MATH 235 (Calculus I)
due Mon 16 Oct 2017

Take-Home Quiz 4: Introduction to derivatives (§2.1, 2.7-3.2)

Directions: This quiz is due on October 16, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. **§2.1 #2** A cardiac monitor is used to measure the heart rate of a patient after surgery. It copiles the number of heartbeats after t minutes. When the data in the table below are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
heartbeats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line.

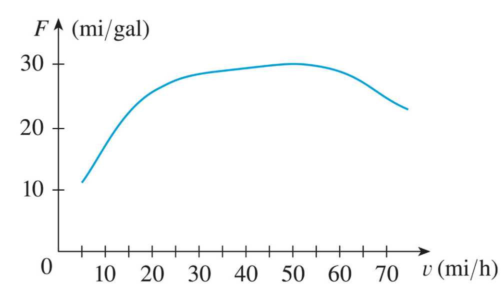
(a) Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t:
- $t = 36$ and $t = 42$
- $t = 38$ and $t = 42$
- $t = 40$ and $t = 42$
- $t = 42$ and $t = 44$

(b) What are your conclusions?

2. **§2.7 #22** If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

3. **§2.7 #26** Sketch the graph of a function f where
- the domain is $(-2, 2)$,
- $f'(0) = -2)$,
- $\lim_{x\to 2^-} f(x) = \infty$,
- f is continuous at all numbers in its domain except ± 1, and
- f is odd.

4. **§2.8 #14** The graph (from the US Department of Energy) shows how driving speed affects gas mileage. Fuel economy F is measured in miles per gallon and speed v is measured in miles per hour.



(a) What is the meaning of the derivative $F'(v)$?

(b) Sketch the graph of $F'(v)$.

(c) At what speed should you drive if you want to save on gas?

5. **§3.1 #72** To show a function $f(x)$ is **differentiable** at a point $x = a$ you must show $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists. The following function g is differentiable except for possibly at $x = 0$ and $x = 2$.

$$g(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \geq 2 \end{cases}$$

   (a) Check the differentiability of g at $x = 0$ and $x = 2$.

   (b) Give a formula for g'.

   (c) Sketch the graphs of g and g' (on the same axes).

6. **§3.1 #80** The general graph of the function $f(x) = ax^2 + bx + c$ is a parabola. Prove that the average of the slopes of the tangent lines to the parabola at the endpoints of any interval $[p, q]$ equals the slope of the tangent line at the midpoint of the interval.

7. **§3.2 #56** Compute $Q'(0)$, where

$$Q(x) = \frac{1 + x + x^2 + xe^x}{1 - x - x^2 - xe^x}.$$

   Hint: Write $Q(x) = \frac{f(x)}{g(x)}$. *Then use* $f(0)$, $f'(0)$, $g(0)$, *and* $g'(0)$ *to compute* $Q'(0)$.

8. **§3.2 #58** A manufacturer produces bolts of fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

   (a) What does it mean, in the context of the problem, to say that $f(2) = 10,000$ and $f'(2) = -350$?

   (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.