Math 115 Quiz 4: $\oint$ 2.5-6 and Barehanded Differentiation

Mon 11 October 2010

Name: ____________________________

You have 25 minutes to complete this quiz. Calculators are OK.

1. **Definitions/Concepts.** (1 pt) Let g be the function defined by

$$g(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ \cos x & \text{if } 0 < x < \frac{\pi}{2} \\ 0 & \text{if } x > \frac{\pi}{2} \end{cases}.$$

Which of the following statements are true? Check all that apply.

(a) g is continuous at $x = 0$ *TRUE*

(b) g is continuous at $x = \frac{\pi}{2}$ *FALSE*

(c) g is differentiable at $x = 0$ *TRUE*

(d) g is differentiable at $x = \frac{\pi}{2}$ *FALSE*

2. **Questions/Problems.** A Purple-Headed Uniquely Nocturnal Chartreuse And Luridly Colored wombat is sighted moving across the diag. Its position, measured in feet from the West Engineering arch, is given as a function of time (in minutes past midnight) in the following table.

t	0	5	10	15	20	25	30
position	0	7	15	27	30	31	218

(a) (4 pts) Estimate the wombat's velocity at $t = 0$, $t = 5$, $t = 10$ and $t = 15$.

At $t = 0$ the velocity is 0, since the wombat hasn't moved yet. For $t = 5$ look at the average velocities from 0 to 5 seconds, and from 5 to 10 seconds.

$$\frac{7-0}{5-0} = \frac{7}{5}$$

$$\frac{15-7}{10-5} = \frac{8}{5}$$

The velocity at $t = 5$ should be somewhere in between so taking the mean of the two numbers above gives an estimated velocity of 1.5 ft/sec. Similarly, at $t = 10$ the velocity is about 2 ft/sec, and at $t = 15$ is about 1.5 ft/sec.

(b) (2 pts) Estimate the wombat's acceleration at $t = 5$ and $t = 10$.

Use the values from part (a), which gives the following table:

t	0	5	10	15
estimated velocity	0	1.5	2	1.5

So the average acceleration at $t = 5$ can be found by looking at the change in average velocity between 0 and 5 seconds, and between 5 and 10 seconds:

$$\frac{1.5-0}{5-0} = \frac{3}{10}$$

$$\frac{2-1.5}{10-5} = \frac{1}{10}$$

so taking the mean, the average acceleration at $t = 5$ seconds is 0.2 ft/sec per sec. Similarly, at $t = 10$ the average acceleration is 0.1 ft/sec per sec.

(c) (1 pt) What do you think happened between $t = 25$ and $t = 30$?
During this time the position jumps dramatically. This means the PHUNCALC wombat's velocity increased dramatically, possibly as a result of being chased by another rodent.

3. **Computations/Algebra.** (2 pts) Use the limit definition of the derivative to compute the following. You *must* show all steps.

$$\frac{d}{dx}\left(\frac{x^2+3}{x^9}\right)$$

The algebra was wayyy too nasty to actually do this by hand. The following answer is what you should include at the very least:

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2+3}{(x+h)^9} - \frac{x^2+3}{x^9}}{h}$$