MATH 2554 (Calculus I)

Dr. Ashley K. Wheeler

University of Arkansas

April 29, 2015

Table of Contents

Monday 27 April (Week 15)

- Computer HW this week: $\oint 5.5$
- Quiz #15 tomorrow (Tues) in drill – take-home due Thurs
- FINAL! Monday 4 May 6-8p OZAR 026. Study the Coordinator's Review Questions.
- dead day review SCEN 101 3-5p bring the Coordinator's Review Questions.
- Thurs 30 April – don't skip drill
- your top 10 quizzes go into your final grade
- BONUS REVIEW see website and MLP – worth up to 2% of final grade

$\oint$ 5.5 Substitution Rule

Idea: Suppose we have $F(g(x))$, where F is an antiderivative of f. Then

$$\frac{d}{dx}\Big[F(g(x))\Big] = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$$

and $\displaystyle\int f(g(x)) \cdot g'(x)\ dx = F(g(x)) + C$

If we let $u = g(x)$, then $du = g'(x)\ dx$. The integral becomes

$$\int f(g(x)) \cdot g'(x)\ dx = \int f(u)\ du.$$

Substitution Rule for Indefinite Integrals

Let $u = g(x)$, where g' is continuous on an interval, and let f be continuous on the corresponding range of g. On that interval,

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

u-Substitution is the Chain Rule, backwards.

Example

Evaluate $\int 8x\cos(4x^2+3)\ dx$.

Solution: Look for a function whose derivative also appears.

$$u(x) = 4x^2 + 3$$

$$\text{and } u'(x) = \frac{du}{dx} = 8x$$

$$\implies du = 8x\ dx.$$

Now rewrite the integral and evaluate. Replace u at the end with its expression in terms of x.

$$\begin{aligned}
\int 8x\cos(4x^2+3)\ dx &= \int \cos(\underbrace{4x^2+3}_{u})\ \underbrace{8x\ dx}_{du} \\
&= \int \cos u\ du \\
&= \sin u + C \\
&= \sin(4x^2+3) + C
\end{aligned}$$

We can even check our answer. By the Chain Rule,

$$\frac{d}{dx}\left(\sin\left(4x^2+3\right)+C\right) = 8x\cos\left(4x^2+3\right).$$

Procedure for Substitution Rule (Change of Variables)

1. Given an indefinite integral involving a composite function $f(g(x))$, identify an inner function $u = g(x)$ such that a constant multiple of $g'(x)$ appears in the integrand.
2. Substitute $u = g(x)$ and $du = g'(x)\ dx$ in the integral.
3. Evaluate the new indefinite integral with respect to u.
4. Write the result in terms of x using $u = g(x)$.

Warning: Not all integrals yield to the Substitution Rule.

Wednesday 29 April (Week 15)

- Computer HW this week: $\oint 5.5$
- Quiz #15 due Thurs
- FINAL! Monday 4 May 6-8p OZAR 026.
  - 2 hours, 20 questions.
  - Study the Coordinator's Review Questions.
  - Study the slides and your class notes BEFORE visiting outside resources.
  - CEA: Email me if you wish to take the exam in SCEN 407. Exam starts at 4p. Reduced distractions in SCEN are not guaranteed.

- Dead Day review SCEN 101 3-5p bring your answers to the Coordinator's Review Questions for feedback
- Thurs 30 April – don't skip drill
- your top 10 quizzes go into your final grade
- BONUS REVIEW see website – worth up to 2% of final grade. Bring to the Final Exam to turn in.

Exercise

Evaluate the following integrals. Check your work by differentiating each of your answers.

- $\displaystyle\int \sin^{10} x \cos x \; dx$
- $\displaystyle -\int \frac{\csc x \cot x}{1 + \csc x} \; dx$
- $\displaystyle\int \frac{1}{(10x-3)^2} \; dx$
- $\displaystyle\int (3x^2 + 8x + 5)^8 (3x+4) \; dx$

Variations on Substitution Rule

There are times when the u-substitution is not obvious or that more work must be done.

Example

Evaluate $\displaystyle\int \frac{x^2}{(x+1)^4}\,dx.$

Solution: Let $u = x + 1$. Then $x = u - 1$ and $du = dx$. Hence,

$$\int \frac{x^2}{(x+1)^4}\,dx = \int \frac{(u-1)^2}{u^4}\,du$$
$$= \int \frac{u^2 - 2u + 1}{u^4}\,du$$

$$
\begin{aligned}
&= \int \left(u^{-2} - 2u^{-3} + u^{-4}\right)\ du \\
&= \frac{-1}{u} + \frac{1}{u^2} + \frac{-1}{3u^3} + C \\
&= \frac{-1}{x+1} + \frac{1}{(x+1)^2} - \frac{1}{3(x+1)^3} + C
\end{aligned}
$$

Exercise

Check it.

This type of strategy works, usually, on problems where you can write u as a linear function of x.

Substitution Rule for Definite Integrals

We can use the Substitution Rule for Definite Integrals in two different ways:

1. Use the Substitution Rule to find an antiderivative F, and then use the Fundamental Theorem of Calculus to evaluate $F(b) - F(a)$.
2. Alternatively, once you have changed variables from x to u, you may also change the limits of integration and complete the integration with respect to u. Specifically, if $u = g(x)$, the lower limit $x = a$ is replaced by $u = g(a)$ and the upper limit $x = b$ is replaced by $u = g(b)$.

The second option is typically more efficient and should be used whenever possible.

Example

Evaluate $\displaystyle\int_0^4 \frac{x}{\sqrt{9+x^2}}\, dx.$

Solution: Let $u = 9 + x^2$. Then $du = 2x\ dx$. Because we have changed the variable of integration from x to u, the limits of integration must also be expressed in terms of u. Recall, u is a function of x (the $g(x)$ in the Chain Rule). For this example,

$$\begin{aligned} x = 0 \quad &\Longrightarrow \quad u(0) = 9 + 0^2 = 9 \\ x = 4 \quad &\Longrightarrow \quad u(4) = 9 + 4^2 = 25 \end{aligned}$$

We had $u = 9 + x^2$ and $du = 2x\ dx \implies \frac{1}{2}du = x\ dx$. So:

$$\begin{aligned}\int_0^4 \frac{x}{\sqrt{9+x^2}}\,dx &= \frac{1}{2}\int_9^{25}\frac{du}{\sqrt{u}}\\ &= \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_9^{25}\\ &= \sqrt{25}-\sqrt{9}\\ &= 5-3=2.\end{aligned}$$

Exercise

Evaluate $\displaystyle\int_0^2 \frac{2x}{(x^2+1)^2}\, dx.$

HW from Section 5.5

Do problems 9–39 odd, 53–63 odd (pp. 363–364 in textbook)

Advice for the FINAL

- Review your notes and the slides first, particularly problems we did in class, then review Quizzes, before visiting outside resources.
- Review the Midterm for an idea of questions the coordinator likes to ask and how they are graded.
- $+C$s, dxs, $\lim$, units, etc. should be included in your answers *or else*. Don't try to round answers unless it is for a story problem, in which case, you should say "approximately".
- "Definition of Derivative" $=$ the definition with limits

- Practice limits and l'Hôpital's Rule so you know which is the quickest technique.
- "Mean Value Theorem for Derivatives" $=$ MVT from $\oint$4.6.
- $\arctan = \tan^{-1}$, etc.
- Use the Continuity Checklist for questions about continuity.
- Use limits for questions about vertical asyptotes and end behavior.
- Know the difference between 1st and 2nd Derivative Tests.

Easter Egg-xercises

Exercise

- Find the 101st derivative of $y = \cos 7x$ at $x = 0$.
- For what values of the constants a and b is $(-1, 2)$ a point of inflection on the curve $y = ax^3 + bx^2 - 8x + 2$?