Quiz 13 Calculus 1

Sections 4.8, 5.1, and 5.2 **Solution**

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1. $\displaystyle\int 3u^{-2} + u^{-1} - 4u^{1/3} + 1\ du = -3u^{-1} + \ln|u| - 3u^{4/3} + u + C$

2. $\displaystyle\int \sin(2y) + \sec^2(y)\ dy = -\frac{\cos(2y)}{2} + \tan(y) + C$

3. $\displaystyle\int \frac{1}{3x-2}\ dx = \frac{\ln|3x-2|}{3} + C$

4. $\displaystyle\int \frac{1}{z^2+4}\ dz = \int \frac{1}{4(z^2/4+1)}\ dz = \frac{1}{4}\int \frac{1}{(z/2)^2+1}\ dz = \frac{1}{2}\arctan\left(\frac{z}{2}\right) + C$

5. We have that
$$f(x) = \int \frac{6}{\sqrt{25-x^2}}\ dx = \int \frac{6}{\sqrt{25(1-x^2/25)}}\ dx = \frac{6}{5}\int \frac{1}{\sqrt{1-(x/5)^2}} = 6\arcsin\left(\frac{x}{5}\right) + C.$$
Since $1 = f(0) = 6\arcsin\left(\frac{0}{5}\right) + C = C$, it must be $C = 1$ and thus $f(x) = 6\arcsin\left(\frac{x}{5}\right) + 1$.

6. We have that $g(x) = \displaystyle\int e^{5x+1}\ dx = \frac{e^{5x+1}}{5} + C$. Since $0 = g(1) = \frac{e^6}{5} + C$, it must be $C = -\frac{e^6}{5}$ and thus $g(x) = \frac{e^{5x+1}}{5} - \frac{e^6}{5}$.

7. If $f(x) \geq 0$, $|f(x)| = f(x)$ and if $f(x) \leq 0$, $|f(x)| = -f(x)$. Thus
$$\int_0^3 |f(x)|\ dx = \int_0^1 |f(x)|\ dx + \int_1^3 |f(x)|\ dx = \int_0^1 f(x)\ dx + \int_1^3 (-f(x))\ dx =$$
$$= \int_0^1 f(x)\ dx - \int_1^3 f(x)\ dx = 4 - (-2) = 6.$$

8.
$$\int_0^3 2f(x) + 1\ dx = \int_0^3 2f(x)\ dx + \int_0^3 1\ dx = 2\int_0^3 f(x)\ dx + 3 =$$
$$= 2\left(\int_0^1 f(x)\ dx + \int_1^3 f(x)\ dx\right) + 3 = 2(4 + (-2)) + 3 = 9.$$

9. The statement is true. There are several reasonable justifications. One is the following. Since the function is constant, it describes a rectangle. If we cut it vertically into rectangles, and we add those together, we will obtain exactly the original rectangle. Thus the area computed with the Riemann sum (the sum of the smaller rectangles) is the same as the area of the original rectangle.

10. We can rewrite $\int_0^1 (e^x - 1)\, dx = e - 2$ as $2 + \int_0^1 (e^x - 1)\, dx = e$. Since $e^x \geq 1$ on $[0, 1]$, $e^x - 1 \geq 0$ on $[0, 1]$ and therefore $\int_0^1 (e^x - 1)\, dx \geq 0$. Thus

$$e = 2 + \int_0^1 (e^x - 1)\, dx \geq 2.$$