UNIT 2, LESSON 3

Product and Quotient Rules

Objectives:

- Use the product and quotient rules to take derivatives
- Solve applied problems involving derivatives.

$u(x) = 2x + 4$ $\quad\quad$ $v(x) = 2x^3$

$u'(x) = 2$ $\quad\quad$ $v'(x) = 6x$

$$Suppose\ f(x) = u(x) \cdot v(x)$$

$$f(x) = (2x + 4)(2x^3) = 4x^4 + 6x^3$$

$$f'(x) = 16x^3 + 18x^2$$

$$NOTICE:\ \ f'(x) \neq u'(\mathrm{x}) \cdot v'(x)$$

Product Rule

If $f(x) = u(x) \cdot v(x)$, and if $u'(x)$ and $v'(x)$ both exist, then

$$f'(x) = u(x) \cdot v'(x) + v(x) \cdot u'(x).$$

(The derivative of a product of two functions is the first function times the derivative of the second plus the second function times the derivative of the first.)

Example:

$$f(x) = (2x + 4)(2x^3) = 4x^4 + 6x^3$$

Find $f'(x)$.

Example:

Let $f(x) = (\sqrt{x} + 3)(x^2 - 5x)$. Find $f'(x)$.

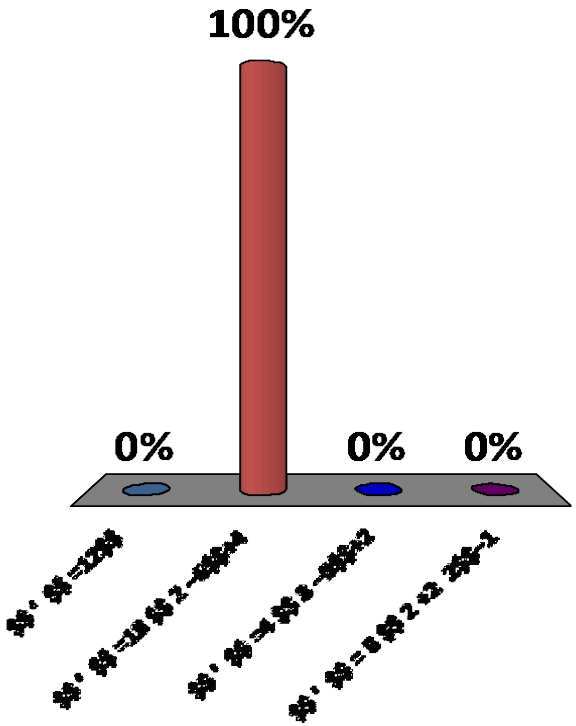
Let $f(x) = (3x^2 + 2)(2x - 1)$. Find $f'(x)$.

A. $f'(x) = 12x$

✓ B. $f'(x) = 18x^2 - 6x + 4$

C. $f'(x) = 4x^3 - 9x + 2$

D. $f'(x) = (3x^2 + 2)(2x - 1)$



Quotient Rule

If $f(x) = u(x)/v(x)$, if all indicated derivatives exist, and if $v(x) \neq 0$, then

$$f'(x) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}.$$

(The derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.)

Example: Let $f(x) = \frac{2x-1}{4x+3}$. Find $f'(x)$.

Example: Let $f(x) = \frac{x^2+7x-2}{x^2-2}$. Find $f'(x)$.

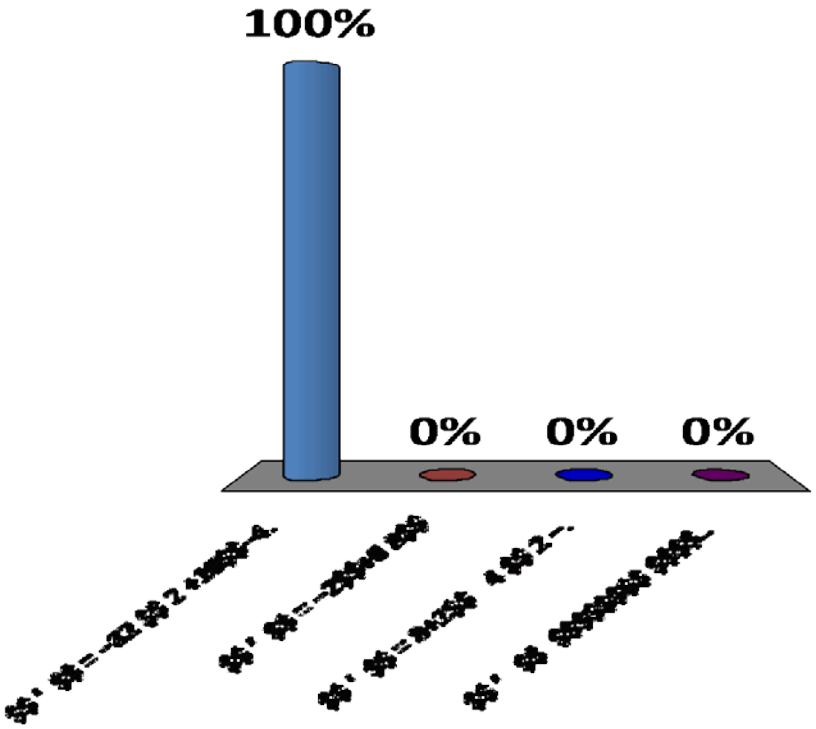
Let $f(x) = \frac{-x^2+8x}{4x^2-5}$. Find $f'(x)$.

A. $f'(x) = \frac{-32x^2+10x-40}{(4x^2-5)^2}$

B. $f'(x) = \frac{-2x+8}{8x}$

C. $f'(x) = \frac{8+2x}{(4x^2-5)^2}$

D. $f'(x)$ *does not exist.*



Example: Let $f(x) = \frac{5x+6}{\sqrt{x}}$. Find $f'(x)$.

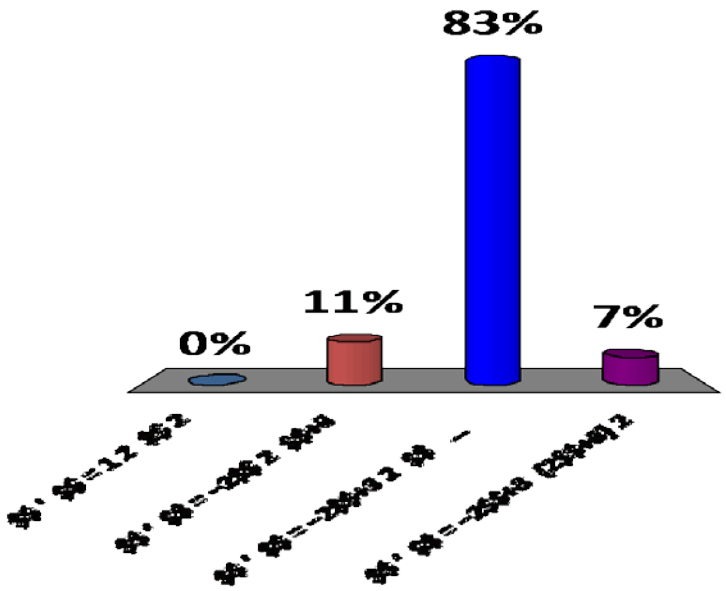
Let $f(t) = \frac{\sqrt{t}}{2t+3}$. Find $f'(t)$.

A. $f'(t) = \frac{1}{2}t^2$

B. $f'(t) = \frac{-2t}{2\sqrt{t+3}}$

C. $f'(t) = \frac{-2t+3}{2\sqrt{t}(2t+3)^2}$

D. $f'(t) = \frac{-2t+3}{(2t+3)^2}$



Example: The total cost (in hundreds of dollars) to produce x units of a product is $C(x) = \frac{8x-3}{4x+1}$. Find the average cost for producing x units.

Example: The total cost (in hundreds of dollars) to produce x units of a product is $C(x) = \frac{8x-3}{4x+1}$. Find the marginal average cost for producing x units.