MATH 2574 (Calculus III)
Spring 2017

Name: SOLUTIONS

Fri 10 Feb 2017

Exam 1: Intro to Multidimensional Calculus (§11.1-11.7, 12.1-12.2)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems. No electronic devices (phones, iDevices, computers, etc) except for a **basic scientific calculator**. On story problems, round to one decimal place. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

In addition, please provide the following data:

Drill Instructor: ____________________

Drill Time: ____________________

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt) ____________________

Good luck!

1. Determine whether the following statements are true or false. You must justify your answer.

(a) **(5 pts)** The domain of the function $u = f(w, x, y, z)$ is a region in $\mathbb{R}^4$.

True. The function has four independent variables.

(b) **(5 pts)** $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$

True. $\vec{u} \times \vec{v}$ is orthogonal to $\vec{v}$ and the dot product of orthogonal vectors is 0.

(c) **(5 pts)** The domain of the function $f(x, y) = 1 - |x - y|$ is $\{(x, y) \mid x \geq y\}$.

False. domain is $\mathbb{R}^2$

(d) **(5 pts)** All level curves of the plane $z = 2x - 3y$ are lines, except for when $z = 0$.

False. $z = z_0 = 2x - 3y$

$\Rightarrow y = \dfrac{2x - z_0}{3} = \dfrac{2}{3}x - \dfrac{z_0}{3}$ ← is a line for all z_0

2. **(18 pts)** Determine an equation of the line ℓ that is perpendicular to the lines

$$\mathbf{r}(t) = \langle -1+3t, 3t, 2t\rangle = \langle -1, 0, 0\rangle + t\langle 3, 3, 2\rangle$$
$$\mathbf{R}(s) = \langle -6+3s, -8+2s, -12+s\rangle = \langle -6, -8, -12\rangle + s\langle 3, 2, 1\rangle$$

and passes through the point P_0 of intersection of the lines $\mathbf{r}$ and $\mathbf{R}$.

To find P_0:

x-comp

$$-1+3t = -6+3s$$
$$3t = -5+3s$$
$$t = -\frac{5}{3}+s$$

$\Rightarrow$

$$3t = -8+2s \quad \text{y-comp.}$$
$$3\left(-\frac{5}{3}+s\right) = -8+2s$$
$$-5+3s = -8+2s$$
$$s = -3$$
$$\Rightarrow t = -\frac{5}{3} - 3 = -\frac{14}{3}$$

Check z-component:

$$2t = -12+s$$
$$2\left(-\frac{14}{3}\right) = -12-3 \Rightarrow \boxed{\text{lines don't intersect}}$$

3. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable functions at $t = 0$ with $\mathbf{u}(0) = \langle 1, 0, 1\rangle$, $\mathbf{u}'(0) = \langle 7, 0, 1\rangle$, $\mathbf{v}(0) = \langle 0, 1, 1\rangle$, $\mathbf{v}'(0) = \langle 1, 3, 2\rangle$. Evaluate the following expressions:

(a) **(6 pts)** $\left.\frac{d}{dt}(\sin(t)\mathbf{u}(t))\right|_{t=0} = \cos(0)\vec{u}(0) + \cancel{\sin(0)\vec{u}'(0)}$

$= \vec{u}(0)$

$\boxed{= \langle 1, 0, 1\rangle}$

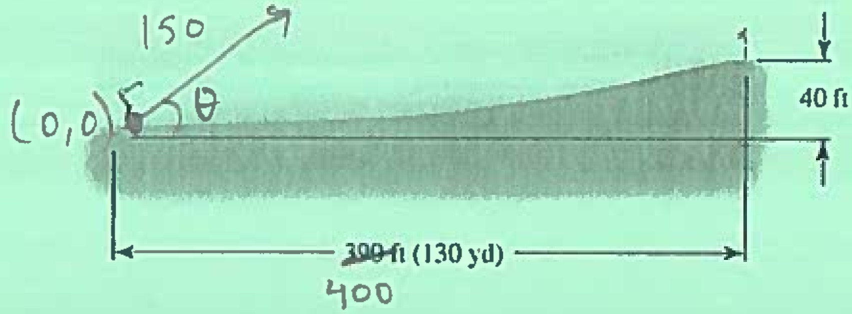
(b) **(6 pts)** $\left.\frac{d}{dt}(\mathbf{u}\cdot\mathbf{v})\right|_{t=0} = \vec{u}'(0)\cdot\vec{v}(0) + \vec{u}(0)\cdot\vec{v}'(0)$

$= \langle 7, 0, 1\rangle\cdot\langle 0, 1, 1\rangle + \langle 1, 0, 1\rangle\cdot\langle 1, 3, 2\rangle$

$= 0 + 0 + 1 + 1 + 0 + 2$

$\boxed{= 4}$

4. A golfer stands 400 ft horizontally from the hole and 40 ft below the hole (see figure).



Suppose the ball is hit with an initial speed of 150 ft/s, at an angle of θ from the ground.

(a) **(12 pts)** Find the acceleration $\mathbf{a}(t)$, velocity $\mathbf{v}(t)$, and position $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ vectors for the trajectory of the ball. The gravitational constant is $g = 32$ ft/s^2.

$$\boxed{\vec{a}(t) = \langle 0, -32\rangle \text{ ft/s}^2}$$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0, -32t\rangle + \vec{C}$$

$$\vec{v}(0) = \langle 150\cos\theta, 150\sin\theta\rangle = \langle 0, -32(0)\rangle + \vec{C}$$

$$\Rightarrow \boxed{\vec{v}(t) = \langle 150\cos\theta, -32t + 150\sin\theta\rangle \text{ ft/s}}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 150\cos\theta\, t, -16t^2 + 150\sin\theta\, t\rangle + \vec{C}$$

$$\vec{r}(0) = \langle 0, 0\rangle = \langle 150\cos\theta(0), -16(0)^2 + 150\sin\theta(0)\rangle + \vec{C}$$

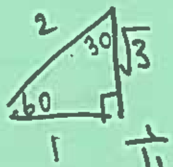
$$\Rightarrow \boxed{\vec{r}(t) = \langle 150\cos\theta\, t, -16t^2 + 150\sin\theta\, t\rangle \text{ ft}}$$

(b) **(6 pts)** Write down a system of two equations to find the two unknowns: (1) time of flight and (2) θ. **Do not** solve the system.

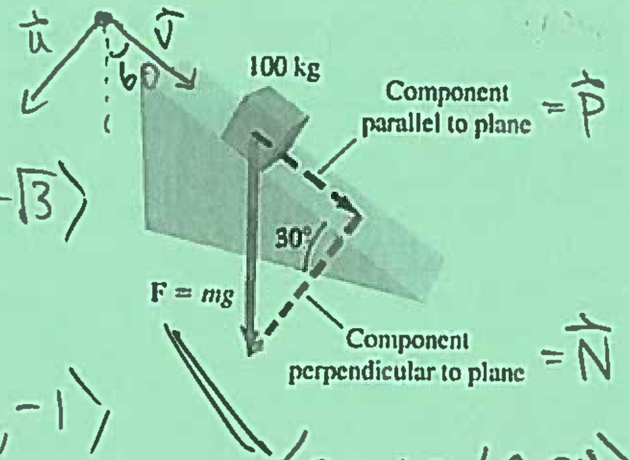
$$\vec{r}(t) = \langle 400, 40\rangle \text{ ft}$$

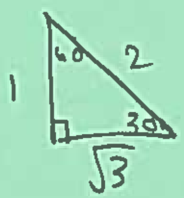
$$\Rightarrow \boxed{\begin{aligned} 150\cos\theta\, t &= 400 \\ -16t^2 + 150\sin\theta\, t &= 40 \end{aligned}}$$

5. **(16 pts)** A 100 kg box rests on a ramp with an incline of 30° to the floor (see figure). Find the components of the force perpendicular to and parallel to the ramp. (The vertical component of the force exerted by an object of mass m is its weight, which is mg, where $g = 9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity.)



$\vec{u} = \langle -1, -\sqrt{3} \rangle$





$\vec{v} = \langle \sqrt{3}, -1 \rangle$

$\langle 0, -100(9.8) \rangle = \langle 0, -980 \rangle$

$$\vec{P} = \mathrm{proj}_{\vec{v}} \vec{F} = \frac{\vec{v}\cdot\vec{F}}{\vec{v}\cdot\vec{v}}\vec{v} = \frac{\sqrt{3}(0) + (-1)(-980)}{\underbrace{(\sqrt{3})^2 + (-1)^2}_{4}} \langle \sqrt{3}, -1 \rangle$$

$$= \left\langle \frac{980}{4}\sqrt{3}, -\frac{980}{4} \right\rangle = \boxed{\langle 245\sqrt{3}, -245 \rangle\ \mathrm{kg\,m/s^2}}$$

$\approx \langle 424.4, -245 \rangle\ \mathrm{kg\,m/s^2}$

$$\vec{N} = \mathrm{proj}_{\vec{u}} \vec{F} = \frac{\vec{u}\cdot\vec{F}}{\vec{u}\cdot\vec{u}}\vec{u}$$

$$= \frac{(-1)(0) + (-\sqrt{3})(-980)}{\underbrace{(-1)^2 + (-\sqrt{3})^2}_{4}} \langle -1, -\sqrt{3} \rangle$$

$$= \langle -245\sqrt{3}, -245(3) \rangle = \boxed{\langle -245\sqrt{3}, -735 \rangle\ \mathrm{kg\,m/s^2}}$$

$\approx \langle -424.4, -735 \rangle\ \mathrm{kg\,m/s^2}$

6. **(15 pts)** Match equations (a)-(f) with the surfaces (A)-(F).

C (a) $y = |x|$ ← sharp corner

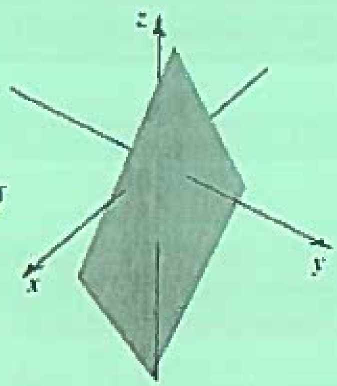
A (b) $3x - 4y - z = 5$ ← plane

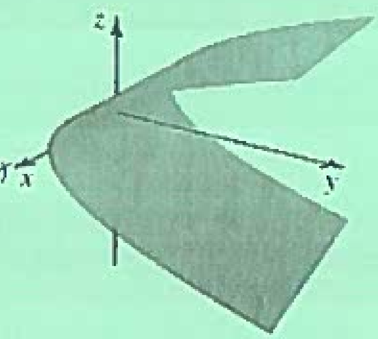
D (c) $y - z^2 = 0$ ← parabolic cylinder

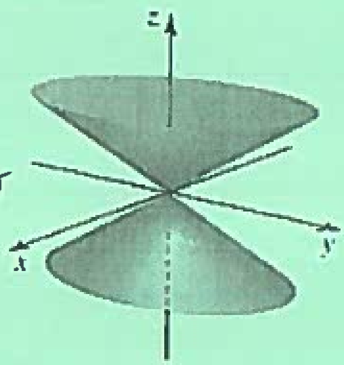
E (d) $4x^2 + \frac{y^2}{4} + z^2 = 1$ ← ellipsoid

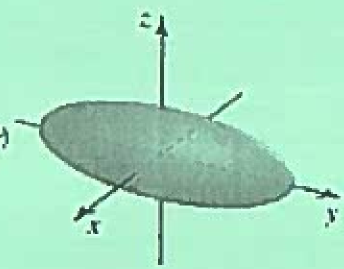
B (e) $x^2 + \frac{y^2}{9} = z^2$ ← cone

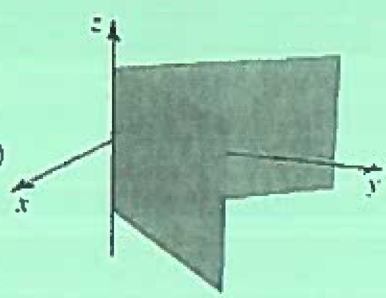
F (f) $x^2 + \frac{y^2}{9} - z^2 = 1$ ← hyperboloid

(A)


(D)


(B)


(E)


(C)


(F)