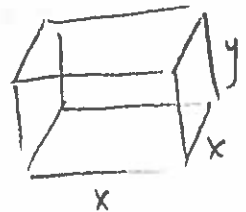


Math 2554 Pop Quiz

Wed 12 Nov 2014

SOLUTIONS

1.  Volume = $V = x^2 y = 12 \text{ m}^3 \Rightarrow y = \frac{12}{x^2} \text{ m}$

Surface Area = $S = 2x^2 + 4xy = 2x^2 + 4x \left(\frac{12}{x^2} \right)$

$$= 2x^2 + \frac{48}{x} \text{ m}^2$$

minimize:

$$\frac{dS}{dx} = 4x - \frac{48}{x^2} = 0 \Rightarrow 4x^3 - 48 = 0 \Rightarrow x = \sqrt[3]{12}$$

$S'(2) < 0$ $S'(3) > 0$ By the 1st Deriv Test, $x = \sqrt[3]{12} \approx 2.289$ gives a min $\Rightarrow y = \frac{12}{(\sqrt[3]{12})^2} = \sqrt[3]{12}$.

Dimensions $\approx 2.289 \text{ m} \times 2.289 \text{ m} \times 2.289 \text{ m}$

2. $S = 2x^2 + 4xy = 20 \text{ m}^2 \Rightarrow y = \frac{20 - 2x^2}{4x} \text{ m}$

$V = x^2 y = x^2 \left(\frac{20 - 2x^2}{4x} \right) = 5x - \frac{1}{2}x^3 \text{ m}^3$

maximize:

$$\frac{dV}{dx} = 5 - \frac{3}{2}x^2 = 0 \Rightarrow x = \pm \sqrt{\frac{10}{3}}$$

$V'(0) > 0$ $V'(2) < 0$

$\leftarrow \begin{array}{c} | & | & | & | \\ -\sqrt{\frac{10}{3}} & 0 & \sqrt{\frac{10}{3}} & 2 \end{array} \rightarrow$

Since x cannot be negative,

$x = \sqrt{\frac{10}{3}} \approx 1.826$ gives a max by the 1st Deriv Test.

$$\Rightarrow y = \frac{20 - 2\left(\sqrt{\frac{10}{3}}\right)^2}{4\sqrt{\frac{10}{3}}} \approx 1.826$$

Dimensions:
 $\approx 1.826 \text{ m} \times 1.826 \text{ m} \times 1.826 \text{ m}$