Quiz 7 Solution Guide

① $f(x) = \ln\left(\frac{x-1}{x+1}\right)$

$$f'(x) = \frac{1}{\frac{x-1}{x+1}} \left(\frac{(1)(x+1) - (x-1)(1)}{(x+1)^2} \right)$$

② $f(x) = (2x^3 - 3x + 12) \cdot 4^x$

$$f'(x) = (6x-3) \cdot 4^x + (2x^3 - 3x + 12)(4^x \ln 4)$$

③ $f(x) = \cos(3^x)$

$$f'(x) = -\sin(3^x)(3^x \ln 3)$$

④ $f(x) = \left(1 + \frac{1}{x}\right)^{2x}$

Must use logarithmic differentiation.

$$\ln(f(x)) = \ln\left(\left(1 + \frac{1}{x}\right)^{2x}\right)$$

$$\ln(f(x)) = 2x \ln\left(1+\frac{1}{x}\right)$$

$$\frac{d}{dx}\left(\ln(f(x))\right) = \frac{d}{dx}\left(2x \ln\left(1+\frac{1}{x}\right)\right)$$

$$\frac{f'(x)}{f(x)} = (2)\ln\left(1+\frac{1}{x}\right) + 2x\left(\frac{1}{1+\frac{1}{x}}\right)\left(-\frac{1}{x^2}\right)$$

$$f'(x) = 2x\ln\left(1+\frac{1}{x}\right)\left[2\ln\left(1+\frac{1}{x}\right) + 2x\left(\frac{1}{1+\frac{1}{x}}\right)\left(\frac{-1}{x^2}\right)\right]$$

5 $f(x) = \sin^{-1}\left(e^{\sin x}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(e^{\sin x}\right)^2}}\left(e^{\sin x}(\cos x)\right)$$

6 $f(x) = \sin\left(\operatorname{arcsec}\left(2w^4\right)\right)$

a) $f'(x) = 0$

b) <u>Note</u>: for $f(x) = \sin\left(\operatorname{arcsec}\left(2x^4\right)\right)$

$$f'(x) = \cos(\operatorname{arcsec}(2x^4))\left(\frac{1}{|2x^4|\sqrt{(2x^4)^2-1}}\right)(8x^3)$$

⑦ $f(x) = \cot^{-1}\left(\frac{1}{x^2+1}\right)$

$$f'(x) = \left(\frac{-1}{1+\left(\frac{1}{x^2+1}\right)^2}\right)\left(\frac{(0)(x^2+1)-(1)(2x)}{(x^2+1)^2}\right)$$

⑧ $f(x) = \arctan(x^2+16)$

$$f'(x) = \left(\frac{1}{1+(x^2+16)^2}\right)(2x)$$

⑨ $f(x) = \dfrac{(x+1)^{\frac{2}{3}}(x-4)^{\frac{5}{2}}}{(5x+3)^{2/3}}$

a) Start using quotient rule...

$$f'(x) = \frac{\frac{d}{dx}\left[(x+1)^{\frac{2}{3}}(x-4)^{\frac{5}{2}}\right](5x+3)^{\frac{2}{3}} - (x+1)^{\frac{2}{3}}(x-4)^{\frac{5}{2}}\left(\frac{2}{3}\right)(5x+3)^{-\frac{1}{3}}(}{\left[(5x+3)^{2/3}\right]^2}$$

$$= \frac{\left[\left(\frac{2}{3}\right)(x+1)^{-\frac{1}{3}}(1)(x-4)^{\frac{5}{2}} + (x+1)^{\frac{2}{3}}\left(\frac{5}{2}\right)(x-4)^{\frac{3}{2}}(1)\right](5x+3)^{\frac{2}{3}} - (x+1)^{2/3}(x-4)^{5/2}\left(\frac{2}{3}\right)(5x+3)^{-1/3}(5}{\left[(5x+3)^{2/3}\right]^2}$$

b) Can use logarithmic differentiation...

$$\ln(f(x)) = \ln\left[\frac{(x+1)^{2/3}(x-4)^{5/2}}{(5x+3)^{2/3}}\right]$$

$$\ln(f(x)) = \frac{2}{3}\ln(x+1) + \frac{5}{2}\ln(x-4) - \frac{2}{3}\ln(5x+3)$$

$$\frac{f'(x)}{f(x)} = \frac{2}{3}\left(\frac{1}{x+1}\right)(1) + \frac{5}{2}\left(\frac{1}{x-4}\right)(1) - \frac{2}{3}\left(\frac{1}{5x+3}\right)(5)$$

$$f'(x) = \left[\frac{(x+1)^{2/3}(x-4)^{5/2}}{(5x+3)^{2/3}}\right]\left(\left(\frac{2}{3}\right)\left(\frac{1}{x+1}\right) + \left(\frac{5}{2}\right)\left(\frac{1}{x-4}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{5x+3}\right)(5)\right)$$

(10) $$\frac{d^{100}}{dx^{100}}\left(2^x\right) = 2^x(\ln 2)^{100}$$

<u>Why?</u>

$$\frac{d}{dx}\left(2^x\right) = 2^x \cdot \ln 2$$

$$\frac{d^2}{dx^2}\left(2^x\right) = \frac{d}{dx}\left(2^x \ln 2\right) = 2^x(\ln 2)(\ln 2)$$

$$= 2^x(\ln 2)^2$$

$$\frac{d^3}{dx^3}\left(2^x\right) = \frac{d}{dx}\left(2^x(\ln 2)^2\right) = 2^x(\ln 2)(\ln 2)^2$$

$$= 2^x(\ln 2)^3$$

$$\frac{d^4}{dx^4}\left(2^x\right) = \frac{d}{dx}\left(2^x(\ln 2)^3\right) = 2^x(\ln 2)(\ln 2)^3$$

$$= 2^x(\ln 2)^4$$

$$\vdots$$