MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Thurs 11 June 2015

Quiz 5: Trig Derivatives and Story Problems (∮3.4-3.5)

Directions: You have 30 minutes to complete this quiz. This quiz is collaborative and open resources.

1. Find $\lim_{x\to 0} \frac{\sin 5x}{3x}$. $\left(\frac{5}{5}\right) = \lim_{x\to 0} \frac{\sin 5x}{5x} \cdot \frac{5}{3}$

   Let $y = 5x$. As $x \to 0$, $y \to 0$.

   $= \frac{5}{3}\left(\lim_{y\to 0} \frac{\sin y}{y}\right) = \boxed{\frac{5}{3}}$

2. Differentiate $y = e^{-x} \csc x$.

   $y' = -e^{-x}\csc x + e^{-x}(-\csc x \cot x)$

   $\boxed{= -e^{-x}\csc x(1+\cot x)}$

3. Find the second derivative of $f(x) = \sin x \cos x$, using only the sum, constant multiple, and product rules.

   $f'(x) = \cos x \cos x + \sin x(-\sin x)$

   $= \cos x \cos x - \sin x \sin x$

   $f''(x) = (-\sin x)\cos x + \cos x(-\sin x) - (\cos x \sin x + \sin x \cos x)$

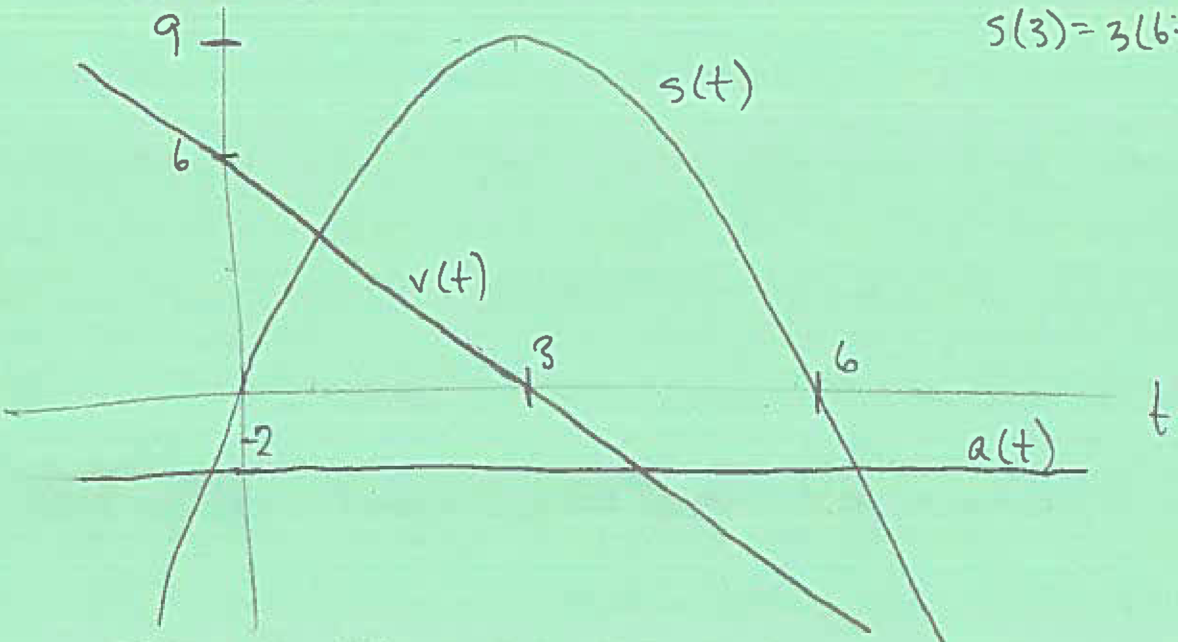
   $\boxed{= -4 \sin x \cos x}$

4. In this problem your answers must include the correct units. Draw your graphs carefully (it's OK if you have to erase and redraw!). Suppose the position (in feet) of an object moving horizontally after t seconds is given by the function $s(t) = 6t - t^2$.

(a) Graph $s(t)$.

$0 = t(6-t)$

$s(3) = 3(6-3) = 9$



(b) Compute and then graph the velocity function, $v(t) = s'(t)$, on the same axes as part (a).

$v(t) = 6 - 2t$

(c) Compute and then graph the acceleration function, $a(t) = v'(t) = s''(t)$, on the same axes as parts (a) and (b).

$a(t) = -2$

(d) When is the object stationary?

$t = 3$ seconds, since $v(3) = 0$.

(e) When is the object moving backwards?

after $t = 3$ seconds, since velocity is negative

(f) When is the object moving forwards?

before $t = 3$ seconds

(g) Determine the velocity at $t = 1$.

$v(1) = 6 - 2(1) = 4$ ft/sec

(h) Determine the acceleration at $t = 1$.

$a(1) = -2$ ft/sec^2

(i) Determine the acceleration when the *velocity* is 0.

-2 ft/sec^2

5. Suppose

$$C(x) = -0.02x^2 + 5x + 10 \qquad \text{and} \qquad p(x) = 100 - 0.1x$$

represent, respectively, the cost of producing x ice cream cones and the sale price per ice cream cone if x of them are sold. The **profit** of selling x ice cream cones is

$$P(x) = xp(x) - C(x)$$

– i.e., the revenue minus the costs. The **average profit per ice cream cone** when x are sold is $\frac{P(x)}{x}$ and the **marginal profit** is $P'(x)$. (The marginal profit approximates the profit obtained by selling one more ice cream cone, given that x have already been sold.)

(a) Find (and simplify) $P(x)$.

$P(x) = xp(x) - C(x)$

$= x(100 - 0.1x) - (-0.02x^2 + 5x + 10)$

$= 100x - 0.1x^2 + 0.02x^2 - 5x - 10$

$\boxed{= -0.08x^2 + 95x - 10}$

(b) Find the average profit per ice cream cone if 500 have been sold.

$\frac{P(500)}{500} = \frac{-0.08(500^2) + 95(500) - 10}{500}$

$= -0.08(500) + 95 - 10/500$

$= -40 + 95 - \frac{1}{50} = 55.02 \boxed{\approx 55 \text{ coins/cone}}$

(c) Find the marginal profit if 500 ice cream cones have been sold.

$P'(x) = -0.16x + 95$

$P'(500) = -0.16(500) + 95$

$= -80 + 95 \boxed{= 15 \text{ coins/cone}}$

(d) **ChAlLeNgE pRoBlEm (worth no credit)** What is the optimal number of ice cream cones to sell for the widest profit margin? What can you conclude about this business model?

Optimize $P'(x)$:

$P''(x) = -0.16$

means the profit margin is always decreasing — selling fewer ice cream cones is more profitable

Conclusion: This is a terrible business model and I should stick to my original plan of selling fro-yo this summer.