Take-Home Quiz 1: Vectors and vector-valued functions (§11.1-11.7)

Directions: This quiz is due on Februrary 3, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. A sum of scalar multiples of two or more vectors (such as $c_1\mathbf{u} + c_2\mathbf{v} + c_3\mathbf{w}$, where c_i are scalars) is called a **linear combination** of the vectors. Express $\langle 4, -8\rangle$ as a linear combination of the vectors $\mathbf{u} = \langle 1, 1\rangle$ and $\mathbf{v} = \langle -1, 1\rangle$.

2. Give a geometric description of the following set of points:
$$x^2 + y^2 + z^2 - 8x + 14y - 18z \geq 65$$

3. Give a geometric description of the set of points (x, y, z) that lie on the intersection of the sphere $x^2 + y^2 + z^2 = 36$ and the plane $z = 6$.

4. Carry out the following steps to determine the (least) distance between the point $P = (0, 2, 6)$ and the line ℓ that is parallel to the $\langle 3, 0, -4\rangle$ and passes through the origin.
   (a) Find any vector $\mathbf{v}$ in the direction of ℓ.
   (b) Find the position vector corresponding to P.
   (c) Find $\text{proj}_{\mathbf{v}}\, \mathbf{u}$.
   (d) Show that $\mathbf{w} = \mathbf{u} - \text{proj}_{\mathbf{v}}\, \mathbf{u}$ is a vector orthogonal to $\mathbf{v}$ whose length is the distance between P and the line ℓ.
   (e) Find $\mathbf{w}$ and $|\mathbf{w}|$. Why is $|\mathbf{w}|$ the least distance between P and ℓ?

5. A particle with a unit negative charge ($q = -1$) enters a constant magnetic field $\mathbf{B} = 5\mathbf{k}$ with a velocity $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$. Find the magnitude and direction of the force on the particle. Make a sketch of the magnetic field, the velocity, and the force.

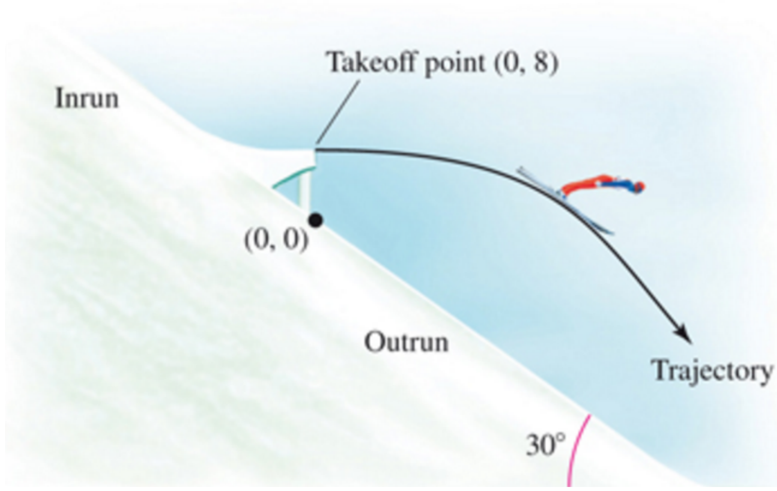
6. Find the area of the triangle with vertices $O = (0, 0, 0)$, $P = (2, 4, 6)$, and $Q = (6, 5, 4)$.

7. Consider the lines
$$\mathbf{r}(t) = \langle 2 + 2t, 8 + t, 10 + 3t\rangle \text{ and}$$
$$\mathbf{R}(s) = \langle 6 + s, 10 - 2s, 16 - s\rangle.$$
   (a) Determine whether the lines intersect (have a common point) and if so, find the coordinates of the point.
   (b) If $\mathbf{r}$ and $\mathbf{R}$ describe the paths of the two particles, do the particles collide? Assume that $t \geq 0$ and $s \geq 0$ measure time in seconds, and that motion starts at $s = t = 0$.

8. Find the unit tangent vector at $t = 0$ for
$$\mathbf{r}(t) = \langle \sin t, \cos t, e^{-t}\rangle, \text{ for } 0 \leq t \leq \pi.$$

9. Consider the curve
$$\mathbf{r}(t) = \langle \sqrt{t}, 1, t\rangle,$$
for $t > 0$. Find all points on the curve at which $\mathbf{r}$ and $\mathbf{r}'$ are orthogonal.

10. The lip of a ski jump is 8 m above the outrun that is sloped at an angle of 30 degrees to the horizontal (see figure).



(a) If the initial velocity of a ski jumper at the lip of the jump is $\langle 40, 0\rangle$ m/s, what is the length of the jump (distance from the origin to the landing point)? Assume only gravity affects the motion.

(b) Assume that air resistance produces a constant horizontal acceleration of 0.15 m/s^2 opposing the motion. What is the length of the jump?

(c) Suppose that the takeoff ramp is titlted upward at an angle of θ, so that the skier's initial velociy is $40\langle\cos\theta, \sin\theta\rangle$ m/s. What value of θ maximizes the length of the jump? Express your answer in degrees and neglect air resistance.