Quiz 8 Calculus 1

Section 3.10 Solutions

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1. Let x and y be the sides and let A be the area. We have $A = xy$, and by the product rule

$$A' = x' \cdot y + x \cdot y'.$$

We have that $x' = y' = 1\text{cm}/s$. After $20s$, $x = 2 + 1 \cdot 20 = 22\text{cm}$ and $y = 4 + 1 \cdot 20 = 24\text{cm}$. Thus

$$A' = 1 \cdot 24 + 22 \cdot 1 = 46\text{cm}^2/s.$$

2. The formula of the volume of the pool is different for the first part (when the bottom is inclined) and for the second part (when the bottom has already been filled). If x is the level of the water, the bottom part that has been filled has length $x \cdot 100/5 = 20x$ (the cross section is a triangle), and thus, as long as $x \leq 5$, the volume is

$$V = \frac{x \cdot (20x)}{2} \cdot 20 = 200x^2.$$

The overall volume of the bottom par is $(5 \cdot 100)/2 \cdot 20 = 5000\text{m}^3$. At a rate of $1\text{m}^3/min$, it takes $5000/60 = 83$ hours to fill the bottom. After 4 hours, we are still filling the bottom. By the chain rule,

$$V' = 400x \cdot x';$$

solving for x' we have

$$x' = \frac{V'}{400x}.$$

After 4 hours, we have filled $4 \cdot 60 = 240\text{m}^3$ of water. If we set

$$240 = 200x^2,$$

we solve for x and find

$$x = \sqrt{240/200} = 1.1\text{m}.$$

If we plug this in the formula for x' (and $V' = 1$) we obtain

$$x' = \frac{1}{400 \cdot 1.1} = 0.0023\text{m}/min.$$

The overall volume of the pool is the volume of the bottom plus the volume of the top. The volume of the bottom is 5000m^3. The volume of the top is

$$1 \cdot 100 \cdot 20 = 2000\text{m}^3.$$

The overall volume is $V = 5000 + 2000 = 7000\text{m}^3$. At a rate of $1\text{m}^3/min$ it takes $7000/60 = 117$ hours to fill the pool.

3. If x is the position of the foot of the ladder and y is the height of the top of the ladder, then

$$x^2 + y^2 = 12^2$$

(the ladder always describes a rectangle triangle). The problem is asking for the value of x when $|x'| = |y'|$. By the chain rule,

$$2x \cdot x' + 2y \cdot y' = 0.$$

We know that $x' = 0.2\text{ft}/s$. Solving for y' we find

$$y' = -\frac{x \cdot x'}{y} = -\frac{x}{y}0.2.$$

Thus $y' = 0.2\text{ft}/s$ if $\frac{x}{y} = 1$, that is, if $x = y$. Since $x^2 + y^2 = 144$, this happens when

$$x^2 + x^2 = 144,$$

that is,

$$x = \sqrt{144/2} = 6\sqrt{2}\text{ft}.$$

4. Let V_1 be the volume of water in the first pool and V_2 be the volume of water in the second pool. If x_1 is the level of the water in the first pool, the amount of water in the first pool is

$$V_1 = x_1 \cdot 25\pi.$$

Similarly, if x_2 is the level of water in the second pool, the amount of water is

$$V_2 = x_2 \cdot 64\pi.$$

The problem is asking for x_2', knowing that $V_1' = V_2'$ and that $x_1' = 0.5\text{m}/min$. If we differentiate V_1 with respect to the time we find

$$V_1' = 25\pi x_1' = 25\pi \cdot .5\text{m}^3/min.$$

Similarly,

$$V_2' = 64\pi x_2'.$$

Solving for x_2' and using that $V_2' = V_1' = 25\pi \cdot .5\text{m}^3/min$, we obtain

$$x_2' = \frac{V_2'}{64\pi} = \frac{25\pi \cdot .5}{64\pi} = \frac{25}{128} = 0.2\text{m}/min.$$

5. The volume of a cone is $1/3\pi r^2 h$, where r is the radius and h is the height. If x is the water level in the cone, the radius is $4x/5$ and the volume of water in the cone is

$$V_{con} = \frac{1}{3}\pi(4x/5)^2 x = \frac{16x^3}{75}\pi.$$

By the chain rule

$$V_{con}' = \frac{16 \cdot 3x^2 \cdot x'}{75}\pi = \frac{16 \cdot x^2 \cdot x'}{25}\pi.$$

We know that $x' = -0.5\text{m}/min$ (the level is dropping, that is the reason for the negative sign). When $x = 1$m we have

$$V'_{con} = \frac{16 \cdot 1^2 \cdot (-.5)}{25}\pi = -.32\pi\text{m}^3/min.$$

If y is the level of water in the cylinder, the amount of water in the cylinder is

$$V_{cyl} = \pi 4^2 y = 16\pi y.$$

Thus

$$V'_{cyl} = 16\pi y'.$$

Solving for y' we have

$$y' = \frac{V'_{cyl}}{16\pi}.$$

Since all the water leaning the cone is falling in the cylinder, $V'_{cyl} = -V'_{con}$. When $x = 1$ we have

$$y' = \frac{V'_{cyl}}{16\pi} = \frac{.32/\pi}{16/\pi} = 0.02\text{m}/min.$$