Math 2554 Quiz 8: § 4.2-4.4 **Name:** KEY
Tues 4 Nov 2014

You have 25 minutes to complete this quiz. Eyes on your own paper and good luck!

1. Let $f(x) = 2 + \cos 2x$, for $-\pi \le x \le \pi$.

   (a) Determine the intervals on which f is concave up or concave down.

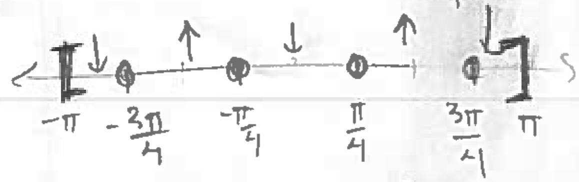
$f'(x) = -2\sin(2x)$

$f''(x) = -4\cos(2x) = 0$

$\cos 2x = 0.$

Cosine is zero at odd multiples of $\frac{\pi}{2}$ so we must have $2x = \frac{\pi}{2} n$ where n is odd.

possible infl pts: $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$



   (b) Identify any inflection points.

f changes concavity at:

$\boxed{x = -\frac{3\pi}{4} \Rightarrow y = 2 + \cos\left(2\left(\frac{-3\pi}{4}\right)\right) = 2}$

$\boxed{x = -\frac{\pi}{4} \Rightarrow y = 2 + \cos\left(2\left(\frac{\pi}{4}\right)\right) = 2}$

continue

Strategic pts to check:

$f''(-\pi) = -4\cos(2(-\pi)) = -4 < 0$

$f''(-\frac{\pi}{2}) = -4\cos\left(2\left(\frac{\pi}{2}\right)\right) = 4 > 0$

$f''(0) = -4\cos(2(0)) = -4 < 0$

$f''\left(\frac{\pi}{2}\right) = -4\cos\left(2\left(\frac{\pi}{2}\right)\right) = 4 > 0$

$f''(\pi) = -4\cos(2\pi) = -4 < 0$

OMIT ~~2. Use the 2nd Derivative Test to identify all local extrema on the function $g(x) = 2x^4 + 9x^3 + 12x^2 - x - 2$.~~

$\boxed{x = \frac{\pi}{4} \Rightarrow y = 2 + \cos\left(2\left(\frac{\pi}{4}\right)\right) = 2}$

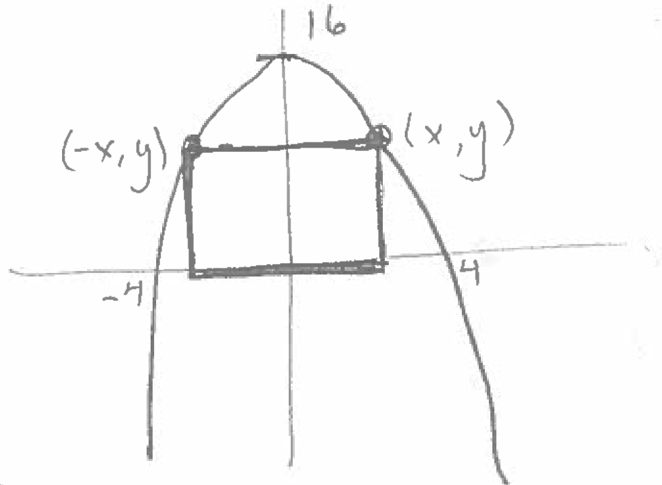
$\boxed{x = \frac{3\pi}{4} \Rightarrow y = 2 + \cos\left(2\left(\frac{3\pi}{4}\right)\right) = 2}$

f is concave up on $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$.

f is concave down on $\left[-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right]$.

MORE ON THE NEXT PAGE →

3. A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 16 - x^2$. What are the dimensions of the rectangle with the maximum area? What is the maximum area?



$y = 16 - x^2$

$A = \text{Area} = 2xy = 2x(16 - x^2)$ From the picture,

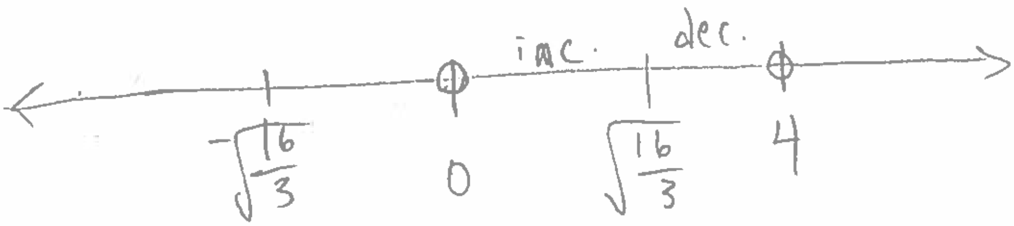
$= 32x - 2x^3$ $\quad 0 < x < 4$

$$\frac{dA}{dx} = 32 - 6x^2 = 0$$

$$32 = 6x^2$$

$$16 = 3x^2$$

$$x = \pm\sqrt{\frac{16}{3}} \approx \pm 2.309$$



1st deriv/strategic pts:

$A'(1) = 32 - 6(1^2) = 26 > 0$

$A'(3) = 32 - \underbrace{6(3^2)}_{54} < 0$

From the 1st deriv test, the maximum area occurs $\longrightarrow$

when $x = \sqrt{\frac{16}{3}}$, so $y = 16 - x^2 = 16 - \frac{16}{3}$

$$= \frac{32}{3}$$

The dimensions of the rectangle are $\boxed{2 \cdot \sqrt{\frac{16}{3}} \times \frac{32}{3}}$

or $\boxed{\approx 4.619 \times 10.667}$