Fri 29 May

- MyLabsPlus (MLP) homeworks due Fridays and Sundays (check each assignment for specific dates and times). All assignments for the term are posted now. You can do them as early and as often as you wish, before their deadlines.

- Read the textbook, too! For some of you it's obvious you've been through the slides beforehand – I'm sure it's been helpful to you and reading the text in advance will be, too.

- Lectures: We are running ahead of schedule on the lectures. I will begin next week's material early. This way you'll have more time to prepare for the Exam (which is next Friday).

Consider the cases where f is not defined past a certain point.

Definition

A function f is **continuous from the left** (or **left-continuous**) at a means

$$\lim_{x \to a^-} f(x) = f(a);$$

a function f is **continuous from the right** (or **right-continuous**) at a means

$$\lim_{x \to a^+} f(x) = f(a).$$

Definition

A function f is **continuous on an interval** I means it is continuous at all points of I.

Notation: Intervals are usually written

$$[a, b], \ (a, b], \ [a, b), \ \text{or} \ (a, b).$$

When I contains its endpoints, "continuity on I" means continuous from the right or left at the endpoints.

Example

Let $f(x) = \begin{cases} x^3 + 4x + 1 & \text{if } x \leq 0 \\ 2x^3 & \text{if } x > 0. \end{cases}$

1. Use the continuity checklist to show that f is not continuous at 0.
2. Is f continuous from the left or right at 0?
3. State the interval(s) of continuity.

(assuming m and n are positive integers and $\frac{n}{m}$ is in lowest terms)

- If m is odd, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points at which f is continuous.

- If m is even, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points a at which f is continuous and $f(a) \geq 0$.

Question

Where is $f(x) = \sqrt[4]{4 - x^2}$ continuous?

Trig Functions: The basic trig functions are all continuous at all points IN THEIR DOMAIN. Note there are points of discontinuity where the functions are not defined – for example, $\tan x$ has asymptotes everywhere that $\cos x = 0$.

Exponential Functions: The exponential functions b^x and e^x are continuous on all points of their domains.

Inverse Functions: If a continuous function f has an inverse on an interval I (meaning if $x \in I$ then $f^{-1}(y)$ passes the vertical line test), then its inverse f^{-1} is continuous on the interval J, which is defined as all the numbers $f(x)$, given x is in I.

Theorem (Intermediate Value Theorem)

Suppose f is continuous on the interval $[a, b]$ and L is a number satisfying

$$f(a) < L < f(b) \quad \text{or} \quad f(b) < L < f(a).$$

Then there is at least one number $c \in (a, b)$, i.e., $a < c < b$, satisfying

$$f(c) = L.$$

Example

Let $f(x) = -x^5 - 4x^2 + 2\sqrt{x} + 5$. Use IVT to show that $f(x) = 0$ has a solution in the interval $(0, 3)$.

2.6 Book Problems

9-23 (odds), 29-37 (odds), 45, 49, 51, 53