Quiz 1: The Idea of Limits

SOLUTIONS
Tues 26 Jan 2016

1. $f(x) = x - x^3 = x(1-x^2) = x(1-x)(1+x)$

(a) i. $\dfrac{f(1.5)-f(1)}{1.5-1} = \dfrac{1.5-(1.5)^3-(1-1^3)}{0.5}$

$= -3.75$

ii. $\dfrac{f(1.05)-f(1)}{1.05-1} = \dfrac{f(1.05)}{0.05} = \dfrac{(1.05)(1-1.05)(1+1.05)}{0.05}$ (-0.05, 2.05)

$= -(1.05)(2.05)$

$= -2.1525$

iii. $\dfrac{f(1.005)-f(1)}{1.005-1} = \dfrac{(1.005)(-0.005)(2.005)}{0.005} = -2.015025$

iv. $\dfrac{f(h)-f(1)}{h-1} = \dfrac{f(h)}{h-1} = \dfrac{h(1-h)(1+h)}{h-1} = -h(1+h)$ $(h \neq 1)$

(b) The slope of the tangent line is approximately -2.

(c) $\lim\limits_{x \to 1^+} \dfrac{f(x)-f(1)}{x-1} \approx -2$

2. $w(z) = z^3 - z^2 = z^2(z-1)$

(a)

z	$w(z)$
0.9	$(0.9)^2(0.9-1) = -0.081$ (-0.1)
0.99	$(0.99)^2(-0.01) = -0.00981$
0.999	$(0.999)^2(-0.001) = -0.00098001$
1.001	$(1.001)^2(0.001) = 0.001002001$
1.01	$(1.01)^2(0.01) = 0.010201$
1.1	$(1.1)^2(0.1) = 0.121$

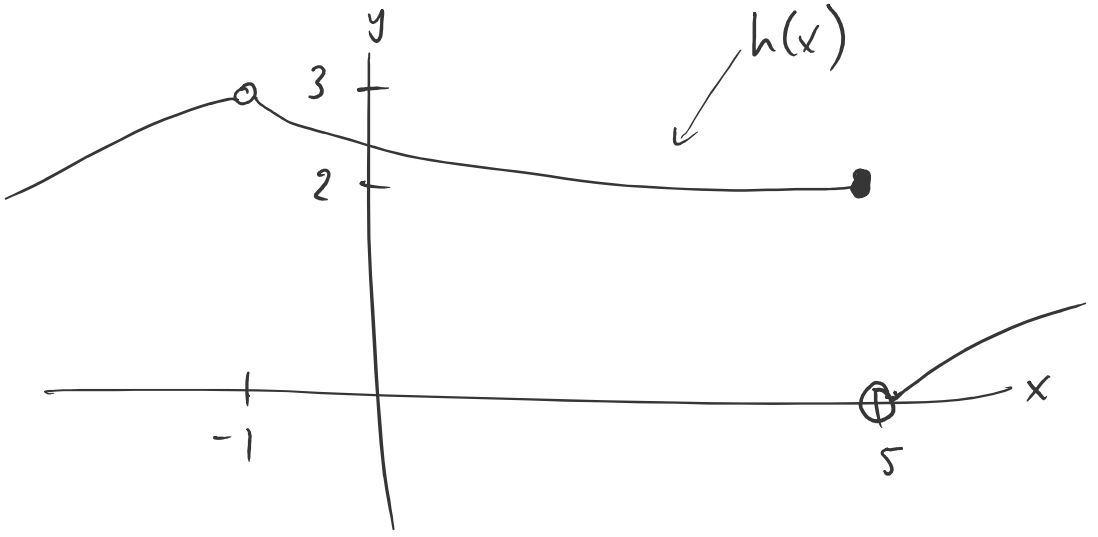
(b) $w(1) \approx 0$ (Check: $1^2(1-1) = 0$ ✓)

(c) $\lim\limits_{z \to 1} w(z) \approx 0$

3. $\lim_{x \to -1} h(x) = 3$ $\quad h(5) = 2$

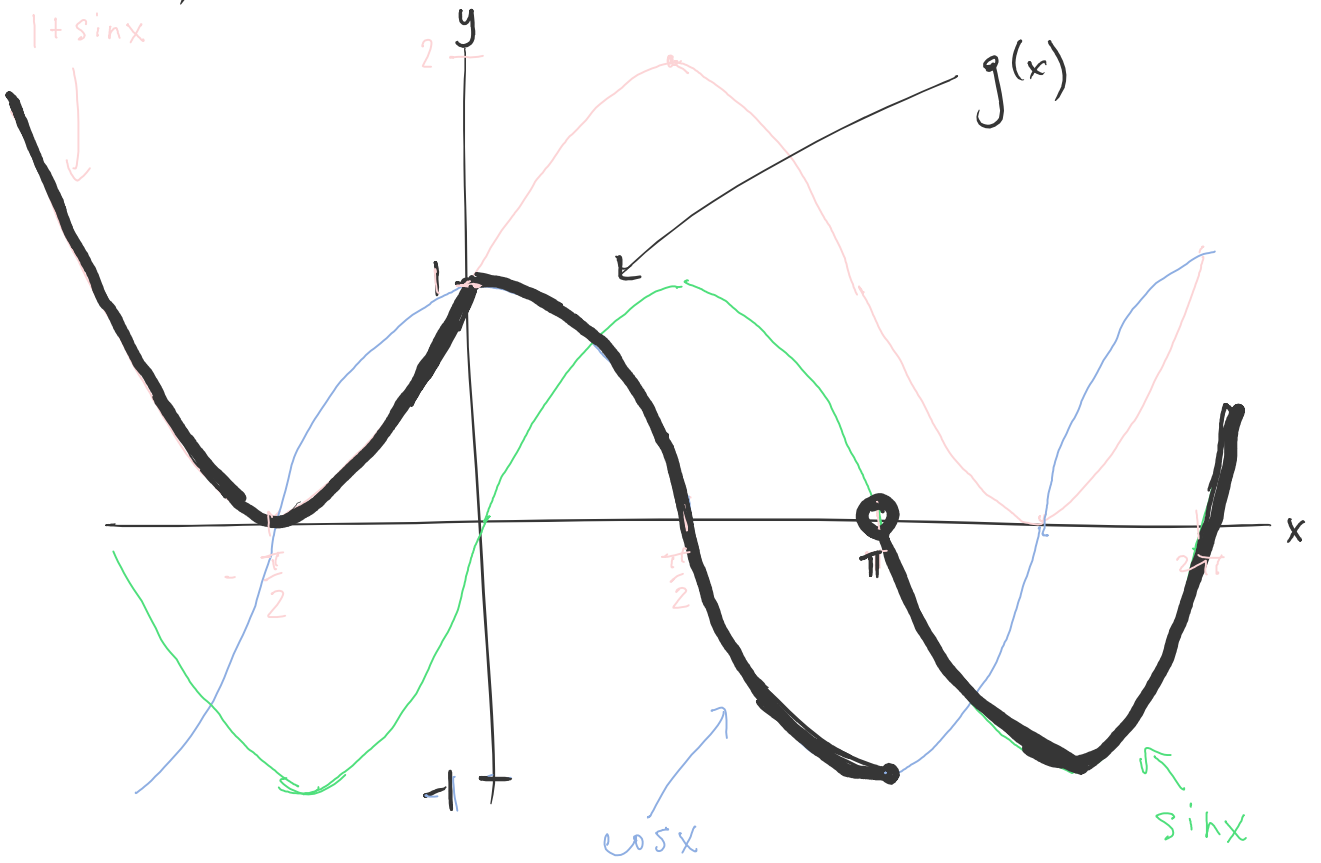
$\lim_{x \to 5^+} h(x) = 0$

$h(-1)$ undef.



4. $g(x) = \begin{cases} 1 + \sin x & x < 0 \\ \cos x & 0 \le x \le \pi \\ \sin x & x > \pi \end{cases}$

(a)



(b) $a = \pi$

(c) $\lim_{x \to a^+} g(x) = -1$

$\lim_{x \to a^-} g(x) = 0$