



# Unit 2, Lesson 4

The Chain Rule

# Objectives:

The lesson focuses on using the chain rule to differentiate three types of functions: a function raised to a power, exponential functions, and logarithmic functions. Students will be able to:

- Break down a composition of two functions into basic functions
- Apply the chain rule to find derivatives of a function raised to a power, exponential functions, and logarithmic functions

# SEPTEMBER 2016

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
26 Quiz IV	27	28	29	30	
U2L4		U2L5			

# OCTOBER 2016

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
3 Quiz V	4	5 Test II Group A	6 Test II Groups A and B	7 Test II Group B	8/9 PCA U3L1 (10/9)
Review For Test II		Review For Test II			

Choose the 3 conditions that must be satisfied for a function to be continuous at a point:

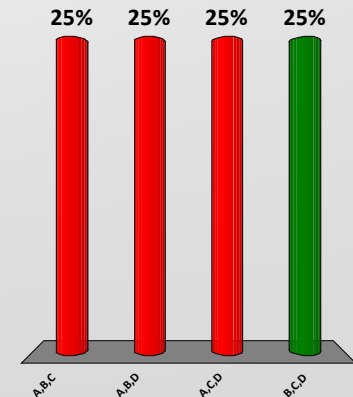
A. A, B, C

B. A, B, D

C. A, C, D

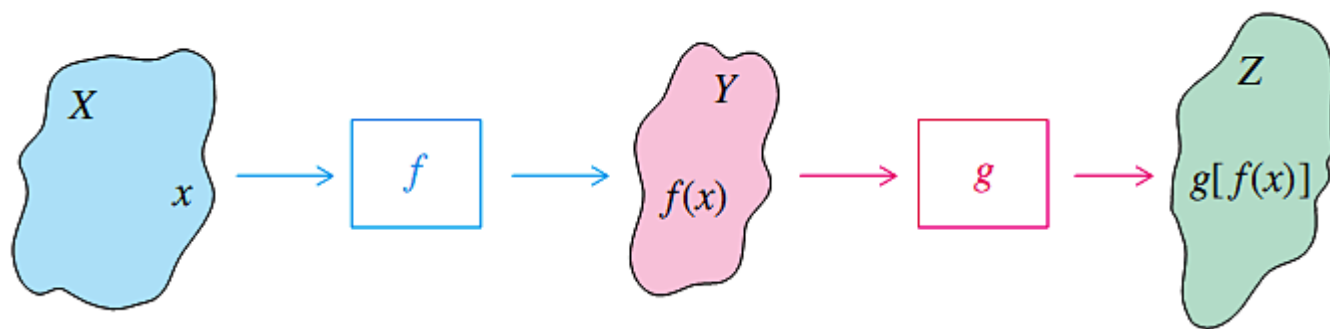
D. B, C, D

- A.  $\lim_{x \rightarrow a} f(x) \neq f(a)$
- B.  $\lim_{x \rightarrow a} f(x) = f(a)$
- C.  $\lim_{x \rightarrow a} f(x)$  exists.
- D.  $f(a)$  is defined.



## Composite Function

Let  $f$  and  $g$  be functions. The **composite function**, or **composition**, of  $g$  and  $f$  is the function whose values are given by  $g[f(x)]$  for all  $x$  in the domain of  $f$  such that  $f(x)$  is in the domain of  $g$ . (Read  $g[f(x)]$  as “ $g$  of  $f$  of  $x$ ”.)



## Chain Rule

If  $y$  is a function of  $u$ , say  $y = f(u)$ , and if  $u$  is a function of  $x$ , say  $u = g(x)$ , then  $y = f(u) = f[g(x)]$ , and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

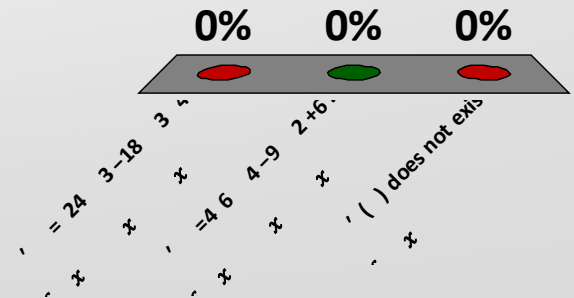
If  $f(x) = (3x^2 - 7)^{2/3}$ , then find  $f'(x)$ .

Let  $f(x) = (6x^4 - 9x^2 + 6)^4$ . Find  $f'(x)$ .

*A.*  $f'(x) = (24x^3 - 18x)^3(4x)^3$

*B.*  $f'(x) = 4(6x^4 - 9x^2 + 6)^3(24x^3 - 18x)$

*C.*  $f'(x)$  does not exist



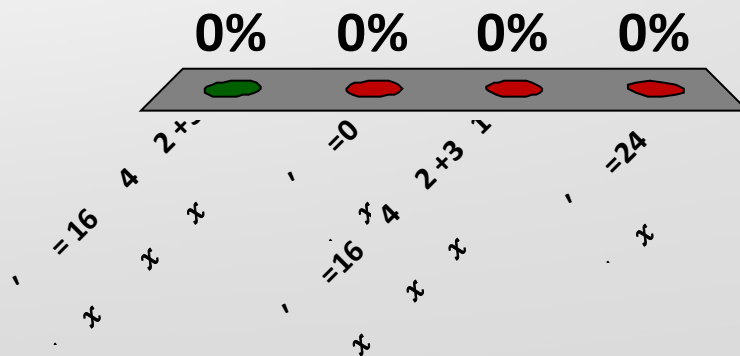
Let  $g(x) = 4\sqrt{4x^2 + 3}$ . Find  $g'(x)$ .

A.  $g'(x) = \frac{16x}{\sqrt{4x^2 + 3}}$

B.  $g'(x) = 0$

C.  $g'(x) = 16x(4x^2 + 3)^{\frac{1}{2}}$

D.  $g'(x) = 24x$





Consider the following table of values of the functions  $f$  and  $g$  and their derivatives at various points.

$x$	1	2	3	4
$f(x)$	1	3	4	2
$f'(x)$	-2	-4	-6	-9
$g(x)$	3	4	2	1
$g'(x)$	$1/9$	$7/9$	$5/9$	$2/9$

Use the table to find  $D_x(f[g(x)])$  at  $x = 3$ .

### Derivative of $e^x$

$$\frac{d}{dx}(e^x) = e^x$$

### Derivative of $a^x$

For any positive constant  $a \neq 1$ ,

$$\frac{d}{dx}(a^x) = (\ln a)a^x.$$

### Derivative of $a^{g(x)}$ and $e^{g(x)}$

$$\frac{d}{dx}(a^{g(x)}) = (\ln a)a^{g(x)}g'(x)$$

and

$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x)$$

Let  $f(x) = e^{-8x}$ . Find  $f'(x)$ .

Let  $f(x) = e^{x^3}$ . Find  $f'(x)$ .

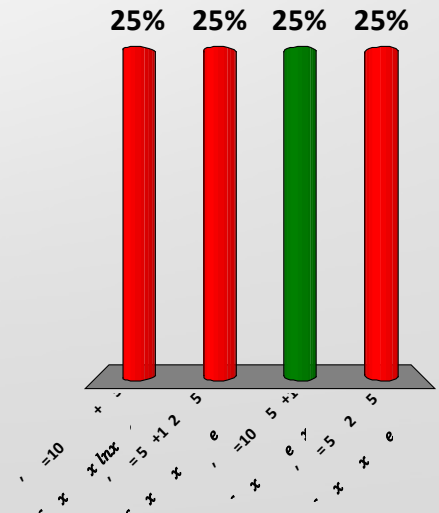
Let  $f(x) = 2e^{5x+1}$ . Find  $f'(x)$ .

*A.*  $f'(x) = 10x(\ln x) + e^5$

*B.*  $f'(x) = (5x + 1)(2)e^{5x}$

*C.*  $f'(x) = 10(e^{5x+1})$

*D.*  $f'(x) = (5x)(2)e^{5x}$



### Derivative of $\ln x$

$$\frac{d}{dx} [\ln x] = \frac{1}{x}$$

### Derivative of $\log_a x$

$$\frac{d}{dx} [\log_a x] = \frac{1}{(\ln a)x}$$

(The derivative of a logarithmic function is the reciprocal of the product of the variable and the natural logarithm of the base.)



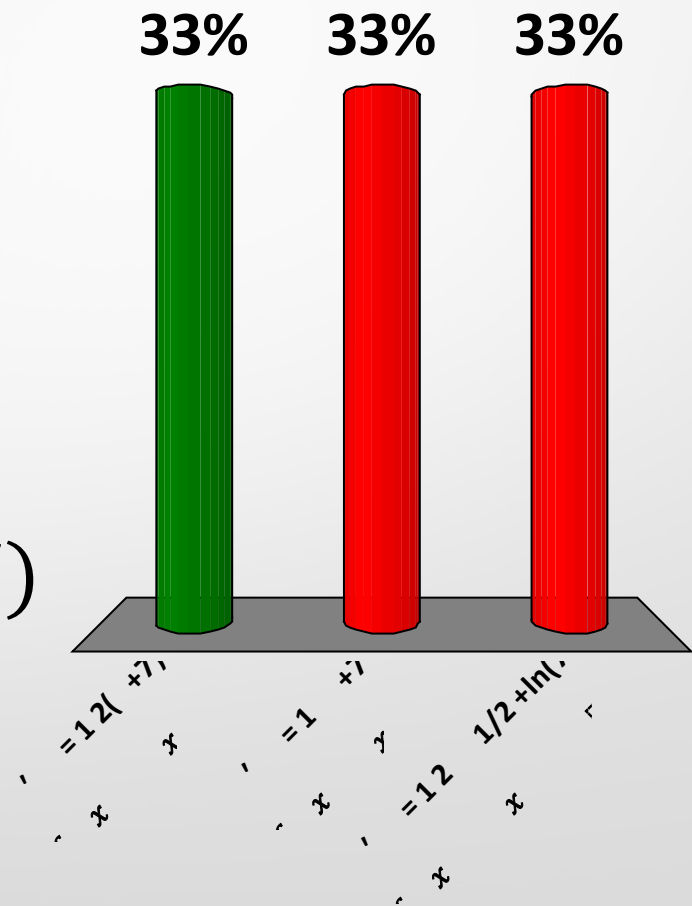
Let  $f(x) = \ln(4 - 3x)$ . Find  $f'(x)$ .

Let  $f(x) = \ln\sqrt{x+7}$ . Find  $f'(x)$ .

*A.*  $f'(x) = \frac{1}{2(x+7)}$

*B.*  $f'(x) = \frac{1}{\sqrt{x+7}}$

*C.*  $f'(x) = \frac{1}{2}x^{1/2} + \ln(7)$



## QUESTION:

- *A friend concludes that because  $y = \ln(6x)$  and  $y = \ln(x)$  both have the same derivative, namely  $\frac{dy}{dx} = \frac{1}{x}$ , then these two functions must be the same.*
- *Is your friend correct? Why or why not?*



## QUESTION:

- If  $f(t)$  give the number of units of a certain product sold by a company after  $t$  days and  $g(x)$  gives the revenue (in dollars) from the sale of  $x$  units of the company's products, what does  $(g \circ f)'(t)$  describe?