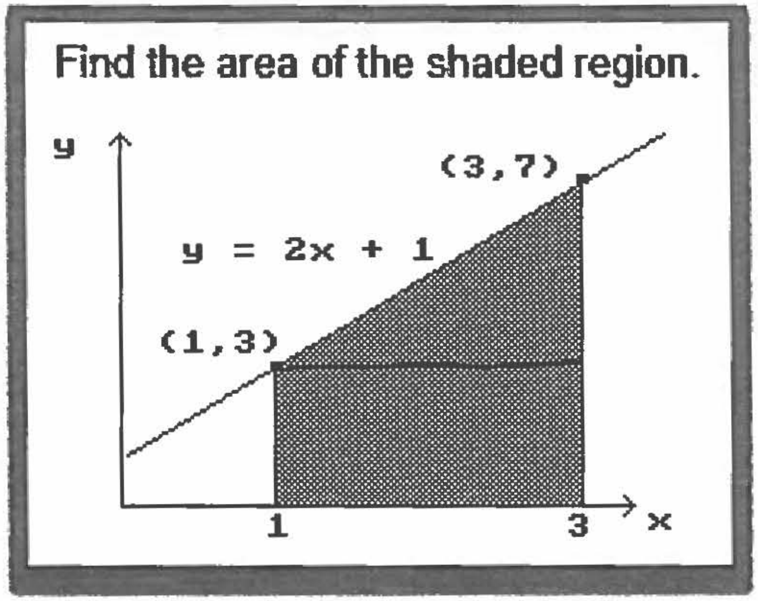


Use geometry:

The triangle has base $= 2$ and height $= 7-3 = 4$

The rectangle has base $= 2$ and height $= 3$

Area $= \frac{1}{2}(2)(4) + 2(3) = \boxed{10}$

Check:

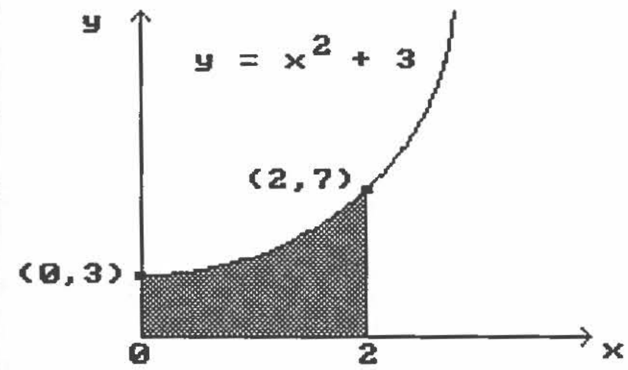
$$\int_1^3 (2x+1)\,dx$$

$$= x^2 + x \Big|_1^3$$

$$= 3^2 + 3 - (1^2 + 1)$$

$$= 9 + 3 - 2 = \boxed{10} \checkmark$$

Find the area of the shaded region.



$$\int_0^2 (x^2+3)\,dx = \frac{x^3}{3} + 3x \Big|_0^2$$

$$= \frac{2^3}{3} + 3(2) - \left(\cancel{\frac{0^3}{3} + 3(0)}\right)$$

$$= \frac{8}{3} + 6 = \boxed{\frac{26}{3}}$$

$\frac{18}{3}$

Example:

Find $\int_1^3 3x^2 dx. = x^3 \Big|_1^3 = 3^3 - 1^3$

$= 27 - 1 = \boxed{26}$

Example:

Find $\int_3^5 (2x^3 - 3x + 4)dx$.

$$= 2\left(\frac{x^4}{4}\right) - 3\left(\frac{x^2}{2}\right) + 4x \Big|_3^5$$

$$= \frac{5^4}{2} - \frac{3}{2}(5^2) + 4(5) - \left[\frac{3^4}{2} - \frac{3}{2}(3^2) + 4(3)\right]$$

$$\boxed{= 256}$$

Compute $\int_0^5 (3x^2 + 2x + 1)\mathrm{dx}$

$= x^3 + x^2 + x \Big|_0^5$

A. 2

B. 17.5

C. 155

D. 210

E. 433

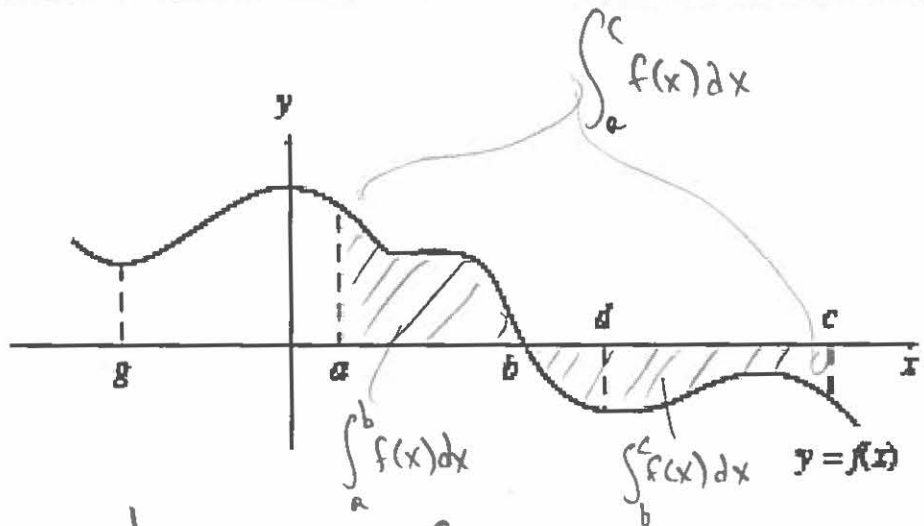
$= 5^3 + 5^2 + 5 - \cancel{(0^3 + 0^2 + 0)}$

$= 125 + 25 + 5$

$= 155$

Assume f(x) is continuous for $g \leq x \leq c$ as shown in the figure. Write an equation relating the three quantities below.

$$\int_a^c f(x)dx,\ \int_a^b f(x)dx,\ \int_b^c f(x)dx$$



$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

$$\int_0^1 x^9 \underbrace{\left(1+x^{10}\right)}_{u}{}^9\,dx \qquad \frac{1}{10}\frac{du}{dx}$$

$u = 1 + x^{10}$

$\frac{du}{dx} = 10x^9$

$$= \int_0^1 \frac{1}{10}\left(\frac{du}{\cancel{dx}}\right)u^9\,\cancel{dx} = \frac{1}{10}\int_0^1 u^9\,du$$

$$= \frac{1}{10}\left(\frac{u^{10}}{10}\right)\Bigg|_{x=0 \Rightarrow u=1+0^{10}=1}^{x=1 \Rightarrow u=1+1^{10}=2}$$

$$= \frac{1}{100}u^{10}\Big|_1^2$$

$$= \frac{2^{10}}{100} - \frac{1^{10}}{100} = \boxed{\frac{1023}{100}}$$

$$\int_{1}^{\frac{e}{2}} \frac{1}{2x}\,dx$$

A. e

B. $2 - 2\ln(2)$

C. $\ln(e^2)$

D. $\ln(e^3)$

E. $\frac{1}{2}\ln\left(\frac{e}{2}\right)$

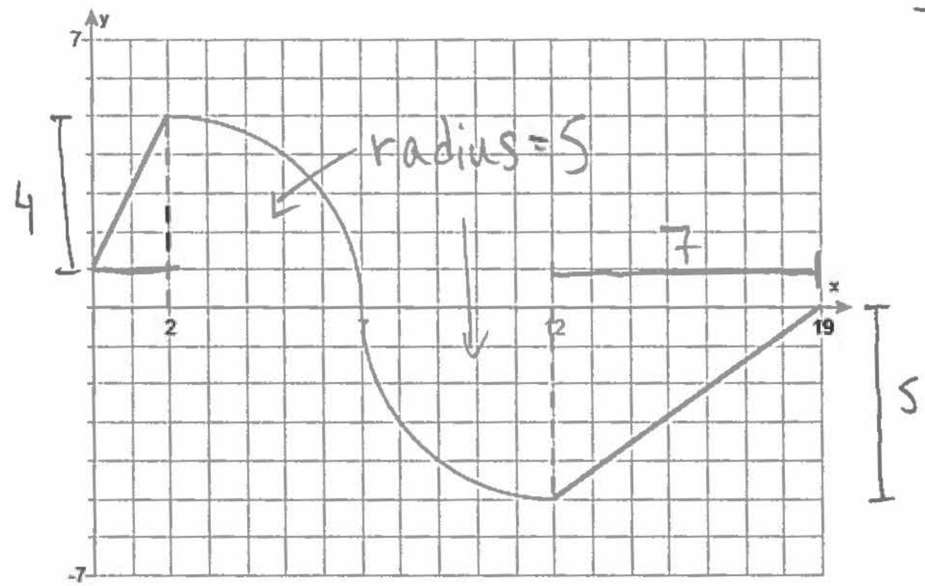
F. $\frac{1}{3}\ln\left(\frac{e}{3}\right)$

$$= \frac{1}{2}\int_{1}^{e/2} \frac{1}{x}\,dx$$

$$= \frac{1}{2}\ln|x|\Big|_{1}^{e/2}$$

$$= \frac{1}{2}\ln\left(\frac{e}{2}\right) - \cancel{\frac{1}{2}\underbrace{\ln(1)}_{=0}}$$

The graph of f(x), shown here, consists of two straight line segments and two quarter circles. Find the value of $\int_0^{19} f(x)dx$.



Use geometry

$$\int_0^{19} f(x)\,dx = (1)(2) + \frac{1}{2}(2)(4)$$

$$+ \cancel{\frac{1}{4}\pi(5)^2} - \cancel{\frac{1}{4}\pi(5^2)}$$

$$- \frac{1}{2}(7)(5)$$

$$= 6 + \frac{35}{2} = \boxed{\frac{47}{2}}$$

$$\int_1^4 (x^2 - 4)dx$$

$$= \frac{x^3}{3} - 4x \Big|_1^4 = \frac{4^3}{3} - 4(4) - \left[\frac{1^3}{3} - 4(1)\right]$$

$$= \frac{64}{3} - 16 - 1 + 4$$

$$\boxed{= \frac{25}{3}}$$

$$\int_0^2 2xe^{x^2}\,dx$$

u

$\frac{du}{dx}$

Bounds: If $x=2$ then
$u=x^2=2^2=4$
If $x=0$ then
$u=x^2=0^2=0$

$$=\int_0^4 e^u\,du = e^u\Big|_0^4 = e^4-e^0 = \boxed{e^4-1}$$

OR:

$$e^u\Big|_{x=0}^{x=2} = e^{x^2}\Big|_0^2 = e^{2^2}-e^{0^2} = \boxed{e^4-1}$$

$$\text{Find } \int_{e^2}^{e^5} \overset{\frac{du}{dx}}{\frac{1}{x\underbrace{\ln(x)}_{u}}}\,dx$$

$x = e^5 \implies u = \ln(e^5) = 5(\ln e)$
$= 5$

$x = e^2 \implies u = \ln(e^2) = 2\ln e$
$= 2$

$$= \int_2^5 \frac{1}{u}\,du = \ln|u| \Big|_2^5$$

$$= \ln(5) - \ln(2)$$

A. $\ln(5) - \ln(\overset{2}{\cancel{3}})$

B. $e^5 - e^{\cancel{3}\,2}$

C. $\ln(\overset{2}{\cancel{3}}) - \ln(5)$

D. $e^5 - e^{\cancel{3}\,2} - \ln(\overset{2}{\cancel{3}}) + \ln(5)$

E. $e^5 - e^{\cancel{3}\,2} + \ln(\overset{2}{\cancel{3}}) - \ln(5)$