Mon 15 Sep 2014

- Tuesday Sept 16: Quiz 3 covers sections 2.6-2.7
- Friday Sept 19: Exam 1 (Chapter 2)
- Preparing for the exam: Go over quizzes. Attend office hours for questions. Do the book problems.

Recall from Chapter 2

Recall from the first day of class, the relationship between secant lines and tangent lines.

We said that the slope of the tangent line at a point is the limit of the slopes of the secant lines as the points get closer and closer.

P = (a, f(a)); Q = (x, f(x)).

Slope of secant line: $\dfrac{f(x)-f(a)}{x-a}$; (average rate of change)

Slope of tangent line: $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$; (instantaneous r.o.c.)

Example

Use the relationship between secant lines and tangent lines, specifically the slope of the tangent line, to find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point P=(1, 5).

Alternate Way of Viewing Tangent Lines

Instead of looking at the points approaching one another, we can also view this as the distance between the points approaching 0. So:

P = (a, f(a)); Q = (a+h, f(a+h)).

Slope of secant line: $\dfrac{f(a+h)-f(a)}{(a+h)-a}=\dfrac{f(a+h)-f(a)}{h}$

Slope of tangent line: $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

Example

Find the equation of a line tangent to the curve

$f(x) = x^2 + 2x + 2$ at the point P=(2, 10).

Definition of Derivative

The slope of the tangent line for the function *f* is a function of *x*, called the derivative of *f*.

The derivative of *f* is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. If $f'(x)$ exists, we say *f* is differentiable at *x*. If *f* is differentiable at every point of an open interval *I*, we say that *f* is differentiable on *I*.

Example

Use the definition of the derivative to find the derivative of the function $f(x) = x^2 + 2x + 2$

Other ways to write derivative

The following are alternative ways of writing the derivative of the function *f* at the point *x:*

$$\frac{dy}{dx}, \frac{df}{dx}, \frac{d}{dx}(f(x)), D_x(f(x)), y'(x)$$

The following are ways to notate the derivative of *f* evaluated at *x* = *a*:

$$f'(a), y'(a), \left.\frac{df}{dx}\right|_{x=a}, \left.\frac{dy}{dx}\right|_{x=a}$$

Graphing the Derivative

The graph of the derivative is essentially the graph of the collection of slopes of the tangent lines of a graph. If you just have a graph (without an equation for the graph), the best you can do is approximate the graph of the derivative.

Simple checklist:

1. Note where $f'(x) = 0$
2. Note where $f'(x) > 0$ (what does this look like?)
3. Note where $f'(x) < 0$ (what does this look like?)

Differentiability and Continuity

Key points about the relationship between differentiability and continuity

- If f is differentiable at a, then f is continuous at a.
- If f is not continuous at a, then f is not differentiable at a.
- **NOTE:** f can be continuous but not differentiable.

When is a function not differentiable at a point?

A function *f* is <u>not</u> differentiable at *a* if at least one of the following conditions holds:

1. *f* is not continuous at *a*.
2. *f* has a corner at *a* (Why does this make *f* not differentiable?)
3. *F* has a vertical tangent at *a* (Why does this make *f* not differentiable?)

Wed 17 Sep 2014

Friday Sept 19: Exam 1 (Chapter 2)

- Preparing for the exam: Go over quizzes. Attend office hours for questions. Do the book problems.

Recall:

If a function g is not continuous at $x = a$, then g

A. Must be undefined at $x = a$.
B. Is not differentiable at $x = a$.
C. Has an asymptote at $x = a$.
D. All of the above.
E. A and B only

HW from section 3.1

- Do problems 11-12, 19-20, 23-26, 31-33, 35-36, 39-43, 45 all (pgs. 131-133 in textbook)

- **NOTE:** We "do not know" any rules for differentiation yet (e.g., power rules, chain rules, etc.). In this section, you are strictly using the definition of the derivatives and definition of slope of tangent lines we have derived!!!

Review for Exam 1, Covering Chapter 2

- 2.1: The Idea of Limits
  - Understand the relationship between average velocity & instantaneous velocity, and secant and tangent lines
  - Be able to compute average velocities and use the idea of a limit to approximate instantaneous velocities
  - Be able to compute slopes of secant lines and use the idea of a limit to approximate the slope of the tangent line
- 2.2: Definitions of Limits
  - Know the definition of a limit
  - Be able to use a graph or a table to determine a limit
  - Know the relationship between one- and two-sided limits

Review for Exam 1, Covering Chapter 2

- 2.3: Techniques for Computing Limits
  - Know and be able to compute limits using analytical methods (e.g. limit laws, additional techniques)
  - Know the Squeeze Thm and be able to use this theorem to determine limits
- 2.4: Infinite Limits
  - Be able to use a graph, a table, or analytical methods to determine infinite limits
  - Know the definition of a vertical asymptote and be able to determine whether a function has vertical asymptotes
- 2.5 Limits at Infinity
  - Be able to find limits at infinity and horizontal asymptotes
  - Know how to compute the limits at infinity of rational functions

Review for Exam 1, Covering Chapter 2

- 2.6: Continuity
  - Know the definition of continuity and be able to apply the continuity checklist
  - Be able to determine the continuity of a function (including those with roots) on an interval
  - Be able to apply the Intermediate Value Thm to a function
- 2.7: Precise Definitions of Limits
  - Understand the δ, ε relationship for limits
  - Be able to use a graph or analytical methods to find a value for

$\delta > 0$ given an $\varepsilon > 0$ (including finding symmetric intervals)

On Exam Day:

- Show up as on time as possible. The exam is designed for 50 minutes.
- Bring pencils and erasers.
- Permissible Calculators: TI-30X, slide rule, abacus
- Sit according to section.
- You are allowed one 3x5 inch notecard, one side only! Using anything else will get it confiscated (and you get in trouble for cheating).