Section 3.2 – The Exponential Function

1. Find the derivative of each of the following functions. You may assume that p and q are constants.

   (a) $f(x) = \pi^x + x^\pi$

   $f'(x) = (\ln \pi)\pi^x + \pi x^{\pi - 1}$

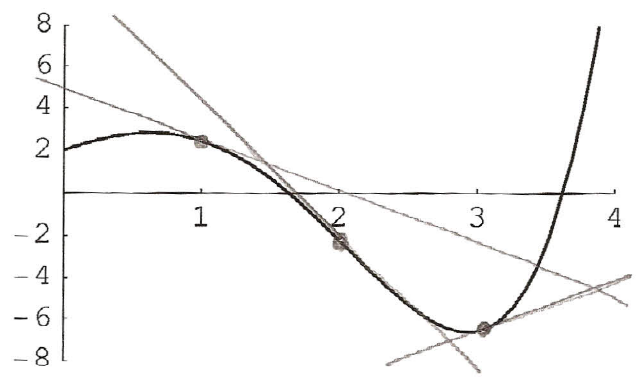
   (b) $y = qx - \dfrac{1}{\sqrt[3]{x}} + 5^x - \mathrm{e}$

   $y' = q + \dfrac{1}{3x^{4/3}} + 5\ln 5$

   (c) $g(t) = \left(\dfrac{p}{2}\right)^t + \dfrac{p}{t^2}$

   $g'(t) = \left(\ln\left(\dfrac{p}{2}\right)\right)\left(\dfrac{p}{2}\right)^t - \dfrac{2p}{t^3}$

2. Let $f(x) = 2\mathrm{e}^x - 3x^2\sqrt{x}$, whose graph is given to the right. Sketch in the tangent lines to $f(x)$ at $x = 1, x = 2$, and $x = 3$, and calculate their slopes.



$f'(x) = 2e^x - 7.5x\sqrt{x}$

$f'(1) = 2e - 7.5$

$\doteq -2.063$

$f'(2) = 2e^2 - 15\sqrt{2}$

$\doteq -6.435$

$f'(3) = 2e^3 - 22.5\sqrt{3}$

$\doteq 1.200$

3. Let $f(x) = 1 + 2e^x - 3x$.

(a) Find the equation of the tangent line to f at $x = 0$.

$f'(0) = 2e^0 - 3 = -1$

$f(0) = 1 + 2e^0 - 0 = 3$

$y - 3 = -1(x - 0)$

$y = -x + 3$

(b) At what value(s) of x does f have a horizontal tangent line? Give answer(s) in exact form and as a decimal approximation.

Horizontal tangent occurs when $f'(x) = 0$.

$f'(x) = 2e^x - 3 = 0$

$e^x = \frac{3}{2}$

$x = \ln\left(\frac{3}{2}\right)$

$\doteq 0.405$

4. The population of the world, P (in billions), is well-approximated by the function $P = 5.6(1.0117)^t$, where t represents the number of years after the beginning of 1994.

(a) What is the population of the world at the beginning of 1999? How fast is the population of the world growing at the beginning of 1999? Include units, and make it clear which answer is which.

The population at the beginning of 1999 is

$P(5) = 5.6(1.0117)^5$

$\doteq 5.935$ billion

The rate of population growth at the beginning of 1999 is

$P'(5) = (\ln(1.0117)) \cdot 5.6(1.0117)^5$

$\doteq 0.069$, or 69 million/year

(b) In what year will the population of the world be growing by 100 million people per year?

Happens when $P'(t) = 0.1$

$= (\ln(1.0117)) \cdot 5.6(1.0117)^t$

$\ln\left(\frac{0.1}{(\ln(1.0117)) \cdot 5.6}\right) = t\ln(1.0117)$

$t \doteq 36.849$

So in 2030.