Quiz 4
In Class Group Quiz

Calculus 1
TA & Drill Time:

Names: Key

On this quiz, you will work in groups of 2-3 students to answer the following questions. Each group will submit one paper, and that paper will contain the names of all persons who worked in that group. Circle the name of the student for whom the quiz belongs.

In this quiz, **you will be using the <u>definition of the derivative</u>** [$f'(x)$] of a function $f(x)$ in terms of the following limit: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.
Be sure to use proper limit notation when working all limit problems.

1. Let $f(x) = x^2 + 3$. Determine the following:

a. $f(1+h)$

$$f(1+h) = (1+h)^2 + 3$$
$$= (h^2 + 2h + 1) + 3$$
$$= \underline{h^2 + 2h + 4}$$

b. $\lim_{h \to 0}\left[\frac{f(1+h) - f(1)}{h}\right] = \lim_{h \to 0} \frac{(1+h)^2 + 3 - (1+3)}{h}$

$$= \lim_{h \to 0} \frac{h^2 + 2h + 4 - 4}{h}$$
$$= \lim_{h \to 0} \frac{h(h+2)}{h}$$
$$= \lim_{h \to 0} h + 2 = \underline{2}$$

c. $\lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h \to 0} \frac{(x+h)^2 + 3 - [x^2+3]}{h}$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$
$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$
$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} 2x + h = \underline{2x}$$

2. Evaluate $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ when $f(x) = 5x$.

$$\lim_{h \to 0} \frac{5(x+h) - 5x}{h} = \lim_{h \to 0} \frac{5x + 5h - 5x}{h} = \lim_{h \to 0} \frac{5h}{h} = \lim_{h \to 0} 5 = \underline{5}$$

Now that you've worked with the definition of the derivative, we will now use this definition to establish several rules for calculating simple derivatives. For all problems below, you **MUST** use the definition of the derivative to determine each answer (even if you already know the derivative rules from prior experience! You can use your prior knowledge to ensure your answers are correct).

3. Use the definition of the derivative to find the derivative of the following functions:

a. $f(x) = 7$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{7 - 7}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = \underline{0}$$

b. $f(x) = -1000$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-1000 - (-1000)}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = \underline{0}$$

c. Based on your responses to a-b, what can you assume about the derivative of any constant function $f(x) = c$, where c is a constant?

For any constant function $f(x) = c$, $f'(x) = 0$.

4. Use the definition of the derivative to find the derivative of the following functions:

a. $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \to 0} 2x + h = 2x$$

b. $f(x) = x^3$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$$= 3x^2$$

c. Based on your responses to a-b, generalize this rule for any power function $f(x) = x^n$. (i.e., what can you say about the derivative of $f(x) = x^n$?) This is called the Power Rule.

For any function $f(x) = x^n$, $f'(x) = nx^{n-1}$

5. Use the definition of the derivative to find the derivative of the following functions (you can use your work from #4 where applicable):

a. $f(x) = 3x^2$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h} = \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-3x^2}{h}$$

$$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2-3x^2}{h} = \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} \frac{h(6x+3h)}{h}$$

$$= \lim_{h\to 0} 6x+3h = 6x,$$

b. $f(x) = 3x^4$

$$\lim_{h\to 0} \frac{f(x+h)-F(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^4-3x^4}{h}$$

$$= \lim_{h\to 0} \frac{3(x^4+4x^3h+6x^2h^2+4xh^3+h^4)-3x^4}{h} = \lim_{h\to 0} \frac{3x^4+12x^3h+18x^2h^2+12xh^3+3h^4-3x^4}{h}$$

$$= \lim_{h\to 0} \frac{12x^3h+18x^2h^2+12xh^3+3h^4}{h} = \lim_{h\to 0} \frac{h(12x^3+18x^2h+12xh^2+3h^3)}{h}$$

$$= \lim_{h\to 0} 12x^3+18x^2h+12xh^2+3h^3 = 12x^3.$$

c. Based on your responses to a-b, generalize this rule for any function $f(x) = cx^n$, where c is a constant. (i.e., what can you say about the derivative of $f(x) = cx^n$?) This is called the Constant Multiple Rule.

For any function $f(x) = cx^n$, $f'(x) = cnx^{n-1}$.

6. Use the definition of the derivative to find the derivative of the following functions:

a. $f(x) = x^2 + x$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+x+h-x^2-x}{h} = \lim_{h\to 0} \frac{2xh+h^2+h}{h} = \lim_{h\to 0} \frac{h(2x+h+1)}{h}$$

$$= \lim_{h\to 0} 2x+h+1 = 2x+1.$$

b. $f(x) = 2x^2 + 4x + 3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 4(x+h) + 3 - (2x^2+4x+3)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+4x+4h+3-2x^2-4x-3}{h} = \lim_{h\to 0} \frac{2x^2+4xh+2h^2+4x+4h+3-2x^2-4x-3}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+4h}{h} = \lim_{h\to 0} \frac{h(4x+2h+4)}{h} = \lim_{h\to 0} 4x+2h+4 = 4x+4.$$

c. Based on your responses to a-b, generalize this rule for any function $f(x) = g(x) + h(x)$, where *g(x)* and *h(x)* are functions. (i.e., what can you say about the derivative of $f(x) = g(x) + h(x)$?) This is called the Sum Rule.

For all functions $f(x) = g(x) + h(x)$, $f'(x) = g'(x) + h'(x)$.