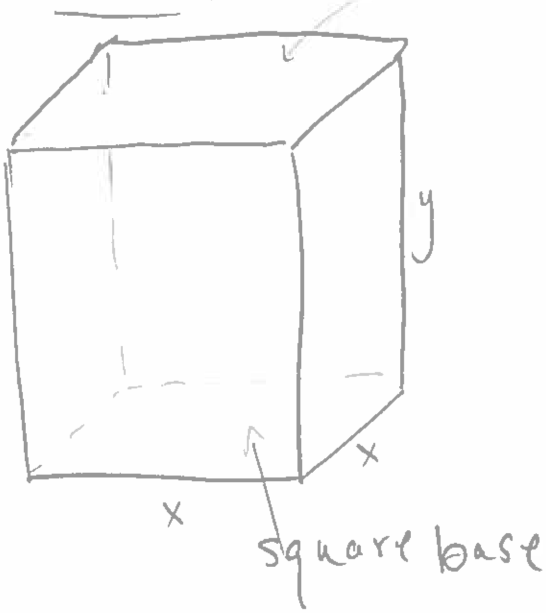
# Slides Exercises Solutions (Fri 26 June)

- An open rectangular box with square base is to be made from 48 ft$^2$ of material. What dimensions will result in a box with the largest volume?

Solution.




Objective:

Maximize Volume

$$V = x^2 y$$

Constraint:

The surface area is 48 ft.

$$48 = \underbrace{x^2}_{\text{base}} + \underbrace{4xy}_{\text{four walls}}$$

$$\rightarrow y = \frac{48 - x^2}{4x} = \frac{12}{x} - \frac{x}{4}$$

one variable:

$$V(x) = x^2\left(\frac{12}{x} - \frac{x}{4}\right) = 12x - \frac{1}{4}x^3$$

$$V'(x) = 12 - \frac{3}{4}x^2 = 0$$

$$\Rightarrow x^2 = 16$$

$$x = 4$$

For a domain, the smallest $x$ can be is zero. The biggest is when $y = 0$, so $x = \sqrt{48}$.

Check for a max:

$$V(0) = 12(0) - \frac{1}{4}(0^3) = 0$$

$$V(\sqrt{48}) = 12\sqrt{48} - \frac{1}{4}(\sqrt{48})^3$$

$$= 12\sqrt{48} - \frac{48}{4}\sqrt{48} = 0$$

$$V(4) = 12(4) - \frac{1}{4}(4^3)$$
$$= 48 - 4^2 = 32$$

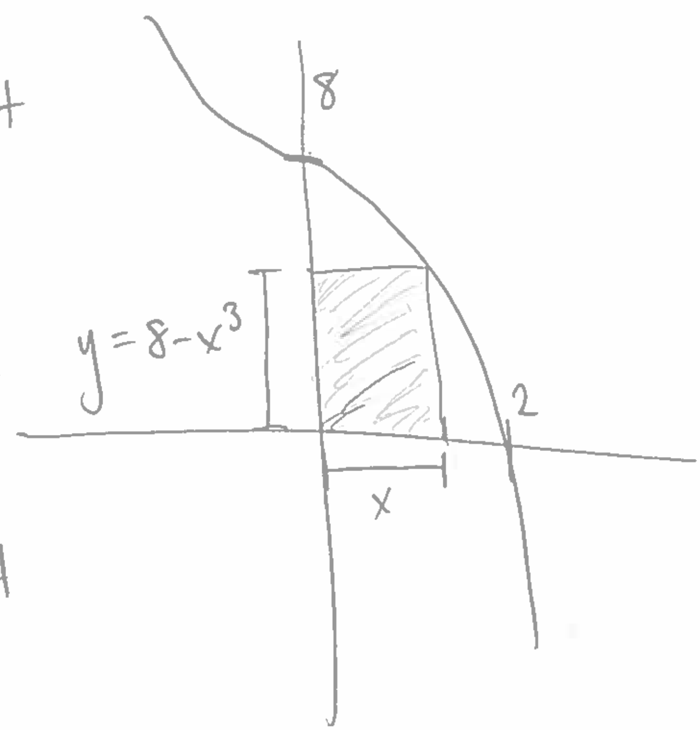
So $\boxed{x = 4\text{ ft}}$ gives a max and we have

$$\boxed{y = \frac{12}{4} - \frac{4}{4} = 3 - 1 = 2\text{ ft}}$$

- Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the x-axis, the y-axis, and the graph of $y = 8 - x^3$

Solution.

To draw the picture, first consider the graph of $x^3$. The negative sign flips it upside down and then adding 8 shifts it up 8 units.




To find the x-intercept, set
$$0 = 8 - x^3$$
$$\Rightarrow x = 2.$$

If $x$ is the length of the rectangle, then we have a formula for the height, it's just the y-value of the graph, $y = 8 - x^3$.

Objective: Maximize area of the rectangle.

$$A = xy$$
$$= x(8-x^3) = 8x - x^4$$

The constraint is the formula $y = 8 - x^3$, along with the conditions

$$0 \le x \le 2$$
$$0 \le y \le 8$$

from the picture.

$$\frac{dA}{dx} = 8 - 4x^3 = 0$$
$$\Rightarrow x = \sqrt[3]{2}$$

Check for a max:

$$A(0) = 0(8-0^3) = 0$$
$$A(2) = 2(8-2^3) = 0$$
$$A(\sqrt[3]{2}) = \sqrt[3]{2}\left(8-(\sqrt[3]{2})^3\right) = 6\sqrt[3]{2}$$

so $\boxed{x = \sqrt[3]{2}}$ gives a max and we have

$$\boxed{y = 8 - (\sqrt[3]{2})^3 = 6}$$