Mon 15 June

- How was last week? We will have a Chain Rule quiz to find out.
- Questions about the exam, last week's materials, etc.?
- Midterm this Friday. Stay tuned for more info.
- Quiz solutions are up, but please bear with me on getting the grading done.

Mon 15 June (cont.)

- If you're interested, here is what I was up to last week: http://www.ams.org/programs/research-communities/mrc-15. The last program like this (specific to my field) was around 5 years ago. Very rewarding but I missed you all!

1. Week 4: 15-19 June
   - Monday 15 June

∮3.7 Implicit Differentiation

- Higher Order Derivatives
- Power Rule for Rational Exponents
- Book Problems

$\oint$ 3.7 Implicit Differentiation

Up to now, we have calculated derivatives of functions of the form $y = f(x)$, where y is defined **explicitly** in terms of x. In this section, we examine relationships between variables that are **implicit** in nature, meaning that y either is not defined explicitly in terms of x or cannot be easily manipulated to solve for y in terms of x.

The goal of **implicit differentiation** is to find a single expression for the derivative directly from an equation of the form $F(x, y) = 0$ without first solving for y.

Example

Calculate $\frac{dy}{dx}$ directly from the equation for the circle

$$x^2 + y^2 = 9.$$

Solution: To remind ourselves that x is our independent variable and that we are differentiating with respect to x, we can replace y with $y(x)$:

$$x^2 + (y(x))^2 = 9.$$

Now differentiate each term with respect to x:

$$\frac{d}{dx}(x^2) + \frac{d}{dx}((y(x))^2) = \frac{d}{dx}(9).$$

By the Chain Rule, $\frac{d}{dx}((y(x))^2) = 2y(x)y'(x)$ (Version 2), or $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ (Version 1). So

$$\begin{aligned} 2x + 2y\frac{dy}{dx} &= 0 \\ \implies \frac{dy}{dx} &= \frac{-2x}{2y} \\ &= -\frac{x}{y}. \end{aligned}$$

The derivative is a function of x and y, meaning we can write it in the form

$$F(x,y) = -\frac{x}{y}.$$

To find slopes of tangent lines at various points along the circle we just plug in the coordinates. For example, the slope of the tangent line at (0,3) is

$$\left.\frac{dy}{dx}\right|_{(x,y)=(0,3)} = -\frac{0}{3} = 0.$$

The slope of the tangent line at $(1, 2\sqrt{2})$ is

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,2\sqrt{2})} = -\frac{1}{2\sqrt{2}}.$$

Question

The following functions are implicitly defined:

- $x^2 + y^2 = 9$
- $x + y^3 - xy = 4$
- $\cos(x - y) + \sin y = \sqrt{2}$

For each of these functions, how would you find $\frac{dy}{dx}$?

Exercise

Find $\frac{dy}{dx}$ for $xy + y^3 = 1$.

Exercise

Find an equation of the line tangent to the curve $x^4 - x^2y + y^4 = 1$ at the point $(-1, 1)$.

Example

Find $\dfrac{d^2y}{dx^2}$ if $xy + y^3 = 1$.

Implicit differentiation also allows us to extend the power rule to rational exponents: Assume p and q are integers with $q \neq 0$. Then

$$\frac{d}{dx}(x^{\frac{p}{q}}) = \frac{p}{q}x^{\frac{p}{q}-1}$$

(provided $x \geq 0$ when q is even and $\frac{p}{q}$ is in lowest terms).

Exercise

Prove it.

3.7 Book Problems

5-21 (odds), 27-45 (odds)