

Math 115 Quiz 10: § 5.1-3 Summing
Rectangles

Wed 8 December 2010

Name: _____

You have 30 minutes to complete this quiz. Make your variables clear and consistent (so if you want to say, for example, $\frac{dy}{dx}$, you should also mention $y = f(x)$, or “ y is a function of x ”). Calculators are OK.

1. **Definitions/Concepts.** (1 pt each) True or False? No explanation necessary.

- (a) For an increasing function, the left-hand sum on a given interval with a given number of subdivisions is always less than the right-hand sum.

TRUE

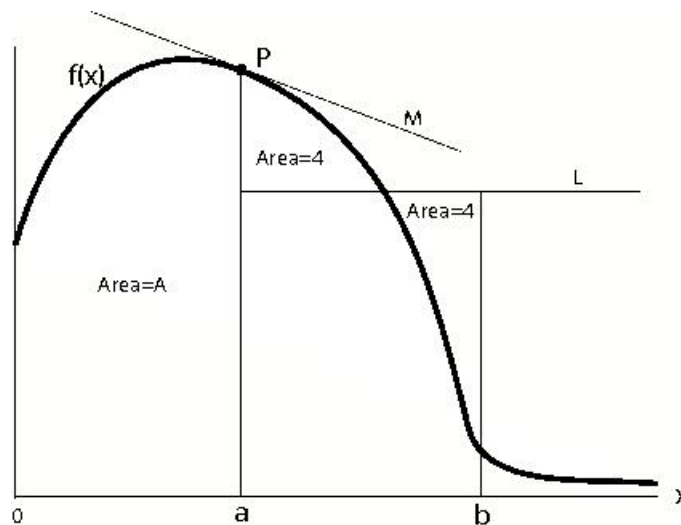
- (b) A 4-term left-hand Riemann sum approximation cannot give the exact value of a definite integral.

FALSE

- (c) The units for an integral of a function $f(x)$ are the same as the units for $f(x)$.

FALSE

2. **Questions/Problems.** Below you will write expressions for each of various quantities indicated on the graph of $f(x)$. Your expressions may involve integrals or derivatives. For example, if asked for the “ x -coordinate of the point P ,” you would write “ a ”.



- a) (1 pt) The height (above the x -axis) of the point P .

$f(a)$

b) (1 pt) The slope of the line M.

$$f'(a)$$

c) (1 pt) The size of the area A.

$$\int_0^a f(x)dx$$

d) (1 pt) The height of the line L.

$$\frac{4}{b-a}$$

OR (for extra credit)

$$\frac{\int_a^b f(x)dx}{b-a}$$

3. Computations/Algebra.

(a) (1 pt) If $F(t) = \frac{1}{2} \sin t^2$, find $F'(t)$.

$$F'(t) = t \cos t^2$$

OR (this problem was a typo – I meant to put $F(t) = \frac{1}{2} (\sin t)^2$)

$$F'(t) = \sin t \cos t$$

(b) Find $\int_{0.2}^{0.4} \sin t \cos t dt$ two ways:

(i) (1 pt) Numerically. Use $n = 4$ subdivisions then average the left and right -hand sums. Tip for showing your work: Use Σ -notation.

In this case $\Delta t = 0.05$ so the average will be

$$\begin{aligned} & \frac{1}{2} [0.05 (\sin 0.2 \cos 0.2 + \sin 0.25 \cos 0.25 + \sin 0.3 \cos 0.3 + \sin 0.35 \cos 0.35) + \\ & \quad 0.05 (\sin 0.25 \cos 0.25 + \sin 0.3 \cos 0.3 + \sin 0.35 \cos 0.35 + \sin 0.2 \cos 0.2)] \\ & \approx 0.0560 \end{aligned}$$

(ii) (1 pt) Using the Fundamental Theorem of Calculus.

$$\frac{1}{2} \sin 0.4^2 - \frac{1}{2} \sin 0.2^2 \approx 0.0597$$

OR, if you noticed the typo,

$$\frac{1}{2} (\sin 0.4)^2 - \frac{1}{2} (\sin 0.2)^2 \approx 0.0561$$