UNIT 4, LESSON 2

Substitution

Power Rule

For any real number $n \neq -1$,

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

(The antiderivative of $f(x) = x^n$ for $n \neq -1$ is found by increasing the exponent n by 1 and dividing x raised to the new power by the new value of the exponent.)

Indefinite Integrals of Exponential Functions

$$\int e^x dx = e^x + C$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C, \quad k \neq 0$$

For $a > 0$, $a \neq 1$:

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int a^{kx} dx = \frac{a^{kx}}{k(\ln a)} + C, \quad k \neq 0$$

(The antiderivative of the exponential function e^x is itself. If x has a coefficient of k, we must divide by k in the antiderivative. If the base is not e, we must divide by the natural logarithm of the base.)

Indefinite Integral of x^{-1}

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

(The antiderivative of $f(x) = x^n$ for $n = -1$ is the natural logarithm of the absolute value of x.)

$$\int \sqrt{t}\, dt =$$

A. Does not exist

B. $\frac{2}{3} t^{3/2} + C$

C. $\frac{1}{2} t^{-3/2} + C$

$$\int \left(\frac{-5}{x} + e^{-2x}\right) dx =$$

A. $3x + 2 + c$

B. $\frac{5}{2x^{-2}} + 2e^{-2x} + c$

C. $-5\ln|x| + 2e^{-2x} + c$

D. $-5\ln|x| - \frac{1}{2}e^{-2x} + c$

Recall:

- The Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(3x^2 + 2)^4 =$$

Substitution Method (u substitution)

We begin by choosing a u from our integrand to make the substitution. For us, our u will generally be

1. The expression under a radical or being raised to a power
2. The expression in the denominator
3. The exponent on e

The goal is to be able to write the integral entirely in terms of the new variable u.

After substitution, the integral should be easier to evaluate. That is, it can be integrated using one of the basic rules.

Example: Find $\int 8x(4x^2 + 8)^6 dx$.

Example: Find $\int x^3\sqrt{3x^4+10}dx$

Example: Find $\int \frac{x+3}{x^2+6x}\,dx$

Find $\int 25x^2 e^{3x^3+2}\ dx$

Find $\int 6x(3x^2+4)^7 dx$.

A. $6(3x^2+4)^8 + c$

B. $\frac{(3x^2+4)^8}{8} + c$

C. $18x^3 + 4x + c$

Find $\int x^2\sqrt{x^3+1}dx$

A. $\frac{2}{9}(x^3+1)^{3/2}+c$

B. $\frac{1}{3}(x^3+1)^{-1/2}+c$

C. $\frac{2}{3}(x^3+1)^{2/3}+c$

$$\int \frac{24x+4}{6x^2+2x}\,dx$$

A. $\ln(6x^2+2x)+C$

B. $\frac{1}{2}\ln(6x^2+2x)+C$

C. $2\ln(6x^2+2x)+C$

D. $24\ln(6x^2+2x)+C$

E. $\frac{1}{4}\ln(6x^2+2x)+C$

The marginal revenue (in thousands of dollars) from the sale of x MP3 players is given by

$$R'(x) = 4x(x^2 + 27{,}000)^{-2/3}.$$

Find the total revenue function if the revenue from 125 players is $29,591.