Tues 23 June

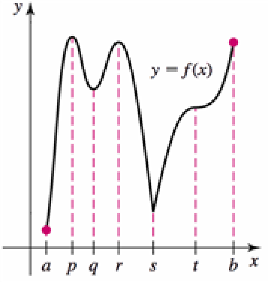
- Grades...
  - Attendance: There is a correlation between good attendance and good scores.
  - MLP: This is the most direct way you can improve your grade. Theoretically, everyone should have an A in this area. Do what you have to – start the problems early, read ahead, come to office hours, work with a friend, etc.
  - Don't skip quizzes. Right now there aren't enough to drop any so everyone's lowest scores are hurting them.

Tues 23 June (cont.)

  - The syllabus contains the information to compute your grade. For the exams, the curve information is on the course webpage; scale your letter grade range to 10 points to get an exact percentage. You are always welcome to email me or come to office hours to get information about your grade, including help with computing it.
- 2 quizzes this week
  - Quiz 7 on Wednesday: open resources + collaborative
  - Quiz 8, stay tuned...
- next Monday Quiz: Optimization ($\oint$4.4) with Related Rates sprinkled in. Closed book. Not collaborative.

Exercise

Use the graph below to identify the points on the interval $I = [a, b]$ at which local and absolute extreme values occur.



Question

Based on the previous graph, how is the derivative related to where the local extrema occur?

Answer: Local extrema occur where the derivative either does not exist or is equal to 0.

Definition

An interior point c of the domain of f at which $f'(c) = 0$ or $f'(c)$ fails to exist is called a **critical point** of f.

Local Extreme Point Theorem

Theorem (Local Extreme Point Theorem)

If f has a local minimum or maximum value at c and $f'(c)$ exists, then $f'(c) = 0$. **(Converse is not true!)**

It is possible for $f'(c) = 0$ or $f'(c)$ not to exist at a point, yet the point not be a local min or max.

Question

What is an example?

Critical points only provide candidates for local extrema; they do not guarantee that the points are local extrema.

Two facts help us in the search for absolute extrema:

- Absolute extrema in the interior of an interval are also local extrema, which occur at critical points of f.
- Absolute extrema may occur at the endpoints of f.

Assume that the function f is continuous on $[a, b]$. To find absolute extrema, use the following procedure:

1. Locate the critical points c in (a, b), where $f'(c) = 0$ or $f'(c)$ does not exist. Again, these points are candidates for absolute extrema.
2. Evaluate f at the critical points and at the endpoints of $[a, b]$.
3. Choose the largest and smallest values of f from 2. for the absolute max and min values, respectively.

Exercise

Given $f(x) = (x+1)^{4/3}$ on $[-8, 8]$, determine the critical points and the absolute extreme values of f.

4.1 Book Problems

11-25 (odds), 31-45 (odds)

- **Note:** So far, we only know how to find absolute extrema from an equation. Techniques for locating local extrema, given an equation, come in later sections.

Definition

Suppose a function f is defined on an interval I.

(a) f is **increasing** on I means for any two points x_1, x_2 in I, with $x_2 > x_1$, we have $f(x_2) > f(x_1)$.

(b) f is **decreasing** on I means for any two points x_1, x_2 in I, with $x_2 > x_1$, we have $f(x_1) > f(x_2)$.

We can rephrase the definition from the previous slide in the following way:

Definition

Suppose f is continuous on an interval I and differentiable at every interior point of I.

(a) f is **increasing** on I means $f'(x) > 0$ for all interior points of I.

(b) f is **decreasing** on I means $f'(x) < 0$ for all interior points of I.

Example

Sketch a function that is continuous on $(-\infty, \infty)$ that has the following properties:

- $f'(-1)$ is undefined;
- $f'(x) > 0$ on $(-\infty, -1)$;
- $f'(x) < 0$ on $(-1, \infty)$.

Example

Find the intervals on which

$$f(x) = 3x^3 - 4x + 12$$

is increasing and decreasing.

First Derivative Test

The **First Derivative Test** is used to find local extrema.

Suppose that f is continuous on an interval that contains a critical point c and assume f is differentiable on an interval containing c, except perhaps at c itself.

- If f' changes sign from positive to negative as x increases through c, then f has a **local maximum** at c.
- If f' changes sign from negative to positive as x increases through c, then f has a **local minimum** at c.
- If f' does not change sign at c (from positive to negative or vice versa), then f has no local extreme value at c.

Exercise

If $f(x) = 2x^3 + 3x^2 - 12x + 1$, identify the critical points on the interval $[-3, 4]$, and use the First Derivative Test to locate the local maximum and minimum values. What are the absolute max and min?

Note: Again, the First Derivative Test does NOT test for increasing/decreasing, only local max/min. Use it on critical points.

Absolute Extremes on any Interval

The Extreme Value Theorem (recall, from Section 4.1) stated that we were guaranteed extreme values only on closed intervals.

However: Suppose f is continuous on an interval I that contains only one local extremum, at $(x =)c$.

- If it is a local minimum, then $f(c)$ is the absolute minimum of f on I.
- If it is a local maximum, then $f(c)$ is the absolute maximum of f on I.

Definition (concavity)

Let f be differentiable on an open interval I.

- f is **concave up** on I means f' is increasing on I.
- f is **concave down** on I means f' is decreasing on I.

Definition

Suppose f is continuous at c and f changes concavity at $x = c$ (from up to down, or vice versa). The point c is called an **inflection point**.

Test for Concavity

Suppose that f'' exists on an interval I.

- If $f'' > 0$ on I, then f is concave up on I.
- If $f'' < 0$ on I, then f is concave down on I.
- If c is a point of I at which $f''(c) = 0$ and f'' changes signs at c, then f has an inflection point at c.

In other words, just as the first derivative f' told us whether the function f was increasing or decreasing, the second derivative f'' tells us whether f' is increasing or decreasing.

Second Derivative Test

Just like with the First Derivative Test, the **Second Derivative Test** locates local extrema.

Suppose that f'' is continuous on an open interval containing c with $f'(c) = 0$.

- If $f''(c) > 0$, then f has a local minimum at c.
- If $f''(c) < 0$, then f has a local maximum at c.
- If $f''(c) = 0$, then the test is inconclusive.

See the Recap of Derivative Properties (Figure 4.36 in the text) for a summary.

Exercise

Let $f(x) = 2x^3 - 6x^2 - 18x$.

(a) Determine the intervals on which f is concave up or down, and identify any inflection points.

(b) Locate the critical points, and use the Second Derivative Test to determine whether they correspond to local minima or maxima, or whether the test is inconclusive.

4.2 Book Problems

11-35 (odds), 47-61 (odds), 67