Section 2.4 – Interpretations of the Derivative

1. Let $f(p)$ represent the daily demand for San Francisco '49ers T-shirts when the price for a shirt is p dollars. In other words, $f(p)$ gives the number of shirts purchased daily if the selling price is p dollars.

   (a) Is f increasing or decreasing? decreasing

   (b) What are the units of p, $f(p)$, and $f'(p)$? \$, shirts, shirts/\$

   (c) Explain, in terms of shirts and dollars, the practical meaning of the following:

   i. $f(20) = 150$ means when a shirt costs \$20, the daily demand is 150 shirts.

   ii. $f'(20) = -5$ means if the price of a shirt increases from \$20 to \$21, the daily demand decreases by about 5 shirts.

   iii. $f(30)$ is the daily demand for \$30 shirts.

   (d) Let d represent demand. Then $d = f(p)$, so the function f takes ___p___ as an input and gives ___d___ as an output. On the other hand, the inverse function f^{-1} takes ___d___ as an input and gives ___p___ as an output, so $f^{-1}($_d_$) =$ _p_.

   (e) Give practical interpretations of $f(25)$ and $f^{-1}(25)$.

   f(25) is the daily demand for \$25 shirts. $f^{-1}(25)$ is the price of shirts when daily demand is 25.

2. (Taken from *Hughes-Hallett, et. al.*) If t is the number of years since 1993, the population, P, of China, in billions, can be approximated by the function

$$P = f(t) = 1.15(1.014)^t.$$

   (a) Calculate and interpret $f(6)$ in the context of this problem. $f(6) \doteq 1.250$ means in 1999, the population is about 1.25 billion.

   (b) Use the table method to estimate $\frac{dP}{dt}$ at $t = 6$, and give an interpretation of this number in the context of this problem. $f'(6) \doteq 0.0174$ means in 1999 the population will increase by about 17.4 million.

3. Between noon and 6 p.m., the temperature in a town rises continually, but rises at its quickest around 3 p.m., and slowest around noon and 6 p.m.

   (a) Sketch a possible graph of $H = f(t)$, where H is the temperature in the town (in degrees Fahrenheit) and t represents the time (in hours) after 12:00 noon.

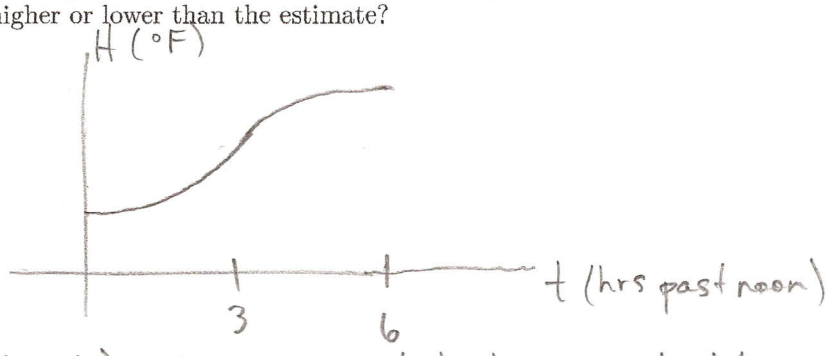
   (b) Explain, in terms of degrees and hours, what each of the following represents:

   (i) $f'(2)$ (ii) $f'(3) = 7$ $f(4) = 40$ $f'(4) = 1$

   (c) Use the statements given in parts (iii) and (iv) from above to estimate the temperature in the town at 5:30 p.m. Is the actual temperature higher or lower than the estimate?

h	$\frac{f(6+h)-f(6)}{h}$
-0.001	0.0174
-0.0001	0.0174
0.001	0.0174

(a)



(b)(i) $f'(2)$ is approximately how much the temperature will increase from 2-3p

(ii) $f'(3)=7$ means between 3p and 4p, the temperature will increase by about 7°F.

(iii) $f(4)=40$ means it is 40°F at 4p.

(iv) $f'(4)=1$ means from 4-5p the temperature will decrease by about 1°F.

(c) $f(5.5) \doteq 40+1.5=41.5°$ will be higher than the actual temp, since the rate of increase is getting smaller.