MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Tues 14 July 2015

Quiz 12: Integrals (∮5.4-5.5)

Directions: You have 30 minutes to complete this quiz. Collaborative and open book.

1. Evaluate the following integral. If possible, use symmetry.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Big(\underbrace{\cos(2x)}_{\text{even}} + \underbrace{\cos x \sin x}_{\text{odd}} - \underbrace{3\sin(x^5)}_{\text{odd}}\Big)\ dx$$

$$= 2\int_0^{\pi/2} \cos(2x)\,dx$$

$$= 2\left(\frac{1}{2}\sin(2x)\right)\Bigg|_0^{\pi/2} = \sin\left(2\cdot\frac{\pi}{2}\right) - \sin(0) = 0 - 0 \boxed{= 0}$$

2. Find the point(s) at which the given function equals its average value on the given interval.

$$f(x) = \frac{\pi}{4}\sin x \quad \text{on} \quad [0,\pi].$$

average: $\bar{f} = \dfrac{1}{\pi - 0}\displaystyle\int_0^{\pi} \frac{\pi}{4}\sin x\,dx$

$$= \frac{1}{4}\int_0^{\pi} \sin x\,dx = -\frac{1}{4}\cos x\Big|_0^{\pi}$$

$$= -\frac{1}{4}\big(-1-(1)\big) = -\frac{1}{4}(-2) = \frac{1}{2}$$

Solve for "c":

$$\frac{\pi}{4}\sin(c) = \frac{1}{2}$$

$$\sin(c) = \frac{2}{\pi}$$

$$\boxed{\begin{aligned} c &= \arcsin\left(\frac{2}{\pi}\right) \approx 0.69, \\ &\pi - \arcsin\frac{2}{\pi} \approx 2.45 \end{aligned}}$$

in the interval

3. Find the area of the region bounded by the graph of

$$f(x) = \frac{x}{\sqrt{x^2-9}}$$

and the x-axis between $x = 4$ and $x = 5$.

$\int_4^5 \frac{x}{\sqrt{x^2-9}}\,dx$ $\quad u = x^2 - 9$ $\quad \Rightarrow u(4) = 4^2 - 9 = 7$

$du = 2x\,dx$ $\quad u(5) = 5^2 - 9 = 16$

$\frac{du}{2} = x\,dx$

$= \int_7^{16} \frac{du}{2\sqrt{u}} = \frac{1}{2}\int_7^{16} u^{-1/2}\,du = \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\Big|_7^{16} = 16^{1/2} - 7^{1/2} = \boxed{4 - \sqrt{7}}$

4. Evaluate the following indefinite integrals.

(a) $\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$ $\quad u = \sqrt{x} + 1$

$du = \frac{1}{2}x^{-1/2}\,dx = \frac{dx}{2\sqrt{x}}$

$= \int u^4\,du = \frac{u^5}{5} + C \boxed{= \frac{(\sqrt{x}+1)^5}{5} + C}$

(b) $\int (x+1)\sqrt{3x+2}\,dx$ $\quad u = 3x + 2 \Rightarrow x = \frac{u-2}{3}$

$= \int \left(\frac{u-2}{3} + 1\right)\sqrt{u}\,\frac{du}{3}$ $\quad du = 3\,dx$ $\quad \frac{du}{3} = dx$

$= \frac{1}{3}\int \left(\frac{u-2+3}{3}\right)\sqrt{u}\,du$

$= \frac{1}{9}\int (u+1)\sqrt{u}\,du = \frac{1}{9}\int \left(u^{3/2} + u^{1/2}\right)du = \frac{1}{9}\left(\frac{u^{5/2}}{\frac{5}{2}} + \frac{u^{3/2}}{\frac{3}{2}} + C\right)$

$\boxed{= \frac{2}{45}(3x+2)^{5/2} + \frac{2}{27}(3x+2)^{3/2} + C}$