U3L5

Multivariable Functions

Suppose a company produces two products. One unit of Product A costs $25 to produce. One unit of Product B costs $12 to produce.

The total cost would be a function of two independent variables.

$$x = \#\ units\ of\ Product\ A$$
$$y = \#units\ of\ Product\ B$$

$$C(x, y) = 25x + 12y$$

$x = \#\, units\ of\ Product\ A$
$y = \#units\ of\ Product\ B$

$$C(x, y) = 25x + 12y$$

Find the total cost when 10 units of Product A and 12 units of Product B are produced.

$$C(10,12) =$$

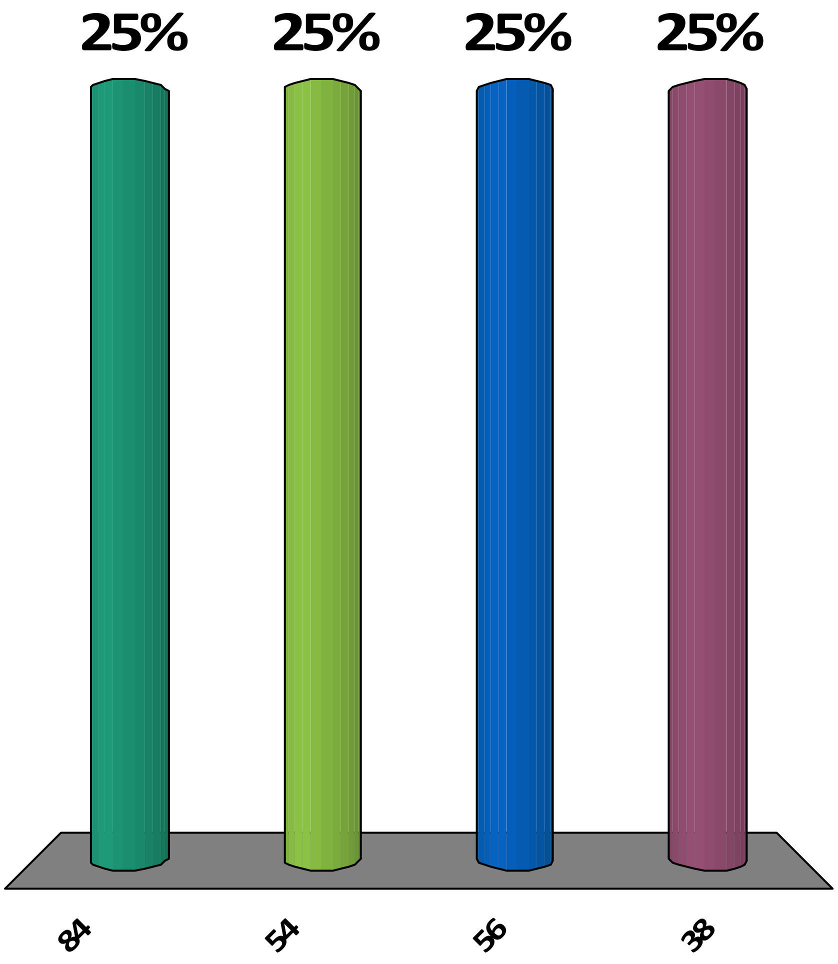
Function of Two or More Variables

The expression $z = f(x, y)$ is a **function of two variables** if a unique value of z is obtained from each ordered pair of real numbers (x, y). The variables x and y are **independent variables**, and z is the **dependent variable**. The set of all ordered pairs of real numbers (x, y) such that $f(x, y)$ exists is the **domain** of f; the set of all values of $f(x, y)$ is the **range**. Similar definitions could be given for functions of three, four, or more independent variables.

Let $f(x, y, z) = \frac{1}{2}x - 3y + z^2$. *Find* $f(6,2,4)$.

Let $\boldsymbol{f(x,y) = x^2 - 2xy + y^3}$.
Find $\boldsymbol{f(-2,4)}$.

A. 84
B. 54
C. 56
D. 38



Partial Derivatives (Informal Definition)

The **partial derivative of *f* with respect to *x*** is the derivative of *f* obtained by treating *x* as a variable and *y* as a constant.

The **partial derivative of *f* with respect to *y*** is the derivative of *f* obtained by treating *y* as a variable and *x* as a constant.

Partial Derivatives (Formal Definition)

Let $z = f(x, y)$ be a function of two independent variables. Let all indicated limits exist. Then the partial derivative of f with respect to x is

$$f_x(x,y) = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h},$$

and the partial derivative of f with respect to y is

$$f_y(x,y) = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}.$$

If the indicated limits do not exist, then the partial derivatives do not exist.

Let $f(x,y) = 2x^2y^3 + 6x^5y^4$. Find $f_x(x,y)$ and $f_y(x,y)$.

Let $g(x, y) = 7x^2y^2 + x^2 + y^2$. Find $g_x(x, y)$ and $g_y(x, y)$.

Let $f(x,y)=e^{3x^2y}$. Find $f_x(x,y)$ and $f_y(x,y)$.

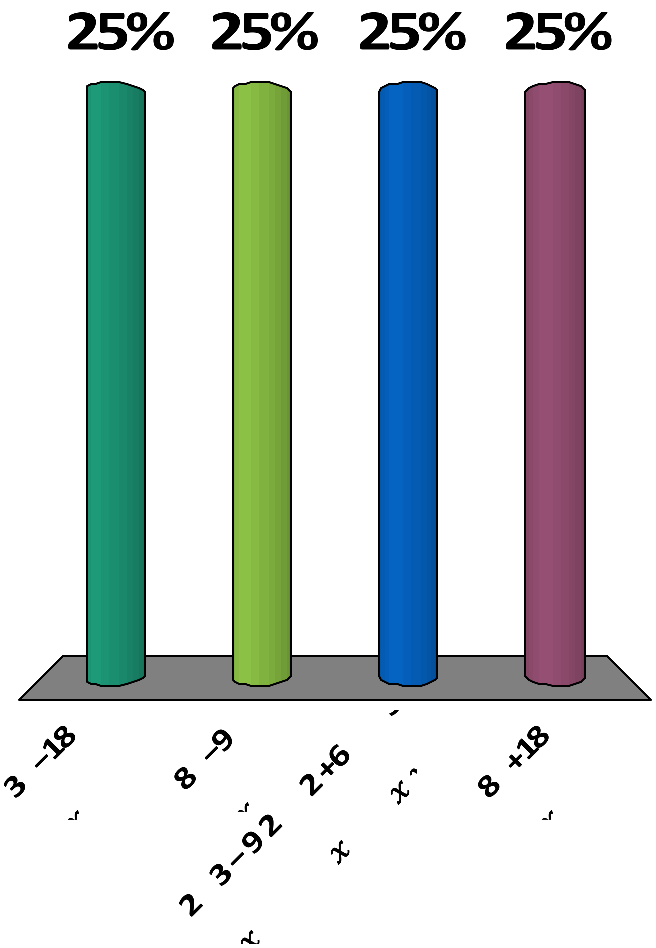
Let $\boldsymbol{f(x,y) = 4x^2 - 9xy + 6y^3}$.
Find $\boldsymbol{f_x(x,y)}$.

A. $3x - 18y$

B. $8x - 9y$

C. $2x^3 - \frac{9}{2}x^2 + 6xy^3$

D. $8x + 18y$



Second-Order Partial Derivatives

For a function $z = f(x, y)$, if the indicated partial derivative exists, then

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = f_{xx}(x, y) = z_{xx} \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} = f_{yy}(x, y) = z_{yy}$$

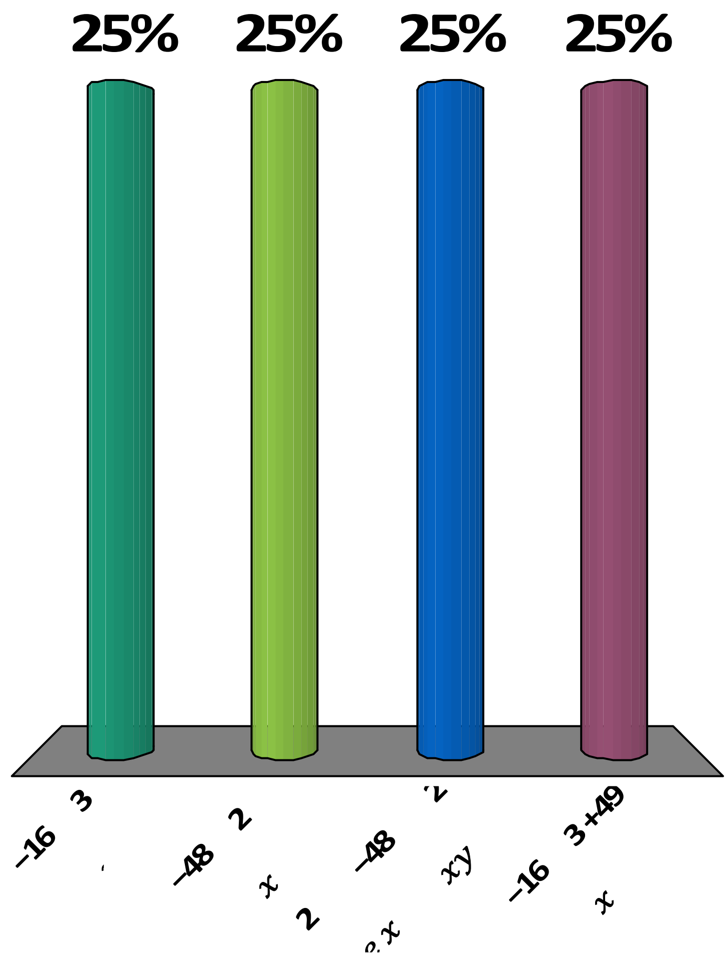
$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y \partial x} = f_{xy}(x, y) = z_{xy} \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y} = f_{yx}(x, y) = z_{yx}$$

$$\textit{Find } f_{xx} \textit{ and } f_{xy}$$

$$f(x, y) = -4x^3 - 3x^2y^3 + 2y^2$$

Let $f(x,y,z) = 2x^2yz^2 + 3xy^2 - 4yz.$
Find $f_{xz}(x,y,z)$ *and* $f_{yz}(x,y,z).$

Let $\boldsymbol{f(x,y) = 2e^{x} - 8x^{3}y^{2}}$.
Find $\boldsymbol{f_{xx}(x,y)}$.



A. $-16x^3$

B. $-48x^2y$

C. $2e^x - 48xy^2$

D. $-16x^3 + 49$

E. $2e^x$

A company that manufactures computers has determined that its production function is given by

$$\boldsymbol{P(x,y) = 0.1xy^2\, ln(2x+3y+2)},$$

where x is the size of the labor force (measured in work-hours per week) and y is the amount of capital (measured in units of $1000) invested. Find the marginal productivity of labor when x=50 and y=20.

Let $p(x, y) = 8x^2 - 16xy + 3y^2 - 32x + 52y - 4$. Find all (x, y) such that $p_x(x, y)$ and $p_y(x, y) = 0$.