MATH 2554 (Calculus I)
Summer 2015

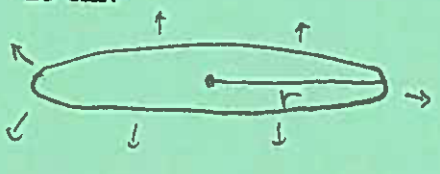
Name: SOLUTIONS

Mon 29 June 2015

Quiz 9: Optimization (§4.4) plus Related Rates

Directions: You have 40 minutes to complete this quiz. Make sure you include units and **answer the question.** This quiz is closed book and you must work alone.

1. Suppose that when a circular plate of metal is heated in an oven, its radius increases at a rate of 0.2 cm/min. At what rate is the plate's area increasing when the radius is 25 cm?



Know: $\frac{dr}{dt} = 0.2 \text{ cm/min}$

WTF: $\left.\frac{dA}{dt}\right|_{r=25\text{cm}}$ — area

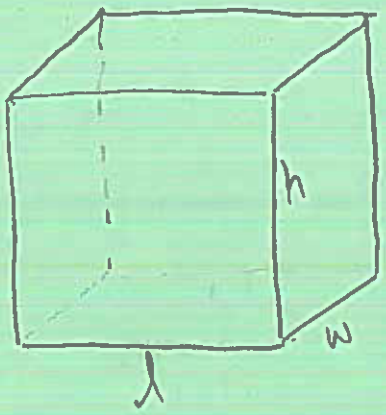
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=25\text{cm}} = 2\pi(25\text{cm})(0.2\text{ cm/min})$

$\approx 31.4 \text{ cm/min}$

2. Suppose an airline policy states that all baggage must be box-shaped with a sum of length, width, and height not exceeding 108 in. What are the dimensions and volume of a square-based box with the greatest volume under these conditions?



Objective: Maximize Volume

$V = lwh$

Constraints:

$l = w$ (square base)

$l + w + h = 2w + h = 108$ in.

$\Rightarrow h = 108 - 2w$

and $0 \leq w \leq 54$.

$V(w) = w^2(108 - 2w)$

$= 108w^2 - 2w^3$

$V'(w) = 2(108)w - 6w^2 = 0$

$6w(36 - w) = 0$

$\Rightarrow w = 36$

Dimensions:
36 in × 36 in × 36 in

Volume:
$36^3 = 46656$ in^3

Check for a max:

$V(0) = 0^2(108 - 2(0)) = 0$

$V(36) = 36^2(108 - 2(36)) = 36^3$ ← max

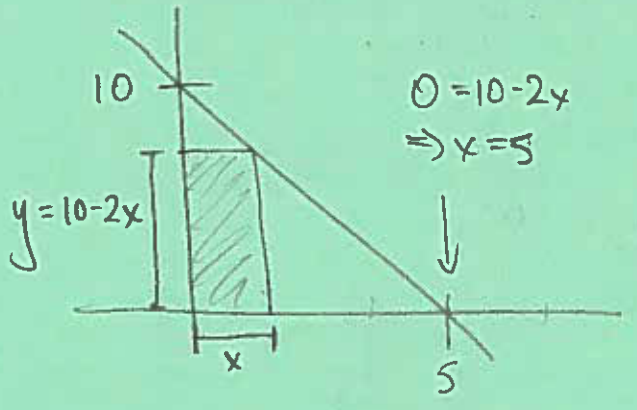
$V(54) = 54^2(108 - 2(54)) = 0$

$\Rightarrow l = w = 36$

$h = 108 - 2(36)$

$= 36$

3. A rectangle is constructed with one side on the positive x-axis, one side on the positive y-axis, and one vertex on the line $y = 10 - 2x$. What dimensions maximize the area of the rectangle? What is the maximum area?



Objective: Maximize area

$A = xy$

Constraints: picture, and

$0 \le x \le 5$.

$A(x) = x(10-2x)$

$= 10x - 2x^2$

$A'(x) = 10 - 4x = 0$

$\Rightarrow x = \frac{5}{2}$

Dimensions:

$\frac{5}{2} \times 5$

Area: $\frac{25}{2} = 12.5$

Check it's a max:

$A(0) = 0(10-2(0)) = 0$

$A\left(\frac{5}{2}\right) = \frac{5}{2}\left(10-2\left(\frac{5}{2}\right)\right) = \frac{25}{2}$ ← max

$\Rightarrow y = 10-2\left(\frac{5}{2}\right) = 5$

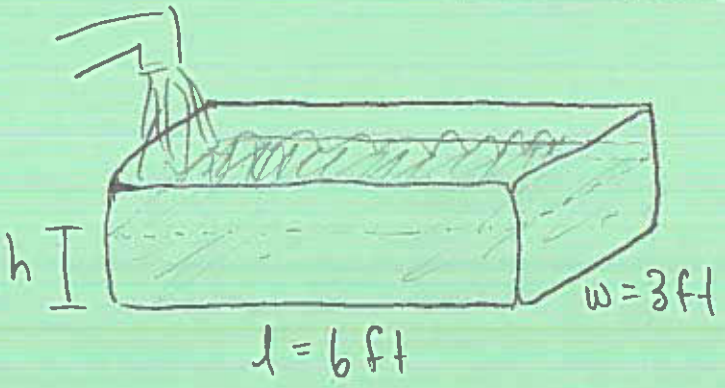
$A(5) = 5(10-2(5)) = 0$

OR, 2nd Derivative Test:

$A''(x) = -4$ means the function $A(x)$ is always concave down so its only critical point is the max.

OR, $A(x)$ is an upside-down parabola; the critical point is its vertex so must be the max.

4. A rectangular bathtub that is 3 ft wide and 6 ft long is being filled with water. How fast is the water level rising if water is filling the tub at a rate of 0.7 ft^3/min?



Know: $V = lwh$ ← Constant

$\frac{dV}{dt} = 0.7\ ft^3/min$

WTF: $\frac{dh}{dt}$

$$\frac{dV}{dt} = lw\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{1}{lw}$$

$$= \frac{(0.7\ ft^3/min)}{(6\ ft)(3\ ft)}$$

$$\approx 0.039\ ft/min$$

(OR $\approx 0.47\ in/min$)