MATH 2574 (Calculus III)
due Wed 8 Feb 2017

Take-Home Quiz 2: Multivariable functions (§12.1-12.2)

Directions: This quiz is due on February 8, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. There is always more than one way to parametrize a given curve. However, there is one "correct" way, and that is by the curve's arc length.

   (a) Given a parametrized curve $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$, the formula for its length from $t = a$ to $t = b$ is given by
   $$s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\ dt.$$
   For each of the following parametrizations of the unit circle, find the arc length.
   i. $\mathbf{r}_1(t) = \langle \cos t, \sin t\rangle$, $0 \le t \le 2\pi$.
   ii. $\mathbf{r}_2(t) = \langle \cos 2t, \sin 2t\rangle$, $0 \le t \le \pi$.

   (b) In part (a), $\mathbf{r}_1$ gives the correct parametrization, in the sense that the length of the curve drawn is equal to the length of time (t) elapsed. We say that $\mathbf{r}_1$ is **parametrized by arc length**. In general, the arc length of a curve $\mathbf{r}(t)$ is a function of time $t \ge a$, given by
   $$s(t) = \int_a^t \sqrt{x'(u)^2 + y'(u)^2 + z'(u)^2}\ du.$$
   Find $s(t)$ for each of the parametrizations in part (a).

   (c) What is $\dfrac{ds}{dt}$? (Hint: Use the Fundamental Theorem of Calculus.)

   (d) Give a condition on $\mathbf{r}'(t)$ that guarantees $\mathbf{r}(t)$ is parametrized by arc length.

2. Consider the following plane and curve:
   $$y = 2x + 1$$
   $$\mathbf{r}(t) = \langle 10\cos t, 2\sin t, 1\rangle, \text{ for } 0 \le t \le 2\pi$$
   Find the point(s) where the plane and curve intersect, if any exist.

3. Find an equation of the plane passing through $(0, -2, 4)$ that is orthogonal to the planes $2x+5y-3z = 0$ and $-x + 5y + 2z = 8$.

4. For each of the following quadric surfaces,
   - Name the surface (see Table 12.1 in the text).
   - Find the intercepts with the three coordinate axes, when they exist.
   - Find the equations of the xy-, zx-, and yz-traces, when they exist.
   - Sketch a graph of the surface.

   (a) $4y^2 + z^2 = x^2$
   (b) $y = \dfrac{x^2}{16} - 4z^2$
   (c) $z = \dfrac{x^2}{4} + \dfrac{y^2}{9}$

5. Suppose you make a one-time deposit of P dollars into a savings account that earns interest at an annual rate of $p\%$ compounded continuously. The balance in the account after t years is

$$B(P, r, t) = Pe^{rt}, \text{ where } r = \frac{p}{100}$$

(so for example, if the annual interest rate is 4%, then $r = 0.04$). Let the interest rate be fixed at $r = 0.04$.

(a) Find the set of all points (P, t) that satisfy $B = 2000$. This curve gives all deposits P and times t that result in a balance of \$2000.

(b) Repeat part (a) with $B = 500, 1000, 1500, 2500$. Draw the resulting level curves of the balance function.

(c) In general, on one level curve, if t increases, then does P increase or decrease?