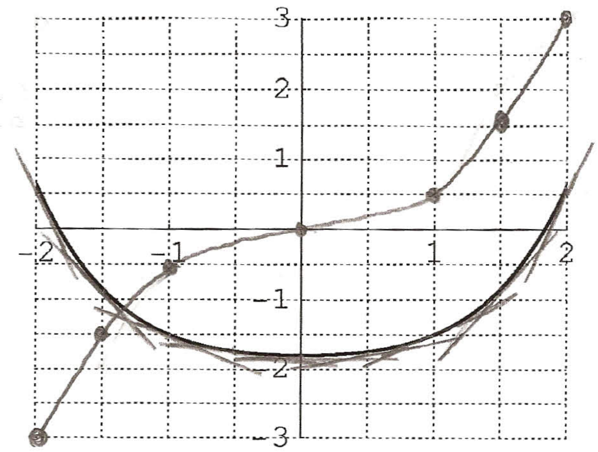
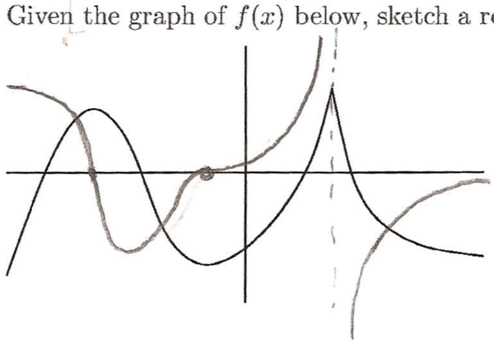
Section 2.3 – The Derivative Function

1. To the right you are given the graph of a function $f(x)$. With a straightedge, draw in tangent lines to $f(x)$ at every half unit and estimate their slope from the grid. Use your slopes to draw a graph of $f'(x)$ on the same set of axes.

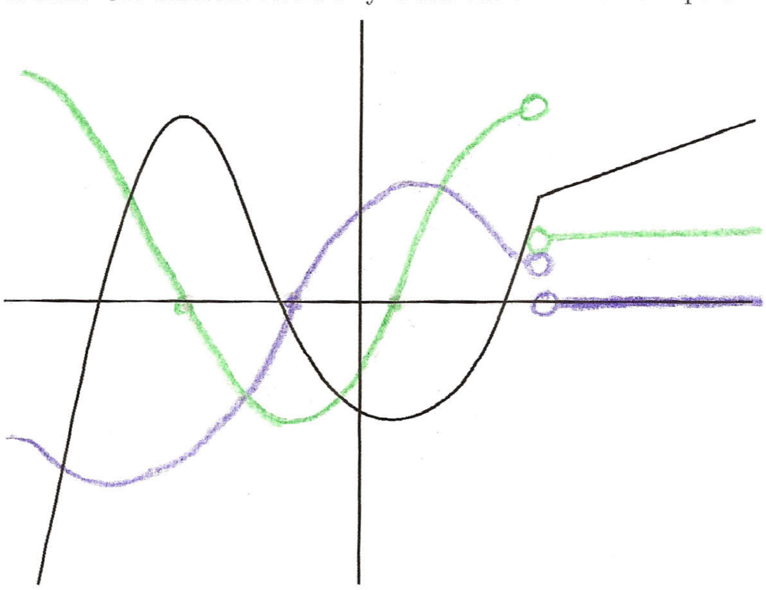


2. Given the graph of $f(x)$ below, sketch a rough graph of $f'(x)$.

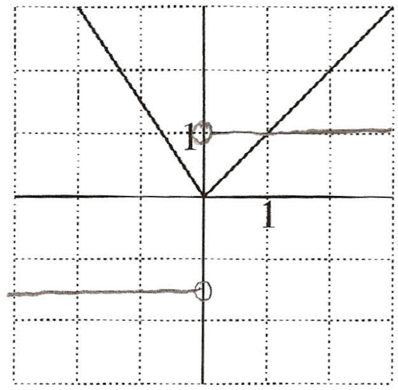


Notes: $f'(x) = 0$ (i.e. $f'(x)$ crosses the x-axis) when slope is zero.
$f'(x) > 0$ (i.e. $f'(x)$ is above the x-axis) when slope is positive.
$f'(x) < 0$ (i.e. $f'(x)$ is below the x-axis) when slope is negative.
If f has a cusp (sharp point) at x, then f' is undefined there.

3. Given below is the graph of a function $y = f(x)$. Sketch a rough graph of $f'(x)$ and of $f''(x)$ on the same set of axes. Use different colors so you can tell the functions apart.



4. Given the graph of $f(x)$ below, sketch an accurate graph of $f'(x)$.



5. Use algebra (that is, use the **limit definition** of the derivative) to find a formula for $f'(x)$ for each of the following functions.

(a) $f(x) = \frac{2}{x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h\to 0} \frac{2x - 2(x+h)}{x(x+h)h}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{x(x+h)\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-2}{x^2+xh} = \frac{-2}{\lim\limits_{h\to 0}(x^2+xh)} = \frac{-2}{x^2+x\left(\lim\limits_{h\to 0} h\right)} = \frac{-2}{x^2}$$

(b) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{\left(\lim\limits_{h\to 0} x+h\right)}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x+\lim\limits_{h\to 0} h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$