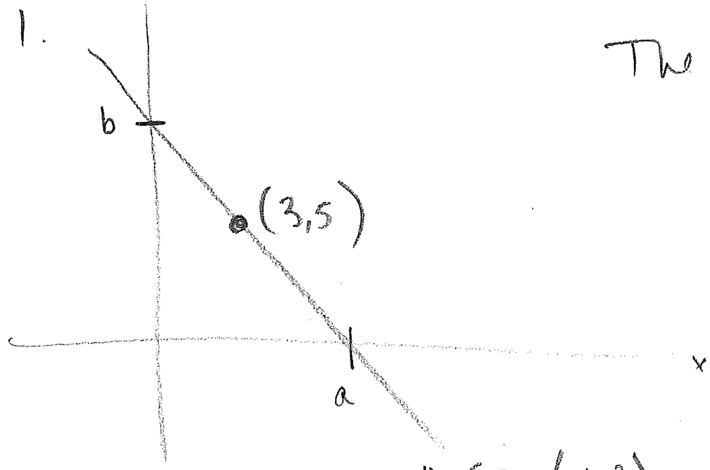
Take-Home Quiz 8

Math 235 (Calc I)

Fall 2017

SOLUTIONS

1.



$y-5=m(x-3)$

The area of the triangle $A=\frac{1}{2}ab$ must be minimized. It depends on the x- and y-intercepts:

$$0-5=m(x-3)=mx-3m$$

$$-5+3m=mx$$

$$\Rightarrow x=\frac{-5+3m}{m}$$

$$y-5=m(0-3)$$

$$y-5=-3m \Rightarrow y=-3m+5$$

The area is a function of m:

$$A(m)=\frac{1}{2}\left(\frac{-5+3m}{m}\right)(-3m+5)=\frac{1}{2}\left(\frac{-5}{m}+3\right)(-3m+5)$$

Optimize:

$$A'(m)=\frac{1}{2}\left(\frac{5}{m^2}\right)(-3m+5)+\frac{1}{2}\left(\frac{-5}{m}+3\right)(-3)$$

$$=-\cancel{\frac{15}{2}\left(\frac{1}{m}\right)}+\frac{25}{2}\left(\frac{1}{m^2}\right)+\cancel{\frac{15}{2}\left(\frac{1}{m}\right)}-\frac{9}{2}=0$$

$$\Rightarrow \frac{25}{\cancel{2}}\left(\frac{1}{m^2}\right)=\frac{9}{\cancel{2}}$$

$$\frac{25}{9}=m^2 \Rightarrow m=\pm\frac{5}{3}$$

$\longrightarrow$

Choose the negative value since the slope must be negative in order to form a triangle.

Use the 1st D Test to check $m = -\frac{5}{3}$ is a minimum:

A is dec. $\qquad\qquad\qquad$ A is inc.

$$\longleftarrow \quad -2 \quad\quad \overset{\emptyset}{-\tfrac{5}{3}} \quad\quad -1 \quad \longrightarrow$$

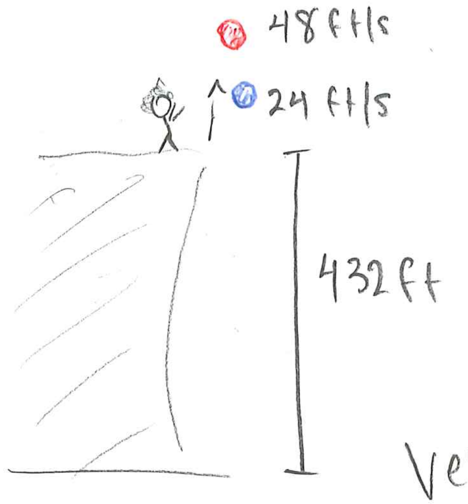
$$A'(-2) = \frac{25}{2}\left(\frac{1}{(-2)^2}\right) - \frac{9}{2}\left(\frac{4}{4}\right) = \frac{25-36}{8} < 0$$

$$A'(-1) = \frac{25}{2}\left(\frac{1}{(-1)^2}\right) - \frac{9}{2} = \frac{25-9}{2} > 0$$

The domain of m is $(-\infty, 0)$ and so there are no endpoints to check. The function $A(m)$ changes from decreasing to increasing at $m = -\frac{5}{3}$ so the critical point is a global minimum.

$\Rightarrow$ ans: $\boxed{m = -\frac{5}{3}}$

2.



(a) In the diagram, the positive direction is up, so acceleration as a function of time is given by $a(t) = -32$.

Velocity is given by

$$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C.$$

At time $t=0$, the red ball is moving up at 48 ft/s. This means

$$v_{red}(0) = -32(0) + C = 48$$

$$\Rightarrow C = 48$$

The position of the red ball is given by

$$h_{red}(t) = \int v_{red}(t)\,dt = \int (-32t + 48)\,dt$$

$$= -32\frac{t^2}{2} + 48t + C.$$

Wheeler tosses the red ball from 432 ft at $t=0$:

$$h_{red}(0) = -16(0)^2 + 48(0) + C = 432$$

$$\Rightarrow C = 432.$$

This means the height of the red ball after t seconds is $\boxed{h_{red}(t) = -16t^2 + 48t + 432}$

→

(b) The blue ball also has velocity $v(t) = -32t + C$, but at $t=1$, its velocity is 24 ft/s:

$$v(1) = -32(1) + C = 24$$

$$\Rightarrow C = 24 + 32 = 56.$$

The velocity function for the blue ball is

$$v_{blue}(t) = -32t + 56.$$

Integrate to find the height function:

$$h_{blue}(t) = \int (-32t + 56)\,dt = -\overset{16}{\cancel{32}}\frac{t^2}{\cancel{2}} + 56t + C$$

At $t=1$, the blue ball is 432 ft high.

$$h_{blue}(1) = -16(1)^2 + 56(1) + C = 432$$

$$\Rightarrow C = 432 + 16 - 56 = 392.$$

The height of the blue ball t seconds after Wheeler tosses the red ball is

$$\boxed{h_{blue}(t) = -16t^2 + 56t + 392}$$

(c) If/when the balls pass each other, they will have the same height:

$$h_{red}(t) = h_{blue}(t)$$

$$\cancel{-16t^2} + 48t + 432 = \cancel{-16t^2} + 56t + 392$$

⟶

$$40 = 8t$$

→ $\boxed{\text{Yes}}$, they pass each other at $\boxed{t = 5 \text{ seconds}}$

(d) The ball hits the ground when the height is 0:

$$h_{red}(t) = -16t^2 + 48t + 432 = 0$$

$$-16(t^2 - 3t - 27) = 0$$

$$3\sqrt{1 - 4(1)(-3)}$$

$$\text{Q-formula: } t = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-27)}}{2(1)}$$

$$= \frac{3 \pm 3\sqrt{13}}{2} \approx 6.908 \text{ sec.}$$

$$h_{blue}(t) = -16t^2 + 56t + 392 = 0$$

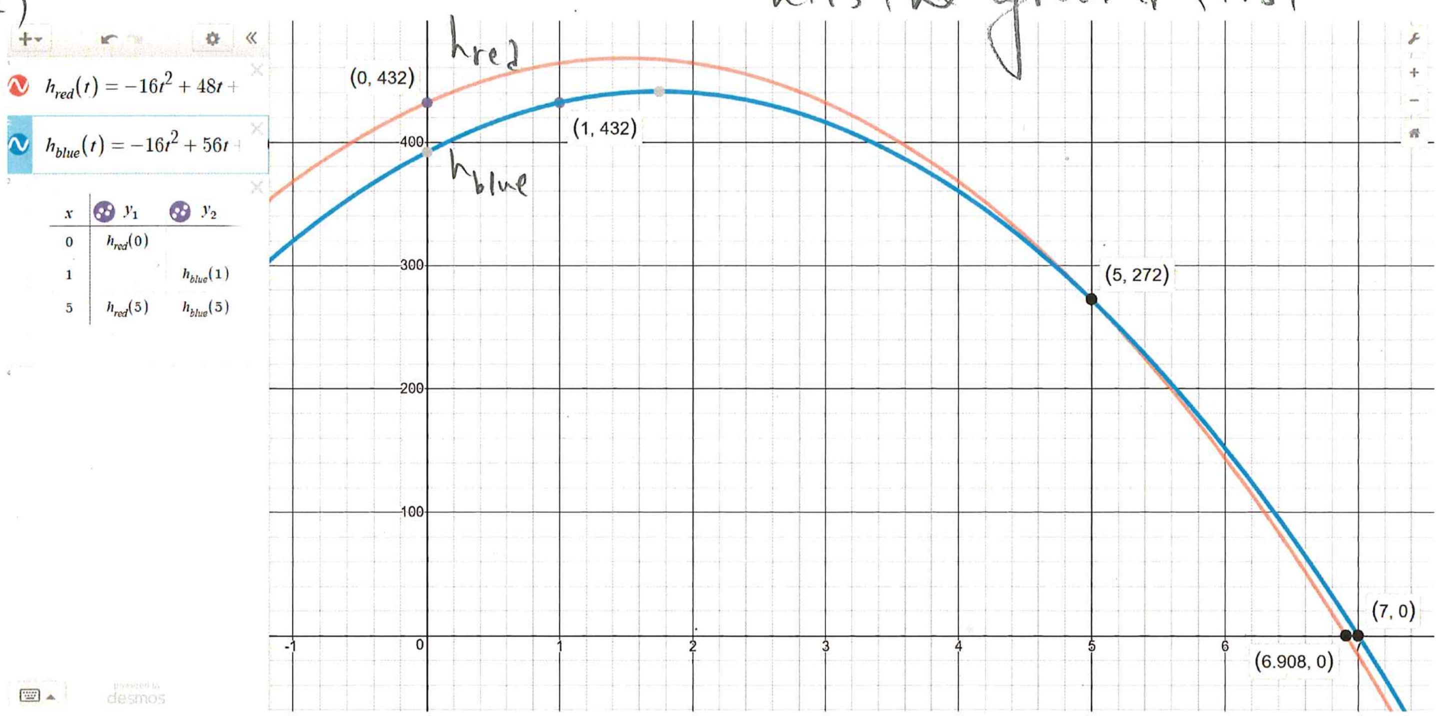
$$= -4(4t^2 - 14t - 98) = 0$$

$$(t - 7)(4t + 14) = 0$$

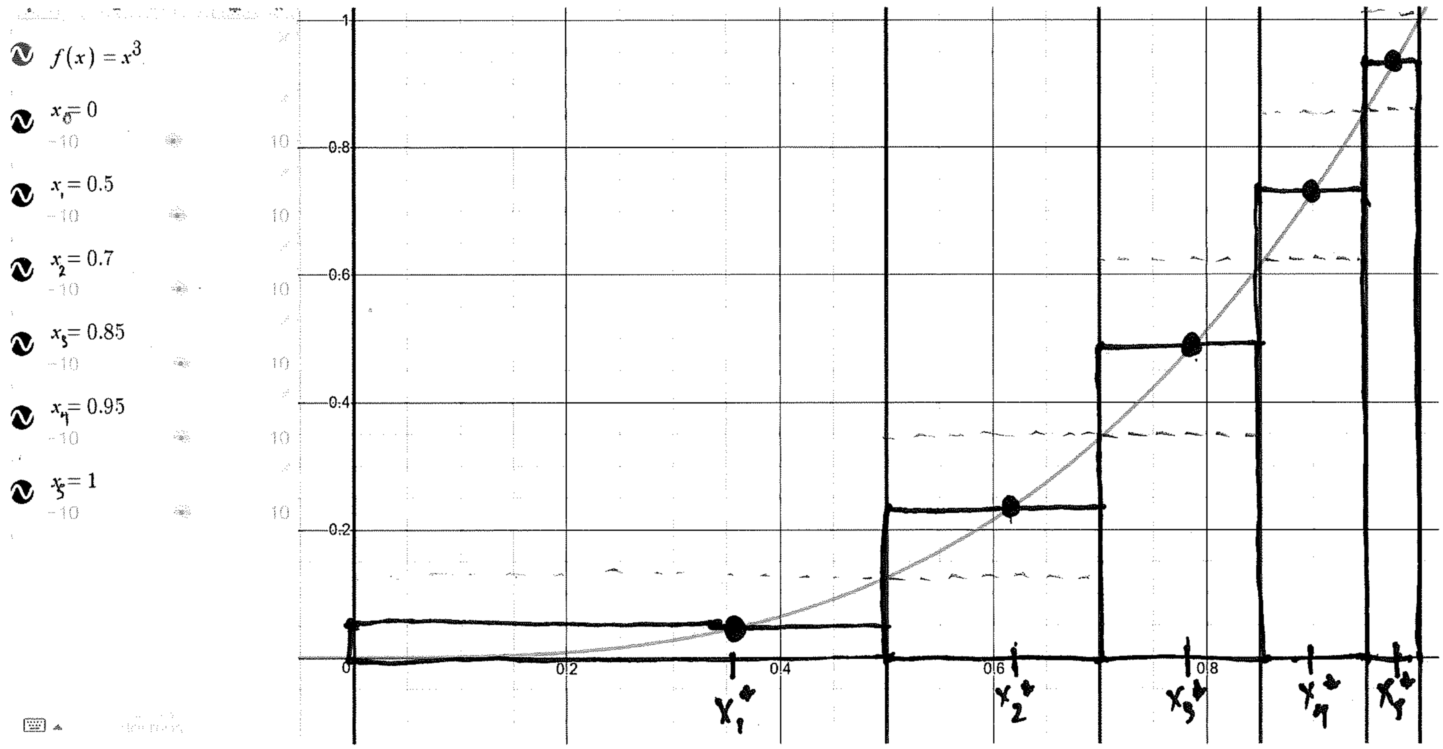
$$t = 7 \text{ sec.}$$

The sooner time is $t \approx 6.908$ sec so the $\boxed{\text{red ball}}$ hits the ground first.

(e)



3. (a)



(b)

k	x_k	x_k^*	$f(x_k^*)$	Δx_k
0	0	—	—	—
1	0.5	0.355	$(0.355)^3$	0.5
2	0.7	0.62	$(0.62)^3$	0.2
3	0.85	0.78	$(0.78)^3$	0.15
4	0.95	0.9	$(0.9)^3$	0.1
5	1	0.975	$(0.975)^3$	0.05

(c)

$$\sum_{k=1}^{5} f(x_k^*)\Delta x_k = \underbrace{(0.355)^3(0.5)}_{\text{area of rectangle \#1}} + \underbrace{(0.62)^3(0.2)}_{\text{area of rectangle \#2}} + (0.78)^3(0.15)$$

$$+ (0.9)^3(0.1) + (0.975)^3(0.05)$$

$$\boxed{= 0.260460806}$$

*Note: The exact area is $\int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1 = \frac{1^4}{4} - \frac{0^4}{4} = \boxed{\frac{1}{4}}$

4. (a) $u = \sin x$

$du = \cos x\,dx$

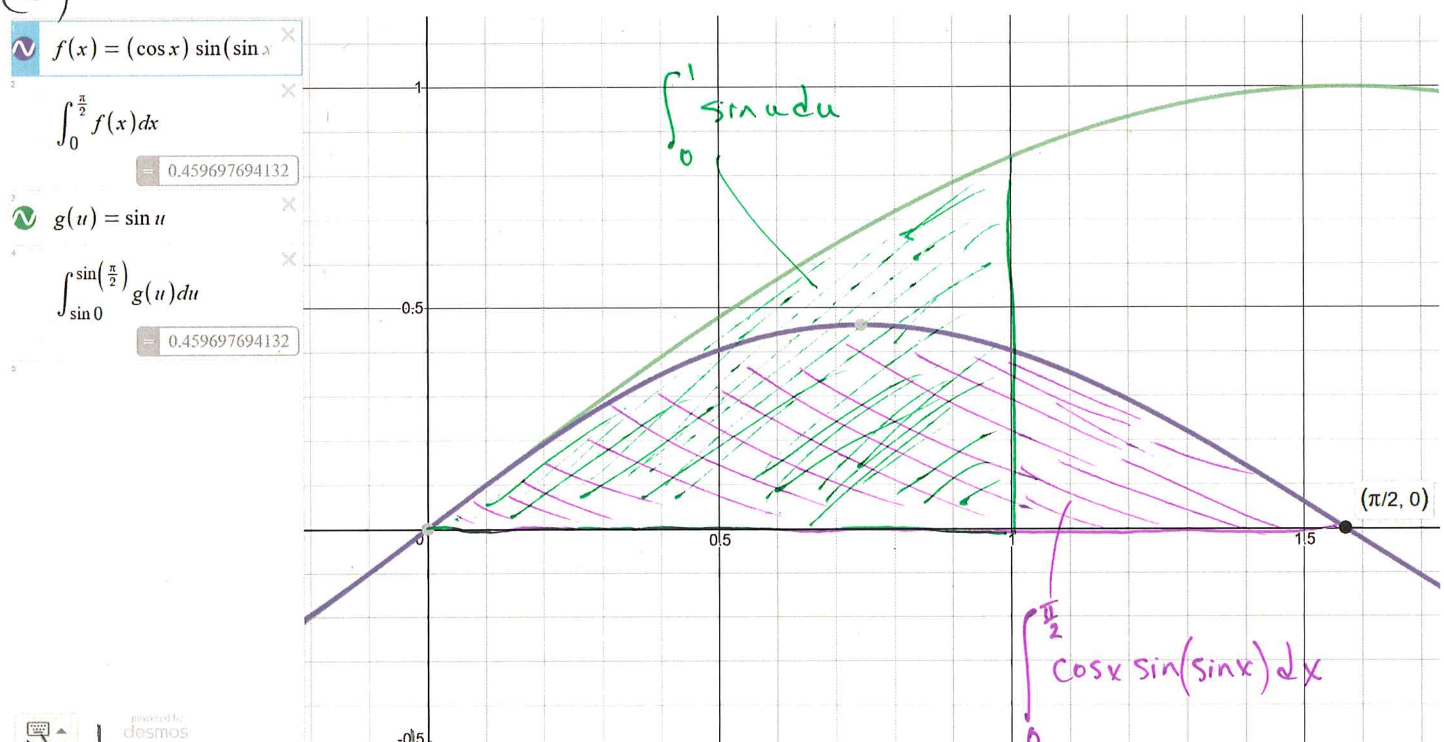
(b) When $x = 0$, $u = \sin(0) = 0$

$x = \frac{\pi}{2}$, $u = \sin\left(\frac{\pi}{2}\right) = 1$

The integral becomes:

$$\boxed{\int_0^1 \sin(u)\,du}$$

(c)



(d) $\int_0^1 \sin u\,du = -\cos u\Big|_0^1$

$= -\cos(1) - (-\cos(0)) = 1 - \cos(1) \approx 0.460$

(compare to the computations on desmos)