Survey of Calculus Final Exam Review
December 5, 2016

1. Determine each of the following limits algebraically.
   (a) $\lim_{x \to 4} \dfrac{x-4}{\sqrt{x}-2}$
   (b) $\lim_{y \to 3} \left[\dfrac{1}{y-3} \left(\dfrac{1}{y} - \dfrac{1}{3} \right) \right]$
2. Let $f(x) = \frac{1}{5}x^2 - 2x$.
   (a) Determine the average rate of change of $f(x)$ on the interval $(2, 10)$.
   (b) The Mean Value Theorem tells us that there is some value of x in the interval $(2, 10)$ at which the *instantaneous* rate of change of $f(x)$ is equal to its *average* rate of change over that interval. Find that value of x.
   (c) Sketch a graph of f and illustrate on your graph the average and instantaneous rates of change found above.
3. Determine the derivative of each of the following functions.
   (a) $f(x) = \ln \sqrt{1+2x}$
   (b) $y = x^2 e^{x^3}$
   (c) $g(z) = (3z^2 - 4)^{97}$
   (d) $y = \dfrac{x^3 - 4x + 5}{x^2 + 9}$
   (e) $k(t) = \dfrac{t}{\ln t}$
4. Let $f(t) = t^2$, $g(t) = t^3$, $h(t) = e^t$, and $k(t) = \ln(t)$. For any value of $t > 1$, which function is changing at the greatest rate? At the smallest rate?
5. A post office can accept a package for mailing only if the sum of its length and its girth (the circumference of its cross section) is at *most* 100 in. What is the maximum volume of a rectangular box with square cross section that can be mailed?
6. A wire of length 100 cm is to be cut into two pieces. Once piece is bent into a circle, and the other piece is bent into a square. Where should the cut be made in order to maximize the sum of the areas enclosed by the square and the circle? Where should the cut be made to minimize that sum?

7. The absolute value function is defined as follows:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$$

Use the definition of the derivative (i.e, $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$) to answer the following question: is $|x|$ differentiable at $x = 0$? Why or why not?

8. Evaluate each of the following indefinite integrals.

(a) $\displaystyle\int x\sqrt{2-3x^2}dx$

(b) $\displaystyle\int \frac{t}{\sqrt{2t^2+1}}dt$

(c) $\displaystyle\int \frac{x^2-1}{\sqrt{x}}dx$

(d) $\displaystyle\int \frac{3}{x+1}dx$

(e) $\displaystyle\int e^{3x} - 5x^2 dx$

9. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals.

(a) $\displaystyle\int_0^3 \frac{e^x}{3+2e^x}dx$

(b) $\displaystyle\int_1^{\sqrt{5}} x(x-2)(x-4)dx$

(c) $\displaystyle\int_1^8 bx^3 - cx^{\frac{1}{3}}dx$, where b and c are real constants

10. Determine the area of each of the following regions.

(a) The region bounded by the curves $y = \sqrt{\frac{x}{2}+1}$, $y = \sqrt{1-x}$, and $y = 0$.

(b) The region bounded by the curves $y = 4\sqrt{2x}$, $y = 2x^2$, and $y = -4x + 6$.