UNIT 2, LESSON 2

Basic Rules and Higher Order Derivatives

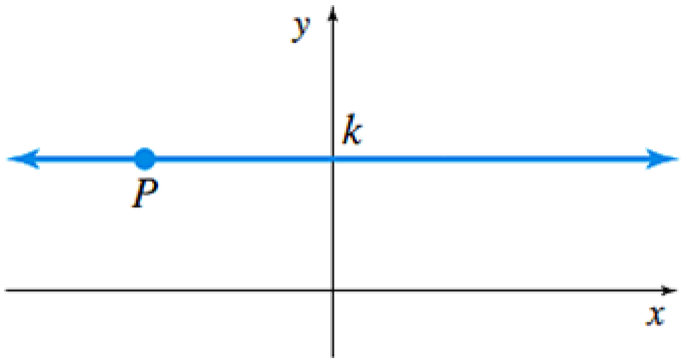
Objectives:

- Use formulas to take derivatives of polynomial, radical, exponential, and logarithmic functions.
- Relate the first derivative to velocity and the second derivative to acceleration.

Notations for the Derivative

The derivative of $y = f(x)$ may be written in any of the following ways:

$$f'(x), \qquad \frac{dy}{dx}, \qquad \frac{d}{dx}[f(x)], \qquad \text{or} \qquad D_x[f(x)].$$



$$f(x) = k$$

$$f'(x) = \lim_{h \to 0} \frac{k - k}{h} = \lim_{h \to 0} 0 = 0$$

Constant Rule

If $f(x) = k$, where k is any real number, then

$$f'(x) = 0.$$

(The derivative of a constant is 0.)

$$f(x) = x$$

$$f'(x) = \lim_{h\to 0}\frac{x+h-x}{h} = \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0} 1 = 1$$

$$f(x) = x^2$$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} =$$

$$= \lim_{h\to 0}\frac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h) = 2x$$

$$f(x) = x^3$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3}{h}$$

$$\lim_{h \to 0} \frac{h(3x^2 + 3xh)}{h} = \lim_{h \to 0} (3x^2 + 3h) = 3x^2$$

Derivative of $f(x) = x^n$		
Function	n	**Derivative**
$f(x) = x$	1	$f'(x) = 1 = 1x^0$
$f(x) = x^2$	2	$f'(x) = 2x = 2x^1$
$f(x) = x^3$	3	$f'(x) = 3x^2$
$f(x) = x^4$	4	$f'(x) = 4x^3$
$f(x) = x^{-1}$	-1	$f'(x) = -1 \cdot x^{-2} = \dfrac{-1}{x^2}$
$f(x) = x^{1/2}$	$1/2$	$f'(x) = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2x^{1/2}}$

Power Rule

If $f(x) = x^n$ for any real number n, then

$$f'(x) = nx^{n-1}.$$

(The derivative of $f(x) = x^n$ is found by multiplying by the exponent n and decreasing the exponent on x by 1.)

If $f(t) = \dfrac{1}{\sqrt{t}}$, find $f'(t)$.

$$f(t) = \frac{1}{\sqrt{t}} = \frac{1}{t^{1/2}} = t^{-1/2}$$

$$f'(t) = -\frac{1}{2}t^{-3/2} = -\frac{1}{2t^{3/2}} = -\frac{1}{2\sqrt{t^3}} = -\frac{1}{2t\sqrt{t}}$$

If $f(x) = \frac{1}{\sqrt[4]{x}}$, then find $f'(x)$.

A. $f'(x)$ *does not exist*

B. $f'(x) = \frac{-1}{(4x)\sqrt[4]{x}}$

C. $f'(x) = \frac{-1}{(2x^2)\sqrt{x}}$

D. $f'(x) = \frac{-1}{8x\sqrt{x}}$

Constant Times a Function

Let k be a real number. If $g'(x)$ exists, then the derivative of $f(x) = k \cdot g(x)$ is

$$f'(x) = k \cdot g'(x).$$

(The derivative of a constant times a function is the constant times the derivative of the function.)

Sum or Difference Rule

If $f(x) = u(x) \pm v(x)$, and if $u'(x)$ and $v'(x)$ exist, then

$$f'(x) = u'(x) \pm v'(x).$$

(The derivative of a sum or difference of functions is the sum or difference of the derivatives.)

Example:

If $f(x) = 2x^3 + 3x^2 - 4x + 3$, then find $f'(x)$.

Example:

$$\text{If } h(t) = -3t^2 + 2\sqrt{t} + \frac{5}{t^4} - 7, \text{ find } h'(t).$$

$$h(t) = -3t^2 + 2t^{1/2} + 5t^{-4} - 7.$$

$$h'(t) = -6t + 2(\frac{1}{2}t^{-1/2}) + 5(-4t^{-5})$$

$$= -6t + t^{-1/2} - 20t^{-5}$$

$$= -6t + \frac{1}{\sqrt{t}} - \frac{20}{t^5}$$

If $f(x) = \frac{x^3+5}{x}$, then find $f'(x)$.

A. $f'(x) = \frac{3x^2}{x}$

B. $f'(x) = 3x^2$

C. $f'(x) = 2x - \frac{5}{x^2}$

D. $f'(x) = 2x + 5$

Marginal Analysis:

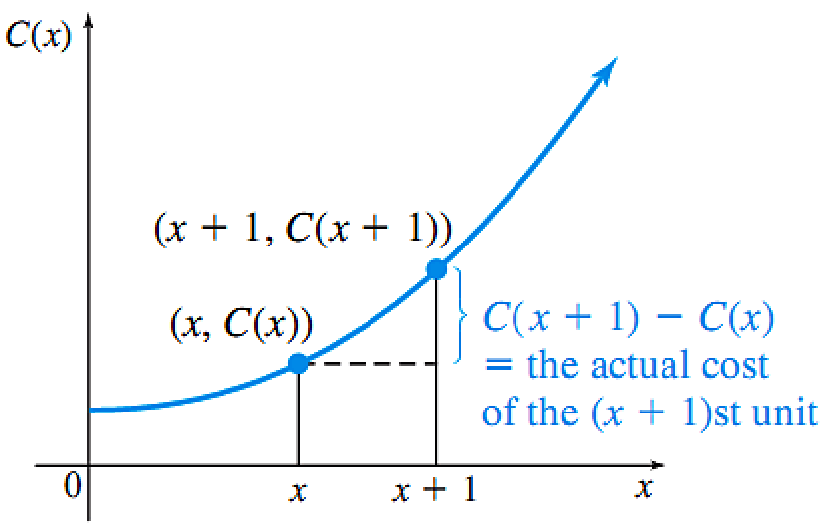
In business and economics, the rates of change of variables such as cost, revenue, and profit are important considerations.

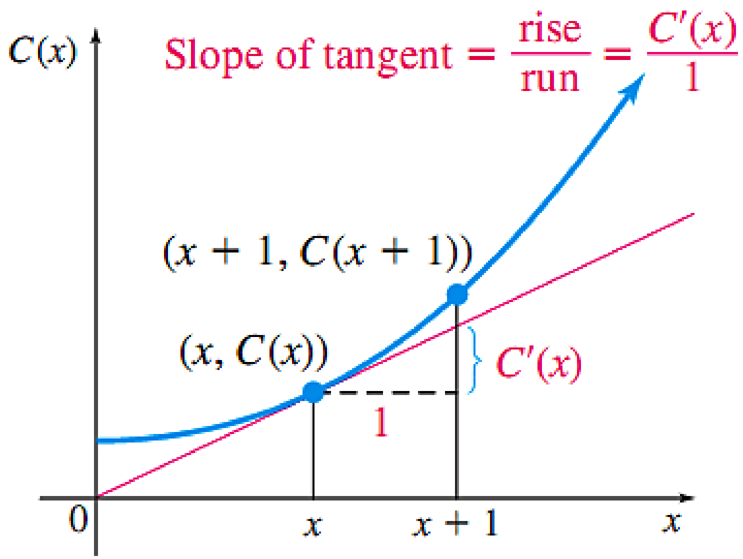
Marginal cost refers to the rate of change of cost.

Marginal revenue refers to__________________________.

Marginal profit refers to ____________________________.

Marginal Cost:





We think of the marginal cost function as being the cost of production one more unit.

Example:

Suppose that the total cost in hundreds of dollars to produce x thousand cases of a beverage is given by:

$$C(x) = 4x^2 + 100x + 500$$

a) Find the marginal cost when $x = 5$.

a) Find the marginal cost when $x = 300$

The demand function for a certain product is given by $p = \frac{50{,}000 - q}{25000}$. Find the marginal revenue when $q = 10{,}000\, units$ and p is in dollars.
(Recall that $revenue = quantity\ x\ price$)

A. $1.20 per unit
B. $1.35 per unit
C. $120 per unit
D. $135 per unit

The number of Americans (in thousands) who are expected to be over 100 years old can be approximated by the function $f(t) = 0.00943t^3 - 0470t^2 + 11.085t + 23.441$ where t is the number of years after 2000.

Find a formula giving the rate of change of the number of Americans over 100 years old.

A. $f'(t) = 0.02829t^2 - 0.940t + 11.085$
B. $f'(t) = 0.134t + 5$
C. $f'(t) = 0.02829t^2 + 0.940t + 11.085$