Unit 1, Lesson 4
Rates of Change

Rates of Change

OBJECTIVES:

- Compute the average rate of change of a function between two points.
- Set up and simplify the difference quotient for functions.
- Find instantaneous rates of change using limits.

There are two types of rate of change we're going to be working with. What is the difference between average rate of change and instantaneous rate of change?

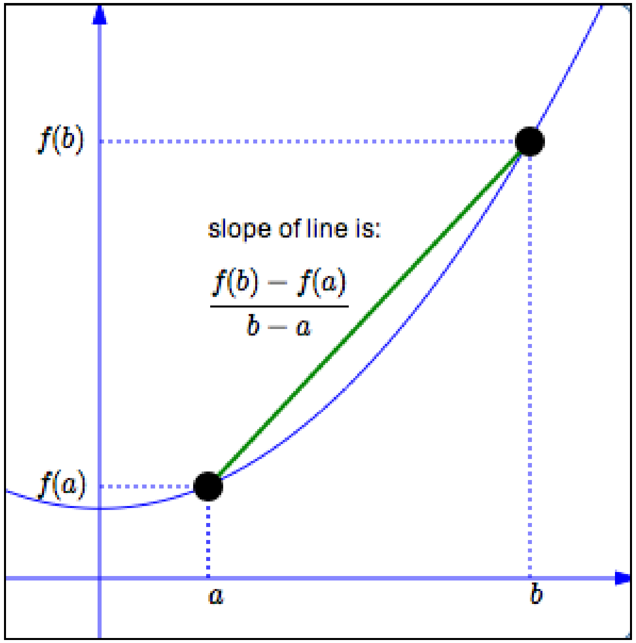
Average rate of change is over an interval.

Instantaneous rate of change is at a point.

Average Rate of Change

The **average rate of change** of $f(x)$ with respect to x for a function f as x changes from a to b is

$$\frac{f(b) - f(a)}{b - a}.$$

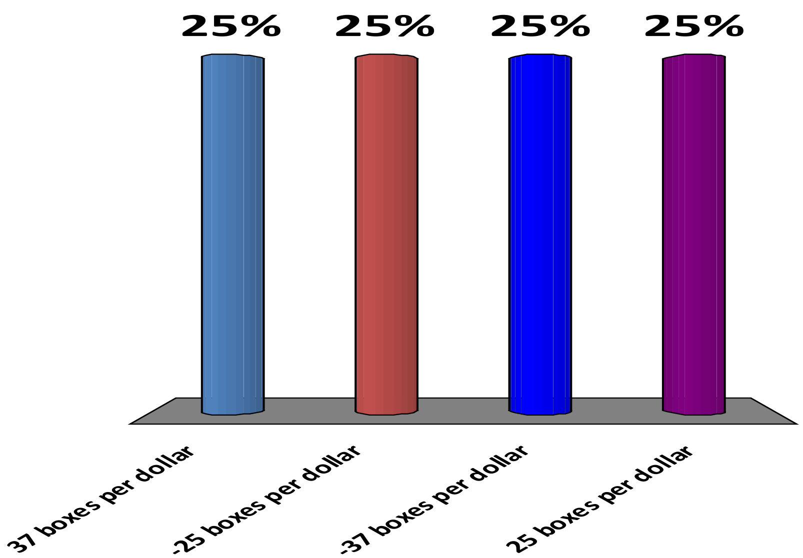


Example:

Suppose that the total profit in hundreds of dollars from selling x items is given by $P(x) = 2x^2 - 5x + 6$. Find the average rate of change of profit from $x = 2$ to $x = 4$.

Suppose customers in a hardware store are willing to buy $N(p)$ boxes of nails at p dollars per box, as given by
$N(p) = 80 - 5p^2,\ 1 \le p \le 4$. Find the average rate of change of demand for a change of price from \$2 to \$3.

A. 37 boxes per dollar
B. -25 boxes per dollar
C. -37 boxes per dollar
D. 25 boxes per dollar



Instantaneous Rate of Change

The **instantaneous rate of change** for a function f when $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided this limit exists.

Instantaneous Rate of Change (Alternate Form)

The **instantaneous rate of change** for a function f when $x = a$ can be written as

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a},$$

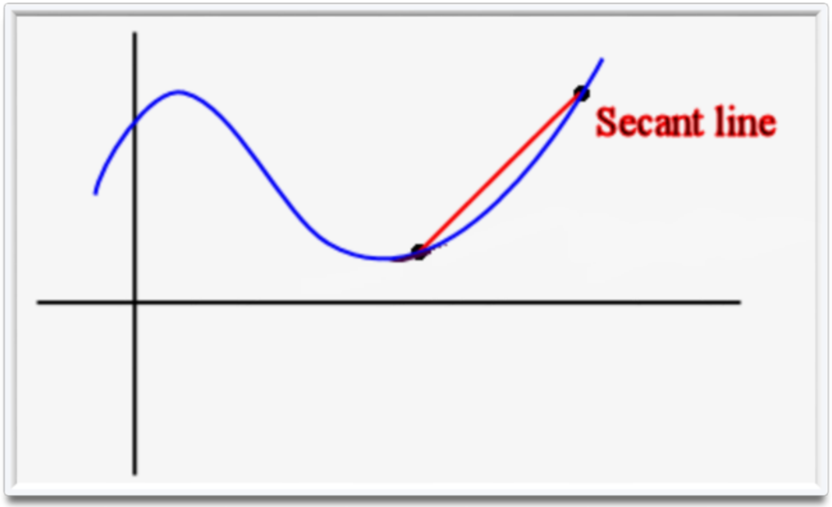
provided this limit exists.

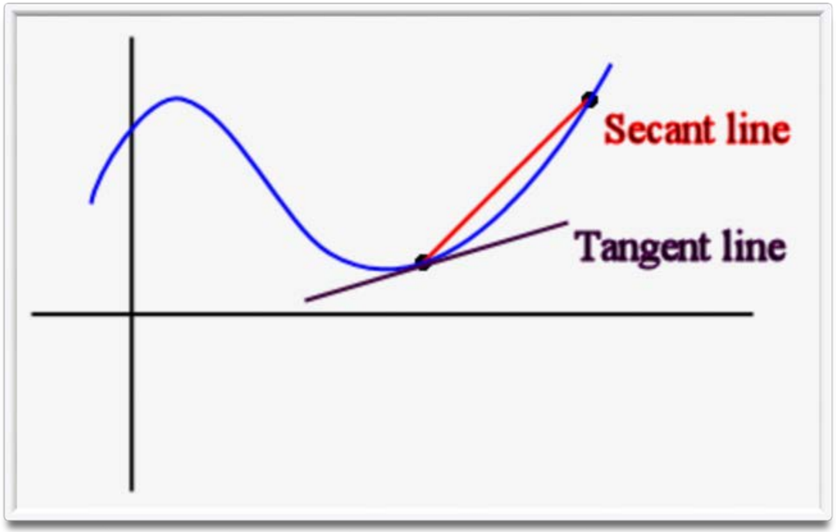
Example:

The distance in feet of an object from a starting point is given by $s(t) = 2t^2 - 5t + 40$, where t is time in seconds.

a) Find the average velocity of the object from 2 seconds to 4 seconds.

b) Find the instantaneous velocity at 4 seconds.


Secant line


Secant line
Tangent line

Difference Quotient:

$$\frac{f(x+h)-f(x)}{h}$$

Example:

Find and simplify the difference quotient:

$$f(x) = x^2 + 2$$

Example:

A company determines that the cost in dollars to manufacture x cases of their product is given by:

$C(x) = 100 + 15x - x^2 \;\; for\; 0 \leq x \leq 7.$

Find and interpret the instantaneous rate of change of cost with respect to the number of cases produced when just one case is produced.

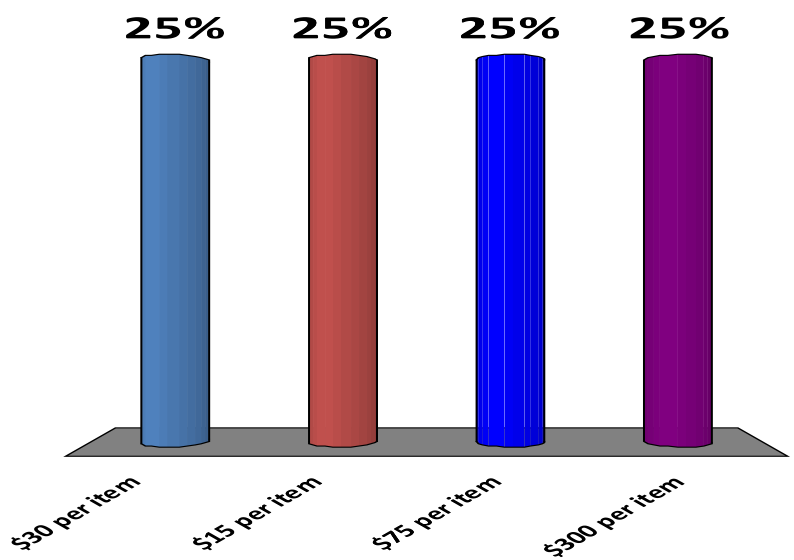
Definition:

Marginal Cost—the instantaneous rate of change of the cost function.

It is the cost of producing one additional item.

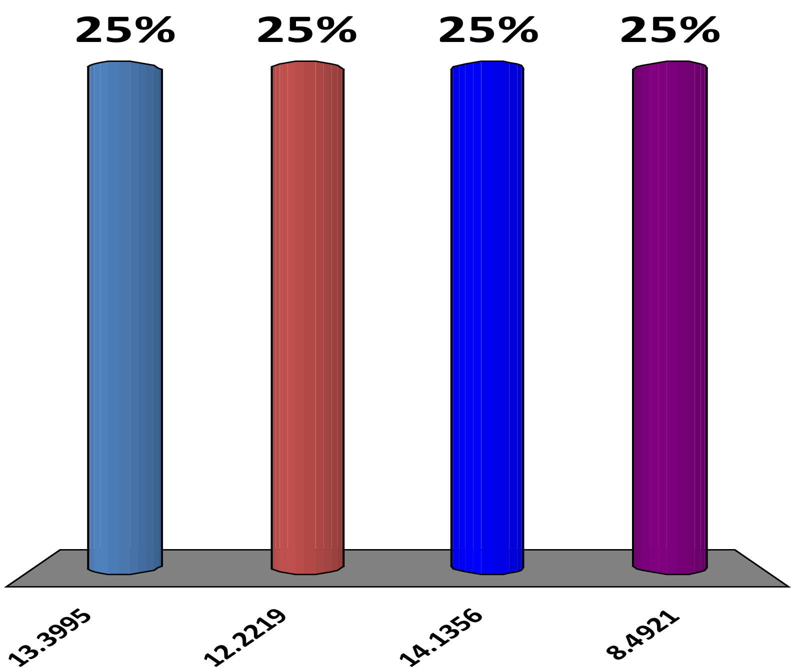
Suppose that the total profit in hundreds of dollars from selling x items is given by $P(x) = 2x^2 - 5x + 6$. Find the instantaneous rate of change of profit when $x = 2$.

A. \$30 per item
B. \$15 per item
C. \$75 per item
D. \$300 per item



Find the average rate of change of the function $y = e^x$ between $x = 0$ and x=4.

A. 13.3995
B. 12.2219
C. 14.1356
D. 8.4921



In summary

Average rate of change:

Rate of change over an interval

Change in y over change in x

Slope of a secant line

Instantaneous rate of change:

Rate of change at a point

Slope of the tangent line

(For us) Marginal cost, profit, revenue, etc.