# Section 3.4 – The Chain Rule

Part I - Find the indicated derivatives.

1. Find $\frac{d^2y}{dx^2}$ where $y = 2x - \frac{1}{\sqrt[3]{x}} + 3^x - e$. $\frac{dy}{dx} = 2 + \frac{1}{3}x^{-4/3} + \ln(3)3^x$ ; $\frac{d^2y}{dx^2} = -\frac{4}{9}x^{-7/3} + (\ln 3)^2 \cdot 3^x$

2. Find $f'(3)$ where $f(z) = \frac{z^2+1}{\sqrt{z}}$. $f'(3) = \frac{\sqrt{3}(2(3)) - (3^2+1)\frac{1}{2}3^{-1/2}}{(\sqrt{3})^2} = \frac{6\sqrt{3} - 5(3^{-1/2})}{3} = \frac{18-5}{3\sqrt{3}} = \frac{13}{3\sqrt{3}} \doteq 2.502$

3. Find $h'(y)$ where $h(y) = (3y^2+7y)(2(1.3)^y+5)$ $h'(y) = (6y+7)(2(1.3)^y+5) + (3y^2+7y)((\ln 1.3)2(1.3)^y)$

4. Find $h'(2)$ where $h(x) = 2f(x) \cdot g(x)$ and $f(2) = 7$, $f'(2) = -2$, $g(2) = -1$, $g'(2) = 3$.

$h'(2) = 2f'(2)g(2) + 2f(2)g'(2) = 2(-2)(-1) + 2(7)(3) = 46$

Part II - Given the table below, find the indicated derivatives at $x = 1$ and $x = -2$ where possible.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 1 | 3 | −2 | −1 |
| −2 | −2 | −5 | 1 | 7 |

1. $\frac{d}{dx}[(f(x))^2 - 3g(x^2)]$ at $x = 1$ $= 2f(1)f'(1) - 3g'(1^2)\cdot 2(1) = 2(1)(3) - 3(-1)\cdot 2 = 12$

2. $\frac{d}{dx}[f(x)\cdot g(x)]$ at $x = 1$ $= f'(1)g(1) + f(1)g'(1) = 3(-2) + 1(-1) = -7$

3. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x = -2$ $= \frac{g(-2)f'(-2) - f(-2)g'(-2)}{g(-2)^2} = \frac{1(-5) - (-2)(7)}{1} = 9$

4. $\frac{d}{dx}[f(g(x))]$ at $x = 1$

$= f'(g(1))\cdot g'(1) = f'(-2)(-1) = 5$

5. $\frac{d}{dx}[g(f(x))]$ at $x = -2$

$= g'(f(1))\cdot f'(1) = g'(1)\cdot f'(1) = (-1)(3) = -3$

6. $\frac{d}{dx}[g(g(x))]$ at $x = -2$

$= g'(g(-2))\cdot g'(-2) = g'(1)\,g'(-2) = (-1)\cdot 7 = -7$

7. $\frac{d}{dx}[f(g(4-6x))]$ at $x = 1$

$f'(g(4-6(1)))\cdot g'(4-6(1))\cdot(-6)$

$= f'(g(-2))\cdot g'(-2)\cdot(-6)$

$= f'(1)\cdot g'(-2)(-6)$

$= 3\cdot 7\cdot(-6)$

$= -126$

8. $\frac{d}{dx}[(g(x))^2]$ at $x = 1$

$= 2g(1)\cdot g'(1)$

$= 2(-2)(-1)$

$= +4$

## Section 3.4 – Differentiation Practice

USE THE VALUES IN THE FOLLOWING TABLE TO ANSWER THE QUESTIONS BELOW.

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ | $f'(x)$ | $g'(x)$ | $h'(x)$ | $f''(x)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | -1 | 4 | -5 | 0 |
| 1 | 3 | 2 | 1 | 3 | -2 | -4 | -4 |
| 2 | 1 | 0 | 3 | -2 | 3 | 2 | 1 |
| 3 | 2 | 3 | 0 | 4 | 2 | -3 | 2 |

1. Determine if $y = f(x)g(x)$ has a horizontal tangent at $x = 1$.

$y'(1) = f'(1)g(1) + f(1)g'(1) = 3\cdot 2 + 3\cdot(-2) = 0$, so yes.

2. Determine if $y = h(g(x))$ is increasing or decreasing at $x = 3$.

$y'(3) = h'(g(3))\cdot g'(3) = h'(3)\cdot 2 = -6$; decreasing.

3. Find the equation of the tangent line to $y = f(g(x))$ at $x = 2$.

$y'(2) = f'(g(2))\,g'(2) = f'(0)\cdot 3 = -3$

$y(2) = f(g(2)) = f(0) = 0$

$y - 0 = -3(x-2)$

$y = -3x + 6$

4. Find $u'(1)$ if $u(x) = \sqrt{h(x)+3}$

$u'(1) = \frac{1}{2}(h(1)+3)^{-1/2}\cdot h'(1) = \frac{1}{2}(1+3)^{-1/2}\cdot(-4) = -1$

5. Determine if $y(x) = (f(x))^2$ is concave up or down at $x = 1$.

$y'(x) = 2f(x)f'(x)$

$y''(1) = 2f'(1)f(1) + 2f(1)f''(1) = 2\cdot3\cdot3 + 2\cdot3\cdot(-4) = -6$; concave down

6. Find the slope of $y = \frac{g(x)}{x^3}$ at $x = 2$.

$y'(2) = \frac{2^3 g'(2) - g(2)\cdot 3(2)^2}{(2^3)^2} = \frac{8\cdot 3 - 0\cdot 12}{64} = \frac{24}{64} = \frac{3}{8}$

7. Find $\frac{dy}{dx}$ for $y = f(g(3))$.

$\frac{dy}{dx} = f'(g(3))\cdot g'(3) = 4\cdot 2 = 8$

8. Find $u'(4)$ if $u(x) = h(\sqrt{x})$.

$u'(4) = h'(\sqrt{4})\cdot \frac{1}{2}4^{-1/2} = 2\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

9. Find the slope of the tangent line to $y = e^{g(x)}$ at $x = 0$.

$y'(0) = g'(0)e^{g(0)}$

$= 4e$