Math 115 Quiz 3: $\oint$ 2.2-4 (Practice Probs for Exam 1) **Mon 4 October 2010**

Name: ____________________________

You have 15 minutes to complete this quiz. No calculators allowed.

1. **Definitions/Concepts.** (1 pt each) You are teaching an introductory calculus course at Michigan State, and your students have just turned in a homework assignment on interpretations of the derivative. You must read all their explanations and decide whether they make sense. Some excerpts from their solutions are given below. Mark each correct or sensible statement with a '+' and each incorrect or nonsensical statement with a '-.'

   Let $g(v)$ denote the fuel efficiency (in miles per gallon) of a car travelling at v miles per hour.

   __+__ (a) The statement $g'(55) = -0.5$ means tha t when you use one more gallon of gasoline, the time you can drive at 55 mph decreases by about a half hour.

   __+__ (b) If $g'(v) < 0$ for $v \geq 45$, then the car's fuel efficiency will be lower at 65 mph than at 45 mph.

   __−__ (c) If $g'(65) = g'(55) = 0$, then the car's fuel efficiency is constant on the interval $55 \leq v \leq 65$.

   No information is given about the fuel efficiency between 55 and 65 mph.

   __−__ (d) Since the units of $g'(v)$ are hours per gallon, the units of $g''(v)$ are hours per gallon squared.

   The units are hours per gal per mph.

2. **Questions/Problems.** (2 pts each) Suppose that the line tangent to the graph of $f(x)$ at $x = 3$ passes through the points $(1, 2)$ and $(5, -4)$.

   (a) Find $f'(3)$.

   This is just the slope of the line tangent to the graph at $x = 3$, which can be computed using the two points given:

$$\begin{aligned} f'(3) &= \frac{-4-2}{5-1} \\ &= \frac{-6}{4} \\ &= \frac{-3}{2}. \end{aligned}$$

   (b) Find $f(3)$.

   When $x = 3$ the function and the tangent line intersect. So, to find $f(3)$, use the equation for the tangent line. Using one of the given points and the slope computed above:

$$\begin{aligned} y - y_0 &= \frac{-3}{2}(x - x_0) \\ y - 5 &= \frac{-3}{2}(x - 1) \\ y &= \frac{-3}{2}x + \frac{3}{2} + 5 \\ &= \frac{-3}{2}x + \frac{7}{2}. \end{aligned}$$

Therefore,

$$\begin{aligned} f(3) &= \frac{-3}{2}(3) + \frac{7}{2} \\ &= \frac{-9}{2} + \frac{7}{2} \\ &= \frac{-2}{2} \\ &= -1. \end{aligned}$$

(c) Estimate the value of $f(2.9)$.

The derivative at $x = 3$ gives a linear approximation to the function $f(x)$ near $x = 3$. So use the tangent line to approximate $f(2.9)$.

$$\begin{aligned} f(2.9) &\approx \frac{-3}{2}(2.9) + \frac{7}{2} \\ &= \frac{-1.7}{2} \\ &= -0.85 \end{aligned}$$

So $f(2.9) \approx -0.85$.

3. **Computations/Algebra.** *– none this week –*