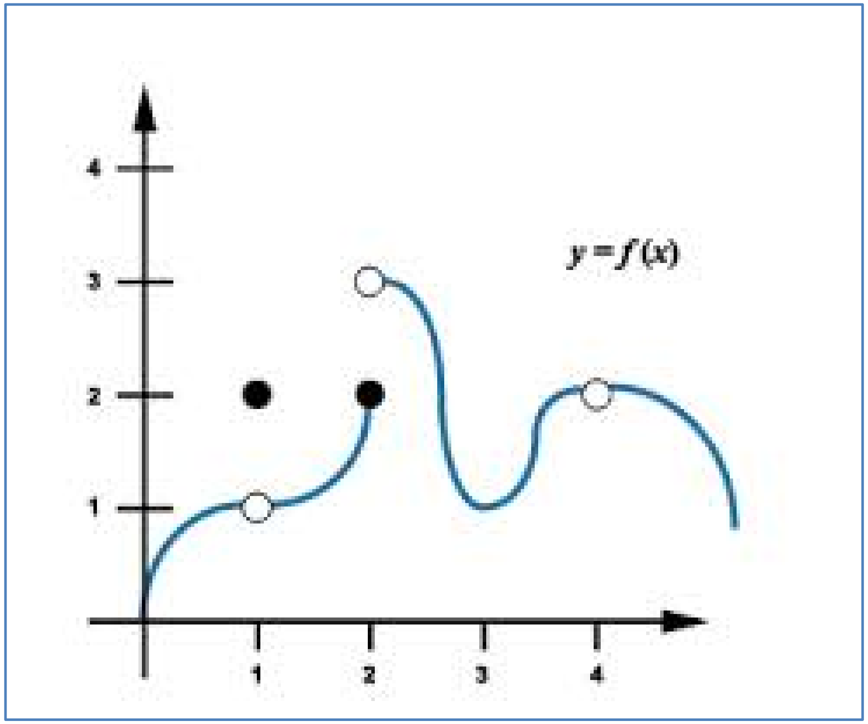
Unit 1, Lesson 2
Graphical and Tabular/Numerical Limits

Graphical and Tabular/Numerical Limits.

OBJECTIVES:

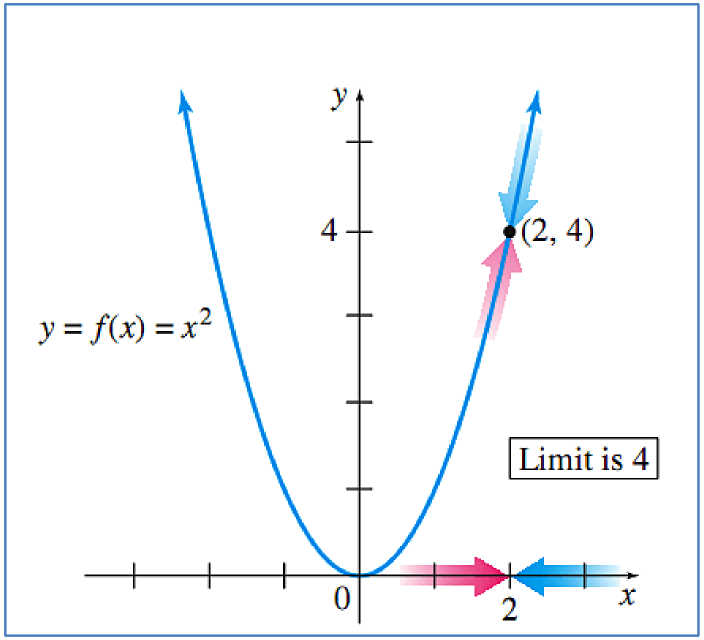
- Evaluate limits by way of tables and graphs.
- Determine the existence of and find limits at real numbers.
- Use rules of limits.

Limits are a tool that helps us to describe the behavior of a function as x values approach a particular number.



What happens to $f(x) = x^2$ when x is really close to 2, but not necessarily equal to 2?

	x approaches 2 from left →				↓	← x approaches 2 from right			
x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
$f(x)$	3.61	3.9601	3.996001	3.99960001	4	4.00040001	4.004001	4.0401	4.41
	$f(x)$ approaches 4 →				↑	← $f(x)$ approaches 4			



We denote this by

$$\lim_{x \to 2} x^2 = 4$$

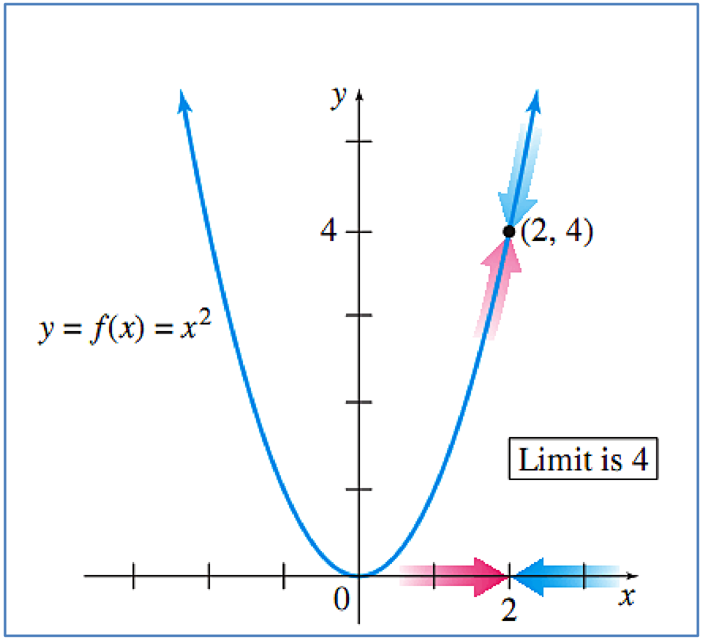
and we say

"the limit of $f(x) = x^2$ as $x \to 2$ is 4.

This is an example of a **two-sided limit.**

One Sided Limits

- *Limit from the left:* $\lim_{x \to 2^-} f(x) = 4$
- *Limit from the right:* $\lim_{x \to 2^+} f(x) = 4$



NOTE: The two sided limit can only exist if both one-sided limits exist and are equal to one another.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} f(x)$$

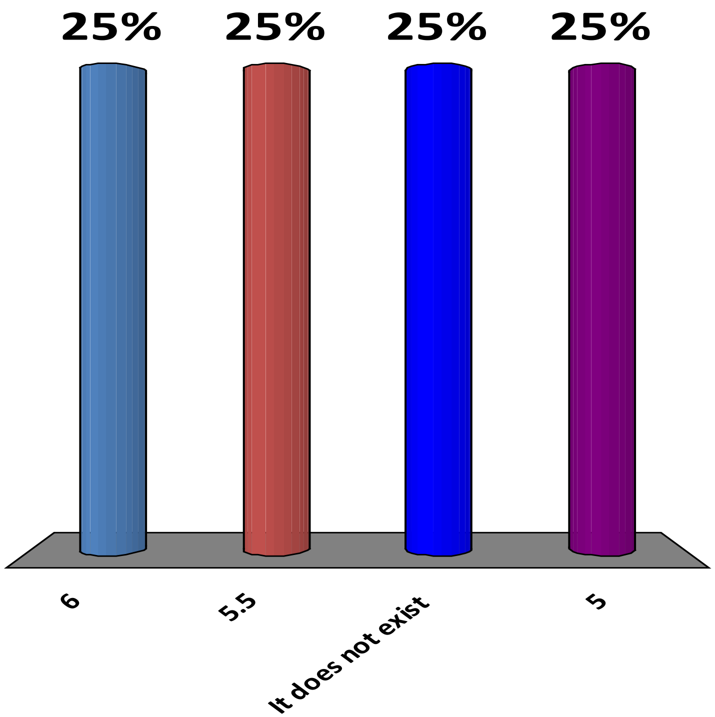
If $\lim_{x \to 2^-} f(x) = 5$ and $\lim_{x \to 2^+} f(x) = 6$, then $\lim_{x \to 2} f(x) =$?

A. 6

B. 5.5

C. It does not exist

D. 5



Limit of a Function

Let f be a function and let a and L be real numbers. If

1. as x takes values closer and closer (but not equal) to a on both sides of a, the corresponding values of $f(x)$ get closer and closer (and perhaps equal) to L; and
2. the value of $f(x)$ can be made as close to L as desired by taking values of x close enough to a;

then L is the **limit** of $f(x)$ as x approaches a, written

$$\lim_{x \to a} f(x) = L.$$

Find $\lim_{x \to 2} g(x)$ where $g(x) = \frac{x^3 - 2x^2}{x - 2}$.

x approaches 2 from left | x approaches 2 from right

x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
$g(x)$	3.61	3.9601	3.996001	3.99960001		4.00040001	4.004001	4.0401	4.41

$f(x)$ approaches 4 | Undefined | $f(x)$ approaches 4

$$\lim_{x \to 2^-} g(x) = 4$$

$$\lim_{x \to 2^+} g(x) = 4$$

$$\lim_{x \to 2} g(x) = 4$$

Use a table to find

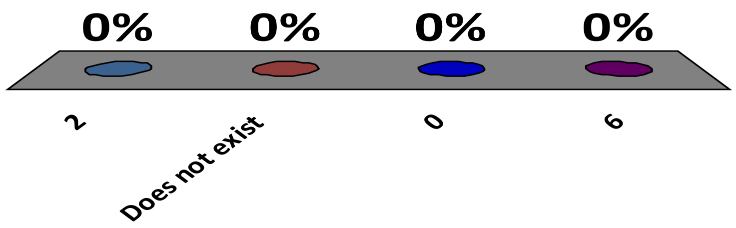
$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

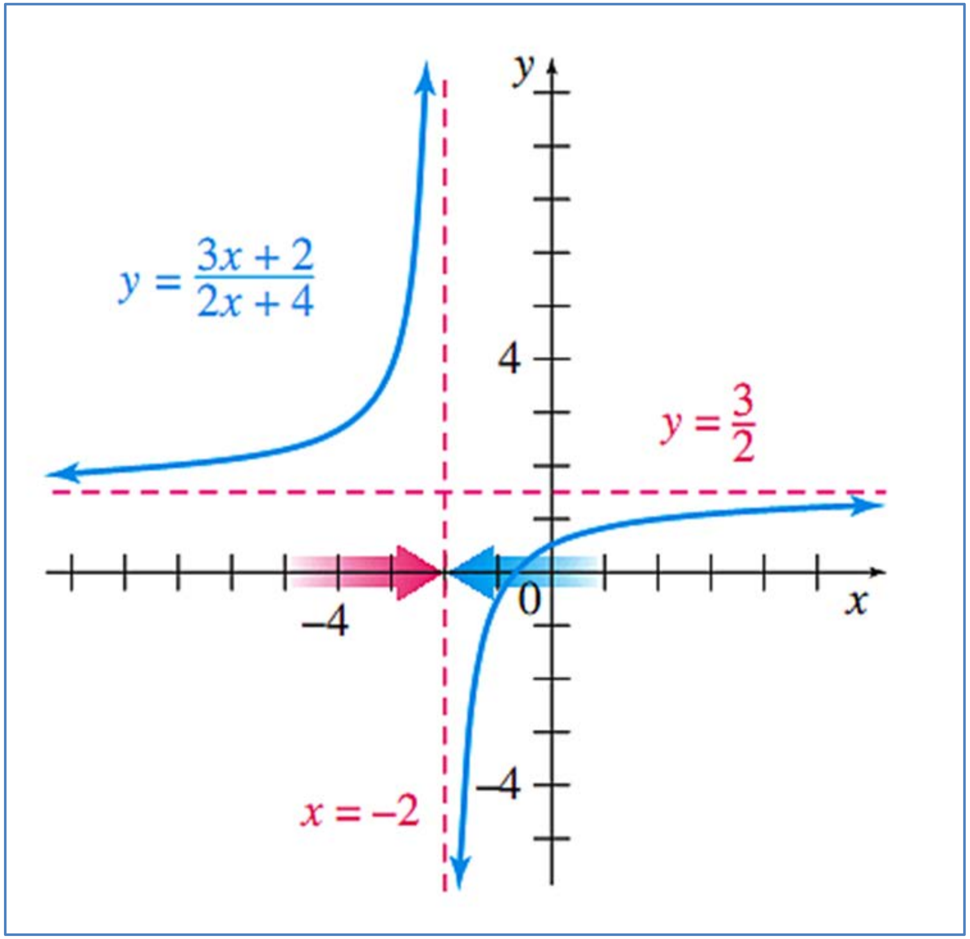
A. 2

B. Does not exist

C. 0

D. 6





$$\lim_{x \to -2^-} \frac{3x+2}{2x+4} = \infty$$

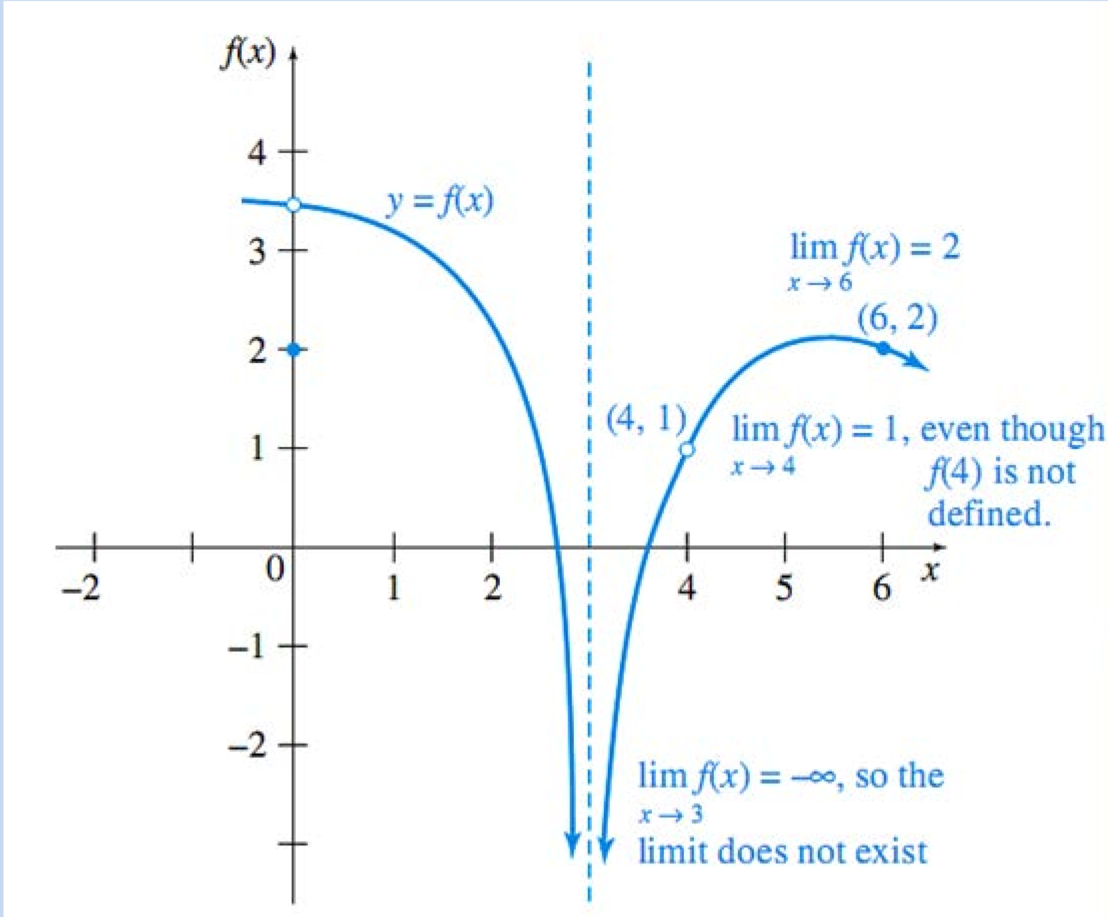
$$\lim_{x \to -2^+} \frac{3x+2}{2x+4} = -\infty$$

$$\lim_{x \to -2} \frac{3x+2}{2x+4} =$$

Existence of Limits

The limit of f as x approaches a may not exist.

1. If $f(x)$ becomes infinitely large in magnitude (positive or negative) as x approaches the number a from either side, we write $\lim_{x \to a} f(x) = \infty$ or $\lim_{x \to a} f(x) = -\infty$. In either case, the limit does not exist.
2. If $f(x)$ becomes infinitely large in magnitude (positive) as x approaches a from one side and infinitely large in magnitude (negative) as x approaches a from the other side, then $\lim_{x \to a} f(x)$ does not exist.
3. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, and $L \neq M$, then $\lim_{x \to a} f(x)$ does not exist.



f(x)
y = f(x)
$\lim_{x \to 6} f(x) = 2$
(6, 2)
(4, 1)
$\lim_{x \to 4} f(x) = 1$, even though $f(4)$ is not defined.
$\lim_{x \to 3} f(x) = -\infty$, so the limit does not exist
x

Limits at Infinity

For any positive real number n,

$$\lim_{x \to \infty} \frac{1}{x^n} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{1}{x^n} = 0.^*$$

Finding Limits at Infinity

If $f(x) = p(x)/q(x)$, for polynomials $p(x)$ and $q(x)$, $q(x) \neq 0$, $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$ can be found as follows.

1. Divide $p(x)$ and $q(x)$ by the highest power of x in $q(x)$.
2. Use the rules for limits, including the rules for limits at infinity,

$$\lim_{x \to \infty} \frac{1}{x^n} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{1}{x^n} = 0,$$

to find the limit of the result from step 1.

$$\text{Find } \lim_{x\to\infty}\frac{2x^2+3x-4}{6x^2-5x+7}.$$

Solution: Here, the highest power of x is x^2, which is used to divide each term in the numerator and denominator.

$$\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{4}{x^2}}{\frac{6x^2}{x^2}-\frac{5x}{x^2}+\frac{7}{x^2}}=\lim_{x\to\infty}\frac{2+\frac{3}{x}-\frac{4}{x^2}}{6-\frac{5}{x}+\frac{7}{x^2}}$$

$$=\frac{2}{6}=\frac{1}{3}$$

$$\lim_{x\to\infty}\frac{1}{x^n}=0$$

$$\lim_{x\to\infty} \frac{2x^2 - 1}{3x^4 + 2} =$$

$$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^4} - \frac{1}{x^4}}{\frac{3x^4}{x^4} + \frac{2}{x^4}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x^2} - \frac{1}{x^4}}{3 + \frac{2}{x^4}} = 0$$

Question:

If $f(1) = 5$, then must $\lim_{x \to 1} f(x)$ exist?

If indeed the limit does exist, then must $\lim_{x \to 1} f(x) = 5$?

Explain your answer.