$s = f(t)$

= position of an object launched into the air, as a function of time

1. Highest value?
   - biggest s-value
   - Look at $s'(t)$ (velocity): Find the t-value where it goes from positive to negative and plug it into s
2. How long to hit the ground?
   - Look for t-values where $s(t) = 0$.
3. Speed on impact?
   - Find $|s'|$ when $s(t) = 0$.

$$p(t) = -0.27t^2 + 101t + 7055$$

$$0 \le t \le 10$$

1. $\frac{\Delta p}{\Delta t} = \frac{p(a+\Delta t) - p(a)}{\Delta t}$ $\quad a = 0$, $\Delta t = 10 - 0 = 10$

$$= \frac{p(10) - p(0)}{10}$$

$$= \frac{-0.27(10)^{\cancel{2}} + 101(\cancel{10}) + \cancel{7055} - [0 + 0 + \cancel{7055}]}{1\cancel{0}}$$

$= 101 - 27 = 74$ thousand / year

2. $\lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{p(2+\Delta t) - p(2)}{\Delta t}$ $\quad (a = 1997 - 1995 = 2)$

$$= p'(2)$$

Compute $p'(t) = -0.27(2t) + 101$

$p'(2) = -0.27(4) + 101 = 101 - 1.08$

$= 99.92$ thousand / year

3. The population is growing, because p' is positive when $0 \le t \le 10$. However, the term $-0.54t$ in p' shows as t increases, the population grows more slowly.

$$C(x) = -0.04x^2 + 100x + 800$$

$$0 \le x \le 1000$$

avg cost function:

$$\bar{c}(x) = \frac{C(x)}{x} = \frac{-0.04x^2 + 100x + 800}{x}$$

$$\boxed{= -0.04x + 100 + \frac{800}{x}}$$

marginal cost function:

$$C'(x) = -0.04(2x) + 100$$

$$\boxed{= -0.08x + 100}$$

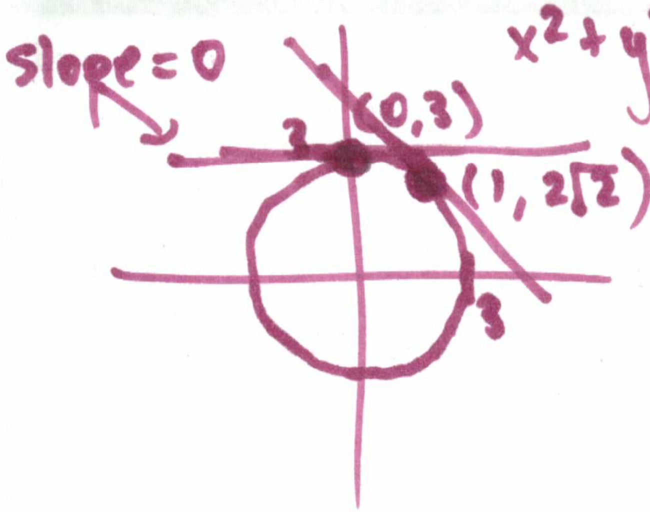
$$\bar{c}(500) = -0.04(500) + 100 + \frac{800}{(500)}$$

$$= -20 + \frac{8}{5} + 100$$

$= \$78.40$ per item, for 500 items

$$C'(500) = -0.08(500) + 100$$

$= -40 + 100 = \$60$, approximately, to produce one more item, having produced 500



We used implicit differentiation to get $\frac{dy}{dx} = \frac{-x}{y}$

$F(\underbrace{x,y})$

Plug in 2 variables instead of the usual one variable (functions $f(x)$)

Plug in both x and y coordinates to get the slope at a point:

$$\underbrace{\left.\frac{dy}{dx}\right|_{(x,y)=(0,3)}}_{\text{How to show you are plugging something into the derivative function, in Leibniz notation}} = -\frac{0}{y} = 0$$

matches the picture

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,2\sqrt{2})} = \frac{-1}{2\sqrt{2}}$$

$$\frac{d}{dx}\left[x^4 - x^2y + y^4 = 1\right] \quad \text{point } (-1,1).$$

$$4x^3 - \left[2xy + x^2\frac{dy}{dx}\right] + 4y^3\frac{dy}{dx} = 0$$

$$(-x^2 + 4y^3)\frac{dy}{dx} = -4x^3 + 2xy$$

$$\frac{dy}{dx} = \frac{-4x^3 + 2xy}{-x^2 + 4y^3}$$

Slope: $\left.\frac{dy}{dx}\right|_{(x,y)=(-1,1)} = \frac{-4(-1)^3 + 2(-1)(1)}{-(-1)^2 + 4(1)^3}$

$$= \frac{4-2}{-1+4} = \frac{2}{3}$$

tangent line:

$$\boxed{y - 1 = \frac{2}{3}(x+1)}$$

$$\frac{d^2}{dx^2}\left(xy + y^3 = 1\right)$$

$$\frac{d}{dx}\left[\frac{d}{dx}\left(xy + y^3 = 1\right)\right]$$

$$\frac{d}{dx}\left[(1)y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0\right] \rightarrow \frac{dy}{dx} = \frac{-y}{x+3y^2}$$

$$\frac{d}{dx}y + \frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{d}{dx}\left(3y^2\frac{dy}{dx}\right) = 0$$

Product Rule (Product Rule)

$$\frac{dy}{dx} + \left((1)\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right) + \left(6y\frac{dy}{dx}\right)\frac{dy}{dx} + (3y^2)\left(\frac{d^2y}{dx^2}\right) = 0$$

Solve for $\frac{d^2y}{dx^2}$:

$$(x+3y^2)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 6y\left(\frac{dy}{dx}\right)^2 = 0$$

$$\frac{d^2y}{dx^2} = \frac{-2\frac{dy}{dx} - 6y\left(\frac{dy}{dx}\right)^2}{x+3y^2}$$

$$= \frac{-2\left(\frac{-y}{x+3y^2}\right) - 6y\left(\frac{-y}{x+3y^2}\right)^2}{x+3y^2}$$

(common denominator)

$$\boxed{= \frac{2y(x+3y^2) - 6y^3}{(x+3y^2)^2}}$$

$$\frac{d}{dx}\left(x^{p/q}\right) = ? \quad x \geq 0,\ q \text{ even}$$

Write $y = x^{p/q}$. The trick is to raise both sides to the q^{th} power, then find $\frac{dy}{dx}$.

- $\left[y = x^{p/q}\right]^q$
- $\frac{d}{dx}\left(y^q = x^p\right)$

$$q y^{q-1} \frac{dy}{dx} = p x^{p-1}$$

$$\frac{dy}{dx} = \frac{p x^{p-1}}{q y^{q-1}}$$

$\longrightarrow$

$$= \frac{p x^{p-1}}{q\left(x^{p/q}\right)^{q-1}}$$

$$= \frac{p}{q} x^{p-1-\frac{p}{q}(q-1)}$$

$$= \frac{p}{q} x^{\cancel{p}-1-\cancel{p}+\frac{p}{q}}$$

$$= \frac{p}{q} x^{\frac{p}{q}-1}$$

Conclusion: Power Rule works for rational exponents.