Group Work Review for Exam III Solutions

Fall 2016
Survey of Calculus

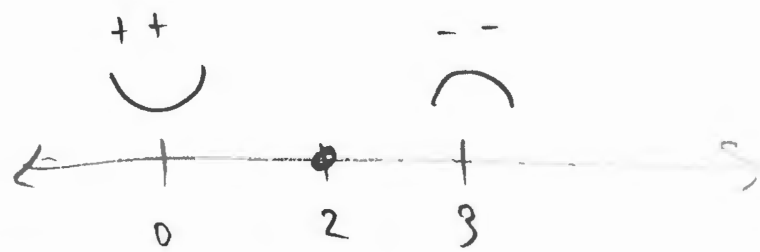
1. $f'(x) = \frac{3}{7}x^{-4/7} + e^x \Rightarrow \boxed{f''(x) = -\frac{12}{49}x^{-11/7} + e^x}$

2. $f'(x) = -6x^2 + 24x + 171$

$f''(x) = -12x + 24 = 0$

$x = 2$

$\boxed{\text{concave up: } (-\infty, 2)}$



$f''(0) = -12(0) + 24 > 0$

$f''(3) = -12(3) + 24 < 0$

3. Want $x + y = 48$ (constraint)

and $P(x,y) = xy$ maximized (objective)

$y = 48 - x \Rightarrow P(x) = x(48 - x) = 48x - x^2$

$P'(x) = 48 - 2x = 0$

$x = 24 \Rightarrow y = 48 - 24 = 24$

Check that $x = 24$ is a max:

$P''(x) = -2 < 0 \Rightarrow$ max for all x

$\boxed{\text{numbers are } 24, 24}$

$\longrightarrow$

4. $f'(x) = \frac{2}{\cancel{7}}(9+7x)^{-5/7}(\cancel{7}) = 0$

$$= \frac{2}{(9+7x)^{5/7}} = 0 \text{ never}$$

$\Rightarrow \boxed{\text{no relative extrema}}$

5. $f'(x) = 9x^2 - 6x - 3 = 0$

$3(3x^2 - 2x - 1) = 0$

$(3x+1)(x-1)$

$\Rightarrow x = \frac{-1}{3}, 1$ are cps

not in domain

$f(-1) = 3(-1)^3 - 3(-1)^2 - 3(-1) + 8$

$-3 \quad -3 \quad +3 + 8$

$= 6$

$f\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right)^3 - 3\left(\frac{-1}{3}\right)^2 - 3\left(\frac{-1}{3}\right) + 8$

$\frac{-3}{27} \quad -\frac{3}{9} \quad + 1 \quad + 8$

$\frac{-1}{9} \qquad \frac{81}{9}$

max $\to = \frac{83}{9} \leftarrow$ larger than $\frac{81}{9} = 9$

$f(0) = 3(0)^3 - 3(0)^2 - 3(0) + 8$

$= 8$

min

absolute max at $x = -\frac{1}{3}, y = \frac{83}{9}$

absolute min at $x = 0, y = 6$

6. $f_x(x,y) = e^{2x+2y+42}(2)$

$f_{xx}(x,y) = 2e^{2x+2y+12}(2)$

$= \boxed{4e^{2x+2y+12}}$

$\longrightarrow$

7. $f_x(x,y) = 4x^3 - 4x = 0 \Rightarrow 4x(x^2-1) = 0$

$x = \pm 1, 0$

$f_y(x,y) = 2y - 4 = 0$

$\Rightarrow y = -2 \quad \Rightarrow \text{cps}: (-1,-2), (1,-2), (0,-2)$

$$D(x,y) = \begin{vmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 12x^2-4 & 0 \\ 0 & 2 \end{vmatrix}$$

$$= (12x^2-4)(2) = 24x^2 - 8$$

$D(-1,-2) = 24(-1)^2 - 8 > 0$

and $f_{xx}(-1,2) = 12(-1)^2 - 4 > 0$

$\Rightarrow \boxed{\text{min at } (x,y) = (-1,-2)}$

$D(1,-2) = 24(1)^2 - 8 > 0$

and $f_{xx} > 0 \Rightarrow \boxed{\text{min at } (x,y) = (1,2)}$

$D(0,0) = 24(0)^2 - 8 < 0$

$\Rightarrow \boxed{\text{saddle point at } (x,y) = (0,0)}$