MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Wed 10 June 2015

Quiz 4: Derivative Shortcuts (§3.2-3.3)

Directions: You have 40 minutes to complete this quiz. Open resources and you may collaborate. Draw graphs carefully (it's OK if you have to erase and redraw!).

$x^2 - \frac{2}{3} = 0$

1. Given $f(x) = -3x^2 + 2$, find the equation of the tangent line at $x = 1$. Then, draw a well-labeled graph that includes $f(x)$ and your tangent line on the same axes.

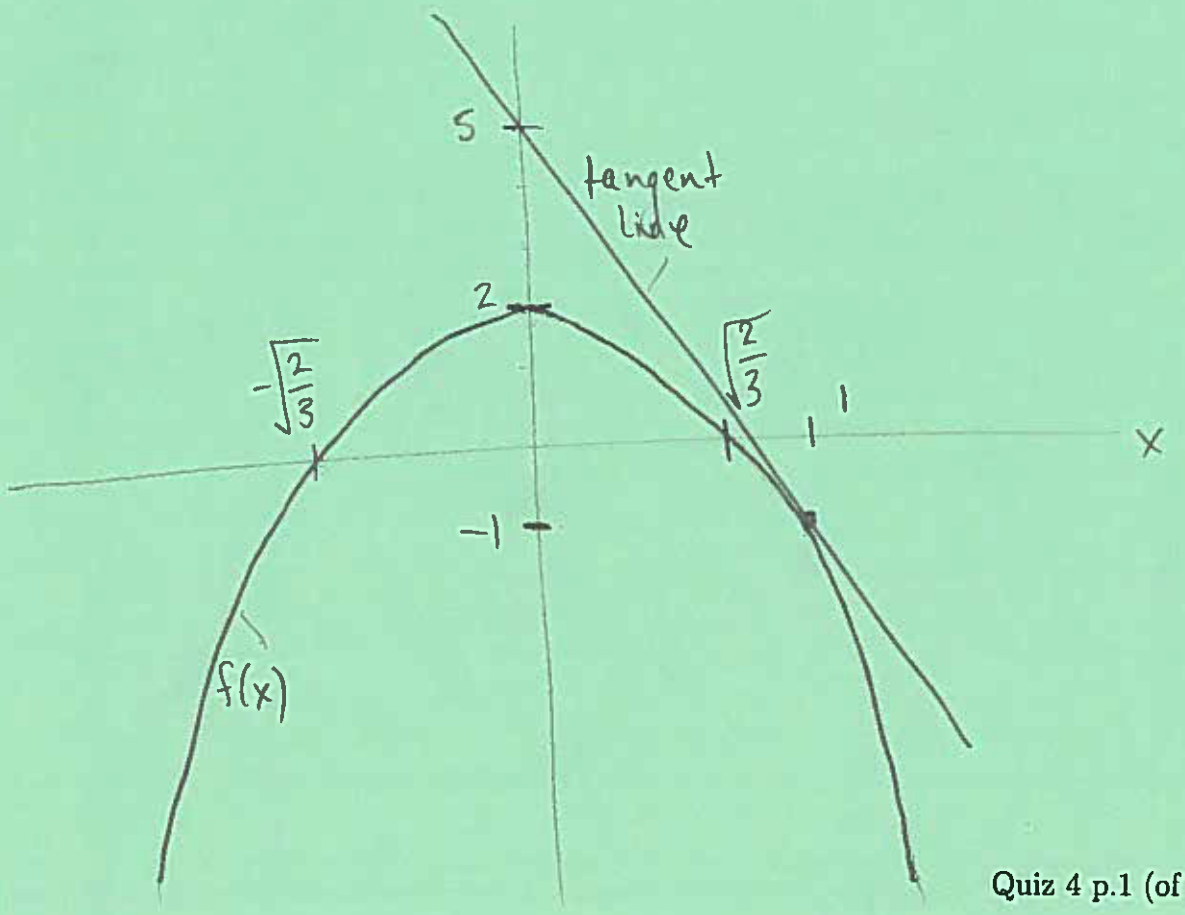
$f'(x) = -6x \qquad f'(1) = -6(1) = -6$

$f(1) = -3(1^2) + 2 = -1$

tangent line:

$y + 1 = -6(x - 1)$

$y = -6x + 5$



2. Suppose the derivative of the function f exists and assume $f(3) = 1$ and $f'(3) = 4$. Let $g(x) = x^2 + f(x)$. Find an equation of the line tangent to $y = g(x)$ at $x = 3$.

$g'(x) = 2x + f'(x)$ $\quad g'(3) = 2(3) + f'(3)$

$= 6 + 4 = 10$

$g(3) = 3^2 + f(3)$

$= 9 + 1 = 10$

tangent line:

$y - 10 = 10(x - 3)$

3. Using the following information:

$$f(2) = 1 \quad f'(2) = 2 \quad g(2) = 5 \quad g'(2) = 3$$

compute $\dfrac{d}{dx}\left(\dfrac{xf(x)}{g(x)}\right)\Big|_{x=2}$.

$$= \frac{g(x)\left((1)f(x) + xf'(x)\right) - xf(x)g'(x)}{g(x)^2}\Bigg|_{x=2}$$

$$= \frac{g(2)\left((1)f(2) + 2f'(2)\right) - 2f(2)g'(2)}{g(2)^2}$$

$$= \frac{5\left(1 + 2(2)\right) - 2(1)(3)}{5^2} = \frac{5(5) - 6}{25} = \frac{19}{25}$$

4. Given the function $f(t) = 6\sqrt{t} - 4t^3 + 9$, we can compute its derivative, showing every step, as follows:

$$\begin{aligned} f'(t) &= \frac{d}{dt}(6\sqrt{t}) + \frac{d}{dt}(-4t^3) + \frac{d}{dt}9 && \text{(Sum Rule)} \\ &= 6\frac{d}{dt}\sqrt{t} - 4\frac{d}{dt}t^3 + 0 && \text{(Constant Multiple + Constant Rule)} \\ &= 6\left(\frac{1}{2}t^{-\frac{1}{2}}\right) - 4(3t^2) && \text{(Power Rule)} \\ &= \frac{3}{\sqrt{t}} - 12t^2 && \text{(Simplify)} \end{aligned}$$

Using the back of the page if necessary, compute the derivative of

$$y = \frac{w^4 + 5we^{\frac{1}{4}w} + 1}{w^2}$$

using the same format for your work. (In other words, show. Every. STEP.)

First, rewrite $y = w^2 + \frac{5e^{\frac{1}{4}w}}{w} + \frac{1}{w^2}$.

$y' = \frac{d}{dw}w^2 + \frac{d}{dw}\left(\frac{5e^{\frac{1}{4}w}}{w}\right) + \frac{d}{dw}\left(\frac{1}{w^2}\right)$ (Sum Rule)

$= 2w + 5\frac{d}{dw}\left(\frac{e^{\frac{1}{4}w}}{w}\right) - 2w^{-3}$ (Power + Constant Multiple Rule)

$= 2w + 5\left(\frac{w\frac{d}{dw}e^{\frac{1}{4}w} - e^{\frac{1}{4}w}\frac{d}{dw}w}{w^2}\right) - \frac{2}{w^3}$ (Quotient Rule)

$= 2w + 5\left(\frac{w\left(\frac{1}{4}e^{\frac{1}{4}w}\right) - e^{\frac{1}{4}w}(1)}{w^2}\right) - \frac{2}{w^3}$ (Exponent + Power Rule)

$= \frac{2w^4 + \frac{5}{4}w^2e^{\frac{1}{4}w} - 5we^{\frac{1}{4}w} - 2}{w^3}$ (Common Denominator + Simplify)