MATH 2554 (Calculus I)

Dr. Ashley K. Wheeler

University of Arkansas

February 4, 2015

Table of Contents

Monday 2 February (Week 4)

- EXAM #1: Friday 6 February
  - in class
  - covers up to and including $\oint 3.1$

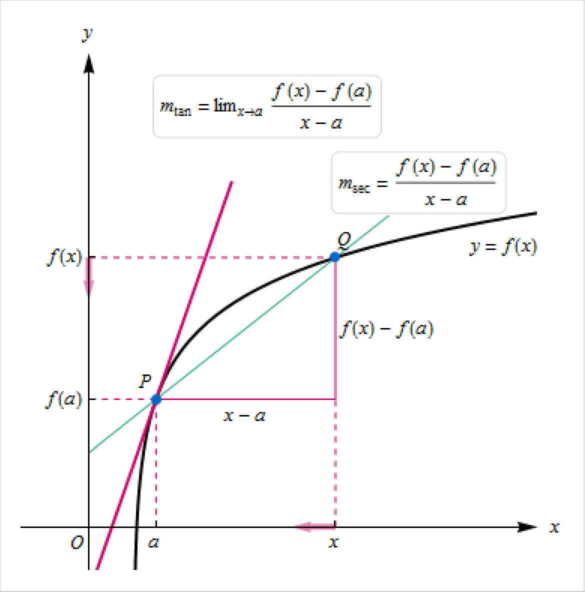
∮ 3.1 Introducing the Derivative

Recall from Ch 2: We said that the slope of the tangent line at a point is the limit of the slopes of the secant lines as the points get closer and closer.

- slope of secant line: $\dfrac{f(x)-f(a)}{x-a}$ (avg. rate of change)
- slope of tangent line: $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ (instantaneous rate of change)

Change location
of point Q

x



Example

Use the relationship between secant lines and tangent lines, specifically the slope of the tangent line, to find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point $P = (1, 5)$.

In the preceding example, we considered two points

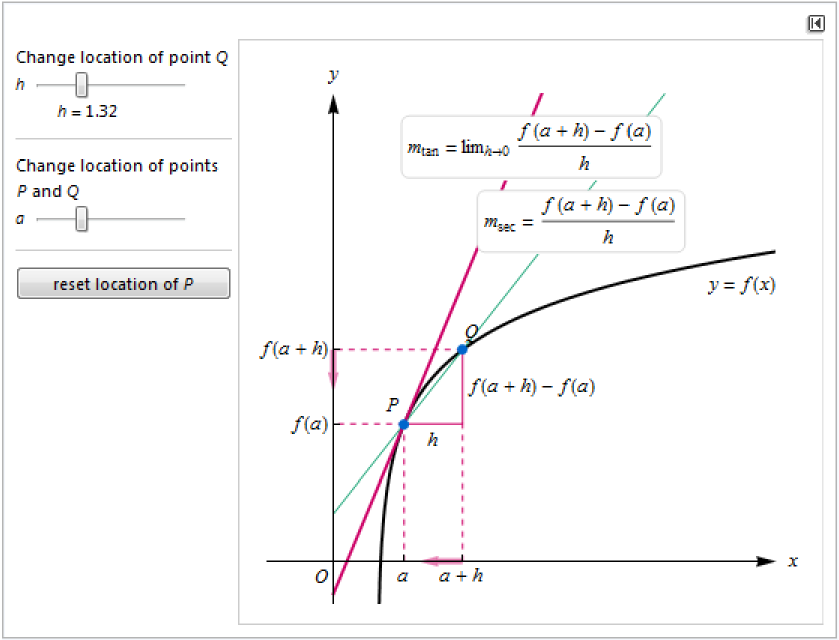
$$P = (a, f(a)) \quad \text{and} \quad Q = (x, f(x))$$

that were getting closer and closer together.

Instead of looking at the points approaching one another, we can also view this as the distance h between the points approaching 0. For

$$P = (a, f(a)) \quad \text{and} \quad Q = (a+h, f(a+h)),$$

- slope of secant line: $\dfrac{f(a+h)-f(a)}{(a+h)-a} = \dfrac{f(a+h)-f(a)}{h}$
- slope of tangent line: $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$


Change location of point Q
h
h = 1.32
Change location of points
P and Q
a
reset location of P
y
x
O
$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
$m_{\sec} = \frac{f(a+h) - f(a)}{h}$
y = f(x)
Q
P
f(a + h)
f(a)
f(a + h) − f(a)
h
a
a + h

Example

Find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point $P = (2, 10)$.

Derivative Defined as a Function

The slope of the tangent line for the function f is a function of x, called the derivative of f.

Definition

The *derivative* of f is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. If $f'(x)$ exists, we say f is *differentiable* at x. If f is differentiable at every point of an open interval I, we say that f is differentiable on I.

Use the definition of the derivative to find the derivative of the function $f(x) = x^2 + 2x + 2$.

Leibniz Notation

A standard notation for change involves the Greek letter Δ.

$$\frac{f(a+h)-f(a)}{h} = \frac{f(x+\Delta x)}{\Delta x} = \frac{\Delta y}{\Delta x}.$$

Apply the limit:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Other Notation

The following are alternative ways of writing $f'(x)$ (i.e., the derivative as a function of x):

$$\frac{dy}{dx} \qquad \frac{df}{dx} \qquad \frac{d}{dx}\left(f(x)\right) \qquad D_x(f(x)) \qquad y'(x)$$

The following are ways to notate the derivative of f evaluated at $x = a$:

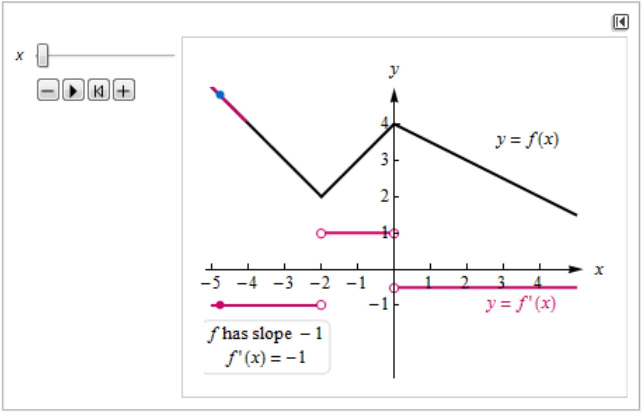
$$f'(a) \qquad y'(a) \qquad \left.\frac{df}{dx}\right|_{x=a} \qquad \left.\frac{dy}{dx}\right|_{x=a}$$

Graphing the Derivative

The graph of the derivative is the graph of the collection of slopes of tangent lines of a graph. If you just have a graph (without an equation for the graph), the best you can do is approximate the graph of the derivative.

Simple checklist:

1. Note where $f'(x) = 0$.
2. Note where $f'(x) > 0$. (What does this look like?)
3. Note where $f'(x) < 0$. (What does this look like?)



Differentiability vs. Continuity

Key points about the relationship between differentiability and continuity:

- If f is differentiable at a, then f is continuous at a.
- If f is not continuous at a, then f is not differentiable at a.
- f can be continuous at a, but not differentiable at a.

A function f is not differentiable at a if at least one of the following conditions holds:

1. f is not continuous at a.
2. f has a corner at a. (Why does this make f not differentiable?)
3. f has a vertical tangent at a. (Why does this make f not differentiable?)

HW from Section 3.1

Do problems 11–12, 19–20, 23–26, 31–33, 35–36, 39–43, 45, 49–52 (pp. 131–133 in textbook)

NOTE: You do not know any rules for differentiation yet (e.g., Power Rule, Chain Rule, etc.) In this section, you are strictly using the definition of the derivative and the definition of slope of tangent lines we have derived.

Wednesday 4 February (Week 4)

- No quiz this week.
- 2 quizzes next week:
    - Tues 10 Feb in-drill group quiz (Quiz 4)
    - Thurs 12 Feb usual weekly take-home quiz (Quiz 5)
- EXAM #1: Friday 6 February
    - in class
    - covers up to and including $\oint 3.1$
    - one 3×5 inch notecard, one-sided only, allowed
    - only the approved calculator allowed (see syllabus)

Exam #1 Review

- $\oint 2.1$ The Idea of Limits
  - Understand the relationship between average velocity & instantaneous velocity, and secant and tangent lines
  - Be able to compute slopes of secant lines and use the idea of a limit to approximate the slope of the tangent line
- $\oint 2.2$ Definitions of Limits
  - Know the definition of a limit
  - Know the relationship between one- and two-sided limits

- $\oint 2.3$ Techniques for Computing Limits
  - Know and be able to compute limits using analytical methods (e.g., limit laws, additional techniques)
  - Know the Squeeze Theorem and be able to use it to determine limits

Example

Evaluate $\displaystyle\lim_{x\to 0} x \sin \frac{1}{x}$.

- $\oint 2.4$ Infinite Limits
  - Know the definition of a vertical asymptote and be able to determine whether a function has vertical asymptotes
- $\oint 2.5$ Limits at Infinity
  - Be able to find limits at infinity and horizontal asymptotes
  - Know how to compute the limits at infinity of rational functions

Example

Determine the end behavior of $f(x)$. If there is a horizontal asymptote, then say so. Next, identify any vertical asymptotes. If $x = a$ is a vertical asymptote, then evaluate $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$.

$$f(x) = \frac{2x^3 + 10x^2 + 12x}{x^3 + 2x^2}$$

- $\oint 2.6$ Continuity
  - Know the definition of continuity and be able to apply the continuity checklist
  - Be able to determine the continuity of a function (including those with roots) on an interval
  - Be able to apply the Intermediate Value Theorem to a function

Example

Determine the value for a that will make $f(x)$ continuous.

$$f(x) = \begin{cases} \frac{x^2+3x+2}{x+1} & x \neq -1 \\ a & x = -1 \end{cases}$$

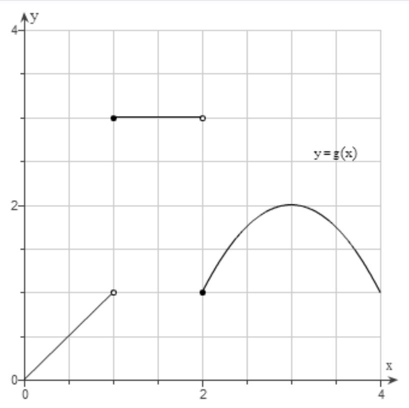
Example

Show that $f(x) = 2$ has a solution on the interval $(-1, 1)$, with

$$f(x) = 2x^3 + x.$$

- $\oint 2.7$ Precise Definition of Limits
  - Understand the δ, ϵ relationship for limits
  - Be able to use a graph or analytical methods to find a value for $\delta > 0$ given an $\epsilon > 0$ (including finding symmetric intervals)

In this example, the two-sided limits at $x = 1$ and $x = 2$ do not exist.



Example

Use the graph to find the appropriate δ.

(a) $|g(x) - 2| < \frac{1}{2}$ whenever $0 < |x - 3| < \delta$

(b) $|g(x) - 1| < \frac{3}{2}$ whenever $0 < |x - 2| < \delta$

- $\oint 3.1$ Introducing the Derivative
  - Know the definition of a derivative and be able to use this definition to calculate the derivative of a given function
  - Be able to determine the equation of a line tangent to the graph of a function at a given point
  - Know the 3 conditions for when a function is not differentiable at a point, and why these three conditions make a function not differentiable at the given point

Example

(a) Use the limit definition of the derivative to find an equation for the line tangent to $f(x)$ at a, where

$$f(x) = \frac{1}{x}; \qquad a = -5.$$

(b) Using the same $f(x)$ from part (a), find a formula for $f'(x)$ (using the limit definition).

(c) Plug -5 into your answer for (b) and make sure it matches your answer for (a).

Other Study Tips

- Brush up on algebra, especially radicals.
- If your answer is something like $\sqrt{2}$, don't plug that into your calculator, just leave it as is.
- When in doubt, show steps. See the document camera notes to get an idea of what's expected.
- You will be punished for wrong notation; e.g., the limit symbol.
- Read the question! Several students always lose points because they didn't answer the question or they didn't follow directions.
- Do the book problems.
- Look at the pictures in the book and the interactive applets on MLP.