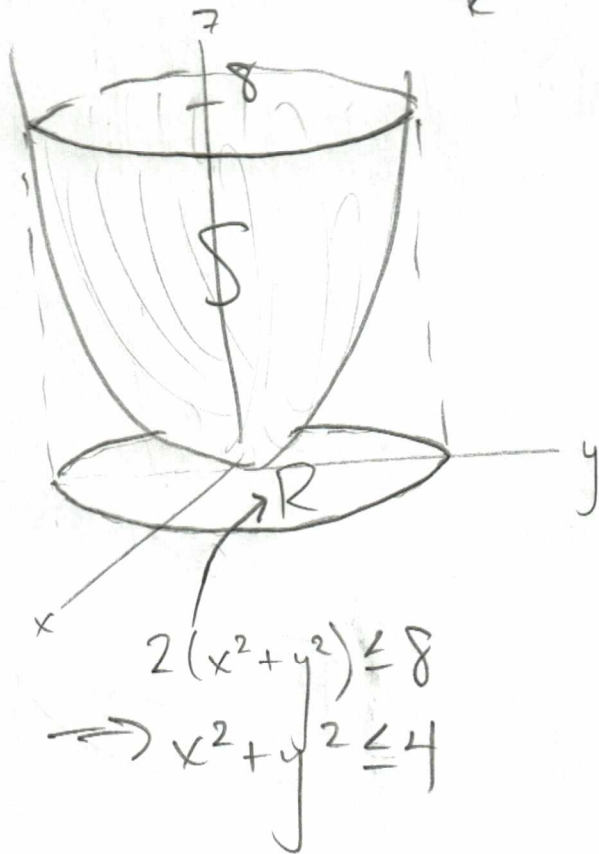


§14.6 #32 | $z = 2(x^2 + y^2)$; $0 \leq z \leq 8$

$$\text{area} = \iint_S dS = \iint_R |\vec{r}_x \times \vec{r}_y| dx dy$$



$$\vec{r}(x, y) = \langle x, y, 2(x^2 + y^2) \rangle$$

$$\vec{r}_x = \langle 1, 0, 4x \rangle$$

$$\vec{r}_y = \langle 0, 1, 4y \rangle$$

$$\Rightarrow \vec{r}_x \times \vec{r}_y = \langle -4x, -4y, 1 \rangle$$

$$|\vec{r}_x \times \vec{r}_y| = \sqrt{(-4x)^2 + (-4y)^2 + 1^2} = \sqrt{16x^2 + 16y^2 + 1}$$

$$\iint_R \sqrt{16x^2 + 16y^2 + 1} dx dy \leftarrow \text{use polar coords}$$

$$= \int_0^{2\pi} \int_0^2 r \sqrt{16r^2 + 1} dr d\theta$$

Put $u = 16r^2 + 1$

$$du = 32r dr$$

$$\Rightarrow \frac{1}{32} du = r dr$$



$$u(0) = 16(0)^2 + 1 = 1$$

$$u(2) = 16(2)^2 + 1 = 65$$

$$= \frac{1}{32} \int_0^{2\pi} \int_1^{65} u^{1/2} du d\theta$$

$$= \frac{1}{32} \left(\frac{2}{3} \right) \int_0^{2\pi} (65^{3/2} - 1^{3/2}) d\theta$$

$$= \frac{1}{48} (65\sqrt{65} - 1) (2\pi) = \frac{(65\sqrt{65} - 1)\pi}{24}$$