Theory: Graph

22 minutes 5 / 9 problems solved

Start practicing

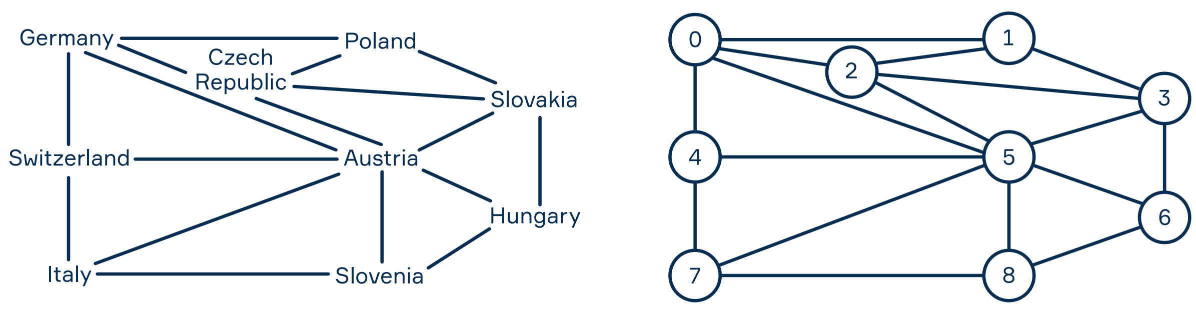
Many real-life problems can be naturally and concisely depicted as a collection of objects and links between them. Such representation is known as a **graph**. A graph can be considered as a set of objects (usually named as **nodes** or **vertices**) and links (**edges**) that connect these objects with each other.

For example, consider the problem of coloring a political map. Here, we need to assign a color to each country taking an obvious restriction into account: two neighbors should have different colors. To get rid of the unnecessary details of a map, we may represent each country as a node and connect two countries with a link if they are neighbors.

Consider the following example:

Source: www.mathsisfun.com

Here is a partial map of Europe. To get a more precise view of the borders, we represent each country as a node and connect two countries if they are neighbors. This results in the following graph:



The left figure corresponds to a map with borders represented as links between the countries, and the right figure is a graph with each country substituted by a node. Thus, we reduce the problem of coloring a political map to the problem of coloring the nodes of a graph. Although the example above is quite simple, such representation would be indispensable for larger maps: it allows us not only to keep the important details but also to automate the solution to this problem.

Consider another example where graphs might be useful. Assume that you need to drive from one point in your city to another. Chances are, you'd probably rely on services like Google Maps or MapQuest. We all use them, but have you ever thought about how they work under the hood? A possible way is to represent the city as a graph with nodes corresponding to

Current topic:

Graph

Topic depends on:

Data structures

Topic is required for:

Tree

Basic terminology

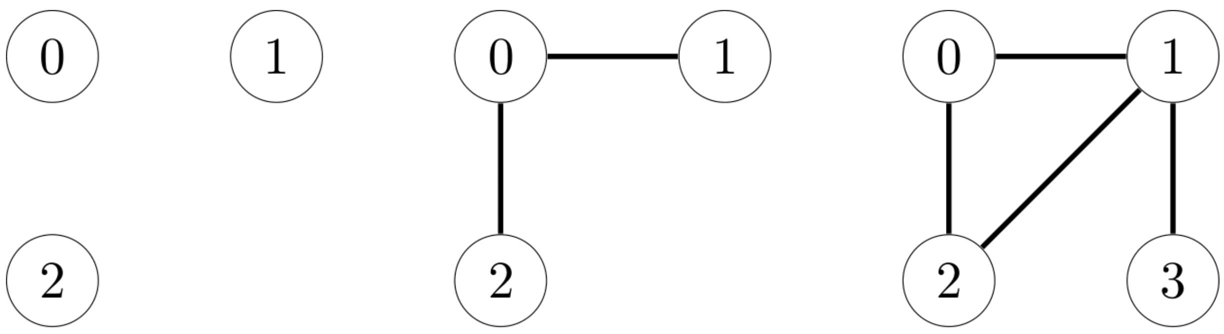
Table of contents:

intersections and edges showing the roads. Then, in terms of graphs, our question is: what is the shortest path between the start node and the destination node?

Map coloring and finding the shortest path are not the only problems where graphs might come in handy. For instance, the World Wide Web can also be represented as a graph with nodes (sites) and edges (corresponding links between these sites). In social networks, people can be considered as nodes and a link between two people shows that one of them follows the other. In other words, a graph is a convenient structure that is suitable for modeling various real-life problems.

§1. Formal definition

Using formal language is essential when it comes to working with graphs conveniently, being able to store them on a computer, and describing algorithms for their processing. Formally, a graph G is a pair of two sets: $G = (V, E)$. Here, V is a set of nodes and E is a set of edges. Each edge is a pair of nodes connected by this edge. Consider the following examples:



G_1 G_2 G_3

Here we have three graphs. The first graph G_1 consists of three nodes and has no edges. So,

$G_1 = (V_1, E_1),\ V_1 = \{0, 1, 2\},\ E_1 = \emptyset$

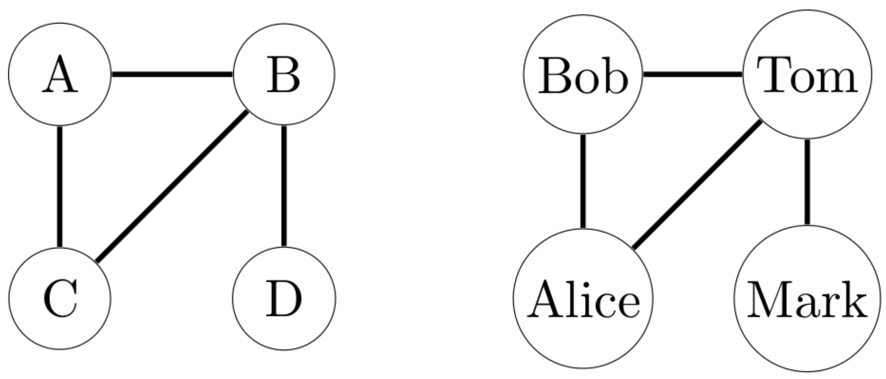
The second graph G_2 is the same as G_1 but contains two edges:

$G_2 = (V_2, E_2),\ V_2 = \{0, 1, 2\},\ E_2 = \{\{0, 1\}, \{0, 2\}\}$

The third graph G_3 consists of four nodes and four edges:

$G_3 = (V_3, E_3),\ V_3 = \{0, 1, 2, 3\},\ E_3 = \{\{0, 1\}, \{0, 2\}, \{1, 2\}, \{1, 3\}\}$

The nodes in the graphs above are labeled as numbers, but labels might actually be of different types. See the following examples:



G_4 G_5

The graph $G_4 = (V_4, E_4)$ consists of four nodes with labels $V_4 = \{A, B, C, D\}$ and edges $E_4 = \{\{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}\}$. The graph $G_5 = (V_5, E_5)$ represents a group of people, where edges represent acquaintance:

$V_5 = \{Bob, Alice, Tom, Mark\},$

$E_5 = \{\{Bob, Alice\}, \{Bob, Tom\}, \{Alice, Tom\}, \{Tom, Mark\}\}.$

Depending on a particular problem you need to solve, different types of labels might be convenient.

§2. Summary

A graph is a convenient structure that can be used to model many real-life objects and processes. In this topic, we considered several examples where graphs might be useful and learned how to describe this structure formally. Yet in order to efficiently solve real problems, you need to know more about graphs. In the following topics, we will learn some terminology used to describe graphs, consider how to store them on a computer, and then learn some efficient algorithms for their processing.

Report a typo

Thanks for your feedback!

Start practicing

Comments (3) Hints (0) Useful links (1) Show discussion