Thurs 9 July

- Exam 3 redo
  - You may collaborate and use resources, including office hours.
  - The deadline to submit is 4pm Friday. NO EXCEPTIONS.
  - If you don't redo your problems, the grade sticks.
- FINAL! is next Thursday, in class. Covers 3.10-5.5.
- Quiz 11 Friday (tomorrow) covers 4.8,5.2, some 5.3.
- this weekend: all MLP assignments reopened; close on Wed night

$\oint$ 5.4 Working with Integrals

Recall the definition of an even function,

$$f(-x) = f(x),$$

and of an odd function,

$$f(-x) = -f(x).$$

These properties are just examples of ways we can simplify integrals.

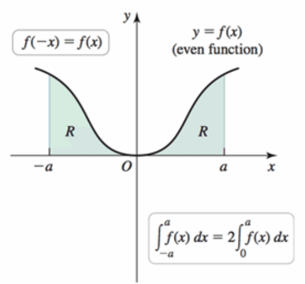
Integrating Even Functions

Even functions are symmetric about the y-axis. So

$$\int_{-a}^{0} f(x)\ dx = \int_{0}^{a} f(x)\ dx$$

i.e., the area under the curve to the left of the y-axis is equal to the area under the curve to the right.

Integrating Even Functions (cont.)



Hence, $\displaystyle\int_{-a}^{a} f(x)\;dx = 2\int_{0}^{a} f(x)\;dx$ for even functions.

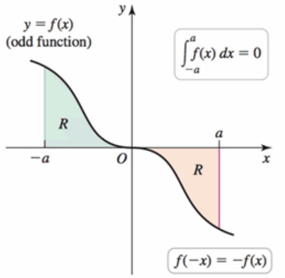
Integrating Odd Functions

On the other hand, odd functions have 180° rotation symmetry about the origin. So

$$\int_{-a}^{0} f(x)\ dx = -\int_{0}^{a} f(x)\ dx$$

i.e., the area under the curve to the left of the origin is the negative of the area under the curve to the right of the origin.

Integrating Odd Functions (cont.)



Hence, $\int_{-a}^{a} f(x)\;dx = 0$ for odd functions.

Exercise

Evaluate the following integrals using the properties of even and odd functions:

1. $\displaystyle\int_{-4}^{4} (3x^2 - x)\ dx$
2. $\displaystyle\int_{-1}^{1} (1 - |x|)\ dx$
3. $\displaystyle\int_{-\pi}^{\pi} \sin x\ dx$

Average Value of a Function

To find the average of $f(x)$ between points a and b, we can estimate by choosing y-values $\overline{x}_k$. If we take n of them, then the average is:

$$\overline{f} \approx \frac{f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)}{n}$$

Average Value of a Function (cont.)

Since $n = \frac{b-a}{\Delta x}$, we have

$$\begin{aligned}
\frac{f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)}{n} &= \frac{f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)}{\left(\frac{b-a}{\Delta x}\right)} \\
&= \frac{1}{b-a}\left(f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)\right)\Delta x \\
&= \frac{1}{b-a}\sum_{k=1}^{n} f(\overline{x}_k)\Delta x.
\end{aligned}$$

The estimate gets more accurate, the more y-values we take. We let $n \to \infty$.

Average Value of a Function (cont.)

Then the average value of an integrable function f on the interval $[a, b]$ is

$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\ dx.$$

Exercise

Find the average value of the function $f(x) = x(1-x)$ on the interval $[0, 1]$.

Theorem

If f is continuous on $[a,b]$, then there is at least one point c in $[a,b]$ such that

$$f(c) = \overline{f} = \frac{1}{b-a}\int_a^b f(x)\ dx.$$

In other words, the horizontal line $y = \overline{f} = f(c)$ intersects the graph of f for some point c in $[a,b]$. (See Figure 5.54)

Exercise

Find or approximate the point(s) at which $f(x) = x^2 - 2x + 1$ equals its average value on $[0, 2]$.

5.4 Book Problems

7-27 (odds), 31-35 (odds)