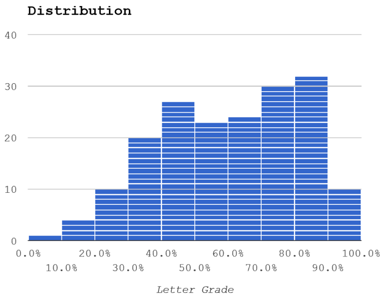
Fri 18 Mar

- Midterm: Take your raw score out of 140, instead of 150, for the curve.



- Office hours 1030-1230 today.

§4.2 What Derivatives Tell Us

Definition

Suppose a function f is defined on an interval I.

- We say that f is **increasing** on I if $f(x_2) > f(x_1)$ whenever x_1 and x_2 are in I and $x_2 > x_1$.
- We say that f is **decreasing** on I if $f(x_2) < f(x_1)$ whenever x_1 and x_2 are in I and $x_2 > x_1$.

How is it related to the derivative?

Suppose f is continuous on an interval I and differentiable at every interior point of I.

- If $f'(x) > 0$ for all interior points of I, then f is increasing on I.
- If $f'(x) < 0$ for all interior points of I, then f is decreasing on I.

Example

Sketch a function that is continuous on $(-\infty, \infty)$ that has the following properties:

- $f'(-1)$ is undefined;
- $f'(x) > 0$ on $(-\infty, -1)$;
- $f'(x) < 0$ on $(-1, 4)$;
- $f'(x) > 0$ on $(4, \infty)$.

Example

Find the intervals on which

$$f(x) = 3x^3 - 4x + 12$$

is increasing and decreasing. If you graph f and f' on the same axes, what do you notice?

First Derivative Test

Suppose that f is continuous on an interval that contains a critical point c and assume f is differentiable on an interval containing c, except perhaps at c itself.

- If f' changes sign from positive to negative as x increases through c, then f has a **local maximum** at c.
- If f' changes sign from negative to positive as x increases through c, then f has a **local minimum** at c.
- If f' does not change sign at c (from positive to negative or vice versa), then f has **no** local extreme value at c.

NOTE: The First Derivative Test does NOT test for increasing/decreasing, only local max/min. Use it on critical points.

Exercise

If $f(x) = 2x^3 + 3x^2 - 12x + 1$, identify the critical points on the interval $[-3, 4]$, and use the First Derivative Test to locate the local maximum and minimum values. What are the absolute max and min?

The Extreme Value Theorem (cf., Section 4.1) stated that we were guaranteed extreme values only on closed intervals.

However: Suppose f is continuous on an interval I that contains only one local extremum at $(x =)c$.

- If it is a local minimum, then $f(c)$ is the absolute minimum of f on I.
- If it is a local maximum, then $f(c)$ is the absolute maximum of f on I.

Just as the first derivative f' told us whether the function f was increasing or decreasing, the second derivative f'' also tells us whether f' is increasing or decreasing.

Definition

Let f be differentiable on an open interval I.

- If f' is increasing on I, then f is **concave up** on I.
- If f' is decreasing on I, then f is **concave down** on I.

Definition

If f is continuous at c and f changes concavity at c (from up to down, or vice versa), then f has an **inflection point** at c.

Test for Concavity

Suppose that f'' exists on an interval I.

- If $f'' > 0$ on I, then f is concave up on I.
- If $f'' < 0$ on I, then f is concave down on I.
- If c is a point of I at which $f''(c) = 0$ and f'' changes signs at c, then f has an inflection point at c.

Example

What would a function with the following properties look like?

1. $f' > 0$ and $f'' > 0$
2. $f' > 0$ and $f'' < 0$
3. $f' < 0$ and $f'' > 0$
4. $f' < 0$ and $f'' < 0$

Suppose that f'' is continuous on an open interval containing c with $f'(c) = 0$.

- If $f''(c) > 0$, then f has a local minimum at c.
- If $f''(c) < 0$, then f has a local maximum at c.
- If $f''(c) = 0$, then the test is inconclusive.

Exercise

Given $f(x) = 2x^3 - 6x^2 - 18x$

(a) Determine the intervals on which it is concave up or concave down, and identify any inflection points.

(b) Locate the critical points, and use the 2nd Derivative Test to determine whether they correspond to local minima or maxima, or whether the test is inconclusive.

4.2 Book Problems

11–47 (odds), 53–81 (odds)