Power Rule

For any real number $\boxed{n \neq -1}$, ← Why? because of

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C.$$

(The antiderivative of $f(x) = x^n$ for $n \neq -1$ is found by increasing the exponent n by 1 and dividing x raised to the new power by the new value of the exponent.)

$$\int x^{\overset{n}{5}}\,dx = \frac{x^{5+1}}{5+1} + C = \boxed{\frac{x^6}{6} + C}$$

Check:

Derivative of $\frac{x^6}{6} + C$

is $\frac{d}{dx}\left(\frac{x^6}{6}\right) + \frac{d}{dx}(C)$ ← constant

$= \frac{1}{6}(6x^5) + 0$

$= x^5$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-3+1}}{-3+1} + C$$

$$= \boxed{\frac{x^{-2}}{-2} + C}$$

Constant Multiple Rule and Sum or Difference Rule

If all indicated integrals exist,

$$\int k \cdot f(x)\,dx = k\int f(x)\,dx, \qquad \text{for any real number } k,$$

and

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$$

(The antiderivative of a constant times a function is the constant times the antiderivative of the function. The antiderivative of a sum or difference of functions is the sum or difference of the antiderivatives.)

$1 = x^0$

Find $\int (6x^2 + 8x - 9)dx.$ $= 6\int x^2 dx + 8\int x\,dx - 9\int 1\,dx$

$= 6\left(\frac{x^3}{3}\right) + C_1 + 8\left(\frac{x^2}{2}\right) + C_2 - 9(x) + C_3$

$= 2x^3 + 4x^2 - 9x + C$

(sum of three constants is still a constant)

$$\int \frac{x^2+1}{\sqrt{x}}\,dx =$$

$$= \int \left(\frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}} \right) dx = \int \left(x^{2-1/2} + x^{-1/2} \right) dx$$

$$= \int \left(x^{3/2} + x^{-1/2} \right) dx$$

$$= \frac{x^{3/2+1}}{3/2+1} + \frac{x^{-1/2+1}}{-1/2+1} + C$$

$$= \frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} + C$$

$$\boxed{\frac{2x^{5/2}}{5} + 2x^{1/2} + C}$$

$$\int (x^2 - 1)^2\, dx =$$

$$= \int (x^2-1)(x^2-1)\,dx = \int \left[(x^2)^2 + (-1)(x^2) + (-1)(x^2) + (-1)(-1)\right] dx$$

$$= \int (x^4 - 2x^2 + 1)\,dx$$

$$\boxed{= \frac{x^5}{5} - \frac{2x^3}{3} + x + C}$$

Suppose a publishing company has found that the marginal cost at a level of production of x thousand books is given by

$$C'(x) = \frac{50}{\sqrt{x}}$$

and that the fixed cost (the cost before the first book can be produced) is \$25,000. Find the cost function $C(x)$.

$$C(x) = \int C'(x)\,dx = \int 50x^{-1/2}\,dx = \frac{50x^{1/2}}{1/2} + C = 100x^{1/2} + C$$

Knowing the fixed cost means we can actually find the correct "C" for this problem. Fixed cost means cost when x = thousands of books = 0.

That means (fixed cost) $= C(0) = 100(0)^{1/2} + C = \$25{,}000$

Solve for C:

$C = \$25{,}000$

Therefore the cost function is

$$\boxed{C(x) = 100\sqrt{x} + 25000}$$

Note: These type of problems, where you have the information to find C, are called initial value problems.