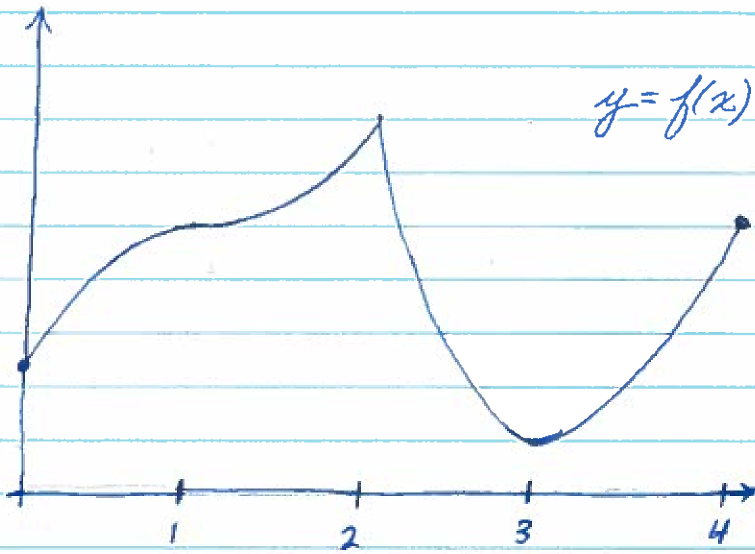
1.



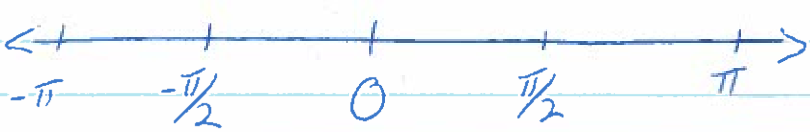
2. $T(x) = \cos^2 x$ on $[-\pi, \pi]$

$T'(x) = 2\cos x(-\sin x)$

$-2\cos x \sin x = 0$ so

$\cos x = 0$ or $\sin x = 0$

$\Rightarrow x = -\pi/2, \pi/2$ $\Rightarrow x = 0, -\pi, \pi$



In quadrant I, $T'(x) < 0$
In quadrant II, $T'(x) > 0$
In quadrant III, $T'(x) < 0$
In quadrant IV, $T'(x) > 0$

$T(x)$ is increasing on $(-\pi/2, 0)$ and $(\pi/2, \pi)$;
decreasing on $(-\pi, -\pi/2)$ and $(0, \pi/2)$

3. $T'(x) = -2\cos x \sin x$

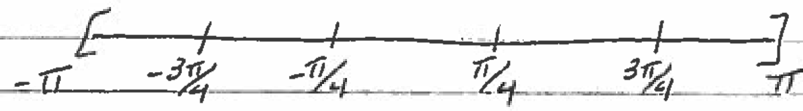
$T''(x) = -2((-\sin x)\sin x + \cos x(\cos x))$

$= -2(-\sin^2 x + \cos^2 x) = -2\cos 2x$

$-2\cos 2x = 0 \Rightarrow 2x = \pi/2 + k\pi$
where k is an integer

$\Rightarrow x = \pi/4 + k\pi/2$

so $x = \pi/4, -\pi/4, 3\pi/4,$ or $-3\pi/4$



$T''(-7\pi/8) = -2\cos(-7\pi/4) = -2/\sqrt{2} < 0$
$T''(-\pi/2) = -2\cos(-\pi) = 2 > 0$
$T''(0) = -2 < 0$
$T''(\pi/2) = 2 > 0$
$T''(7\pi/8) = -2/\sqrt{2} < 0$

So $T(x)$ is concave up on
$(-3\pi/4, -\pi/4)$ and $(\pi/4, 3\pi/4)$;

concave down on
$(-\pi, -3\pi/4)$, $(-\pi/4, \pi/4)$, and $(3\pi/4, \pi)$

4. $g'(x) = \dfrac{(1)(x^2+1)^2 - x(2(x^2+1)\cdot 2x)}{(x^2+1)^4}$

$= \dfrac{(x^2+1) - 4x^2}{(x^2+1)^3} = \dfrac{1-3x^2}{(x^2+1)^3}$

$(x^2+1)^3 \neq 0$

$1 - 3x^2 = 0 \Rightarrow 3x^2 = 1$

$\Rightarrow x^2 = 1/3 \Rightarrow x = \pm 1/\sqrt{3}$

Critical Points: $1/\sqrt{3}$, $-1/\sqrt{3}$

$g(-2) = \dfrac{-2}{(4+1)^2} = -2/25 = -0.08$

$g(1/\sqrt{3}) = \frac{1}{\sqrt{3}} / ((1/3+1))^2 = \frac{1}{\sqrt{3}}(3/4)^2$

$= 9/16\sqrt{3} \approx 0.325$

$g(-1/\sqrt{3}) = -\frac{1}{\sqrt{3}} / (1/3+1)^2 = -\frac{1}{\sqrt{3}}(3/4)^2$

$= -9/16\sqrt{3} \approx -0.325$

$g(2) = 2/(4+1)^2 = 0.08$

Absolute max of $9/16\sqrt{3} = \dfrac{3\sqrt{3}}{16}$
at $x = 1/\sqrt{3}$

Absolute min of $-9/16\sqrt{3} = \dfrac{3\sqrt{3}}{16}$
at $x = -1/\sqrt{3}$

5. $g'(x) = xe^{-x/2}(-1/2) + e^{-x/2}$

$= e^{-x/2}(-x/2 + 1)$

$g'(x) = 0 \implies -x/2 + 1 = 0$

$\implies x = 2$

Critical point: 2

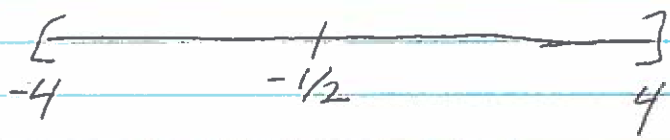
$g(0) = 0$
$g(2) = 2e^{-1} = 2/e \approx 0.736$
$g(5) = 5e^{-5/2} \approx 0.410$

Absolute max of $2/e$ at $x = 2$
Absolute min of 0 at $x = 0$

6. $h'(x) = -2x - 1$

$-2x - 1 = 0 \Rightarrow x = -\frac{1}{2}$

Critical Point: $-\frac{1}{2}$



$h'(0) = -1 < 0 \quad h'(-1) = 1 > 0$

Local max at $x = -\frac{1}{2}$
No local min

7. $k'(x) = \frac{1}{x^2+1} - 3x^2$

$= \frac{1}{x^2+1} - \frac{3x^2(x^2+1)}{x^2+1} = \frac{1-3x^4-3x^2}{x^2+1}$

$x^2+1 \neq 0$

$-3x^4-3x^2+1 = 0$; Let $u = x^2$, so

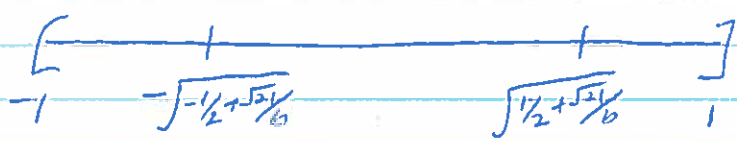
$-3u^2-3u+1=0$

$u = \frac{-(-3) \pm \sqrt{(-3)^2-4(-3)(1)}}{2(-3)} = \frac{3 \pm \sqrt{21}}{-6}$

$= -\frac{1}{2} \pm \frac{\sqrt{21}}{6}$

$x = \pm\sqrt{u} = \pm\sqrt{-\frac{1}{2}+\frac{\sqrt{21}}{6}}$ since $-\frac{1}{2}-\frac{\sqrt{21}}{6} < 0$

$x \approx \pm 0.514$



$k'(-0.75) = \frac{1}{(-\frac{3}{4})^2+1} - 3\left(\frac{3}{4}\right)^2 = \frac{1}{25/16} - \frac{27}{16}$

$= \frac{16}{25} - \frac{27}{16} < 0$

$k'(0) = 1 > 0$; $k'(0.75) = k'(-0.75) < 0$

Local max at $x = \sqrt{\frac{1}{2}+\frac{\sqrt{21}}{6}}$
Local min at $x = -\sqrt{\frac{1}{2}+\frac{\sqrt{21}}{6}}$

8. $\alpha'(x) = \frac{1}{3}(x+1)^2$

$\frac{1}{3}(x+1)^2 = 0 \Rightarrow x = -1$

Critical Point: -1

$\alpha''(x) = \frac{2}{3}(x+1) \qquad \alpha''(-1) = 0$

Test is inconclusive

9. $\beta'(x) = -40x + 10x^3$

$-40x + 10x^3 = 0 \Rightarrow 10x(-4 + x^2) = 0$

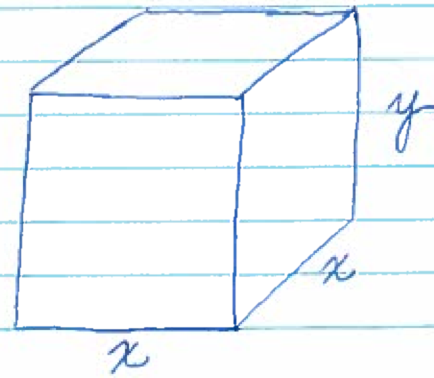
Critical Points: $0, 2, -2$

$\beta''(x) = -40 + 30x^2$

$\beta''(0) = -40$
$\beta''(2) = -40 + 120 = 80$
$\beta''(-2) = 80$

Local max at $x = 0$
Local min at $x = 2, -2$

10. $x^2y = 80 \Rightarrow$
$y = 80/x^2$



Surface area:

$S(x) = 2x^2 + 4xy$

$= 2x^2 + \frac{320}{x}$

$S'(x) = 4x - \frac{320}{x^2}$

$4x - \frac{320}{x^2} = 0 \Rightarrow 4x = \frac{320}{x^2}$

$\Rightarrow 4x^3 = 320 \Rightarrow x^3 = 80$

so $x = \sqrt[3]{80} = 2\sqrt[3]{10}$ is the critical point

$2\sqrt[3]{10}$

$S'(1) = 4 - 320 < 0$
$S'(20) = 80 - \frac{320}{400} > 0$

so S has a local minimum at $x = 2\sqrt[3]{10}$. Since this is the only extremum on $(0, \infty)$ and S is continuous on $(0, \infty)$, this is an absolute minimum.

Min. value: $S(2\sqrt[3]{10}) = 24 \cdot 10^{2/3}$ ft^2
Dimensions: $x = y = 2\sqrt[3]{10}$