Example:

Suppose that the total profit in hundreds of dollars from selling x items is given by $P_f(x) = 2x^2 - 5x + 6$. Find the average rate of change of profit from $x = \underset{a}{2}$ to $x = \underset{b}{4}$.

$$\frac{P(4)-P(2)}{4-2} = \frac{2\overset{32}{(4)^2} \overset{-20}{- 5(4)} + \cancel{6} - \left(2(2)^2 - 5(2) + \cancel{6}\right)}{2}$$

$$= \frac{14}{2} = 7 \Rightarrow \boxed{\$700 \text{ per item}}$$

Suppose customers in a hardware store are willing to buy $N(p)$ boxes of nails at p dollars per box, as given by $N(p) = 80 - 5p^2, 1 \leq p \leq 4$. Find the average rate of change of demand for a change of price from \$2 to \$3.

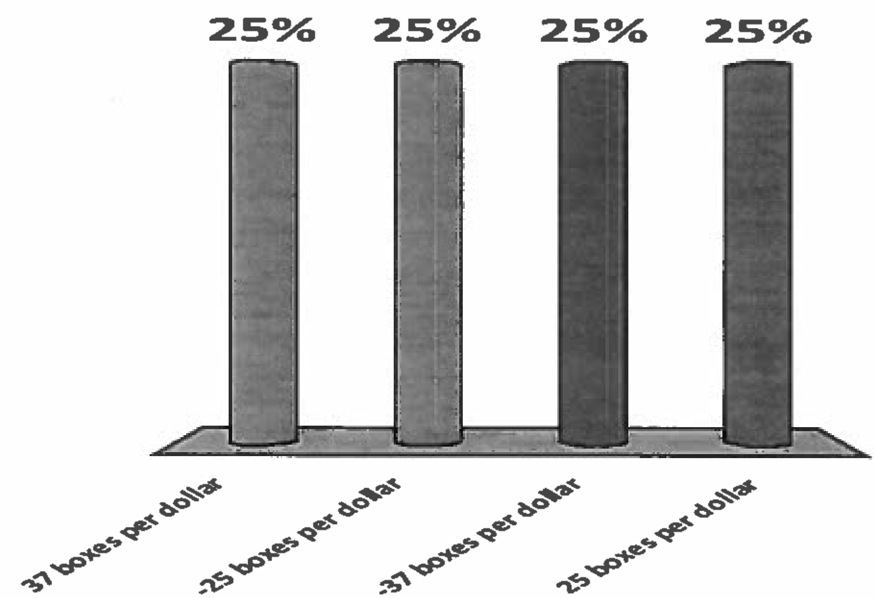
- see next page for work -

A. 37 boxes per dollar

B. -25 boxes per dollar

C. -37 boxes per dollar

D. 25 boxes per dollar



$$\frac{N(3)-N(2)}{3-2} = \frac{\cancel{80} - 5(3)^2 - \left(\cancel{80} - 5(2)^2\right)}{1}$$

$$= -45 + 20$$

$$= -25$$

$\Rightarrow$ $\boxed{\text{25 fewer boxes per dollar}}$

Suppose that the total profit in hundreds of dollars from selling x items is given by $P(x) = 2x^2 - 5x + 6$. Find the instantaneous rate of change of profit when $x = 2$.

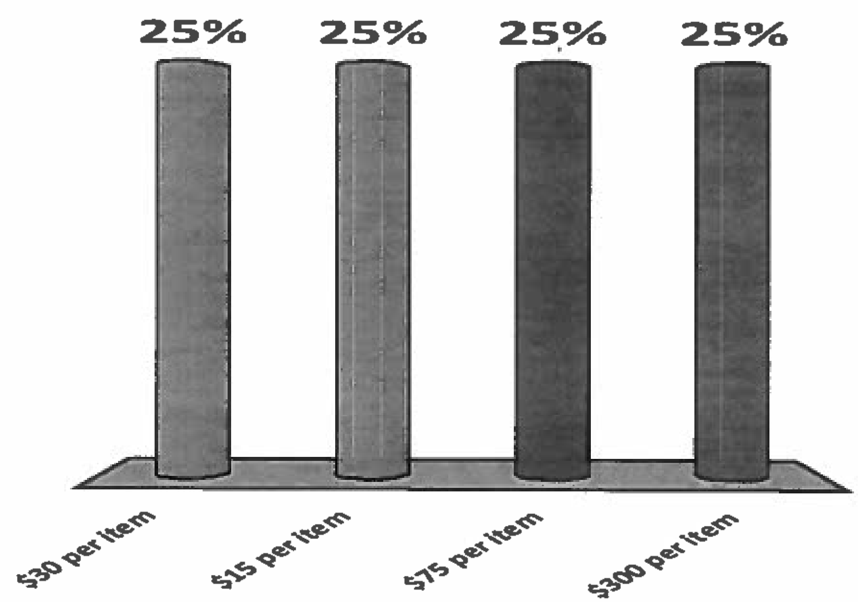
—see next page for work—

A. \$30 per item

B. \$15 per item

C. \$75 per item

D. \$300 per item



$$\lim_{x\to 2} \frac{P(x)-P(2)}{x-2}$$

$$=\lim_{x\to 2} \frac{2x^2-5x+\cancel{6}-(2\overset{(8}{(2)^2}-\overset{-10)}{5(2)}+\cancel{6})}{x-2}$$

$$=\lim_{x\to 2} \frac{2x^2-5x+2}{x-2} \quad \overset{2x^2-x-4x+2\ \checkmark}{(2x-1)(x-2)}$$

$$=\lim_{x\to 2} \frac{(2x-1)\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x\to 2}(2x-1)$$

$$=2(2)-1$$

$$=3$$

$\Rightarrow$ $\boxed{\text{\$300 per item}}$

OR

$$\lim_{h\to 0} \frac{P(2+h)-P(2)}{h}$$

$$=\lim_{h\to 0} \frac{2(2+h)^2-\overset{-10-5h}{5(2+h)}+\cancel{6}-(\overset{(8}{2(2)^2}-\overset{-10)}{5(2)}+\cancel{6})}{h}$$

$$=\lim_{h\to 0} \frac{2(2)^2+4(2)h+2h^2-10-5h+2}{h}$$

$$=\lim_{h\to 0} \frac{\cancel{8}+8h+2h^2-\cancel{8}-5h}{h}$$

$$=\lim_{h\to 0} \frac{3\cancel{h}+2h^2}{\cancel{h}} = \lim_{h\to 0}(3+2h)$$

$$=3+2(0)=3$$

$\Rightarrow$ $\boxed{\text{\$300 per item}}$

Find the average rate of change of the function $y = e^x$ between $x = 0$ and x=4.

$$\frac{e^4 - e^0}{4-0} = \frac{e^4 - 1}{4} \approx 13.3995$$

A. 13.3995

B. 12.2219

C. 14.1356

D. 8.4921