Survey of Calculus Unit I
Practice Problems

F2016
-SOLUTIONS-

1. $11x^3+31x^2-6x = x(11x^2+31x-6)$

$$= \boxed{x(11x-2)(x+3)}$$

Check: $x(11x-2)(x+3)$

$x(11x^2-2x+33x-6)$

$=11x^3+31x^2-6x$

2. $\sqrt{x}\,x^2 = x^{1/2}\cdot x^2 = x^{\frac{1}{2}+2} = \boxed{x^{5/2}}$

3. $\left(\frac{x^3y}{z^7}\right)^{-2} = \frac{x^{-6}y^{-2}}{z^{-8}} = \boxed{\frac{z^8}{x^6y^2}}$

4. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2(x+h)^2+3-(2x^2+3)}{h}$ ($x^2+2xh+h^2$)

$$= \lim_{h\to 0}\frac{2x^2+4xh+h^2-2x^2}{h}$$

$$= \lim_{h\to 0}\frac{4xh+h^2}{h} = \lim_{h\to 0}(4x+h)$$

$$= 4x+0 = \boxed{4x}$$

→

5. $\boxed{f(x) = \frac{1}{x-3}}$

OR any $f(x)$ where any of the following hold:

- f is not defined at $x=3$
- $\lim_{x\to 3} f(x)$ does not exist
- $f(3) \neq \lim_{x\to 3} f(x)$

(Note, these are exactly the conditions on the Continuity Checklist.)

6. a. $\lim_{x\to 2} f(x) = 3$

b. $\lim_{x\to 0^-} f(x) = -1$

c. $\lim_{x\to 0^+} f(x) = 1$

d. Yes, because $f(2) = 3 = \lim_{x\to 2} f(x)$, by part a.

e. No, because $\lim_{x\to 0^-} f(x) = -1 \neq 1 = \lim_{x\to 0^+} f(x)$,

and so $\lim_{x\to 0} f(x)$ does not exist.

7. $\dfrac{f(5)-f(2)}{5-2} = \dfrac{3(5)^2-5-\overbrace{\left(3(2)^2-2\right)}^{19}}{3}$

$= \dfrac{75-5-10}{3} = \dfrac{60}{3} = \boxed{20}$

8. $\lim\limits_{x\to 2} \dfrac{f(x)-f(2)}{x-2} = \lim\limits_{x\to 2} \dfrac{3x^2-x-\overbrace{\left(3(2)^2-2\right)}^{10}}{x-2}$

$= \lim\limits_{x\to 2} \dfrac{3x^2-x-10}{x-2}$ $(3x+5)(x-2)$ $= 3x^2+5x-6x-10$ ✓

$= \lim\limits_{x\to 2} \dfrac{(3x+5)(x-2)}{x-2}$

$= \lim\limits_{x\to 2} (3x+5) = 3(2)+5$

$= \boxed{11}$

OR

$\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} \dfrac{\overbrace{3(2+h)^2}^{4+4h+h^2}-(2+h)-10}{h}$

$= \lim\limits_{h\to 0} \dfrac{12+12h+3h^2-2-h-10}{h}$

$= \lim\limits_{h\to 0} \dfrac{11h+3h^2}{h} = \lim\limits_{h\to 0}(11+3h)$

$= 11+3(0) = \boxed{11}$