27 Aug 2014

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Section 2.1

- How would you define, and then differentiate between, the following pairs of terms?
  - Instantaneous velocity vs. average velocity?
  - Tangent line vs. secant line?

- An object is launched into the air, and its position (in feet) at any time *t* (in seconds) is given by the equation:

$$s(t) = -4.9t^2 + 30t + 20$$

1. Compute the average velocity of the object over the following time intervals: [1, 3], [1, 2], [1, 1.5]
2. As your interval gets shorter, what do you notice about the average velocities? What do you think would happen if we computed the average velocity of the object over the interval [1, 1.2]? [1, 1.1]? [1, 1.05]?

- An object is launched into the air, and its position (in feet) at any time *t* (in seconds) is given by the equation:

$$s(t) = -4.9t^2 + 30t + 20$$

3. How could you use the average velocities to estimate the instantaneous velocity at t = 1?
4. What do the average velocities you computed on #1 represent on the graph of s(t)?

What happens to the relationship between instantaneous velocity and average velocity as the time interval gets shorter?

What is the relationship between the secant lines and the tangent lines as the time interval gets shorter?

The instantaneous velocity at $t = 1$ is the limit of the average velocities as t approaches 1.

The slope of the tangent line at (1, 45.1) is the limit of the slopes of the secant lines as

t approaches 1.

HW from section 2.1

- Do problems 1-3, 7, 9, 11, 13, 17, and 21

Section 2.2

Based on your everyday experiences, how would you define a “limit”?

Based on your mathematical experiences, how would you define a “limit”?

How do your definitions above compare or differ?

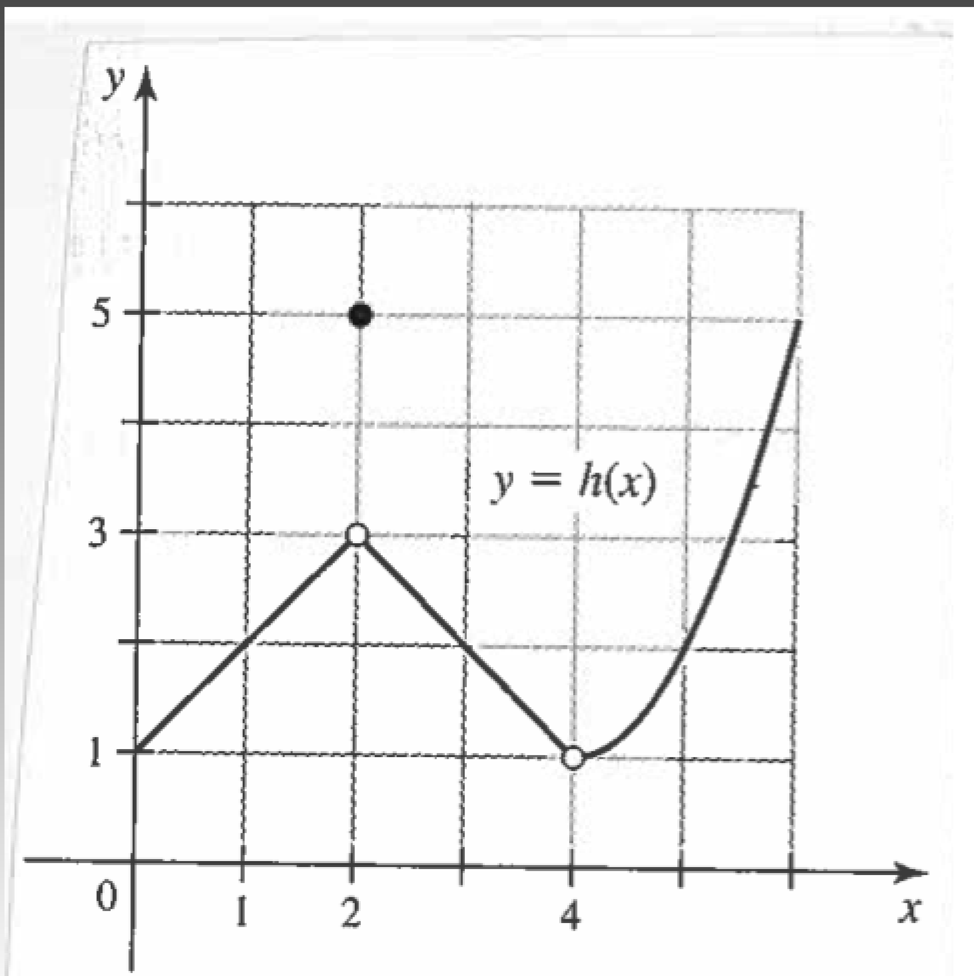
Definition of a Limit of a Function

Suppose the function *f* is defined for all *x* near *a* except possibly at *a*. If *f(x)* is arbitrarily close to *L* (as close to *L* as we like) for all *x* sufficiently close (but not equal) to *a*, we write

$$\lim_{x \to a} f(x) = L$$

and say the limit of *f(x)* as *x* approaches *a* equals *L.*

Determining Limits from a Graph



Determine the following:

1. h(1)
2. h(2)
3. h(4)
4. $\lim_{x\to 2} h(x)$
5. $\lim_{x\to 4} h(x)$
6. $\lim_{x\to 1} h(x)$

Question:

Does $\lim_{x \to a} f(x)$ always equal *f(a)*?

Determining Limits from a Table

Suppose $f(x) = \dfrac{x^2 + x - 20}{x - 4}$

Create a table of values of *f(x)* when

x=3.9; 3.99; 3.999; and

x=4.1; 4.01; 4.001

What can you conjecture about $\lim_{x \to 4} f(x)$?

Definitions of One-Sided Limits

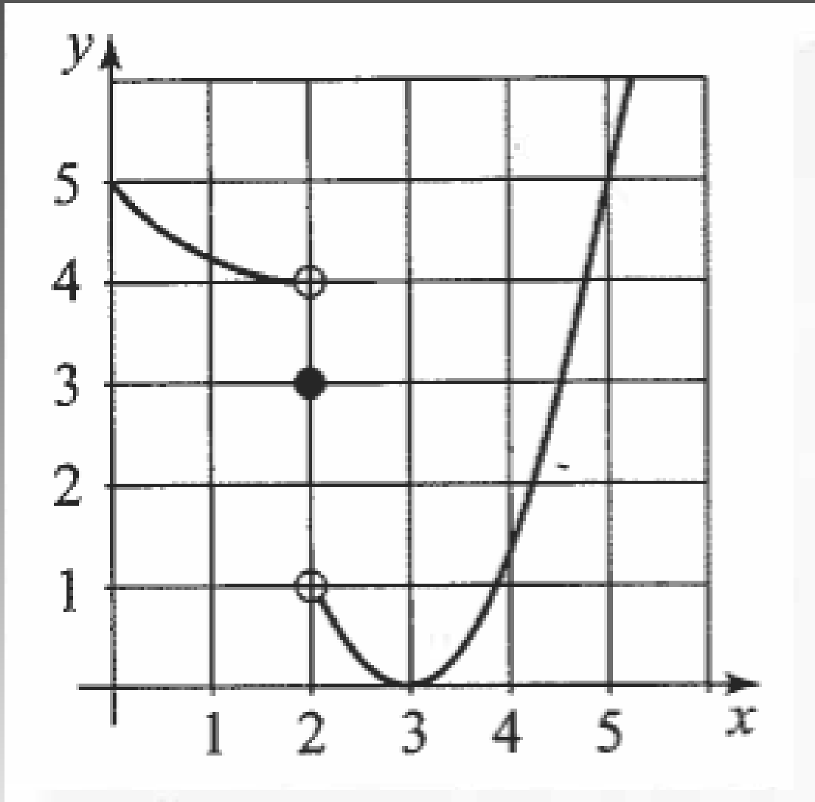
Notice in the previous example we can approach $f(x)$ from both sides as

x approaches a (e.g., when $x > a$ and when

$x < a$). Up to this point we have been working with two-sided limits; however, from some functions it makes sense to examine one-sided limits.

Determining One- and Two-Sided Limits

Determine the following:



1. g(2)

2. $\lim_{x\to 2^+} g(x)$

3. $\lim_{x\to 2^-} g(x)$

4. $\lim_{x\to 2} g(x)$

Theorem regarding Relationship Between One- and Two-Sided Limits

Assume *f* is defined for all *x* near *a* except possibly at *a*. Then $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^-} f(x) = L$.

HW from section 2.2

- Do problems 1-4, 10, 12, 16, 18, 20, and 25 (pgs. 61-63 in textbook)

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29 Aug 2014 (cont.)

- Structure of the course: Expect lectures and drill to be the way they have been this week. The course is designed in a way that forces you to be more proactive in your learning.
- Diverse levels of Calculus exposure. If you have not seen something before, then ask (instructor, friend, Google, etc.). If everything seems familiar to you then check on your neighbors. You may find, through explaining to someone else, that you don't understand the material as well as you thought you did.
- Resources: tutoring, other Cal I courses (Google searches can pull up lots of lecture notes from other semesters, other schools, etc.), other Cal books, etc.

Section 2.3

Given the function *f(x) = 4x+7,* find $\lim_{x \to -2} f(x)$

- Graphically
- Numerically (e.g., using a table of values near -2);
- A direct computational method of your choosing

Compare and contrast these methods—which is most favorable?

Many times, analytical or computational methods are much more efficient and effective. This section provides various laws and techniques in determining limits. For example:

Limits of Linear Functions: Let *a, b,* and *m* be real numbers. For linear functions $f(x) = mx + b$,

$$\lim_{x \to a} f(x) = f(a) = ma + b$$

Limit Laws

Assume $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. The following properties hold, where *c* is a real number and

m, n > 0 and are integers.

Sum: $\lim_{x \to a}[f(x)+g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

Difference: $\lim_{x \to a}[f(x)-g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

Constant Multiple: $\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$

Product: $\lim_{x \to a}[f(x)g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)]$

Limits of Polynomials and Rational Functions

(Example where you can plug in "a" to get the limit) Assume *p* and *q* are polynomials and *a* is a constant.

1. Polynomials: $\lim_{x \to a} p(x) = p(a)$

2. Rational Functions (assuming q(a) is non-zero): $\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$

Exercises

Use the limit laws to evaluate the following limits

1. $\lim\limits_{x\to 1}\left[\dfrac{4f(x)g(x)}{h(x)}\right]$, given that $\lim\limits_{x\to 1} f(x)=5$, $\lim\limits_{x\to 1} g(x)=-2$,
$\lim\limits_{x\to 1} h(x)=-4$

2. $\lim\limits_{x\to 3}\dfrac{4x^2+3x-6}{2x-3}$

3. $\lim\limits_{x\to 1^-} g(x)$ and $\lim\limits_{x\to 1^+} g(x)$, given $g(x)=\begin{cases} x^2 & x\le 1 \\ x+2 & x>1 \end{cases}$

Additional Techniques

There are times that direct substitution fails:

Evaluate the following limits:

1. $\lim_{t \to 2} \dfrac{3t^2 - 7t + 2}{2 - t}$

2. $\lim_{h \to 0} \dfrac{\dfrac{1}{5+h} - \dfrac{1}{5}}{h}$

Squeeze Theorem

A final method for evaluating limits involves the relationships of functions with each other.

Theorem: Assume the functions *f, g,* and *h* satisfy $f(x) \leq g(x) \leq h(x)$ for all values of *x* near *a*, except possibly at *a*. If $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$

HW from section 2.3

- Do problems 12-30 (every 3rd problem), 31, 33, 37-47 odds, 51, 53, 61-65 odds (pgs. 73-75 in textbook)