Wed 15 July

- MLP assignments are ALL reopened. Deadline's been extended, you have until Thursday night to complete them.
- FINAL! Thursday, in class. Covers $\oint$3.10-5.5.
- Grades: Here is how to compute your grade
  - Homework: MLP average out of 100 points.
  - Quizzes: For each quiz, find the percentage. For example, if you got 11/13 on the last quiz, the percentage is 84.6%. A skipped quiz is a zero. Take the average of your top 10 percentages, out of 100 points.

Wed 15 July (cont.)

- Exam 1: Here is how to adjust your score with the curve (out of 125 points):

$$\frac{125}{100}\left(\frac{4}{5}\left(\frac{100}{125} * \text{your score} - 70.4\right) + 77\right)$$

- Midterm (out of 175 points):

$$\frac{175}{100}\left(\frac{9.05}{11}\left(\frac{100}{175} * \text{your score} - 71.4\right) + 77\right)$$

- Exam 3: your score out of 125 points
- Divide the total by 625 to get your current percentage.

Final (Exam #4) Review

- $\oint$ 4.4 Optimization Problems
  - Be able to solve optimization problems that maximize or minimize a given quantity.
  - Be able to identify and express the constraints and objective function in an optimization problem.
  - Be able to determine your interval of interest in an optimization problem (e.g., what range of x-values are you searching for your extreme points?)
  - **As to formulas, the same comment made above with respect to formulas for related rates problems applies here as well.**

Final (Exam #4) Review (cont.)

Exercise

What two nonnegative real numbers a and b whose sum is 23 will

(a) minimize $a^2 + b^2$?

(b) maximzie $a^2 + b^2$?

Final (Exam #4) Review (cont.)

Exercise

Squares with sides of length x are cut out of each corner of a 3 ft by 4 ft cardboard rectangle. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.

Final (Exam #4) Review (cont.)

- $\oint$ 4.5 Linear Approximation and Differentials
  - Be able to find a linear approximation for a given function.
  - Be able to use a linear approximation to estimate the value of a function at a given point.
  - Be able to use differentials to express how the change in x (dx) impacts the change in y (dy).

Final (Exam #4) Review (cont.)

- $\oint$ 4.6 Mean Value Theorem (for Derivatives)
  - Know and be able to state Rolle's Thm and the Mean Value Thm, including knowing the hypotheses and conclusions for both.
  - Be able to apply Rolle's Thm to find a point in a given interval.
  - Be able to apply the MVT to find a point in a given interval.
  - Be able to use the MVT to find equations of secant and tangent lines.

Final (Exam #4) Review (cont.)

Exercise (s)

Determine whether the Mean Value Theorem (or Rolle's Theorem) applies to the following functions. If it does, then find the point(s) guaranteed by the theorem to exist.

(1) $f(x) = \sin(2x)$ on $\left[0, \frac{\pi}{2}\right]$

(2) $g(x) = \ln(2x)$ on $[1, e]$

(3) $h(x) = 1 - |x|$ on $[-1, 1]$

Final (Exam #4) Review (cont.)

Exercise (s)

(4) $j(x) = x + \frac{1}{x}$ on $[1, 3]$

(5) $k(x) = \frac{x}{x+2}$ on $[-1, 2]$

Final (Exam #4) Review (cont.)

- $\oint$ 4.7 L'Hôpital's Rule
  - Know how to use L'Hôpital's Rule, including knowing under what conditions the Rule works.
  - Be able to apply L'Hôpital's Rule to a variety of limits that are in indeterminate forms (e.g., $0/0$, ∞/∞, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0).
  - Be able to use L'Hôpital's Rule to determine the growth rates of two given functions.
  - Be aware of the pitfalls in using L'Hôpital's Rule.
  - PRACTICE THESE. Some of the book problems have non-obvious algebra tricks that simplify an otherwise crazy problem.

Final (Exam #4) Review (cont.)

- $\int$ 4.8 Antiderivatives
  - Know the definition of an antiderivative and be able to find one or all antiderivatives of a function.
  - Be able to evaluate indefinite integrals, including using known properties of indefinite integrals (i.e., Power Rule, Constant Multiple Rule, Sum Rule).
  - Know how to find indefinite integrals of the six trig functions, of e^{ax}, of $\ln x$, and of the three inverse trig functions listed in the notes.
  - Be able to solve initial value problems to find specific antiderivatives.
  - Be able to use antiderivatives to work with motion problems.

Final (Exam #4) Review (cont.)

- $\int$ 5.1 Approximating Areas under Curves
  - Be able to use rectangles to approximate area under the curve for a given function.
  - Know how to calculate left Riemann sums, right Riemann sums, and midpoint Riemann sums for a function.
  - Be able to sum a series of numbers written in sigma notation. You need to know these common sums:

$$\sum_{k=1}^{n} c = cn \quad \text{and} \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

  - Be able to identify whether a given Riemann sum written in sigma notation is a left, right, or midpoint sum.

Final (Exam #4) Review (cont.)

- $\int$ 5.2 Definite Integrals
  - Be able to compute left, right, or midpoint Riemann sums for curves that have negative components, and understand the concept of net area.
  - Be able to evaluate a definite integral using geometry or a given graph.
  - Know the properties of definite integrals and be able to use them to evaluate a definite integral.

Final (Exam #4) Review (cont.)

Exercise

Suppose

$$\int_1^4 f(x)\,dx = 8 \qquad \text{and} \qquad \int_1^6 f(x)\,dx = 5.$$

Evaluate the following integrals:

(a) $\displaystyle\int_1^4 (-3f(x))\;dx$

(b) $\displaystyle\int_6^4 12f(x)\,dx$

(c) $\displaystyle\int_4^6 (f(x)+3x)\;dx$

Final (Exam #4) Review (cont.)

- $\oint$ 5.3 Fundamental Theorem of Calculus
  - Understand the concept of an area function, and be able to evaluate an area function as x changes.
  - Know the two parts of the Fundamental Theorem of Calculus and its significance (i.e., the inverse relationship between differentiation and integration).
  - Use the FTC to evaluate definite integrals or simplify given expressions.

Final (Exam #4) Review (cont.)

Exercise

Evaluate each:

(a) $\displaystyle\int_0^{\ln 8} e^x \, dx$

(b) $\displaystyle\frac{d}{dx}\int_x^0 \frac{dp}{p^2+1}$

(c) the net area of the region bounded between the x-axis and the function $f(x) = x(x-2)(x-4)$

(d) $\displaystyle\frac{d}{dy}\int_2^{y^3} (t^2+t+1) \, dt$

Final (Exam #4) Review (cont.)

- $\int$ 5.4 Working with Integrals
  - Be able to integrate even and odd functions knowing the "shortcuts" provided by these functions' characteristics.
  - Be able to find the average value of a function.
  - Know the Mean Value Theorem for Integrals and be able to use it to find points associated with the average value of a function.

Final (Exam #4) Review (cont.)

Exercise

Find the point(s) at which the function

$$f(x) = 1 - |x|$$

equals its average value on the interval $[-1, 1]$. Then draw the picture of $f(x)$, labelling the points and the average value you computed.

Final (Exam #4) Review (cont.)

- $\oint$ 5.5 Substitution Rule
  - Definite integrals.
  - Indefinite integrals.
  - Change of variables.

Exercise (s)

Evaluate, using substitution:

1. $\displaystyle\int \frac{y}{\sqrt{y-4}}\,dy$

Final (Exam #4) Review (cont.)

Exercise (s)

2. $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

3. $\displaystyle\int_0^1 2x(4 - x^2)\, dx$

4. $\displaystyle\int_1^{e^2} \frac{\ln x}{x}\, dx$

Tips for Studying Efficiently and Effectively

- Given today's lists of materials you should know for the exam, if you see a topic you don't know then go back to the slides covering that topic first.
- Review slides for days you missed.
- Redo the quizzes until you can get a perfect score without looking at the key.
- Book problems. Do those problems with the same attention and care you put into Exam #3.
- If you spent 10 hours on Exam #3 then spend at least that much time studying for the final.

Exercise (s)

1. Find the 101st derivative of $y = \cos 7x$ at $x = 0$.
2. For what values of the constants a and b is $(-1, 2)$ a point of inflection on the curve $y = ax^3 + bx^2 - 8x + 2$?
3. Evaluate:
$$\int_u^v (\cos t)\, g'(\sin t)\; dt.$$