If the slope of the tangent line to the graph of f at $x = 2$ is positive, which of the following are true?

A. $1 + 1 = 7$

B. $f'(2) = 0$

C. $f'(2)$ is positive

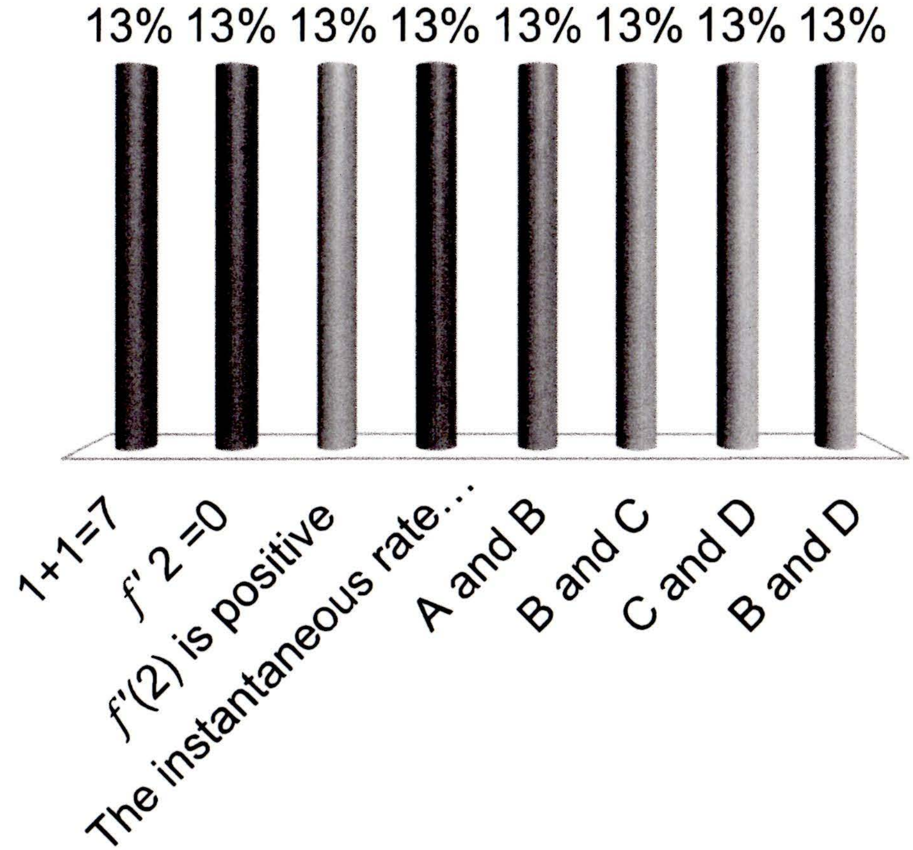
D. The instantaneous rate of change of f at $x = 2$ is positive.

E. A and B

F. B and C

G. C and D

H. B and D



Find the intervals in which the following function is increasing or decreasing.

$$f(x) = x^3 + 3x^2 - 9x + 1$$

$$f'(x) = 3x^2 + 6x - 9 = 0$$

$$= 3(x^2 + 2x - 3) = 0$$

$$= 3(x+3)(x-1) = 0$$

$$f'(-4) = 3(-4+3)(-4-1) > 0$$

+ − −

$$f'(0) = 3(0+3)(0-1) < 0$$

+ + −

$$f'(2) = 3(2+3)(2-1) > 0$$

+ + +

increasing:

$(-\infty, -3), (1, \infty)$

decreasing:

$(-3, 1)$

Find the intervals for which the following function increases and decreases:

$$f(x) = \frac{x-1}{x+1}$$

$\cancel{x}+1 \quad -\cancel{x}+1$

$$f'(x) = \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2} > 0 \text{ for all } x \text{ except } -1$$

<u>increasing</u>:

$(-\infty,-1), (-1,\infty)$

<u>decreasing</u>:

$\emptyset$

Given the function
$f(x) = -x^3 - 2x^2 + 15x + 10,$
find the critical numbers.

$f'(x) = -3x^2 - 4x + 15 = 0$

Q-formula: 16 + 180

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(15)(-3)}}{2(-3)}$$

$$= \frac{4 \pm \sqrt{196}}{-6} = \frac{4 \pm 14}{-6}$$

$$= \frac{2 \pm 7}{-3} = \boxed{-3, \frac{5}{3}}$$

A. $x = 2 \quad x = \frac{2}{3}$

B. $x = 5 \quad x = -3$

C. $x = \frac{2}{3} \quad x = -\frac{5}{3}$

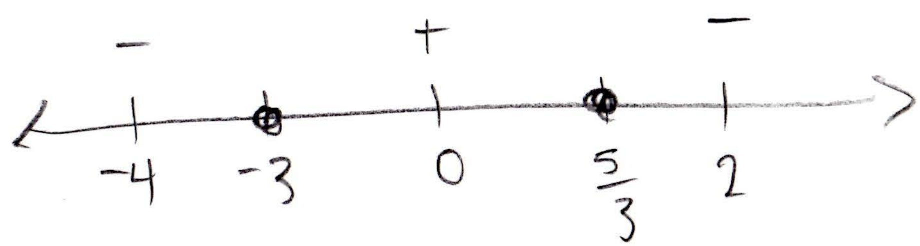
D. $x = \frac{5}{3} \quad x = -3$

Determine where the function

$f(x) = -x^3 - 2x^2 + 15x + 10,$

is increasing. Give your answer in interval notation.

– see last slide for CP's –



A. $\left(-3, \frac{5}{3}\right)$

B. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

C. $(-\infty, \infty)$

D. The function never increases.

$f'(-4) = -3(-4)^2 - 4(-4) + 15 < 0$

$-3(16) + 16 + 15$

$f'(0) = -3(0)^2 - 4(0) + 15 > 0$

$0 \quad 0 + 15$

$f'(2) = -3(2)^2 - 4(2) + 15 < 0$

$-12 - 8 + 15$

Determine where the function

$f(x) = -x^3 - 2x^2 + 15x + 10$,

is decreasing. Give your answer in interval notation.

— see last slide for work —

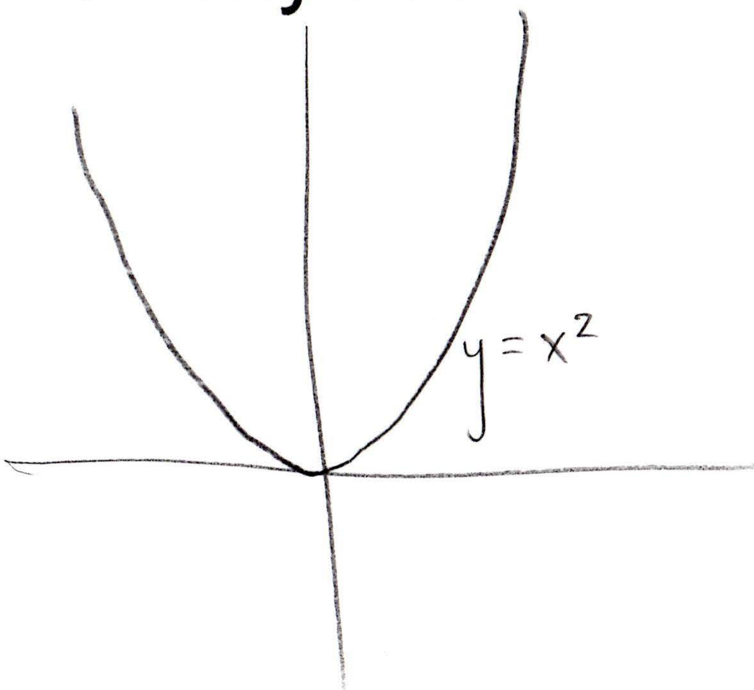
A. $\left(-3, \frac{5}{3}\right)$

B. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

C. $(-\infty, \infty)$

D. The function never decreases.

A friend looks at the graph of $y = x^2$ and observes that if you stare at the origin, the graph increases whether you go left or right, so the graph is increasing everywhere. Is your friend correct? Why or why not?



Yes, but the friend's observation is not enough information to be able to tell.

We also need to know $x=0$ is the only critical point.

Find the critical numbers for $f(x) = \dfrac{x}{x+1}$.

$$f'(x) = \frac{\overset{x+1}{(x+1)(1)} - \overset{-x}{x(1)}}{(x+1)^2} = \frac{1}{(x+1)^2}$$ ← never zero (but is undefined at $x=-1$)

A. $x = 0$

B. $x = -1$

C. $x = 0\,, x = -1$

D. $x = \ln(2)$

E. There are no critical numbers.