Wed 3 June

- Reminder: Do the book problems! If you are having software issues with the text, go to the 2nd floor of SCEN. Either Nick Shapter (sp?) or someone in the testing labs can help you fix it.
- Look at old Wheeler Cal I materials for studying. Do practice problems completely (write your solution exactly the way you would on an exam). Don't study alone. Review slides and Quiz solutions, especially for material you may have missed. Work through slides and problems you didn't understand the first time through (you'll surprise yourself!).

Wed 3 June (cont.)

- Once we exhaust the review slides, unless there are review questions we will start Week 3 topics.
- Exam 1 on Friday
  - covers up to $\oint 3.1$
  - syllabus-approved calculator (though you probably won't need any at all)
  - 50 min. Class will start 30 min late to enforce that timeframe.
- sub for Week 3
  - The sub may or may not have a different teaching style but the slides are posted, regardless.
  - Expect 2 quizzes next week.
  - Stay on top of book problems and MLP assignments.

1 Week 2: 1-5 June
- Wednesday 3 June

Exam #1 Review
- Other Study Tips

∮3.2 Rules of Differentiation
- Constant Functions
- Power Rule
- Constant Multiple Rule
- Sum Rule
- Exponential Functions
- Higher-Order Derivatives
- Book Problems

Exam #1 Review

- $\oint$2.6 Continuity
  - Know the definition of continuity and be able to apply the continuity checklist
  - Be able to determine the continuity of a function (including those with roots) on an interval
  - Be able to apply the Intermediate Value Theorem to a function

Exam #1 Review (cont.)

Example

Determine the value for a that will make $f(x)$ continuous.

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1} & x \neq -1 \\ a & x = -1 \end{cases}$$

Example

Show that $f(x) = 2$ has a solution on the interval $(-1, 1)$, with

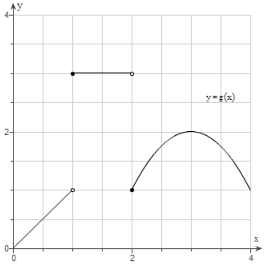
$$f(x) = 2x^3 + x.$$

Exam #1 Review (cont.)

- $\oint$ 2.7 Precise Definition of Limits
  - Understand the δ, ϵ relationship for limits
  - Be able to use a graph or analytical methods to find a value for $\delta > 0$ given an $\epsilon > 0$ (including finding symmetric intervals)

Exam #1 Review (cont.)

Example



Use the graph to find the appropriate δ.

(a) $|g(x) - 2| < \frac{1}{2}$ whenever

$$0 < |x - 3| < \delta$$

(b) $|g(x) - 1| < \frac{3}{2}$ whenever

$$0 < |x - 2| < \delta$$

In this example, the two-sided limits at $x = 1$ and $x = 2$ do not exist.

Exam #1 Review (cont.)

- $\oint$3.1 Introducing the Derivative
  - Know the definition of a derivative and be able to use this definition to calculate the derivative of a given function
  - Be able to determine the equation of a line tangent to the graph of a function at a given point
  - Know the 3 conditions for when a function is not differentiable at a point, and why these three conditions make a function not differentiable at the given point

Exam #1 Review (cont.)

Example

(a) Use the limit definition of the derivative to find an equation for the line tangent to $f(x)$ at a, where

$$f(x) = \frac{1}{x}; \quad a = -5.$$

(b) Using the same $f(x)$ from part (a), find a formula for $f'(x)$ (using the limit definition).

(c) Plug -5 into your answer for (b) and make sure it matches your answer for (a).

Other Study Tips

- Brush up on algebra, especially radicals.
- If your answer is something like $\sqrt{2}$, don't plug that into your calculator, just leave it as is.
- When in doubt, show steps. Defer to class notes and old exams to get an idea of what's expected.
- You will be punished for wrong notation; e.g., the limit symbol.
- Read the question! Several students always lose points because they didn't answer the question or they didn't follow directions.
- Do the book problems.
- Look at the pictures in the book and the interactive applets on MLP.

1 Week 2: 1-5 June
- Wednesday 3 June

Exam #1 Review
- Other Study Tips

∮3.2 Rules of Differentiation
- Constant Functions
- Power Rule
- Constant Multiple Rule
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$\oint$ 3.2 Rules of Differentiation

Recall the definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(as a function of x, i.e., a formula).
And, for any particular point a, we have

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Constant Functions

The constant function $f(x) = c$ is a horizontal line with a slope of 0 at every point. This is consistent with the definition of the derivative:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{c - c}{h} \\ &= \lim_{h \to 0} 0 = 0. \end{aligned}$$

Therefore, for constant functions, $\frac{d}{dx} c = 0$.

Fact: For any positive integer n, we can factor

$$x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n-1}).$$

For example, when $n = 2$, we get

$$x^2 - a^2 = (x-a)(x+a),$$

which is the difference of squares formula.

Power Rule, cont.

Suppose $f(x) = x^n$ where n is a positive integer. Then at a point a,

$$
\begin{aligned}
f'(a) &= \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{x^n - a^n}{x-a} \\
&= \lim_{x\to a} \frac{(x-a)(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n-1})}{x-a} \\
&= (a^{n-1} + a^{n-2}\cdot a + \cdots + a\cdot a^{n-2} + a^{n-1}) = na^{n-1}.
\end{aligned}
$$

Using the formula for the derivative as a function of x, one can show $\frac{d}{dx}(x^n) = nx^{n-1}$.

Constant Multiple Rule

Consider a function of the form $cf(x)$, where c is a constant. Just like with limits, we can factor out the constant:

$$\begin{aligned}
\frac{d}{dx}[cf(x)] &= \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h} \\
&= \lim_{h\to 0} \frac{c[f(x+h) - f(x)]}{h} = c\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= cf'(x)
\end{aligned}$$

Therefore, $\frac{d}{dx}[cf(x)] = cf'(x)$.

Sum Rule

Sums of functions also behave under the same limit laws when we differentiate:

$$
\begin{aligned}
\frac{d}{dx}[f(x)+g(x)] &= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\left[\frac{[f(x+h)-f(x)]}{h}+\frac{[g(x+h)-g(x)]}{h}\right] \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= f'(x)+g'(x)
\end{aligned}
$$

So if f and g are differentiable at x,

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x).$$

The Sum Rule can be generalized for more than two functions to include n functions.

Note: Using the Sum Rule and the Constant Multiple Rule produces the Difference Rule:

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x).$$

Exercise

Using the differentiation rules we have discussed, calculate the derivatives of the following functions. Note which rule(s) you are using.

1. $y = x^5$
2. $y = 4x^3 - 2x^2$
3. $y = -1500$
4. $y = 3x^3 - 2x + 4$

Let $f(x) = b^x$, where $b > 0$, $b \neq 1$. To differentiate at 0, we write

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{b^x - b^0}{x} = \lim_{x \to 0} \frac{b^x - 1}{x}.$$

It is not obvious what this limit should be. However, consider the cases $b = 2$ and $b = 3$. By constructing a table of values, we can see that

$$\lim_{x \to 0} \frac{2^x - 1}{x} \approx 0.693 \quad \text{and} \quad \lim_{x \to 0} \frac{3^x - 1}{x} \approx 1.099.$$

So, $f'(0) < 1$ when $b = 2$ and $f'(0) > 1$ when $b = 3$. As it turns out, there is a particular number b, with $2 < b < 3$, whose graph has a tangent line with slope 1 at $x = 0$. In other words, such a number b has the property that

$$\lim_{x \to 0} \frac{b^x - 1}{x} = 1.$$

Question

What number is it?

Answer: This number is $e = 2.718281828459\ldots$ (known as the Euler number). The function $f(x) = e^x$ is called the **natural exponential function**.

Now, using $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, we can find the formula for $\frac{d}{dx}(e^x)$:

$$
\begin{aligned}
\frac{d}{dx}(e^x) &= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} \\
&= e^x \left(\lim_{h\to 0} \frac{e^h - 1}{h} \right) \\
&= e^x \cdot 1 = e^x
\end{aligned}
$$

Exercise

(a) Find the slope of the line tangent to the curve $f(x) = x^3 - 4x - 4$ at the point $(2, -4)$.

(b) Where does this curve have a horizontal tangent?

If we can write the derivative of f as a function of x, then we can take *its* derivative, too. The derivative of the derivative is called the **second derivative** of f, and is denoted f''.

In general, we can differentiate f as often as needed. If we do it n times, the nth derivative of f is

$$f^{(n)}(x) = \frac{d^n f}{dx^n} = \frac{d}{dx}[f^{(n-1)}(x)].$$

3.2 Book Problems

3-45 (x3)

- For these problems, use only the rules we have derived so far.