Section 3.9 – Linear Approximation and the Derivative

1. Show that $\frac{1}{\sqrt{x+1}} \approx 1 - \frac{x}{2}$ near $x = 0$.

near $x=0$,

$$f(x) \doteq f(0) + f'(0)x$$

$$= 1 + -\frac{1}{2}(0+1)^{-3/2}x$$

$$= 1 - \frac{1}{2}x$$

2. (a) Show that $1 + kx$ is the local linearization of $(1+x)^k$ at $x = 0$.

near $x=0$,

$$L(x) = (1+0)^k + k(1+0)^{k-1}x$$

$$= 1 + kx$$

(b) Someone claims that the square root of 1.1 is about 1.05. Without using a calculator, do you think this estimate is about right? **Hint:** Use the linearization you calculated in part (a).

Using (a), $k = \frac{1}{2}$, $x = 0.1$. So

$$L(0.1) = 1 + \frac{1}{2}0.1$$

$$= 1.05.$$

yes