Wed 3 Sep 2014

- Thurs 4 Sep Quiz in drill, covers 2.1-2.3 (material from last week)
- Sun 7 Sep: Computer HW #1 due
- Clickers start next week. Buy the clicker, give Dr. Wheeler the “device ID” number

Squeeze Theorem

A final method for evaluating limits involves the relationships of functions with each other.

Theorem: Assume the functions *f, g,* and *h* satisfy $f(x) \leq g(x) \leq h(x)$ for all values of *x* near *a*, except possibly at *a*. If $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$

Squeeze Theorem

Example:

Draw a graph of the inequality

$$-|x| \le x^2 \ln x^2 \le |x|$$

What is the $\lim\limits_{x \to 0} x^2 \ln x^2$?

HW from section 2.3

- Do problems 12-30 (x3), 31, 33, 37-47 odds, 51, 53, 61-65 odds (pgs. 73-75 in textbook)

Exercise

We have examined a number of laws and methods to evaluate limits. Consider the following limit:

$$\lim_{x \to 0} \frac{1}{x}$$

How would you evaluate this limit?

In the next two sections, we examine limit scenarios involving infinity. The two situations are:

- **Infinite limits**; (as the independent variable ‘x’ approaches a finite number, the dependent variable ‘y’ becomes arbitrarily large or small)

- **Limits at infinity**; (as the independent variable ‘x’ approaches an arbitrarily large or small number, the dependent variable ‘y’ approaches a finite number)

Definition of Infinite Limits

Suppose *f* is defined for all *x* near *a*. If *f(x)* grows arbitrarily large for all *x* sufficiently close (but not equal) to *a*, we write $\lim_{x \to a} f(x) = \infty$ and say that the limit of *f(x)* as *x* approaches *a* is infinity.

If *f(x)* is negative and grows arbitrarily large in magnitude for all *x* sufficiently close (but not equal) to *a*, we write $\lim_{x \to a} f(x) = -\infty$ and say that the limit of *f(x)* as *x* approaches *a* is negative infinity.

In both cases, the limit does not exist.

Exercise

Use a graph and a table of values to evaluate the following limits:

Given $f(x) = \dfrac{1}{x^2 - x}$, determine:

$$\lim_{x \to 0^+} f(x) \qquad \lim_{x \to 1^+} f(x) \qquad \lim_{x \to 0^-} f(x) \qquad \lim_{x \to 1^-} f(x)$$

What are the locations called where *f(x)* has infinite limits?

Definition of Vertical Asymptotes

If $\lim_{x \to a} f(x) = \pm\infty$, $\lim_{x \to a^+} f(x) = \pm\infty$, or $\lim_{x \to a^-} f(x) = \pm\infty$

then the line *x* = *a* is called a vertical asymptote of *f*.

Determining Infinite Limits Analytically

We have seen how to use tables and graphs to determine infinite limits, but we can also use number sense as a basis for analytical approaches to determining infinite limits.

Exercise: Given $f(x)=\dfrac{3x-4}{x+1}$, determine $\lim\limits_{x\to -1^-} f(x)$ and $\lim\limits_{x\to -1^+} f(x)$ using number sense (and without a graph or table!)

Exercise

What is/are the vertical asymptote(s) of the following function?

$$f(x) = \frac{3x^2 - 48}{x+4}$$

What is the $\lim_{x \to -4} f(x)$? Does that correspond to your earlier answer?

HW from section 2.4

- Do problems 7-10, 15, 17-26, 36-37 (pgs. 81-84 in textbook)

Fri 5 Sep 2014

- comp.uark.edu/~ashleykw/CalI2014/2554f14.html
- MLP: FOLLOW THE INSTRUCTIONS ON THE SYLLABUS. Then if that doesn't work, go to MRTC (2^{nd} floor of SCEN)
- Exam 1 in two weeks, on Fri 19 Sep. Will cover Sections 2.1-2.7

In sections 2.4 and 2.5, we examine limit scenarios involving infinity. The two situations are:

- **Infinite limits**; (as the independent variable ‘x’ approaches a finite number, the dependent variable ‘y’ becomes arbitrarily large or small)
- **Limits at infinity**; (as the independent variable ‘x’ approaches an arbitrarily large or small number, the dependent variable ‘y’ approaches a finite number)

Here we examine the “end behavior” of functions.

Exercise

Evaluate the following functions at the given points:
x=100; 1000; 10000; -100; -1000; -10000

$$f(x)=\frac{4x^2+3x-2}{x^2+2}$$

$$f(x)=-2+\frac{\cos x}{\sqrt[3]{x}}$$

What is your conjecture about $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$?

What are these limits called?

Definition of Limits at Infinity and Horizontal Asymptotes

If *f(x)* becomes arbitrarily close to a finite number *L* for all sufficiently large and positive x, then we write $\lim_{x \to \infty} f(x) = L$. We say the limit of *f(x)* as *x* approaches infinity is *L*.

In this case the line *y = L* is a horizontal asymptote of *f*. The limit at negative infinity, $\lim_{x \to -\infty} f(x) = M$

is defined analogously and in this case the horizontal asymptote is *y = M*.

It is possible for a limit to be both an infinite limit and a limit at infinity.

Question: What happens to $f(x) = x^n$ as *x* approaches infinity? What happens as *x* approaches negative infinity?

Definition of Infinite Limits at Infinity

If *f(x)* becomes arbitrarily large as *x* becomes arbitrarily large, then we write $\lim_{x\to\infty} f(x)=\infty$

The limits $\lim_{x\to\infty} f(x)=-\infty$, $\lim_{x\to-\infty} f(x)=\infty$, and

$\lim_{x\to-\infty} f(x)=-\infty$ are defined similarly.

Limits at Infinity of Powers and Polynomials

Let n be a positive integer and let p be the polynomial,

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ where $a_n \neq 0$.

1. $\lim_{x \to \pm\infty} x^n = \infty$, when n is even.

2. $\lim_{x \to \infty} x^n = \infty$ and $\lim_{x \to -\infty} x^n = -\infty$ when n is odd.

3. $\lim_{x \to \pm\infty} \frac{1}{x^n} = \lim_{x \to \pm\infty} x^{-n} = 0$

4. $\lim_{x \to \pm\infty} p(x) = \infty$ or $-\infty$, depending on the degree of the polynomial and the sign of the leading coefficient, a_n

End Behavior and Asymptotes of Rational Functions

Suppose $f(x)=\dfrac{p(x)}{q(x)}$ is a rational function, where

$$p(x)=a_m x^m + a_{m-1}x^{m-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$$q(x)=b_m x^m + b_{m-1}x^{m-1} + \ldots + b_2 x^2 + b_1 x + b_0$$

where $a_m \neq 0$ and $b_n \neq 0$.

1. If *m* < *n*, then $\lim_{x\to\pm\infty} f(x)=0$, and *y = 0* is a horizontal asymptote of *f*.

2. If *m* = *n*, then $\lim_{x\to\pm\infty} f(x)=\dfrac{a_m}{b_n}$, and $y=\dfrac{a_m}{b_n}$ is a horizontal asymptote of *f*.

End Behavior of Algebraic and Transcendental Functions

Finally we examine the end behavior of rational functions. How do these functions behave as $x \to \pm\infty$?

$$f(x) = \frac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}}$$

$$g(x) = \cos x$$

$$h(x) = e^x$$

HW from section 2.5

- Do problems 9-10, 13-35 odds, 39, 43, 45, 53 (pgs. 92-94 in textbook)