MATH 2554 (Calculus I)
Summer 2015

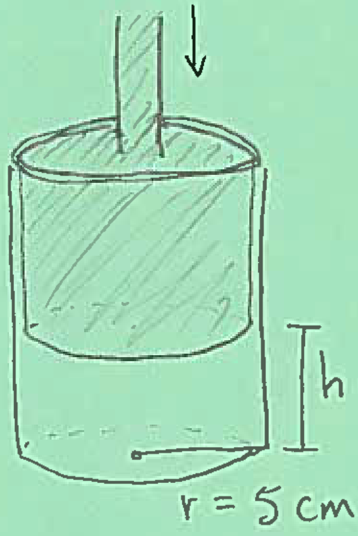
Name: SOLUTIONS

Wed 24 June 2015

Quiz 7: Related Rates (§3.10)

Directions: You have 45 minutes to complete this quiz. This quiz is open book and collaborative.

1. **(#10)** A piston is seated at the top of a cylindrical chamber with radius 5 cm when it starts moving into the chamber at a constant speed of 3 cm/sec. What is the rate of change of the volume of the cylinder when the piston is 2 cm from the base of the chamber?



Know: $\frac{dh}{dt} = -3 \text{ cm/sec}$

WTF: $\left.\frac{dV}{dt}\right|_{h=2\text{cm}}$

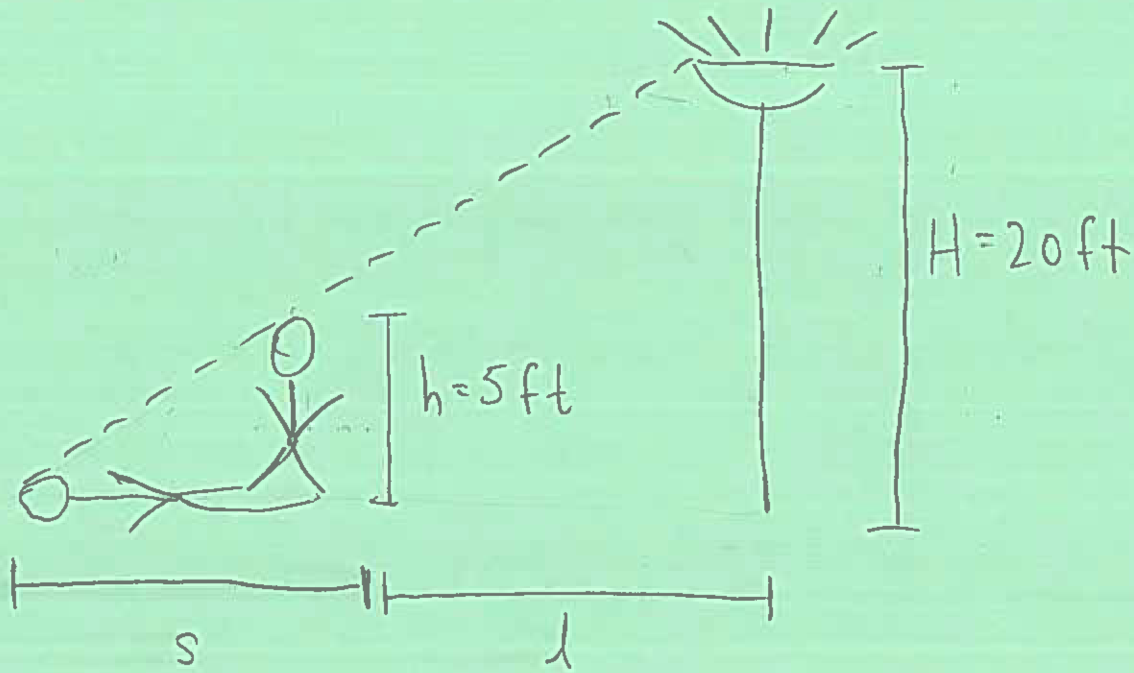
(V = volume)

$V = \pi r^2 h$ (r is constant)

$$\left.\frac{dV}{dt}\right|_{h=2} = \pi r^2 \left.\frac{dh}{dt}\right|_{h=2} = \pi(5\text{cm}^2)\left(-3\,\frac{\text{cm}}{\text{sec}}\right)$$

$$= -75\pi \approx \boxed{-235.6\ \frac{\text{cm}^3}{\text{sec}}}$$

2. **(#19(a))** A five foot tall woman walks at 8 ft/sec toward a street light that is 20 ft tall. What is the rate of change of the length of her shadow when she is 15 ft from the street light?



Know: $\frac{dl}{dt} = -8$ ft/sec   WTF: $\left.\frac{ds}{dt}\right|_{l=15\text{ ft}}$

For an equation involving s, use similar triangles: $\frac{H}{s+l} = \frac{h}{s} \Rightarrow Hs = h(s+l)$
$= hs + hl$

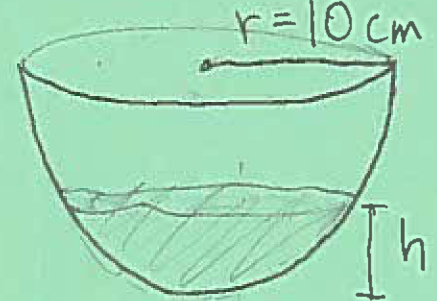
$\Rightarrow s = \underbrace{\left(\frac{h}{H-h}\right)}_{\text{constant}} l$

$$\left.\frac{ds}{dt}\right|_{l=15} = \left(\frac{h}{H-h}\right)\left.\frac{dl}{dt}\right|_{l=15} = \left(\frac{5\text{ ft}}{20\text{ ft} - 5\text{ ft}}\right)(-8\text{ ft/sec})$$

$\boxed{\approx -2.7 \text{ ft/sec}}$

3. (#24(a)) A hemispherical bowl with a radius of 10 cm is filled with fruit punch at a rate of 3 cm^3/sec. How fast is the punch level rising when it is 5 cm deep?

Fact: The volume of a cap of thickness h sliced from a sphere of radius r is



$$V = \frac{1}{3}\pi h^2(3r - h).$$

$$= \pi r h^2 - \frac{1}{3}\pi h^3$$

(r is constant)

Know: $\frac{dV}{dt} = 3 \text{ cm}^3/\text{sec}$ (V = volume)

WTF: $\left.\frac{dh}{dt}\right|_{h=5 \text{ cm}}$

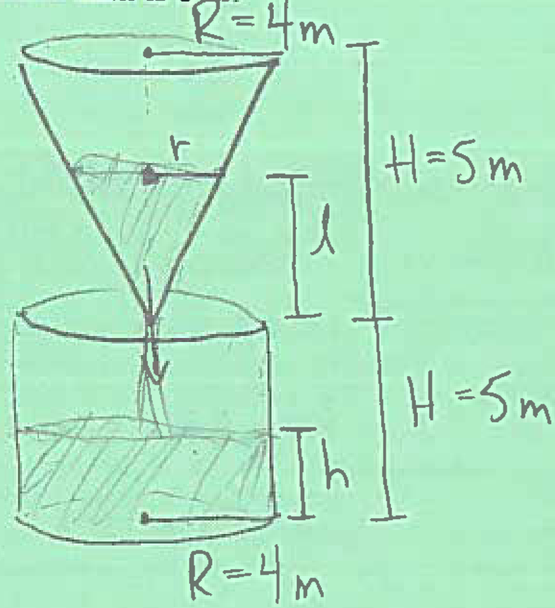
$$\frac{dV}{dt} = 2\pi r h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt} = 3 \text{ cm}^3/\text{sec}$$

$$(2\pi r h - \pi h^2)\frac{dh}{dt} = 3$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=5\text{cm}} = \frac{3 \text{ cm}^3/\text{sec}}{2\pi(10\text{cm})(5\text{ cm}) - \pi(5\text{cm})^2}$$

$$= \frac{3}{75\pi} \boxed{\approx 0.0127 \text{ cm}/\text{sec}}$$

4. **(#38(a))** A conical tank with an upper radius of 4 m and a height of 5 m drains water into a cylindrical tank with a radius of 4 m and a height of 5 m. If the water level in the conical tank drops at a rate of 0.5 m/min, at what rate does the water level in the cylindrical tank rise when the water level in the conical tank is 3 m?



Know: $\frac{dl}{dt} = -0.5 \text{ m/min}$

WTF: $\left.\frac{dh}{dt}\right|_{l=3\text{ m}}$

The volume of the cone decreases at the same rate the volume of the cylinder increases.

$$V_{cone} = \frac{1}{3}\pi r^2 l$$

$$= \frac{1}{3}\pi\left(\frac{R}{H}l^2\right)l$$

Using similar triangles,

$$\frac{H}{l} = \frac{R}{r} \Rightarrow r = \underbrace{\frac{R}{H}}_{\text{Constant}} l$$

$$\frac{dV_{cone}}{dt} = \pi \underbrace{\frac{R^2}{H^2}}_{\text{constant}} l^2 \frac{dl}{dt}$$

$$V_{cylinder} = \pi R^2 h$$

$$\frac{dV_{cylinder}}{dt} = \pi R^2 \frac{dh}{dt} = -\frac{dV_{cone}}{dt}$$

Solve for $\frac{dh}{dt}$:

$$\cancel{\pi}\cancel{R^2}\frac{dh}{dt} = -\cancel{\pi}\frac{\cancel{R^2}}{H^2} l^2 \frac{dl}{dt}$$

$$\left.\frac{dh}{dt}\right|_{l=3} = -\frac{(3m)^2}{(5m)^2}\left(-\frac{1}{2}\text{ m/min}\right)$$

$$\boxed{= 0.18 \text{ m/min}}$$