Take-Home Quiz #6 — Math 235 (Calc I), Fall 2017

SOLUTIONS

1. (a) $L(x) = f'(a)(x-a) + f(a)$ $\quad a=0$

$= f'(0)(x-0) + f(0)$ $\quad f'(x) = e^x\cos x + e^x(-\sin x) = e^x(\cos x - \sin x)$

$= e^0(\cos 0 - \sin 0)x + e^0\cos 0$

$\boxed{= x+1}$

(b) desmos: $\sim -0.763 < x < 1.5706$

(c) $dy = f'(x)\,dx = e^x(\cos x - \sin x)\,dx$

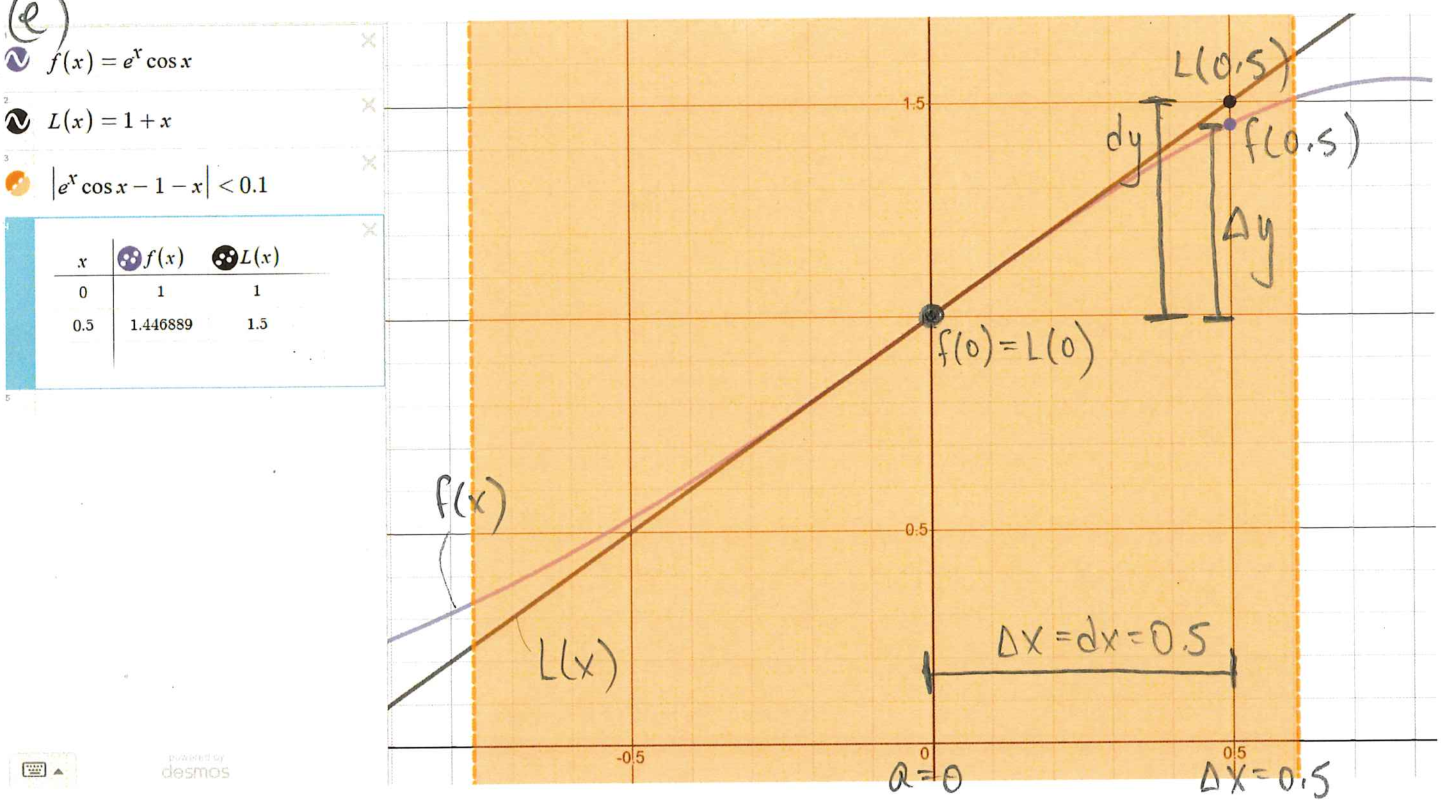
(d) $\Delta x = 0.5 \Rightarrow dx = \Delta x = 0.5$, $x = a + \Delta x = 0 + 0.5 = 0.5$

$dy = f'(0.5)\Delta x = e^{0.5}(\cos 0.5 - \sin 0.5)\cdot 0.5 \approx \boxed{0.328}$

$\Delta y = f(x) - f(a)$

$= f(0.5) - f(0) = e^{0.5}\cos 0.5 - 1 \approx \boxed{0.447}$

(e)



2. (a) $f'(x) = 2(x-1)$

$\Rightarrow L_f(x) = 2(0-1)(x-0) + (0-1)^2$

$\boxed{= -2x+1}$

$g'(x) = -2e^{-2x}$

$\Rightarrow L_g(x) = -2e^{-2(0)}(x-0) + e^{-2(0)}$

$\boxed{= -2x+1}$

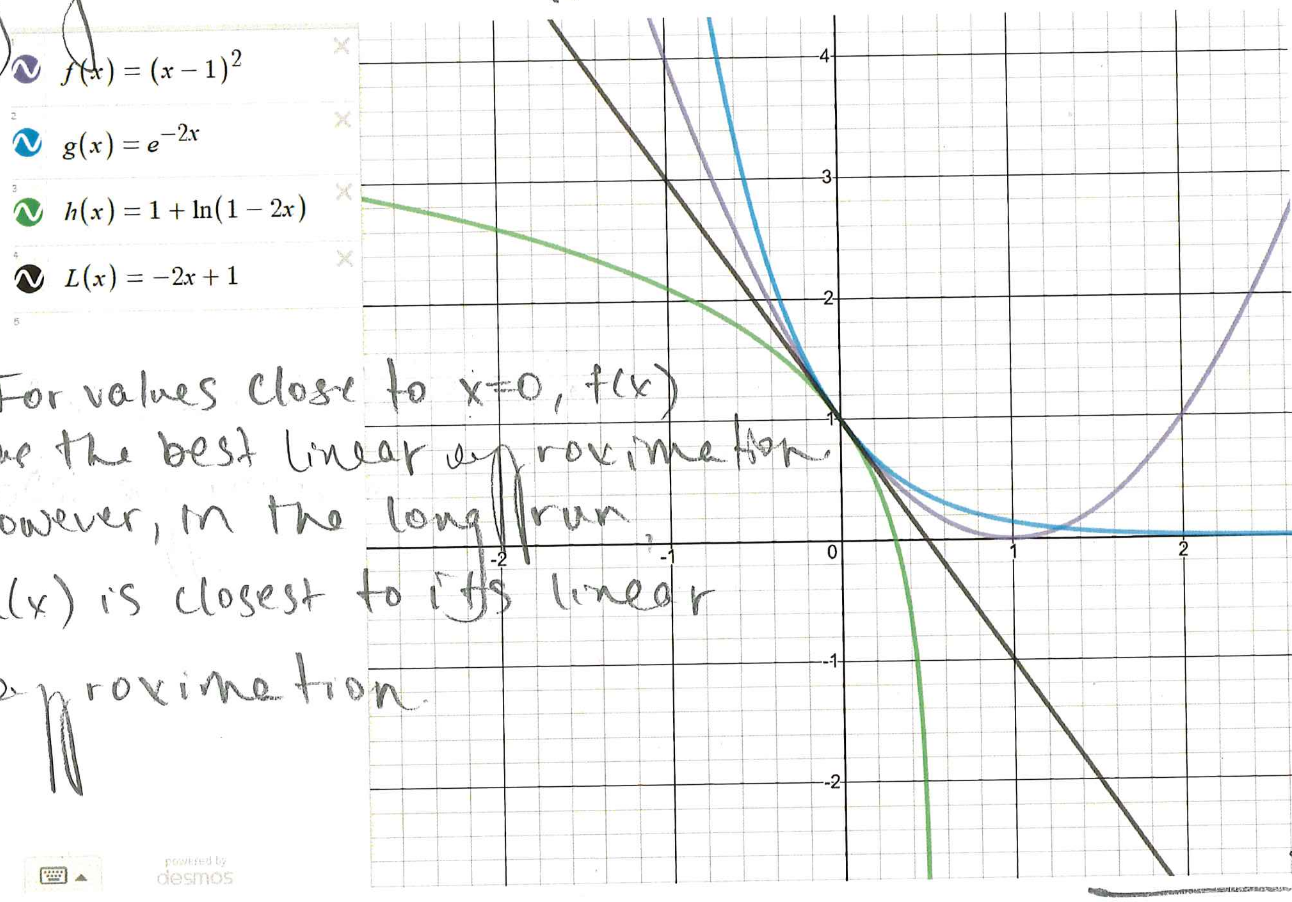
$h'(x) = \dfrac{-2}{1-2x}$

$\Rightarrow L_h(x) = \dfrac{-2}{1-2(0)}(x-0) + 1 + \ln(1-2(0))$

$\boxed{= -2x+1}$

All three functions happen to have the same tangent line at $x=0$.

(b)



For values close to $x=0$, $f(x)$ has the best linear approximation. However, in the long run $h(x)$ is closest to it's linear approximation.

3. (a) 4θ and $\tan\theta$ both range over all real numbers so the image of $g(\theta)$ is $\mathbb{R}$.

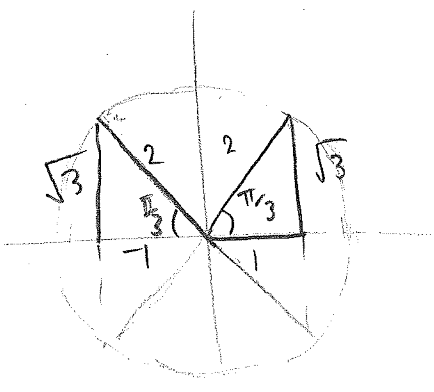
(b) $g'(\theta) = 4 - \sec^2\theta$

$$= 4 - \frac{1}{\cos^2\theta}$$

is undefined when $\cos\theta = 0$, but these points are not in the domain for g.

$$g'(\theta) = 0 = 4 - \frac{1}{\cos^2\theta}$$

$$\Rightarrow \frac{1}{\cos^2\theta} = 4 \Rightarrow \frac{1}{4} = \cos^2\theta \Rightarrow \cos\theta = \pm\frac{1}{2}$$



$\Rightarrow \theta = \pm\frac{\pi}{3} \pm$ multiples of 2π

(when $\cos\theta = \frac{1}{2}$)

$\theta = \pm\frac{2\pi}{3} \pm$ multiples of 2π

(when $\cos\theta = -\frac{1}{2}$)

Critical points: $\left(\frac{\pi}{3} \pm 2n\pi,\ 4\left(\frac{\pi}{3} + 2n\pi\right) + \tan\left(\frac{\pi}{3} + 2n\pi\right)\right)$,

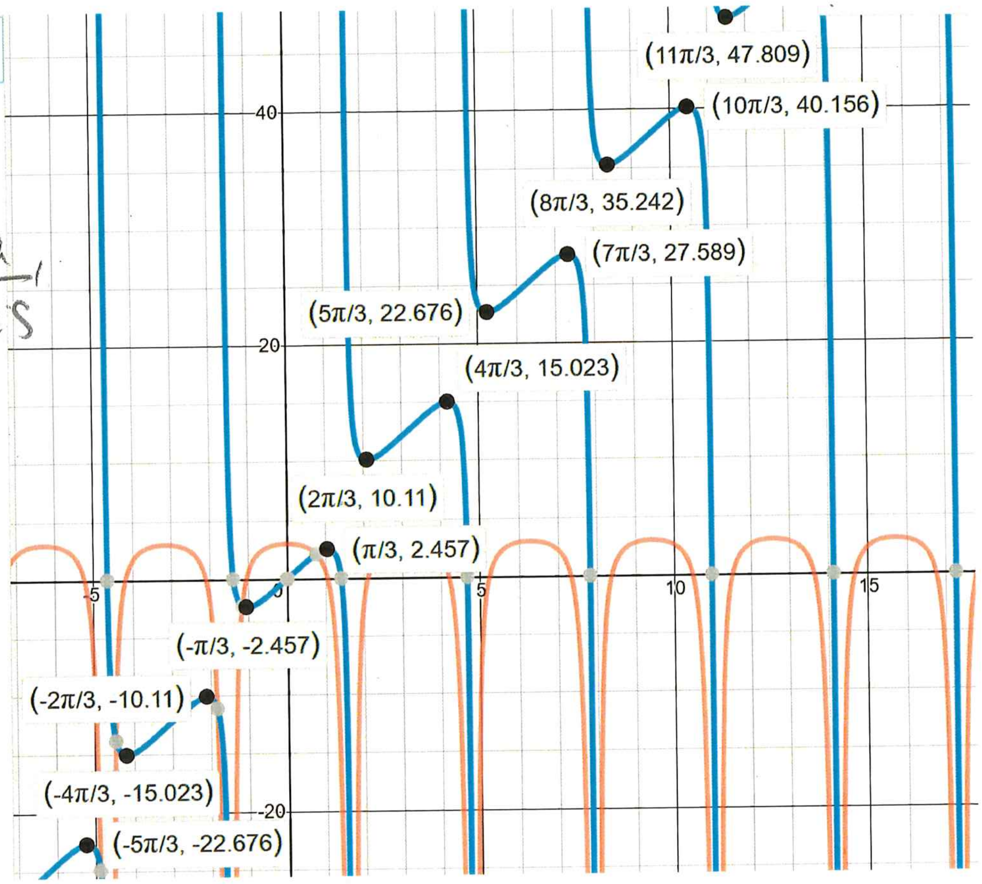
$$= \boxed{\left(\frac{\pi}{3} \pm 2n\pi,\ \frac{4\pi}{3} \pm 8n\pi + \frac{\sqrt{3}}{2}\right)}$$

$$\boxed{\left(\frac{-\pi}{3} \pm 2n\pi,\ -\frac{4\pi}{3} \mp 8n\pi - \frac{\sqrt{3}}{2}\right)},\ \boxed{\left(\frac{2\pi}{3} \pm 2n\pi,\ \frac{8\pi}{3} \pm 8n\pi + \frac{\sqrt{3}}{2}\right)},$$

$$\boxed{\left(\frac{-2\pi}{3} \pm 2n\pi,\ -\frac{8\pi}{3} \mp 8n\pi - \frac{\sqrt{3}}{2}\right)}$$

(c) $g(x) = 4x - \tan x$

$y = g'(x)$



No global extrema, since the range is $\mathbb{R}$.

Local minima occur at

$\theta = \frac{2\pi}{3} \pm n\pi$

Local maxima occur at

$\theta = \frac{\pi}{3} \pm n\pi$

4. Let $v(t)$ = car's speed at time t.

Since v is continuous, the Mean Value Theorem says for some $t=c$ between 2_p and $2{:}10_p$, the derivative, a.k.a. acceleration of the car, is

$$v'(c) = \frac{v(2{:}10_p) - v(2_p)}{(10 \text{ minutes})}$$

$$= \frac{50\text{mph} - 30\text{mph}}{\left(\frac{1}{6}\text{ hours}\right)} = \frac{20 \text{ miles}/\text{hour}}{\frac{1}{6}\text{ hours}}$$

$$= 120 \frac{\text{miles}}{\text{hour}^2}.$$