$$\int \sqrt{t}\, dt = \int t^{1/2}\, dt$$

$$= \frac{t^{3/2}}{3/2} + C$$

$$= \frac{2}{3} t^{3/2} + C$$

A. Does not exist

B. $\frac{2}{3} t^{3/2} + C$

C. $\frac{1}{2} t^{-3/2} + C$

[redacted]

$$\int \left(\frac{-5}{x} + e^{-2x} \right) dx = -5\int \frac{1}{x}\,dx + \int e^{-2x}\,dx$$

$$= -5\left(\ln|x| + C\right) + \left(\frac{-1}{2}e^{-2x} + C\right)$$

$$= -5\ln|x| - \frac{1}{2}e^{-2x} + C$$

A. $3x + 2 + c$

B. $\dfrac{5}{2x^{-2}} + 2e^{-2x} + c$

C. $-5\ln|x| + 2e^{-2x} + c$

D. $-5\ln|x| - \dfrac{1}{2}e^{-2x} + c$

<u>Check:</u>

$$\frac{d}{dx}\left(-5\ln|x| - \frac{1}{2}e^{-2x}\right)$$

$$= -5\left(\frac{1}{x}\right) - \frac{1}{2}(-2)e^{-2x}$$

$$= \frac{-5}{x} + e^{-2x} \checkmark$$

Recall:

- The Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(3x^2+2)^4 =$$

$$= 4(3x^2+2)^3(6x) = 24x(3x^2+2)^3$$

Idea: u-Sub is the Chain Rule backwards

Example: Find $\int \underbrace{8x}_{\frac{du}{dx}}(\underbrace{4x^2 + 8}_{u})^6 dx$.

$$= \int \frac{du}{d\cancel{x}} u^6 d\cancel{x} = \int u^6 du$$

$$= \frac{u^7}{7} + C$$

$$\boxed{= \frac{(4x^2+8)^7}{7} + C}$$

* Look for a function sitting next to its derivative

Example: Find $\int \underbrace{x^3}_{\frac{1}{12}\left(\frac{du}{dx}\right)}\sqrt{\underbrace{3x^4 + 10}_{u}}\,dx$

because $\frac{d}{dx}(3x^4+10) = 12x^3$

$= \int \frac{1}{12}\sqrt{u}\,du$

$= \frac{1}{12}\left(\frac{2}{3}u^{3/2}\right) + C$

$\boxed{= \frac{1}{18}(3x^4+10)^{3/2} + C}$

Check:

$\frac{d}{dx}\left(\frac{1}{18}(3x^4+10)^{3/2} + C\right)$

$= \frac{1}{\cancel{18}}\left(\frac{\cancel{3}}{2}\right)(3x^4+10)^{1/2}(\cancel{12}x^3)$

$= x^3\sqrt{3x^4+10}$ ✓

Example: Find $\int \frac{x+3}{x^2+6x}\,dx$

$\frac{1}{2}\frac{du}{dx}$ because

u

$\frac{d}{dx}\left(x^2+6x\right)$

$= 2x+6$

$= 2(x+3)$

$= \int \frac{1}{2}\left(\frac{1}{u}\right) du$

$= \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln\left|x^2+6x\right| + C}$

Check:

$\frac{d}{dx}\left(\frac{1}{2}\ln\left|x^2+6x\right| + C\right)$

$= \frac{1}{2}\left(\frac{1}{x^2+6x}\right)(2x+6)$

$= \frac{1}{\cancel{2}}\left(\frac{1}{x^2+6x}\right)\cancel{2}(x+3)$

$= \frac{x+3}{x^2+6x}$ ✓

Find $\int \underbrace{25x^2}_{\frac{25}{9}\left(\frac{du}{dx}\right)} e^{\overbrace{3x^3+2}^{u}}\,dx$

(b/c $\frac{d}{dx}(3x^3+2) = 9x^2$)

$$= \frac{25}{9}\int e^u\,du = \frac{25}{9}e^u + C$$

$$\boxed{= \frac{25}{9}e^{3x^3+2} + C}$$

Check:

$$\frac{d}{dx}\left(\frac{25}{9}e^{3x^3+2} + C\right)$$

$$= \frac{25}{\cancel{9}}e^{3x^3+2}(\cancel{9}x^2)$$

$$= 25x^2e^{3x^3+2} \checkmark$$

Find $\int \underbrace{6x}_{\frac{du}{dx}}(\underbrace{3x^2+4}_{u})^7\,dx.$ $= \int u^7\,du$

$= \dfrac{u^8}{8} + C$

$\boxed{= \dfrac{(3x^2+4)^8}{8} + C}$

A. $6(3x^2+4)^8 + c$

B. $\dfrac{(3x^2+4)^8}{8} + c$

C. $18x^3 + 4x + c$

Find $\int \underbrace{x^2}_{\left(\frac{1}{3}\frac{du}{dx}\right)} \sqrt{\underbrace{x^3+1}_{u}}\,dx$ $= \int \frac{1}{3}\sqrt{u}\,du$

$= \frac{1}{3}\frac{u^{3/2}}{3/2} + C$

$= \frac{2}{9}(x^3+1)^{3/2} + C$

A. $\frac{2}{9}(x^3+1)^{3/2}+c$

B. $\frac{1}{3}(x^3+1)^{-1/2}+c$

C. $\frac{2}{3}(x^3+1)^{2/3}+c$

$$\int \frac{24x + 4}{\boxed{6x^2 + 2x}} dx \leftarrow u$$

$$\Rightarrow \frac{du}{dx} = 12x + 2$$

A. $\ln(6x^2 + 2x) + C$

B. $\frac{1}{2}\ln(6x^2 + 2x) + C$

C. $2\ln(6x^2 + 2x) + C$

D. $24\ln(6x^2 + 2x) + C$

E. $\frac{1}{4}\ln(6x^2 + 2x) + C$

$$= \int \frac{2}{u} du = 2\ln|u| + C$$

$$= 2\ln|6x^2 + 2x| + C$$

The marginal revenue (in thousands of dollars) from the sale of x MP3 players is given by

$$R'(x) = 4x(x^2 + 27{,}000)^{-2/3}.$$

Find the total revenue function if the revenue from 125 players is \$29,591.

$$R(x) = \int R'(x)\,dx = \int \underbrace{4x}_{2\frac{du}{dx}} \underbrace{(x^2+27000)}_{u}^{-2/3}\,dx$$

$$= 2\int u^{-2/3}\,du = 2\frac{u^{1/3}}{\frac{1}{3}} + C$$

$$= 6(x^2+27000)^{1/3} + C$$

Find C:

$$R(125) = 29591$$

$$= 6((125)^2 + 27000)^{1/3} + C$$

$$\Rightarrow C = 29591 - 6((125)^2+27000)^{1/3}$$

$$\approx 29381.41$$

so $\boxed{R(x) = 6(x^2+27000)^{1/3} + 29381.41}$