Section 3.6 – The Chain Rule and Inverse Functions

1. Calculate $\frac{d}{dx}\left(\arcsin(e^{x^2})\right)$.

$$\frac{d}{dx}\left(\sin\left(\arcsin e^{x^2}\right)\right) = 1$$

$$\cos\left(\arcsin e^{x^2}\right) \cdot \frac{d}{dx} \arcsin e^{x^2} \cdot 2xe^{x^2} = 1$$

$$\frac{d}{dx} \arcsin e^{x^2} = \frac{1}{2xe^{x^2} \cos\left(\arcsin e^{x^2}\right)}$$

$$= \frac{1}{2xe^{x^2}\sqrt{1-e^{2x^2}}}$$

2. Calculate $\frac{d}{dx}\left(x \arctan(x^3)\right)$.

$$= \arctan x^3 + x \cdot \frac{1}{1+x^6} \cdot 3x^2$$

$$= \arctan x^3 + \frac{3x^3}{1+x^6}$$

3. Find all points on the curve $f(x) = \ln(x^2 - 4x + 5)$ where the tangent line is horizontal.

$$f'(x) = \frac{2x-4}{x^2-4x+5} = 0$$

$$2x = 4$$

$$x = 2$$

4. Air is being blown into a spherical balloon at a constant rate of 30 cm^3 per second. Find the rate at which the radius of the balloon is increasing when the radius is 1 cm and the rate at which the radius is increasing when the radius is 2 cm.

$V = $ volume

$= \frac{4}{3}\pi r^3$

$r = r(t)$

$$\frac{dV}{dt} = 4\pi r^2 \cdot 2r \cdot \frac{dr}{dt} = 30 \text{ cm}^3/\text{sec}$$

$$\frac{dr}{dt} = \frac{30}{6\pi r^3}$$

$r = 1$ cm

rate is $\frac{30}{6\pi(1)^3} = \frac{30}{6\pi} \doteq 1.592$ cm/sec

$r = 2$ cm

rate is $\frac{30}{6\pi(2)^3} = \frac{30}{48\pi} \doteq 0.199$ cm/sec