§2.3 slide

$$\lim_{x\to 0} x\sin\frac{1}{x}$$

$$x\left[-1 \le \sin\left(\frac{1}{x}\right) \le 1\right]$$

$$\lim_{x\to 0}\left[-|x| \le x\sin\frac{1}{x} \le |x|\right]$$

$$0 \le \lim_{x\to 0} x\sin\left(\frac{1}{x}\right) \le 0$$

By the Squeeze Thm,

$$\lim_{x\to 0} x\sin\frac{1}{x} = 0.$$

☆ Question: For $\lim_{x\to 0} x^2\sin\frac{1}{x}$, why don't you need the absolute value symbols?

§2.4-2.5 slides

$$f(x) = \frac{2x^3 + 10x^2 + 12x}{x^3 + 2x^2}$$

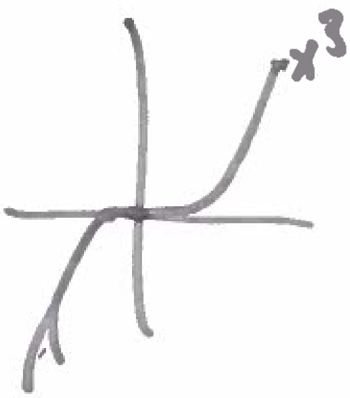
$$\lim_{x\to\infty} f(x)\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right) = \lim_{x\to\infty} \frac{2 + \cancel{\frac{10}{x}}^{\to 0} + \cancel{\frac{12}{x^2}}^{\to 0}}{1 + \cancel{\frac{2}{x}}^{\to 0}}$$

$$= 2$$

$$\lim_{x\to-\infty} f(x)\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right) = \lim_{x\to-\infty} \frac{2 + \frac{10}{x} + \frac{12}{x^2}}{1 + 2/x} = 2$$

* Use this technique when $x \to \pm\infty$ (highest power of denominator)

horizontal asymp. $y = 2$



$$\lim_{x\to-\infty} -\frac{1}{x^3}$$

Vertical Asymptotes:

Factor first!

$$f(x) = \frac{2x^3 + 10x^2 + 12x}{x^3 + 2x^2}$$

$$= \frac{2x(x^2+5x+6)}{x^2(x+2)} = \frac{2\cancel{x}(x+3)\cancel{(x+2)}}{x^{\cancel{2}}\cancel{(x+2)}}$$

Check v.a. for $x=0$:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{2(x+3)}{x} = \infty$$

pos

pos

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{2(x+3)}{x} = -\infty$$

pos

neg

* Make sure at least one of these is not a #.

*For horizontal asymptotes there is no need to factor. For vertical asymptotes, you should factor.

---

§2.6 slides

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1} & x \neq -1 \\ a & x = -1 \end{cases}$$

Use the Continuity Checklist:

1. $f(-1) = a$ (so f is defined at -1)

2. $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} \dfrac{(x+2)\cancel{(x+1)}}{\cancel{x+1}} = 1$

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} x+2 = 1$

$\longrightarrow$

(So 2-sided limit exists)

③ Must have

$$f(-1) = \lim_{x \to -1} f(x)$$

$$a = 1$$

So $\boxed{a = 1}$.

---

§2.6 slides

IVT Problem:

$$f(x) = 2x^3 + x$$

From the Theorem, $a = -1$, $b = 1$, $L = 2$.

First, is $f(x)$ continuous on the interval $(-1, 1)$? Yes, since polynomials are continuous everywhere.

$\longrightarrow$

Find the endpoints:

$$f(-1) = 2(-1)^3 + (-1) = -2-1 = -3$$

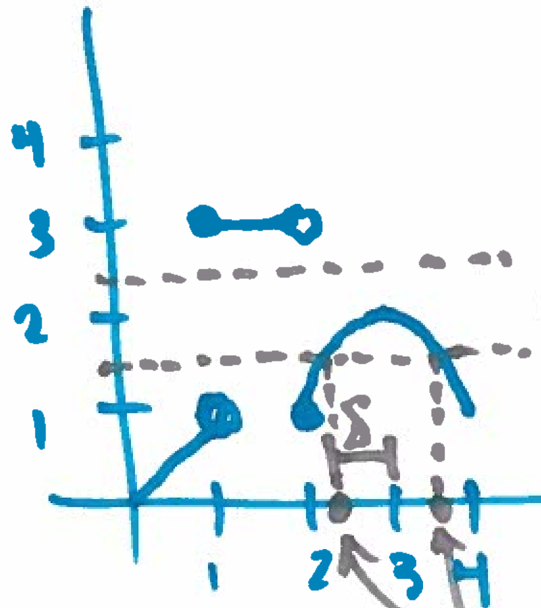
$$f(1) = 2(1)^3 + (1) = 3$$

Since $\overset{-3}{f(-1)} < 2 < \overset{3}{f(1)}$,

by the Intermediate Value Theorem,

$f(x)=2$ has a solution in $(-1,1)$.

§2.7 slide



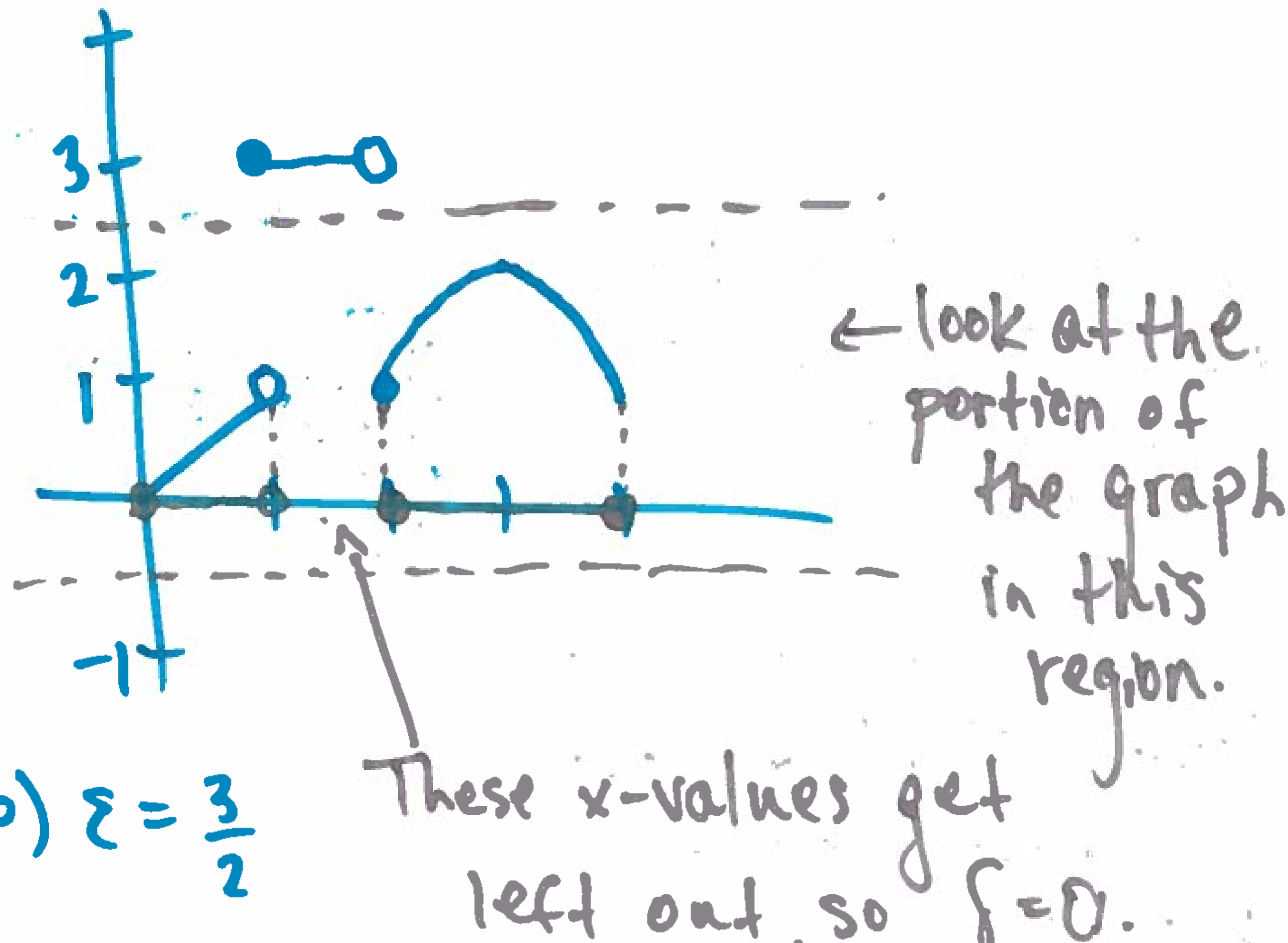
← look for the portion of the function that appears in this region

Since we don't have the formula for the function, and it's not linear, it is not immediate what the x-values are.

However, they are centered around $x=3$, so the δs on each side are equal.

(a) $\varepsilon = \frac{1}{2}$

⟶



(b) $\varepsilon = \frac{3}{2}$

But we need
$0 < |x-2| < \delta$.

The point is, $\lim_{x \to 2} g(x)$ does not exist. So the ε-δ stuff does not work.

☆ Understand how to find δs and how to tell when they don't exist.

(a) Since the problem asks about a particular point on the graph, use

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

negatives cancel

$$\lim_{x\to -5} \frac{\frac{1}{x}-\frac{1}{(-5)}}{x-(-5)} = \lim_{x\to -5} \frac{\frac{5+x}{5x}}{x+5}$$

common denominator

$$= \lim_{x\to -5} \frac{\cancel{5+x}}{5x(\cancel{x+5})}$$

$$= \lim_{x\to -5} \frac{1}{5x} = -\frac{1}{25}$$

Equation for tangent line:

$$y-f(-5)=\left(\frac{-1}{25}\right)(x-(-5))$$

→

Simplify:

$$y - \left(\frac{1}{-5}\right) = -\frac{1}{25}(x+5)$$

$$\boxed{y + \frac{1}{5} = -\frac{1}{25}(x+5).}$$

(b) Since the problem asks for a formula, use

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{\cancel{x} - (x+h)}{x(x+h)}}{h}$$

common denominator

$$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h)\cancel{h}} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

→

(c) $f'(-5) = \dfrac{-1}{(-5)^2} = \dfrac{-1}{25}$ ✓.