Math 2554 Exam 2
Friday 27 February 2015

Name: SOLUTIONS

Calculus I
Exam 2: Derivatives

Please provide the following data:

Drill Instructor: ______________________________

Drill Time: ______________________________

Student ID or clicker #: ______________________________

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard, one side only, is allowed. No graphing calculators. No programmable calculators. No electronic devices except for the approved calculators (so no phones, iDevices, computers, etc). If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: **(1 pt)**

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Good luck!

1. **(3 pts each)** For each function $g(x)$, find the value of $g'(3)$ using the data given below.

$$f(1) = 6 \qquad f(3) = 2 \qquad f(6) = 5 \qquad f(9) = -3$$
$$f'(1) = -2 \qquad f'(3) = 4 \qquad f'(6) = -1 \qquad f'(9) = 1$$

(a) $g(x) = f(3) + 10f(2x)$

$g'(x) = 0 + 10f'(2x)(2)$

$g'(3) = 10f'(2(3))(2)$

$= 20(-1) = -20$

(b) $g(x) = \dfrac{f(x)}{x}$

$g'(x) = \dfrac{xf'(x) - f(x)(1)}{x^2}$

$g'(3) = \dfrac{3(4) - 2(1)}{3^2} = \dfrac{10}{9}$

(c) $g(x) = (f(x))^3$

$g'(x) = 3f(x)^2 f'(x)$

$g'(3) = 3(2)^2(4) = 48$

(d) **EXTRA CREDIT** $g(x) = f(x^2)$

$g'(x) = 2x f'(x^2)$

$g'(3) = 2(3)(1)$

$= 6$

2. **(8 pts)** Evaluate $\lim_{x\to 0} \frac{\sin 3x}{x}$. $\left(\frac{3}{3}\right)$

$$= \lim_{x\to 0} \frac{3\sin 3x}{3x}$$

Let $y = 3x$. As $x \to 0$, $y \to 0$.

$$= 3\lim_{y\to 0} \frac{\sin y}{y}$$

$$= 3(1)$$

$$= 3$$

3. **(5 pts each)** Suppose a company produces x items at a cost $C(x)$.

(a) Write the formula for **average cost** and say in words what it means.

$$\bar{C}(x) = \frac{C(x)}{x}$$

price per item to produce x items

(b) Write the formula for **marginal cost** and say in words what it means..

$$C'(x) = \lim_{\Delta x\to 0} \frac{C(x+\Delta x) - C(x)}{\Delta x}$$

approximate price to produce another item having produced x already

4. **(8 pts each)** For each function find $\frac{d^2y}{dx^2}$.

(a) $x^4 + y^4 = 64$

$$\frac{d^2}{dx^2}\left(x^4 + y^4 = 64\right)$$

$$\frac{d}{dx}\left(4x^3 + 4y^3\frac{dy}{dx} = 0\right) \Longrightarrow \frac{dy}{dx} = \frac{-4x^3}{4y^3} = \frac{-x^3}{y^3}$$

$$12x^2 + 12y^2\frac{dy}{dx}\left(\frac{dy}{dx}\right) + 4y^3\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-12x^2 - 12y^2\left(\frac{dy}{dx}\right)^2}{4y^3} = \frac{-3x^2 - 3y^2\left(\frac{-x^3}{y^3}\right)^2}{4y^3}\,\,{}_4 = \frac{-3x^2y^4 + 3x^6}{4y^7}$$

(b) $e^{2y} + x = y$

$$\frac{d^2}{dx^2}\left(e^{2y} + x = y\right)$$

$$\frac{d}{dx}\left(2e^{2y}\frac{dy}{dx} + 1 = \frac{dy}{dx}\right) \Longrightarrow \frac{dy}{dx} = \frac{1}{1-2e^{2y}}$$

$$4e^{2y}\frac{dy}{dx}\left(\frac{dy}{dx}\right) + 2e^{2y}\frac{d^2y}{dx^2} + 0 = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{4e^{2y}\left(\frac{dy}{dx}\right)^2}{1-2e^{2y}} = \frac{4e^{2y}}{(1-2e^{2y})^3}$$

5. **(4 pts each)** Let $F = f + g$ and $G = 3f - g$, where the graphs of f and g are shown in Figure 1.

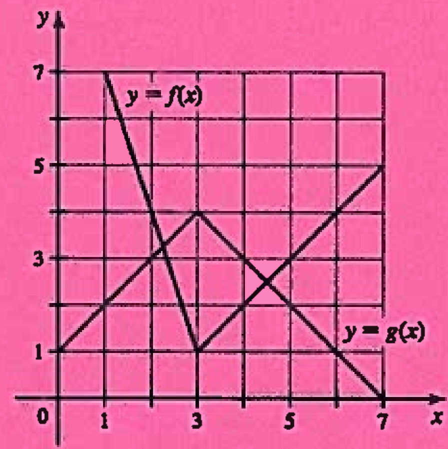

Figure 1: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

Find the following derivatives:

(a) $F'(1) = f'(1) + g'(1)$

$= -3 + 1$

$= -2$

(b) $G'(1) = 3f'(1) - g'(1)$

$= 3(-3) - (1)$

$= -10$

(c) $F'(5) = f'(5) + g'(5)$

$= 1 + (-1)$

$= 0$

(d) $G'(5) = 3f'(5) - g'(5)$

$= 3(1) - (-1)$

$= 4$

6. **(3 pts each)** Find the derivative of each of the following functions:

(a) $f(x) = \dfrac{(x-1)(2x^2-1)}{x^3-1}$ $= \dfrac{\cancel{(x-1)}(2x^2-1)}{\cancel{(x-1)}(x^2+x+1)}$

$$f'(x) = \frac{(x^2+x+1)(4x) - (2x^2-1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{4x^3+4x^2+4x - (4x^3+2x^2-2x-1)}{(x^2+x+1)^2}$$

$$= \frac{2x^2+6x+1}{(x^2+x+1)^2}$$

derivative is not defined when $x=1$

(b) $f(x) = \dfrac{x+1}{x^2e^{2x}}$

$$f'(x) = \frac{x^2e^{2x}(1) - (x+1)(2xe^{2x} + x^2(2e^{2x}))}{(x^2e^{2x})^2}$$

$$= \frac{x^2e^{2x} - 2x^2e^{2x} - 2x^3e^{2x} - 2xe^{2x} - 2x^2e^{2x}}{x^4e^{4x}}$$

$$= \frac{xe^{2x}(x - 2x - 2x^2 - 2 - 2x)}{x^4e^{4x}}$$

$$= \frac{-3x-2x^2-2}{x^3e^{2x}}$$

(c) $y = \sin x + \cos x$

$$y' = \cos x - \sin x$$

(d) $y = 5x^2 + \cos x$

$$y' = 10x - \sin x$$

(e) $y = \dfrac{(x^2-1)\sin x}{\sin x + 1}$

$$y' = \frac{(\sin x + 1)\left[2x\sin x + (x^2-1)(\cos x)\right] - (x^2-1)\sin x(\cos x)}{(\sin x + 1)^2}$$

$$= \frac{2x(\sin x)^2 + (x^2-1)\sin x\cos x + 2x\sin x + (x^2-1)\cos x - (x^2-1)\sin x\cos x}{(\sin x + 1)^2}$$

$$= \frac{2x(\sin x)^2 + 2x\sin x + (x^2-1)\cos x}{(\sin x + 1)^2}$$