Student Name

① $$\lim_{x\to -3} \frac{x+9}{x^2\sqrt[3]{5x+7}} = \frac{(-3)+9}{(-3)^2\left(\sqrt[3]{5(-3)+7}\right)}$$

$$= \frac{6}{-18} \quad \blacksquare$$

② $$\lim_{t\to 5} \frac{\frac{1}{5} - \frac{5}{t^2}}{t-5}$$

$$= \lim_{t\to 5} \frac{1(t^2) - 5(5)}{5t^2(t-5)}$$

$$= \lim_{t\to 5} \frac{t^2 - 25}{5t^2(t-5)}$$

$$= \lim_{t\to 5} \frac{(t+5)\cancel{(t-5)}^{\,1}}{5t^2\cancel{(t-5)}}$$

$$= \frac{(5)+5}{5(5)^2}$$

$$= \frac{10}{125} \quad \blacksquare$$

③ $\lim_{x \to 0} x \cos\left(\frac{1}{x}\right)$

Soln.

$$\left|\cos\left(\tfrac{1}{x}\right)\right| \leq 1$$

$$|x| \cdot \left|\cos\left(\tfrac{1}{x}\right)\right| \leq |x|$$

$$\left|x \cos\left(\tfrac{1}{x}\right)\right| \leq |x|$$

So, $-|x| \leq x\cos\left(\frac{1}{x}\right) \leq |x|$

$\lim_{x \to 0} -|x| = 0$ & $\lim_{x \to 0} |x| = 0.$

By The Squeeze Theorem

$$\lim_{x \to 0} x\cos\left(\tfrac{1}{x}\right) = 0 \blacksquare$$

④ $\lim_{h \to 4} \dfrac{h^2 - h - 12}{h^2 - 2h - 8}$

$$= \lim_{h \to 4} \frac{(h+3)\cancel{(h-4)}^{1}}{(h+2)\cancel{(h-4)}}$$

$$= \lim_{h \to 4} \frac{h+3}{h+2} = \frac{4+3}{4+2} = \frac{7}{6} \blacksquare$$

⑤ $$\lim_{x \to 2} \frac{2x^2 - 8}{x + 2} = \frac{2(2)^2 - 8}{(2) + 2} = 0$$

⑥ $$\lim_{x \to 6} \frac{x^2 + 3x - 10}{x - 6} = \lim_{x \to 6} \frac{(x+5)(x-2)}{x-6}$$

As $x \to 6^+$: $x^2 + 3x - 10 \to 44$; $x - 6 \to 0$ and is positive.

we write:

$$\lim_{x \to 6^+} \frac{x^2 + 3x - 10}{x - 6} = \infty$$

As $x \to 6^-$: $x^2 + 3x - 10 \to 44$; $x - 6 \to 0$ & is negative.

we write:

$$\lim_{x \to 6^-} \frac{x^2 + 3x - 10}{x - 6} = -\infty$$

So, $\lim_{x \to 6} \frac{x^2 + 3x - 10}{x - 6}$ DNE

⑦ $$\lim_{x \to 6} \frac{\sqrt{x-6} + 2}{\sqrt{x+3} - 6} = \frac{\sqrt{(6) - 6} + 2}{\sqrt{(6) + 3} - 6} = \frac{2}{-3}$$

⑧ $$\lim_{t\to 2} \frac{4t^4-64}{t-2} = \lim_{t\to 2} \frac{4(t^4-16)}{t-2}$$

$$= \lim_{t\to 2} \frac{4(t^2+4)(t^2-4)}{t-2}$$

$$= \lim_{t\to 2} \frac{4(t^2+4)(t+2)\cancel{(t-2)}}{\cancel{t-2}} \to 1$$

$$= 4\left((2)^2+4\right)\left((2)+2\right)$$

$$= 128 \quad \blacksquare$$

⑨ $$\lim_{x\to 0} \frac{a-\sqrt{a^2-x^2}}{x^2}\left(\frac{a+\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}\right)$$

$$= \lim_{x\to 0} \frac{a^2-\left(\sqrt{a^2-x^2}\right)^2}{x^2\left(a+\sqrt{a^2-x^2}\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{a^2-a^2}^{\,0}+\cancel{x^2}}{\cancel{x^2}\left(a+\sqrt{a^2-x^2}\right)} \to 1$$

$$= \lim_{x\to 0} \frac{1}{a+\sqrt{a^2-x^2}}$$

$$= \frac{1}{a+\sqrt{a^2-(0)^2}} = \frac{1}{2a} \quad \blacksquare$$

(10) $\dfrac{2x^2+3x-5}{x^2-7x+6} = \dfrac{(2x+5)\cancel{(x-1)}}{(x-6)\cancel{(x-1)}}$

f has a potential vertical asymptote $x=6$.

As $x \to 6^+$: $2x^2+3x-5 \to 85$,

$x^2-7x+6 \to 0$ and is positive.

We write:

$$\lim_{x\to 6^+} \frac{2x^2+3x-5}{x^2-7x+6} = \infty .$$

By the definition, $x=6$ is a vertical asymptote of f.