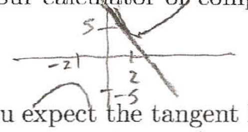
# Chapter 3 – Derivative Practice and Summary

1. (a) Find the equation of the tangent line to $f(x) = x + \frac{4}{x}$ at the point $(1, 5)$.

$f'(x) = 1 - \frac{4}{x^2}$ ; $f'(1) = 1 - 4 = -3$ ; $y - 5 = -3(x-1)$

$y = -3x + 8$

(b) Use your calculator or computer to graph $f(x)$ and the tangent line you found to check your work.



(c) Do you expect the tangent line approximation to $f(x)$ at $x = 1$ to be an over- or under-estimate? Why?

An overestimate, since the slope is negative and the line lies under the graph of $f(x)$.

(d) Use the tangent line to estimate the value of the function at $x = 1.1$.

$f(1.1) \doteq -3(1.1) + 8 = 4.7$

(e) Compare the actual value of $f(1.1)$ to your estimate in part (d). Does your result confirm your prediction in part (c)?

$f(1.1) = 1.1 + \frac{4}{1.1} \doteq 4.736$ ; yes

2. Find the first derivative of the following functions.

(a) $y = \dfrac{\tan x}{x}$ $\quad y' = \dfrac{x\sec^2 x - \tan x}{x^2}$

(b) $y = \sin(e^x)$ $\quad y' = e^x \cos e^x$

(c) $y = e^{\sqrt{x}}$ $\quad y' = \dfrac{1}{2\sqrt{x}} e^{\sqrt{x}}$

(d) $y = \ln(\cos(\theta^2))$ $\quad y' = \dfrac{-2\theta\sin(\theta^2)}{\cos(\theta^2)} = -2\theta\tan(\theta^2)$

3. Find $\frac{dy}{dx}$ by implicit differentiation: $x^4 + y^4 = 16$.

$4x^3 + 4y^3 y' = 16$

$\dfrac{dy}{dx} = \dfrac{16 - 4x^3}{4y^3}$

4. Find an equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.

$2yy' = 3x^2(2-x) + x^3(-1)$

$y' = \dfrac{3x^2(2-x) + x^3(-1)}{2y}$

@ $(1,1)$:

slope $= \dfrac{3(1)(2-1) + (1^3)(-1)}{2(1)} = 1$

$y - 1 = x - 1$

$y = x$