UNIT 3, LESSON 2

Absolute and Relative Extrema

If $f'(x) > 0$ for all x in an interval (a, b), then…

A. f is increasing on (a, b).
B. f is decreasing on (a, b).
C. f' is increasing on (a, b).
D. f' is decreasing on (a, b).

Objectives:

The first derivative test is introduced as a means of finding the maximums and minimums of a function.

Students will be able to:

- Find absolute extrema on a closed interval.
- Find relative extrema using the first derivative test.
- Solve application problems.

Relative Maximum or Minimum

Let c be a number in the domain of a function f. Then $f(c)$ is a **relative** (or **local**) **maximum** for f if there exists an open interval (a, b) containing c such that

$$f(x) \leq f(c)$$

for all x in (a, b).

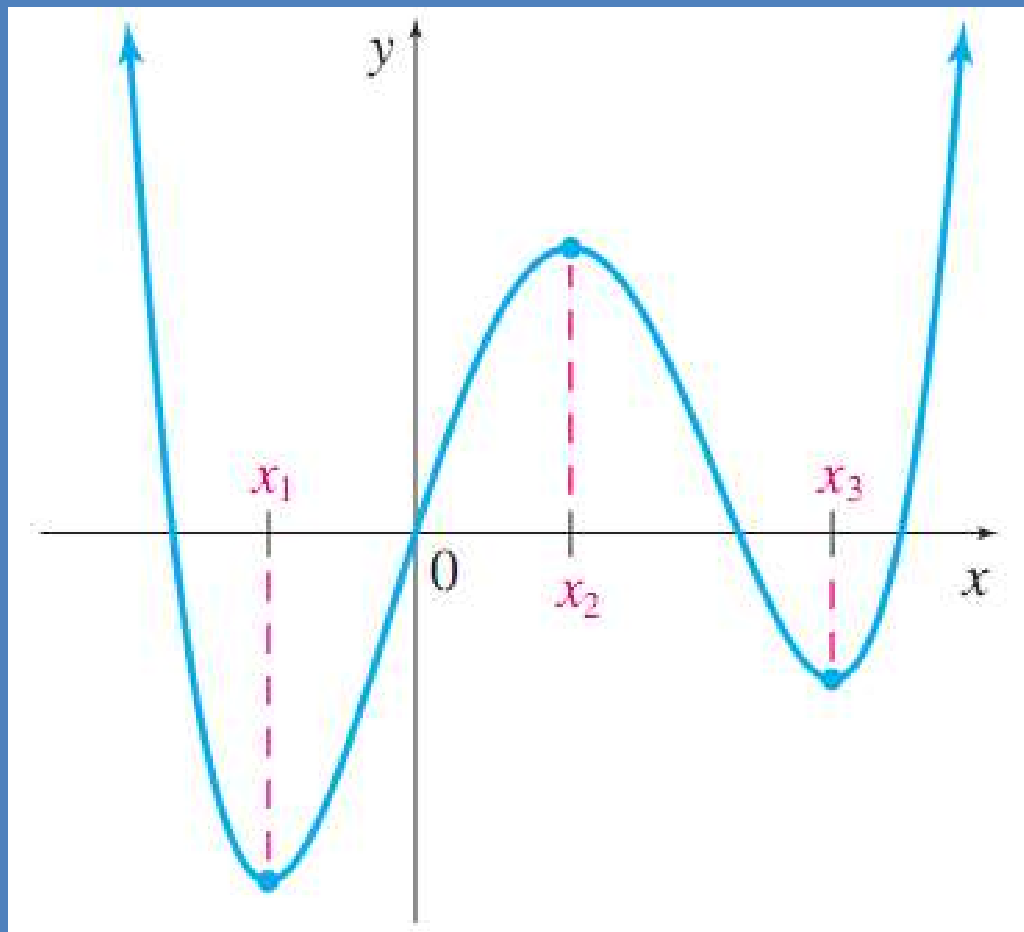
Likewise, $f(c)$ is a **relative** (or **local**) **minimum** for f if there exists an open interval (a, b) containing c such that

$$f(x) \geq f(c)$$

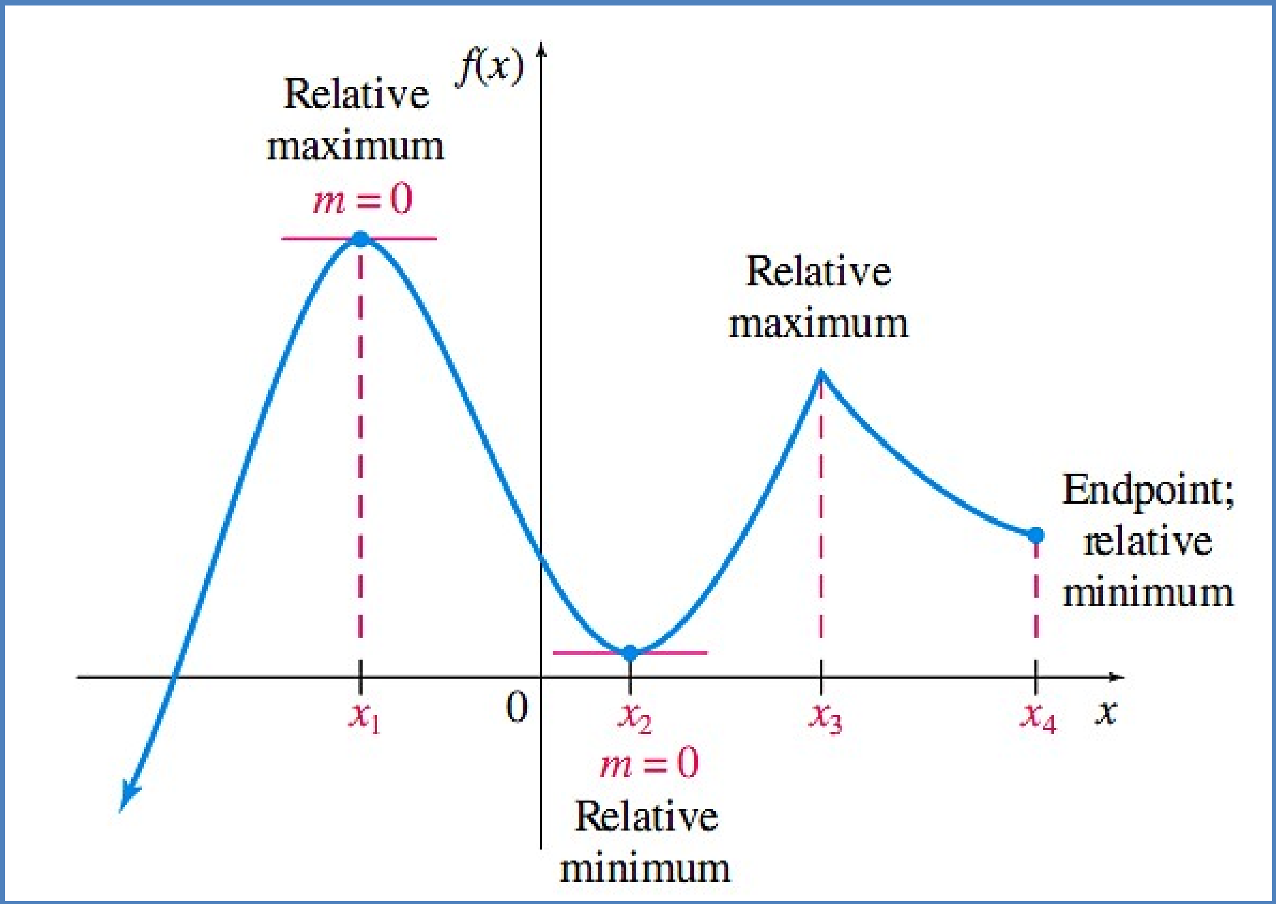
for all x in (a, b). *(continued)*

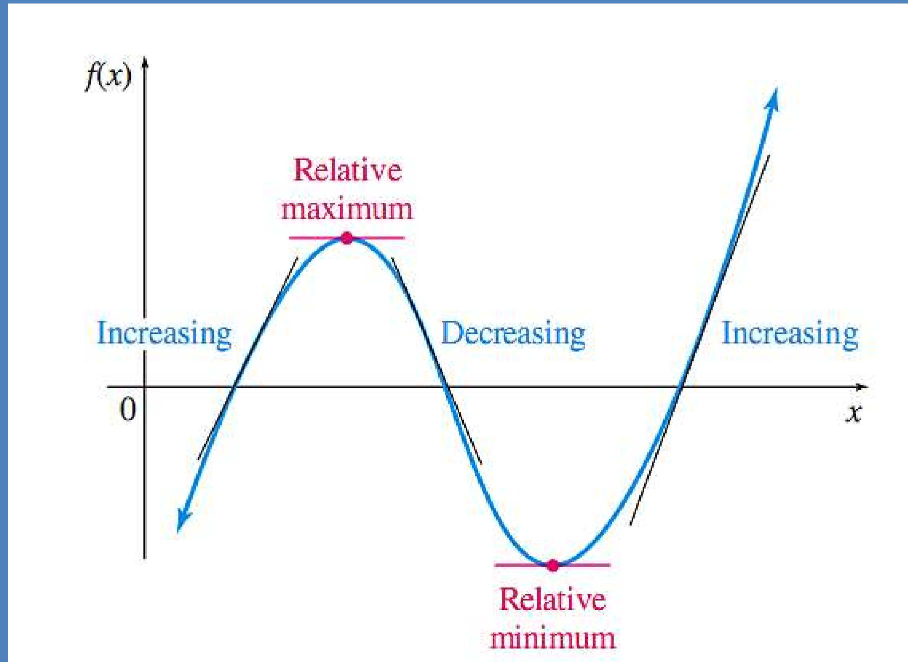
A function has a **relative** (or **local**) **extremum** (plural: **extrema**) at c if it has either a relative maximum or a relative minimum there.

If c is an endpoint of the domain of f, we only consider x in the half-open interval that is in the domain.*


y
x
0
x_1
x_2
x_3

If a function f has a relative extremum at c, then c is a critical number or c is an endpoint of the domain.





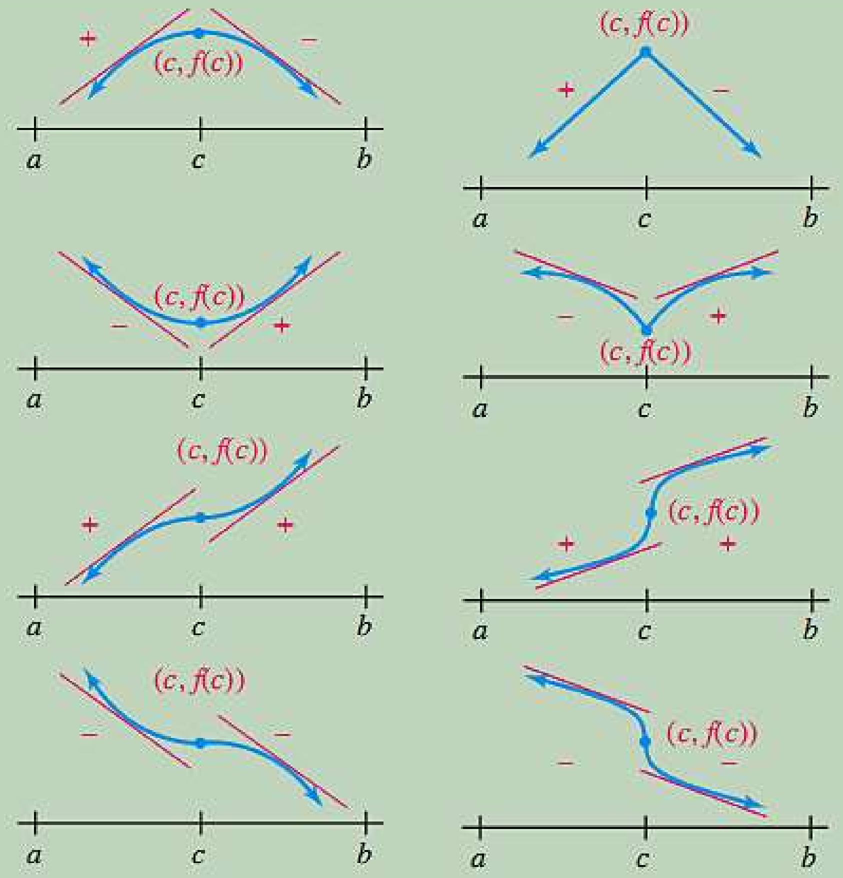
First Derivative Test

Let c be a critical number for a function f. Suppose that f is continuous on (a, b) and differentiable on (a, b) except possibly at c, and that c is the only critical number for f in (a, b).

1. $f(c)$ is a relative maximum of f if the derivative $f'(x)$ is positive in the interval (a, c) and negative in the interval (c, b).
2. $f(c)$ is a relative minimum of f if the derivative $f'(x)$ is negative in the interval (a, c) and positive in the interval (c, b).

Relative Extrema

$f(x)$ has:	Sign of f' in (a, c)	Sign of f' in (c, b)	Sketches
Relative maximum	+	−	
Relative minimum	−	+	
No relative extrema	+	+	
No relative extrema	−	−	



Example: Find all relative extrema.

$$f(x) = -x^3 - 2x^2 + 15x + 10$$

Find all relative extrema for the following function: $f(x) = 2x^3 - 3x^2 - 72x + 15$

A. Rel. max @ $x = -3$
   Rel. min @ $x = 4$
B. Rel. max @ $x = -4$
   Rel. min @ $x = 3$
C. Rel. max @ $x = 4$
   Rel. min @ $x = -3$

Absolute Maximum or Minimum

Let f be a function defined on some interval. Let c be a number in the interval. Then $f(c)$ is the **absolute maximum** of f on the interval if

$$f(x) \leq f(c)$$

for every x in the interval, and $f(c)$ is the **absolute minimum** of f on the interval if

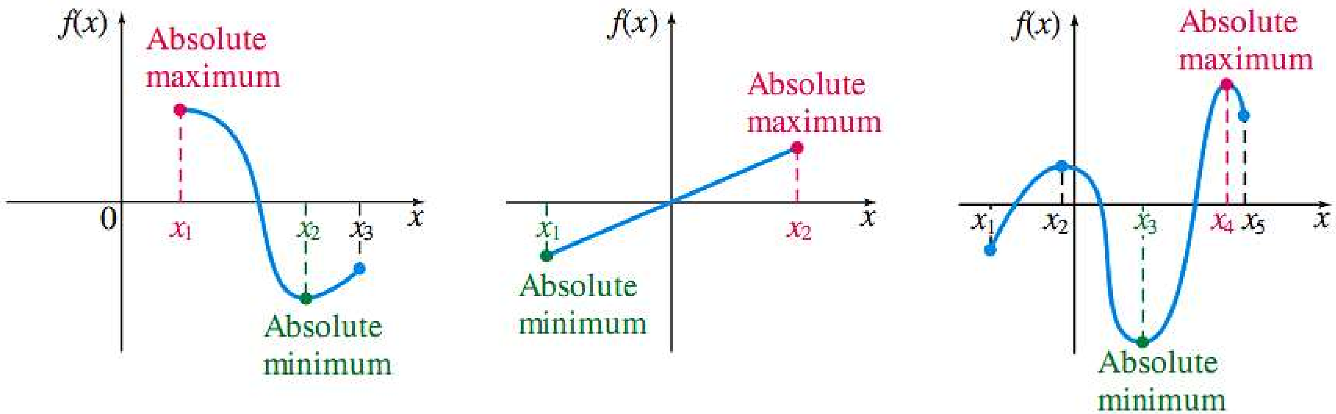
$$f(x) \geq f(c)$$

for every x in the interval.

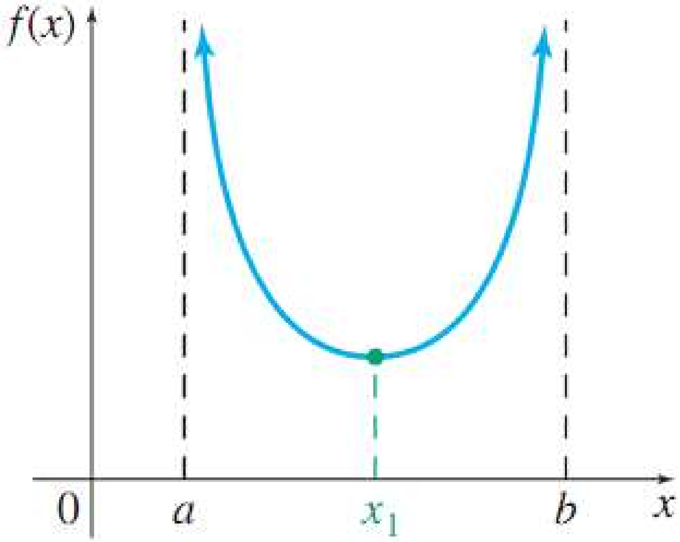
A function has an **absolute extremum** (plural: **extrema**) at c if it has either an absolute maximum or an absolute minimum there.

Extreme Value Theorem

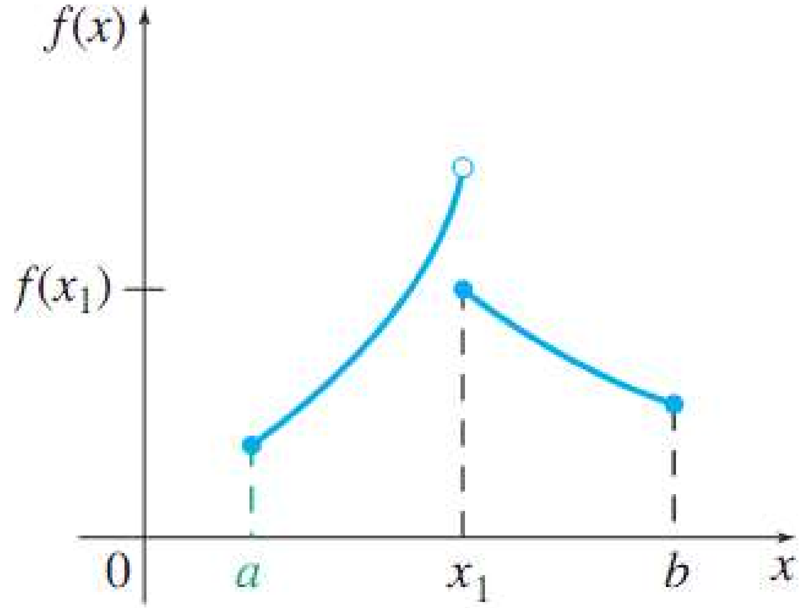
A function f that is continuous on a closed interval $[a, b]$ will have both an absolute maximum and an absolute minimum on the interval.


f(x)
Absolute
maximum
0
x₁
x₂
x₃
x
Absolute
minimum
f(x)
Absolute
maximum
x₁
x₂
x
Absolute
minimum
f(x)
Absolute
maximum
x₁
x₂
x₃
x₄
x₅
x
Absolute
minimum

(a) (b) (c)



(a)



(b)

Finding Absolute Extrema

To find absolute extrema for a function f continuous on a closed interval $[a, b]$:

1. Find all critical numbers for f in (a, b).
2. Evaluate f for all critical numbers in (a, b).
3. Evaluate f for the endpoints a and b of the interval $[a, b]$.
4. The largest value found in Step 2 or 3 is the absolute maximum for f on $[a, b]$, and the smallest value found is the absolute minimum for f on $[a, b]$.

Find the absolute extrema of the function $f(x) = 3x^{2/3} - 3x^{5/3}$ on the interval [0,8].

Critical Point Theorem

Suppose a function f is continuous on an interval I and that f has exactly one critical number in the interval I, located at $x = c$.

If f has a relative maximum at $x = c$, then this relative maximum is the absolute maximum of f on the interval I.

If f has a relative minimum at $x = c$, then this relative minimum is the absolute minimum of f on the interval I.

Example: The total profit $P(x)$(in thousands of dollars) from a sale of x thousand units of a new product is given by $P(x) = \ln(-x^3 + 3x^2 + 144x + 1)$ where $0 \leq x \leq 10$. Find the number of units that should be sold in order to maximize the total profit. What is the maximum profit?

The U.S. and Canadian exchange rate changes daily. The value of the U.S. dollar (in Canadian dollars) between 2000 and 2010 can be approximated by the function

$$f(t) = 0.00316t^3 - 0.047t^2 + .114t + 1.47$$

where t is the number of years since 2000. Based on this approximation, in what year during this period did the value of the U.S. dollar reach its absolute minimum?

A. 2005

B. 2006

C. 2007

D. 2008

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$$f(t) = 0.00316t^3 - 0.047t^2 + .114t + 1.47$$

where t is the number of years since 2000. What is the minimum value of the dollar during this period?

A. \$1.00 Canadian

B. \$1.02 Canadian

C. \$.95 Canadian

D. \$.98 Canadian