MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Fri 26 June 2015

Quiz 8: Graphing (∮4.3)

Directions: You have 30 minutes to complete this quiz. This quiz is closed book and collaborative. The goal of this problem is to produce a graph of the function

$$f(x) = xe^{-x}$$

from scratch.

(1) Find the domain for $f(x)$.

all real numbers ($\mathbb{R}$)

(2) Is $f(x)$ even, odd, or neither? You must justify your answer.

$$f(-x) = -xe^{-(-x)} = -xe^{x}$$

$$\neq \pm f(x)$$

so neither

(3) Find $f'(x)$ and $f''(x)$. You are not required to simplify.

$$f'(x) = (1)e^{-x} + x(-e^{-x})$$

$$= e^{-x}(1-x)$$

$$f''(x) = (-e^{-x})(1-x) + e^{-x}(-1)$$

$$= e^{-x}(-1+x-1)$$

$$= e^{-x}(-2+x)$$

(4) Find the critical points. If there are none, then say why.

$f'(x) = e^{-x}(1-x) = 0$

never 0

$\boxed{\Rightarrow x = 1}$

(f' is defined everywhere)

(5) Find the possible inflection points. If there are none, then say why.

$f''(x) = e^{-x}(-2+x) = 0$

$\boxed{\Rightarrow x = 2}$

(6) What are the intervals where $f(x)$ is increasing? What are the intervals where $f(x)$ is decreasing?

$f'(x) = e^{-x}(1-x) > 0$

always positive

$\Rightarrow 1 > x$

f is increasing on $(-\infty, 1)$

$f'(x) < 0$

$\Rightarrow 1 < x$

f is decreasing on $(1, \infty)$

(7) What are the intervals where $f(x)$ is concave up? What are the intervals where $f(x)$ is concave down?

$f''(x) = e^{-x}(x-2) > 0$

always +

$\Rightarrow x > 2$

f is concave up on $(2, \infty)$

$f''(x) < 0$

$\Rightarrow x < 2$

f is concave down on $(-\infty, 2)$

(8) Find the local extrema and inflection points. You must justify your answers. If there are no extrema or inflection points you should also say why.

From (6), f' changes sign at $x=1$. By the 1^{st} Derivative Test, $x=1$ gives a local max.

From (7), f'' changes sign at $x=2$, so $x=2$ gives an inflection point.

(9) Find the vertical asymptotes, using the limit definition of a vertical asymptote. If there are no vertical asymptotes, then say so.

Since f is defined for all x, there are no vertical asymptotes.

(10) Determine the end behavior.

Hint: By l'Hôpital's Rule (§4.7),

$$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{1}{e^x}.$$

For $\lim_{x\to-\infty} f(x)$ *you do not need l'Hôpital's Rule.*

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{e^x} \quad (\text{l'Hôpital's Rule})$$

$$= 0 \leftarrow \text{horizontal asymptote at } y=0.$$

$$\lim_{x\to-\infty} f(x) = \left(\lim_{x\to-\infty} x\right)\left(\lim_{x\to-\infty} e^{-x}\right)$$

$$= -\infty \cdot \infty$$

$$= -\infty$$

(11) Find the y-intercepts and x-intercepts, if there are any.

$\underline{y\text{-intercept:}}$

$$y = 0e^{-0} = 0 \cdot 1 = 0$$

$\underline{x\text{-intercept:}}$

$$0 = xe^{-x}$$

$$\Rightarrow x = 0.$$

(12) Using all the information above, draw a well-labeled graph of $f(x)$. Your picture should be consistent with your answers to parts (1)-(11).