Mon 4 Apr

- Exam 3: Friday. Covers §3.10-4.6

§4.6 Mean Value Theorem

In this section, we examine the Mean Value Theorem, one of the "big ideas" that provides the basis for much of calculus.

Before we get to the mean Value Theorem, we examine Rolle's Theorem, where the property $f(a) = f(b)$ holds, for some function $f(x)$ defined on an interval $[a, b]$.

Question

If you have two points $(a, f(a))$ and $(b, f(b))$, with the property that $f(a) = f(b)$, what might this look like?

Theorem (Rolle's Theorem)

Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$. Then there is at least one point c in (a, b) such that $f'(c) = 0$.

Essentially what Rolle's Theorem concludes is that at some point(s) between a and b, f has a horizontal tangent.

Question

Note the hypotheses in this theorem: f is continuous on $[a, b]$ and differentiable on (a, b). Why are these important?

Exercise

Determine whether Rolle's Theorem applies to the function $f(x) = x^3 - 2x^2 - 8x$ on the interval $[-2, 0]$.

- If it doesn't, find an interval for which Rolle's Thm could apply to that function.
- If it does, what is the "c" value so that $f'(c) = 0$?

Theorem (Mean Value Theorem (MVT))

If f is continuous on a closed interval $[a, b]$ and differentiable on (a, b), then there is at least one point c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

See Figure 4.68 on p. 276 for a visual justification of MVT.

The slope of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$ is

$$\frac{f(b) - f(a)}{b - a}.$$

MVT says that there is a point c on f where the tangent line at c (whose slope is $f'(c)$) is parallel to this secant line.

Question

Suppose you leave Fayetteville for a location in Fort Smith that is 60 miles away. If it takes you 1 hour to get there, what can we say about your speed? If it takes you 45 minutes to get there, what can we say about your speed?

Example

Let $f(x) = x^2 - 4x + 3$.

1. Determine whether the MVT applies to $f(x)$ on the interval $[-2, 3]$.
2. If so, find the point(s) that are guaranteed to exist by the MVT.

Example

How many points c satisfy the conclusion of the MVT for $f(x) = x^3$ on the interval $[-1, 1]$? Justify your answer.

Theorem (Zero Derivative Implies Constant Function)

If f is differentiable and $f'(x) = 0$ at all points of an interval I, then f is a constant function on I.

Theorem (Functions with Equal Derivatives Differ by a Constant)

If two functions have the property that $f'(x) = g'(x)$ for all x of an interval I, then $f(x) - g(x) = C$ on I, where C is a constant.

Theorem (Intervals of Increase and Decrease)

Suppose f is continuous on an interval I and differentiable at all interior points of I.

- *If $f'(x) > 0$ at all interior points of I, then f is increasing on I.*
- *If $f'(x) < 0$ at all interior points of I, then f is decreasing on I.*

4.6 Book Problems

7-14, 17-24