Tues 30 June

- Exam 3 on Thursday is 50 minutes, covers $\oint$3.10-4.6. $\oint$4.7 will be next week.
- MLP homeworks are both due Sunday. Happy 4th!
- tomorrow: review for Exam 3

4.6 Mean Value Theorem

Theorem (Rolle's Theorem)

Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$. Then there is at least one point c in (a, b) such that $f'(c) = 0$.

Theorem (Mean Value Theorem (MVT))

If f is continuous on a closed interval $[a, b]$ and differentiable on (a, b), then there is at least one point c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

See Figure 4.68 in your text for a visual justification of MVT. The slope of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$ is

$$\frac{f(b) - f(a)}{b - a}.$$

MVT says that there is a point c on f where the tangent line at c (whose slope is $f'(c)$) is parallel to this secant line.

Example

Let $f(x) = x^2 - 4x + 3$.

(a) Determine whether the MVT applies to $f(x)$ on the interval $[-2, 3]$.

(b) If so, find the point(s) that are guaranteed to exist by the MVT.

Example

How many points c satisfy the conclusion of the MVT for $f(x) = x^3$ on the interval $[-1, 1]$? Justify your answer.

Consequences of MVT

Theorem (Zero Derivative Implies Constant Function)

If f is differentiable and $f'(x) = 0$ at all points of an interval I, then f is a constant function on I.

Theorem (Functions with Equal Derivatives Differ by a Constant)

If two functions have the property that $f'(x) = g'(x)$ for all x of an interval I, then $f(x) - g(x) = C$ on I, where C is a constant.

Theorem (Intervals of Increase and Decrease)

Suppose f is continuous on an interval I and differentiable at all interior points of I.

- *If $f'(x) > 0$ at all interior points of I, then f is increasing on I.*
- *If $f'(x) < 0$ at all interior points of I, then f is decreasing on I.*

4.6 Book Problems

7, 10, 11, 13, 15, 17, 20-22, 24-26, 29