Math 2554 Quiz 3: $\oint$ 2.6-2.7 **Name:** KEY
Tues 16 Sep 2014

You have 20 minutes to complete this quiz. Eyes on your own paper and good luck!

1. **Definitions/Concepts.**

   (a) (3 pts) The function f is **continuous at the point** a means it satisfies the Continuity Checklist:

   SOLUTION:

   1. $f(a)$ is defined (a is in the domain of f).
   2. $\lim_{x\to a} f(x)$ exists.
   3. $\lim_{x\to a} f(x) = f(a)$ (the value of f equals the limit of f at a).

   (b) (2 pts) "The limit of $f(x)$ as x approaches a equals L" means that for any positive number ϵ, there is another positive number δ such that

   SOLUTION: $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

2. **Questions/Problems.** Suppose $\lim_{x\to 3} f(x) = 4$, where f is the function in Figure 1.

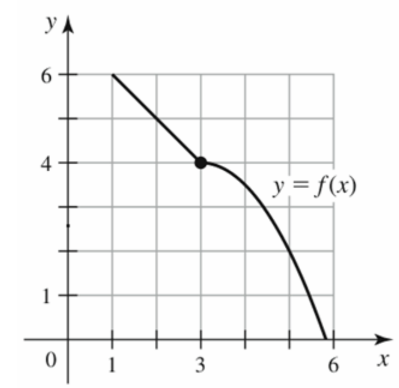

Figure 1: $f(x)$ (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*, p. 116)

What must δ equal in order to satisfy $|f(x) - 4| < \epsilon$ whenever $0 < |x - 3| < \delta$, for

(a) (1 pt) $\epsilon = 2$?

SOLUTION: 2

(b) (1 pt) $\epsilon = \frac{1}{2}$?

SOLUTION: $\frac{1}{2}$

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(c) (1 pt) Write a formula for δ in terms of ϵ that works, once ϵ gets small enough.

SOLUTION: $\delta = \epsilon$

(d) (ChAlLeNgE pRoBlEm) Justify your answer to (c).

SOLUTION: To the left of $x = 3$, $f(x)$ is linear, so $\delta \sim \epsilon$. To the right of $x = 3$, $f(x)$ is quadratic, so $\delta \sim \sqrt{\epsilon}$. Once ϵ gets smaller than 1, $\sqrt{\epsilon} > \epsilon$. This means the smaller value for δ will be on the linear side of $x = 3$.

3. **Computations/Algebra.** (2 pts) Let

$$g(x) = \begin{cases} \frac{x^2+3x+2}{x+1} & x \neq -1 \\ k & x = -1 \end{cases}.$$

Using the Continuity Checklist, find the value of k that makes g continuous at the point -1.

SOLUTION: In the Continuity Checklist, $g(-1)$ is defined (as k). We compute,

$$\begin{aligned} \lim_{x\to -1} g(x) &= \lim_{x\to -1} \frac{x^2+3x+2}{x+1} \\ &= \lim_{x\to -1} \frac{(x+2)(x+1)}{x+1} \\ &= \lim_{x\to -1} x+2 \\ &= (-1)+2 = 1 \quad \text{exists;} \end{aligned}$$

and finally we set

$$g(-1) = k = \lim_{x\to -1} g(x) = 1.$$