MATH 2554 (Calculus I)
Spring 2016

Name: _____

Fri 12 Feb 2016

Exam 1: Limits (§2.1-3.1)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: **(1 pt)** _____

Good luck!

1. **(10 pts)** Let $f(x) = \dfrac{x+7}{x^4 - 49x^2}$. Identify all vertical asymptotes for f (or if there are none, say so and why). Then, for each vertical asymptote a, find $\lim\limits_{x \to a^+} f(x)$ and $\lim\limits_{x \to a^-} f(x)$.

2. **(10 pts)** Determine the end behavior of $f(x) = \dfrac{x+1}{\sqrt{9x^2+x}}$. If there are any horizontal asymptotes then identify them.

3. **(3 pts ea)** Evaluate the following limits analytically:

(a) $\lim_{y \to 3} \dfrac{\sqrt{3y+16}-5}{y-3}$

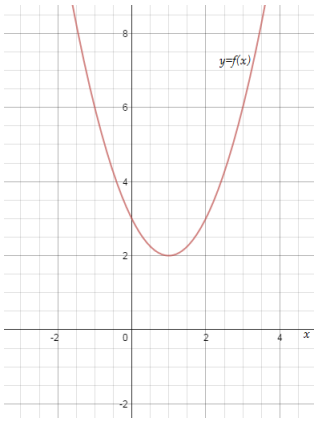
(b) $\lim_{x \to 0} (2x^{-8} + 4x^3)$

(c) $\lim_{x \to \infty} \pi e^{-x}$

(d) $\lim_{t \to -1} f(t)g(t)$, given that $\lim_{t \to -1} f(t) = 2$ and $\lim_{t \to -1} g(t) = 8$.

4. **(10 pts)** Use the Intermediate Value Theorem to show $f(x) = 4x^3 - 6x^2 + 3x - 2$ must cross the line $y = 10$ in the interval $(1, 2)$.

5. **(3 pts ea)** Let $f(x) = x^2 - 2x + 3$. Below is a graph of $f(x)$, drawn at `desmos.com`.



(a) Use the graph to find a number $\delta > 0$ such that if $|x - 1| < \delta$ then $|f(x) - 2| < 1$. If no such number exists, then say so.

(b) If, when we use smaller and smaller values $\epsilon < 1$, we can always find a corresponding value $\delta > 0$, as in (a), then we will have proved that

$$\lim_{x \to ?} f(x) = ? \quad \text{(rewrite the limit, with the ?s filled in).}$$

(c) For *any* $\epsilon > 0$, find $\delta > 0$ so that $|f(x) - 2| < \epsilon$ whenever $0 < |x - 1| < \delta$. *Hint: Your answer will be an expression with ϵs in it.*

6. When computing derivatives in this problem you must use the limit definitions. Given the function,

$$s(t) = \sqrt{5t}$$

(a) **(5 pts)** write the formula for the slope of the secant line joining the points $(a, s(a))$ and $(b, s(b))$;

(b) **(5 pts)** find $s'(1)$;

(c) **(3 pts)** write the equation of the line tangent to $s(t)$ at $t = 1$.

7. **(5 pts)** Find constants b and c in the polynomial $p(x) = x^2 + bx + c$ so that

$$\lim_{x \to 2} \frac{p(x)}{x - 2} = 6.$$

8. **(5 pts)** Determine the interval(s) of continuity for

$$f(x) = \frac{x + 2}{x^2 - 4}.$$