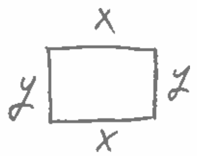
Survey of Calculus Optimization Practice
October 27, 2016

1. A carpenter is building a rectangular shed with a fixed perimeter of 52 feet. What are the dimensions of the largest shed that can be built?



$A = xy$

$52 = 2x + 2y$

$y = 26 - x$

$A = x(26 - x)$

$= -x^2 + 26x$

$A' = -2x + 26$

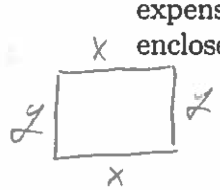
$0 = -2x + 26$

$2x = 26$

$x = 13 \rightarrow y = 26 - 13 = 13$

13 ft $\times$ 13 ft

2. A fence must be built to enclose a rectangular area of $20,000 \text{ ft}^2$. Fencing costs \$1 per foot for the two sides facing north and south. The sides facing east and west use a more expensive fencing which costs \$2 per foot. Find the cost of the least expensive fence that encloses the desired area.



$yx = 20,000$

$c = 2x + 4y$

$y = \frac{20,000}{x}$

$c = 2x + 4\left(\frac{20,000}{x}\right)$

$= 2x + 80,000x^{-1}$

$c' = 2 - 80,000x^{-2}$

$0 = 2 - 80,000x^{-2}$

$40,000 = x^2$

$x = 200, \; y = \frac{20,000}{200} = 100$

200 ft $\times$ 100 ft

so $c = 2(200) + 4(100) = \$800$

3. The llama population of a certain area can be modeled using the function $L(t) = 7te^{-t/13}$, where t is the number of years after 2015 and $L(t)$ is measured in hundreds of llamas. In what year will the llama population in the area reach its maximum? (Round your answer to the nearest year.)

$L'(t) = 7t\left(e^{-t/13} \cdot \frac{-1}{13}\right) + 7e^{-t/13}$

$0 = \frac{-7}{13}te^{-t/13} + 7e^{t/13}$

$0 = -7e^{-t/13}\left(\frac{1}{13}t - 1\right)$

$\frac{1}{13}t - 1 = 0$

$t = 13$

13 yrs. after 2015 is 2028

4. The American Pre-Ground Coffee Company wants to manufacture cylindrical aluminum coffee cans with a volume of 1,250 cm^2. What should the radius and height of the container be to minimize the amount of alluminum used?

$1250 = \pi r^2 h$

Surface area $= 2\pi r^2 + 2\pi r h$

$h = \frac{1250}{\pi r^2}$

$S = 2\pi r^2 + 2\pi r\left(\frac{1250}{\pi r^2}\right) = 2\pi r^2 + 2500 r^{-2}$

$S' = 4\pi r - 2500 r^{-3}$

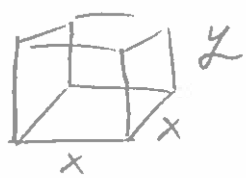
$0 = 4\pi r - 2500 r^{-3}$

$0 = 4r(\pi - 625 r^{-4})$

$\cancel{r = 0}$ $\quad r = \frac{5}{\sqrt[4]{\pi}}$

$\to h = \frac{1250}{\pi\left(\frac{5}{\sqrt[4]{\pi}}\right)^2} = \frac{1250}{\pi \cdot \frac{25}{\pi^{1/2}}} = \frac{1250}{\pi^{1/2} \cdot 25} = 50\pi$

5. Suppose you are constructing an open-top rectangular box with a square base and a volume of $32 in^3$. What dimensions of the box will maximize the surface area?



$V = 32 = x^2 y \to y = \frac{32}{x^2}$

$S = 4xy + x^2$

$S = 4x\left(\frac{32}{x^2}\right) + x^2$

$= 128x^{-1} + x^2$

$S' = -128x^{-2} + 2x$

$0 = -128x^{-2} + 2x$

$0 = 2x(-64x^{-3} + 1)$

$\cancel{x = 0}$ $\quad -64x^{-3} + 1 = 0$

$x^{-3} = \frac{1}{64}$

$x = 4$

$\to y = \frac{32}{4^2} = 2$

4 in x 4 in x 2 in.