Mon 1 June

- first two MLP deadlines have passed
- Quiz feedback:
  - Quiz 1 median: 18.5/21
  - Quiz 2 median: 13/20
  - Still early in the term – If you are ever below the median get help (friend, tutor, office hours, internet) right away
- Quiz 3 on Wed with review on Thurs. If there are no review questions then we will start Week 3.
- Exam 1 on Friday
  - covers up to $\oint$3.1
  - syllabus-approved calculator (though you probably won't need any at all)
  - 50 min. Class will start 30 min late to enforce that timeframe.
- sub for Week 3

1. Week 2: 1-5 June
   - Monday 1 June

$\oint$2.7 Precise Definitions of Limits

- Seeing ϵs and δs on a Graph
- Finding a Symmetric Interval
- Book Problems

$\oint$3.1 Introducing the Derivative

$\oint$ 2.7 Precise Definitions of Limits

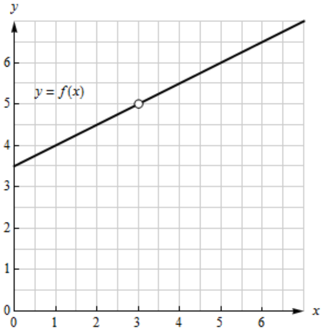
Assume that $f(x)$ exists for all x in some open interval (open means: neither of the endpoints not included) containing a, except possibly at a. "The limit of $f(x)$ as x approaches a is L", i.e.,

$$\lim_{x \to a} f(x) = L,$$

means for any $\epsilon > 0$ there exists $\delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta.$$

Question



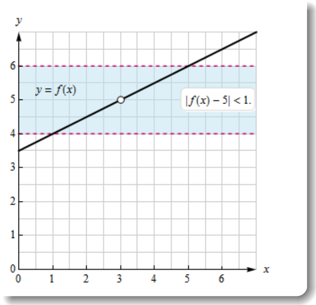
Using the graph, for each $\epsilon > 0$, determine a value of $\delta > 0$ to satisfy the statement

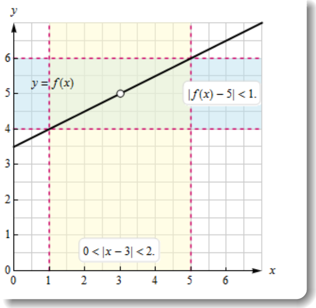
$$|f(x) - 5| < \epsilon \quad \text{whenever} \quad 0 < |x - 3| < \delta.$$

- $\epsilon = 1$
- $\epsilon = 0.5.$

Seeing ϵs and δs on a Graph, cont.

When $\epsilon = 1$:

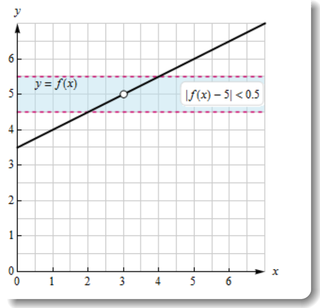


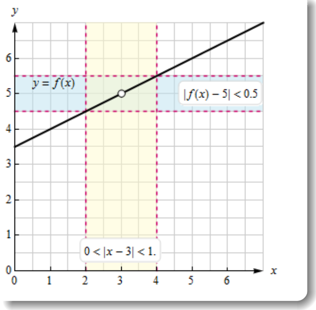


$\ldots\ \delta = 2$

Seeing ϵs and δs on a Graph, cont.

When $\epsilon = 0.5$:



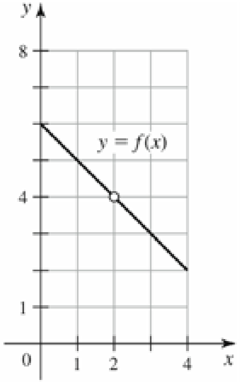


$\ldots\ \delta = 1$

The ϵs and δs give a way to visualize computing the limit, and prove it exists. As the ϵs get smaller and smaller, we want there to always be a δ. In this example,

$$\lim_{x \to 3} f(x) = 5.$$

Exercise



Using the graph, for each $\epsilon > 0$, determine a value of $\delta > 0$ to satisfy the statement

$$|f(x) - 4| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta.$$

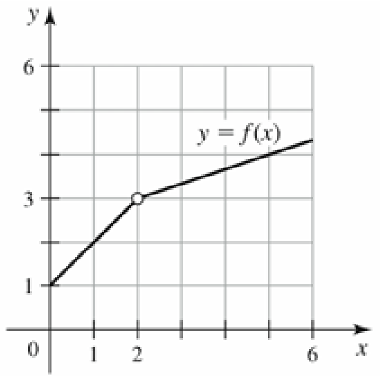
- $\epsilon = 1$
- $\epsilon = 0.5$.

Question

When finding an interval $(a - \delta, a + \delta)$ around the point a, what happens if you compute two different δs?

Answer: To obtain a symmetric interval around a, use the smaller of the two δs as your distance around a.

Exercise



The graph of $f(x)$ shows

$$\lim_{x \to 2} f(x) = 3.$$

For $\epsilon = 1$, find the corresponding value of $\delta > 0$ so that

$$|f(x) - 3| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta.$$

Exercise

Let $f(x) = x^2 - 4$. For $\epsilon = 1$, find a value for $\delta > 0$ so that

$$|f(x) - 12| < \epsilon \quad \text{whenever} \quad 0 < |x - 4| < \delta.$$

In this example, $\lim_{x \to 4} f(x) = 12$.

2.7 Book Problems

1-7, 9-18

1 Week 2: 1-5 June
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∮2.7 Precise Definitions of Limits
- Seeing ϵs and δs on a Graph
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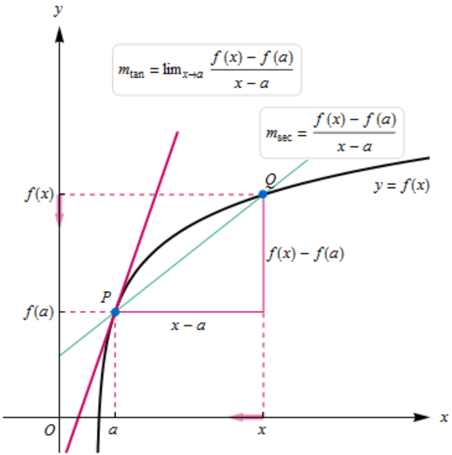
∮3.1 Introducing the Derivative

$\oint$ 3.1 Introducing the Derivative

Recall from Ch 2: We said that the slope of the tangent line at a point is the limit of the slopes of the secant lines as the points get closer and closer.

- slope of secant line: $\dfrac{f(x) - f(a)}{x - a}$ (average rate of change)
- slope of tangent line: $\displaystyle\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ (instantaneous rate of change)



y
$m_{\tan} = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$
$m_{\sec} = \dfrac{f(x) - f(a)}{x - a}$
Q
$y = f(x)$
$f(x)$
$f(x) - f(a)$
P
$f(a)$
$x - a$
O
a
x
x

Example

Use the relationship between secant lines and tangent lines, specifically the slope of the tangent line, to find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point $P = (1, 5)$.