MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Thurs 9 July 2015

Quiz 10: L'Hôpital's Rule (∮4.7) and Riemann Sums (∮5.1)

Directions: You have 30 minutes to complete this quiz. Collaborative and open book.

1. $\lim_{x\to 2\pi} \frac{x\sin x + x^2 - 4\pi^2}{x - 2\pi}$ $\quad \frac{0}{0}$

$$= \lim_{x\to 2\pi} \frac{(1)\sin x + x\cos x + 2x}{1} = \sin(2\pi) + 2\pi\cos(2\pi) + 2(2\pi)$$

L'Hôp

$$\boxed{= 6\pi}$$

2. $\lim_{x\to \pi/2} \frac{2\tan x}{\sec^2 x} = \lim_{x\to\frac{\pi}{2}} 2\frac{\sin x}{\cos x}\cdot\frac{\cos^{\cancel{2}} x}{1}$

$$= \lim_{x\to\frac{\pi}{2}} 2\sin x\cos x = 2\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)$$

$$\boxed{= 0}$$

(L'Hôpital's Rule not needed)

$0 \cdot \infty$

3. $\lim_{x\to 0^+}(\sin x)\sqrt{\frac{1-x}{x}} = \lim_{x\to 0^+} \frac{\sin x\sqrt{1-x}}{\sqrt{x}}$ $\frac{0}{0}$

L'Hôp

$$= \lim_{x\to 0^+} \frac{\cos x\sqrt{1-x} + \sin x\left(\frac{1}{2}\right)(1-x)^{-1/2}(-1)}{\frac{1}{2}x^{-1/2}}$$

$$= \lim_{x\to 0^+} 2\sqrt{x}\left(\cos x\sqrt{1-x} - \frac{\sin x}{2\sqrt{1-x}}\right)$$

$$= 2\sqrt{0}\left(\cos(0)\sqrt{1-0} - \frac{\sin(0)}{2\sqrt{1-0}}\right) = 0\cdot(1-0)\ \boxed{= 0}$$

$\infty - \infty$

4. $\lim_{x\to\infty}\left(x - \sqrt{x^2+1}\right) = \lim_{x\to\infty} x - \sqrt{x^2\left(1+\frac{1}{x^2}\right)}$

$$= \lim_{x\to\infty} x - x\sqrt{1+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} x\left(1-\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \lim_{x\to\infty} \frac{1-\sqrt{1+\frac{1}{x^2}}}{\frac{1}{x}}$$ let $t = \frac{1}{x}$.

$$= \lim_{t\to 0^+} \frac{1-\sqrt{1+t^2}}{t}$$ $\frac{0}{0}$

L'Hôp

$$= \lim_{t\to 0^+} \frac{-\frac{1}{2}(1+t^2)^{-1/2}\cdot 2t}{1}\ \boxed{= 0}$$

5. $\lim_{x\to 0}(1+4x)^{\frac{3}{x}}$ $\rightsquigarrow$ Put $L = \ln\left(\lim_{x\to 0}(1+4x)^{3/x}\right)$

$$= \lim_{x\to 0} \ln\left((1+4x)^{3/x}\right)$$

$$= \lim_{x\to 0} \frac{3}{x}\ln(1+4x)$$

$$= \lim_{x\to 0} \frac{3\ln(1+4x)}{x} \quad \frac{0}{0}$$

L'Hôp $\longrightarrow$ $$= \lim_{x\to 0} \frac{3\cdot\frac{1}{1+4x}\cdot 4}{1}$$

$$= \lim_{x\to 0} \frac{12}{1+4x} = 12$$

$$\Rightarrow \lim_{x\to 0}(1+4x)^{\frac{3}{x}} \boxed{= e^{12}}$$

6. Write down the left, right, and midpoint Riemann sums approximating the area under the curve

$$f(x) = \frac{1}{x} \quad \text{on the interval} \quad [1,6]$$

using four rectangles. Your answers should be in Σ-notation. You don't have to compute the sums.

$a = 1 \quad n = 4 \quad \Delta x = \frac{b-a}{n} = \frac{5}{4}$

$b = 6$

left: $\sum_{k=1}^{4} f\left(1+(k-1)\frac{5}{4}\right)\frac{5}{4}$

(left)

$$= \sum_{k=1}^{4} \frac{\frac{5}{4}}{1+(k-1)\frac{5}{4}} = \sum_{k=1}^{4} \frac{5}{4+5(k-1)} \boxed{= \sum_{k=1}^{4} \frac{5}{5k-1}}$$

right: $\sum_{k=1}^{4} f\left(1 + k \cdot \frac{5}{4}\right)\frac{5}{4} = \sum_{k=1}^{4} \frac{\frac{5}{4}}{1 + \frac{5}{4}k}$

$$\boxed{= \sum_{k=1}^{4} \frac{5}{4 + 5k}}$$

(right)

midpoint: $\sum_{k=1}^{4} f\left(1 + \left(k - \frac{1}{2}\right) \cdot \frac{5}{4}\right) \cdot \frac{5}{4}$

$$= \sum_{k=1}^{4} \frac{\frac{5}{4}}{1 + \left(k - \frac{1}{2}\right)\frac{5}{4}} = \sum_{k=1}^{4} \frac{5}{4 + 5\left(k - \frac{1}{2}\right)}$$

$$= \sum_{k=1}^{4} \frac{10}{8 + 10k - 5}$$

$$\boxed{= \sum_{k=1}^{4} \frac{10}{3 + 10k}}$$

(midpoint)