Orders of groups and elements

1. Orders of groups and elements

Order of an element

Order of a group

Order of an element

Definition 1

Let G denote a group with $g \in G$.

(a) *g has* **infinite order** *means all the non-negative powers of g are distinct.*

(b) *g has* **order** *d means d is the smallest positive integer such that $g^d = g^0 = 1$. We say g has* **finite order**.

We write $|g|$ or $\mathrm{ord}(g)$ to denote the order of g.

Question (cf. Problem 54)

What is the order of each element in $\mathbb{Z}_{12}$?

Exercise 1 (cf. Problem 53)

In $G = (\mathrm{GL}(2,\mathbb{R}), \cdot)$ find the order of each of the following elements:

$$g = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad h = \begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix} \qquad j = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Exercise 2 (cf. Problem 55)

Find the order of each element in the sandpile group $\mathcal{S}(\Gamma)$ corresponding to the graph in Figure **??**.

Order of a group

Definition 2

The **order** *of a group G is its cardinality as a set, $|G|$.*

Caution: The *dihedral group D_n of order n* has group order not n, but $2n$.

Exercise 3

Find the order of the *symmetric group of order n*, S_n (it's not n).

Theorem 1 (Lagrange's Theorem)

Given a finite (order) group G, the order of every subgroup of G divides $|G|$. $\square$

Question

Verify Lagrange's Theorem for $\mathbb{Z}_{12}$.