Math 2603 Final
due Wed 17 Dec 2014

Name: ______________________________

Discrete Math Final

Please provide the following data:

Drill Time: ______________________________

Student ID: ______________________________

Exam Instructions:

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: ____

Good luck!

1. (a) Prove or disprove: for all sets A, B, C,

$$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B.$$

(b) Prove: For any integer n, if n is even then n^2 is even.

2. Prove or disprove: For all nonnegative integers n and r with $r + 1 \leq n$,

$$\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}.$$

3. Prove the following statements:

   (a) The product of any nonzero rational number and any irrational number is irrational.

   (b) Use the previous result and the fact that $\sqrt{3}$ is irrational to show that $\sqrt{75}$ is irrational.

4. Use induction to prove $6 \cdot 7^n - 2 \cdot 3^n$ is divisible by 4, for all $n \geq 1$.

5. Let I denote the following relation on $\mathbb{R}$:

$$\text{For all } x, y \in \mathbb{R},\ x\mathrm{I}y \text{ if and only if } x - y \text{ is an integer.}$$

Prove or disprove:

(a) I is reflexive.

(b) I is symmetric.

(c) I is antisymmetric.

(d) I is transitive.

(e) I is an equivalence relation.

(f) I is a partial order.

6. Let P, Q, R denote propositions. Prove or disprove:

$$(P \to Q) \to R \equiv P \vee (R \to Q).$$

7. Prove that for all integers $n \geq 1$,

$$\sum_{k=1}^{3n} (4k+3) = 3n(6n+5).$$

8. Suppose the midterm consists of 12 distinct, prewritten questions, where five are easy, three are hard, and four are medium.

   (a) How many ways can the questions be ordered so that questions 1 and 2 are both hard?

   (b) How many ways can the questions be ordered so that all the easy ones are first, then the medium ones, then the hard ones?

   (c) How many ways can the questions be ordered so that none of the first 5 questions are hard?

   (d) How many ways can the questions be ordered so that no two consecutive questions are easy?

9. (a) Draw an example of a connected graph of order 6 that has both an Eulerian cycle that is not Hamiltonian, and a Hamiltonian cycle that is not Eulerian.

(b) Suppose S, T are sets with $|S| = 10$ and $|T| = 8$. Which is larger, $|\mathcal{P}(S \times T)|$ or $|S \times \mathcal{P}(T)|$ and why?

10. For each of the following functions the domain is $\mathbb{Z} \times \mathbb{Z}$ and the codomain is $\mathbb{Z}$. Determine whether each function is one-to-one, onto, or both. You must prove your answers.

    (a) $f(m, n) = m - n$

    (b) $f(m, n) = mn$

    (c) $f(m, n) = n^2 + 1$