MATH 2574 (Calculus III)
Fall 2015

Name: _____

Thurs 8 Oct 2015

Quiz 6: Directional Derivatives ($\oint$12.4-12.6)

Directions: You have 30 minutes to complete this quiz. You may collaborate.

1. The level curves of the surface $z = x^2 + y^2$ are circles in the xy-plane centered at the origin.

   (a) **(1 pt)** In the xy-plane, draw and label the three level curves corresponding to $z_0 = 1, \sqrt{2},$ and 4.

   (b) **(2 pts)** On the same picture, draw the gradient vector at the point $(1, 1)$.

   (c) **(2 pts)** Write down the gradient vector: $\langle \dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y} \rangle =$

2. **(2 pts)** Compute the directional derivative of $h(x, y)$, in the direction of the vector $\mathbf{v}$, at the point $(\ln 2, \ln 3)$, where

$$\mathbf{v} = \langle 1, 1 \rangle \quad \text{and} \quad h(x, y) = e^{-x-y}.$$

   Hint: Make sure your answer has the correct magnitude!

3. Suppose our Sun is centered at the origin in $\mathbb{R}^3$ and some other star is a distance r away, at the coordinates (x, y, z). The **gravitational potential** between the two stars (or any two objects) is the function

$$V(r) = \frac{-GMm}{r},$$

where G is the gravitational constant, and in this case m denotes the mass of the Sun and M denotes the mass of the other star.

(a) **(1 pt)** Write down V as a function of $x, y,$ and z.

(b) **(2 pts)** The gravitational force between the two stars is the vector-valued function

$$\mathbf{F} = -\nabla V(x, y, z).$$

Write down the magnitude $|\mathbf{F}|$ as a function of r.