MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Wed 3 June 2015

Quiz 3: Formal Definitions of Continuity, Limits, and Derivatives (∮2.6-3.1)

Directions: You have 30 minutes to complete this quiz. This quiz is closed book but you may collaborate with each other.

1. Find the numbers a and b that will make f continuous for all x. (*Reminder:* In order to claim something is continuous at a point you must use the Continuity Checklist.)

$$f(x) = \begin{cases} 2x + a & x \leq 0 \\ x^2 + 1 & 0 < x \leq 2 \\ bx - 2 & x > 2 \end{cases}$$

f is a polynomial on each of its pieces so the only points to check for continuity are 0 and 2

√List:

1. $f(0) = 2(0) + a = a$

2. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2x + a = a$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 + 1 = 1$

For 2-sided to exist, need $\boxed{a = 1} = f(0)$.

1. $f(2) = 2^2 + 1 = 5$

2. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 + 1 = 5$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} bx - 2 = 2b - 2$

For 2-sided to exist, need $5 = 2b - 2$ $\Rightarrow \boxed{b = \frac{7}{2}}$

3. $f(2) = 5 = \lim_{x \to 2} f(x)$ (when $b = \frac{7}{2}$)

2. You may use whichever (limit) definition of the derivative you prefer for the following questions (you are not allowed to use any derivative shortcuts yet). Given $f(x) = x^2+3$,

(a) find $f'(1)$;

$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^2-\cancel{3}-(1^2-\cancel{3})}{x-1}$$

$$= \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$= \lim_{x\to 1} x+1 \boxed{= 2}$$

(b) find a formula for $f'(x)$;

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+\cancel{3}-(x^2+\cancel{3})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2x\cancel{h}+h^2-\cancel{x^2}}{\cancel{h}} = \lim_{h\to 0} 2x+h \boxed{= 2x}$$

(c) use your answer to (a) to write the equation of the line tangent to $f(x)$ at $x = 1$.

$$y - f(1) = 2(x-1)$$

$$f(1) = 1^2+3 = 4$$

$$\Rightarrow \boxed{y-4 = 2(x-1)}$$

3. Does the function
$$f(x) = 2x^5 - 8x^3 + 5x^2 + 3x - 5$$
cross the horizontal line $y = -4$ for some x in the interval $(0, 1)$? Justify your answer – specifically, if there is an important theorem you are using then you must name it and show why you can use it in this situation.

f is a polynomial so is continuous everywhere, particularly on $[0,1]$.

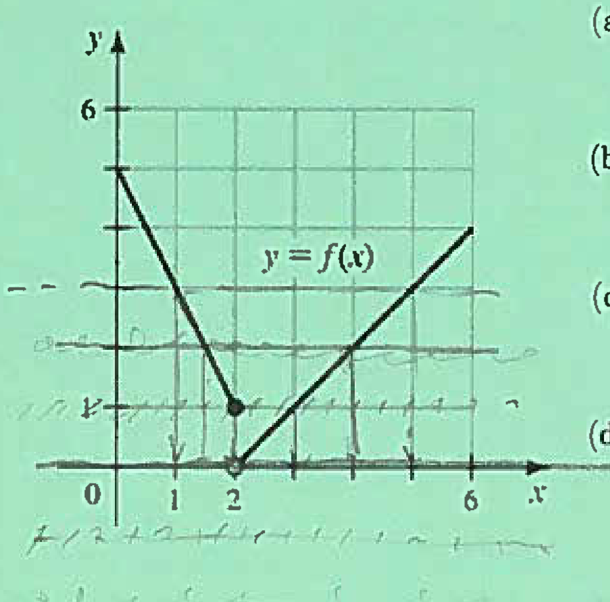
$f(0) = 2(0^5) - 8(0^3) + 5(0^2) + 3(0) - 5 = -5$

$f(1) = 2(1^5) - 8(1^3) + 5(1^2) + 3(1) - 5$

$= 2 - 8 + \cancel{5} + 3 - \cancel{5} = -3$

Since $-3 < -4 < -5$, by the Intermediate Value Theorem there exists c between 0 and 1, where $f(c) = -4$.

4. This problem gives an example where the ϵ-δ technique fails. Using the figure, for each statement, determine the appropriate value of $\delta > 0$. If no such δ exists, say why.



(a) $|f(x) - 1| < 2$ whenever $0 < |x - 2| < \delta$

$\delta = 1$

(b) $|f(x) - 1| < 1$ whenever $0 < |x - 2| < \delta$

$\delta = \frac{1}{2}$

(c) $|f(x) - 0| < 2$ whenever $0 < |x - 2| < \delta$

$\delta = \frac{1}{2}$

(d) $|f(x) - 0| < 1$ whenever $0 < |x - 2| < \delta$

δ does not exist

(b/c $\lim_{x \to 2} f(x)$ DNE)