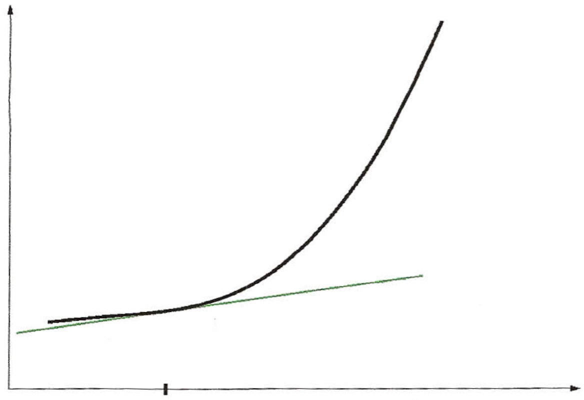
Section 2.2 – The Derivative at a Point



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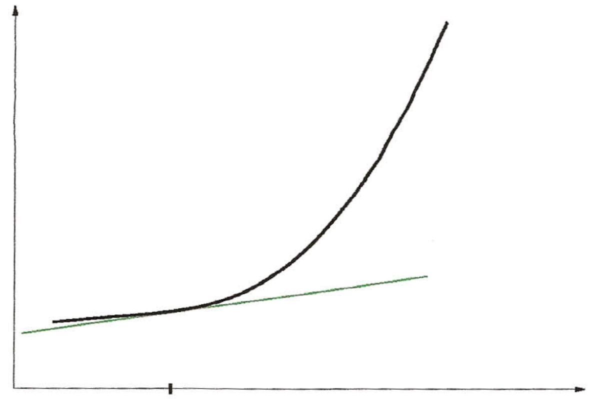

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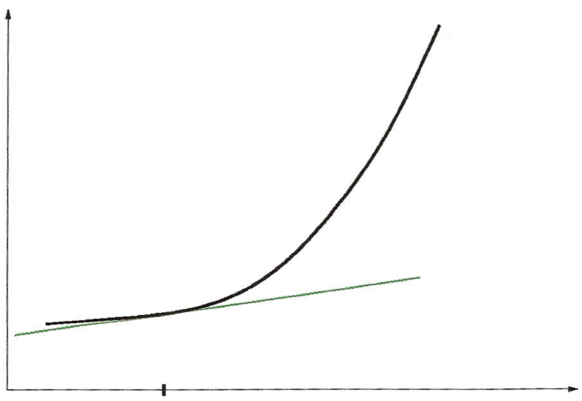


a

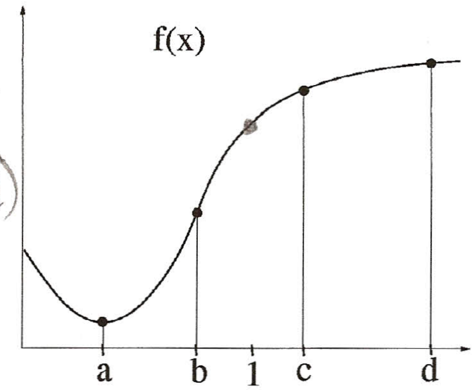
Definition. The *average rate of change* of f over the interval a to $a+h$ is given by

$$\frac{f(a+h)-f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

Definition. The instantaneous rate of change of f at a, called the *derivative of f at a*, is given by

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

1. Given to the right is the graph of a function f.



(a) Rank the following quantities in order from smallest to largest: $f(0)$, $f(a)$, $f(b)$, $f(1)$, $f(c)$, $f(d)$

$f(a) < f(0) < f(b) < f(1) < f(c) < f(d)$

(b) Rank the following quantities in order from smallest to largest: $f'(0)$, $f'(a)$, $f'(b)$, $f'(1)$, $f'(c)$, $f'(d)$

$f'(0) < f'(a) < f'(d) < f'(c) < f'(1) < f'(b)$

2. Let $f(x) = \ln x$. Estimate $f'(3)$ accurate to 2 decimal places.

$$f'(3) = \lim_{h \to 0} \frac{\ln(3+h) - \ln 3}{h}$$

$$f'(3) \doteq 0.33$$

h	-0.001	-0.0001	0.0001
difference quotient	0.333	0.333	0.333

3. Let $f(x) = x^2 - 3x - 2$.

(a) Use algebra to find $f'(x)$ at $x = 2$.

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 3(2+h) - 2 - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h + h^2 - 6 - 3h - 2 + 4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h} + h^{\cancel{2}}}{\cancel{h}} = \lim_{h \to 0} 1 + h$$

$$= 1 + \lim_{h \to 0} h = 1.$$

(b) Find the equation of the tangent line to f at $x = 2$.

slope = 1

Use the point $(2, -4)$

$y + 4 = x - 2$

$y = x - 6$