Math 2603 Exam 1 **Name:** _____
Wed 24 Sep 2014

Discrete Math
Exam 1 (Ch. 1-2: Set theory, logic, proofs)

Please provide the following data:

Drill Time: _____

Student ID: _____

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard is allowed. No graphing calculators. No programmable calculators. No phones, iDevices, computers, etc. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: _____

Good luck!

1. Truth tables and valid arguments.

   (a) Fill in the truth table:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \rightarrow (\neg R)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

   (b) Using table (a) explain if the argument below is valid.

$$\begin{array}{l} P \wedge Q \\ Q \\ \hline \therefore (P \wedge Q) \rightarrow (\neg R) \end{array}$$

   (c) Determine if the following argument is valid. You may extend the table in (a) if necessary, but either way, you must justify your answer.

$$\begin{array}{l} P \wedge Q \rightarrow \neg R \\ P \vee \neg Q \\ \neg Q \rightarrow P \\ \hline \therefore R \end{array}$$

2. Let $P =$ "If $\dfrac{6}{d} \in \mathbb{Z}$, where $d \in \mathbb{Z}$, then $d = 3$."

   (a) Is P a proposition?

   (b) What is $\neg P$?

   (c) Give the contrapositive of P.

   (d) Give the converse of P.

   (e) Is P true or false? (Prove or give a counterexample.)

3. For a set S let $\mathcal{P}(S)$ denote its power set.

   (a) Prove $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$.

   (b) Disprove $\mathcal{P}(X \cup Y) \subseteq \mathcal{P}(X) \cup \mathcal{P}(Y)$.

4. Prove by contrapositive: If $x^2 \in \mathbb{R} \setminus \mathbb{Q}$, then $x \in \mathbb{R} \setminus \mathbb{Q}$.

5. True/False. If true, use induction to prove it. If false, give a counterexample.

   (a) $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = \dfrac{(-1)^{n+1} n(n+1)}{2}$

(b) For all $n \in \mathbb{Z}_{>0}$, $11^n - 6$ is divisible by 5.

6. **cHaLlEnGe PrObLeM** Prove: Given any two rational numbers r and s with $r < s$, there is a rational number between r and s.

7. **EXTRA CREDIT** Use the quotient-remainder theorem, with $d = 3$, to prove that the product of any three consecutive integers is divisible by 3.