

Math 2554 Exam 2
Friday 27 February 2015

Name: _____

SOLUTIONS

Calculus I

Exam 2: Derivatives

Please provide the following data:

Drill Instructor: _____

Drill Time: _____

Student ID or clicker #: _____

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard, one side only, is allowed. No graphing calculators. No programmable calculators. No electronic devices except for the approved calculators (so no phones, iDevices, computers, etc). If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt)

Good luck!

1. (3 pts each) For each function $g(x)$, find the value of $g'(3)$ using the data given below.

$$\begin{array}{llll} f(1) = 6 & f(3) = 2 & f(6) = 5 & f(9) = -3 \\ f'(1) = -2 & f'(3) = 4 & f'(6) = -1 & f'(9) = 1 \end{array}$$

(a) $g(x) = f(3) + 10f(2x)$

$$g'(x) = 0 + 10f'(2x)(2)$$

$$g'(3) = 10f'(2(3))(2)$$

$$= 20(-1) = -20$$

(b) $g(x) = \frac{f(x)}{x}$

$$g'(x) = \frac{x f'(x) - f(x)(1)}{x^2}$$

$$g'(3) = \frac{3(4) - 2(1)}{3^2} = \frac{10}{9}$$

(c) $g(x) = (f(x))^3$

$$g'(x) = 3f(x)^2 f'(x)$$

$$g'(3) = 3(2)^2(4) = 48$$

(d) **EXTRA CREDIT** $g(x) = f(x^2)$

$$g'(x) = 2x f'(x^2)$$

$$g'(3) = 2(3)(1)$$

$$= 6$$

2. (8 pts) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \left(\frac{3}{3}\right)$

$$= \lim_{x \rightarrow 0} 3 \frac{\sin 3x}{3x}$$

$$= 3 \lim_{y \rightarrow 0} \frac{\sin y}{y}$$

$$= 3(1)$$

$$= 3$$

Let $y = 3x$. As $x \rightarrow 0$,
 $y \rightarrow 0$.

3. (5 pts each) Suppose a company produces x items at a cost $C(x)$.

(a) Write the formula for average cost and say in words what it means.

$$\bar{C}(x) = \frac{C(x)}{x}$$

price per item to produce x items

(b) Write the formula for marginal cost and say in words what it means..

$$C'(x) = \lim_{\Delta x \rightarrow 0} \frac{C(x + \Delta x) - C(x)}{\Delta x}$$

approximate price to produce another
item having produced x already

4. (8 pts each) For each function find $\frac{d^2y}{dx^2}$.

(a) $x^4 + y^4 = 64$

$$\frac{d^2}{dx^2} (x^4 + y^4 = 64)$$

$$\frac{d}{dx} \left(4x^3 + 4y^3 \frac{dy}{dx} = 0 \right) \Rightarrow \frac{dy}{dx} = \frac{-4x^3}{4y^3} = -\frac{x^3}{y^3}$$

$$12x^2 + 12y^2 \frac{dy}{dx} \left(\frac{dy}{dx} \right) + 4y^3 \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-12x^2 - 12y^2 \left(\frac{dy}{dx} \right)^2}{4y^3} = \frac{-3x^2 - 3y^2 \left(-\frac{x^3}{y^3} \right)^2}{4y^3} = \frac{-3x^2y^4 + 3x^6}{4y^7}$$

(b) $e^{2y} + x = y$

$$\frac{d^2}{dx^2} (e^{2y} + x = y)$$

$$\frac{d}{dx} \left(2e^{2y} \frac{dy}{dx} + 1 = \frac{dy}{dx} \right) \Rightarrow \frac{dy}{dx} = \frac{1}{1 - 2e^{2y}}$$

$$4e^{2y} \frac{dy}{dx} \left(\frac{dy}{dx} \right) + 2e^{2y} \frac{d^2y}{dx^2} + 0 = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{4e^{2y} \left(\frac{dy}{dx} \right)^2}{1 - 2e^{2y}} = \frac{4e^{2y}}{(1 - 2e^{2y})^3}$$

5. (4 pts each) Let $F = f + g$ and $G = 3f - g$, where the graphs of f and g are shown in Figure 1.

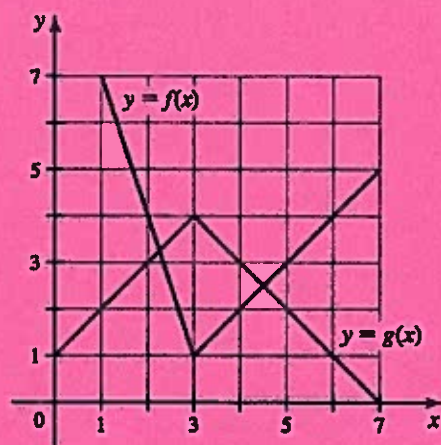


Figure 1: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

Find the following derivatives:

$$\begin{aligned} \text{(a) } F'(1) &= f'(1) + g'(1) \\ &= -3 + 1 \\ &= -2 \end{aligned}$$

$$\begin{aligned} \text{(b) } G'(1) &= 3f'(1) - g'(1) \\ &= 3(-3) - (1) \\ &= -10 \end{aligned}$$

$$\begin{aligned} \text{(c) } F'(5) &= f'(5) + g'(5) \\ &= 1 + (-1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{(d) } G'(5) &= 3f'(5) - g'(5) \\ &= 3(1) - (-1) \\ &= 4 \end{aligned}$$

6. (3 pts each) Find the derivative of each of the following functions:

$$(a) f(x) = \frac{(x-1)(2x^2-1)}{x^3-1} = \frac{(x-1)(2x^2-1)}{(x-1)(x^2+x+1)}$$

$$f'(x) = \frac{(x^2+x+1)(4x) - (2x^2-1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{4x^3 + 4x^2 + 4x - (4x^3 + 2x^2 - 2x - 1)}{(x^2+x+1)^2}$$

$$= \frac{2x^2 + 6x + 1}{(x^2+x+1)^2}$$

derivative is
not defined
when $x = 1$

$$(b) f(x) = \frac{x+1}{x^2 e^{2x}}$$

$$f'(x) = \frac{x^2 e^{2x}(1) - (x+1)(2x e^{2x} + x^2(2e^{2x}))}{(x^2 e^{2x})^2}$$

$$= \frac{x^2 e^{2x} - 2x^2 e^{2x} - 2x^3 e^{2x} - 2x e^{2x} - 2x^2 e^{2x}}{x^4 e^{4x}}$$

$$= \frac{x e^{2x} (x - 2x - 2x^2 - 2 - 2x)}{x^4 e^{4x}}$$

$$= \frac{-3x - 2x^2 - 2}{x^3 e^{2x}}$$

(c) $y = \sin x + \cos x$

$$y' = \cos x - \sin x$$

(d) $y = 5x^2 + \cos x$

$$y' = 10x - \sin x$$

(e) $y = \frac{(x^2 - 1)\sin x}{\sin x + 1}$

$$y' = \frac{(\sin x + 1) \left[2x \sin x + (x^2 - 1) \cos x \right] - (x^2 - 1) \sin x (\cos x)}{(\sin x + 1)^2}$$

$$= \frac{2x(\sin x)^2 + (x^2 - 1) \sin x \cos x + 2x \sin x + (x^2 - 1) \cos x - (x^2 - 1) \sin x \cos x}{(\sin x + 1)^2}$$

$$= \frac{2x(\sin x)^2 + 2x \sin x + (x^2 - 1) \cos x}{(\sin x + 1)^2}$$

The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters. The document outlines various methods for collecting and organizing data, including the use of spreadsheets and databases. It also highlights the need for regular audits and reviews to ensure the integrity of the information.

The second part of the document focuses on the analysis and interpretation of the collected data. It describes how statistical techniques can be applied to identify trends and patterns in the data. The document also discusses the importance of context in interpreting the results, noting that data should always be viewed in light of the specific circumstances and objectives of the study.

The third part of the document addresses the communication of findings. It stresses the importance of presenting the results in a clear and concise manner, using appropriate visual aids such as charts and graphs. The document also discusses the need for transparency in reporting, including the disclosure of any limitations or potential biases in the study.

Finally, the document concludes with a summary of the key points and a call to action. It encourages the reader to adopt the principles and practices outlined in the document to ensure the highest quality of their work. The document also provides a list of references for further reading and a glossary of key terms.