Wed 27 May

- comp.uark.edu/~ashleykw/Cal1Sum2015/cal1sum15.html
  Course website. All information is here, including a link to MLP, lecture slides, administrative information, solutions to graded work, etc.
- MyLabsPlus (MLP) has the graded homework. Due Fridays and Sundays (check each assignment for specific dates and times). All assignments for the term are posted now. You can do them as early as you wish, before their deadlines.

Wed 27 May (cont.)

- Lecture slides are available on the course website. I'll try to have the week's slides posted in advance but the individual lectures might not be posted until right before class.

  Don't try to take notes from the slides. Instead, print out the slides beforehand or else follow along on your tablet/phone/laptop. You should, however, take notes when we do exercises during lecture (which is frequent). We will always review those solutions on the document camera. Document camera notes are reserved only for those who attended lecture that day. :p

- Quizzes may or may not be announced but expect two during a non-exam week and one during an exam week. Lengths of quizzes may vary, including possible take-home quizzes. Quizzes may or may not be collaborative.

Wed 27 May (cont.)

- For old Calculus materials, see the parent page `comp.uark.edu/~ashleykw` and look for links under "Previous Semesters". Last semester's in-class exam solutions are posted there, for example. There are also older versions of the lecture slides if you want to look ahead.

1 Week 1: 26-29 May

- Wednesday 27 May

∮2.3 Techniques for Computing Limits, cont.

- Additional (Algebra) Techniques
- Another Technique: Squeeze Theorem
- Book Problems

∮2.4 Infinite Limits

- Definition of Infinite Limits
- Definition of a Vertical Asymptote
- Book Problems

∮2.5 Limits at Infinity

- Horiztonal Asymptotes
- Infinite Limits at Infinity
- Algebraic and Transcendental Functions
- Book Problems

Additional (Algebra) Techniques

When direct substitution (a.k.a. plugging in a) fails try using algebra:

- Factor and see if the denominator cancels out.

Example

$$\lim_{t \to 2} \frac{3t^2 - 7t + 2}{2 - t}$$

- Look for a common denominator.

Example

$$\lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$$

Another Technique: Squeeze Theorem

This method for evaluating limits uses the relationship of functions with each other.

Theorem (Squeeze Theorem)

Assume $f(x) \leq g(x) \leq h(x)$ for all values of x near a, except possibly at a, and suppose

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L.$$

Then since g is always between f and h for x-values close enough to a, we must have

$$\lim_{x \to a} g(x) = L.$$

Example

(a) Draw a graph of the inequality

$$-|x| \leq x^2 \ln\left(x^2\right) \leq |x|.$$

(b) Compute $\lim_{x\to 0} x^2 \ln\left(x^2\right)$.

2.3 Book Problems

12-30 (x3), 31, 33, 37-47 (odds), 51, 53, 61-65 (odds)

- In general, review your algebra techniques, since knowing them can very often save you a lot of headache.

1 Week 1: 26-29 May

$\oint$ 2.4 Infinite Limits

In the next two sections, we examine limit scenarios involving infinity. The two situations are:

- **Infinite limits:** as x (i.e., the independent variable) approaches a finite number, y (i.e., the dependent variable) becomes arbitrarily large or small

  $$\text{looks like: } \lim_{x \to \text{number}} f(x) = \pm\infty$$

- **Limits at infinity:** as x approaches an arbitrarily large or small number, y approaches a finite number

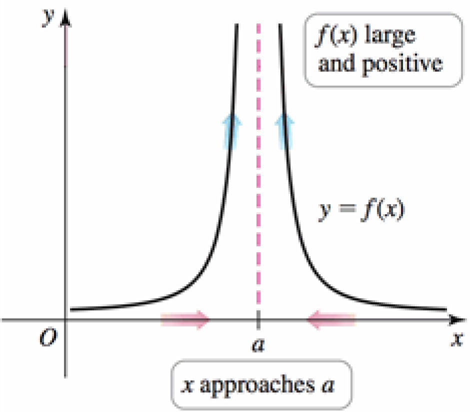
  $$\text{looks like: } \lim_{x \to \pm\infty} f(x) = \text{number}$$

Definition (positively infinite limit)

Suppose f is defined for all x near a. If $f(x)$ grows arbitrarily large for all x sufficiently close (but not equal) to a, we write

$$\lim_{x\to a} f(x) = \infty$$

and say **the limit of** $f(x)$ **as** x **approaches** a **is infinity**.

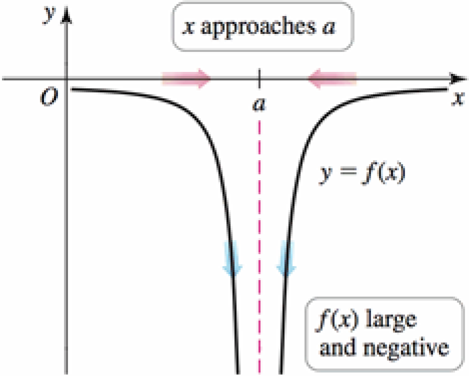

y
f(x) large
and positive
y = f(x)
O
a
x
x approaches a

Definition (negatively infinite limit)

Suppose f is defined for all x near a. If $f(x)$ is negative and grows arbitrarily large in magnitude for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = -\infty$$

and say **the limit of** $f(x)$ **as** x **approaches** a **is negative infinity**.


x approaches a
y
O
a
x
y = f(x)
f(x) large
and negative

The definitions work for one-sided limits, too.

Exercise

Using a graph and a table of values, given $f(x) = \dfrac{1}{x^2 - x}$, determine:

(a) $\lim\limits_{x \to 0^+} f(x)$

(b) $\lim\limits_{x \to 0^-} f(x)$

(c) $\lim\limits_{x \to 1^+} f(x)$

(d) $\lim\limits_{x \to 1^-} f(x)$

Definition

Suppose a function f satisfies at least one of the following:

- $\lim_{x \to a} f(x) = \pm\infty$,
- $\lim_{x \to a^+} f(x) = \pm\infty$
- $\lim_{x \to a^-} f(x) = \pm\infty$

Then the line $x = a$ is called a **vertical asymptote** of f.

Exercise

Given $f(x) = \dfrac{3x-4}{x+1}$, determine, analytically (meaning without using a table or a graph),

(a) $\displaystyle\lim_{x\to -1^+} f(x)$

(b) $\displaystyle\lim_{x\to -1^-} f(x)$

Remember to check for factoring –

Exercise

(a) What is/are the vertical asymptotes of

$$f(x) = \frac{3x^2 - 48}{x + 4}?$$

(b) What is $\lim_{x \to -4} f(x)$?

2.4 Book Problems

7-10, 15, 17-26, 36-37

$\oint$ 2.5 Limits at Infinity

Limits at infinity determine what is called the **end behavior** of a function.

Horizontal Asymptotes

Definition

If $f(x)$ becomes arbitrarily close to a finite number L for all sufficiently large and positive x, then we write

$$\lim_{x \to \infty} f(x) = L.$$

The line $y = L$ is a **horizontal asymptote** of f.

The limit at negative infinity, $\lim_{x \to -\infty} f(x) = M$, is defined analogously and in this case, the horizontal asymptote is $y = M$.

Question

Is it possible for a limit to be both an infinite limit and a limit at infinity? (Yes.)

If $f(x)$ becomes arbitrarily large as x becomes arbitrarily large, then we write

$$\lim_{x\to\infty} f(x) = \infty.$$

(The limits $\lim\limits_{x\to\infty} f(x) = -\infty$, $\lim\limits_{x\to-\infty} f(x) = \infty$, and $\lim\limits_{x\to-\infty} f(x) = -\infty$ are defined similarly.)

Powers and Polynomials: Let n be a positive integer and let $p(x)$ be a polynomial.

- $n =$ even number: $\lim_{x \to \pm\infty} x^n = \infty$
- $n =$ odd number: $\lim_{x \to \infty} x^n = \infty$ and $\lim_{x \to -\infty} x^n = -\infty$

- (again, assuming n is positive)

$$\lim_{x\to\pm\infty}\frac{1}{x^n}=\lim_{x\to\pm\infty}x^{-n}=0$$

- For a polynomial, only look at the term with the highest exponent:

$$\lim_{x\to\pm\infty}p(x)=\lim_{x\to\pm\infty}(\text{constant})\cdot x^n$$

The constant is called the **leading coefficient**, $\mathsf{lc}(p)$. The highest exponent that appears in the polynomial is called the **degree**, $\deg(p)$.

Rational Functions: Suppose $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function.

- If $\deg(p) < \deg(q)$, i.e., the numerator has the smaller degree, then
$$\lim_{x\to\pm\infty} f(x) = 0$$
and $y = 0$ is a horizontal asymptote of f.

- If $\deg(p) = \deg(q)$, i.e., numerator and denominator have the same degree, then
$$\lim_{x\to\pm\infty} f(x) = \frac{\mathrm{lc}(p)}{\mathrm{lc}(q)}$$
and $y = \frac{\mathrm{lc}(p)}{\mathrm{lc}(q)}$ is a horizontal asymptote of f.

- If $\deg(p) > \deg(q)$, (numerator has the bigger degree) then

$$\lim_{x \to \pm\infty} f(x) = \infty \quad \text{or} \quad -\infty$$

  and f has no horizontal asymptote.

- Assuming that $f(x)$ is in reduced form (p and q share no common factors), vertical asymptotes occur at the zeroes of q.

  (This is why it is a good idea to check for factoring and cancelling first!)

Exercise

Determine the end behavior of the following functions (in other words, compute both limits, as $x \to \pm\infty$, for each of the functions):

1. $f(x) = \dfrac{x+1}{2x^2-3}$
2. $g(x) = \dfrac{4x^3-3x}{2x^3+5x^2+x+2}$
3. $h(x) = \dfrac{6x^4-1}{4x^3+3x^2+2x+1}$

Example

Determine the end behavior of the following functions.

1. $f(x) = \dfrac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}}$ (radical signs appear)

2. $g(x) = \cos x$ (trig)

3. $h(x) = e^x$ (exponential)

2.5 Book Problems

9-10, 13-35 (odds), 39, 43, 45, 53