Math 115 Quiz 7: $\oint$ 3.9, 4.1, 4.2 Using First and Second Derivatives **Mon 8 November 2010**
Name: ____________________________

You have 20 minutes to complete this quiz. Make your variables clear and consistent (so if you want to say, for example, $\frac{dy}{dx}$, you should also mention $y = f(x)$, or "y is a function of x"). Calculators are OK.

1. **Definitions/Concepts.**

   (a) (2 pts) Complete this statement: Suppose f is differentiable at a. Then, for values of x near a, the tangent line approximation to $f(x)$ is

   $$f(x) \approx f(a) + f'(a)(x - a).$$

   (b) (1 pt) TRUE or **FALSE**: If the derivative of f is zero at the point $x = a$, then a is either a local maximum or a local minimum.

   For example, if $f(x) = x^3$, then $f'(0) = 0$. However, $x = 0$ is not a local maximum or a local minimum.

2. **Questions/Problems.** (3 pts) For which powers p is $y = x^p$ concave up on the region $x \in (0, \infty)$? Explain.

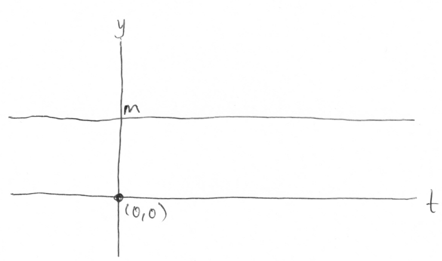
   First, look at the second derivative:

   $$\begin{aligned} y &= x^p \\ y' &= px^{p-1} \\ y'' &= (p-1)px^{p-2}. \end{aligned}$$

   Concave up means $y'' > 0$. When $x \in (0, \infty)$, x^{p-2} can never be negative. So it is necessary to only look at when $(p-1)p > 0$. When p and $p - 1$ are both positive $p - 1$ is smaller than p, so really we only need $p - 1 > 0$. This happens when $p > 1$. When p and $p - 1$ are both negative it is enough that $p < 0$. Hence, for all $p \in (-\infty, 0) \cup (1, \infty)$, $y = x^p$ is concave up on the domain $x \in (0, \infty)$.

   (3 pts) Are exponential functions of the form $y = mc^t$ always increasing if $m > 0$? If yes, say why. If no, give a concrete counterexample (equation and sketch of graph).

   No. For example, if $c = 1$ then $y = m \cdot 1^t = m$ has the following constant graph:



3. **Computations/Algebra.** (1 pt) Find all critical points of the following function. Use the second derivative to tell if each critical point is a local maximum, local minimum, or cannot be determined.

$$f(x) = (x^3 - 8)^4$$

Critical points are where $f' = 0$.

$$\begin{aligned} f'(x) = 0 &= 4(x^3 - 8)^3 \cdot (3x^2) \\ &= 12x^2(x^3 - 8)^3 \end{aligned}$$

So either $x = 0$ or $x^3 - 8 = 0$. Therefore the critical points are $x = 0, 2$. The second derivative is

$$\begin{aligned} f''(x) &= 24x(x^3 - 8)^3 + 12x^2 \cdot 3(x^3 - 8)^2 \cdot (3x^2) \\ &= 24x(x^3 - 8)^3 + 108x^3(x^3 - 8)^2. \end{aligned}$$

Substituting in the critical points,

$$\begin{aligned} f''(0) &= 24(0)((0)^3 - 8)^3 + 108(0)^3((0)^3 - 8)^2 \\ &= 0 \end{aligned}$$

means it cannot be determined if $x = 0$ is an extremum. Similarly,

$$\begin{aligned} f''(2) &= 24(2)((2)^3 - 8)^3 + 108(2)^3((2)^3 - 8)^2 \\ &= 0 \end{aligned}$$

so it cannot be determined if $x = 2$ is an extremum.