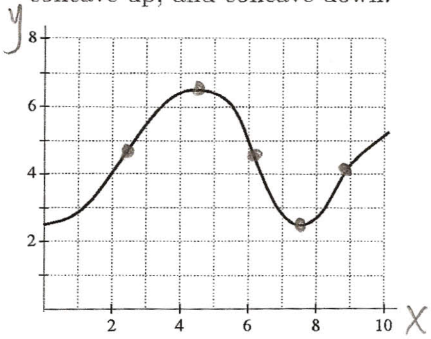
Section 2.5 – The Second Derivative

A function f is said to be...

1. *increasing* on an interval if its graph rises from left to right on that interval.
2. *decreasing* on an interval if its graph falls from left to right on that interval.
3. *concave up* on an interval if its graph is shaped like part, or all, of a right-side up bowl on that interval.
4. *concave down* on an interval if its graph is shaped like part, or all, of an upside down bowl on that interval.

Example 1. Given below is the graph of a function f. Estimate the intervals on which f is increasing, decreasing, concave up, and concave down.



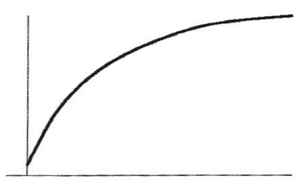
inc $x \in [0, 4.5) \cup (7.5, 10]$

dec $x \in (4.5, 7.5)$

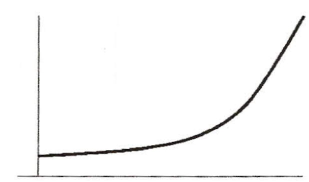
C.U. $x \in [0, 2.5) \cup (6, 9)$

C.D. $x \in (2.5, 6) \cup (9, 10]$

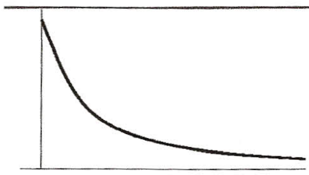
Example 2. Consider the four functions given below.



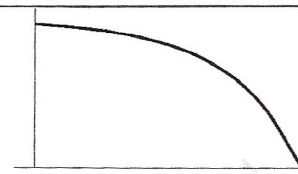
increasing
Concave down



increasing
Concave up



decreasing
Concave up



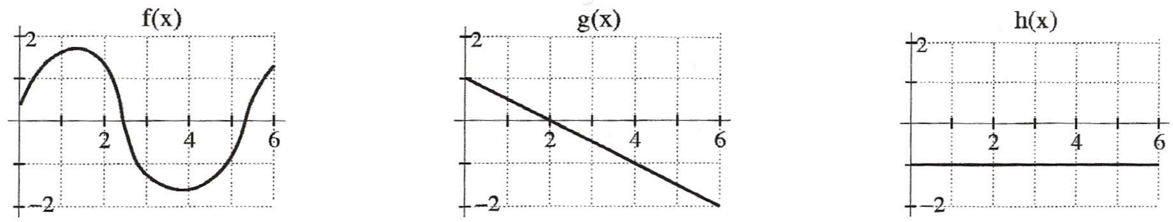
decreasing
Concave down

Summary of Key Facts. Assume that f is a function such that f' and f'' both exist.

1. If f is increasing on an interval, then $f'>0$ on that interval.
2. If f is decreasing on an interval, then $f'<0$ on that interval.
3. If f is concave up on an interval, then $f''>0$ on that interval.
4. If f is concave down on an interval, then $f''<0$ on that interval.

Practice Problems

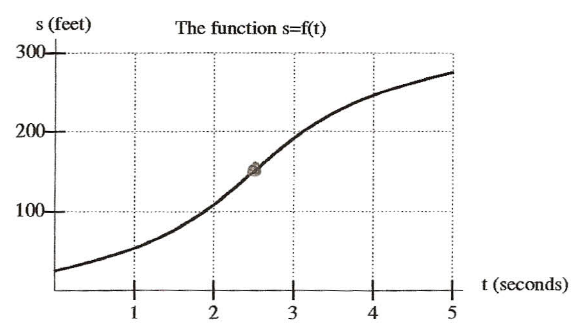
1. Given below are the graphs of three functions: f, g, and h.



Use the graphs to decide whether each of the quantities that follow are positive, negative, or zero.

(a) $f'(1) > 0$
(b) $f''(1) < 0$
(c) $f'(2) < 0$
(d) $f''(2) < 0$
(e) $f'(5) > 0$
(f) $f''(5) > 0$
(g) $g'(3) < 0$
(h) $g''(3) = 0$
(i) $h'(3) = 0$
(j) $h''(3) = 0$
(k) $f(1) > 0$
(l) $h(1) < 0$

2. A South Dakota driver cruising along I-90 speeds up, sees a highway patrol car, and then begins to slow down. A graph of her displacement as a function of time is shown to the right. **Note.** Positive values of s indicate that the driver is east of her starting point, and negative values of s indicate that she is west of her starting point.



(a) On what approximate intervals is $f'(t)$ positive? negative? Interpret the meaning of these intervals in the context of this problem.

$f'(t)>0$ for $t\in[0,5]$, so the driver moves east the entire time.
$f'(t)$ is never negative, i.e., the driver never moves west.

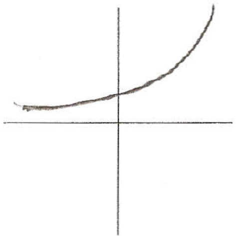
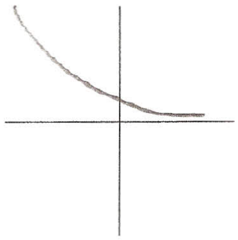
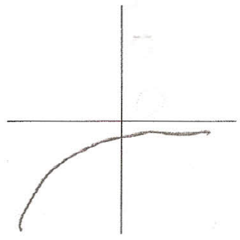
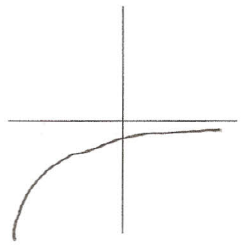
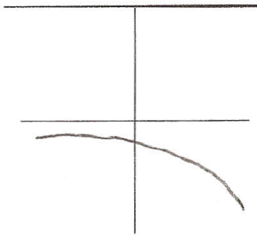
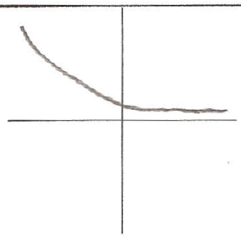
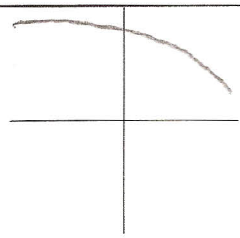
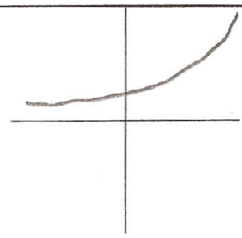
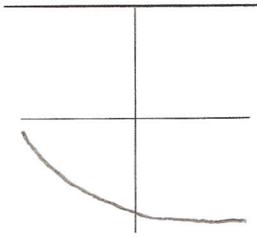
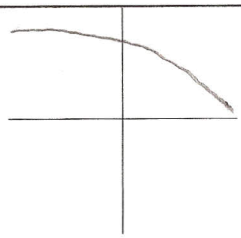
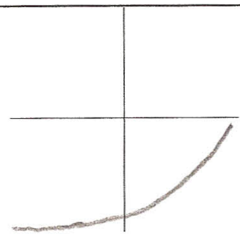
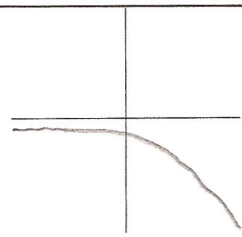
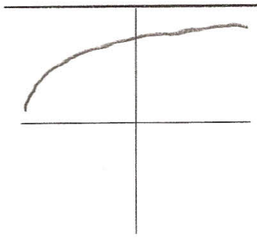
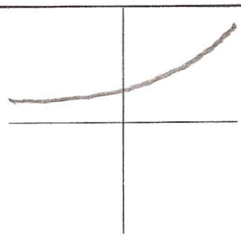
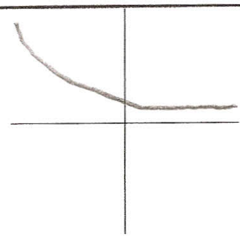
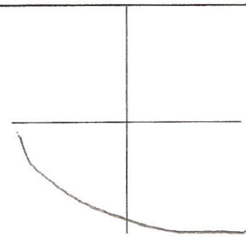
(b) On what approximate intervals is $f''(t)$ positive? negative? Interpret the meaning of these intervals in the context of this problem.

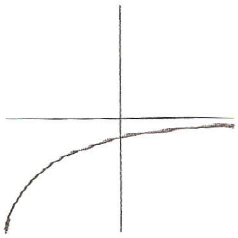
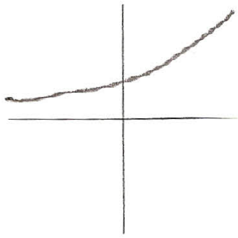
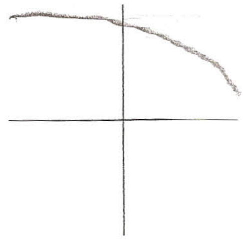
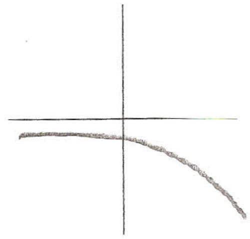
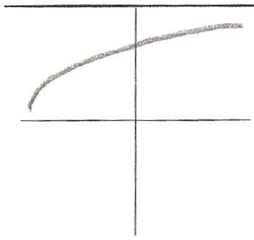
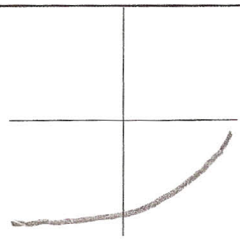
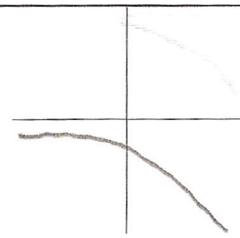
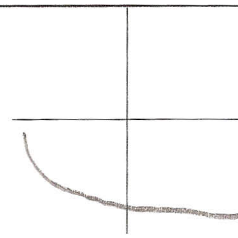
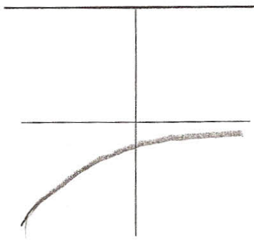
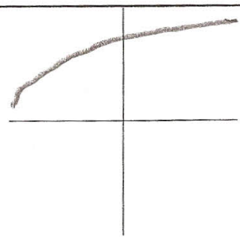
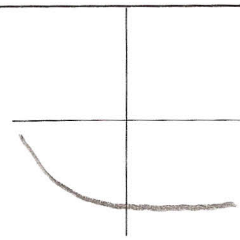
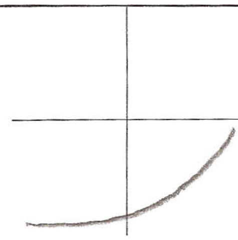
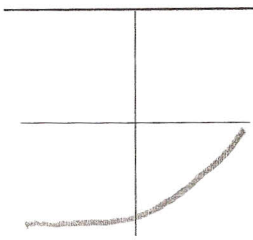
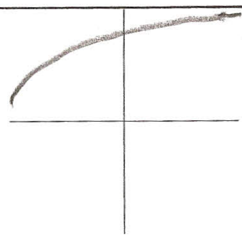
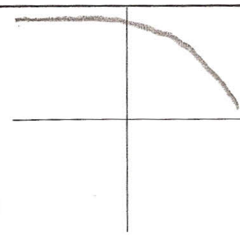
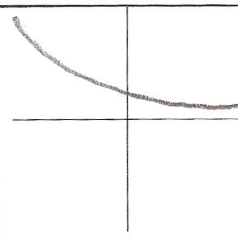
$f''(t)>0$ for $t\in[0,2.5)$, so the driver speeds up during the first 2.5 seconds.

$f''(t)<0$ for $t\in(2.5,5]$ means she slows down in the next 2.5 seconds.

Section 2.5 – Derivatives

In each of the following situations, sketch the graph of a function $f(t)$ that has the indicated properties.

 $f(t)$ is increasing $f(t) > 0$ $f'(t)$ is increasing	 $f(t) > 0$ $f(t)$ is decreasing $f''(t) > 0$	 $f(t)$ is increasing $f''(t) < 0$ $f(t) < 0$	 $f''(t) < 0$ $f'(t) > 0$ $f(t) < 0$
 $f(t) < 0$ $f'(t)$ is decreasing $f'(t) < 0$	 $f'(t)$ is increasing $f(t) > 0$ $f'(t) < 0$	 $f'(t) < 0$ $f''(t) < 0$ $f(t) > 0$	 $f(t)$ is increasing $f(t) > 0$ $f''(t) > 0$
 $f'(t)$ is increasing $f(t)$ is decreasing $f(t) < 0$	 $f(t) > 0$ $f(t)$ is decreasing $f'(t)$ is decreasing	 $f(t) < 0$ $f'(t)$ is increasing $f(t)$ is increasing	 $f''(t) < 0$ $f(t) < 0$ $f(t)$ is decreasing
 $f(t) > 0$ $f(t)$ is increasing $f''(t) < 0$	 $f'(t) > 0$ $f(t) > 0$ $f''(t) > 0$	 $f'(t) < 0$ $f''(t) > 0$ $f(t) > 0$	 $f(t) < 0$ $f''(t) > 0$ $f'(t) < 0$

 $f'(t)$ is decreasing $f(t) < 0$ $f'(t) > 0$	 $f'(t)$ is increasing $f'(t) > 0$ $f(t) > 0$	 $f(t) > 0$ $f''(t) < 0$ $f(t)$ is decreasing	 $f(t)$ is decreasing $f'(t)$ is decreasing $f(t) < 0$
 $f''(t) < 0$ $f(t) > 0$ $f'(t) > 0$	 $f'(t) > 0$ $f(t) < 0$ $f'(t)$ is increasing	 $f(t) < 0$ $f'(t) < 0$ $f''(t) < 0$	 $f'(t)$ is increasing $f'(t) < 0$ $f(t) < 0$
 $f(t) < 0$ $f'(t)$ is decreasing $f(t)$ is increasing	 $f'(t)$ is decreasing $f(t) > 0$ $f(t)$ is increasing	 $f(t)$ is decreasing $f''(t) > 0$ $f(t) < 0$	 $f(t)$ is increasing $f(t) < 0$ $f''(t) > 0$
 $f''(t) > 0$ $f'(t) > 0$ $f(t) < 0$	 $f(t) > 0$ $f'(t)$ is decreasing $f'(t) > 0$	 $f'(t) < 0$ $f'(t)$ is decreasing $f(t) > 0$	 $f(t) > 0$ $f'(t)$ is increasing $f(t)$ is decreasing