If $f'(x) > 0$ for all x in an interval (a, b), then…

A. f is increasing on (a, b).

B. f is decreasing on (a, b).

C. f' is increasing on (a, b).

D. f' is decreasing on (a, b).

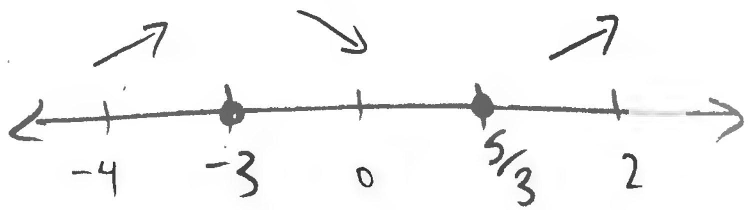
[redacted]

Example: Find all relative extrema.

$$f(x) = -x^3 - 2x^2 + 15x + 10$$

$f'(x) = -3x^2 - 4x + 15 = 0$

Q-formula: $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(-3)(15)}}{2(-3)}$



$= \dfrac{\overset{2}{\cancel{4}} \pm \cancel{2}\sqrt{\overset{49}{4 - (-3)(15)}}}{\cancel{2}(-3)} = \dfrac{2 \pm 7}{-3}$

$= -3, \frac{5}{3}$

↑

CP's

$f'(-4) < 0 \quad f'(0) > 0 \quad f'(2) < 0$

$\boxed{\begin{array}{l} x = -3 \text{ is a relative max} \\ x = \frac{5}{3} \text{ is a relative min} \end{array}}$

Find all relative extrema for the following function: $f(x) = 2x^3 - 3x^2 - 72x + 15$

$f'(x) = 6x^2 - 6x - 72 = 6(x^2 - x - 12) = 0$

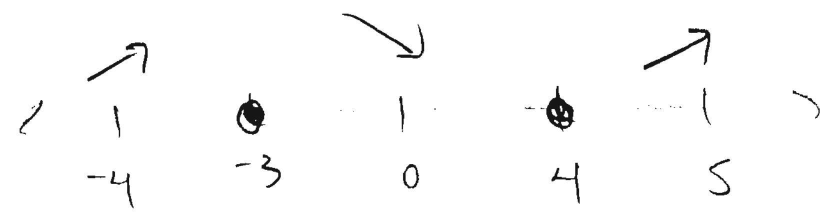
$6(x-4)(x+3) = 0$

CP's: $x = 4, -3$

A. Rel. max @ $x = -3$
   Rel. min @ $x = 4$

B. Rel. max @ $x = -4$
   Rel. min @ $x = 3$

C. Rel. max @ $x = 4$
   Rel. min @ $x = -3$



$f'(-4) = 6(-4-4)(-4+3) > 0$

$f'(0) = 6(0-4)(0+3) < 0$

$f'(5) = 6(5-4)(5+3) > 0$

Finding Absolute Extrema

To find absolute extrema for a function f continuous on a closed interval $[a, b]$:

1. Find all critical numbers for f in (a, b).
2. Evaluate f for all critical numbers in (a, b).
3. Evaluate f for the endpoints a and b of the interval $[a, b]$.
4. The largest value found in Step 2 or 3 is the absolute maximum for f on $[a, b]$, and the smallest value found is the absolute minimum for f on $[a, b]$.

Find the absolute extrema of the function $f(x) = 3x^{2/3} - 3x^{5/3}$ on the interval [0,8].

$$f'(x) = 3\left(\frac{2}{3}\right)x^{-1/3} - 3\left(\frac{5}{3}\right)x^{2/3}$$

$$= \frac{2}{x^{1/3}} - 5x^{2/3} = 0$$

Multiply by $x^{1/3}$:

$$= 2 - 5x = 0$$

$$\Rightarrow x = \frac{2}{5}$$

$$f(0) = 3(0)^{2/3} - 3(0)^{5/3} = 0$$

$$f\left(\frac{2}{5}\right) = 3\left(\frac{2}{5}\right)^{2/3} - 3\left(\frac{2}{5}\right)^{5/3}$$

$$= 3\left(\left(\frac{2}{5}\right)^{2/3} - \left(\frac{2}{5}\right)^{5/3}\right) \approx 3(0.326)$$

$\boxed{\text{max at } x = 2/5}$

$$f(8) = 3(8)^{2/3} - 3(8)^{5/3}$$

$$= 3\left(8^{2/3} - 8^{5/3}\right) \approx 3(-28)$$

$\boxed{\text{min at } x = 8}$

Example: The total profit $P(x)$ (in thousands of dollars) from a sale of x thousand units of a new product is given by $P(x) = \ln(-x^3 + 3x^2 + 144x + 1)$ where $0 \le x \le 10$. Find the number of units that should be sold in order to maximize the total profit. What is the maximum profit?

Find the absolute max of $P(x)$ on $[0,10]$

$$P'(x) = \frac{-3x^2+6x+144}{-x^3+3x^2+144x+1} = 0$$

$$= \frac{-3}{-x^3+3x^2+144x+1}(x^2-2x-48) = 0$$

never 0

CP's

$(x-8)(x+6) = 0 \Rightarrow x = 8, -6 \leftarrow$ not in the domain

$P(0) = \ln(-(0)^3 + 3(0)^2 + 144(0) + 1) = \ln(1) = 0$

$P(8) = \ln(-(8)^3 + 3(8)^2 + 144(8) + 1) = \ln(833) \leftarrow$ max at $\boxed{\text{8 thousand units}}$

$P(10) = \ln(-10^3 + 3(10)^2 + 144(10) + 1) = \ln(741)$

The U.S. and Canadian exchange rate changes daily. The value of the U.S. dollar (in Canadian dollars) between 2000 and 2010 can be approximated by the function

[0,10]

$f(t) = 0.00316t^3 - 0.047t^2 + .114t + 1.47$

where t is the number of years since 2000. Based on this approximation, in what year during this period did the value of the U.S. dollar reach its absolute minimum?

$f'(t) = 3(0.00316)t^2 - 2(0.047)t + 0.114 = 0$

A. 2005

Divide by $3(0.00316)$: $\rightarrow t^2 - \frac{0.094t}{0.00948} + \frac{0.114}{0.00948} = 0$

B. 2006

Q-formula:

$t = \frac{-\left(\frac{-0.094}{0.00948}\right) \pm \sqrt{\left(\frac{-0.094}{0.00948}\right)^2 - 4(1)\left(\frac{0.114}{0.00948}\right)}}{2(1)}$

C. 2007

D. 2008

$f(0) = 1.47$

$f(1.415) \approx 1.55$

$f(8.501) \approx 0.98 \leftarrow$ min

$f(10) = 1.07$

$\approx 1.415, 8.501$

[redacted]

The U.S. and Canadian exchange rate changes daily. The value of the U.S. dollar (in Canadian dollars) between 2000 and 2010 can be approximated by the function

$f(t) = 0.00316t^3 - 0.047t^2 + .114t + 1.47$

where t is the number of years since 2000. What is the minimum value of the dollar during this period?

A. $1.00 Canadian

B. $1.02 Canadian

C. $.95 Canadian

D. $.98 Canadian – see last slide for work –