

## Take-Home Quiz 3: Formal limits and continuity (§2.4-2.6)

**Directions:** This quiz is due on September 25, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. **§2.4 #12** A crystal growth furnace is used in research to determine how best to manufacture crystals used in electronic components for the space shuttle. For proper growth of the crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.155w + 20$$

where  $T$  is the temperature in degrees Celsius and  $w$  is the power input in watts.

- How much power is needed to maintain the temperature at  $200^\circ\text{C}$ ?
  - If the temperature is allowed to vary from  $200^\circ\text{C}$  by up to  $\pm 1^\circ\text{C}$ , what range of wattage is allowed for the input power?
  - In terms of the  $\epsilon, \delta$  definition of  $\lim_{x \rightarrow a} f(x) = L$ , what is  $x$ ? What is  $f(x)$ ? What is  $a$ ? What is  $L$ ? What value of  $\epsilon$  is given? What is the corresponding value of  $\delta$ ?
2. **§2.5 #44** The gravitational force exerted by the planet Earth on a unit mass at a distance  $r$  from the center of the planet is

$$F(r) = \begin{cases} \frac{GM}{R^3} r & \text{if } r < R \\ \frac{GM}{r^2} & \text{if } r \geq R \end{cases}$$

where  $M$  is the mass of Earth,  $R$  is its radius, and  $G$  is the gravitational constant. Is  $F$  a continuous function of  $r$  on the domain  $(0, \infty)$ ? Your justification should invoke the Continuity Checklist.

3. **§2.5 #52** Suppose  $f$  is continuous on  $[1, 5]$  and the only solutions of the equation  $f(x) = 6$  are  $x = 1$  and  $x = 4$ . If  $f(2) = 8$ , explain why  $f(3) > 6$ .
4. **§2.6 #68**

- A tank contains 5000 L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25 L/min. Show that the concentration of salt after  $t$  minutes (in grams per liter) is

$$C(t) = \frac{30t}{200 + t}$$

- What happens to the concentration as  $t \rightarrow \infty$ ?
5. **§2.6 #10** Sketch the graph of an example of a function  $f$  that satisfies all of the following:

- $\lim_{x \rightarrow 3} f(x) = -\infty$
- $\lim_{x \rightarrow \infty} f(x) = 2$
- $f(0) = 0$
- $f$  is even