Mon 8 Dec 2014

- Tues 9 Dec: 5.5 Quiz in drill
- Wed 10 Dec: review
- Thu 11 Dec: last day of class
- Mon 15 Dec, 6-8p: FINAL Hillside 206
- CEA: likely location SCEN 401, starts at 4p, not allowed to leave until after 6p… stay tuned for more information

Substitution Rule

We have seen a few methods to find antiderivatives (e.g., power rule, knowledge of derivatives, etc.). However, for many functions, it is more challenging to find the antiderivative.

Today we examine the substitution rule as a method to integrate.

Integration by Trial and Error

One somewhat inefficient method to find an antiderivative is by trial and error (with a natural check in find the derivative).

EX: $\int \cos(2x+5)$

Integration by Trial and Error

EX: $\int \cos(2x+5)$

Is it sin (2x+5) + C?

Check: $\frac{d}{dx}\sin(2x+5) = 2 \cdot \cos(2x+5)$

How can you use your first attempt to refine your guess?

Integration by Trial and Error

EX: $\int \cos(2x+5)$

So we try $\dfrac{1}{2}\sin(2x+5)+C$

Check: $\dfrac{d}{dx}\left(\dfrac{1}{2}\sin(2x+5)+C\right)=\dfrac{1}{2}\left(2\cdot\cos(2x+5)\right)=\cos(2x+5)$

So $\int \cos(2x+5)=\dfrac{1}{2}\sin(2x+5)+C$

Substitution Rule

Trial and error can work in particular settings, but it is not an efficient strategy and doesn't work with some functions.

However, just as the Chain Rule helped us differentiate complex functions, the substitution rule (based on the Chain Rule) allows us to integrate complex functions.

Substitution Rule

Suppose we have F(g(x)), where *F* is an antiderivative of *f*. Then $\frac{d}{dx}[F(g(x))] = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$.

So $\int f(g(x)) \cdot g'(x) dx = F(g(x)) + C$

If we let u = g(x), then du = g'(x)dx.

So $\int f(g(x)) \cdot g'(x) dx = \int f(u)\, du$: the substitution rule for integrals.

Substitution Rule for Indefinite Integrals

Let *u = g(x)*, where *g'* is continuous on an interval, and let *f* be continuous on the corresponding range of *g*. On that interval,

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

Example

Evaluate $\int 8x\cos(4x^2+3)\,dx$

Solution: Let $u=4x^2+3$. So *du = 8x dx.* So

$$\int 8x\cos(4x^2+3)\,dx = \int \cos(4x^2+3)8x\,dx$$

$$= \int \cos u\,du$$

$$= \sin u + C$$

$$= \sin(4x^2+3) + C$$

So $\int 8x\cos(4x^2+3)\,dx = \sin(4x^2+3)+C$

Procedures for Substitution Rule

1. Given an indefinite integral involving a composite function $f(g(x))$, identify an inner function $u = g(x)$ such that a constant multiple of $g'(x)$ appears in the integrand.
2. Substitute $u = g(x)$ and $du = g'(x)$ in the integral.
3. Evaluate the new indefinite integral with respect to u.
4. Write the result in terms of x using $u = g(x)$.

NOTE: Not all integrals yield to the Substitution Rule.

Exercises

Evaluate the following integrals:

$$\int \sin^{10} x \cos x \, dx$$

$$-\int \frac{\csc x \cot x}{1+\csc x} dx$$

$$\int \frac{1}{10x-3} dx$$

$$\int (3x^2+8x+5)^8 (3x+4) \, dx$$

Variations on Substitution Rule

There are times when the u-substitution is not obvious or that more work must be done.

Example: $\int \frac{x^2}{(x+1)^4}dx$

Variations on Substitution Rule

Example: $\int \frac{x^2}{(x+1)^4}dx$

Let u = x+1. This works for the bottom, but on top we need to replace x^2. So if u = x+1, then x = u-1. In both cases, du = dx. So

$$\int \frac{x^2}{(x+1)^4}dx = \int \frac{(u-1)^2}{u^4}du = \int \frac{u^2-2u+1}{u^4}$$

$$= \int u^{-2} - 2u^{-3} + u^{-4}\,du$$

$$= \frac{u^{-1}}{-1} - \frac{2u^{-2}}{-2} + \frac{u^{-3}}{-3} + C$$

$$= \frac{-1}{u} + \frac{1}{u^2} - \frac{1}{3u^3} + C$$

$$= \frac{-1}{x+1} + \frac{1}{(x+1)^2} - \frac{1}{3(x+1)^3} + C$$

Substitution Rule for Definite Integrals

We can also use the Substitution Rule for Definite Integrals in two different ways:

1. Use the Substitution Rule to find an antiderivative F, and then use the Fundamental Theorem of Calculus to evaluate F(b)-F(a).
2. Once you have changed variables from x to u, you may also change the limits of integration and complete the integration with respect to u. Specifically, if $u = g(x)$, the lower limit $x = a$ is replaced by $u = g(a)$ and the upper limit $x = b$ is replaced by $u = g(b)$.

The second option is typically more efficient and should be used whenever possible.

Example

Evaluate $\int_0^4 \frac{1}{(4x-3)^4}\,dx$

Let u = 4x – 3. Then du = 4 dx. However, now that we are integrating over du instead of dx, we also have to change the interval of integration. If u = 4x – 3,

When x = 0, u = 4(0) – 3 = -3. So u = -3.

When x = 4, u = 4(4) – 3 = 13. So u = 13.

So $\int_0^4 \frac{1}{(4x-3)^4}\,dx = \frac{1}{4}\int_{-3}^{13} \frac{1}{u^4}\,du = \frac{1}{4}\left[\left.\frac{1}{-3u^3}\right|_{-3}^{13}\right] = \ldots$

If the substitution $u = x^2 - 4$ is used on the definite integral $\int_0^6 f(x)dx$, then the new limits of integration are:

A. *u = 0* and *u = -4*

B. *u = -4* and *u = 32*

C. *u = -4* and *u = 6*

D. *u = 0* and *u = 32*

Exercise

Evaluate $\displaystyle\int_0^2 \frac{2x}{(x^2+1)^2}\,dx$

Homework from Section 5.5

Do problems 9-27 odd, 29-39 odd, 53-63 odd (pgs. 363-364).

Wed 10 Dec 2014

- Last day of lecture!
- Exam 3 returned in drill tomorrow
- Monday 15 Dec: Final Exam, 6-8 pm, Hillside 206

Final Preparation

Preparation for final: Be sure to download the study guide for the final. Be prepared to do:

- Integration (power rule, substitution)—you'll have time to check these using differentiation!
- Related Rates
- Optimization
- Use of First and Second Derivative Test
- Derivatives of trig functions, inverse trig functions, log and exponential functions
- Use of derivative to find equations of tangent lines
- Limits (using analytical methods and L'Hopital's)

Final Preparation

Preparation for final:

- In general, anything that is on the study guide is fair game!!!
- WATCH YOUR NOTATION!!!! (e.g., limit notations, derivative notation, integral notation, etc.)
- WATCH YOUR DIRECTIONS!!!!! (e.g., finding limits analytically)
- CHECK YOUR WORK!!!!! (You should have time!!)

Final Preparation

A good place to start is reworking problems from the 4 exams (3 hourly tests plus midterm). This gives you a wide (yet still incomplete) scope of the problems we have done.

Other things you can do to prepare for the final:

- Examine the Study Plan on Mylabsplus to see areas where you struggled on Computer HW's
- Review Completed Paper HW's (or finish paper HW's!)
- Go back over problems worked in class, on quizzes, and on drill exercises

About the Test

- It is cumulative!!! However, the course has built to this point, so expect more from material since the midterm than before.

- Approximately 20 questions in 2 hours

- Expect grades Sat 20 Dec

Evaluating the integral $\int \frac{x}{x^2+1}\,dx$ yields the result

A. $x\tan^{-1}x + C$

B. $\dfrac{\frac{x^2}{2}}{\frac{x^3}{3+x}} + C$

C. $\frac{1}{2}\ln(x^2+1) + C$

D. $\ln x + C$