4.3.15

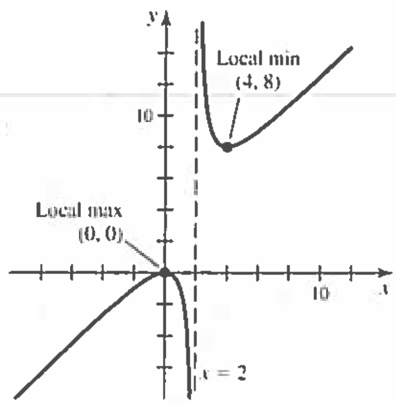
The domain of f is $(-\infty, 2)\cup(2, \infty)$, and there is no symmetry. Note that $\lim_{x\to 2^+} f(x) = \infty$ and $\lim_{x\to 2^-} f(x) = -\infty$, so there is a vertical asymptote at $x = 2$. There isn't a horizontal asymptote, since $\lim_{x\to\pm\infty} f(x) = \pm\infty$.

$f'(x) = \frac{(x-2)(2x)-x^2}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$. This is 0 when $x = 4$ and when $x = 0$.

$f''(x) = \frac{(x-2)^2(2x-4)-(x^2-4x)(2)(x-2)}{(x-2)^4} = \frac{8}{(x-2)^3}$. This is never 0.

Note that $f'(-1) > 0$, $f'(1) < 0$, $f'(3) < 0$ and $f'(5) > 0$. So f is decreasing on $(0,2)$ and on $(2,4)$. It is increasing on $(-\infty, 0)$ and on $(4,\infty)$. There is a local maximum of 0 at $x = 0$ and a local minimum of 8 at $x = 4$.

Note that $f''(x) > 0$ for $x > 2$ and $f''(x) < 0$ for $x < 2$, So f is concave up on $(2,\infty)$ and concave down on $(-\infty, 4)$. There are no inflection points, since the only change in concavity occurs at a vertical asymptote. The only intercept is $(0,0)$.



4.3.16

The domain of f is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$, and there is even symmetry since $f(-x) = \frac{(-x)^2}{(-x)^2-4} = \frac{x^2}{x^2-4} = f(x)$.

Since $\lim_{x\to\pm\infty} \frac{x^2}{x^2-4}\cdot\frac{1/x^2}{1/x^2} = \lim_{x\to\pm\infty} \frac{1}{1-(4/x^2)} = 1$, there is a horizontal asymptote at $y = 1$. Also, since $\lim_{x\to -2^-} f(x) = \infty$, $\lim_{x\to -2^+} f(x) = -\infty$, $\lim_{x\to 2^-} f(x) = -\infty$ and $\lim_{x\to 2^+} f(x) = \infty$, there are vertical asymptotes at $x = -2$ and $x = 2$.

$f'(x) = \frac{(x^2-4)(2x)-x^2(2x)}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$. This is 0 when $x = 0$. $f''(x) = \frac{(x^2-4)^2(-8)-(-8x)(2)(x^2-4)(2x)}{(x^2-4)^4} = \frac{8(3x^2+4)}{(x^2-4)^3}$, which is never 0.

Note that $f'(x) > 0$ on $(-\infty,-2)$ and on $(-2,0)$, while $f'(x) < 0$ on $(0,2)$ and on $(2,\infty)$. So f is increasing on $(-\infty,-2)$ and on $(0,2)$, and is decreasing on $(0,2)$ and on $(2,\infty)$. There is a local maximum of 0 at $x = 0$.

Note also that $f''(x) > 0$ for $x < -2$, and $f''(x) > 0$ for $x > 2$, while $f''(x) < 0$ for $-2 < x < 2$. So f is concave up on $(-\infty,-2)$ and on $(2,\infty)$, while it is concave down on $(-2,2)$. There are no inflection points since the only changes in concavity occur at asymptotes.