

Quiz 3: Trajectories and Arc Length (§11.7-11.8)

Directions: You have 30 minutes to complete this quiz. You may collaborate.

1. (**2 pts**) A cycloid is the path traced by a point on a rolling circle (think of a light on the rim of a moving bicycle wheel). The cycloid generated by a circle of radius a is given by the parametric equation

$$x = a(t - \sin t) \quad y = a(1 - \cos t).$$

- (a) The parameter range $0 \leq t \leq 2\pi$ produces one arch of the cycloid. Compute its length. **Hint:** You might need the half-angle formula

$$\sin^2 \theta = \frac{1}{2} (1 - \cos^2 \theta).$$

- (b) Draw a well-labelled graph of the arch of the cycloid.

2. A golf ball has an initial position

$$\vec{r}(0) = \langle x_0, y_0 \rangle = \langle 0, 0 \rangle = 0\hat{i} + 0\hat{j} \text{ ft}$$

when it is hit at an angle of 30° from the ground and with an initial speed of 150 ft/s. For the following, neglect air resistance and assume gravity is a constant $g = 32 \text{ ft/s}^2$. **You must include units in your answers to receive credit.**

(a) **(1 pt)** The golf ball's acceleration vector is: $\vec{a}(t) =$

(b) **(1 pt)** Its initial speed is: $|\vec{v}(0)| =$

(c) **(1 pt)** Its initial velocity is: $\vec{v}(0) =$

(d) **(1 pt)** The golf ball's velocity vector is $\vec{v}(t) =$

(e) **(1 pt)** The golf ball's position vector is $\vec{r}(t) =$

(f) **(1 pt)** Determine the golf ball's time of flight.

(g) **(1 pt)** How far does the golf ball travel?

(h) **(1 pt)** What is the maximum height of the golf ball?