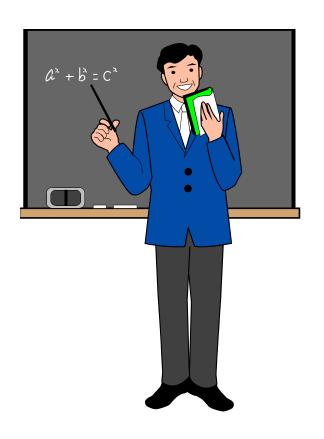# Survey of Calculus

Unit 1, Lesson 1
College Algebra Review

Algebra Review

Properties of Real Numbers

For all real numbers a, b, and c:

1. $a + b = b + a$;   **Commutative properties**
   $ab = ba$;

2. $(a + b) + c = a + (b + c)$;   **Associative properties**
   $(ab)c = a(bc)$;

3. $a(b + c) = ab + ac$.   **Distributive property**

Special Factorizations

$$x^2 - y^2 = (x + y)(x - y) \qquad \text{Difference of two squares}$$

$$x^2 + 2xy + y^2 = (x + y)^2 \qquad \text{Perfect square}$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2) \qquad \text{Difference of two cubes}$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2) \qquad \text{Sum of two cubes}$$

Zero-Factor Property

If a and b are real numbers, with $ab = 0$, then either

$$a = 0 \text{ or } b = 0 \quad \text{(or both)}.$$

Solve $2m^2 + 7m = 15.$

Quadratic Formula
The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Solve $\dfrac{1}{x^2-4}+\dfrac{2}{x-2}=\dfrac{1}{x}$.

Inequality Symbols

$<$ means *is less than*

$>$ means *is greater than*

$\leq$ means *is less than or equal to*

$\geq$ means *is greater than or equal to*

Properties of Inequality

For all real numbers a, b, and c:

1. If $a < b$, then $a + c < b + c$.

2. If $a < b$ and if $c > 0$, then $ac < bc$.

3. If $a < b$ and if $c < 0$, then $ac > bc$.

Solve $3z - 2 > 5z + 7.$

Definition of Exponent

If n is a natural number, then

$$a^n = a \cdot a \cdot a \cdot \cdots \cdot a,$$

where a appears as a factor n times.

Zero and Negative Exponents

If a is any nonzero real number, and if n is a positive integer, then

$$a^0 = 1 \qquad \text{and} \qquad a^{-n} = \frac{1}{a^n}.$$

Properties of Exponents

For any integers m and n, and any real numbers a and b for which the following exist:

1. $a^m \cdot a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(ab)^m = a^m \cdot b^m$

5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Simplify $$\left(\frac{y^2 z^{-4}}{y^{-3} z^4} \right)^{-2}$$

Radicals

If n is an even natural number and $a > 0$, or n is an odd natural number, then

$$a^{1/n} = \sqrt[n]{a}.$$

Properties of Radicals

For all real numbers a and b and natural numbers m and n such that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers:

1. $(\sqrt[n]{a})^n = a$

2. $\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is even} \\ a & \text{if } n \text{ is odd} \end{cases}$

3. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

4. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}} \quad (b \neq 0)$

5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Simplify.

$$\sqrt[3]{27x^6}$$

Divide.

$$\frac{6a + 6}{2a - 12} \div \frac{a^2 - 1}{a^2 - 2a - 24}$$

Find an equation of the line that contains the following pair of points.

(5,1) and (3,4)

Graph the linear equation.

$$4x - 2y = 8$$

Find the domain of the function.

$$f(x) = \frac{10}{x^2 - 25}$$