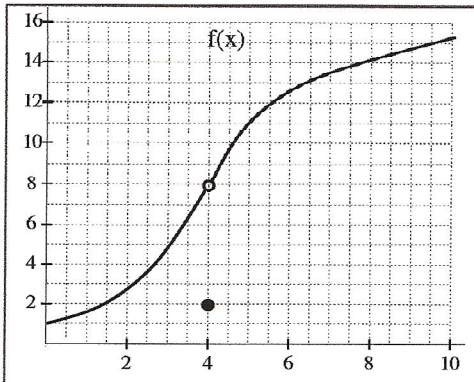
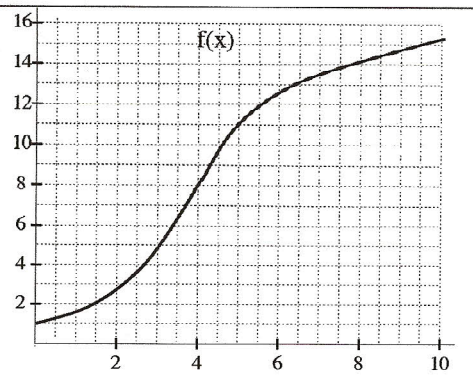
Section 2.6 – Differentiability



$\lim_{x \to 4} f(x) = 8$

$f(4)$ 2

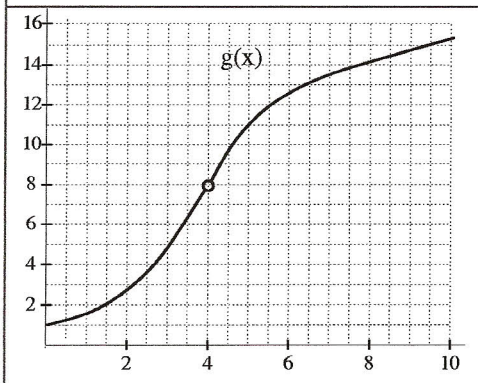
f is not continuous at $x = 4$.



$\lim_{x \to 4} f(x) = 8$

$f(4) = 8$

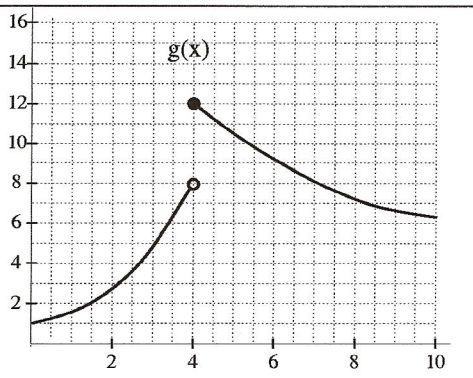
f is continuous at $x = 4$.



$\lim_{x \to 4} g(x) = 8$

$g(4)$ undefined

g is not continuous at $x = 4$.



$\lim_{x \to 4} g(x)$ undefined

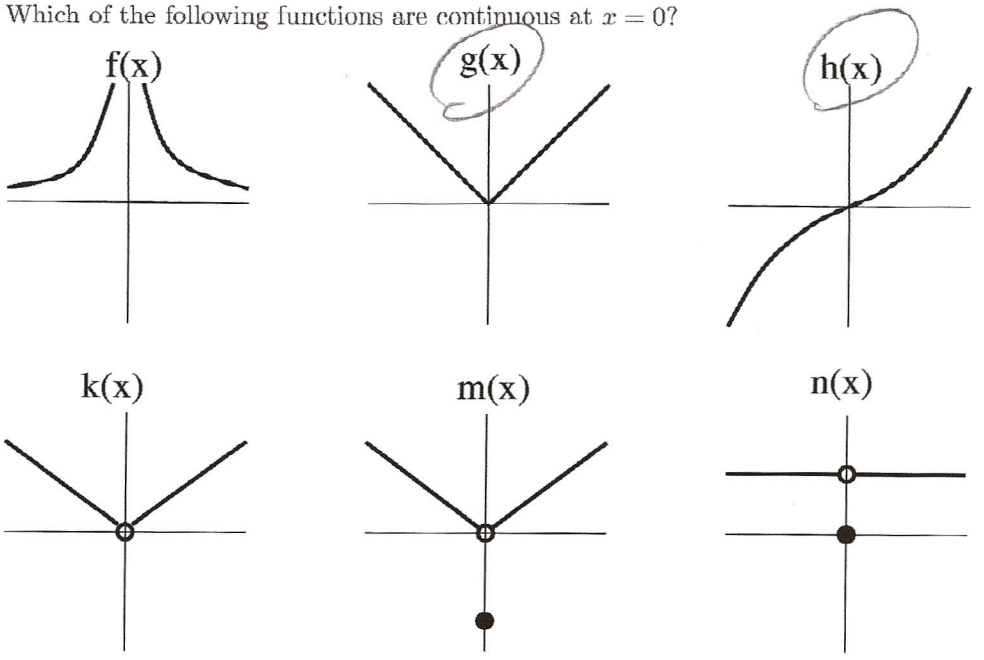
$g(4) = 12$

g is not continuous at $x = 4$.

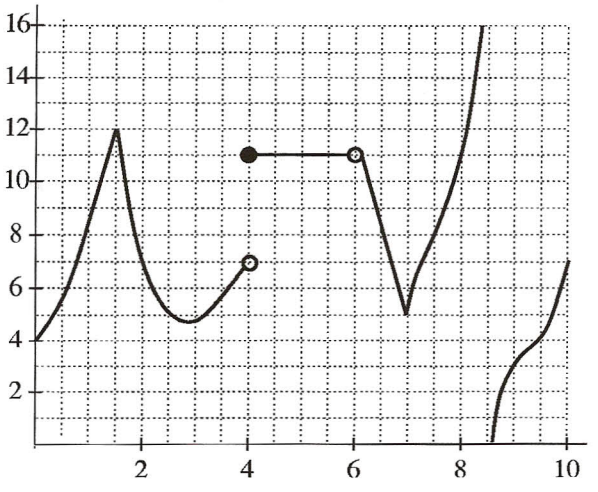
Theorem. The function f is continuous at $x = c$ if f is defined at $x = c$ and if

$$\lim_{x \to c} f(x) = \underline{\quad f(c) \quad}.$$

1. Which of the following functions are continuous at $x = 0$?



2. Consider the function $f(x)$ given below.



(a) At what values of x is f not continuous?

$x = 4, 6, 8.5$

(b) At what values of x is f not differentiable?

$x = 1.5, 4, 6, 7, 8.5$

Section 2.6 – Differentiability

1. A magnetic field, B, is given as a function of the distance, r, from the center of a wire as follows:

$$B = \begin{cases} \frac{r}{r_0} B_0 & \text{for } r \le r_0 \\ \frac{r_0}{r} B_0 & \text{for } r > r_0 \end{cases}$$

(a) Is B continuous at r_0? Explain.

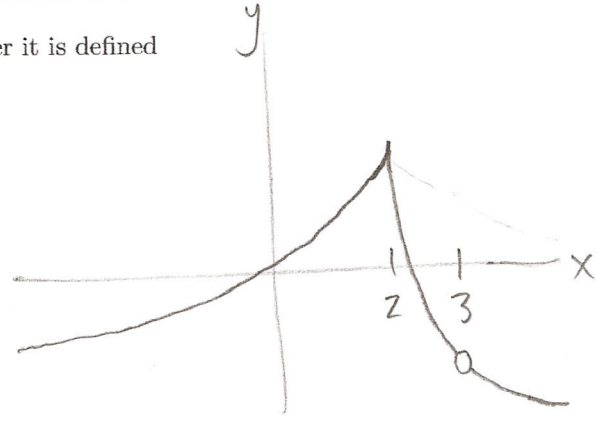
Yes. $B(r_0) = B_0$, and $\lim_{r \to r_0^-} B = B_0 = \lim_{r \to r_0^+} B_0$.

(b) Is B differentiable at r_0? Explain.

No. $\lim_{h \to 0^-} \dfrac{\frac{r_0}{r_0+h} B_0 - B_0}{h} = \dfrac{(h - r_0)B_0}{r_0 h} \neq \lim_{h \to 0^+} \dfrac{\frac{r_0}{r_0+h} B_0 - B_0}{h} = \dfrac{(r_0 - 1)B_0}{h(r_0+h)}$

2. Sketch the graph of $y = f(x)$ if f has the following properties:

- $f(x)$ is continuous everywhere except at 3
- $f(x)$ has a vertical tangent line at 2
- $f(x)$ is not differentiable at 3
- $f''(x) > 0$ wherever it is defined



3. Find the intersection point of the tangent line to $y = x^x$ at 1.1 and the x-axis.

Using the calculator's derivative function for the graph

$y'(1.1) \doteq 1.216$

$1.1^{1.1} \doteq 1.111$

tangent line

$y - 1.111 = 1.216(x - 1.1)$

intersection: $y = 0$

$-1.111 = 1.216x - 1.350$

$x \doteq 0.197$