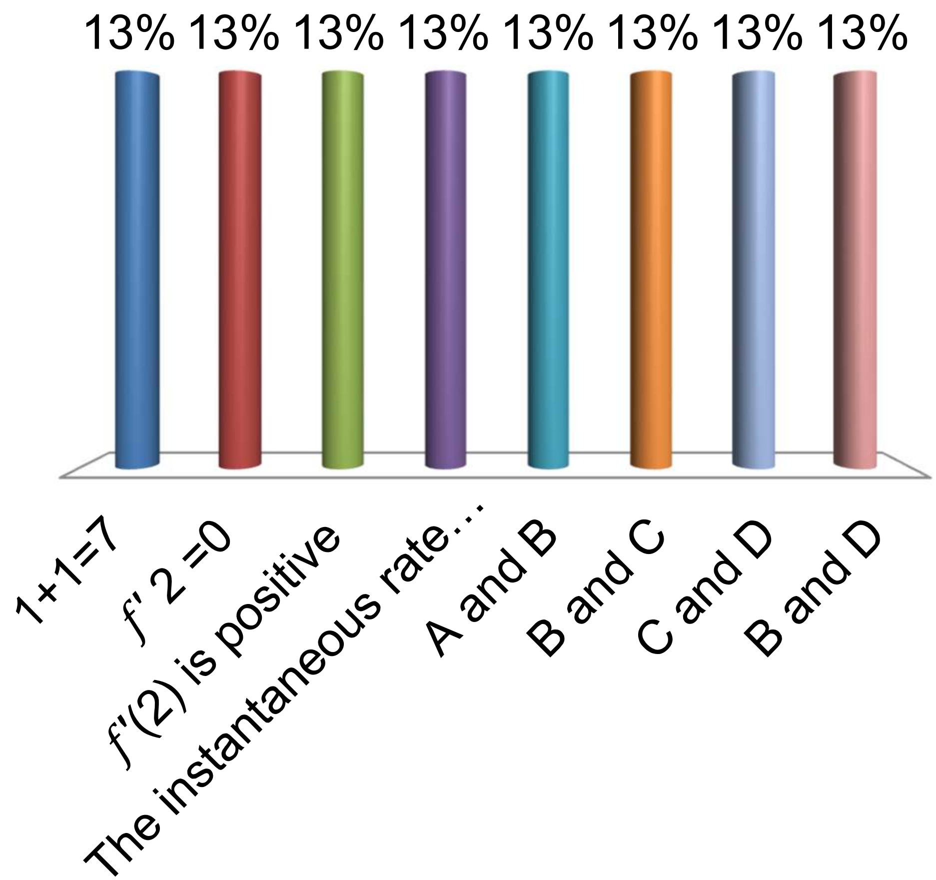
UNIT 3, LESSON 1

Increasing and Decreasing Functions

If the slope of the tangent line to the graph of f at $x = 2$ is positive, which of the following are true?

A. $1 + 1 = 7$
B. $f'(2) = 0$
C. $f'(2)$ is positive
D. The instantaneous rate of change of f at $x = 2$ is positive.
E. A and B
F. B and C
G. C and D
H. B and D



Increasing and Decreasing Functions

Let f be a function defined on some interval. Then for any two numbers x_1 and x_2 in the interval, f is **increasing** on the interval if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_2,$$

and f is **decreasing** on the interval if

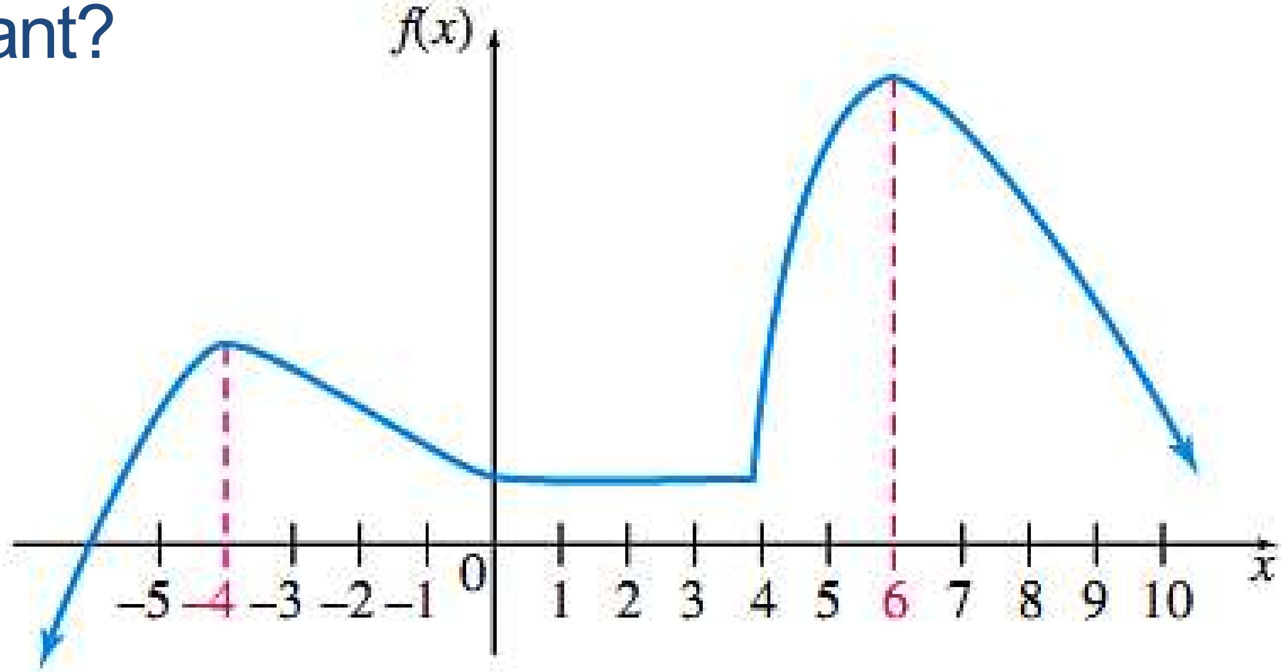
$$f(x_1) > f(x_2) \quad \text{whenever} \quad x_1 < x_2.$$

A function is increasing on an interval if larger x values from that interval result in larger outputs.

A function is decreasing on an interval if larger x values from that interval result in smaller outputs.

A function is **increasing** if the graph goes up from left to right and **decreasing** if the graph goes down from left to right.

Where is the function represented by the following graph increasing or decreasing? Where is the function constant?



Example 1: https://www.desmos.com/calculator/qmlzxd4eml

Example 2: https://www.desmos.com/calculator/q3kowv6rhd

Example 3: https://www.desmos.com/calculator/piz1ivw8w7

Example 4: https://www.desmos.com/calculator/hcels6ivpb

https://www.desmos.com/calculator/su8ik7fxow

Test for Intervals Where $f(x)$ is Increasing and Decreasing

Suppose a function f has a derivative at each point in an open interval; then
if $f'(x) > 0$ for each x in the interval, f is *increasing* on the interval; $\nearrow$
if $f'(x) < 0$ for each x in the interval, f is *decreasing* on the interval; $\searrow$
if $f'(x) = 0$ for each x in the interval, f is *constant* on the interval. $\rightarrow$

This means that when we want to know if a function f is increasing or decreasing, we look at its derivative, f'.

Critical Numbers

The **critical numbers** for a function f are those numbers c in the domain of f for which $f'(c) = 0$ or $f'(c)$ does not exist. A **critical point** is a point whose x-coordinate is the critical number c and whose y-coordinate is $f(c)$.

Applying the Test

1. Locate the critical numbers for f on a number line, as well as any points where f is undefined. These points determine several open intervals.
2. Choose a value of x in each of the intervals determined in Step 1. Use these values to decide whether $f'(x) > 0$ or $f'(x) < 0$ in that interval.
3. Use the test on the previous page to decide whether f is increasing or decreasing on the interval.

Find the intervals in which the following function is increasing or decreasing.

$$f(x) = x^3 + 3x^2 - 9x + 1$$

Find the intervals for which the following function increases and decreases:

$$f(x) = \frac{x-1}{x+1}$$

Given the function
$f(x) = -x^3 - 2x^2 + 15x + 10,$
find the critical numbers.

A. $x = 2 \quad x = \frac{2}{3}$

B. $x = 5 \quad x = -3$

C. $x = \frac{2}{3} \quad x = -\frac{5}{3}$

D. $x = \frac{5}{3} \quad x = -3$

Determine where the function

$f(x) = -x^3 - 2x^2 + 15x + 10,$

is increasing. Give your answer in interval notation.

A. $\left(-3, \frac{5}{3}\right)$

B. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

C. $(-\infty, \infty)$

D. The function never increases.

Determine where the function $f(x) = -x^3 - 2x^2 + 15x + 10$, is decreasing. Give your answer in interval notation.

A. $\left(-3, \frac{5}{3}\right)$

B. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

C. $(-\infty, \infty)$

D. The function never decreases.

A friend looks at the graph of $y = x^2$ and observes that if you stare at the origin, the graph increases whether you go left or right, so the graph is increasing everywhere. Is your friend correct? Why or why not?

Find the critical numbers for $f(x) = \frac{x}{x+1}$.

A. $x = 0$
B. $x = -1$
C. $x = 0\,, x = -1$
D. $x = \ln(2)$
E. There are no critical numbers.