
Training Samples per SNR	20
Training mini-batch Size	120
Training Codewords	All zero codewords with noise
Validation Set Size	50000 per SNR
Validation Codewords	Random codewords with noise

Training Results

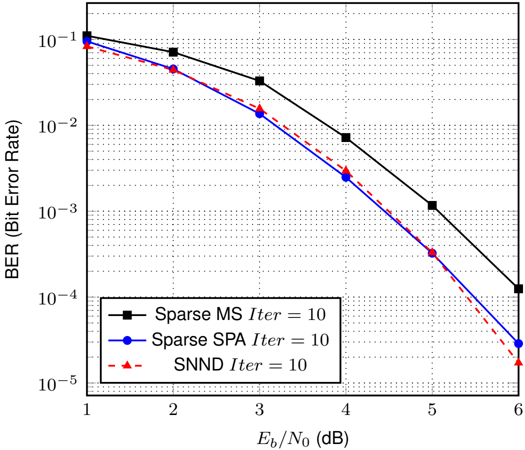


- Converge at around 150 epochs.

Performance Comparison with $N = 64$



Performance Comparison with $N = 128$



Optimizing with Just One Weight

- Constraint multiple weights to one unified α.
- **Initialization**: Initializing α to one provides a good starting point (equivalent to Min-sum):

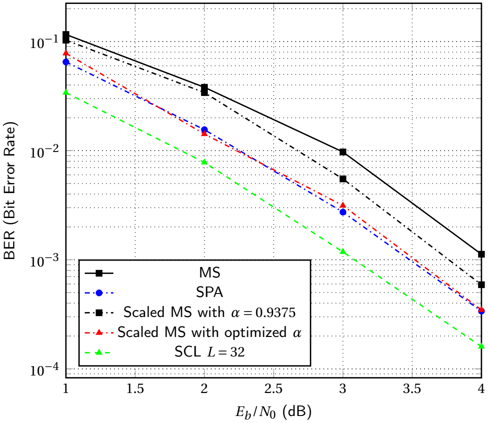
$$x_{i,e=(c,v)} = \prod_{e'} \mathsf{sign}(x_{i-1,e'}) \times \min_{e'}(|x_{i-1,e'}|).$$

- Obtain good parameter through adaptive training instead of greedy searching as in [5].
- Train by unfolding to 10 iterations and test with 50 iterations.

[5] [Yuan and Parhi, *TSP* 2014]

Performance Comparison with $N = 128$

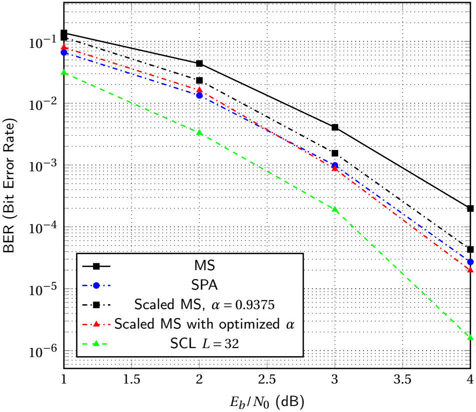
- Trained α is close to 0.85.
- 0.2 dB gain over empirical scaling factor $\alpha = 0.9375$ suggested in [5].



[5] [Yuan and Parhi, *TSP* 2014]

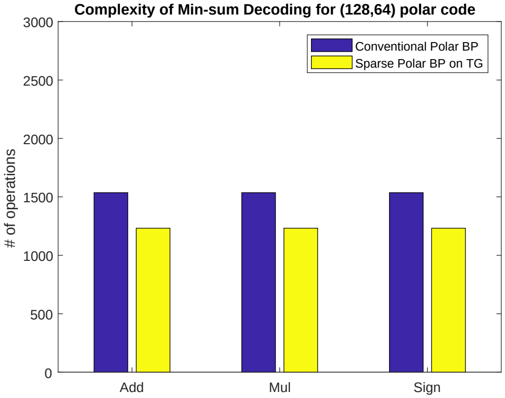
Performance Comparison with $N = 256$

- 0.1 dB gain over empirical scaling factor $\alpha = 0.9375$ suggested in [5].

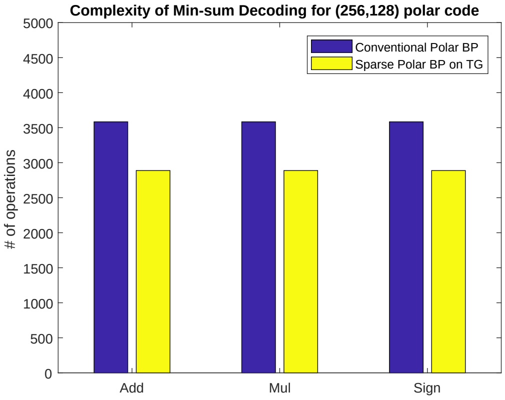


[5] [Yuan and Parhi, *TSP* 2014]

Complexity Comparison



Complexity Comparison



▷ About 25% complexity reduction compared with original polar BP.

Comparison of Decoding Latency

- **Latency on G-matrix factor graph**: $2I \log_2 N$.
- **Latency on sparse Tanner graph**: $2I$ - code length independent.
- **Latency reduction**: $\log_2 N$.

Outline

Conclusion

- **Polar neural network decoder based on Tanner graph.**
  - Reduced decoding complexity
  - Higher parallelism
- **Reduction of training weights.**
  - Restricting training weights to only one
- **Optimizing polar codes on two types of graphs.**
  - Tanner graph
  - Original factor graph

Reference

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Thanks for Your Attention!

Q & A