MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Mon 15 June 2015

Quiz 6: The Chain Rule ($\oint 3.6$)

Directions: You have 30 minutes to complete this quiz. This quiz is closed book, and you must work alone.

Use the Chain Rule to differentiate the following functions (you do not need to simplify):

1. $y = \sin\left(\frac{x}{4}\right)$

$y' = \frac{1}{4}\cos\left(\frac{x}{4}\right)$

2. $y = (e^x)^3$

$y' = 3(e^x)^2 \cdot e^x$

3. $y = e^{\tan t}$

$y' = (\sec^2 t)e^{\tan t}$

4. $y = \sin\left(4x^3 + 3x + 1\right)$

$y' = (12x^2 + 3)\cos(4x^3 + 3x + 1)$

5. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$y' = \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2} \cdot \left(1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\right)$

$\cdot\left(1 + \frac{1}{2}x^{-1/2}\right)$

6. $y = \left(\frac{3x}{4x+2}\right)^5$

$y' = 5\left(\frac{3x}{4x+2}\right)^4 \cdot \frac{(4x+2)(3) - (3x)(4)}{(4x+2)^2}$

7. $y = \csc\left(t^2 + t\right)$

$y' = (2t+1)\left(-\csc(t^2+t)\cot(t^2+t)\right)$

8. $y = \cos^4\left(7x^3\right)$

$y' = 4\cos^3(7x^3)(-\sin(7x^3))(21x^2)$

9. $y = \tan\left(e^{\sqrt{3}x}\right)$

$y' = \sqrt{3}e^{\sqrt{3}x}\sec^2\left(e^{\sqrt{3}x}\right)$

10. $y = e^{x^3}$

$y' = 3x^2 e^{x^3}$

11. Assume f is a differentiable function whose graph passes through the point $(1, 4)$. If $g(x) = f(x^2)$ and the line tangent to the graph of f at $(1, 4)$ is $y = 3x + 1$, determine each of the following:

(a) $g(1) = f(1^2) = f(1) = 4$

(b) $g'(x) = 2x f'(x^2)$

(c) $g'(1) = 2(1) f'(1) = 2 \cdot 3 = 6$

(d) Find an equation of the line tangent to the graph of g when $x = 1$.

$y - 4 = 6(x - 1)$

12. Suppose f is differentiable for all real numbers with $f(0) = -3$, $f(1) = 3$, $f'(0) = 3$, and $f'(1) = 5$. Let $g(x) = \sin(\pi f(x))$. Find the following:

(a) $g'(x) = \pi f'(x) \cos(\pi f(x))$

(b) $g'(0) = \pi f'(0) \cos(\pi f(0))$

$= \pi(3) \cos(\pi(-3)) = -3\pi$

(c) $g'(1) = \pi f'(1) \cos(\pi f(1))$

$= \pi(5) \cos(\pi(3)) = -5\pi$