Suppose you are given the function $f(x) = 3x^2 + 1$ and asked to find the instantaneous rate of change at $x = 1$. Which would give you the correct answer?

A. $f(1)$

B. $f(2)$

C. $f'(1)$

D. $f'(2)$

E. $f(f'(1))$

F. $f(f'(2))$

Suppose $f(x) = 3x^2 + 1$. If you wanted to find the y –coordinate on the graph of f when $x = 2$ what would you compute?

A. $f(1)$
B. $f(2)$
C. $f'(1)$
D. $f'(2)$

[redacted]

f' is a function. Suppose you were asked to find the instantaneous rate of change of f' at $x = 2$. What would you do?

A. Find the derivative of f' and plug in $x = 2$

B. Plug $x = 2$ into f'

C. None of the above.

Notation for Higher Derivatives

The second derivative of $y = f(x)$ can be written using any of the following notations:

$$f''(x), \quad \frac{d^2y}{dx^2}, \quad \text{or} \quad D_x^2[f(x)].$$

The third derivative can be written in a similar way. For $n \geq 4$, the nth derivative is written $f^{(n)}(x)$.

Find $f''(1)$ if $f(x) = 5x^4 - 4x^3 + 3x$.

$f'(x) = 20x^3 - 12x^2 + 3$

$f''(x) = 60x^2 - 24x$

$f''(1) = 60(1)^2 - 24(1) = \boxed{36}$

Find the second derivative for

$f(x) = (x^3 + 1)^2$ $\longrightarrow$ $f'(x) = 2(x^3+1)(3x^2)$

$= 6x^5 + 6x^2$

$f''(x) = 30x^4 + 12x$

A. $f''(x) = 6x(x^3 + 1)^3$

B. $f''(x) = 30x^4 + 12x$

C. $f''(x) = 6x^4 + 12x$

D. $f''(x) = 30x^2 - 12x$

Example: Find all intervals where $f(x) = x^4 - 8x^3 + 18x^2$ is concave upward or downward, and find all inflection points.

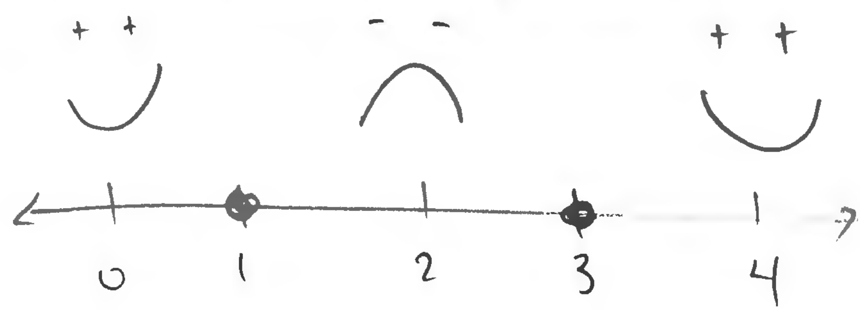
$f'(x) = 4x^3 - 24x^2 + 36x$

$f''(x) = 12x^2 - 48x + 36 = 0$

$12(x^2 - 4x + 3) = 0$

$12(x-3)(x-1) = 0$

$x = 3, 1 \leftarrow$ possible inflection points (pips)



$f''(0) = 12(0-3)(0-1) > 0$

$\quad\quad\quad\quad +\quad\; -\quad\; -$

$f''(2) = 12(2-3)(2-1) < 0$

$\quad\quad\quad\quad +\quad\; -\quad\; +$

$f''(4) = 12(4-3)(4-1) > 0$

$\quad\quad\quad\quad +\quad\; +\quad\; +$

<u>Concave up</u>: $(-\infty, 1), (3, \infty)$

<u>concave down</u>: $(1, 3)$

<u>inflection points</u>: $x = 1, 3$

Example: Find the open intervals where the function $f(x) = \dfrac{4}{x-2}$ is concave upward or concave downward. Find any inflection points.

$$f'(x) = \frac{\cancel{(x-2)(0)} - 4(1)}{(x-2)^2} = \frac{-4}{(x-2)^2}$$

$$= -4(x-2)^{-2}$$

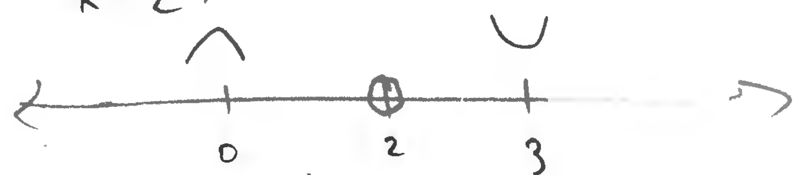
$$f''(x) = -4(-2)(x-2)^{-3}(1)$$

$$= \frac{8}{(x-2)^3} = 0$$

never

$\boxed{\text{No inflection points.}}$

But f(x) is undefined at x = 2:



$$f''(0) = \frac{8}{(0-2)^3} < 0$$

$$f''(3) = \frac{8}{(3-2)^3} > 0$$

$\Rightarrow$ Concave up: $(-\infty, 2)$

Concave down: $(2, \infty)$

Find any open intervals where the function $f(x) = \ln(x^2 + 100)$ is concave upward.

A. $(-10,10)$

B. $(-\infty,-10) \cup (10,\infty)$

C. $(-\infty,-10)$

D. $(10,\infty)$

$$f'(x) = \frac{2x}{x^2+100}$$

$$f''(x) = \frac{\overbrace{(x^2+100)(2)}^{2x^2+200} - \overbrace{2x(2x)}^{-4x^2}}{(x^2+100)^2} = \frac{-2x^2+200}{(x^2+100)^2} = 0$$

$$\Rightarrow \underbrace{\frac{-2}{(x^2+100)^2}}_{\text{never } 0}(x^2-100) = 0$$

$\Rightarrow x = \pm 10 \leftarrow$ pips

$$f''(-11) = \frac{-2}{((-11)^2+100)^2}((-11)^2-100) < 0$$

$$f''(0) = \frac{-2}{(0^2+100)^2}(0^2-100) > 0$$

$$f''(11) = \frac{-2}{(11^2+100)^2}(11^2-100) < 0$$

Example: Use the second derivative test to find where all relative extrema for

$$f(x) = 4x^3 + 7x^2 - 10x + 8$$

$f'(x) = 12x^2 + 14x - 10 = 0$

$\Rightarrow 2(6x^2 + 7x - 5) = 0$

Q-formula: $x = \dfrac{-7 \pm \sqrt{\overset{49+120}{7^2 - 4(6)(-5)}}}{2(6)}$

$f''(x) = 24x + 14$

$= \dfrac{-7 \pm 13}{12} = \dfrac{6}{12}, \dfrac{-20}{12} = \dfrac{1}{2}, \dfrac{-5}{3} \leftarrow$ CP's

2nd Deriv Test:

$f''\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right) + 14 > 0 \Rightarrow$ $x = \frac{1}{2}$ is a min

$f''\left(-\frac{5}{3}\right) = 24\left(-\frac{5}{3}\right) + 14 < 0 \Rightarrow$ $x = \frac{-5}{3}$ is a max

Use the second derivative test to find where the relative extrema occur for $f(x) = 2x^3 - 3x^2 - 72x + 15$.

$f'(x) = 6x^2 - 6x - 72 = 0$

$6(x^2 - x - 12) = 0$

$6(x-4)(x+3) = 0$

$\Rightarrow x = 4, -3 \leftarrow$ CPs

A. Relative max at $x = -3$. Relative min at $x = 4$.

$f''(x) = 12x - 6$

2nd Deriv Test!

$f''(4) = 12(4) - 6 > 0$ min

$f''(-3) = 12(-3) - 6 < 0$ max

B. Relative min at $x = -3$. Relative max at $x = 4$.

C. Relative max at $x = 3$. Relative min at $x = -4$.