Find all critical points for $f(x, y) = 4x^3 + 3xy + 4y^3$.

$f_x(x,y) = 12x^2 + 3y = 0 \Rightarrow y = -\frac{12x^2}{3} = -4x^2$

$f_y(x,y) = 3x + 12y^2 = 0$

$\Rightarrow 3x + 12\underbrace{(-4x^2)^2}_{16x^4} = 0$

$3x(\underbrace{1 + 64x^3}_{x^3 = \frac{-1}{64}}) = 0$

$x = 0, \frac{-1}{4}$

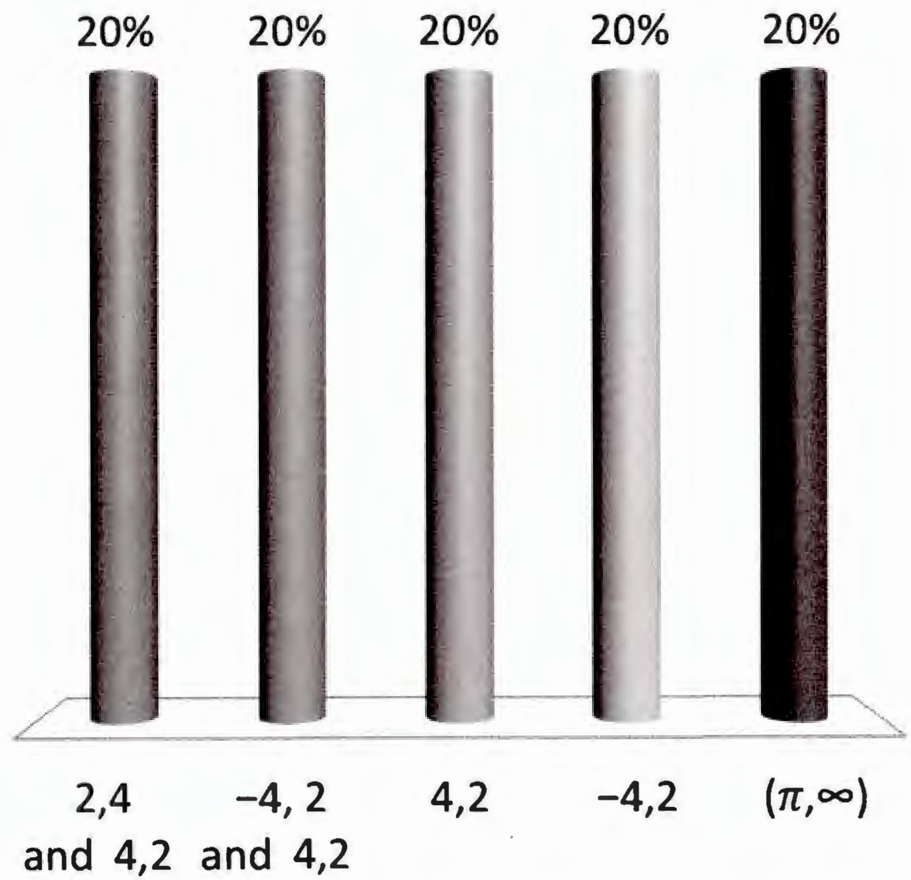
When $x = 0$, $y = -4(0)^2 = 0$

When $x = \frac{-1}{4}$, $y = -4\left(\frac{-1}{4}\right)^2 = -4\left(\frac{1}{16}\right) = \frac{-1}{4}$

$$\boxed{\text{CPs: } (x,y) = (0,0) \text{ and } \left(\frac{-1}{4}, \frac{-1}{4}\right)}$$

Find all critical points for $f(x,y) = 6x^2 + 6y^2 + 6xy + 36x - 5$

A. (2,4) and (4,2)
B. $(-4,2)$ and (4,2)
C. (4,2)
D. $(-4,2)$
E. (π,∞)



$f_x(x,y) = 12x + 6y + 36 = 0$

$f_y(x,y) = 12y + 6x = 0$

$\Rightarrow x = -\frac{12y}{6} = -2y$

$12(-2y) + 6y + 36 = 0$

$-24y + 6y = -36$

$y = \frac{-36}{-18} = 2$

$\Rightarrow x = -2(2) = -4$

Find all points where the function
$f(x,y) = 9xy - x^3 - y^3 - 6$
has any relative maxima or relative minima.

$f_x(x,y) = 9y - 3x^2 = 0$

$f_y(x,y) = 9x - 3y^2 = 0$

$\Rightarrow x = \frac{3y^2}{9} = \frac{1}{3}y^2$

$9y - 3\left(\frac{1}{3}y^2\right)^2 = 0$

$\frac{1}{9}y^4$

$9y - \frac{1}{3}y^4 = 0$

$y\left(9 - \frac{1}{3}y^3\right) = 0$

$y^3 = \frac{9}{\frac{1}{3}} = 27$

$y = 0, 3$

When $y = 0$,
$x = \frac{1}{3}(0)^2 = 0$

When $y = 3$,
$x = \frac{1}{3}(3)^2 = 3$

CPs are $(x,y) = (0,0)$ and $(3,3)$

Discriminant:

$$D(x,y) = \begin{vmatrix} f_{xx}(x,y) & f_{yx}(x,y) \\ f_{xy}(x,y) & f_{yy}(x,y) \end{vmatrix}$$

the bars mean take the determinant of this 2x2 matrix

$f_{xx} = -6x \quad f_{yx} = 9$

$f_{xy} = 9 \quad f_{yy} = -6y$

notice $f_{xy} = f_{yx}$

$$D(0,0) = \begin{vmatrix} -6(0) & 9 \\ 9 & -6(0) \end{vmatrix} = (0)(0) - 9^2 < 0$$

$\Rightarrow$ saddle point

$$D(3,3) = \begin{vmatrix} -6(3) & 9 \\ 9 & -6(3) \end{vmatrix} = (-6(3))^2 - 9^2 > 0$$

$f_{xx}(3,3) = -6(3) < 0$

$\boxed{\Rightarrow \text{max at } (x,y) = (3,3)}$

Find all the local maxima, local minima, and saddle points of the given function:

$$f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$$

$f_x(x,y) = 4x + 3y - 5 = 0$

$f_y(x,y) = 3x + 8y + 2 = 0$

$\Rightarrow x = \frac{-8y-2}{3}$

$4\left(\frac{-8y-2}{3}\right) + 3y - 5 = 0$

$\left(\frac{-32}{3} + 3\right)y - \frac{8}{3} - 5 = 0$

$-\frac{23}{3}y - \frac{23}{3} = 0$

$y = \frac{\frac{23}{3}}{\frac{23}{3}} = -1$

$\Rightarrow x = \frac{-8(-1)-2}{3} = \frac{6}{3} = 2$

CP at $(x,y) = (2,-1)$

Discriminant: $D(x,y) = \begin{vmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{vmatrix}$

$= \begin{vmatrix} 4 & 3 \\ 3 & 8 \end{vmatrix} = \underset{32-9}{4(8) - 3^2} = 23 > 0$

for all (x,y)

and $f_{xx} = 4 > 0$

for all (x,y)

$\Rightarrow$ min at

$\boxed{(x,y) = (2,-1)}$

Find all the local maxima, local minima, and saddle points of the given function:

$$f(x,y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$$

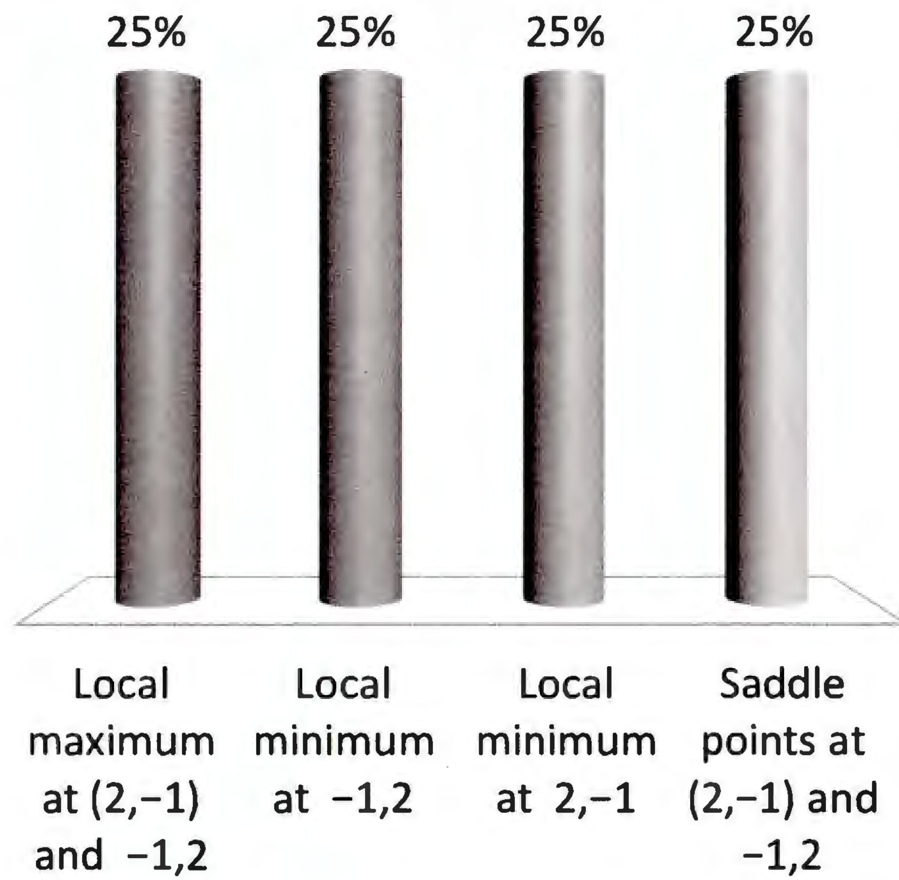
A. Local maximum at $(2,-1)$ and $(-1,2)$

B. Local minimum at $(-1,2)$

C. Local minimum at $(2,-1)$

D. Saddle points at $(2,-1)$ and $(-1,2)$

—See previous slide for work—



Show that $f(x, y) = 1 - x^4 - y^4$ has a relative maximum, even though D in the theorem is 0.

$f_x(x,y) = -4x^3 = 0$

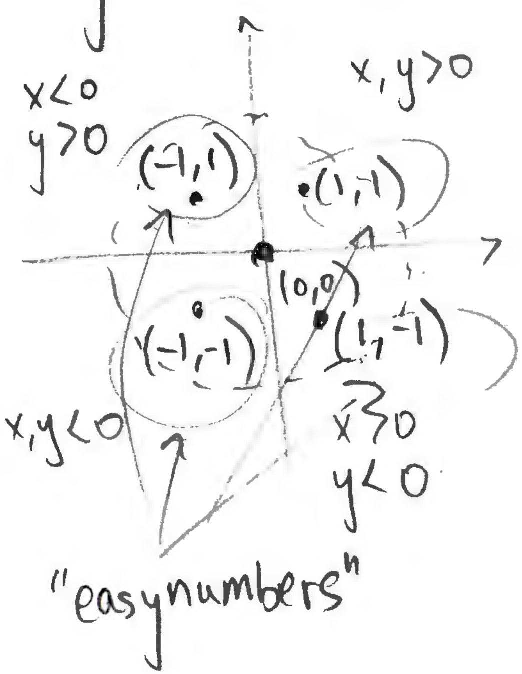
$f_y(x,y) = -4y^3 = 0$

$\Rightarrow$ CP at $(0,0)$

$D(x,y) = \begin{vmatrix} -12x^2 & 0 \\ 0 & -12y^2 \end{vmatrix} = 144x^2y^2$

$D(0,0) = 144(0)^2(0)^2 = 0$

Look at a circular region around the CP:



$f(-1,1) = 1-(-1)^4-(1)^4 = -1$

$1-1-1$

$f(1,1) = 1-(1)^4-(1)^4 = -1$

$f(-1,-1) = 1-(-1)^4-(-1)^4 = -1$

$f(1,-1) = 1-(1)^4-(-1)^4 = -1$

$f(0,0) = 1-0^4-0^4 = 1$

$\Leftarrow$ all < 1

$\Rightarrow$ $f(0,0)$ is larger than all points in the circle $\Rightarrow$ max

Suppose the labor cost (in dollars) for manufacturing a camera can be approximated by

$$L(x,y) = \frac{3}{2}x^2 + y^2 - 5x - 6y - 2xy + 120$$

where x is the number of hours required by a skilled craftsperson and y is the number of hours required by a semiskilled person. Find the values of x and y that minimize the labor cost. Find the minimum labor cost.

$L_x = 2\left(\frac{3}{2}\right)x - 5 - 2y = 0$

$L_y = 2y - 6 - 2x = 0$

$\Rightarrow y = \frac{6+2x}{2} = 3 + x$

$3x - 5 - 2(3+x) = 0$

$-6-2x$

$x - 5 - 6 = 0$

$x = 11$

$\Rightarrow y = 3 + 11 = 14$

Labor cost is minimized at

$\boxed{\begin{array}{l} x = 11 \text{ hours of a skilled person} \\ y = 14 \text{ hours of a semi-skilled person} \end{array}}$

Minimum labor cost is

$L(11,14) = \frac{3}{2}(11)^2 + (14)^2 - 5(11) - 6(14) - 2(11)(14) + 120$

$\boxed{\$50.50}$