Section 3.3 – The Product and Quotient Rules

1. Find the derivative of each of the following functions. You may assume that a, b, c, and d are constants.

(a) $f(x) = (x^2 - \sqrt{x}) \cdot 3^x$

$$f'(x) = \left(2x - \frac{1}{2\sqrt{x}}\right)3^x + (x^2 - \sqrt{x})(\ln 3)3^x$$

$$= \left((\ln 3)x^2 + 2x - (\ln 3)\sqrt{x} - \frac{1}{2\sqrt{x}}\right)3^x$$

(b) $g(x) = \dfrac{ax+b}{cx+d}$

$$g'(x) = \frac{(cx+d)a - (ax+b)c}{(cx+d)^2}$$

$$= \frac{ad - bc}{(cx+d)^2}$$

2. Suppose that f and h are functions and that $f(3) = 2$, $f'(3) = -2$, $h(3) = 1$, and $h'(3) = 4$.

(a) Calculate $m'(3)$, where $m(x) = f(x)h(x)$.

$$m'(3) = f'(3)h(3) + f(3)h'(3)$$
$$= (-2)(1) + (2)(4)$$
$$= 6$$

(b) Calculate $p'(3)$, where $p(x) = \dfrac{f(x)}{x^2 h(x)}$.

$$p'(3) = \frac{3^2 h(3) f'(3) - f(3)\left(2 \cdot 3 h(3) + 3^2 h'(3)\right)}{3^2 h(3)}$$

$$= \frac{9(1)(-2) - 2\left(6(1) + 9(4)\right)}{9 \cdot 1}$$

$$= \frac{-18 - 84}{9} = \frac{-102}{9} = -\frac{34}{3} \approx -11.333$$