Math 2554 Exam 2
Friday 27 February 2015

Name: SOLUTIONS

Calculus I
Exam 2: Derivatives

Please provide the following data:

Drill Instructor: ____________

Drill Time: ____________

Student ID or clicker #: ____________

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard, one side only, is allowed. No graphing calculators. No programmable calculators. No electronic devices except for the approved calculators (so no phones, iDevices, computers, etc). If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: **(1 pt)**

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Good luck!

1. **(3 pts each)** For each function $g(x)$, find the value of $g'(3)$ using the data given below.

$$f(1) = 6 \qquad f(3) = 2 \qquad f(6) = 5 \qquad f(9) = -3$$
$$f'(1) = -2 \qquad f'(3) = 4 \qquad f'(6) = -1 \qquad f'(9) = 1$$

(a) $g(x) = f(3) + 10f(2x)$

constant

Chain Rule

$g'(x) = 0 + 10 f'(2x) \cdot 2$

$g'(3) = 0 + 10 f'(6) \cdot 2$

$= 20(-1) = \boxed{-20}$

(b) $g(x) = \dfrac{f(x)}{x}$

$g'(x) = \dfrac{x f'(x) - f(x)(1)}{x^2}$

$g'(3) = \dfrac{3f'(3) - f(3)}{3^2} = \dfrac{3(4) - 2}{9} = \boxed{\dfrac{10}{9}}$

(c) $g(x) = (f(x))^3$

$g'(x) = 3f(x)^2 \cdot f'(x) \leftarrow$ Chain Rule

$g'(3) = 3f(3)^2 \cdot f'(3)$

$= 3(2)^2(4) = \boxed{48}$

(d) **EXTRA CREDIT** $g(x) = f(x^2)$

$g'(x) = f'(x^2) \cdot 2x \leftarrow$ Chain Rule

$g'(3) = f'(3^2) \cdot 2(3)$

$= f'(9) \cdot 6 = (1)(6) = \boxed{6}$

2. **(8 pts)** Evaluate $\lim_{x \to 0} \frac{\sin 3x}{x}$. $\left(\frac{3}{3}\right)$

$$= \lim_{x \to 0} \frac{3 \sin 3x}{3x}$$

$$= 3 \lim_{\substack{x \to 0 \\ 3x \to 0}} \frac{\sin 3x}{3x} \to 1$$

$$= \boxed{3}$$

3. **(5 pts each)** Suppose a company produces x items at a cost $C(x)$.

(a) Write the formula for **average cost** and say in words what it means.

$\frac{C(x)}{x}$ = average price per item, having produced x items

(b) Write the formula for **marginal cost** and say in words what it means..

$C'(x)$ = approximate cost to produce one more item, having produced x items so far

4. **(8 pts each)** For each function find $\frac{d^2y}{dx^2}$.

$\frac{d}{dx}\Big($(a) $x^4 + y^4 = 64\Big)$

$= 4x^3 + 4y^3\frac{dy}{dx} = 0$

$\Rightarrow 4y^3\frac{dy}{dx} = -4x^3$

$\frac{dy}{dx} = \frac{-4x^3}{4y^3} = \frac{-x^3}{y^3}.$

Quotient Rule:

$\frac{d^2y}{dx^2} = \frac{y^3(-3x^2) - (-x^3)\left(3y^2\frac{dy}{dx}\right)}{(y^3)^2}$

$\boxed{= \frac{-3x^2y^3 + 3x^3y^2\left(\frac{-x^3}{y^3}\right)}{y^6}}$

$= \frac{-3x^2y^4 - 3x^6}{y^7}$ (simplified)

$\frac{d}{dx}\Big($(b) $e^{2y} + x = y\Big)$

$= 2e^{2y}\frac{dy}{dx} + 1 = \frac{dy}{dx}$

$\Rightarrow 1 = \frac{dy}{dx} - 2e^{2y}\frac{dy}{dx}$

$= \frac{dy}{dx}(1 - 2e^{2y})$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1-2e^{2y}} = (1-2e^{2y})^{-1}$

$\frac{d^2y}{dx^2} = -(1-2e^{2y})^{-2}(-4e^{2y})\frac{dy}{dx}$

$\boxed{= (1-2e^{2y})^{-2}(4e^{2y})\left(\frac{1}{1-2e^{2y}}\right)}$

$= \frac{4e^{2y}}{(1-2e^{2y})^3}$ (simplified)

5. **(4 pts each)** Let $F = f + g$ and $G = 3f - g$, where the graphs of f and g are shown in Figure 1.

$F' = f' + g'$ $G' = 3f' - g'$

derivative = slope

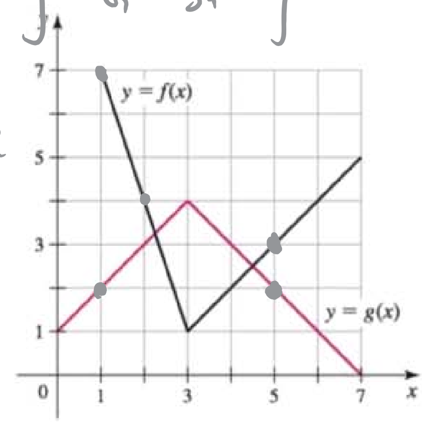

Figure 1: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

Find the following derivatives:

(a) $F'(1) = f'(1) + g'(1)$

$= -3 + 1 = \boxed{-2}$

(b) $G'(1) = 3f'(1) - g'(1)$

$= 3(-3) - 1 = \boxed{-10}$

(c) $F'(5) = f'(5) + g'(5)$

$= 1 - 1 = \boxed{0}$

(d) $G'(5) = 3f'(5) - g'(5)$

$= 3(1) - (-1) = 3 + 1 = \boxed{4}$

6. **(3 pts each)** Find the derivative of each of the following functions:

(a) $f(x) = \dfrac{(x-1)(2x^2-1)}{x^3-1} = \dfrac{\cancel{(x-1)}(2x^2-1)}{\cancel{(x-1)}(x^2+x+1)}$ $(x \neq 1)$

$$\boxed{f'(x) = \frac{(x^2+x+1)(4x)-(2x^2-1)(2x+1)}{(x^2+x+1)^2}}$$

$$= \frac{\cancel{4x^3}+4x^2+4x-\cancel{4x^3}+2x-2x^2+1}{(x^2+x+1)^2}$$

$$= \frac{2x^2+6x+1}{(x^2+x+1)^2}$$ (simplified)

(b) $f(x) = \dfrac{x+1}{x^2e^{2x}}$

$$\boxed{f'(x) = \frac{x^2e^{2x}(1)-(x+1)\left(2xe^{2x}+x^2(2e^{2x})\right)}{(x^2e^{2x})^2}}$$

$$= \frac{\cancel{x^2e^{2x}}-2x^2e^{2x}-2x^3e^{2x}-2xe^{2x}-\cancel{2x^2e^{2x}}}{(x^2e^{2x})^2}$$

$$= \frac{-2x^3e^{2x}-3x^2e^{2x}-2xe^{2x}}{x^4e^{4x}}$$

$$= \frac{-2x^2-3x-2}{x^3e^{2x}}$$ (simplified)

(c) $y = \sin x + \cos x$

$y' = \cos x - \sin x$

(d) $y = 5x^2 + \cos x$

$y' = 10x - \cos x$

(e) $y = \dfrac{(x^2-1)\sin x}{\sin x + 1}$

$$\boxed{y' = \frac{(\sin x + 1)\left(2x\sin x + (x^2-1)\cos x\right) - (x^2-1)\sin x(\cos x)}{(\sin x + 1)^2}}$$

$$= \frac{2x\sin^2 x + 2x\sin x + \cancel{(x^2-1)\sin x}\cos x + (x^2-1)\cos x - \cancel{(x^2-1)\sin x\cos x}}{(\sin x + 1)^2}$$

$$= \frac{2x\sin x(\sin x + 1) + (x^2-1)\cos x}{(\sin x + 1)^2}$$ (simplified)