# UNIT 4, LESSON 1

Antiderivatives

# Objectives:

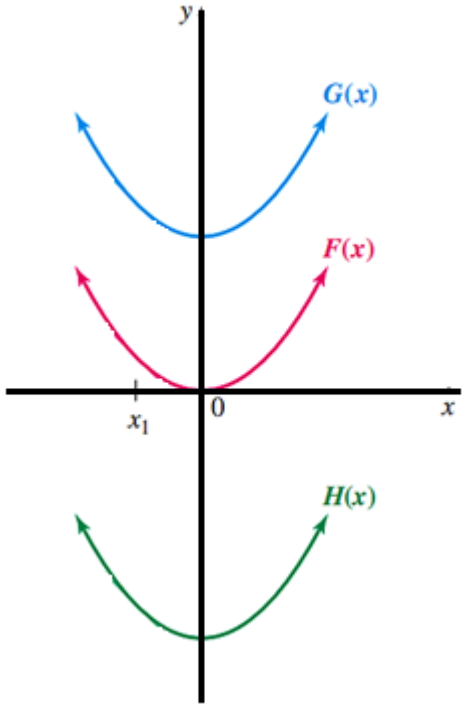
- Calculate indefinite integrals of basic polynomial, radical, and exponential functions.

**Antiderivative**

If $F'(x) = f(x)$, then $F(x)$ is an **antiderivative** of $f(x)$.

Let $F'(x) = 2x$.

Can you find a function $F(x)$ such that $F'(x) = 2x$?




$$F(x) = x^2$$

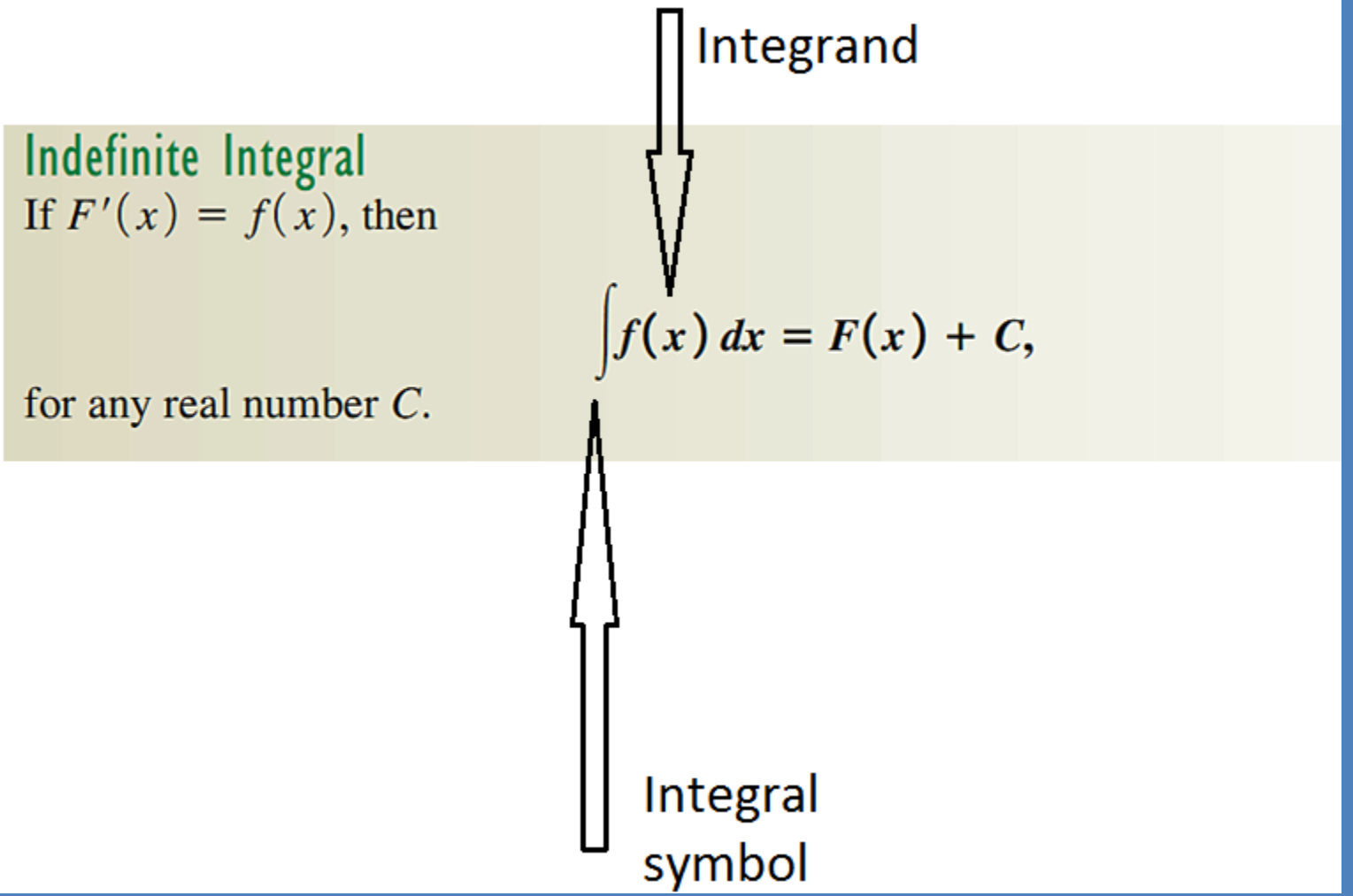
$$G(x) = x^2 + 2$$

$$H(x) = x^2 - 4$$

If $F(x)$ and $G(x)$ are both antiderivatives of a function $f(x)$ on an interval, then there is a constant $C$ such that

$$\boldsymbol{F(x) - G(x) = C.}$$

(Two antiderivatives of a function can differ only by a constant.) The arbitrary real number $C$ is called an integration constant.




For our example: $\int 2x\,dx = x^2 + C$

## Power Rule

For any real number $n \neq -1$,

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C.$$

(The antiderivative of $f(x) = x^n$ for $n \neq -1$ is found by increasing the exponent $n$ by 1 and dividing $x$ raised to the new power by the new value of the exponent.)

$$\int x^5 dx =$$

$$\int \frac{1}{x^3} dx =$$

## Constant Multiple Rule and Sum or Difference Rule

If all indicated integrals exist,

$$\int k \cdot f(x)\,dx = k\int f(x)\,dx, \qquad \text{for any real number } k,$$

and

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$$

(The antiderivative of a constant times a function is the constant times the antiderivative of the function. The antiderivative of a sum or difference of functions is the sum or difference of the antiderivatives.)

Find $\int (6x^2 + 8x - 9)dx.$

$$\int \frac{x^2+1}{\sqrt{x}}\,dx =$$

$$\int (x^2 - 1)^2 \, dx =$$

## Indefinite Integrals of Exponential Functions

$$\int e^x dx = e^x + C$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C, \quad k \neq 0$$

**For $a > 0, a \neq 1$:**

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int a^{kx} dx = \frac{a^{kx}}{k(\ln a)} + C, \quad k \neq 0$$

(The antiderivative of the exponential function $e^x$ is itself. If $x$ has a coefficient of $k$, we must divide by $k$ in the antiderivative. If the base is not $e$, we must divide by the natural logarithm of the base.)

## Indefinite Integral of $x^{-1}$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

(The antiderivative of $f(x) = x^n$ for $n = -1$ is the natural logarithm of the absolute value of $x$.)

Find $\int\left(\frac{3}{x} + e^{-3x}\right)dx.$

Suppose a publishing company has found that the marginal cost at a level of production of $x$ thousand books is given by

$$C'(x) = \frac{50}{\sqrt{x}}$$

and that the fixed cost (the cost before the first book can be produced) is \$25,000. Find the cost function $C(x)$.