**MATH 2554 (Calculus I)**
**Summer 2015**

Name: ______________________

Thurs 2 July 2015

# Exam 3: Using Derivatives ($\oint$3.10-4.6)

Version B

**Exam Instructions:** You have 50 minutes to complete this exam. Follow the directions and answer the question, using boss notation where appropriate. Justification is required for all problems.

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Signature: **(1 pt)** ______________________

Good luck!

1. **(20 pts)** Sketch a graph of a function $f(x)$, continuous on $(-\infty, \infty)$, that satisfies all of the following criteria:

   - $f(-4) = 1$ and $f(2) = -1$
   - $f'(x) < 0$ and $f''(x) < 0$ on $(-\infty, 0)$
   - $f'(x) < 0$ and $f''(x) > 0$ on $(0, 2)$
   - $f'(x) > 0$ and $f''(x) > 0$ on $(2, 4)$
   - $f'(x) > 0$ and $f''(x) < 0$ on $(4, \infty)$

2. (a) **(9 pts)** What are the three hypotheses for Rolle's Theorem?

   (b) **(7 pts)** Given the three hypotheses, what is the conclusion of Rolle's Theorem?

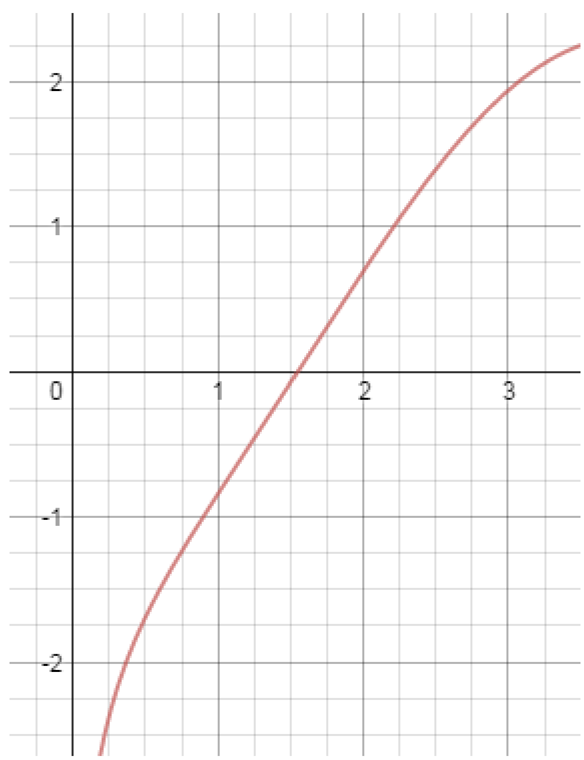
   (c) **(7 pts)** The *Mean Value Theorem* applies to $f(x) = x(x^2 - x - 2)$ on $[-1, 1]$. (You don't have to prove that.) Find the point(s) guaranteed to exist by the Mean Value Theorem.

3. **(7 pts ea)** Let $f(x) = \ln x - \sin(2 - x)$.

(a) Write the equation for the linear approximation to $f(x)$ at $x = 2$.

(b) Use your answer to (a) to approximate $f(1)$.

(c) Below is the graph of $f(x)$, drawn at the website `desmos.com/calculator`. On the same axis, draw your tangent line. Label both $f(1)$ and your approximation from part (b).



4. **(20 pts)** A rectangular flower garden with an area of 32 $\text{m}^2$ is surrounded by a grass border that is 1 m wide on the top and bottom, and 2 m wide on the other two sides. What dimensions of the garden minimize the combined area of the garden and borders? Use the 2nd Derivative Test to justify your answer.

5. **(10 pts ea)** Let $f(x)$ be a function, continuous on $(-\infty, \infty)$, such that

$$f'(x) = \frac{2-2x^2}{1+x^2} \quad \text{and} \quad f''(x) = \frac{-8x}{(1+x^2)^2}.$$

(a) Determine the intervals on which $f(x)$ is increasing and decreasing.

(b) Determine the intervals on which $f(x)$ is concave up and concave down.

6. **(20 pts)** A rectangle initially has dimensions 1 cm by 5 cm. All sides begin increasing in length at a rate of 2 cm/sec. At what rate is the area of the rectangle increasing after 20 sec?