MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Fri 29 May 2015

Quiz 2: Evaluating Limits Analytically (∮2.3-2.5)

Directions: You have 30 minutes to complete this quiz. Work individually.

1. Evaluate the following limits, analytically.

(a) $\lim_{h\to 5} \frac{5h^2-6h+1}{\sqrt{16+4h}+2}$ $= \frac{5(5^2)-6(5)+1}{\sqrt{16+4(5)}+2}$

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$$= \frac{125-30+1}{\sqrt{36}+2} = \frac{96}{8} = 12$$

(b) $\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-3}\left(\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right)$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{x+6-9}} = \sqrt{3+6}+3 = 6$$

(c) $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$ $= 0$

Squeeze Theorem!

For all $x \neq 0$,

$$-1 \leq \cos\frac{1}{x} \leq 1 \Rightarrow -|x| \leq |x|\cos\left(\frac{1}{x}\right) \leq |x|$$

$x\cos\frac{1}{x}$ is always bounded between $\pm x$, when $x \neq 0$. $\lim_{x\to 0} \pm x = 0 \longrightarrow \lim_{x\to 0} x\cos\frac{1}{x} = 0$

2. Where applicable, your justification must involve determining the sign of the numerator and denominator for x-values sufficiently close to 0.

$$h(x) = \begin{cases} \dfrac{x-5}{x} & x < 0 \\ \pi & x \geq 0 \end{cases}$$

Evaluate:

(a) $\lim_{x\to 0^+} h(x) = \lim_{x \to 0^+} \pi = \pi$

(b) $\lim_{x\to 0^-} h(x) = \lim_{x \to 0^-} \dfrac{\overbrace{x-5}^{\text{approaches } -5}}{\underbrace{x}_{\text{approaches } 0\text{, neg}}} = \infty$

(c) $\lim_{x\to 0} h(x)$ DNE, because the one-sided limits are not equal

(d) $h(0) = \pi$

(e) Does h have a vertical asymptote at the line $x = 0$? Explain why or why not.

Yes. Even though $h(0)$ is defined, it is enough that $\lim_{x \to 0^-} h(x) = \infty$, in order to have a vertical asymptote.

3. Use the given information to compute the following limits. Show which limit laws you are using and why you are allowed to use them.

$$\lim_{x\to 1} f(x) = 8 \qquad \lim_{x\to 1} g(x) = 3 \qquad \lim_{x\to 1} h(x) = 2$$

(a) $\lim_{x\to 1}(4+3f(x)) = \lim_{x\to 1} 4 + 3\lim_{x\to 1} f(x)$

$= 4 + 3(8) = 28$

(b) $\lim_{x\to 1} \frac{f(x)g(x)}{h(x)^2} = \frac{\left(\lim_{x\to 1} f(x)\right)\left(\lim_{x\to 1} g(x)\right)}{\left(\lim_{x\to 1} h(x)\right)^2}$ ← OK, b/c $\neq 0$

$= \frac{8(3)}{2^2} = 6$

4. Determine the end behavior for $f(x) = \frac{x^2-4x+2}{x-1}$.

$\lim_{x\to\infty} \frac{x^2-4x+2}{x-1} = \infty$; rational function —

when x gets big $f(x) \approx x$

$\lim_{x\to -\infty} f(x) = -\infty$