MATH 2554 Quiz 9 (Sections 4.1 and 4.2)
Due Tuesday, Mar. 31

This quiz is due Tuesday, Mar. 31 at the beginning of your drill. You may use your brain, notes, book, other humans and any pet of your choice. **Your solutions must be legible, in order, stapled, de-fringed, and with your name on the top right corner of each page.** If you fail to meet any of these requirements you will receive a zero. Each question is worth one point and is all-or-nothing.

1. Sketch the graph of a continuous function on $[0, 4]$ satisfying the following properties:
   - $f'(x) = 0$ at $x = 1$ and $x = 3$
   - $f'(2)$ is undefined
   - f has an absolute maximum at $x = 2$
   - f has neither a local maximum nor a local minimum at $x = 1$
   - f has an absolute minimum at $x = 3$

   It should be clear that your graph satisfies all of the listed properties.

Let $T(x) = \cos^2 x$

2. Use the first derivative to find the intervals on $[-\pi, \pi]$ on which T is increasing and decreasing.
3. Use the second derivative to find the intervals on $[-\pi, \pi]$ on which T is concave up and concave down. (Hint: A good knowledge of trigonometric identities will help you here.)

Find all critical points for each of the following functions. Then, find all absolute extreme values and determine where each occurs.

4. $q(x) = \dfrac{x}{(x^2 + 1)^2}$ on $[-2, 2]$
5. $g(x) = xe^{-x/2}$ on $[0, 5]$

Find all critical points for each of the following functions. Then use the First Derivative Test to locate the local maxima and minima.

6. $h(x) = -x^2 - x + 2$ on $[-4, 4]$
7. $k(x) = \tan^{-1} x - x^3$ on $[-1, 1]$

Find all critical points for each of the following functions. Then use the Second Derivative Test to determine whether they correspond to local minima or local maxima, or whether the test is inconclusive.

8. $\alpha(x) = \dfrac{1}{9}(x + 1)^3 - 5$

9. $\beta(x) = -20x^2 + \frac{5}{2}x^4$

10. Suppose a box has a square base with sides of length x, and its volume is 80 ft^3. Find the absolute minimum of the surface area function. What are the dimensions of the box with minimum surface area?