Exercises $\boxed{\text{SOLUTIONS}}$

§11.3 (a) $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

angle between $\vec{u}$ and $\vec{v}$

$$\Rightarrow \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \frac{\overset{-1}{1(-1)} + \overset{2}{2(1)} + \overset{12}{3(4)}}{\left(\sqrt{\underset{1}{1^2}+\underset{4}{2^2}+\underset{9}{3^2}}\right)\left(\sqrt{\underset{1}{(-1)^2}+\underset{1}{1^2}+\underset{16}{4^2}}\right)}$$

$$= \frac{13}{(\underset{2\cdot 7}{\sqrt{14}})(\underset{2\cdot 9}{\sqrt{18}})} = \boxed{\frac{13}{6\sqrt{7}}}$$

(b) $\text{proj}_{\vec{u}}\vec{v} = |\vec{v}|\cos\theta \dfrac{\vec{u}}{|\vec{u}|}$

$$= \overset{3\sqrt{2}}{\sqrt{18}}\left(\frac{13}{\underset{2}{6}\sqrt{7}}\right)\frac{1}{\underset{(\sqrt{7})(\sqrt{2})}{\sqrt{14}}}\langle 1,2,3\rangle$$

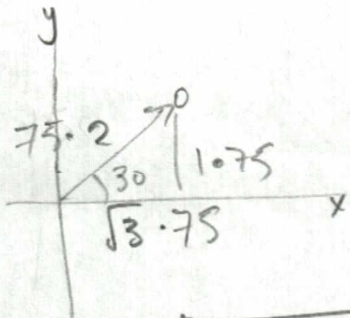
$$= \frac{13}{2(7)}\langle 1,2,3\rangle = \boxed{\left\langle \frac{13}{14}, \frac{13}{7}, \frac{39}{14}\right\rangle}$$

$\longrightarrow$

$\boxed{\S 11.7}$

$$\vec{v}(t) = \int \langle 0, -32 \rangle \, dt$$

$$= \langle 0, -32t \rangle + C$$



$$\vec{v}(0) = \langle 0, -32(0) \rangle + C = \vec{v}_0$$

$$= \langle 75, 75\sqrt{3} \rangle$$

$$\boxed{\Rightarrow \vec{v}(t) = \langle 75, -32t + 75\sqrt{3} \rangle \text{ ft/s}}$$

$$\vec{r}(t) = \int \langle 75, -32t + 75\sqrt{3} \rangle \, dt$$

$$= \langle 75t, -16t^2 + 75\sqrt{3}\,t \rangle + C$$

$$\vec{r}(0) = \langle 75(0), -16(0)^2 + 75\sqrt{3}(0) \rangle + C = \langle x_0, y_0 \rangle$$

$$= \langle 0, 0 \rangle$$

$$\boxed{\Rightarrow \vec{r}(t) = \langle 75t, -16t^2 + 75\sqrt{3}\,t \rangle \text{ ft}}$$

§12.1 | parallel to $\vec{u}$ and $\vec{v}$

$\rightarrow$ let $\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$

$= \langle 0, -3, 3 \rangle$

Plane:

$\cancel{0(x-2)} \boxed{-3(y-3) + 3(z-1) = 0}$

or $-3y + 9 + 3z - 3 = 0$

$\rightarrow \boxed{-3y + 3z = -6}$

$\longrightarrow$

§12.5

$$\left.\frac{\partial z}{\partial s}\right|_{(s,t)=(1,\frac{1}{e})} = \left.\left(\frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}\right)\right|_{(s,t)=(1,\frac{1}{e})}$$

$$= \left.\left(\frac{3}{1+(3x+2y)^2}(2st) + \frac{2}{1+(3x+2y)^2}(\ln t)\right)\right|_{(1,\frac{1}{e})}$$

$$= \frac{3(2)(1)(\frac{1}{e}) + 2\overbrace{\ln(\frac{1}{e})}^{-1}}{1+\left(3(1)^2(\frac{1}{e}) + 2(1)\underbrace{(\ln\frac{1}{e})}_{-1}\right)^2}$$

$$\boxed{= \frac{6e^{-1}-2}{1+(3e^{-1}-2)^2}}$$

$$\left.\frac{\partial z}{\partial t}\right|_{(1,\frac{1}{e})} = \left.\left(\frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}\right)\right|_{(1,e^{-1})}$$

$$= \left.\frac{3(s^2) + 2(\frac{s}{t})}{1+(3x+2y)^2}\right|_{(s,t)=(1,e^{-1})}$$

$$\boxed{= \frac{3+2e}{1+(3e^{-1}-2)^2}}$$

§12.6 (a) Normalize: let $\vec{v} = \frac{\vec{u}}{|\vec{u}|} = \left\langle \frac{12}{13}, \frac{-5}{13} \right\rangle$

$$D_{\vec{v}} f = \nabla f \cdot \vec{v}$$

$$= \langle \underbrace{y\cos(xy)}_{\pi\cos(-\pi)\ =\ -1}, \underbrace{x\cos(xy) - \sin y}_{-\cos(-\pi) - \sin(\pi)\ =\ 1} \rangle \cdot \left\langle \frac{12}{13}, \frac{-5}{13} \right\rangle$$

$$\Rightarrow D_{\vec{v}} f(-1,\pi) = \boxed{-\frac{12}{13}\pi - \frac{5}{13}}$$

(b) Maximal change of f is in the direction of its gradient:

From (a),

$$\nabla f(-1,\pi) = \boxed{\langle -\pi, 1 \rangle}$$

$\longrightarrow$

§12.7 | One can rewrite the function

implicitly: $G(x,y,z) = yz + xz + xy^{-2} - 9 = 0$.

Then $\nabla G = \langle z + y^{-2},\ z - 2xy^{-3},\ y + x \rangle$.

$\qquad 2+(1)^{-2} \qquad 2-2(3)(1)^{-3} \qquad 1+3$

tangent plane:

$$\nabla G(3,1,2) \cdot \langle x-3, y-1, z-2 \rangle = 0$$

$$\Rightarrow 3(x-3) - 4(y-1) + 4(z-2) = 0$$

or $3x - 9 - 4y + 4 + 4z - 8 = 0$

$\qquad 9 - 4 + 8$

$$\Rightarrow \boxed{3x - 4y + 4z = 13}$$

$\longrightarrow$

§12.8 $f_x = ye^{-x-y} + xy(-e^{-x-y}) = 0 = e^{-x-y}(y-xy)$

never 0

$\Rightarrow y - xy = y(1-x) = 0$

$\Rightarrow y = 0$ or $x = 1$

$f_y = \cancel{e^{-x-y}}(x-xy) = 0$ (x and y are interchangeable in $f(x,y)$)

If $y=0$ then $x - x(0) = 0$
$\Rightarrow x = 0$

If $x=1$ then $1 - 1(y) = 0$
$\Rightarrow y = 1$

$\boxed{\text{CPs: } (0,0), (1,1)}$

discriminant: $\begin{vmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{vmatrix}$

$$= \left(-e^{-x-y}(y-xy) + e^{-x-y}(-y)\right)$$
$$\cdot \left(-e^{-x-y}(x-xy) + e^{-x-y}(-x)\right)$$
$$-\left(-e^{-x-y}(y-xy) + e^{-x-y}(1-x)\right)^2$$

$\longrightarrow$

$$= \underbrace{-e^{-x-y}(2y-xy)}_{f_{xx}}\underbrace{(-e^{-x-y})(2x-xy)}_{f_{yy}}$$

$$-\left(-e^{-x-y}(y-xy+x-1)\right)^2$$

D-Test:

$$D(0,0) = -e^0\cancel{(0)}\cancel{(-e^0)(0)} - (-e^0(-1))^2$$

$$= -1 \quad \boxed{\Rightarrow \text{saddle at } (x,y)=(0,0)}$$

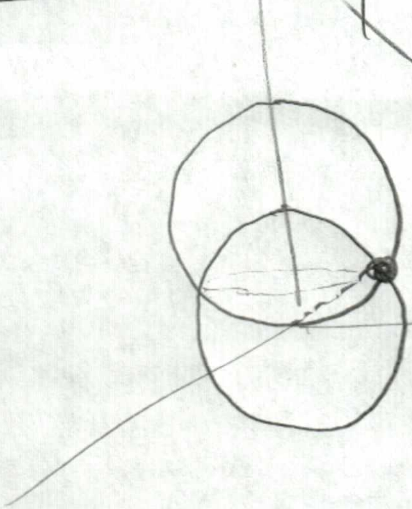
$$D(1,1) = -e^{-2}(2-1)(-e^{-2})(2-1) - \cancel{(-e^{-2}(1-1+1-1))^2}$$

$$= e^{-2} > 0$$

$$f_{xx}(1,1) = -e^{-2}\cancel{(2-1)} < 0$$

$$\boxed{\Rightarrow \text{local max at } (x,y)=(1,1)}$$

§13.5



$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta + \int_0^{2\pi}\int_{\pi/3}^{\pi/2}\int_0^{2\cos\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$2\cos\varphi = 1$$

$$\cos\varphi = \frac{1}{2} \Rightarrow \varphi = \frac{\pi}{3}$$