Find the area of the region enclosed by $y = x^2 - 2x$ and $y = x$ on $[0,4]$.

area=

$$\int_0^3 \underbrace{\left(x-(x^2-2x)\right)}_{-x^2+3x} dx + \int_3^4 \underbrace{(x^2-2x-x)}_{x^2-3x} dx$$

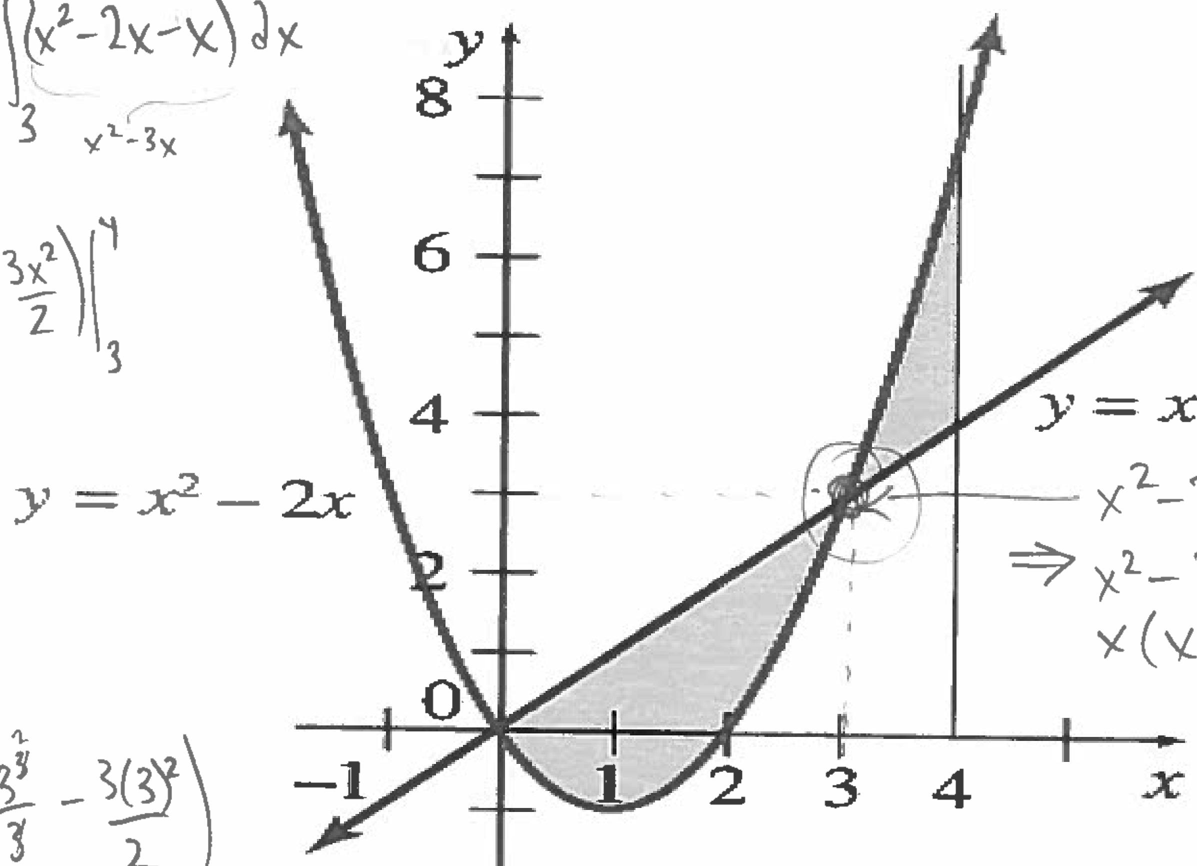
$$= \left(\frac{-x^3}{3} + \frac{3x^2}{2}\right)\Bigg|_0^3 + \left(\frac{x^3}{3} - \frac{3x^2}{2}\right)\Bigg|_3^4$$

$$= \frac{-(3)^3}{3} + \frac{3(3)^2}{2}$$

$$-\left[\frac{-0^3}{3} + \frac{3(0)^2}{2}\right]$$

$$+ \frac{4^3}{3} - \frac{3(4)^2}{2} - \left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)$$

$$= \frac{64}{3} - 24 - 18 + 27 = \boxed{\frac{19}{3}}$$



$x^2 - 2x = x$

$\Rightarrow x^2 - 3x = 0$

$x(x-3) = 0$

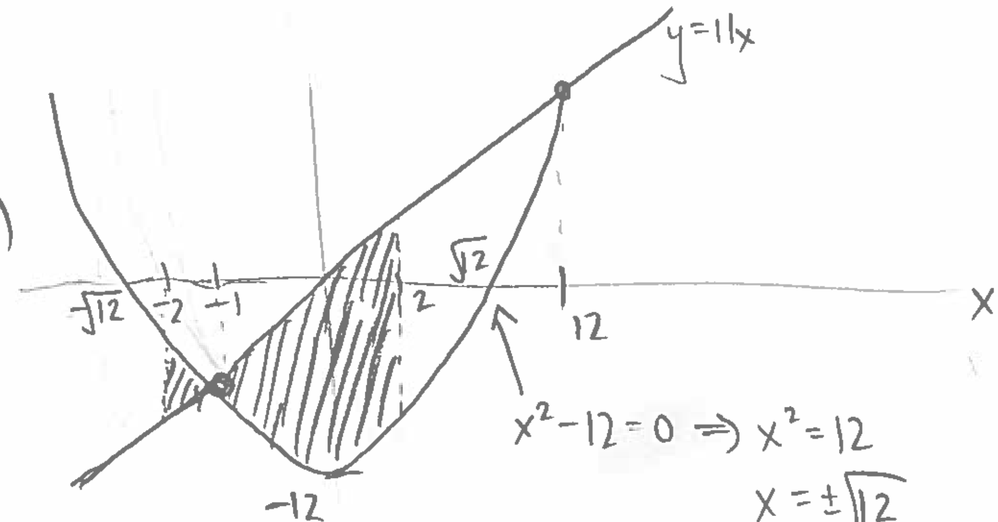
Find the area between $y = 11x$ and $y = x^2 - 12$ on the interval $[-2,2]$.

Check $11x = x^2 - 12$

$0 = x^2 - 11x - 12$

$= (x-12)(x+1)$

$12 \qquad -1$



$x^2 - 12 = 0 \Rightarrow x^2 = 12$

$x = \pm\sqrt{12}$

A. 23.6

B. 134/19

C. 169/3

D. 12/5

$\boxed{\neq \frac{37}{3}}$

$$\text{area} = \int_{-2}^{-1} (x^2 - 12 - 11x)\,dx + \int_{-1}^{2} (11x - (x^2 - 12))\,dx$$

$$= \left(\frac{x^3}{3} - 12x - \frac{11}{2}x^2\right)\Big|_{-2}^{-1} + \left(\frac{11x^2}{2} - \frac{x^3}{3} + 12x\right)\Big|_{-1}^{2}$$

$$= \frac{(-1)^3}{3} - 12(-1) - \frac{11}{2}(-1)^2 - \left[\frac{(-2)^3}{3} - 12(-2) - 11(-2)\right] + \frac{11(2)^2}{2} - \frac{2^3}{3} + 12(2)$$

$-\frac{1}{3} + 12 - \frac{11}{2} \qquad -\frac{8}{3} + 24 \quad + 22 \qquad 22 - \frac{8}{3} + 24$

$$- \left[\frac{11(-1)^2}{2} - \frac{(-1)^3}{3} + 12(-1)\right]$$

$\frac{11}{2} + \frac{1}{3} - 12$

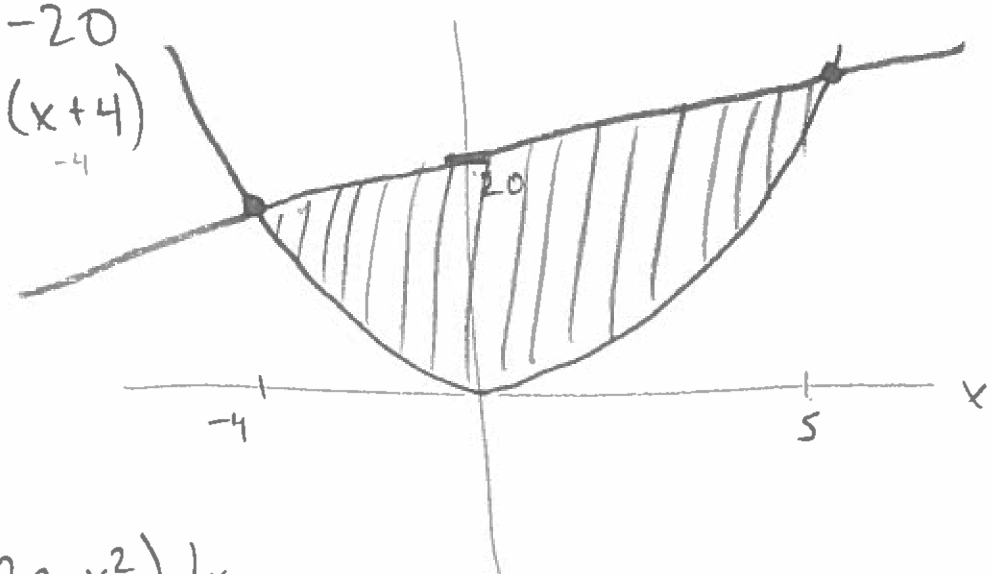
Find the area of the region bounded by the graphs of $y = x + 20$ and $y = x^2$.

$x + 20 = x^2$

$\Rightarrow 0 = x^2 - x - 20$

$= (x-5)(x+4)$

$5 \quad -4$



A. 243/2

B. 167/5

C. 256/7

D. 165/3

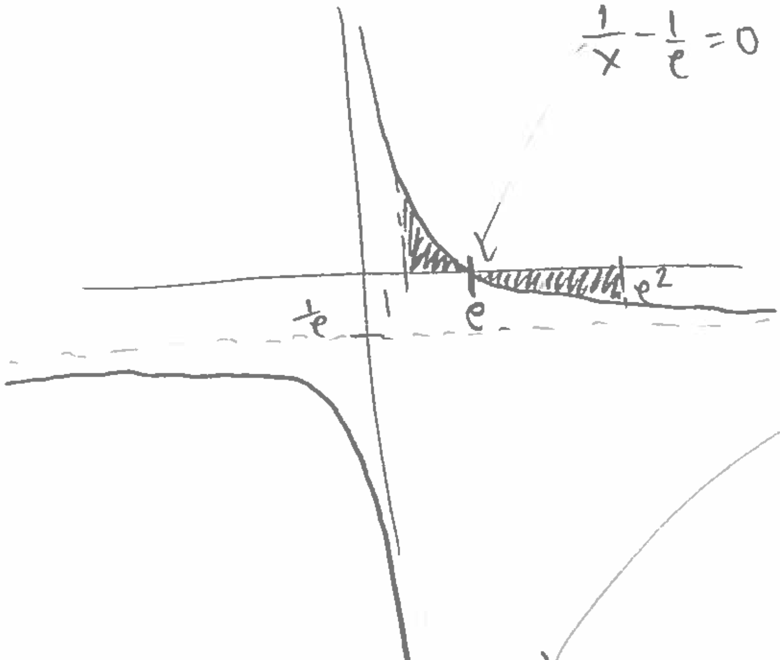
$$\text{area} = \int_{-4}^{5} (x + 20 - x^2)\, dx$$

$$= \frac{x^2}{2} + 20x - \frac{x^3}{3} \Big|_{-4}^{5}$$

$$= \frac{5^2}{2} + 20(5) - \frac{5^3}{3} - \left[\frac{(-4)^2}{2} + 20(-4) - \frac{(-4)^3}{3}\right]$$

$y=0$

Find the area between the x-axis and $f(x)=\frac{1}{x}-\frac{1}{e}$ on the interval $[1, e^2]$.



$\frac{1}{x}-\frac{1}{e}=0 \Rightarrow x=e$

$$\int_1^e \left(\frac{1}{x}-\frac{1}{e}-0\right)dx + \int_e^{e^2}\left(0-\left(\frac{1}{x}-\frac{1}{e}\right)\right)dx$$

$$=\left(\ln(x)-\frac{1}{e}x\right)\Big|_1^e + \left(-\ln(x)+\frac{1}{e}x\right)\Big|_e^{e^2}$$

$$=\ln(e)-\frac{1}{e}(e) - \left[\ln(1)-\frac{1}{e}(1)\right] + -\ln(e^2)+\frac{1}{e}(e^2) - \left[-\ln(e)+\frac{1}{e}(e)\right]$$

$$=\cancel{1}-\cancel{1}-0+\frac{1}{e} + \underbrace{-2\ln(e)}_{=-2} + e+0-1$$

$$\boxed{= e+\frac{1}{e}-3}$$

Q: Why is it OK to leave abs. value off $\ln(x)$ in this problem?

Find the area between $y = 2e^{3x}$ and $y = e^{3x} + e^6$ on the interval [0,3].

$2e^{3x} = e^{3x} + e^6$

$\ln \Rightarrow \left(e^{3x} = e^6\right)$

$3x = 6$

$x = 2$

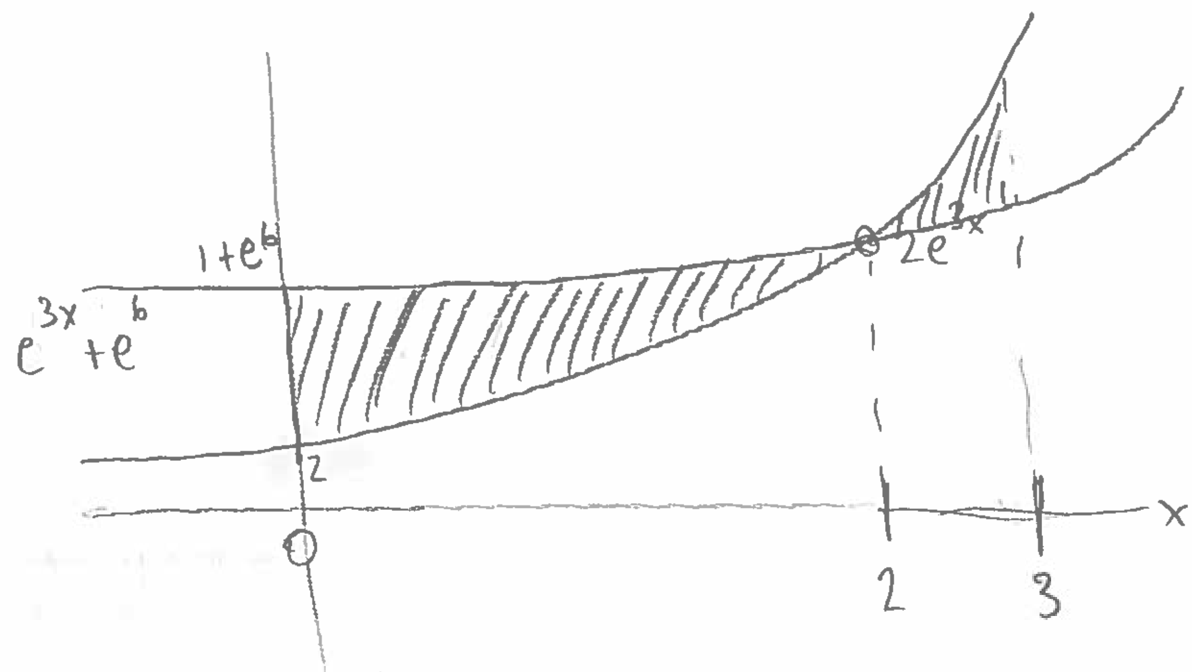
A. $\frac{1}{3}e^6 + \frac{1}{3}e^9 + \frac{1}{3}$

B. $\frac{1}{3}e^6 + \frac{1}{3}e^9 + \frac{9}{30}$

C. $\frac{1}{3}e^6 + \frac{1}{3}e^9 - \frac{1}{3}$

D. $\frac{1}{3}e^6 + \frac{1}{3}e^9 + \frac{1990}{3000}$

E. $\frac{1}{3}e^6 + \frac{1}{3}e^9 \pm \pi \times \infty$



$$\int_0^2 \left(\underbrace{e^{3x} + e^6 - 2e^{3x}}_{-e^{3x}+e^6}\right)dx + \int_2^3 \left(\underbrace{2e^{3x} - e^{3x} - e^6}_{e^{3x}-e^6}\right)dx$$

$$= -\frac{1}{3}e^{3x} + e^6 x \Big|_0^2 + \frac{1}{3}e^{3x} - e^6 x \Big|_2^3$$

$$= -\frac{1}{3}e^6 + e^6(2) - \left[-\frac{1}{3}e^0 + e^6(0)\right] + \frac{1}{3}e^9 - e^6(3) - \left[\frac{1}{3}e^6 - 2e^6\right] = \overbrace{\left(-\frac{1}{3} + 2 - 3 - \frac{1}{3} + 2\right)}^{1/3} e^6 + \frac{1}{3}e^9 + \frac{1}{3}$$