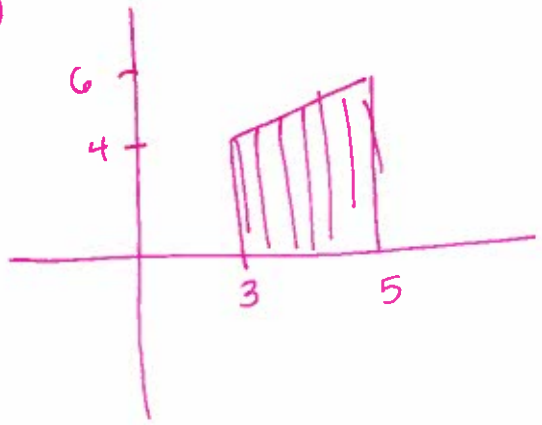
Quiz 12

① 

② $A = \frac{1}{2}(4+6)(2) = 10$

③ a) $n = 4$

$\Delta x = \frac{5-3}{4} = .5 \qquad \bar{x}_k = 3 + .5k$

$$\sum_{k=1}^{4} f(.5k+3)(.5)$$

$$= .5 \sum_{k=1}^{4} (.5k+3) + 1$$

$$= .5 \sum_{k=1}^{4} .5k \; + \; .5 \sum_{k=1}^{4} 4$$

$$= (.5)^2 \sum_{k=1}^{4} k \; + \; .5(4)(4)$$

$= .25\left(\frac{4(4+1)}{2}\right) + 8 \quad = 2.5 + 8 = \boxed{10.5}$

b) $n = 400$

$\Delta x = \frac{5-3}{400} = .005 \qquad \bar{x}_k = 3 + .005k$

$$\sum_{k=1}^{400} f(.005k + 3)(.005)$$

$$= .005 \sum_{k=1}^{400} (.005k + 3) + 1$$

$$= .005 \sum_{k=1}^{400} .005k \; + \; .005 \sum_{k=1}^{400} 4$$

$$= (.005)^2 \sum_{k=1}^{400} k \; + \; .005(4)(400)$$

$$= (.005)^2 \left(\frac{400(401)}{2}\right) + 8$$

$= 2.005 + 8$

$= \boxed{10.005}$

c) $n = 4000$

$\Delta x = \frac{5-2}{4000} = .0005 \qquad \bar{x}_k = 3 + .0005k$

$$\sum_{k=1}^{4000} f(.0005k + 3)(.0005)$$

$$= .0005 \sum_{k=1}^{4000} (.0005k + 3) + 1$$

$$= .0005 \sum_{k=1}^{4000} .0005k + .0005 \sum_{k=1}^{4000} 4$$

$$= (.0005)^2 \sum_{k=1}^{4000} k + (.0005)(4)(4000)$$

$$= (.0005)^2 \left(\frac{4000(4001)}{2} \right) + 8$$

$$= 2.0005 + 8$$

$$= \boxed{10.0005}$$

④ a) $n = 4$

$\Delta x = .5$ $\qquad \bar{x}_k = 3 + (k-1)(.5) = .5k + 2.5$

$$\sum_{k=1}^{4} f(.5k + 2.5)(.5)$$

$$= .5 \sum_{k=1}^{4} (.5k + 2.5) + 1$$

$$= .5 \sum_{k=1}^{4} .5k + 3.5$$

$$= .5 \sum_{k=1}^{4} .5k + .5 \sum_{k=1}^{4} 3.5$$

$$= (.5)^2 \sum_{k=1}^{4} k + .5(3.5)(4)$$

$$= (.5)^2 \left(\frac{4(4+1)}{2} \right) + 7$$

$$= 2.5 + 7 = \boxed{9.5}$$

b) $n = 400$

$\Delta x = .005$ $\quad \bar{x}_k = 3 + (k-1)(.005) = .005k + 2.9$

$$\sum_{k=1}^{400} f(.005k + 2.995)(.005)$$

$$= .005 \sum_{k=1}^{400} (.005k + 2.995) + 1$$

$$= .005 \sum_{k=1}^{400} .005k + .005 \sum_{k=1}^{400} 3.995$$

$$= (.005)^2 \sum_{k=1}^{400} k + .005(3.995)(400)$$

$$= (.005)^2 \left(\frac{400(401)}{2} \right) + 7.99$$

$$= 2.005 + 7.99$$

$$= \boxed{9.995}$$

c) $n = 4000$

$\Delta x = .0005$ $\quad \bar{x}_k = 3 + (k-1)(.0005) = .0005k + 2.9995$

$$\sum_{k=1}^{4000} f(.0005k + 2.9995)(.0005)$$

$$= .0005 \sum_{k=1}^{4000} (.0005k + 2.9995) + 1$$

$$= .0005 \sum_{k=1}^{4000} .0005k + .0005 \sum_{k=1}^{4000} 3.9995$$

$$= (.0005)^2 \sum_{k=1}^{4000} k + .0005(3.9995)(4000)$$

$$= (.0005)^2 \left(\frac{4000(4001)}{2} \right) + 7.999$$

$$= 2.0005 + 7.999 = \boxed{9.9995}$$

⑤ The difference is positive & as n gets larger the difference gets smaller.

⑥ The difference is negative & as n gets larger the magnitude of the difference gets smaller.