MATH 235 (Calculus I)
due Tues 19 Sep 2017

Take-Home Quiz 2: Exponentials, Inverses, and Limits (§1.4-1.5, 2.2-2.3)

Directions: This quiz is due on September 19, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

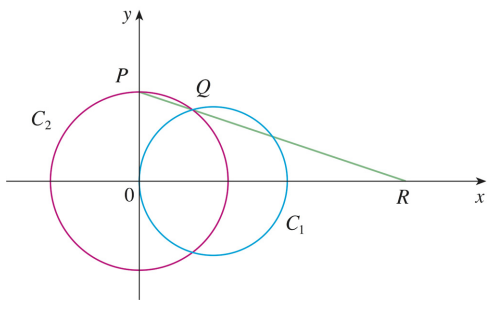
---

1. **§1.4 #32** An isotope of sodium, ^{24}Na, has a half-life of 15 hours. A sample of this isotope has mass 2g.
   (a) How much of the sample remains after 60 hours?
   (b) Find an exponential function modelling the decay of ^{24}Na.
   (c) Estimate the amount (round to three decimal places) of the 2g sample remaining after 4 days.
   (d) Use a graph (e.g., `desmos.com/calculator`) to estimate the number of days (round to one decimal place) it takes for the 2g mass to be reduced to 0.01g.

2. Simplify each expression and explain why (e.g., with a triangle, the unit circle, trig identities).
   (a) **§1.5 #64** $\arctan(-1)$
   (b) **§1.5 #66** $\arccos(\sqrt{3}/2)$
   (c) **§1.5 #70** $\cos(2\arcsin(5/13))$
   (d) **§1.5 #72** $\sin(2\arccos x)$

3. In the theory of relativity, the mass of a particle with speed v is
$$m = f(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
   where m_0 is the rest mass of the particle and c is the speed of light in a vacuum.
   (a) **§1.5 #20** Find the inverse function of f and explain its meaning.
   (b) **§2.2 #54** What happens as $v \to c^-$?
   (c) **§2.3 #56** The Lorentz contraction formula
$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$
   expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest. Find $\lim_{v\to c^-} L$ and interpret the result. Why is a left-hand limit necessary?

4. **§2.3 #66** The figure shows a fixed circle C_1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x-axis.



(a) What is the x-coordinate of R, in terms of r? *(Hint: Find the equation of the line determined by the points P and Q.)*

(b) What happens to R as C_2 shrinks, that is, as $r \to 0^+$? *(Hint: Use your answer to part (a). R does not go to infinity.)*

5. **§2.3 #60** If $\lim_{x\to 0} \dfrac{f(x)}{x^2} = 5$, find the following limits. *(Hint: Rewrite $f(x) = \frac{f(x)}{x^2} \cdot x^2$ and $\frac{f(x)}{x} = \frac{f(x)}{x^2} \cdot x$.)*

   (a) $\lim_{x\to 0} f(x)$

   (b) $\lim_{x\to 0} \dfrac{f(x)}{x}$

6. **§2.3 #52** Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

   (a) Evaluate each of the following, if it exists.

      i. $\lim_{x\to 1^-} g(x)$
      ii. $\lim_{x\to 1} g(x)$
      iii. $g(1)$
      iv. $\lim_{x\to 2^-} g(x)$
      v. $\lim_{x\to 2^+} g(x)$
      vi. $\lim_{x\to 2} g(x)$

   (b) Sketch the graph of g.