

## **Bonus Review**

**Worth up to 2% of your final grade.**

Due May 4, 2015 at 6:00 pm. You may use your book, notes, brain, and any human of your choice, but remember you take the final exam alone! Work **MUST** be legible, organized and STAPLED to the review. If not, I will not grade your review, and you will not earn back any points. **ALL OR NOTHING!**

Evaluate 1-8

1.  $\lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{2x^2 - 2x - 12}$

2.  $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$

3.  $\lim_{x \rightarrow -0} \frac{\tan 5x}{4x}$

4.  $\lim_{x \rightarrow -3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

5.  $\lim_{x \rightarrow 5} h(x)$  where  $h(x) = \begin{cases} x^2 - 16 & x \leq 5 \\ 3 + x & x > 5 \end{cases}$

6.  $\lim_{x \rightarrow 10} \frac{x + 10}{x - 10}$

7.  $\lim_{x \rightarrow -10} \frac{x - 10}{x + 10}$

8.  $\lim_{x \rightarrow 0} \frac{\cos x}{x^2}$

9. State the definition of “f is continuous at  $x=c$ .”

10. State the definition of the derivative of a function  $f(x)$ .

11. The tangent line to the graph of  $y = g(x)$  at the point  $(-1, 4)$  passes through the point  $(3, 6)$ . Find  $g(-1)$  and  $g'(-1)$ .

12. Give an example of a function that is continuous everywhere, but not differentiable at  $x=0$ .

13. Find the equation for the line tangent to the graph of  $y = \sqrt{2x+5}$  at the point  $(2,3)$ .

14. Find the equation of the line tangent to  $f(x) = x^3 + 2$  and parallel to  $3x - y = 4$ .

15. Find the point on the graph of  $h(x) = \sqrt{4x-9}$  where the tangent line to  $h$  also passes through the origin. Find the equation of this line.

16. Find  $k$  such that  $y = 4x - 9$  is tangent to  $f(x) = x^2 - kx$ .

Differentiate 17-24

17.  $f(x) = x^3 \cos 2x$

18.  $f(x) = x^4 - \frac{1}{x^4}$

19.  $f(x) = e^{5x} - \frac{1}{5x}$

20.  $f(x) = \frac{x^3}{\ln x}$

21.  $f(x) = \frac{x^2 + 4}{x^4 - x^2}$

22.  $f(x) = \arcsin(\ln 3x)$

23.  $f(x) = \ln\left(\frac{1}{3}(1-x^2)^{3/2}\right)$

24.  $f(x) = x^{-x^2}$

25.  $f(x) = x^8 \cosh(8x)$

25. Find  $\frac{dy}{dx}$  for  $x^4 + e^{3x}y = 2 + y^2$ .

26. Find the rate of change of the distance between the origin and a moving point of the graph of  $y = \sin x$  if  $dx/dt = 2$  centimeters per second.

27. The radius of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the volume when  $r = 6$  inches and  $r = 25$  inches.
28. At a sand plant and gravel plant, sand is falling off a conveyor belt and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?
29. I throw a ripe tomato straight up in the air with an initial velocity of 64 feet per second. (Initial height being my shoulder.) How fast is the tomato going when it hits the ceiling 8 feet above the level of my shoulder?
30. State the Mean Value theorem. Does it apply to the function  $f(x) = \frac{x+1}{x}$  on the interval  $\left[\frac{1}{2}, 2\right]$ ? If so, find the  $c$  values the MVT guarantees.
31. State Rolle's Theorem. Does it apply to the function  $f(x) = x^2 - 5$  on the interval  $[-3, 3]$ ? If so, find the  $c$  values Rolle's Theorem guarantees.
32. Consider the function  $f(x) = 2x^3 - 9x^2 - 108x + 200$
- Find all relative extrema using the 1<sup>st</sup> derivative test.
  - Find all relative extrema using the 2<sup>nd</sup> derivative test.
  - What intervals is the function increasing, decreasing, concave up, concave down?
  - What are the critical numbers and inflection points for  $f$ ?
33. If  $p(x) = x^2 + 2$  is an antiderivative of  $q(x)$ , then what function of  $x$  is  $q(x)$ ?
34. Find all horizontal and vertical asymptotes of  $y = \frac{x^2 + x - 2}{9x - 6 - 3x^2}$ .

Evaluate 35-37

35.  $\lim_{x \rightarrow \infty} \frac{x^2 + 2}{x^3 - 3}$

36.  $\lim_{x \rightarrow -\infty} \frac{3x^3 + x^2 - 2}{5x^3 - x + 6}$

37.  $\lim_{x \rightarrow \infty} \frac{3(x - \cos x)}{x}$

38. Let  $F(x) = \int_0^x e^{-t^2} dt$  (defined for  $x \geq 0$ ).

a. Find  $F(0)$ .

b. Find  $F'(x)$ .

Evaluate 38-44

38.  $\int \frac{1}{2x+1} dx$

39.  $\int \sin 3x \cos 3x dx$

40.  $\int x^2 \sqrt{4x^3 + 8} dx$

41.  $\int \frac{1}{\sqrt{3-25x^2}} dx$

42.  $\int x e^{-x^2} dx$

43.  $\int \frac{6-6x}{x^2-2x+2} dx$

44.  $\int_1^{10} \left( x^2 - \frac{1}{x \ln x^3} \right) dx$

45. Find the average value of  $f(x) = 4 + \frac{2}{\sqrt{x}}$  on the interval  $[1, 4]$ . Where in this interval is the average value attained?

46. If  $\int_a^b f(x) dx = 0$ , what can you conclude about the function  $f$  over the interval  $[a, b]$ ?

47. Solve the initial value problem  $\frac{dy}{dx} = -2e^{6x}$  given that  $y(0) = 2$ .

48. Find the two positive numbers that satisfy:

a. The product is 192 and the sum is a minimum.

b. The second number is the reciprocal of the first and the sum is a minimum.

49. Find the point on the graph of the function  $f(x) = (x + 1)^2$  that is closest to the point  $(2, 0)$ .

50. Determine the dimensions of a rectangular solid ( with a square base) with a maximum volume if its surface area is 337.5 square centimeters.