Mon 3 Nov 2014

- Tuesday 4 Nov: Quiz 8 (4.2-4.4)
- Friday 7 Nov: Exam 2!!!
  - In class
  - Covers 3.9-4.5
- Today: 4.5
- Wednesday: REVIEW

Linear Approximation and Differentials

In section 4.5, you see the pictures on pg. 268 examining the behavior of a curve and tangent line when you zoom in with a graphing utility.

As you zoom in more and more, the curve appears more like the tangent line.

The idea—that smooth curves appear straighter on smaller scales—is the basis of linear approximations.

Linear Approximation

One of the properties of a function that is differentiable at a point P is that it is locally linear near P (i.e., the curve approaches the tangent line at P).

Therefore, it makes sense to approximate a function with its tangent line, which matches the value and slope of the function at P.

Recall the general equation of a line:

$$y - y_0 = m(x - x_0)$$

Linear Approximation

At the given point P = (a, f(a)), the slope of the line tangent to the curve at that point is f'(a). So:

$$y - f(a) = f'(a)(x - a)$$

$$\square \quad y = f(a) + f'(a)(x - a)$$

This new function is called the linear approximation to f at the point a and is denoted L(x).

*Linear approximations and differentials are the first steps to integral calculus and polynomial approximations seen in later chapters.

Definition of Linear Approximation

Suppose *f* is differentiable on an interval *I* containing the point *a*. The linear approximation to *f* at *a* is the linear function

$$L(x) = f(a) + f'(a)(x - a)$$

for *x* in *I*.

Example of Linear Approximation

Write the equation of the line that represents the linear approximation to $f(x)=\dfrac{x}{x+1}$ at a = 1, and then use the linear approximation to estimate f(1.1).

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Write the equation of the line that represents the linear approximation to $f(x) = \dfrac{x}{x+1}$ at a = 1, and then use the linear approximation to estimate f(1.1).

$f'(x) = \dfrac{1}{(x+1)^2}$, $f(a) = \dfrac{1}{2}$ and $f'(a) = \dfrac{1}{4}$. So

$$L(x) = \frac{1}{2} + \frac{1}{4}(x-1) = \frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(x+1)$$

Example of Linear Approximation

Write the equation of the line that represents the linear approximation to $f(x) = \dfrac{x}{x+1}$ at a = 1, and then use the linear approximation to estimate f(1.1).

Since $L(x) = \dfrac{1}{4}(x+1)$, we can use this to estimate the value at x = 1.1. $f(1.1) \approx L(1.1) = \dfrac{1}{4}(1.1+1) = 0.525$

Note that *f(1.1) = .5238*, so the error is

$$\frac{.525 - .5238}{.5238} \times 100 = 0.23\%$$

Intro to Differentials

Our linear approximation $L(x) = f(a) + f'(a)(x-a)$ can be used to approximate *f(x)* when *a* is fixed and *x* is a nearby point (e.g., $f(x) \approx f(a) + f'(a)(x-a)$).

Notice that, when rewritten,

$$f(x) - f(a) \approx f'(a)(x-a) \Rightarrow \Delta y \approx f'(a)\Delta x$$

So a change in *y* can be approximated by the corresponding change in *x*, magnified or diminished by a factor of f'(a). This states the fact that f'(a) is the rate of change of *y* with respect to *x*.

Intro to Differentials

So if *f* is differentiable on an interval *I* containing the point *a*, then the change in the value of *f* between two points *a* and *a* + Δ x is approximately

$$\Delta y \approx f'(a)\Delta x$$

where *a* + Δ x is in *I*.

Intro to Differentials

We now have two related quantities:

- The change in the function *y = f(x)* as *x* changes from *a* to *a* + Δ x (which we call Δ y)
- The change in the linear approximation *y = L(x)* as *x* changes from *a* to *a* + Δ x (called the differential dy)

So, when the x-coordinate changes from *a* to *a* + Δ x:

- The function change is **exactly** $\Delta y = f(a+\Delta x) - f(a)$
- The linear approximation *L(x)* changes

$$\Delta L = L(a+\Delta x) - L(a)$$

$$= [f(a) + f'(a)(a+\Delta x - a)] - [f(a) + f'(a)(a-a)] = f'(a)\Delta x$$

Intro to Differentials

To distinguish between the change in the function *Δy* and the change in the linear approximation *ΔL*, we define the differentials *dx* and *dy:*

- *dx* is simply the change in *x,* or *Δx*
- *dy* is the change in the linear approximation, which is

$$\Delta L = f'(a)\Delta x$$

SO:

$$\Delta L = dy = f'(a)\Delta x = f'(a)dx \Rightarrow dy = f'(a)dx$$

Definition of Differentials

Let *f* be differentiable on an interval containing *x*.

- A small change in *x* is denoted by the differential *dx*.
- The corresponding change in *y = f(x)* is approximated by the differential *dy = f'(x)dx*; that is,

$$\Delta y = f(x + dx) - f(x) \approx dy = f'(x)dx$$

The use of differentials is critical as we approach integration

Example

Use the notation of differentials [dy = f'(x)dx] to approximate the change in $f(x) = x - x^3$ given a small change *dx*.

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Use the notation of differentials [dy = f'(x)dx] to approximate the change in $f(x) = x - x^3$ given a small change *dx*.

For $f(x) = x - x^3$, we have $f'(x) = 1 - 3x^2$. So $dy = (1 - 3x^2)dx$

Explanation: So a small change *dx* in the variable *x* produces an approximate change of $dy = (1 - 3x^2)dx$

For example, for *x=2*, if we increase *x* from 2 to 2.1, then *dx* =0.1 and $dy = (1 - 3(2)^2)(0.1) = -1.1$.

This means as *x* increases by 0.1, *y* decreases by 1.1.

Homework from Section 4.5

Do problems 7-9 all, 11, 12, 29-38 all (pg. 273).

Wed 5 Nov 2014

- Friday 7 Nov in class: Exam 3 (Covering 3.9 to 4.5)
  - One 3x5’’ notecard, both sides – if you don’t follow the notecard rules, it will be taken away.

Review: 3.9 Inverse Trig Function Derivatives

- Know them.

Review: 3.10 Related Rates

- Know the steps to solving related rates problems, and be able to use them to solve problems given variables and rates of change.
- Be able to solve related rates problems. (If there is a geometry formula you don't think you'll know, then put it on your notecard.)
- Know the difference between a related rates problem and an optimization problem. As you study and do problems, try to identify key words the distinguish each.

Review: 4.1 Maxima and Minima

- Know the definitions of maxima, minima, and what makes these points local or absolute extrema.
  - Endpoints don't count as local extrema, but you do have to check them for absolute extrema (plug into original function).
- Know how to find critical points for a function.
  - Critical points are places where either the derivative is zero, or it doesn't exist.
  - Endpoints are not critical points.
- Given a function on a given interval, be able to find local or absolute extrema.
- Given specified properties of a function, be able to sketch a graph of that function.

Review: 4.2 What Derivatives Tell Us

- Be able to use the FIRST derivative to determine where a function is increasing or decreasing.
- Be able to use the First Derivative Test to identify local maxima or local minima – choose ``strategic” points to plug into f’, then look for where the function changes from increasing to decreasing or vice versa.
- Be able to find critical points, absolute extrema, and inflection points for a function.
- Be able to use the SECOND derivative to determine the concavity of a function – choose ``strategic” points to plug into f’’, then look for where the function changes concavity.
- Be able to use the Second Derivative Test to determine whether a given point is a local max or min.
- Know your Derivative Properties!!! (see p. 242) Also see the Chapter 2 handouts from comp.uark.edu/~ashleykw/115Fall2010/Handouts/handouts.html to supplement.

Review: 4.3 Graphing Functions

- Be able to find specific characteristics of a function that are spelled out in the graphing guidelines on pg. 248 -- e.g., know how to find x- and y-intercepts (precal), vertical/horizontal asymptotes (chapter 2), critical points, inflection points, intervals of concavity and increasing/decreasing, etc.
- Note: for the domain of the function, if a point is not in the domain then it doesn't count as a critical point or an inflection point. However, when you draw your number line you should still indicate the ``naughty" points and check intervals.
- Be able to use these specific characteristics of a function to sketch a graph of the function.

Review: 4.4 Optimization

- Be able to solve optimization problems that maximize or minimize a given quantity.
- Be able to identify and express the constraints and objective function in an optimization problem.
- Be able to determine your interval of interest in an optimization problem (e.g., what range of x values are you searching for your extreme points?)
- See the Chapter 4 handouts from comp.uark.edu/~ashleykw/115Fall2010/Handouts/handouts.html for supplements.
- General tips for story problems: Draw a picture with labels that are clear, concise, and consistent. Your picture might not be perfect the first time but that's why you have an eraser. Also, every step you do in the problem, make sure there is a point to it. Keep track of units and **answer the question**. You'll lose points on the exam if you mess up units.

Review: 4.5 Linear Approximations and Differentials

- Be able to find a linear approximation for a given function.
- Be able to use a linear approximation to estimate the value of a function at a given point.
- Be able to use differentials to express how the change in x (dx) impacts the change in y (dy)

Other Tips:

- READ THE BOOK! As you saw on Exam 1, many questions were taken verbatim from the book – some of them examples already worked out, not exercises!
- Practice. Do the assigned problems in the book as if they are exam problems – show your work, time yourself, check your answers with a friend.
- Quizzes 7,8,9 from comp.uark.edu/~ashleykw/115Fall2010/115f10.html for more supplements.
- You don’t necessarily have to memorize things. Make a draft of your notecard so that you can use its space the most efficiently.
- Do the easier problems first on the exam – note, easy is subjective!
- Take your time on the exam. Look at all the questions first and then budget your time for each one. There will be ~6 questions. The exam is ~7 pages long.