Thurs 28 May

- MyLabsPlus (MLP) homeworks due Fridays and Sundays (check each assignment for specific dates and times). All assignments for the term are posted now. You can do them as early as you wish, before their deadlines.
- Lecture slides are available on the course website. I'll try to have the week's slides posted in advance but the individual lectures might not be posted until right before class.
- Read the textbook, too! For some of you it's obvious you've been through the slides beforehand – I'm sure it's been helpful to you and reading the text in advance will be, too.

Thurs 28 May (cont.)

- Quiz tomorrow will cover $\oint$2.3-2.5. It might be collaborative/open resources, depending on how far we get today, but come prepared to work alone.
- A blank copy of Quiz 1, plus another copy with its solutions, are now posted on the course webpage.
- Lectures: We are running ahead of schedule on the lectures. I will begin next week's material early. This way you'll have more time to prepare for the Exam (which is next Friday).

Remember to check for factoring –

Exercise

(a) What is/are the vertical asymptotes of

$$f(x) = \frac{3x^2 - 48}{x + 4}?$$

(b) What is $\lim_{x \to -4} f(x)$?

2.4 Book Problems

7-10, 15, 17-26, 36-37

Limits at infinity determine what is called the **end behavior** of a function.

Definition

If $f(x)$ becomes arbitrarily close to a finite number L for all sufficiently large and positive x, then we write

$$\lim_{x \to \infty} f(x) = L.$$

The line $y = L$ is a **horizontal asymptote** of f.

The limit at negative infinity, $\lim_{x \to -\infty} f(x) = M$, is defined analogously and in this case, the horizontal asymptote is $y = M$.

Infinite Limits at Infinity

Question

Is it possible for a limit to be both an infinite limit and a limit at infinity? (Yes.)

If $f(x)$ becomes arbitrarily large as x becomes arbitrarily large, then we write

$$\lim_{x\to\infty} f(x) = \infty.$$

(The limits $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$, and $\lim_{x\to-\infty} f(x) = -\infty$ are defined similarly.)

Powers and Polynomials: Let n be a positive integer and let $p(x)$ be a polynomial.

- $n =$ even number: $\lim\limits_{x \to \pm\infty} x^n = \infty$
- $n =$ odd number: $\lim\limits_{x \to \infty} x^n = \infty$ and $\lim\limits_{x \to -\infty} x^n = -\infty$

- (again, assuming n is positive)

$$\lim_{x\to\pm\infty} \frac{1}{x^n} = \lim_{x\to\pm\infty} x^{-n} = 0$$

- For a polynomial, only look at the term with the highest exponent:

$$\lim_{x\to\pm\infty} p(x) = \lim_{x\to\pm\infty} (\text{constant}) \cdot x^n$$

  The constant is called the **leading coefficient**, $\text{lc}(p)$. The highest exponent that appears in the polynomial is called the **degree**, $\deg(p)$.

Rational Functions: Suppose $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function.

- If $\deg(p) < \deg(q)$, i.e., the numerator has the smaller degree, then
$$\lim_{x \to \pm\infty} f(x) = 0$$
and $y = 0$ is a horizontal asymptote of f.

- If $\deg(p) = \deg(q)$, i.e., numerator and denominator have the same degree, then
$$\lim_{x \to \pm\infty} f(x) = \frac{\text{lc}(p)}{\text{lc}(q)}$$
and $y = \frac{\text{lc}(p)}{\text{lc}(q)}$ is a horizontal asymptote of f.

- If $\deg(p) > \deg(q)$, (numerator has the bigger degree) then

$$\lim_{x \to \pm\infty} f(x) = \infty \quad \text{or} \quad -\infty$$

and f has no horizontal asymptote.

- Assuming that $f(x)$ is in reduced form (p and q share no common factors), vertical asymptotes occur at the zeroes of q.

(This is why it is a good idea to check for factoring and cancelling first!)

Exercise

Determine the end behavior of the following functions (in other words, compute both limits, as $x \to \pm\infty$, for each of the functions):

1. $f(x) = \dfrac{x+1}{2x^2-3}$
2. $g(x) = \dfrac{4x^3-3x}{2x^3+5x^2+x+2}$
3. $h(x) = \dfrac{6x^4-1}{4x^3+3x^2+2x+1}$

Example

Determine the end behavior of the following functions.

1. $f(x) = \dfrac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}}$ (radical signs appear)
2. $g(x) = \cos x$ (trig)
3. $h(x) = e^x$ (exponential)

2.5 Book Problems

9-10, 13-35 (odds), 39, 43, 45, 53

$\oint$ 2.6 Continuity

Informally, a function f is "continuous at $x = a$" means for x-values anywhere close enough to a the graph can be drawn without lifting a pencil. In other words, no holes, breaks, asymptotes, etc.

Definition

A function f is **continuous** at a means

$$\lim_{x \to a} f(x) = f(a).$$

If f is not continuous at a, then a is a **point of discontinuity**.

Continuity Checklist

In order to claim something is continuous, you must verify all three:

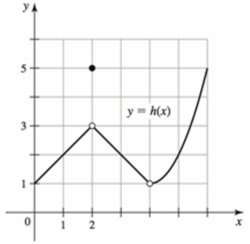
1. $f(a)$ is defined (i.e., a is in the domain of f – no holes, asymptotes).
2. $\lim_{x\to a} f(x)$ exists. You must check both sides and make sure they equal the same number.
3. $\lim_{x\to a} f(x) = f(a)$ (i.e., the value of f equals the limit of f at a).

Question

What is an example of a function that satisfies this condition?

Example

- Where are the points of discontinuity of the function below?
- Which aspects of the checklist fail?



recall (Continuity Checklist):

1. function is defined
2. the two-sided limit exists
3. 2. = 1.

If f and g are continuous at a, then the following functions are also continuous at a. Assume c is a constant and $n > 0$ is an integer.

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\dfrac{f}{g}$, provided $g(a) \neq 0$
6. $[f(x)]^n$

From the rules above, we can deduce:

1. Polynomials are continuous for all $x = a$.
2. Rational functions are continuous at all $x = a$ except for the points where the denominator is zero.
3. If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ is continuous at a.

Continuity on an Interval

Consider the cases where f is not defined past a certain point.

Definition

A function f is **continuous from the left** (or **left-continuous**) at a means

$$\lim_{x \to a^-} f(x) = f(a);$$

a function f is **continuous from the right** (or **right-continuous**) at a means

$$\lim_{x \to a^+} f(x) = f(a).$$