Quiz 9: L'Hôpital's Rule and Antiderivatives **MATH 2554 (Calculus I)**
§4.7, 4.9 **due Tues 19 Apr 2016**

Directions: This quiz is due on Tuesday, 19 April, 2016 at the beginning of your drill. You may use your brain, notes, book, or other humans to complete your work. **Your solutions must be on a separate sheet of paper, in order, stapled, de-fringed, and legible with your name on the top right corner of the first page.** If you fail to meet any of these requirements, you will receive a zero. Each question is worth one point, and will be graded as correct or not correct (all or nothing).

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1. **(1 pt ea)** Compute the following limits:

   (a) $\lim_{\theta\to 0}(\csc\theta - \cot\theta)$

   (b) $\lim_{u\to 1}\frac{u^{10}-1}{12u-12}$

2. The compound interest formula is $A(t) = A_0\left(1+\frac{r}{n}\right)^{nt}$, where

$$\begin{aligned} A_0 &= \text{ the initial dollar amount, called the } \textbf{principal}, \\ r &= \text{ the annual, or } \textbf{nominal}, \text{ interest rate (expressed as a decimal)}, \\ n &= \text{ the number of times interest is compounded per year, and} \\ t &= \text{ years.} \end{aligned}$$

   (a) In practice, interest is typically compounded yearly ($n = 1$), half-yearly ($n = 2$), quarterly ($n = 4$), monthly ($n = 12$), weekly ($n = 52$), or daily ($n = 365$). In theory, n can become arbitrarily large.

   (1 pt) How does the compound interest forumla change when $n \to \infty$?

   (b) Any annual interest rate r with compound frequency n can be expressed as a continuously compounded interest rate $r_0 = n\ln\left(1+\frac{r}{n}\right)$.

   (1 pt) Which credit card contract is better for the credit card company: 12% annual interest rate compounded monthly, or 13% annual interest rate compounded yearly?

3. **(1 pt)** Sometimes L'Hôpital's Rule just won't work. Try it with the following limit:

$$\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}}$$

   What should the limit be (and why)?

4. **(1 pt)** Exponential functions will always grow faster than power functions. To see why, find $\lim\limits_{x\to\infty}\dfrac{b^x}{x^n}$, assuming $b > 0$.

   Hint: Since b and n are not given, compute the limit for a few small cases first, e.g., $n = 2, 3, 4$, and then find the pattern.

5. Here you will derive the position function $s(t)$ for an object tossed into the air. Recall that the velocity function is $v(t) = s'(t)$ and the acceleration function is $a(t) = s''(t)$. Assume "up" is the positive direction. Acceleration is due to gravity, g (e.g., -9.8 m/s^2 or -16 ft/s^2). The initial velocity of the object is written $v(0) = v_0$ and the height from which the object is tossed is $s(0) = h$.

   (a) **(1 pt)** To get the position function, solve the initial value problem: Find $s(t)$, given $s''(t) = g$, $s'(0) = v_0$, and $s(0) = h$.

   (b) **(1 pt)** If we include air resistance, the formula gets more complicated. The velocity function in this case is

$$v(t) = \frac{-mg}{\beta} + \left(\frac{mg}{\beta} + v_0\right) e^{\frac{-\beta}{m}t},$$

   where m is the mass of the object and β is a constant called the **drag coefficient**. Using $s(t) = \int v(t)\, dt$ and $s(0) = h$, rewrite the position function with air resistance.

6. The constant of integration C matters. Let $f(x) = 2x$.

   (a) **(1 pt)** Evaluate $\int f(x)\, dx$.

   (b) **(1 pt)** Let F denote your answer to (a). Solve each of the initial value problems

   - $F(0) = 8$
   - $F(1) = 1$

   and draw the graphs for both on the same axes.