Math 115 Extra Credits **Fall 2010**

Points from extra credit go to your total quiz score.

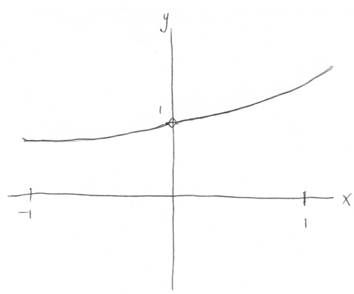
Week 3 (2 pts) p. 59 # 34

(a) Here, $f(x) = \frac{e^x - 1}{x}$.

x	$f(x)$ (to 5 decimal places)
-0.1	0.95163
-0.01	0.99502
-0.001	0.99950
-0.0001	1.00000
0.0001	1.00010
0.001	1.00050
0.01	1.00502
0.1	1.05170

(b) Conjecture: $\lim_{x\to 0} f(x) = 1.00005$, which is between $f(-0.0001)$ and $f(0.0001)$.

(c) The function is defined everywhere except at $x = 0$.



(d) From the table in (a), $x \in [-0.01, 0.01]$ is a suitable interval.

13 Oct (1 pt) Differentiate the following with respect to x.

1. $y = 6$; $y' = 0$
2. $y = 5x$; $y' = 5$
3. $y = 21x^3 + 9x$; $y' = 63x^2 + 9$
4. $y = x^2 + 2x - 1$; $y' = 2x + 2$
5. $y = a_n x^n + a_{n-1}x^{n-1} + \ldots a_1 x + a_0$; $y' = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1$

27 Oct (1 pt) Use the quotient rule to find

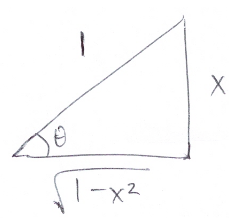
$$\frac{d}{dx}\left(\tan x = \frac{\sin x}{\cos x}\right).$$

$$\begin{aligned}
\frac{d}{dx}\tan x &= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \\
&= \sec^2 x.
\end{aligned}$$

1 Nov (1 pt) Compute $\frac{d}{dx}\arcsin x$ using the right triangle.

Since $\sin(\arcsin x) = x$, implicit differentiation gives

$$\begin{aligned}
1 &= \frac{d}{dx}\sin(\arcsin x) \\
&= \cos(\arcsin x)\cdot\frac{d}{dx}\arcsin x \\
\frac{1}{\cos(\arcsin x)} &= \frac{d}{dx}\arcsin x.
\end{aligned}$$



In the right triangle, $\sin\theta = x$ so $\arcsin x = \theta$. Then

$$\begin{aligned}
\frac{d}{dx}\arcsin x &= \frac{1}{\cos(\arcsin x)} \\
&= \frac{1}{\cos\theta} \\
&= \frac{1}{\frac{\sqrt{1-x^2}}{1}} \\
&= \frac{1}{\sqrt{1-x^2}}.
\end{aligned}$$

2 Dec (1 pt) Using $\Delta x = 0.05$, find the left-hand sum of

$$\int_0^1 e^{-x^2}\,dx.$$

There are $n = \frac{1-0}{0.05} = 20$ subdivisions, so the left-hand sum is given by

$$\Delta x \sum_{k=0}^{19} e^{-x_k^2}$$

or

$$0.05 \sum_{k=0}^{19} e^{-(0.05k)^2}.$$

The terms of the sum are determined, to three decimal places, by the table:

k	0	1	2	3	4
$x_k = 0.05k$	0	0.998	0.990	0.978	0.961
k	5	6	7	8	9
$x_k = 0.05k$	0.939	0.913	0.885	0.852	0.817
k	10	11	12	13	14
$x_k = 0.05k$	0.779	0.739	0.698	0.655	0.613
k	15	16	17	18	19
$x_k = 0.05k$	0.570	0.527	0.486	0.449	0.406
k	20				
$x_k = 0.05k$	0.368				

Then the left-hand sum is about 0.762. Compare to the actual value of the integral, which is about 0.747.

6 Dec (1 pt) Use any resource (world wide web, textbook, friend, etc.) to explain why

$$\sum_{k=1}^{\infty} \frac{1}{2^k} = 1.$$

Choose any tiny number and call it ϵ. If there is an integer N such that

$$1 - \sum_{k=1}^{n} \frac{1}{2^n} < \epsilon$$

no matter what value of n, as long as $n \geq N$, then it means each term of the sum will bring the total closer to 1. And, we can get as close to 1 as we want to. Since the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$ goes to 0, one of those numbers, say $\frac{1}{2^N}$, is smaller than ϵ. But

$$1 - \frac{1}{2^N} = \sum_{k=1}^{N} \frac{1}{2^k}$$

means that

$$\begin{aligned} 1 - \sum_{k=1}^{N} \frac{1}{2^k} &= \frac{1}{2^N} \\ &< \epsilon. \end{aligned}$$

Therefore, the result is true.