Take-Home Quiz 8: Optimization and integrals (§4.7, 4.9, 5.1-5.5)

Directions: This quiz is due on December 5, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. **§4.7 #56** The equation of a line with slope m passing through the point $(3, 5)$ is given by

$$y - 5 = m(x - 3).$$

   If m is negative, then that line will cut out a triangle in the first quadrant. Find the value of m that minimizes the area of the triangle.

2. **§4.9 #68** Prof Wheeler tosses a red ball from the edge of a 432 ft cliff at an upward speed of 48 ft/s. One second later, she tosses a blue ball off the edge of the cliff at an upward speed of 24 ft/s.

   (a) Recall, the acceleration force due to gravity is 32 ft/s^2. Use it to find the height, $h_{\text{red}}(t)$ of the red ball at time t seconds (assume Wheeler tosses the ball at $t = 0$). *Hint: This is done in Example 7.*

   (b) Find the height $h_{\text{blue}}(t)$ of the blue ball, t seconds after Wheeler has tossed the first ball. In other words, assume Wheeler tosses the blue ball at $t = 1$.

   (c) Do the balls ever pass eachother? If so, at what time?

   (d) Which ball hits the ground first?

   (e) Sketch or print a graph of h_{red} and h_{blue} on the same axes.

3. **§5.1 #26** *How well can you estimate the area under the curve $f(x) = x^3$ from 0 to 1, using five approximation rectangles?*

   Before calculus was invented, people estimated the area under a curve by drawing rectangles to get an approximate shape, and then adding up the areas of the rectangles. The more rectangles, the better the approximation.

   (a) Sketch or print the graph of $f(x)$ illustrating your choice of how to draw the five rectangles. *Hint: For a better estimate, your rectangles don't all have to be the same size!*

   (b) In the following table, k enumerates the rectangles you've drawn. For example, x_2 and x_3 are the endpoints of Rectangle No. 3. The width of Rectangle No. 3 is $\Delta x_3 = x_3 - x_2$. The number you choose to plug into $f(x)$ for the height of Rectangle No. 3 is x_3^*, and must be between x_2 and x_3.

   Fill in the table to correspond to your choice of rectangles.

k	x_k	x_k^*	$f(x_k^*)$	Δx_k^*
0	0	-		
1				
2				
3				
4				
5	1			

   (c) The area is $\approx \displaystyle\sum_{k=1}^{5} f(x_k^*)\Delta x_k^* = \,?$

4. **§5.5 #62** Consider the integral $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\ dx$.

(a) This integral requires u-substitution to solve. Find u and du.

(b) Rewrite the integral in terms of u, <u>including the bounds</u>.

(c) Sketch or print a graph of the integrands of both integrals on the same axes. Under each curve, shade the area the integrals compute.

(d) Compute the integral you wrote from part (b). Use a calculator or online integrator (`desmos` will do it, for example) to check your answer equals the original integral.