# *UNIT 2, LESSON 1*

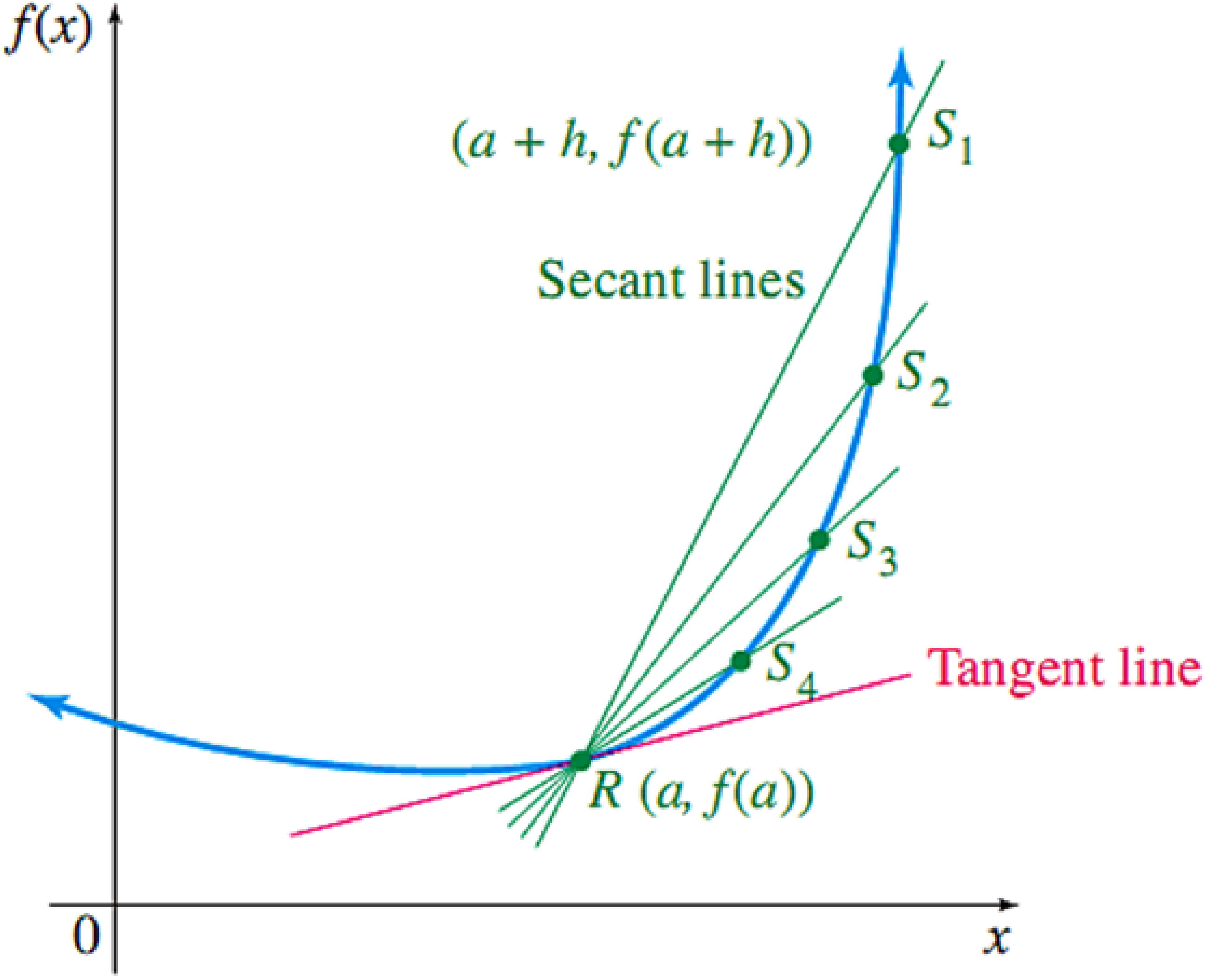
## LIMIT DEFINITION OF THE DERIVATIVE

# Limit Definition of the Derivative

This section defines the derivative in terms of the limit of the difference quotient.  Graphically, this is seen as the slope of the tangent line.

OBJECTIVES:

- Express the derivative of a function as a limit
- Evaluate this limit of the difference quotient algebraically
- Write the equation of the tangent line at a given point


$f(x)$
$(a + h, f(a + h))$
$S_1$
Secant lines
$S_2$
$S_3$
$S_4$
Tangent line
$R\ (a, f(a))$
0
$x$

## Slope of the Tangent Line

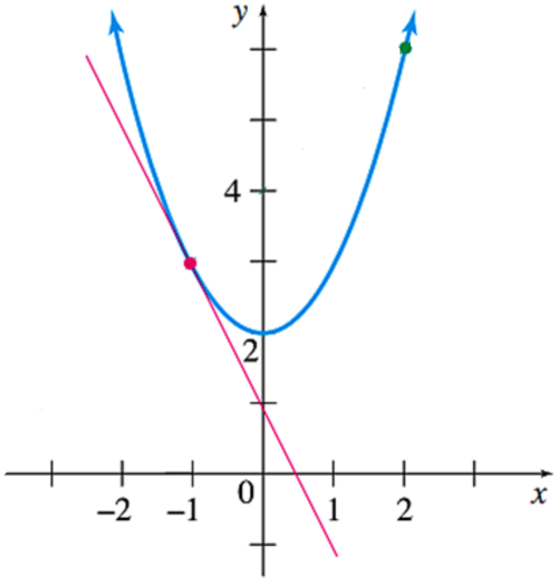
The **tangent line** of the graph of $y = f(x)$ at the point $(a, f(a))$ is the line through this point having slope

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided this limit exists. If this limit does not exist, then there is no tangent at the point.

Find the slope and equation of the tangent line to the graph of $f(x) = x^2 + 2$ at $x = -1$.

Slope of tangent line = $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$



## Derivative

The **derivative** of the function $f$ at $x$ is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided this limit exists.

If $x$ is a value in the domain of $f$ and if $f'(x)$ exists, then $f$ is ***differentiable*** at $x$.

## Finding $f'(x)$ from the Definition of Derivative

The four steps used to find the derivative $f'(x)$ for a function $y = f(x)$ are summarized here.

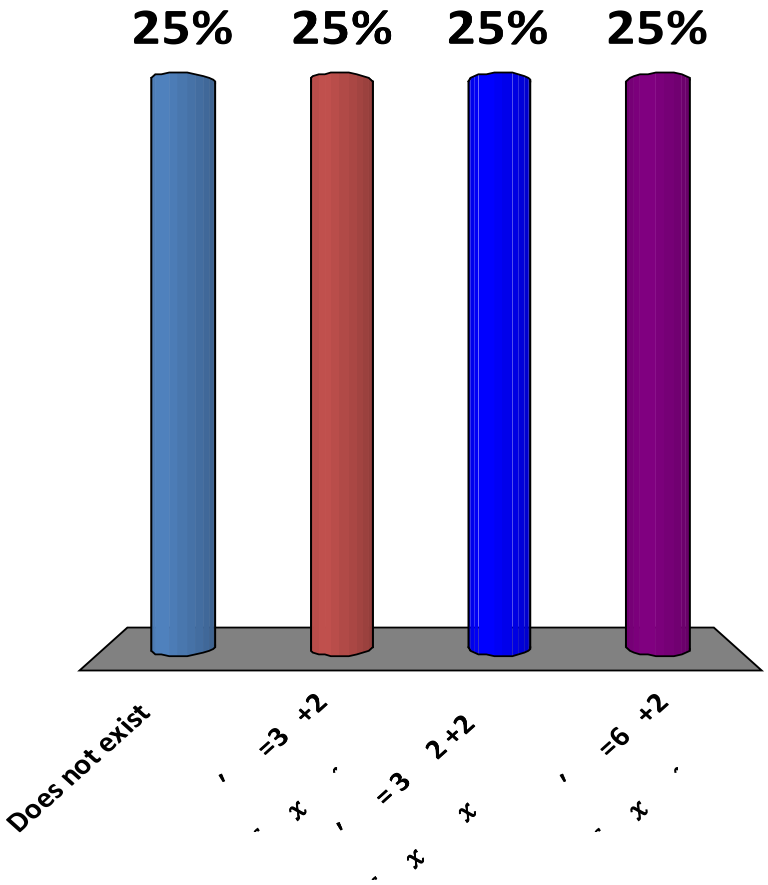
1. Find $f(x + h)$.
2. Find and simplify $f(x + h) - f(x)$.
3. Divide by $h$ to get $\dfrac{f(x + h) - f(x)}{h}$.
4. Let $h \to 0$; $f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$, if this limit exists.

Let $f(x) = x^2$. Find the derivative.

Let $f(x) = 2x^3 + 4x$. Find $f'(x)$. Then find $f'(2)$.

# Let $f(x) = 3x^2 + 2x + 1$. Find $f'(x)$.

A. Does not exist
B. $f'(x) = 3x + 2$
C. $f'(x) = \frac{3x^2+2}{h}$
D. $f'(x) = 6x + 2$



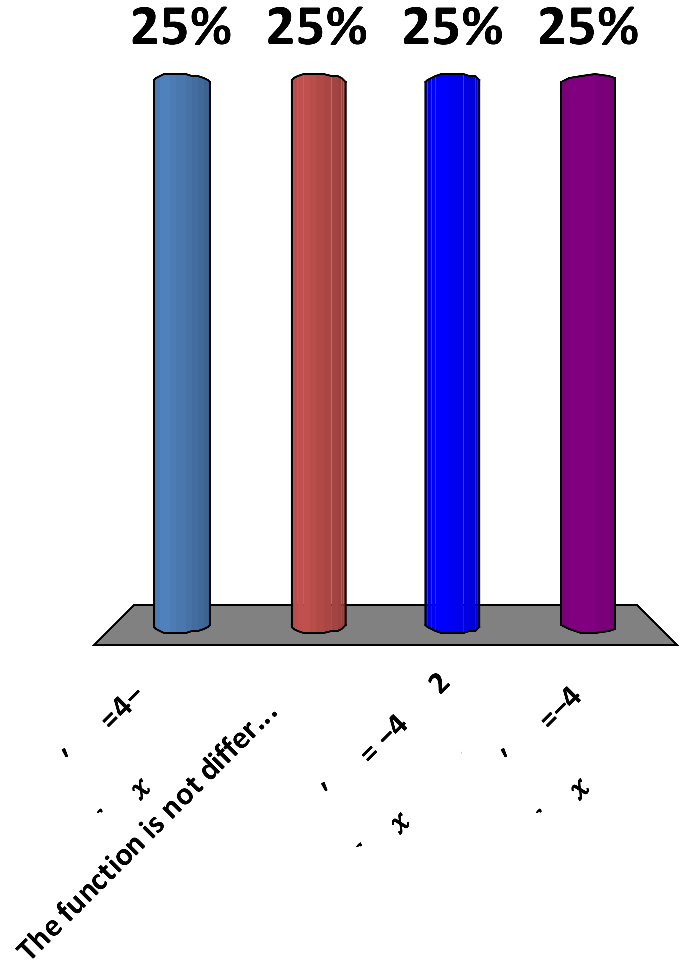
# Let $f(x) = \frac{4}{x}$. Find $f'(x)$.

A. $f'(x) = 4 - x$

B. The function is not differentiable

C. $f'(x) = \frac{-4}{x^2}$

D. $f'(x) = -4x$



## Existence of the Derivative

The derivative exists when a function $f$ satisfies *all* of the following conditions at a point.

1. $f$ is continuous,
2. $f$ is smooth, and
3. $f$ does not have a vertical tangent line.

The derivative does *not* exist when *any* of the following conditions are true for a function at a point.

1. $f$ is discontinuous,
2. $f$ has a sharp corner, or
3. $f$ has a vertical tangent line.

The cost in dollars to manufacture $x$ graphing calculators is given by $C(x) = -0.005x^2 + 20x + 150$ when $0 \leq x \leq 2000$. Find the rate of change of cost with respect to the number manufactured when 100 calculators are made.

Find the equation of the tangent line to $f(x) = 6 - x^2$ when $x = -1$.