Mon 8 Sep 2014

- Quiz 2 Thurs 11 Sep covers sections 2.4-2.5
- Quiz 3 Tues 16 Sep covers sections 2.6-2.7
- EXAM 1 Fri 19 Sep covers 2.1-2.7
- comp.uark.edu/~ashleykw/CalI2014/2554f14.html
- CLICKERS used for attendance now

Question

- Informally, what does it mean to be continuous? Where do you often hear this term?

Definition of Continuity

Informal Definition: A function f is continuous at $x = a$ if the graph of f contains no holes or breaks at $x = a$. In other words, the graph near $x = a$ can be drawn without lifting a pencil.

Formal Definition: A function f is continuous at a

if $\lim_{x \to a} f(x) = f(a)$. If f is not continuous at a, then a is a point of discontinuity.

Continuity Checklist

The following principles must hold for a function *f* to be continuous at $x = a$:

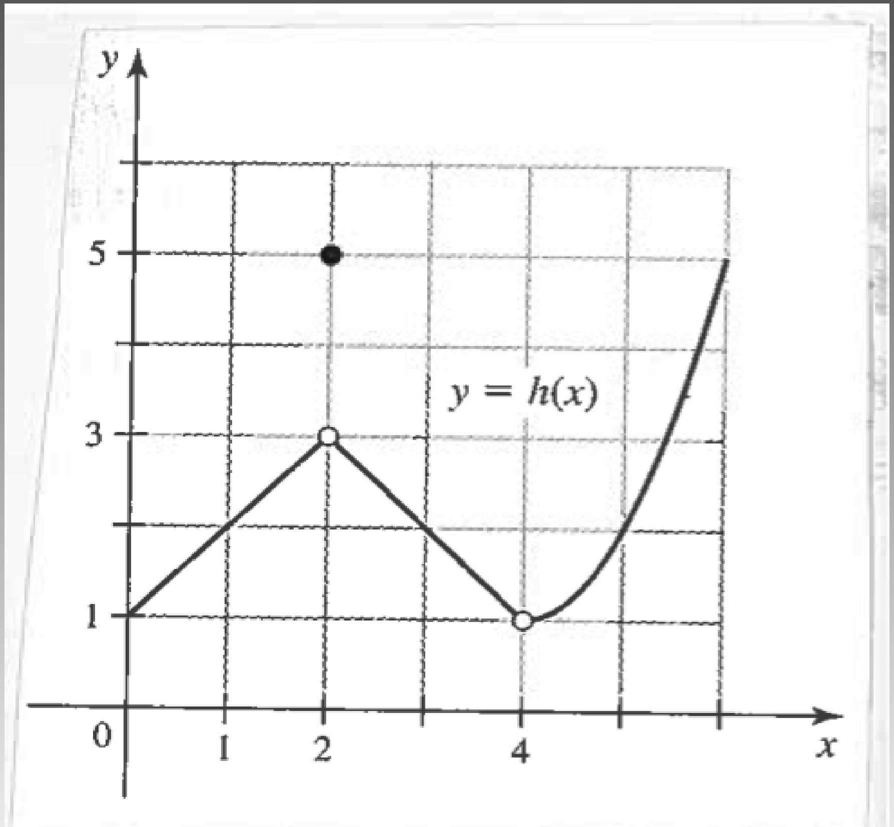
1. *f(a)* is defined (e.g., *a* is in the domain of *f*)

2. $\lim_{x \to a} f(x)$ exists

3. $\lim_{x \to a} f(x) = f(a)$ (e.g., the value of *f* equals the limit of *f* at *a*)

Example

- Where are the points of discontinuity of the function below? Which aspects of the checklist fail?



Continuity Rules

If *f* and *g* are continuous at *a*, then the following functions are also continuous at *a*. Assume *c* is a constant and $n > 0$ is an integer.

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $[f(x)]^n$
6. f / g, provided $g(a) \neq 0$

Other Continuity Rules

1. A polynomial function is continuous for all x

2. A rational function a function of the form $\frac{p}{q}$,
where p and q are polynomials) is continuous for all x for which $q(x) \neq 0$

3. If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ is continuous at a.

Continuity on an Interval

We can use the previous continuity checklists to examine the behavior of functions on a fixed interval, in particular at the endpoints.

Suppose a function f is continuous for all x in the interval (a, b). To determine the continuity at $x = a$ and at $x = b$, we introduce the concepts of left-continuity and right continuity.

Continuity on an Interval

A function *f* is continuous from the left (or left-continuous) at *a* if

$$\lim_{x \to a^-} f(x) = f(a)$$

A function *f* is continuous from the right (or right-continuous) at *a* if

$$\lim_{x \to a^+} f(x) = f(a)$$

Overall, a function *f* is continuous on an interval *I* if it is continuous at all points of *I*. If *I* contains its endpoints, continuity on *I* means continuous from the right or left at the endpoints.

Example

Let $f(x) = \begin{cases} x^3 + 4x + 1 & x \le 0 \\ 2x^3 & x > 0 \end{cases}$

1. Use the continuity checklist to show that *f* is not continuous at 0.
2. Is *f* continuous from the left or right at 0?
3. State the interval(s) of continuity

Continuity with Roots

Assume that *m* and *n* are positive integers with no common factors.

If *m* is an odd integer, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points at which *f* is continuous.

If *m* is an even integer, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points *a* at which *f* is continuous and *f(a) > 0*.

Example: Where is $f(x)=\sqrt[4]{4-x^2}$ continuous?

Continuity with Transcendental Functions

Trig Function: The basic trig functions (e.g., sin x, cos x, tan x, csc x, sec x, cot x) are all continuous at all points in their domain. (Note there are points of discontinuity where the functions are not defined)

Exponential Functions: The exponential functions b^x and e^x are continuous on all points of their domains.

Inverse Functions: If a continuous function *f* has an inverse on an interval *I*, then its inverse f^{-1} is also continuous (on the interval consisting of the point *f(x)*, where *x* is in *I*).

Wed 10 Sep 2014

- Quiz 2 tomorrow covers sections 2.4-2.5
- Quiz 3 Tues 16 Sep covers sections 2.6-2.7
- EXAM 1 Fri 19 Sep covers 2.1-2.7
- comp.uark.edu/~ashleykw/CalI2014/2554f14.html
- If you have trouble accessing the website, go to the math.uark.edu directory, click on “Ashley Wheeler” and you will get to a page with the link “Personal Webpage”. Click on that link, then scroll to the link for “Calculus I” under the heading “Courses I'm teaching”
- CLICKERS used for attendance now – no clicker, no excuse

Intermediate Value Theorem

Theorem: Suppose *f* is continuous on the interval [a,b] and *L* is a number between *f(a)* and *f(b)*. Then there is at least one number *c* in *(a,b)* satisfying *f(c) = L.*

In general, this theorem allows us to find a *y*-value (*'L'*) in between two other *y*-values (*f(a)* and *f(b)*) in the interval [a,b].

Example: Use the Intermediate Value Theorem to show that

$f(x) = -x^5 - 4x^2 + 2\sqrt{x} + 5$ has a root in the interval (0,3) (i.e., a point x=c, between 0 and 3, where f(c)=0).

HW from section 2.6

- Do problems 9-23 odds, 29-37 odds, 45, 49, 51, 53 (pgs. 103-105 in textbook)

Moving Toward a Precise Definition of Limits

So far in our dealings with limits, we have used informal terms such as "sufficiently close" and "arbitrarily large". Now we will formalize what these terms mean mathematically.

Recall: $|f(x)-L|$ and $|x-a|$ refer to the distance between *f(x)* and *L* (or between *x* and *a*).

Also recall that when we worked informally with limits, we wanted *x* to approach *a*, but not necessarily equal *a*. Likewise, we wanted *f* to be arbitrarily close to *L*, but not necessarily equal *L*.

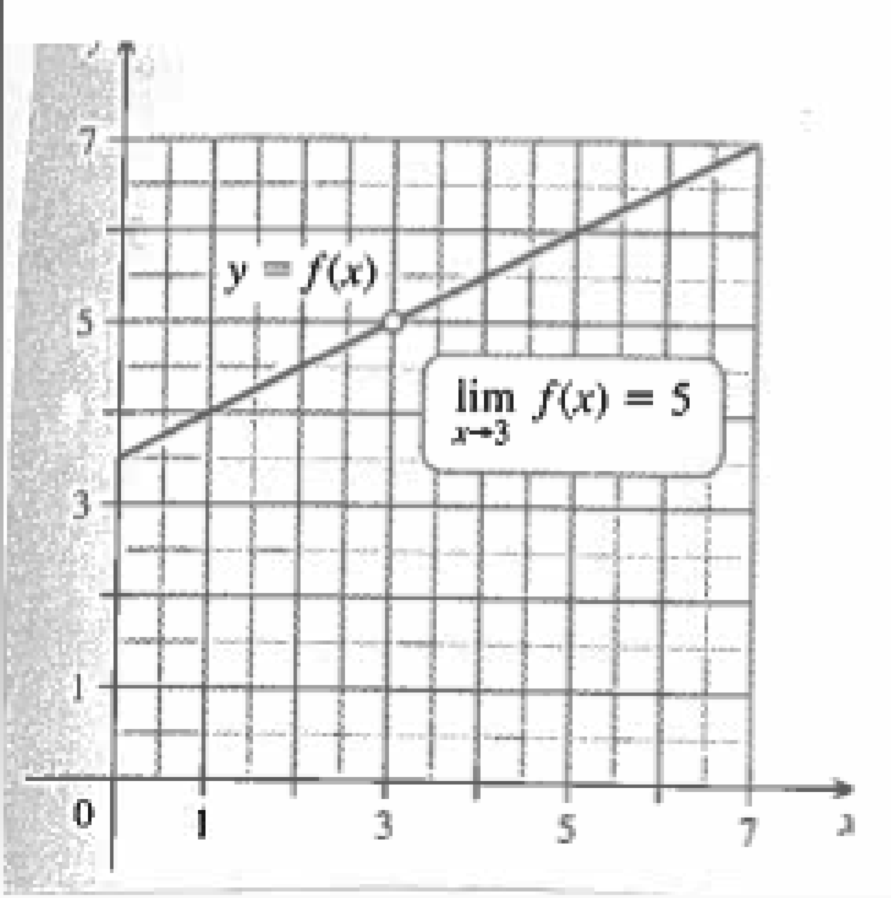
Introducing δ and ε

Recall when we worked informally with limits, we tried to get our function *f(x)* closer and closer to *L* as *x* got closer and closer to *a.* For example, we may want the distance between *f(x)* and *L* to be less than 1 (e.g., $|f(x)-L|<1$). For this to happen, how close does *x* have to be to *a*?

What if we want *f(x)* and *L* to be less than 0.5? 0.1? 0.01? In each case, how close does *x* and *a* have to be?

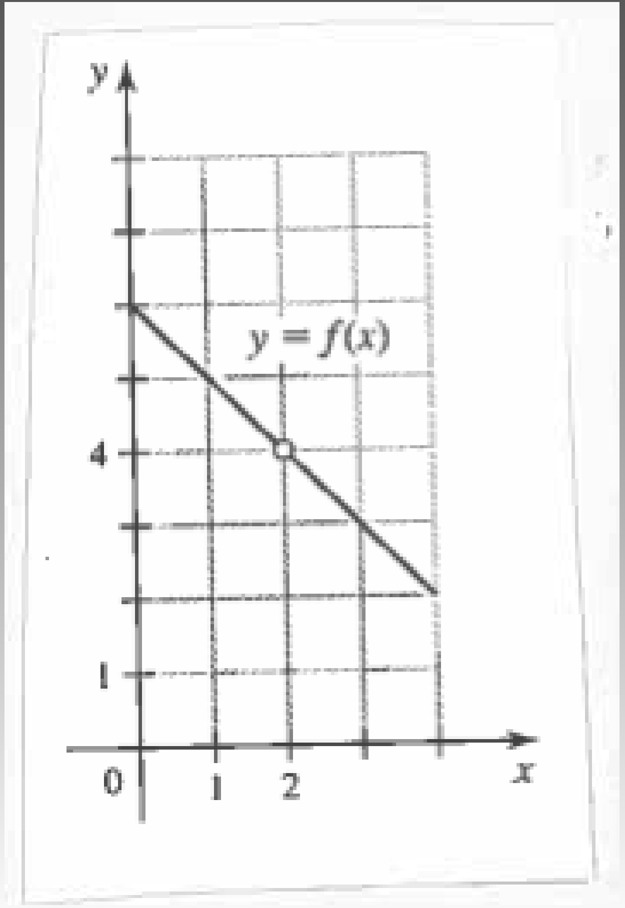
Finding δ's from a graph

Using the graph, for each ε > 0, determine a value of δ > 0 to satisfy the statement $|f(x)-5|<\varepsilon$ whenever $0<|x-3|<\delta$. Let ε = 1 and ε = 0.5



Exercise: Finding δ's from a graph

Using the graph, for each ε > 0, determine a value of δ > 0 to satisfy the statement $|f(x)-4|<\varepsilon$ whenever $0<|x-2|<\delta$. Let ε = 1 and ε = 0.5



Fri 12 Sep 2014

- Tuesday Sept 16: Quiz 3
- Friday Sept 19: Exam 1 (Chapter 2)

Formal Definition of a Limit

Assume that *f(x)* exists for all *x* in some open interval containing *a,* except possibly at *a*. We say that the limit of *f(x)* as *x* approaches *a* is *L,* written

$$\lim_{x \to a} f(x) = L$$

if for **any** number ε > 0 there is a corresponding number δ > 0 such that

$$|f(x) - L| < \varepsilon \quad \text{whenever} \quad 0 < |x - a| < \delta$$

Example

Let $f(x) = x^2 - 4$ and note that $\lim_{x \to 4} f(x) = 12$.

For ε = 1 and ε = .5, find a value for δ > 0 (using either a graph or analytical methods) so that

$|f(x) - 12| < \varepsilon$ whenever $0 < |x - 4| < \delta$

Finding a Symmetric Interval

When finding an interval around a $(a\text{-}\delta, a+\delta)$, you may find that the interval is not symmetric around a. To obtain a symmetric interval around a, use the smaller of the two δ*'s* as your distance around a.

HW from section 2.7

- Do problems 9, 11-18 all (pgs. 115-117 in textbook)

Recall from Chapter 2

Recall from the first day of class, the relationship between secant lines and tangent lines.

We said that the slope of the tangent line at a point is the limit of the slopes of the secant lines as the points get closer and closer.

P = (a, f(a)); Q = (x, f(x)).

Slope of secant line: $\dfrac{f(x)-f(a)}{x-a}$; (average rate of change)

Slope of tangent line: $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$; (instantaneous r.o.c.)

Alternate Way of Viewing Tangent Lines

Instead of looking at the points approaching one another, we can also view this as the distance between the points approaching 0. So:

P = (a, f(a)); Q = (a+h, f(a+h)).

Slope of secant line: $\dfrac{f(a+h)-f(a)}{(a+h)-a}=\dfrac{f(a+h)-f(a)}{h}$

Slope of tangent line: $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

Definition of Derivative

The slope of the tangent line for the function *f* is a function of *x*, called the derivative of *f*.

The derivative of *f* is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. If $f'(x)$ exists, we say *f* is differentiable at *x*. If *f* is differentiable at every point of an open interval *I*, we say that *f* is differentiable on *I*.

Alternate Notation

For historical and practical reasons, several notations for the derivative are used. A standard notation for change involves the Greek uppercase letter Δ (delta). So

$$\frac{f(a+h)-f(a)}{h}=\frac{f(x+\Delta x)}{\Delta x}=\frac{\Delta y}{\Delta x}$$

Additionally:

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{dy}{dx}$$

Other ways to write derivative

The following are alternative ways of writing the derivative of the function *f* at the point *x:*

$$\frac{dy}{dx}, \frac{df}{dx}, \frac{d}{dx}(f(x)), D_x(f(x)), y'(x)$$

The following are ways to notate the derivative of *f* evaluate

$$f'(a), y'(a), \left.\frac{df}{dx}\right|_{x=a}, \left.\frac{dy}{dx}\right|_{x=a}$$

Graphing the Derivative

The graph of the derivative is essentially the graph of the collection of slopes of the tangent lines of a graph. If you just have a graph (without an equation for the graph), the best you can do is approximate the graph of the derivative.

Simple checklist:

1. Note where $f'(x) = 0$
2. Note where $f'(x) > 0$ (what does this look like?)
3. Note where $f'(x) < 0$ (what does this look like?)

Differentiability and Continuity

Key points about the relationship between differentiability and continuity

- If f is differentiable at a, then f is continuous at a.
- If f is not continuous at a, then f is not differentiable at a.

NOTE: f can be continuous but not differentiable.

When is a function not differentiable at a point?

A function *f* is <u>not</u> differentiable at *a* if at least one of the following conditions holds:

1. *f* is not continuous at *a*.
2. *f* has a corner at *a* (Why does this make *f* not differentiable?)
3. *F* has a vertical tangent at *a* (Why does this make *f* not differentiable?)

Click in Your Response

If a function g is not continuous at $x = a$, then g

A. Must be undefined at $x = a$.

B. Is not differentiable at $x = a$.

C. Has an asymptote at $x = a$.

D. All of the above.

E. A and B only

HW from section 3.1

- Do problems 11-12, 19-20, 23-26, 31-33, 35-36, 39-43, 45 all (pgs. 131-133 in textbook)

- **NOTE:** We do not know any rules for differentiation yet (e.g., power rules, chain rules, etc.). In this section, you are strictly using the definition of the derivatives and definition of slope of tangent lines we have derived!!!