MATH 235 (Calculus I)
Fall 2017

Name: _____

Mon 11 Dec 2017

Final Exam

Exam Instructions: You have 120 minutes to complete this exam. **Justification** is required for all problems. **Notation** matters! You will also be penalized for **missing/incorrect units** and **rounding errors**.

No electronic devices (phones, iDevices, computers, etc) except for a basic scientific calculator. On story problems, round to one decimal place. If you do not have a calculator, then write down the exact formula you would plug in to a calculator to get the answer.

If you finish early then you may leave, UNLESS there are less than 10 minutes left. To prevent disruption, if you finish with less than 10 minutes remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of James Madison University.

Signature: **(1 pt)** _____

Good luck!

Question	Points	Score
1	20	
2	4	
3	8	
4	2	
5	3	
6	12	
7	3	
8	3	
9	9	
10	4	
11	2	
12	4	
13	4	
14	5	
15	3	
16	3	
17	2	
18	4	
19	2	
20	2	
Total:	99	

1. Determine whether each statement is TRUE or FALSE. You will not receive full credit without justification of your answer.

   (a) **(2 pts)** If $f(s) = f(t)$, then $s = t$.

   (b) **(2 pts)** If $x > 0$, then $(\ln x)^6 = 6 \ln x$.

   (c) **(2 pts)** If x is any real number, then $\sqrt{x^2} = x$.

   (d) **(2 pts)** If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$, then $\lim_{x \to 0} (f(x) - g(x)) = 0$.

   (e) **(2 pts)** If f is continuous on $[-1, 1]$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number c such that $|c| < 1$ and $f(c) = \pi$.

   (f) **(2 pts)** If f and g are differentiable, then $\dfrac{d}{dx}(f(x)g(x)) = f'(x)g'(x)$.

   (g) **(2 pts)** If f is differentiable, then $\dfrac{d}{dx}\sqrt{f(x)} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

   (h) **(2 pts)** $\dfrac{d}{dx}(\ln 10) = \dfrac{1}{10}$

   (i) **(2 pts)** If f' is continuous on $[1, 3]$, then $\displaystyle\int_1^3 f'(v)\, dv = f(3) - f(1)$.

   (j) **(2 pts)** If $\displaystyle\int_0^1 f(x)\, dx = 0$, then $f(x) = 0$ for all $0 \leq x \leq 1$.

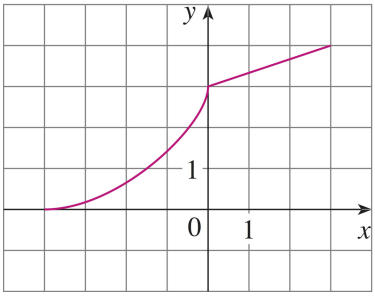
2. Consider the function $g(x) = \sqrt{16 - x^4}$.

   (a) **(2 pts)** Find the domain of g.

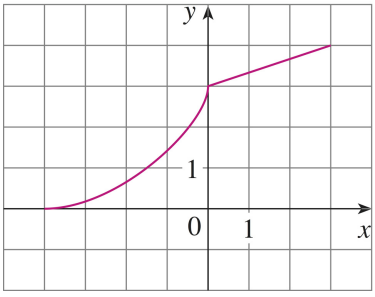
   (b) **(2 pts)** Find the range of g.

3. In each part of this question, the graph of f is given. Sketch the given transformation on the same axes.

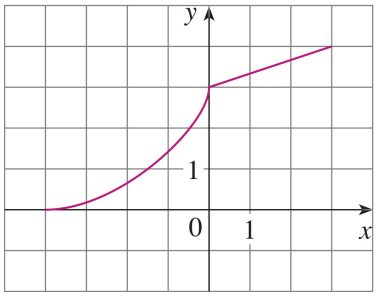
   (a) **(2 pts)** $y = 2 - f(x)$



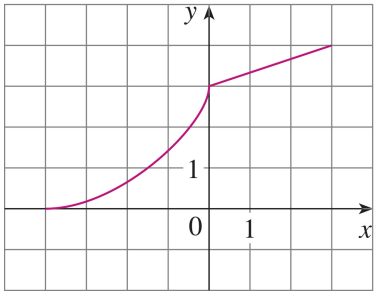
   (b) **(2 pts)** $y = f^{-1}(x)$



   (c) **(2 pts)** $y = \frac{1}{2}f(x) - 1$



   (d) **(2 pts)** $y = f^{-1}(x + 3)$



4. **(2 pts)** A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12,000 to produce 1500 toaster ovens a week. Express the cost as a function of the number of toaster ovens produced per week, assuming that it is linear.

5. **(3 pts)** Given $f(x) = \dfrac{x+1}{2x+1}$, find the inverse function $f^{-1}(x)$.

6. Evaluate each of the limits. You must your work and/or justify your answers.

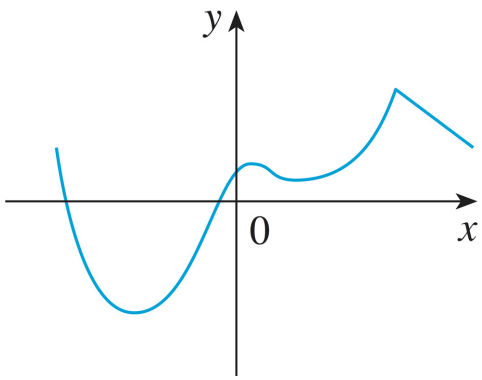
   (a) **(3 pts)** $\lim\limits_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3}$

   (b) **(3 pts)** $\lim\limits_{v\to 4^+} \dfrac{4-v}{|4-v|}$

   (c) **(3 pts)** $\lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2-9}}{2x-6}$

   (d) **(3 pts)** $\lim\limits_{x\to \infty} e^{x-x^2}$

7. **(3 pts)** The graph of f is shown. Sketch the graph of f' on the same axes.



8. **(3 pts)** Use implicit differentiation of the equation $xe^y = y \sin x$ to find $\dfrac{dy}{dx}$.

9. Find y'. Do not simplify.

   (a) **(2 pts)** $y = e^{x \sec x}$

   (b) **(2 pts)** $y = 10^{\tan \pi\theta}$

   (c) **(2 pts)** $y = \arctan(\arcsin x)$

   (d) **(3 pts)** $y = \sin^2\left(\cos\sqrt{\pi x}\right)$

10. Find f' in terms of g'.

    (a) **(1 pt)** $f(x) = g(x^2)$

    (b) **(1 pt)** $f(x) = g(g(x))$

    (c) **(1 pt)** $f(x) = e^{g(x)}$

    (d) **(1 pt)** $f(x) = g(\ln x)$

11. **(2 pts)** Find h', given that $h(x) = \sqrt{\dfrac{f(x)}{g(x)}}$.

12. *In this problem, round to one decimal place. If you do not have a calculator, write the exact number that you would plug in to a calculator to get the answer.* Cobalt-60 has a half-life of 5.24 years.

    (a) **(2 pts)** Find the mass that remains from a 100-mg sample after 20 years.

    (b) **(2 pts)** How long would it take for the 100-mg sample to decay to 1 mg?

13. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). The contents of the cup also form a cone shape with radius r and height h. The volume of the contents of the cup is given by the formula $V = \dfrac{1}{3}\pi r^2 h$.

    (a) **(2 pts)** Use similar triangles to find an equation relating r and h. You may find it helpful to draw a diagram of the cup with some water in it.

(b) **(2 pts)** If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

14. Let $y = ax^3 + bx^2$.

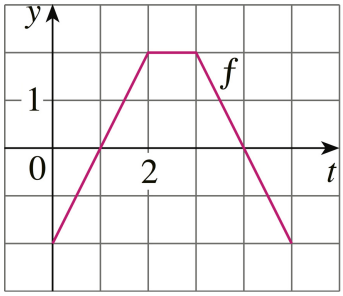
(a) **(3 pts)** For what values of the constants a and b is $(1, 3)$ a possible point of inflection of the curve $y = ax^3 + bx^2$? *Hint: To solve this problem you need two equations to find the two unknowns.*

(b) **(2 pts)** Is $(1, 3)$ an actual inflection point of the curve? You *must* justify your answer.

15. **(3 pts)** Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible. *You must provide justification for why your critical point is indeed a max.*

16. **(3 pts)** A particle is moving with acceleration function $a(t) = \sin t + 3\cos t$. Its initial velocity is $v(0) = 2$ and its initial position is $s(0) = 0$. Find its position function $s(t)$.

17. **(2 pts)** The graph of f consists of the three line segments shown. If $g(x) = \int_0^x f(t)\,dt$, find $g(4)$ and $g'(4)$.



18. Evaluate each of the integrals.

(a) **(2 pts)** $\displaystyle\int_1^{10} \frac{x}{x^2-4}\,dx$

(b) **(2 pts)** $\displaystyle\int \sin x \cos(\cos x)\,dx$

19. **(2 pts)** Find the derivative of the function

$$g(x) = \int_1^{\sin x} \frac{1-t^2}{1+t^4}\, dt.$$

20. **(2 pts)** Suppose h is a function such that $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, $h''(2) = 13$, and h'' is continuous everywhere. Evaluate $\displaystyle\int_1^2 h''(u)\, du$.