U4L3

Fundamental Theorem of Calculus

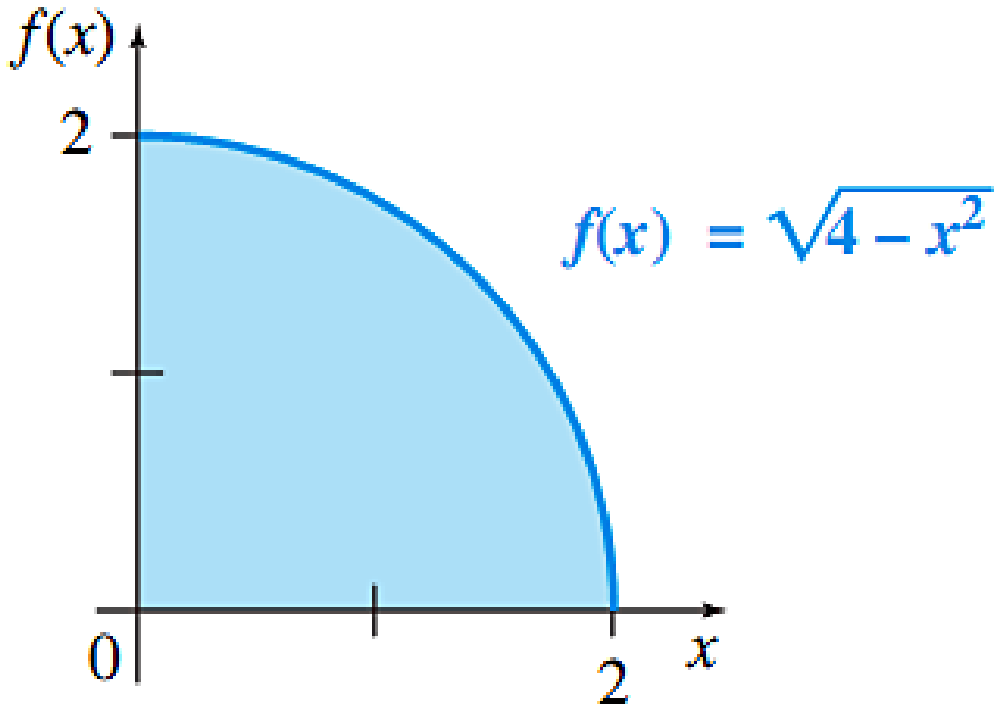
Objectives:

This lesson will continue with integration and relate the previous lesson to finding the area beneath any function. We will introduce the Fundamental Theorem of Calculus. Students will be able to:

- Evaluate definite integrals to find net area between a curve and the x-axis using the FTC.

- Use basic integration properties to solve graphical net area problems

Suppose we want to find the area between the graph of $f(x) = \sqrt{4 - x^2}$ and the x-axis.



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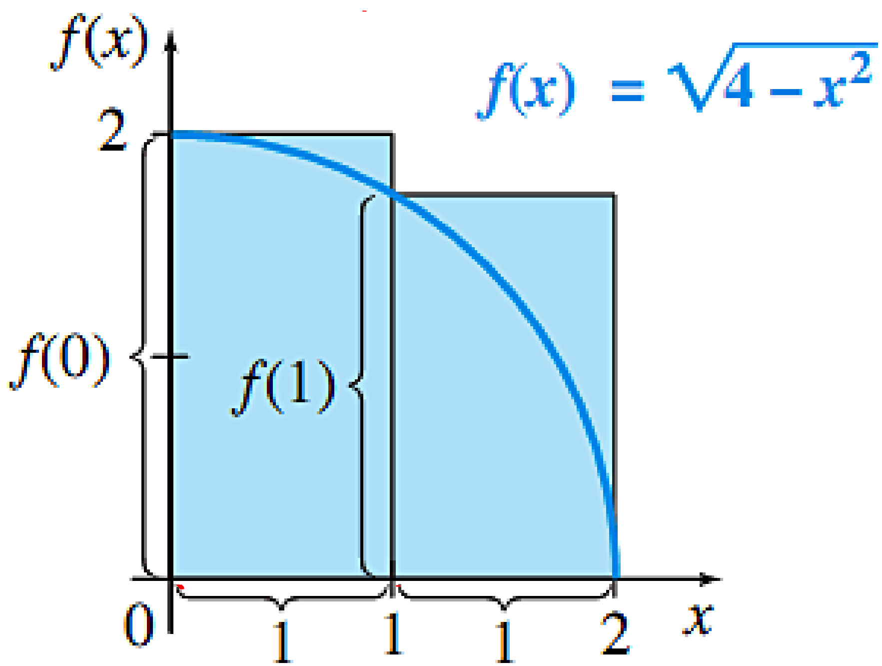

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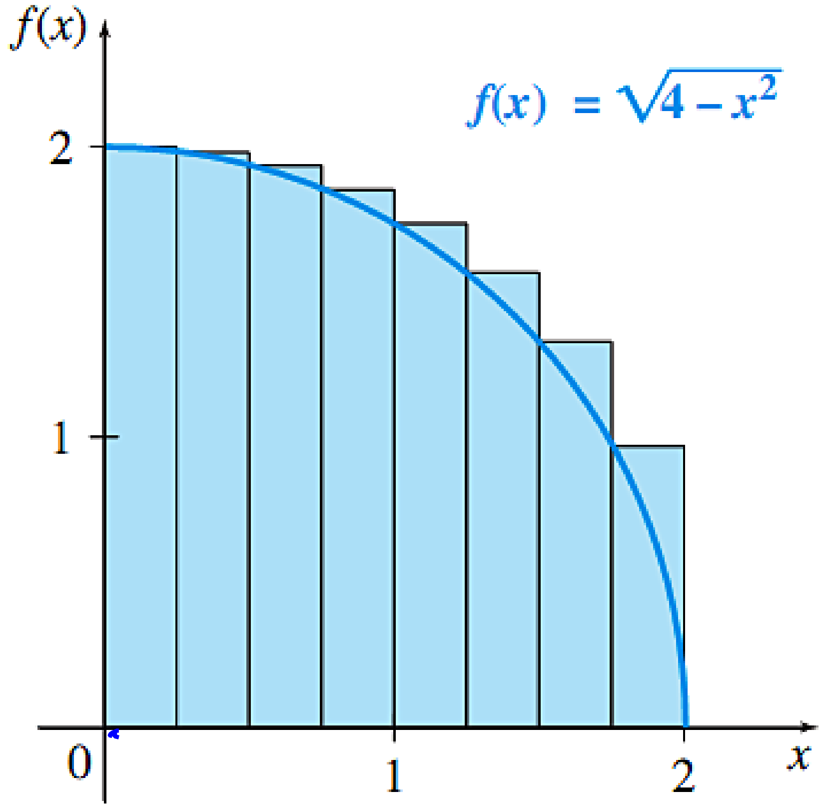

The Definite Integral

If f is defined on the interval $[a, b]$, the **definite integral** of f from a to b is given by

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

provided the limit exists, where $\Delta x = (b - a)/n$ and x_i is *any* value of x in the ith interval.*

https://www.desmos.com/calculator/tgyr42ezjq

Riemann sums

desmos

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1 $f(x) = \sin(2x) + \frac{x}{3}$

2 Endpoints, number of intervals, and method

3 left endpoint

4 $a = -1$

-10 10

5 right endpoint

6 $b = 3$

-10 10

7 number of intervals

8 $n = 12$

1 100

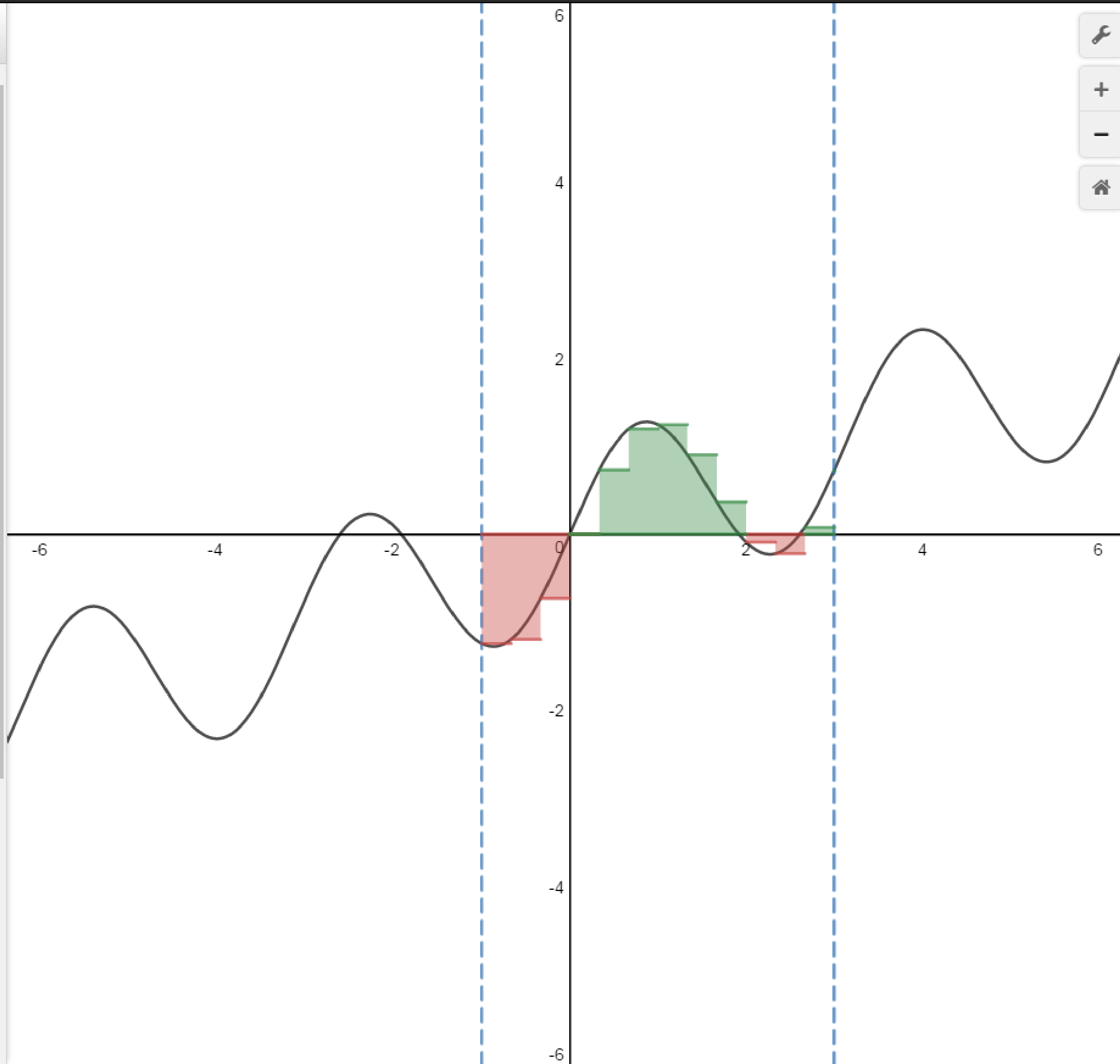
9 choice of method: set c=0 for left-hand sum, c=1 for right-hand sum, c=0.5 for midpoint sum

10 $c = 0$

0 1

11 Integral approximation

$n = 1$



Fundamental Theorem of Calculus

Let f be continuous on the interval $[a, b]$, and let F be *any* antiderivative of f. Then

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b.$$

Properties of Definite Integrals

If all indicated definite integrals exist,

1. $\int_a^a f(x)\,dx = 0;$

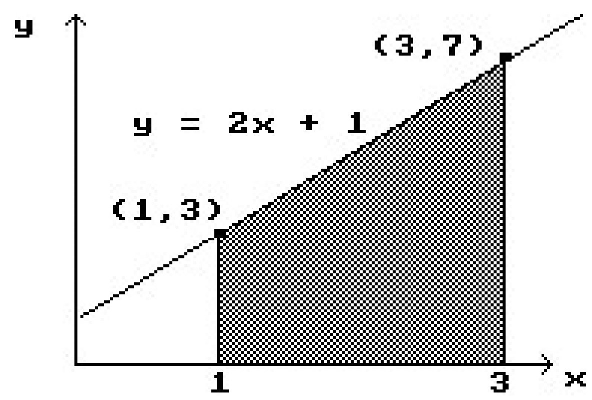
2. $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$ for any real constant k (constant multiple of a function);

3. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ (sum or difference of functions);

4. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for any real number c;

5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

Find the area of the shaded region.



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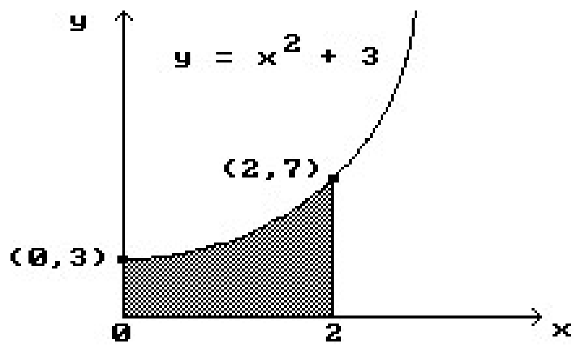


Example:

Find $\int_{1}^{3} 3x^2 dx$.

Example:

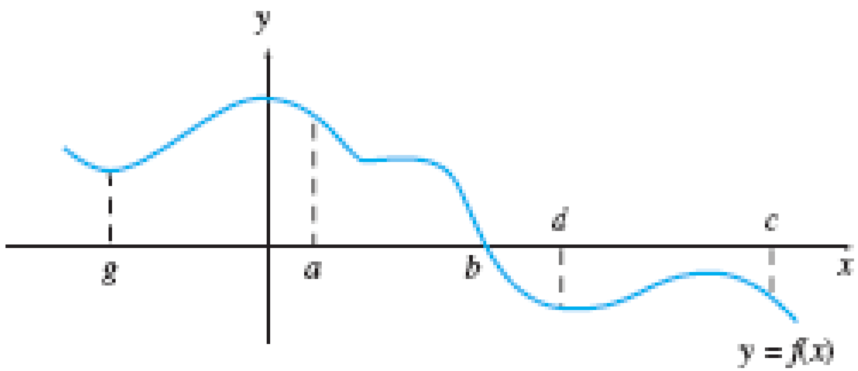
Find $\int_{3}^{5}(2x^3-3x+4)dx.$

Compute $\int_0^5 3x^2 + 2x + 1\,\mathrm{dx}$

A. 2
B. 17.5
C. 155
D. 210
E. 433

Assume f(x) is continuous for $g \le x \le c$ as shown in the figure . Write an equation relating the three quantities below.

$$\int_a^c f(x)dx,\ \int_a^b f(x)dx,\ \int_b^c f(x)dx$$



$$\int_{0}^{1} x^{9}\left(1+x^{10}\right)^{9}dx$$

$$\int_{1}^{\frac{e}{2}} \frac{1}{2x}\,dx$$

A. e

B. $2 - 2\ln(2)$

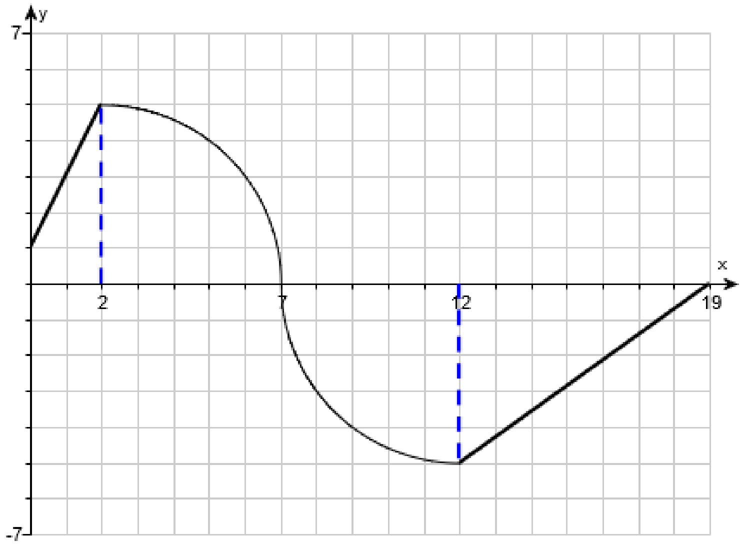
C. $\ln(e^2)$

D. $\ln(e^3)$

E. $\frac{1}{2}\ln\left(\frac{e}{2}\right)$

F. $\frac{1}{3}\ln\left(\frac{e}{3}\right)$

The graph of f(x), shown here, consists of two straight line segments and two quarter circles. Find the value of $\int_{0}^{19} f(x)dx$.



Using the TI-83 or TI-84 to compute definite integrals.

Your calculator can be used to compute definite integrals. However, it will not necessarily give you the exact value of the integral. In situations where we are only looking for a decimal approximation, this can be sufficient, but when an exact answer is required we will often still need to use the fundamental theorem.

$$\int_1^4 (x^2 - 4)\,dx$$

$$\int_0^2 2xe^{x^2}\, dx$$

Find $\int_{e^2}^{e^5} \frac{1}{x\ln(x)}\, dx$

A. $\ln(5) - \ln(3)$

B. $e^5 - e^3$

C. $\ln(3) - \ln(5)$

D. $e^5 - e^3 - \ln(3) + \ln(5)$

E. $e^5 - e^3 + \ln(3) - \ln(5)$