

Take-Home Quiz 2: Multivariable functions (§12.1-12.2)

Directions: This quiz is due on February 8, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another’s work is unacceptable, regardless of whether you document it.

1. There is always more than one way to parametrize a given curve. However, there is one “correct” way, and that is by the curve’s arc length.
- (a) Given a parametrized curve $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$, the formula for its length from $t = a$ to $t = b$ is given by

$$s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt.$$

For each of the following parametrizations of the unit circle, find the arc length.

- i. $\mathbf{r}_1(t) = \langle \cos t, \sin t \rangle$, $0 \leq t \leq 2\pi$.
ii. $\mathbf{r}_2(t) = \langle \cos 2t, \sin 2t \rangle$, $0 \leq t \leq \pi$.
- (b) In part (a), \mathbf{r}_1 gives the correct parametrization, in the sense that the length of the curve drawn is equal to the length of time (t) elapsed. We say that \mathbf{r}_1 is **parametrized by arc length**. In general, the arc length of a curve $\mathbf{r}(t)$ is a function of time $t \geq a$, given by

$$s(t) = \int_a^t \sqrt{x'(u)^2 + y'(u)^2 + z'(u)^2} du.$$

Find $s(t)$ for each of the parametrizations in part (a).

- (c) What is $\frac{ds}{dt}$? (Hint: Use the Fundamental Theorem of Calculus.)
(d) Give a condition on $\mathbf{r}'(t)$ that guarantees $\mathbf{r}(t)$ is parametrized by arc length.
2. Consider the following plane and curve:

$$y = 2x + 1$$
$$\mathbf{r}(t) = \langle 10 \cos t, 2 \sin t, 1 \rangle, \text{ for } 0 \leq t \leq 2\pi$$

Find the point(s) where the plane and curve intersect, if any exist.

3. Find an equation of the plane passing through $(0, -2, 4)$ that is orthogonal to the planes $2x + 5y - 3z = 0$ and $-x + 5y + 2z = 8$.
4. For each of the following quadric surfaces,
- Name the surface (see Table 12.1 in the text).
 - Find the intercepts with the three coordinate axes, when they exist.
 - Find the equations of the xy -, zx -, and yz -traces, when they exist.
 - Sketch a graph of the surface.

(a) $4y^2 + z^2 = x^2$

(b) $y = \frac{x^2}{16} - 4z^2$

(c) $z = \frac{x^2}{4} + \frac{y^2}{9}$

5. Suppose you make a one-time deposit of P dollars into a savings account that earns interest at an annual rate of $p\%$ compounded continuously. The balance in the account after t years is

$$B(P, r, t) = Pe^{rt}, \text{ where } r = \frac{p}{100}$$

(so for example, if the annual interest rate is 4%, then $r = 0.04$). Let the interest rate be fixed at $r = 0.04$.

- (a) Find the set of all points (P, t) that satisfy $B = 2000$. This curve gives all deposits P and times t that result in a balance of \$2000.
- (b) Repeat part (a) with $B = 500, 1000, 1500, 2500$. Draw the resulting level curves of the balance function.
- (c) In general, on one level curve, if t increases, then does P increase or decrease?