

Quiz 8: Graphing (§4.3)

Directions: You have 30 minutes to complete this quiz. This quiz is closed book and collaborative. The goal of this problem is to produce a graph of the function

$$f(x) = xe^{-x}$$

from scratch.

(1) Find the domain for $f(x)$.

(2) Is $f(x)$ even, odd, or neither? You must justify your answer.

(3) Find $f'(x)$ and $f''(x)$. You are not required to simplify.

(4) Find the critical points. If there are none, then say why.

(5) Find the possible inflection points. If there are none, then say why.

(6) What are the intervals where $f(x)$ is increasing? What are the intervals where $f(x)$ is decreasing?

(7) What are the intervals where $f(x)$ is concave up? What are the intervals where $f(x)$ is concave down?

(8) Find the local extrema and inflection points. You must justify your answers. If there are no extrema or inflection points you should also say why.

- (9) Find the vertical asymptotes, using the limit definition of a vertical asymptote. If there are no vertical asymptotes, then say so.

- (10) Determine the end behavior.

Hint: By l'Hôpital's Rule (§4.7),

$$\lim_{x \rightarrow \infty} x e^{-x} = \lim_{x \rightarrow \infty} \frac{1}{e^x}.$$

For $\lim_{x \rightarrow -\infty} f(x)$ you do not need l'Hôpital's Rule.

(11) Find the y -intercepts and x -intercepts, if there are any.

(12) Using all the information above, draw a well-labeled graph of $f(x)$. Your picture should be consistent with your answers to parts (1)-(11).