Take-Home Quiz 4

SOLUTIONS

Math 236 (Calc II)

Fall 2017

1. $a_1 = 1$

$a_2 = a_1 + 2 = 1 + 2 = \underline{3}$

$a_3 = a_2 + 2 = \underline{5}$

$a_4 = a_3 + 2 = \underline{7}$

$a_5 = a_4 + 2 = \underline{9}$

Closed form:

$a_k = 1 + 2(k-1)$

$= 1 + 2k - 2$

$\boxed{= 2k - 1}$

2. (a) Each term a_k can be written as a fraction $\frac{a_k \cdot 10^{k-1}}{10^{k-1}}$. For example, $a_1 = \frac{a_1 \cdot 10^0}{10^0}$;

$a_2 = \frac{a_2 \cdot 10}{10} = \frac{31}{10}$

$a_3 = \frac{314}{100}$

The k^{th} term has $k-1$ decimal places, so multiplying by 10^{k-1} makes it an integer.

(b) The sequence is increasing because the k^{th} term adds a multiple of $\frac{1}{10^{k-1}}$ to the last.

(c) An upper bound is any number larger than π, such as 4. The least upper bound is π.

(d) This sequence consists of rational numbers but its least upper bound, π, is not rational.

3.
(a) i. Newton's method for $f(x) = x^2 - a$ gives

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^2 - a}{2x_k} = \frac{2}{2}x_k - \frac{1}{2}x_k + \frac{a}{2x_k}$$

$$= \frac{1}{2}x_k + \frac{a}{2x_k} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right).$$

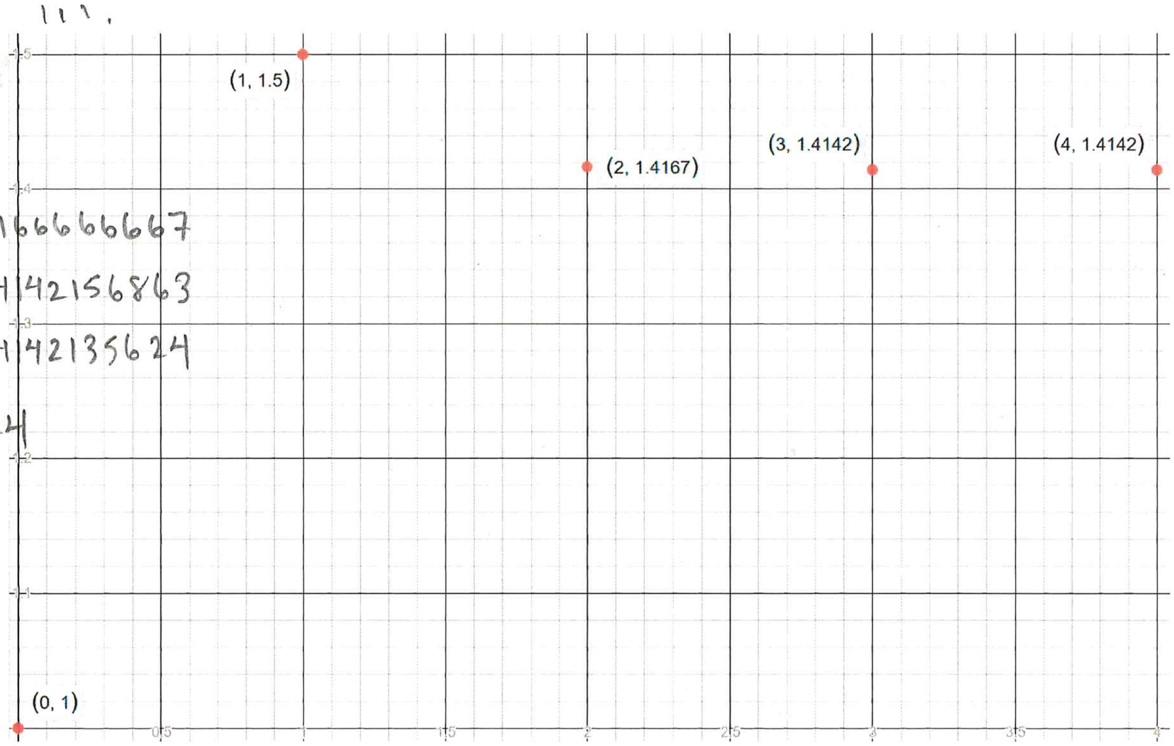
ii.

x_1	y_1
0	$x_0 =$ 1
1	$x_1 = \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2}$
2	$x_2 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{\frac{3}{2}}\right) \approx 1.416666667$
3	$x_3 = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{\frac{17}{12}}\right) \approx 1.414215686$
4	$x_4 = \frac{1}{2}\left(\frac{577}{408} + \frac{2}{\frac{577}{408}}\right) \approx 1.414213562$

$\sqrt{2} \approx 1.414213562$

4 terms

iii.



(b) Newton's method fails if $f'(x_0) = 0$ because division by zero occurs: $x_1 = x_0 + \dfrac{f(x_0)}{f'(x_0)}$.

4. From the recursion formula, we can write

$$a_{k+1} = \frac{2336}{2337} a_k$$

$$= \frac{2336}{2337}\left(\frac{2336}{2337} a_{k-1}\right)$$

$$= \frac{2336}{2337}\left(\frac{2336}{2337}\left(\frac{2336}{2337} a_{k-2}\right)\right)$$

$$\vdots$$

$$= \left(\frac{2336}{2337}\right)^{k+1} \underbrace{a_{(k+1)-(k+1)}}_{a_0 = 10 \text{ grams.}}$$

After 100 years, there will be

$$a_{100} = \left(\frac{2336}{2337}\right)^{100} \cdot 10 \approx \boxed{9.581 \text{ grams}}$$