UNIT 3, LESSON 4

Optimization

- Identify the objective function.
- Identify constraints
- Solve applied problems involving maximizing or minimizing a function.

Solving an Applied Extrema Problem

1. Read the problem carefully. Make sure you understand what is given and what is unknown.
2. If possible, sketch a diagram. Label the various parts.
3. Decide on the variable that must be maximized or minimized. Express that variable as a function of *one* other variable.
4. Find the domain of the function.
5. Find the critical points for the function from Step 3.
6. If the domain is a closed interval, evaluate the function at the endpoints and at each critical number to see which yields the absolute maximum or minimum. If the domain is an open interval, apply the critical point theorem when there is only one critical number. If there is more than one critical number, evaluate the function at the critical numbers and find the limit as the endpoints of the interval are approached to determine if an absolute maximum or minimum exists at one of the critical points.

Example: Find two nonnegative numbers x and y for which $2x + y = 30$ and xy^2 is maximized.

Find two nonnegative numbers x and y for which $x + 3y = 30$ such that x^2y is maximized.

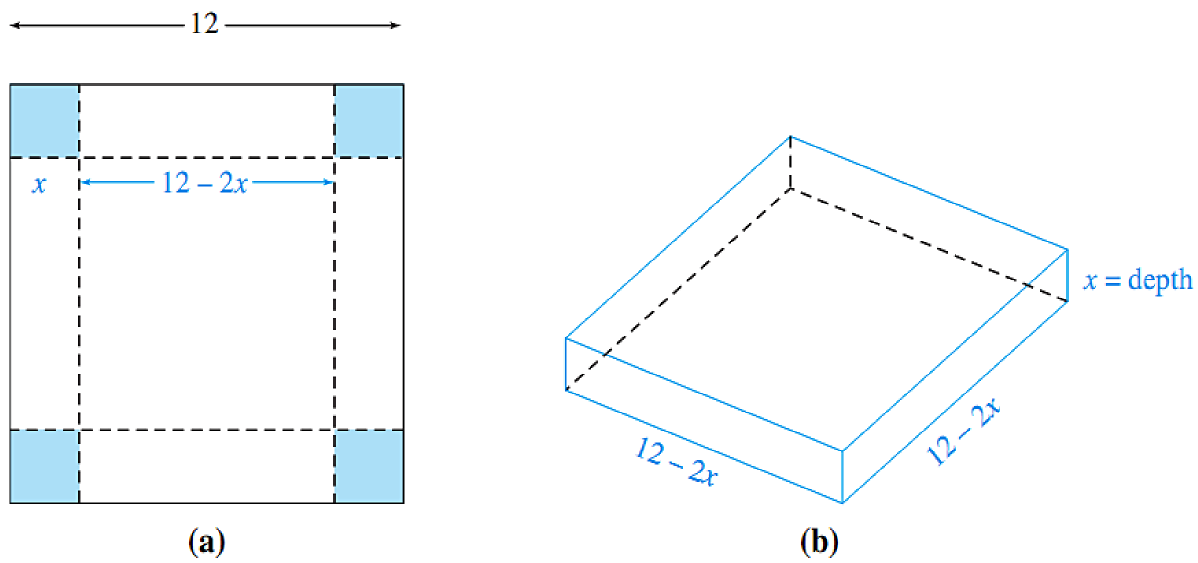
A. $x = 20, y = \frac{10}{3}$

B. $x = 10, y = \frac{20}{3}$

C. $x = \frac{5}{3},\ y = 20$

D. $x = 10, y = \frac{5}{3}$

An open box is to be made by cutting a square from each corner of a $12in\ by\ 12in$ piece of metal and then folding up the sides. What size square should be cut from each corner to produce a box of maximum volume?



Suppose you are constructing an open-top rectangular box with a square base and a volume of $32in^3$. What dimensions of the box will maximize the surface area?

A carpenter is building a rectangular shed with a fixed perimeter of 52 feet. What are the dimensions of the largest shed that can be built?

A. 12ft x 20ft
B. 13ft x 26ft
C. 12ft x 13ft
D. 13ft x 13ft

A fence must be built to enclose an area of 20,000 ft^2. Fencing costs \$1 per foot for the two sides facing North and South and \$2 per foot for the sides facing East and West. Find the cost of the least expensive fence.

A. $\$\pi$
B. \$800
C. \$100
D. \$200

The llama population of a certain area can be modeled using the function $L(t) = 7te^{\frac{-t}{13}}$ where t is the number of years after 2015 and $L(t)$ is hundreds of llamas. In what year will the area be populated by the most llamas?

A. 2015

B. 2013

C. 2030

D. 2028