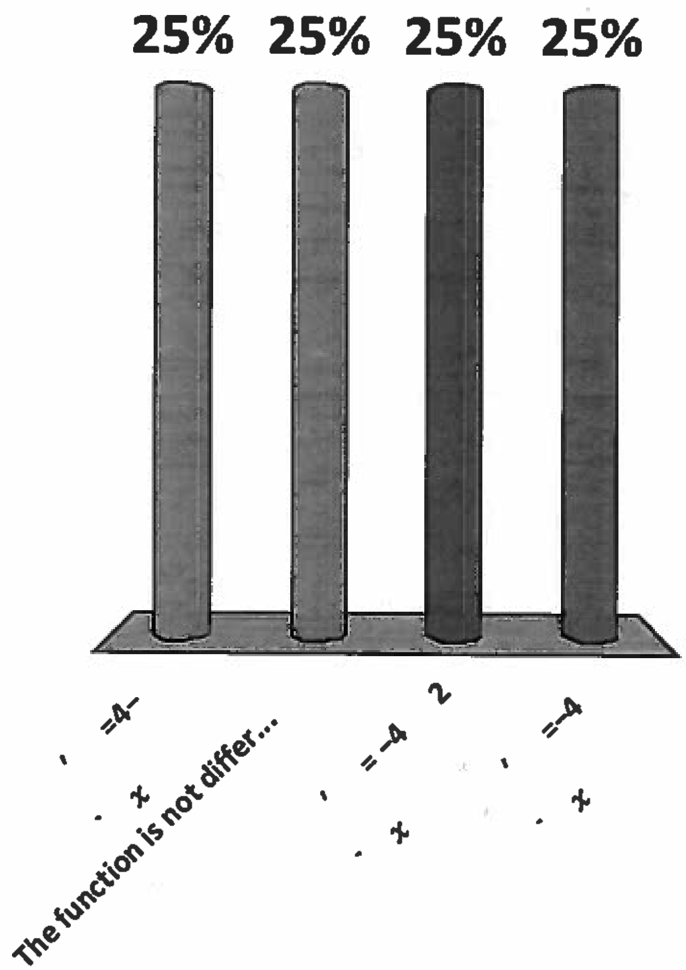
Let $f(x) = \frac{4}{x}$. Find $f'(x)$.

A. $f'(x) = 4 - x$

B. The function is not differentiable

C. $f'(x) = \frac{-4}{x^2}$ (see next page)

D. $f'(x) = -4x$



$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\frac{4}{x+h} - \frac{4}{x}}{h} \right)$$

$$= \lim_{h\to 0} \frac{\frac{4x - 4(x+h)}{(x+h)(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4x} - \cancel{4x} - 4h}{(x+h)(x)h}$$

$$= \lim_{h\to 0} \frac{-4\cancel{h}}{(x+h)x\cancel{h}} = \boxed{\frac{-4}{x^2}}$$

The cost in dollars to manufacture x graphing calculators is given by $C(x) = -0.005x^2 + 20x + 150$ when $0 \leq x \leq 2000$. Find the rate of change of cost with respect to the number manufactured when 100 calculators are made.

$$C'(100) = \lim_{h \to 0} \frac{C(100+h) - C(100)}{h}$$

$$= \lim_{h \to 0} \left(\frac{-0.005(\cancel{100^2} + 200h + h^2) + 20(\cancel{100}+h) + \cancel{150}}{h} - \frac{\left[-0.005(\cancel{100^2}) + 20(\cancel{100}) + \cancel{150}\right]}{h} \right)$$

$$= \lim_{h \to 0} \frac{-0.005(200\cancel{h} + h^{\cancel{2}}) + 20\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} (-1 - 0.005h + 20)$$

$= -1 + 20 =$ $\boxed{\text{\$19 more for the 101st calculator having produced 100 calculators}}$

Find the equation of the tangent line to $f(x) = 6 - x^2$ when $x = -1$. $\leftarrow x_0$

$$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1} \frac{\cancel{6} - x^2 - (\cancel{6} - (-1)^2)}{x+1}$$

$$= \lim_{x \to -1} \frac{-x^2 + 1}{x+1} \qquad = -(x^2-1)$$

$$= \lim_{x \to -1} \frac{-\cancel{(x+1)}(x-1)}{\cancel{x+1}}$$

$$= \lim_{x \to -1} -(x-1) = -(-1-1)$$

$$= 2 \leftarrow m$$

$$f(-1) = 6 - (-1)^2$$

$$= 6 - 1 = 5 \leftarrow y_0$$

tangent line:

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 5 = 2(x+1)}$$

OR

$$y = 2x + 2 + 5$$

$$\Rightarrow \boxed{y = 2x + 7}$$