Find $\lim\limits_{x\to 5} \dfrac{x^2-3x-10}{x-5}$.

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}} = \lim_{x\to 5} (x+2) = 5+2 = \boxed{7}$$

b/c $x+2$ is a polynomial.

Remarks:

- For rational functions, only factor if $x \to$ number. If $x \to \pm\infty$ then divide the numerator and denominator by the highest x-power that appears in the denominator.
- $\dfrac{x^2-3x-10}{x-5}$ and $x+2$ are <u>not</u> the same function!!! But they are equal for all x "sufficiently close" to 2 (see ⑦ in the Limit Laws).
- You must include $\lim\limits_{x\to 5}(\ \ )$ in every step of the problem until you actually take the limit.

Conjugate Trick

$$\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36} = \lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}\left(\frac{\sqrt{x}+6}{\sqrt{x}+6}\right) = \lim_{x\to 36}\frac{\cancel{x-36}}{(\cancel{x-36})(\sqrt{x}+6)}$$

$$= \lim_{x\to 36}\frac{1}{\sqrt{x}+6} = \frac{\lim_{x\to 36} 1}{\sqrt{\lim_{x\to 36} x}+\lim_{x\to 36} 6} = \frac{1}{\sqrt{36}+6} = \boxed{\frac{1}{12}}$$

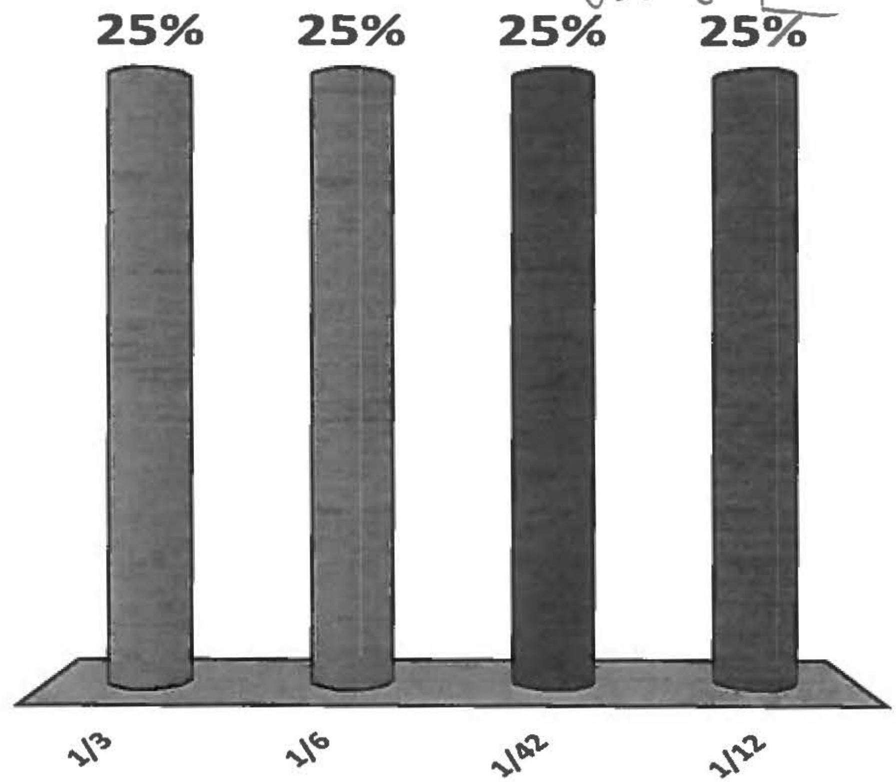
This is a difference of squares: $(\sqrt{x})^2-6^2=(\sqrt{x}-6)(\sqrt{x}+6)$

A. 1/3

B. 1/6

C. 1/42

D. 1/12



True or False:

If $\lim_{x \to c} f(x) = L$ and $f(c) = L$,
then $f(c)$ is continuous at c.

True: This is the definition of continuous.

A rational function can have infinitely many x-values at which it is not continuous.

False: Rational functions are discontinuous when the denominator is zero. Such points are the roots of a polynomial $q(x)$, and there cannot be more roots than $\deg(q)$, which is a finite number.

Find all values of x where the piecewise function is discontinuous.

$$f(x) = \begin{cases} 5x-4 & \text{if } x<0 \\ x^2 & \text{if } 0 \le x \le 3 \\ x+6 & \text{if } x>3 \end{cases}$$

possible discontinuities are at $x=0,3$ because each piece of $f(x)$ is a polynomial, so is continuous at all the points in between.

Use the Continuity ✓List:

$c=0$:

① $f(c)=f(0)$
$=0^2=0$

② $\lim_{x\to c} f(x)$ exists if the 2-sided limits do:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(5x-4) = 5(0)-4 = -4$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0^2 = 0$

not equal

So $\lim_{x\to c} f(x) = \lim_{x\to 0} f(x)$ does not exist.

$c=3$:

① $f(c)=f(3)=3^2=9$

② $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 = 3^2 = 9$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+6 = 3+6 = 9$

③ $\lim_{x\to c} f(x) = 9 = f(c)$ ✓

$\Rightarrow f$ is cont. at $x=3$

$\boxed{f \text{ is discontinuous at } x=0}$

Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 & x > 2 \end{cases}$$

Use the Continuity $\checkmark$List for $c=2$:

A. 2

B. 3

C. 5

D. 1

① $f(2) = 2^3 = 8$

② $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^3 = 2^3 = 8$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax^2 = a(2^2) = 4a.$

$\Rightarrow \lim_{x \to 2} f(x) = 8 = 4a$

$\Leftrightarrow \boxed{a = 2}$

③ $\lim_{x \to 2} f(x) = 8 = f(2)$ ✓