1. $\lim_{x\to 0} \frac{\sin(6x)}{\sin x} = \lim_{x\to 0} \frac{\sin(6x)}{x} \Big/ \frac{\sin x}{x}$

$= \left(\lim_{x\to 0} \frac{6\sin(6x)}{6x}\right) \Big/ \left(\lim_{x\to 0} \frac{\sin x}{x}\right) = 6/1 = 6$

2. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) =$

$\frac{(-\sin x)\sin x - (\cos x)\cos x}{(\sin x)^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= -1/\sin^2 x = -\csc^2 x$

3. $s(t) = 2t^3 - 21t^2 + 60t$

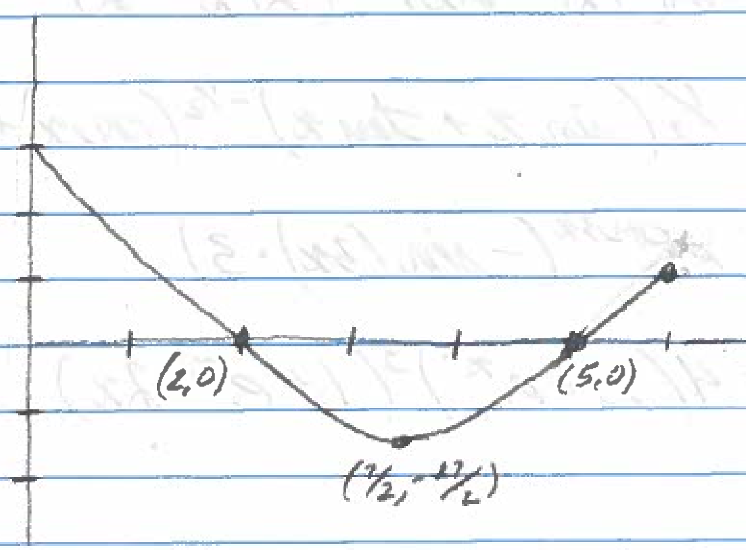
$v(t) = s'(t) = 6t^2 - 42t + 60$

$= 6(t^2 - 7t + 10) = 6(t-5)(t-2)$

$-\frac{b}{2a} = \frac{-(-7)}{2(1)} = 7/2$

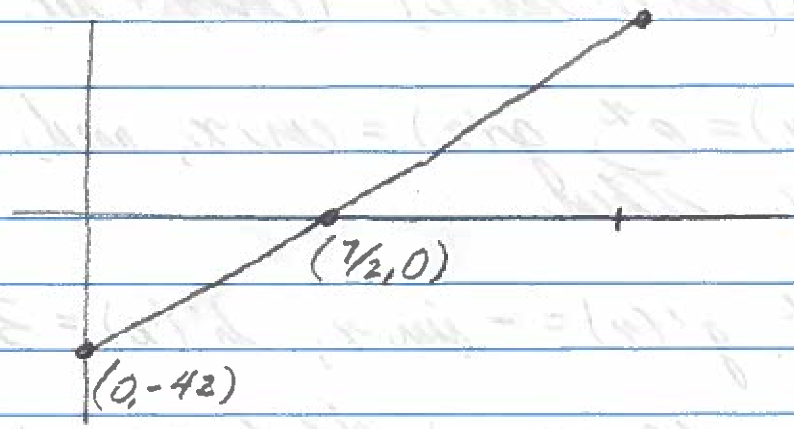
$v(7/2) = 6(49/4 - 49/2 + 10) = 6(-49/4 + 10)$

$= 6(-9/4) = -54/4 = -27/2$



4. The object is stationary at $t=2$ and at $t=5$ because its velocity is 0 at those times.
The object attains its minimum velocity when $t = 7/2$ because that is where the vertex of the graph of v occurs.

5. $v(t) = 6t^2 - 42t + 60$
$a(t) = v'(t) = 12t - 42$



6. $\frac{dy}{dx} = \frac{d}{dx}(1+\cos x)(\sin x)$
$+ (1+\cos x)\cdot \frac{d}{dx}(\sin x)$

$= (-\sin x)(\sin x) + (1+\cos x)(\cos x)$

7. Let $f(x) = x^{-2}$ and $g(x) = 7x^3 - 2x$. Then

$$f'(x) = -2x^{-3}, \quad g'(x) = 21x^2 - 2,$$

and

$$dy/dx = f'(g(x)) \cdot g'(x) = -2(7x^3 - 2x)^{-3}(21x^2 - 2)$$

8. Let $f(x) = \sqrt{x}$ and $g(x) = \sin x + \tan x$. Then

$$f'(x) = \tfrac{1}{2}x^{-1/2}, \quad g'(x) = \cos x + \sec^2 x,$$

and

$$dy/dx = \tfrac{1}{2}(\sin x + \tan x)^{-1/2}(\cos x + \sec^2 x)$$

9. Let $f(x) = e^x$, $g(x) = \cos x$, and $h(x) = 3x$. Then

$$f'(x) = e^x, \quad g'(x) = -\sin x, \quad h'(x) = 3,$$

and

$$dy/dx = f'(g(h(x))) \cdot d/dx(g(h(x)))$$

$$= f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

$$= e^{\cos 3x}(-\sin 3x) \cdot 3$$

10. Let $f(x) = x^4$ and $g(x) = x + e^{x^2}$.
Then

$$f'(x) = 4x^3 \quad \text{and}$$

$$g'(x) = 1 + \tfrac{d}{dx}(e^{x^2}).$$

We must again use the chain rule,
so let $p(x) = e^x$ and $q(x) = x^2$.
Now

$$p'(x) = e^x \quad \text{and} \quad q'(x) = 2x,$$

so $\tfrac{d}{dx}(e^{x^2}) = e^{x^2} \cdot 2x$. This means

$$g'(x) = 1 + e^{x^2}(2x)$$

so $\tfrac{dy}{dx} = 4(x + e^{x^2})^3(1 + e^{x^2} \cdot 2x)$

Solutions to Quiz 6

1. We rewrite the limit, and make use of the limit $\lim_{x\to 0}\frac{\sin x}{x}=1$.

2. We begin by using the trigonometric identity $\cot x=\frac{\cos x}{\sin x}$. After that, we apply the quotient rule to find the derivative. Our final result is due to the trig identity $\frac{1}{\sin x}=\csc x$.

3. The velocity function is the derivative of the position function. In this case, the velocity function v is a quadratic function, so we factor it to find its roots. We can also find the coordinates of the vertex since the x-coordinate is always equal to $-\frac{b}{2a}$. With this information, we are able to sketch a graph of the function.

5. We differentiate the velocity function to find the acceleration function, a. In this case a is linear, so its graph is a straight line.

6. In problem 6 we apply the product rule.

7-10. In the last four problems we apply the chain rule to find the derivatives. In problems 7 and 8, we determine our outside function (f) and our inside function (g) before taking their derivatives and then applying the chain rule. Problems 9 and 10 require two applications of the chain rule. In problem 9, we have a composition of three functions, which we call f, g, and h. In problem 10, we have a composition of two functions, but one of the terms of the inside function is itself a composition of two functions.