Fri 24 Oct 2014

- Midterms: no curve. Lots of A's! ☺

Maxima and Minima

- In Chapter 4, we look at many uses of the derivative beyond its use in studying rates of change (e.g., section 3.5).

- In the first couple of sections of Chapter 4, we examine the graphs of functions and what the derivative can tell us about the graph’s behavior and characteristics.

Definition of Absolute Maxima and Minima

Let f be defined on an interval I containing c.

- f has an absolute maximum value on I at c if

  $f(c) \geq f(x)$ for every x in I.

- f has an absolute minimum value on I at c if

  $f(c) \leq f(x)$ for every x in I.

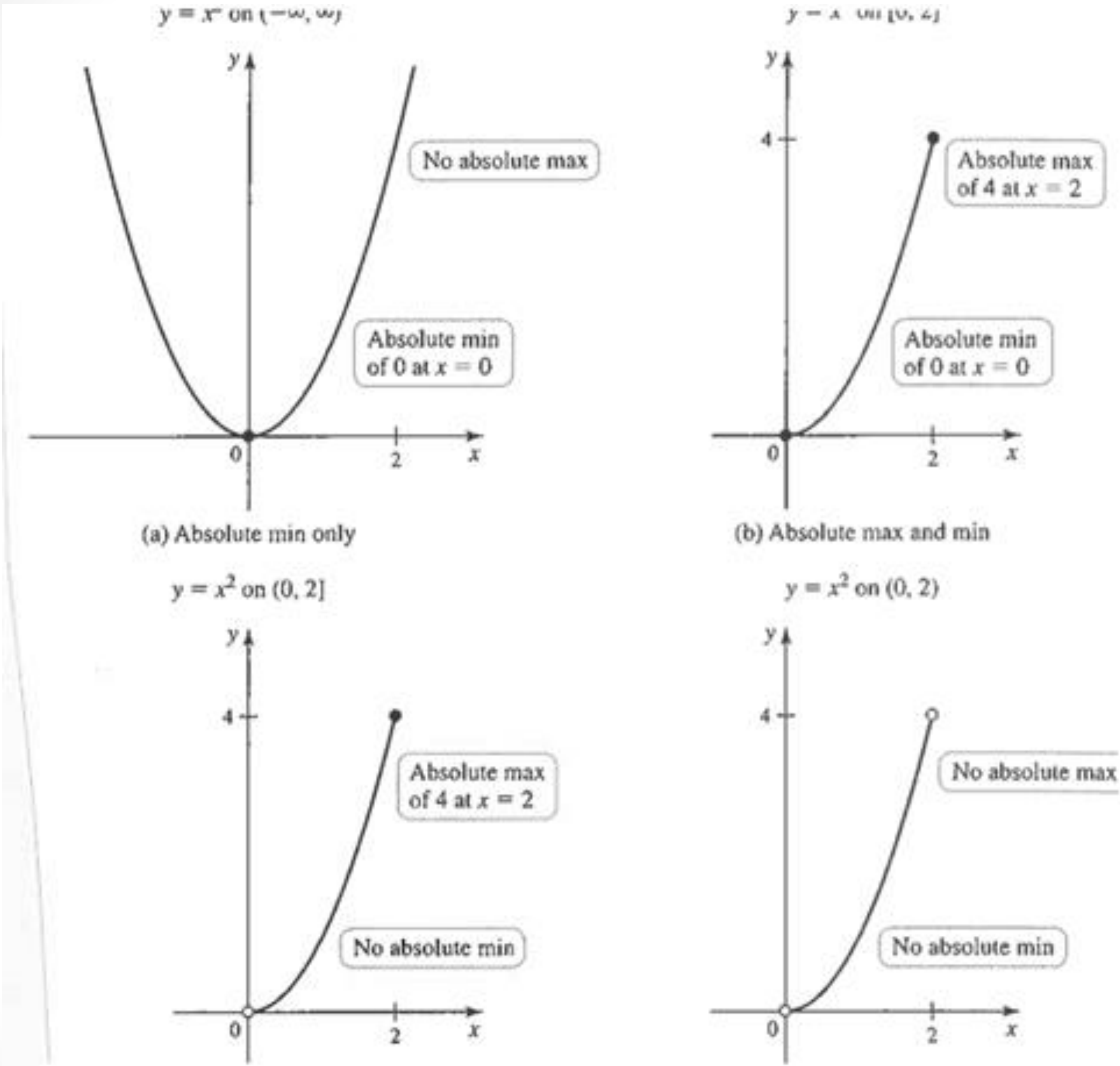
The existence and location of absolute extreme values depend on the function and the interval of interest.

Extreme Value Theorem

THM: A function that is continuous on a closed interval [a, b] has an absolute maximum value and an absolute minimum value on that interval.

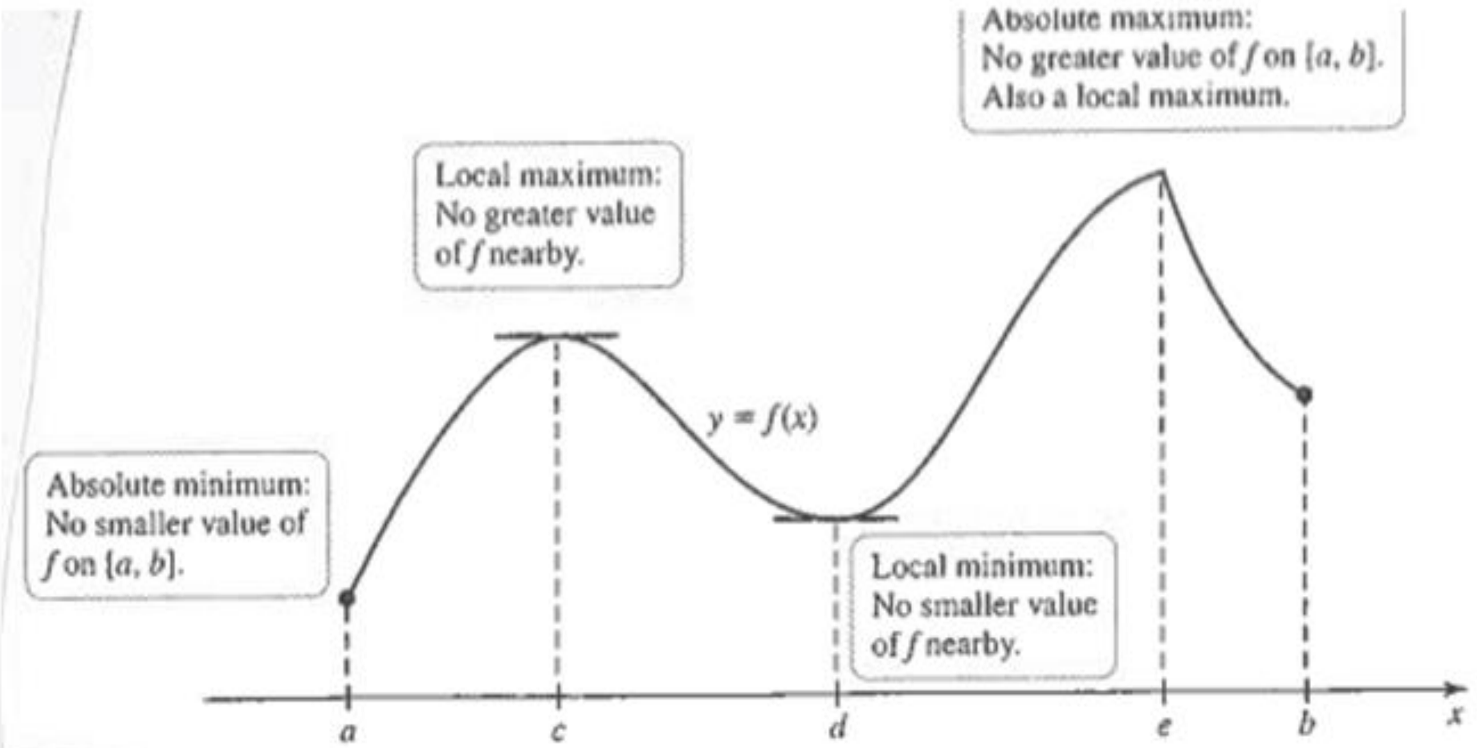
The EVT provides the criteria that ensures absolute extrema:

- the function must be continuous on the interval of interest;
- the interval of interest must be closed and bounded.


No absolute max
Absolute min of 0 at x = 0
(a) Absolute min only
Absolute max of 4 at x = 2
Absolute min of 0 at x = 0
(b) Absolute max and min
y = x² on (0, 2]
Absolute max of 4 at x = 2
No absolute min
y = x² on (0, 2)
No absolute max
No absolute min

Local Minima and Maxima

- Beyond absolute extrema, a graph may have a number of peaks and dips throughout its interval of interest.



Definition of Local Minima and Maxima

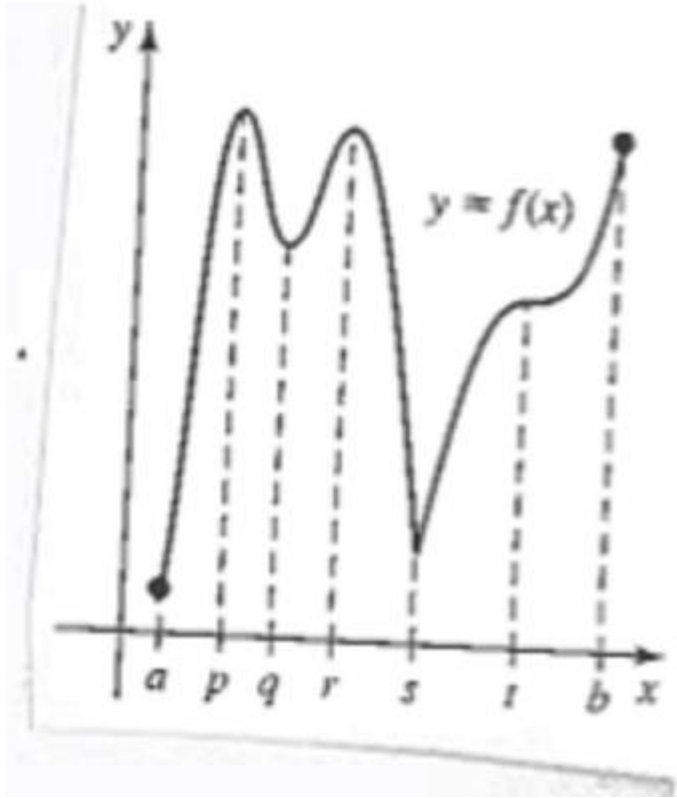
Suppose *I* is an interval on which *f* is defined and *c* is an interior point of *I*.

- If $f(c) \geq f(x)$ for all x in some open interval containing c, then $f(c)$ is a local maximum value of f.

- If $f(c) \leq f(x)$ for all x in some open interval containing c, then $f(c)$ is a local maximum value of f.

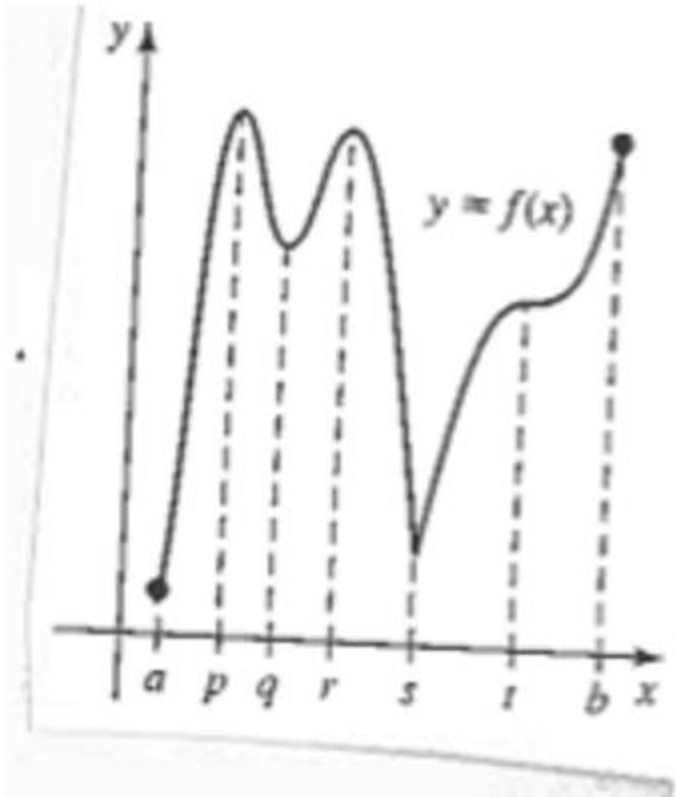
Exercise

Use the graph below to identify the points on the interval [a, b] at which local and absolute extreme values occur.



Critical Points

Based on the previous graph, how is the derivative related to where the local extrema occur?



Critical Points

Based on the previous graph, how is the derivative related to where the local extrema occur?

Local extrema occur where the derivative either does not exist or where the derivative = 0.

An interior point *c* of the domain of *f* at which $f'(c) = 0$ or $f'(c)$ fails to exist is called a **critical point** of *f*.

Local Extreme Point Theorem

- **THM:** If f has a local minimum or maximum value at c and $f'(c)$ exists, then $f'(c) = 0$. (Converse is not true)

- It is possible for $f'(c) = 0$ or $f'(c)$ to not exist at a point, yet the point not be a local min or max. Therefore, critical points provide candidates for local extrema, but do not guarantee that the points are local extrema (see pg. 227 for examples)

Procedures for Locating Absolute Min and Max

Two facts help us in the search for absolute extrema:

- Absolute extrema in the interior of an interval are also local extrema, which occur at critical points of f.
- Absolute extrema may occur at the endpoints of f.

Procedures for Locating Absolute Min and Max

Assume the function *f* is continuous on the closed interval [a, b].

1. Locate the critical points *c* in (a, b), where $f'(c) = 0$ or $f'(c)$ does not exist. These points are candidates for absolute extrema.
2. Evaluate *f* at the critical points and at the endpoints of [a, b].
3. Choose the largest and smallest values of *f* from Step 2 for the absolute maximum and minimum values, respectively.

NOTE: In this section, given an equation, we can identify critical points and absolute extrema, but not local extrema. Techniques for locating local extrema come in later sections.

On the interval [-2, 2], the function $f(x) = x^4$

A. has an absolute maximum but no local maxima

B. has an absolute maximum at an interior point of the interval

C. has no local or absolute extrema

D. has a local minimum but no absolute minimum

Exercise

Given the function $f(x) = (x+1)^{\frac{4}{3}}$ on the interval [-8, 8], determine the critical points and the absolute extreme values of *f*.

Homework from Section 4.1

Do problems 11-25 odd, 31-45 odd (pgs. 229-230).