# Quiz 1: The Idea of Limits (§2.1-2.2)

**Directions:** You have 30 minutes to complete this quiz. This quiz is open resources.

1. Given the equation $f(x) = x^3 - x^2$,

    (a) determine the slope of the secant line between the following $x$-coordinates:

        i. $[1, 1.5]$

    $$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{(1.5)^3 - (1.5)^2 - (1^3 - 1^2)}{0.5}$$
    $$\boxed{= 2.25}$$

        iii. $[1, 1.005]$

    $$\frac{(1.005)^3 - (1.005)^2 - 0}{0.005}$$
    $$\boxed{= 1.010025}$$

        ii. $[1, 1.05]$

    $$\frac{(1.05)^3 - (1.05)^2 - (1^3 - 1^2)}{0.05}$$
    $$\boxed{= 1.1025}$$

        iv. $[1, h]$, assuming $h > 1$

    $$\frac{f(h) - f(1)}{h - 1} = \frac{h^3 - h^2 - (1^3 - 1^2)}{h - 1}$$
    $$= \frac{h^2(h-1)}{h-1} \boxed{= h^2}$$

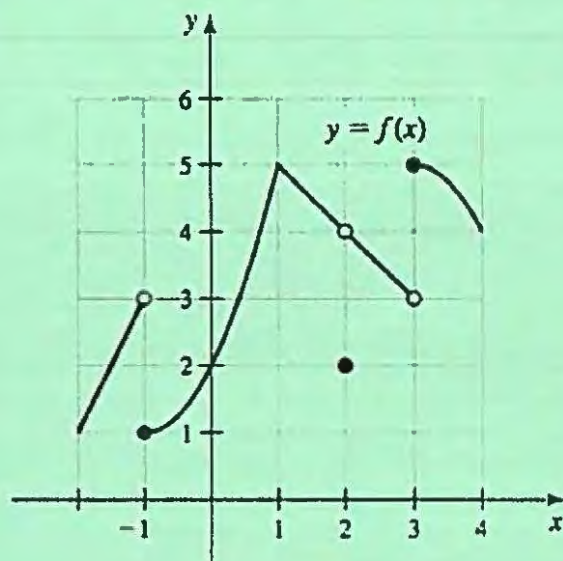    (b) then use your answers from (a) to estimate the slope of the tangent line to $f(x)$ at $x = 1$;

    slope $\approx 1$; using (a)iv., squaring numbers closer to 1 gives answers even closer to 1.

    (c) using the limit symbol, how would you express your conclusion in part (b)?

    $$\lim_{h \to 1^+} \frac{f(h) - f(1)}{h - 1} = \lim_{h \to 1^+} \frac{h^3 - h^2}{h - 1}$$
    $$= \lim_{h \to 1^+} h^2 = 1$$

2. Use the graph of $f$ in the figure below to find the following values. If it is not possible, then say so.



(a) $f(-1) = 1$

(b) $\lim\limits_{x \to -1^-} f(x) = 3$

(c) $\lim\limits_{x \to 3^+} f(x) = 5$

(d) $\lim\limits_{x \to -1^+} f(x) = 1$

(e) $\lim\limits_{x \to 3} f(x)$ DNE

3. Sketch the graph of a function satisfying all of the following:

- $f(2) = 4$
- $f(-1) = 0$
- $\lim\limits_{x \to 2^+} f(x) = -3$
- $\lim\limits_{x \to 2^-} f(x) = 5$