Cal III Quiz

Name: Solutions
drill time: _____

1. Given the force field $F(x,y,z) = \langle x, y, z \rangle$, find the work required to move an object along the line segment from $(1,1,1)$ to $(1,2,4)$.

Parametrize the line segment

$\vec{r}(t) = \langle 1, 1+t, 1+3t \rangle = \vec{F} = \langle x(t), y(t), z(t) \rangle$

$0 \le t \le 1$

$\Rightarrow \vec{r}\,'(t) = \langle 0, 1, 3 \rangle$

$$\text{Work} = \int_{\text{line segment}} \vec{F} \cdot \vec{r}\,'(t)\,dt = \int_0^1 [1(0) + (1+t)(1) + (1+3t)(3)]\,dt$$

$1 + t + 3 + 9t$

$$= \int_0^1 (4 + 10t)\,dt = 4t + \frac{10t^2}{2}\Big|_0^1$$

$= 4 + 5 = \boxed{9}$

0 term vanis

2. Find the scalar integral $\int_C x\,ds$ where C is the curve $y = x^2$ from $(0,0)$ to $(2,4)$.

Write $\vec{r}(t) = \langle t, t^2 \rangle$ $0 \le t \le 2$; $\vec{r}\,'(t) = \langle 1, 2t \rangle$

$|\vec{r}\,'(t)| = \sqrt{1 + (2t)^2} = \sqrt{1 + 4t^2}$

$$\Rightarrow \int_C x\,ds = \int_0^2 t\,|\vec{r}\,'(t)|\,dt$$

$$= \int_0^2 t\sqrt{1+4t^2}\,dt$$

Put $u = 1 + 4t^2$

$du = 8t\,dt$

$\rightarrow \frac{1}{8}du = t\,dt$

$u(2) = 1 + 4(2)^2 = 17$

$u(0) = 1 + 4(0)^2 = 1$

$$= \frac{1}{8}\int_1^{17} \sqrt{u}\,du = \frac{1}{8}\left(\frac{2}{3}u^{3/2}\Big|_1^{17}\right)$$

$$= \boxed{\frac{1}{12}\left(17\sqrt{17} - 1\right)}$$