

Take-Home Quiz 1: Vectors and vector-valued functions (§11.1-11.7)

Directions: This quiz is due on February 3, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

1. A sum of scalar multiples of two or more vectors (such as $c_1\mathbf{u} + c_2\mathbf{v} + c_3\mathbf{w}$, where c_i are scalars) is called a **linear combination** of the vectors. Express $\langle 4, -8 \rangle$ as a linear combination of the vectors $\mathbf{u} = \langle 1, 1 \rangle$ and $\mathbf{v} = \langle -1, 1 \rangle$.

2. Give a geometric description of the following set of points:

$$x^2 + y^2 + z^2 - 8x + 14y - 18z \geq 65$$

3. Give a geometric description of the set of points (x, y, z) that lie on the intersection of the sphere $x^2 + y^2 + z^2 = 36$ and the plane $z = 6$.

4. Carry out the following steps to determine the (least) distance between the point $P = (0, 2, 6)$ and the line ℓ that is parallel to the $\langle 3, 0, -4 \rangle$ and passes through the origin.

(a) Find any vector \mathbf{v} in the direction of ℓ .

(b) Find the position vector corresponding to P .

(c) Find $\text{proj}_{\mathbf{v}} \mathbf{u}$.

(d) Show that $\mathbf{w} = \mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u}$ is a vector orthogonal to \mathbf{v} whose length is the distance between P and the line ℓ .

(e) Find \mathbf{w} and $|\mathbf{w}|$. Why is $|\mathbf{w}|$ the least distance between P and ℓ ?

5. A particle with a unit negative charge ($q = -1$) enters a constant magnetic field $\mathbf{B} = 5\mathbf{k}$ with a velocity $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$. Find the magnitude and direction of the force on the particle. Make a sketch of the magnetic field, the velocity, and the force.

6. Find the area of the triangle with vertices $O = (0, 0, 0)$, $P = (2, 4, 6)$, and $Q = (6, 5, 4)$.

7. Consider the lines

$$\mathbf{r}(t) = \langle 2 + 2t, 8 + t, 10 + 3t \rangle \text{ and}$$

$$\mathbf{R}(s) = \langle 6 + s, 10 - 2s, 16 - s \rangle.$$

(a) Determine whether the lines intersect (have a common point) and if so, find the coordinates of the point.

(b) If \mathbf{r} and \mathbf{R} describe the paths of the two particles, do the particles collide? Assume that $t \geq 0$ and $s \geq 0$ measure time in seconds, and that motion starts at $s = t = 0$.

8. Find the unit tangent vector at $t = 0$ for

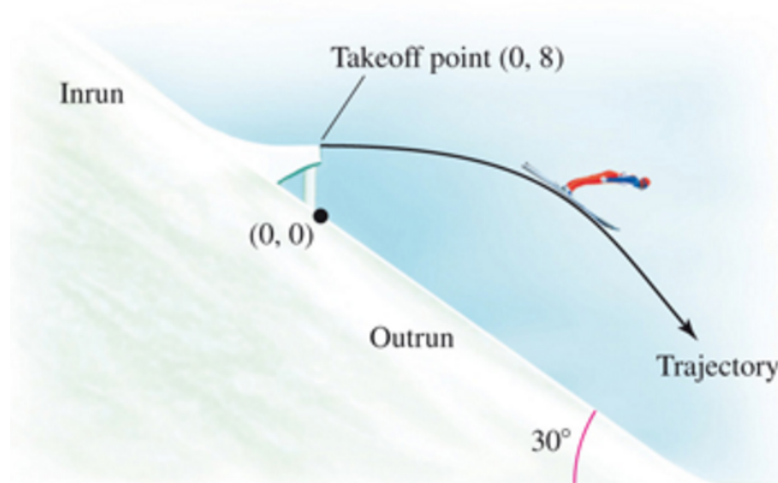
$$\mathbf{r}(t) = \langle \sin t, \cos t, e^{-t} \rangle, \text{ for } 0 \leq t \leq \pi.$$

9. Consider the curve

$$\mathbf{r}(t) = \langle \sqrt{t}, 1, t \rangle,$$

for $t > 0$. Find all points on the curve at which \mathbf{r} and \mathbf{r}' are orthogonal.

10. The lip of a ski jump is 8 m above the outrun that is sloped at an angle of 30 degrees to the horizontal (see figure).



- (a) If the initial velocity of a ski jumper at the lip of the jump is $\langle 40, 0 \rangle$ m/s, what is the length of the jump (distance from the origin to the landing point)? Assume only gravity affects the motion.
- (b) Assume that air resistance produces a constant horizontal acceleration of 0.15 m/s^2 opposing the motion. What is the length of the jump?
- (c) Suppose that the takeoff ramp is tilted upward at an angle of θ , so that the skier's initial velocity is $40\langle \cos \theta, \sin \theta \rangle$ m/s. What value of θ maximizes the length of the jump? Express your answer in degrees and neglect air resistance.