Wed 20 Jan

Welcome to Cal I!

- comp.uark.edu/~ashleykw/Cal1Spring2016/cal1spr16.html
  Course website. All information is here, including a link to MLP, lecture slides, administrative information, etc. You should have already seen the **syllabus** by now.
- MyLabsPlus (MLP) has the graded homework. Solutions to Quizzes and Drill exercises will be posted there, under "Menu $\rightarrow$ Course Tools $\rightarrow$ Document Sharing".

Wed 20 Jan (cont.)

- Lecture slides are available on the course website. I'll try to have the week's slides posted in advance but the individual lectures might not be posted until right before class. **Don't try to take notes from the slides.** Instead, print out the slides beforehand or else follow along on your tablet/phone/laptop. You should, however, take notes when we do exercises during lecture.
- For old Calculus materials, see the parent page `comp.uark.edu/~ashleykw` and look for links under "Previous Semesters".

§2.1 The Idea of Limits

Question

How would you define, and then differentiate between, the following pairs of terms?

- instantaneous velocity vs. average velocity?
- tangent line vs. secant line?

(Recall: What is a tangent line and what is a secant line?)

Example

An object is launched into the air. Its position s (in feet) at any time t (in seconds) is given by the equation:

$$s(t) = -4.9t^2 + 30t + 20.$$

(a) Compute the average velocity of the object over the following time intervals: $[1, 3]$, $[1, 2]$, $[1, 1.5]$

(b) As your interval gets shorter, what do you notice about the average velocities? What do you think would happen if we computed the average velocity of the object over the interval $[1, 1.2]$? $[1, 1.1]$? $[1, 1.05]$?

Example, cont.

An object is launched into the air. Its position s (in feet) at any time t (in seconds) is given by the equation:

$$s(t) = -4.9t^2 + 30t + 20.$$

(c) How could you use the average velocities to estimate the instantaneous velocity at $t = 1$?

(d) What do the average velocities you computed in 1. represent on the graph of $s(t)$?

Question

What happens to the relationship between instantaneous velocity and average velocity as the time interval gets shorter?

Answer: The instantaneous velocity at $t = 1$ is the limit of the average velocities as t approaches 1.

Question

What about the relationship between the secant lines and the tangent lines as the time interval gets shorter?

Answer: The slope of the tangent line at $(1, 45.1 = s(1))$ is the limit of the slopes of the secant lines as t approaches 1.

2.1 Book Problems

1-3, 7-13, 15, 21, 25, 27, 29

Even though book problems aren't turned in, they're a very good way to study for quizzes and tests (wink wink wink).

§2.2 Definition of Limits

Question

- Based on your everyday experiences, how would you define a "limit"?
- Based on your mathematical experiences, how would you define a "limit"?
- How do your definitions above compare or differ?

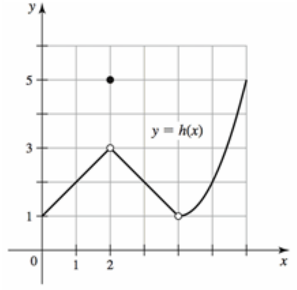
Definition (limit)

Suppose the function f is defined for all x near a, except possibly at a. If $f(x)$ is arbitrarily close to L (as close to L as we like) for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = L$$

and say **the limit of** $f(x)$ **as** x **approaches** a **equals** L.

Exercise



Determine the following:

(a) $h(1)$

(b) $h(2)$

(c) $h(4)$

(d) $\lim_{x \to 2} h(x)$

(e) $\lim_{x \to 4} h(x)$

(f) $\lim_{x \to 1} h(x)$

Question

Does $\lim_{x \to a} f(x)$ always equal $f(a)$?

(Hint: Look at the example from the previous slide!)

Exercise

Suppose $f(x) = \dfrac{x^2 + x - 20}{x - 4}$.

(a) Create a table of values of $f(x)$ when

$$x = 3.9, 3.99, 3.999, \text{ and}$$
$$x = 4.1, 4.01, 4.001$$

(b) What can you conjecture about $\lim_{x \to 4} f(x)$?

Up to this point we have been working with two-sided limits; however, for some functions it makes sense to examine one-sided limits.

Notice how in the previous example we could approach $f(x)$ from both sides as x approaches a, i.e., when $x > a$ and when $x < a$.

Definition (right-hand limit)

Suppose f is defined for all x near a with $x > a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a with $x > a$, we write

$$\lim_{x \to a^+} f(x) = L$$

and say **the limit of** $f(x)$ **as** x **approaches** a **from the right equals** L.

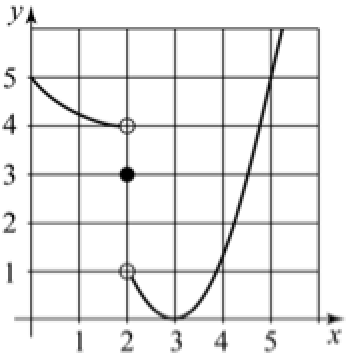
Definition (left-hand limit)

Suppose f is defined for all x near a with $x < a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a with $x < a$, we write

$$\lim_{x \to a^-} f(x) = L$$

and say **the limit of** $f(x)$ **as** x **approaches** a **from the left equals** L.

Exercise



Determine the following:

(a) $g(2)$

(b) $\lim_{x \to 2^+} g(x)$

(c) $\lim_{x \to 2^-} g(x)$

(d) $\lim_{x \to 2} g(x)$

Theorem

If f is defined for all x near a except possibly at a, then $\lim_{x\to a} f(x) = L$ if and only if both $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^-} f(x) = L$.

In other words, the only way for a two-sided limit to exist is if the one-sided limits equal the same number (L).

2.2 Book Problems

1-4, 7, 9, 11, 13, 19, 23, 29, 31