

Math 2603 Exam 3
Mon 24 Nov 2014

Name: _____

Discrete Math
Exam 3 (Ch. 6-8 as we have covered)

Please provide the following data:

Drill Time: _____

Student ID: _____

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard, two-sided, is allowed. No graphing calculators. No programmable calculators. No phones, iDevices, computers, etc. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: _____

Good luck!

1. (a) How many strings can be formed using all of the letters

A S S I S T A N T S H I P

- (b) How many such strings have all the S's consecutive?

- (c) How many such strings from (a) have no consecutive S's?

2. (a) The Binomial Theorem (BT) states $(x + y)^n =$

(b) Using the BT, write down the coefficient for x^2yz in the expansion of $(2x + y - z)^4$.

(c) Prove that

$$\left(\frac{m}{m+n}\right)^m \left(\frac{n}{m+n}\right)^n \cdot \binom{m+n}{m} < 1$$

for all $m, n \in \mathbb{Z}_{>0}$. *Hint:* Consider the term for $k = m$ in the BT expansion of $(x + y)^{m+n}$ for appropriate x and y .

3. Professor Euclid is paid every other Friday. Using the Pigeonhole Principle, prove that in one year's time, there will be some month where she got paid three times. To get credit, you MUST state which version of the Principle Pigeonhole you used and how you applied it in solving the problem.

4. How many positive integer solutions are there of

$$x_1 + x_2 + x_3 = 20?$$

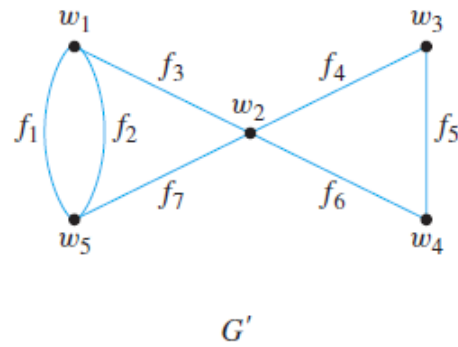
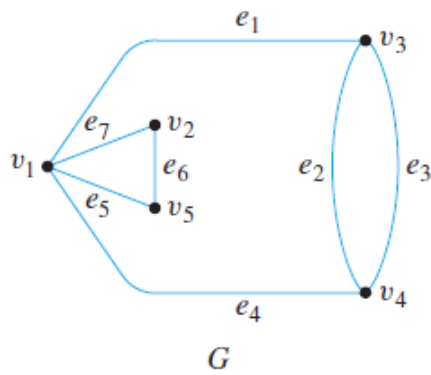
5. (a) Draw the complete bipartite graph $K_{2,4}$, with vertices and edges legibly labeled.

(b) Write the adjacency matrix for $K_{2,4}$.

(c) Write the incidence matrix for $K_{2,4}$.

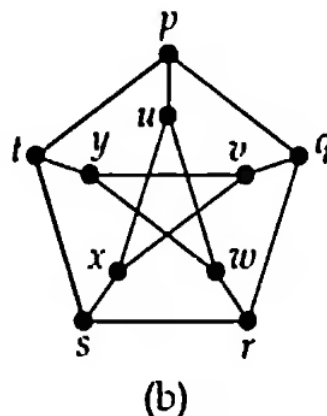
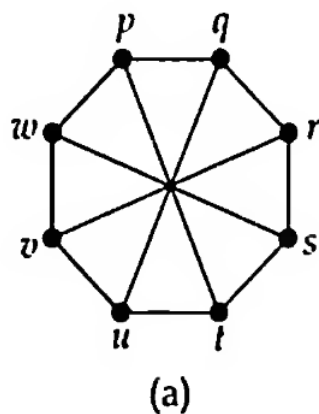
(d) Does $K_{2,4}$ have an Euler cycle? If yes, then list the ordering of edges that give one. If no, then prove why not.

6. Are the following¹ graphs G , G' isomorphic? If so then, exhibit an isomorphism. If not, then state an invariant not shared by the two graphs. If the invariant you cite was not mentioned in class then you must prove it is actually an invariant.



¹Image Credit: Epp, Susanna. *Discrete Mathematics with Applications*. Cengage Learning, 2010. p. 677.

7. Choose one of the following² graphs, (a) or (b), to consider. NOTE: All vertices are labeled.



Is the graph you chose planar? Prove your answer: If it is planar then redraw it without any edges overlapping; if it is not planar then exhibit, by series reduction if necessary, a subgraph homeomorphic to $K_{3,3}$ or K_5 .

²Image Credit: Aldous, Joan M. and Wilson, Robin J. *Graphs and Applications: An Introductory Approach*. Springer-Verlag London, 2000. p. 262.

8. The Fibonacci sequence is defined by the recurrence relation

$$f_n = f_{n-1} + f_{n-2},$$

for $n \geq 3$.

(a) How many initial conditions should there be and what are they?

(b) **EXTRA CREDIT** Solve the relation to get an explicit formula for f_n .

9. **EXTRA CREDIT** Find a formula for the probability that out of n millennials, at least two will have the same birthday, where “same birthday” means same month, date, AND year. Millennials are those born in the years 1980-1995. Assume no leap years in that time period.

10. **cHaLIEnGe PrObLeM** Find such a formula, only this time, taking leap years into account.