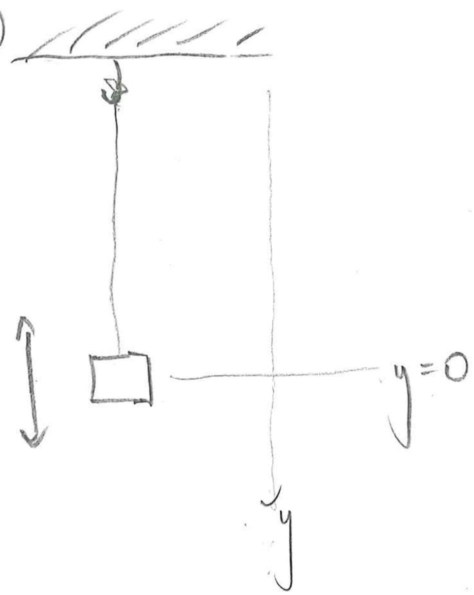
Take-Home Quiz No. 5

Math 235 (Calc I)

Fall 2017

SOLUTIONS

1. (a)



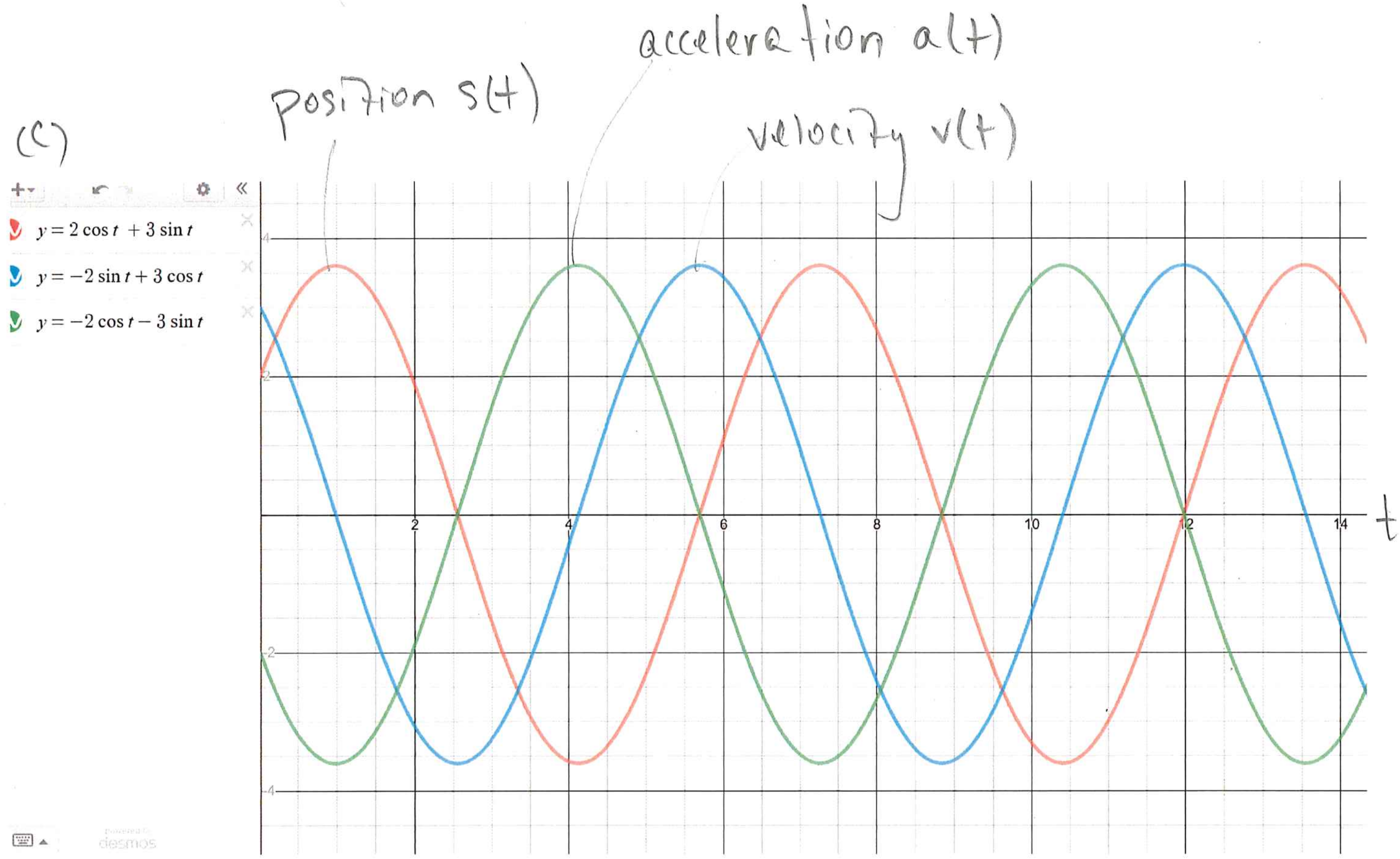
(b) $v(t) = s'(t)$

$$\boxed{= -2\sin t + 3\cos t}$$

$a(t) = v'(t)$

$$\boxed{= -2\cos t - 3\sin t}$$

(c)



(d) The mass passes through equilibrium for the first time at t_0, the smallest non-negative solution to the equation

$$s(t)=0.$$

Using the graph on desmos.com, this is

$\boxed{t_0 \approx 2.554 \text{ seconds}}$

(e) The distance is given by the amplitude of the position function, and also where the velocity is zero. From the graph, this is $\boxed{\approx 3.606 \text{ cm}}$

(f) Speed is greatest whenever $a(t)=0$. From the graph, the first time is at $\boxed{\approx 2.554 \text{ sec}}$ and then <u>every ~ 3.141 sec</u>. These times also coincide with whenever the mass is at equilibrium.

2.(a) First find the slope of the tangent line:

$$\frac{d}{dx}\left(x^2 - xy + y^2 = 3\right)$$

$$2x - (1)y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-2x}{2y-x} \Rightarrow m = \frac{dy}{dx}\Big|_{(-1,1)} = \frac{(1)-2(-1)}{2(1)-(-1)} = \frac{3}{3} = 1$$

The normal line is

$$y - 1 = -1(x-(-1)) \Rightarrow y = -x$$

Intersection points are given by:

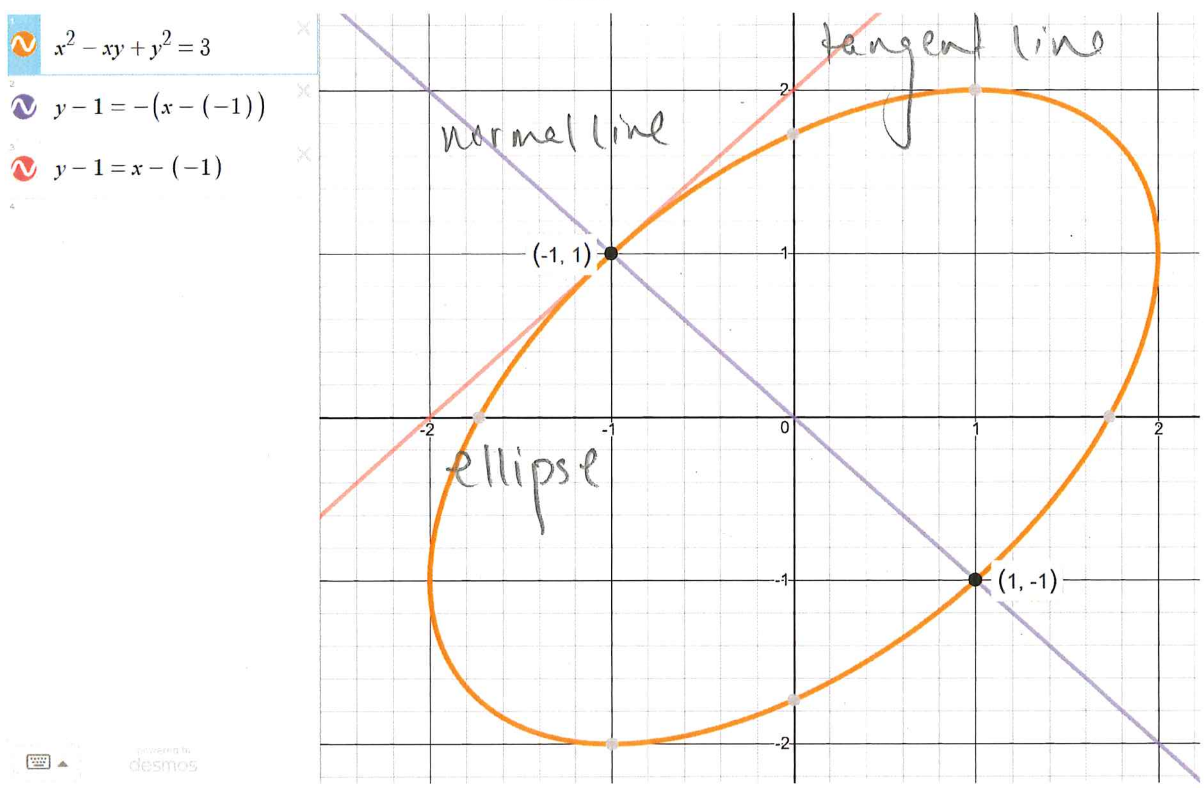
$$x^2 - x(-x) + (-x)^2 = 3$$

$$3x^2 = 3$$

$$x^2 = 1 \Rightarrow x = \pm 1$$

Since $y = -x$, these are $\boxed{(1,-1)}$ and $(-1,1)$

(b)



3. (a) Set up a system of 2 equations to find 2 unknowns. We are given $n(0)=20$ and $n'(0)=12$. The formula for $n'(t)$ is

$$n'(t)=\frac{(1+be^{-0.7t})(0)-a(-0.7be^{-0.7t})}{(1+be^{-0.7t})^2}$$

$$=\frac{0.7abe^{-0.7t}}{(1+be^{-0.7t})^2}$$

Solve:

$$20=\frac{a}{1+be^{-0.7(0)}}=\frac{a}{1+b}\Rightarrow a=20(1+b)$$

$$12=\frac{0.7ab\cancel{e^{-0.7(0)}}}{(1+b\cancel{e^{-0.7(0)}})^2}=\frac{0.7ab}{(1+b)^2}$$

$$=\frac{0.7\left(20\cancel{(1+b)}\right)b}{(1+b)^2} \qquad b\neq -1$$

$$\Rightarrow 12=\frac{0.7(20)b}{1+b}$$

$$12+12b=0.7(20)b$$

$$12=(0.7(20)-12)b\Rightarrow \boxed{b=\frac{12}{0.7(20)-12}=6}$$

$\longrightarrow$

$$\Rightarrow \boxed{\begin{aligned} a &= 20(1+b) \\ &= 20(1+6) = 140 \end{aligned}}$$

(b) Long run behavior is given by

$$\lim_{t\to\infty} n(t) = \lim_{t\to\infty} \frac{140}{1+6e^{-0.7t}}$$

$$= \lim_{t\to\infty} \frac{140}{1+\underbrace{\frac{6}{e^{0.7t}}}_{0}} = 140$$

$\Rightarrow$ The population stabilizes at 140 cells.

4. Using the half-life of ^{14}C,

$$\frac{1}{2} = e^{k(5730)}$$

$$\ln\left(\frac{1}{2}\right) = k(5730) \Rightarrow k = \frac{\ln\left(\frac{1}{2}\right)}{5730}.$$

Thus, given an initial ^{14}C mass of m_0, the amount remaining after t years of radioactive decay is

$$m(t) = m_0 e^{t\left(\frac{\ln\left(\frac{1}{2}\right)}{5730}\right)}$$

$\longrightarrow$

(a) $m(68{,}000{,}000) = m_0 \boxed{e^{(68{,}000{,}000)\left(\frac{\ln\left(\frac{1}{2}\right)}{5730}\right)}}$

According to keisan.casio.com/calculator, this is $\boxed{\approx 3.69 \times 10^{-3573}}$.

(b) Solve for t, given $m(t) = 0.1\%$ of m_0
$= 0.001 m_0$

$$0.001 m_0 = m_0 e^{t\left(\frac{\ln\left(\frac{1}{2}\right)}{5730}\right)}$$

$$\Rightarrow \ln(0.001) = t\left(\frac{\ln\left(\frac{1}{2}\right)}{5730}\right)$$

$$\Rightarrow t = \frac{(5730)\ln(0.001)}{\ln\left(\frac{1}{2}\right)}$$

$$\boxed{\approx 57{,}100 \text{ years}}$$

5. Over an hour-long period, the temperature decreased by $32.5 - 30.3 = 2.2°C$. The surrounding temperature is $T_s = 20°C$, so Newton's Law of Cooling says that at 230p,

$\longrightarrow$

$$\left.\frac{dT}{dt}\right|_{T=30.3} \approx k(30.3-20) = -2.2$$

$$\Rightarrow k = \frac{-2.2}{13.3}$$

The temperature of the body t hours after its expiration is

$$T(t) = (T(0)-T_s)e^{kt} + T_s$$

$$\Rightarrow T(t)-T_s = (T(0)-T_s)e^{kt}$$

$$\frac{T(t)-T_s}{T(0)-T_s} = e^{kt}$$

$$\ln\left(\frac{T(t)-T_s}{T(0)-T_s}\right) = kt \Rightarrow t = -\frac{13.3}{2.2}\ln\left(\frac{T(t)-20}{37.0-20}\right),$$

assuming the body was normal temperature when it died. This means at 130p,

$$t = \left(-\frac{13.3}{2.2}\ln\left(\frac{32.5-20}{17}\right)\text{hours}\right)\left(\frac{60\text{ min}}{\text{hour}}\right) \approx 122.9\text{ min}.$$

This is ~2 hours and 2.9 minutes after the body died, so the murder took place at $\boxed{1128a}$.