Section 3.1 – Powers and Polynomials

1. Find the derivative of each of the following functions. You may assume that p and q are constants.

   (a) $y = 3x^2 + 2x + 1$

   $y' = 6x + 2$

   (b) $y = 5x^2 + \frac{1}{x}$

   $y' = 10x - \frac{1}{x^2}$

   (c) $f(t) = \dfrac{t^2 + pt + q}{\sqrt{t}}$

   $= t^{3/2} + pt^{1/2} + qt^{-1/2}$

   $f'(t) = \frac{3}{2}t^{1/2} + \frac{p}{2}t^{-1/2} - \frac{q}{2}t^{-3/2}$

2. Find the equation of the tangent line to the graph of the function $f(x) = 1/x$ at $x = 2$.

   $f'(2) = \frac{-1}{(2)^2} = \frac{-1}{4}$

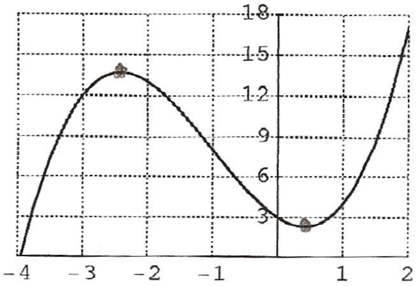
   $f(2) = \frac{1}{2}$

   Then

   $y - \frac{1}{2} = -\frac{1}{4}(x-2)$

   $y = -\frac{1}{4}x + 1$

3. Given to the right is the graph of the function $f(x) = x^3 + 3x^2 - 3x + 3$.



(a) Graphically estimate the value(s) of x at which f has a horizontal tangent line. Then, use derivatives to find more accurate estimates.

$x \doteq -2.5, 0.5$

$f'(x) = 3x^2 + 6x - 3 = 0$

$x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4-4(-1)}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2} \doteq -2.414, 0.414$

(b) Find all values of x at which the tangent line to f is parallel to the line $y = 6x + 6$.

Want $f'(x) = 6 = 3x^2 + 6x - 3$

$2 = x^2 + 2x - 1$

$0 = x^2 + 2x - 3$

$= (x+3)(x-1)$

So $x = -3, 1$

4. (Taken from *Hughes-Hallett, et. al.*) At a time t seconds after it is thrown up in the air, a tomato is at a height of $f(t) = -4.9t^2 + 25t + 3$ meters.

(a) What is the average velocity of the tomato during the first 2 seconds? Give units.

$\frac{f(2) - f(0)}{2 - 0} = 15.2 \text{ m/s}$

(b) Find the instantaneous velocity of the tomato at $t = 2$. Give units.

$f'(2) = -9.8(2) + 25 = 5.4 \text{ m/s}$

(c) What is the acceleration at $t = 2$?

$f''(2) = -9.8 \text{ m/s}^2$

(d) How high does the tomato go?

f is an upside-down parabola, so its maximum height is when $f'(x) = 0 = -9.8x + 25$

$x \doteq 2.551$

so $f(2.551) \doteq 34.888$ m