

## Quiz 3: Formal Definitions of Continuity, Limits, and Derivatives (§2.6-3.1)

**Directions:** You have 30 minutes to complete this quiz. This quiz is closed book but you may collaborate with each other.

1. Find the numbers  $a$  and  $b$  that will make  $f$  continuous for all  $x$ . (*Reminder:* In order to claim something is continuous at a point you must use the Continuity Checklist.)

$$f(x) = \begin{cases} 2x + a & x \leq 0 \\ x^2 + 1 & 0 < x \leq 2 \\ bx - 2 & x > 2 \end{cases}$$

2. You may use whichever (limit) definition of the derivative you prefer for the following questions (you are not allowed to use any derivative shortcuts yet). Given  $f(x) = x^2 + 3$ ,

(a) find  $f'(1)$ ;

(b) find a formula for  $f'(x)$ ;

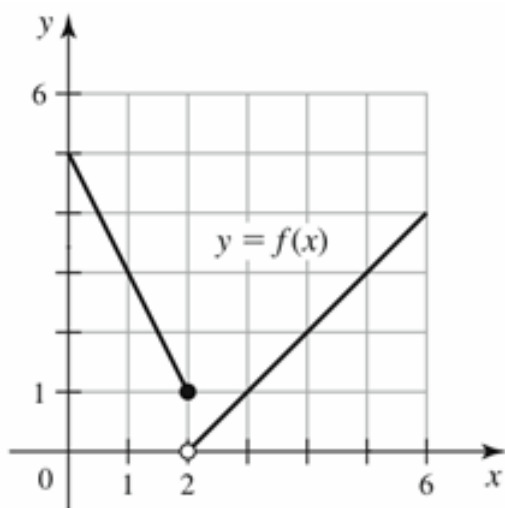
(c) use your answer to (a) to write the equation of the line tangent to  $f(x)$  at  $x = 1$ .

3. Does the function

$$f(x) = 2x^5 - 8x^3 + 5x^2 + 3x - 5$$

cross the horizontal line  $y = -4$  for some  $x$  in the interval  $[0, 1]$ ? Justify your answer – specifically, if there is an important theorem you are using then you must name it and show why you can use it in this situation.

4. This problem gives an example where the  $\epsilon$ - $\delta$  technique fails. Using the figure, for each statement, determine the appropriate value of  $\delta > 0$ . If no such  $\delta$  exists, say why.



(a)  $|f(x) - 1| < 2$  whenever  $0 < |x - 2| < \delta$

(b)  $|f(x) - 1| < 1$  whenever  $0 < |x - 2| < \delta$

(c)  $|f(x) - 0| < 2$  whenever  $0 < |x - 2| < \delta$

(d)  $|f(x) - 0| < 1$  whenever  $0 < |x - 2| < \delta$