Math 115 Quiz 10: $\oint$ 5.1-3 Summing Rectangles **Wed 8 December 2010**
Name: _____

You have 30 minutes to complete this quiz. Make your variables clear and consistent (so if you want to say, for example, $\frac{dy}{dx}$, you should also mention $y = f(x)$, or "y is a function of x"). Calculators are OK.
Part 2.(b) is worth 2 points. All other lines are worth 1 point apiece.

1. **Definitions/Concepts.** – *none this week* –

2. **Questions/Problems.**

   (a) Suppose that a company (called All Things Food) has hired you as a consultant. You are to help them save their failing product, "Big J's Bar-B-Q Ice Cream." You have discovered that their cost and revenue functions (in dollars) are:

   $$C(q) = 100 + 2q \qquad \text{and} \qquad R(q) = 15q^{0.75},$$

   where q is the number of ice cream containers produced.

   a) What is the product's fixed cost?

   $100

   b) Last year, All Things Food produced 2400 containers of Big J's Bar-B-Q Ice Cream. What was their profit?

   $$\begin{aligned} \pi(2400) &= -(100 + 2(2400)) + 15(2400)^{.75} \\ &= \$243.39 \end{aligned}$$

   c) Find formulas for marginal cost and marginal revenue, and evaluate at $q = 2400$.

   $MC(q) = 2$

   $MC(2400) = \$2/\text{container of ice cream}$

   $MR(q) = (.75)15q^{-.25} = 11.25q^{-.25}$

   $MR(2400) = 11.25(2400)^{-.25} = \$1.61/\text{container of ice cream}$

d) Big J wants to increase production to do better this year. Based on the marginal revenue and marginal cost *at this point* ($q = 2400$), explain whether Big J's strategy is sound.

At $q = 2400$ the marginal cost is greater than the marginal revenue. This means further increasing production would reduce profit. Therefore Big J's strategy is not sound.

e) What production level will maximize the profit available to the company?
To maximize profit, set

$$\pi'(q) = 11.25q^{-.25} - 2 = 0.$$

The only critical point is at $q \approx 1001$. Notice $\pi'(q)$ is monotonically decreasing on the domain $q \geq 0$. In particular, π' must go from positive to negative at the critical point. There for the critical point is a maximum. The production level to maximize profit is $q = 1001$ containers of ice cream.

(b) The metal frame of a rectangular box has a square base. The horizontal rods in the base are made out of one metal and the vertical rods are made out of a different metal. If the horizontal rods expand at a rate of 0.001 cm/hr and the vertical rods expand at a rate of 0.002 cm/hr, at what rate is the volume of the box expanding when the base has an area of 9 cm^2 and the volume is 180 cm^3?

Let x denote the length of a horizontal rod in cm, y denote the length of a vertical rod in cm, and t denote time in hrs. Then the given data is

$$\frac{dx}{dt} = 0.001 \text{ cm/hr}, \frac{dy}{dt} = 0.002 \text{ cm/hr}.$$

Let $V = x^2y$ denote the volume. The volume of the box is expanding at the rate

$$\frac{dV}{dt} = 2x\frac{dx}{dt}y + x^2\frac{dy}{dt}.$$

When the base has area 9 cm^2, $x = 3$. Then since $V = 180$, this implies $y = 20$. Substituting all the numerical values gives

$$\begin{aligned} \frac{dV}{dt} &= 2(3)(0.001)(20) + (3)^2(0.002) \\ &\approx 0.138 \text{ cm}^3/\text{hr}. \end{aligned}$$

3. **Computations/Algebra.** – *none this week* –