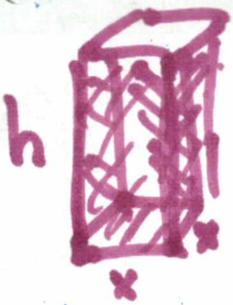


$48 ft^2 = x^2 + 4xh$ ← constraint (use picture)

$$\frac{48-x^2}{4x} = h$$

$0 \le x \le \sqrt{48}$

$V = x^2 h$ ← objective function

$$= x^2\left(\frac{12}{x} - \frac{x}{4}\right) = 12x - \frac{1}{4}x^3$$

$$V'(x) = 12 - \frac{3}{4}x^2 = 0$$

$$-\frac{3}{4}x^2 = -12$$

$$x^2 = 16$$

$x = 4$ is a CP

Endpts:

$V(4) = 12(4) - \frac{1}{4}(4^3) = 32 ft^3$

$V(0) = 0$

$V(\sqrt{48}) = 48\left(\frac{12}{\sqrt{48}} - \frac{\sqrt{48}}{4}\right) = (\sqrt{48})12 - 12\sqrt{48} = 0$

OR

$$V(\sqrt{48}) = 48\left(\frac{12}{\sqrt{48}} - \frac{\sqrt{48}}{4}\right) = 0$$

cross multiply!
48-48

So the max volume is $32ft^2$, when $x=4ft$. That means

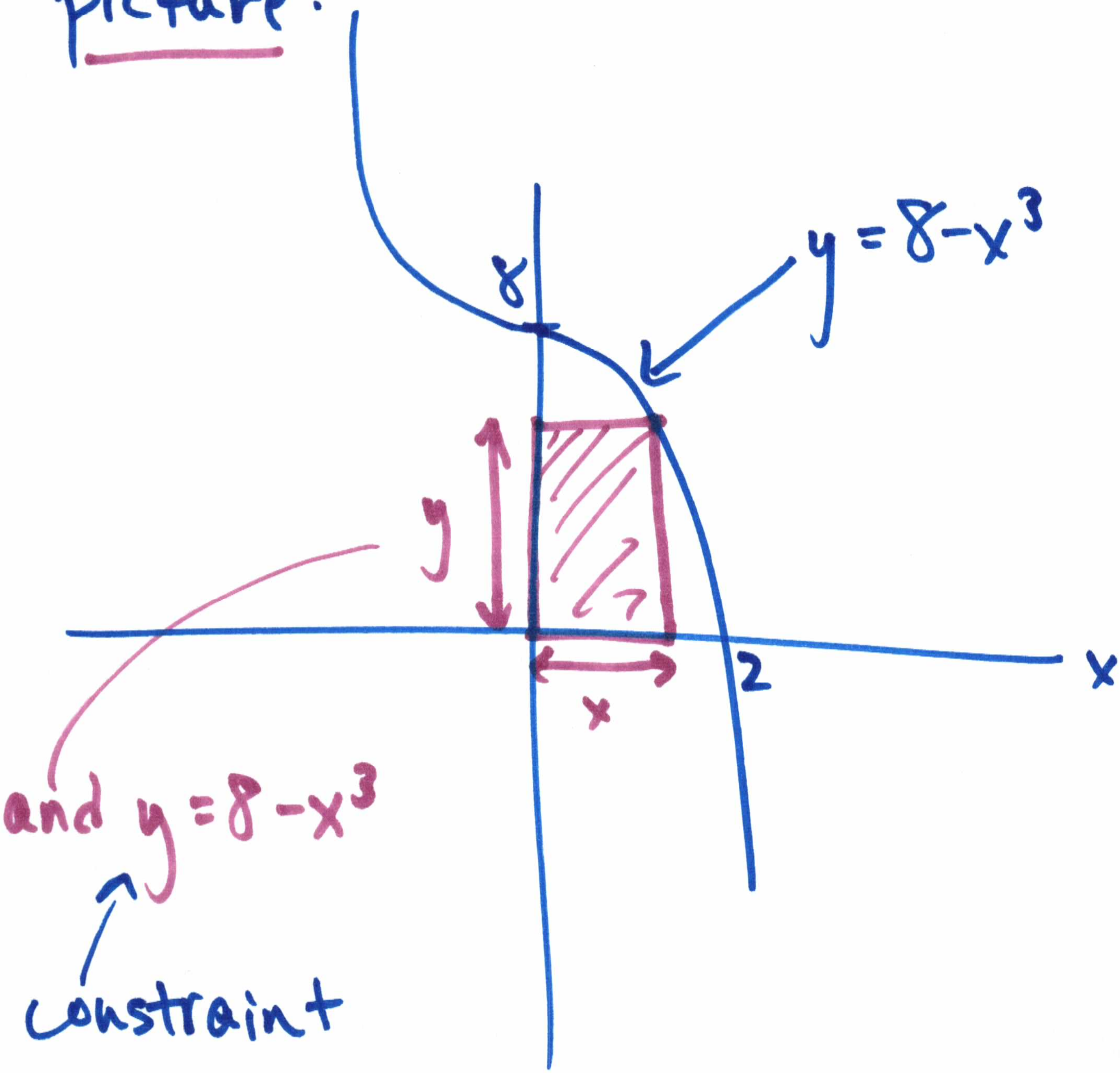
$$h = \frac{12}{x} - \frac{x}{4} = \frac{12}{(4)} - \frac{(4)}{4} = 2ft$$

and to answer the question, the dimensions are

$$\boxed{4ft \times 4ft \times 2ft.}$$

Rectangle inscribed in x-axis, y-axis, and $y = 8 - x^3$. Max area?

picture:



Objective: $A = xy = x(8 - x^3)$

$= 8x - x^4$

$A'(x) = 8 - 4x^3 = 0$

$\Rightarrow x = \sqrt[3]{2}$

Does it give a max or min? We must compare to the endpoints. From the picture, $0 \le x \le 2$.

Absolute extreme:

$A(0) = 0(8 - 0^3) = 0$

max

$A(\sqrt[3]{2}) = \sqrt[3]{2}\left(8 - (\sqrt[3]{2})^3\right)$

$= \sqrt[3]{2}(8-2) = \boxed{6\sqrt[3]{2}}$

$A(2) = 2(8 - 2^3) = 0.$

The max area is $6\sqrt[3]{2}$ and the dimensions are $x = \sqrt[3]{2}$, $\longrightarrow$

$$y = 8 - x^3 = 8 - \left(\sqrt[3]{2}\right)^3$$
$$= 6.$$