MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Fri 10 July 2015

# Quiz 11: Antiderivatives ($\oint$4.8, 5.2, 5.3)

**Directions:** You have 30 minutes to complete this quiz. Collaborative and open book.

1. A mass oscillates up and down on the end of a spring. Find its position $s$ relative to the equilibrium position if its acceleration is

$$a(t) = \sin(\pi t),$$

and its initial velocity and position are $v(0) = 3$ and $s(0) = 0$, respectively.

$$v(t) = \int a(t)\,dt = \int \sin(\pi t)\,dt$$

$$= -\frac{1}{\pi}\cos(\pi t) + C$$

$$v(0) = -\frac{1}{\pi}\cos(\pi(0)) + C = 3$$

$$\Rightarrow C = 3 + \frac{1}{\pi}$$

$$s(t) = \int v(t)\,dt = \int\left(-\frac{1}{\pi}\cos\pi t + 3 + \frac{1}{\pi}\right)dt$$

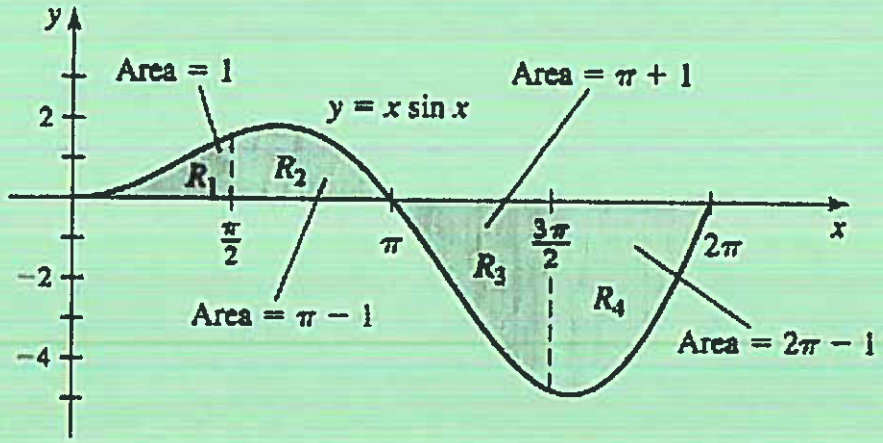
$$= -\frac{1}{\pi}\left(\frac{1}{\pi}\sin(\pi t)\right) + \left(3 + \frac{1}{\pi}\right)t + C$$

$$s(0) = -\frac{1}{\pi^2}\sin(\pi(0)) + \left(3 + \frac{1}{\pi}\right)(0) + C = 0$$

$$\Rightarrow C = 0$$

$$\Rightarrow \boxed{s(t) = -\frac{1}{\pi^2}\sin(\pi t) + \left(3 + \frac{1}{\pi}\right)t}$$

2. Use the picture to evaluate:




(a) $\int_0^{\frac{3\pi}{2}} x \sin x \, dx = 1 + (\pi - 1) - (\pi + 1)$

$= 1 + \pi - 1 - \pi - 1$

$= -1$

(b) $\int_{\frac{\pi}{2}}^{2\pi} x \sin x \, dx = \pi - 1 - (\pi + 1) - (2\pi - 1)$

$= \pi - 1 - \pi - 1 - 2\pi + 1$ $= -1 - 2\pi$

3. Use the Fundamental Theorem of Calculus to simplify the following expressions:

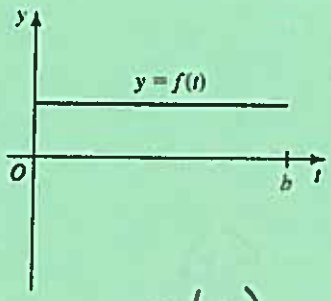
(a) $\frac{d}{dy} \int_{y^2}^{100} \frac{dw}{w^2+1} = -\frac{d}{dy} \int_{100}^{y^2} \frac{dw}{w^2+1}$

$= -\frac{1}{(y^2)^2+1} \cdot 2y$ $= \frac{-2y}{y^4+1}$

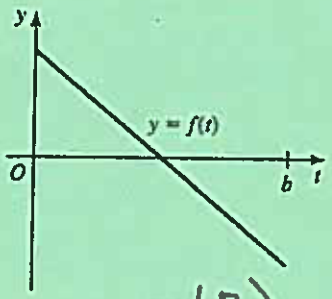
(b) $\frac{d}{dx} \int_1^x e^t \, dt$

$= e^x$

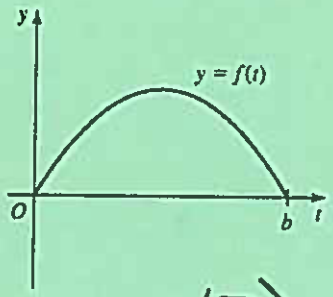
4. Match the functions $f$ whose graphs are given in (a)-(d) with the area functions $A(x) = \int_0^x f(t)\,dt$, whose graphs are given in (A)-(D).



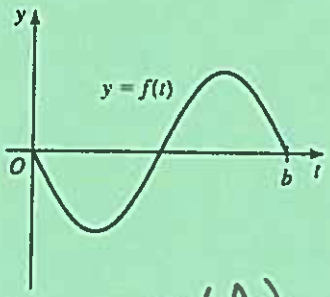

(a) (C)



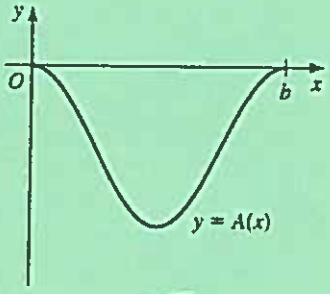

(b) (B)



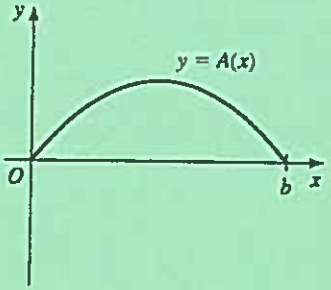

(c) (D)



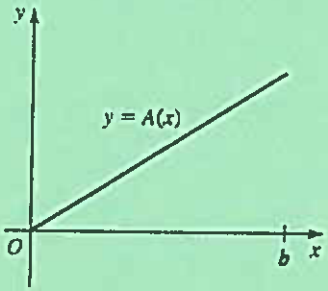

(d) (A)



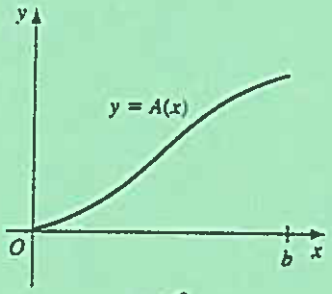

~~(A)~~




~~(B)~~




~~(C)~~




~~(D)~~