Take-Home Quiz 5: Miscellaneous (§10.1-10.3, 12.3, 12.6-12.7, 13.1, 13.5)

---

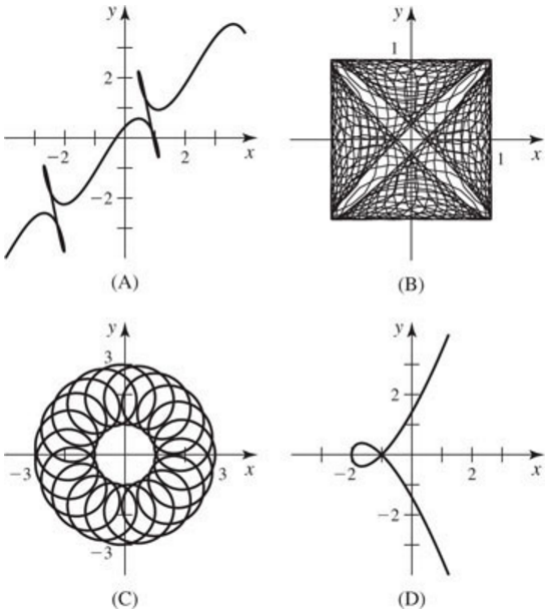
1. **10.1 #74** Match equations (a)-(d) with graphs (A)-(D). Explain your reasoning.

   (a) $x = t^2 - 2,\ y = t^3 - t$

   (b) $x = \cos(t + \sin 50t),\ y = \sin(t + \cos 50t)$

   (c) $x = t + \cos 2t,\ y = t - \sin 4t$

   (d) $x = 2\cos t + \cos 20t,\ y = 2\sin t + \sin 20t$



2. **10.3 #16, 18, 20** Find the points at which the following polar curves have a horizontal or a vertical tangent line.

   (a) $r = 2 + 2\sin\theta$

   (b) $r = 3 + 6\sin\theta$

   (c) $r = \sec\theta$

3. **12.3 #74** Find the value of a for which f is continuous at all points in $\mathbb{R}^2$.

$$f(x,y) = \begin{cases} \dfrac{1 + 2xy - \cos xy}{xy} & \text{if } xy \neq 0 \\ a & \text{if } xy = 0 \end{cases}$$

4. **13.1 #52** Find the value of $a > 0$ such that the average value of the function

$$f(x,y) = x + y - 8$$

   over the region $R = \{(x,y) \mid 0 \leq x \leq a,\ 0 \leq y \leq a\}$ is zero.

5. **10.2 #89** The equations

$$r = a + b\cos\theta \qquad \text{and} \qquad r = a + b\sin\theta$$

describe curves known as **limaçons**.

- If $|a| = |b|$ then the limaçon is a **cardiod**.
- If $|a| < |b|$ then the limaçon has an inner loop.
- If $|b| < |a| < 2|b|$ then the limaçon has a dent or dimple.
- If $|a| > 2|b|$ then the limaçon is oval-shaped.

Match equations (a)-(f) with the limaçons in figures (A)-(F).

(a) $r = -1 + \sin\theta$

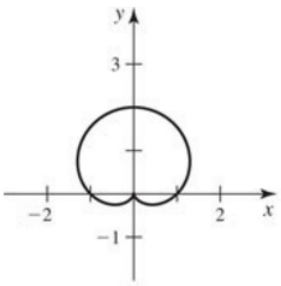
(b) $r = 2 + \sin\theta$

(c) $r = 1 + 2\sin\theta$

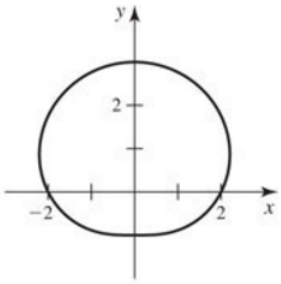
(d) $r = -1 + 2\cos\theta$

(e) $r = 1 - 2\cos\theta$

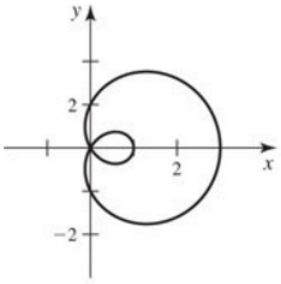
(f) $r = 1 + \frac{2}{3}\sin\theta$



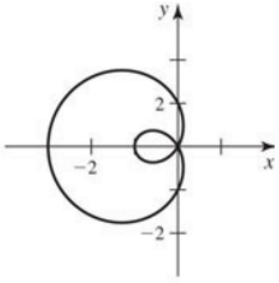
(A)



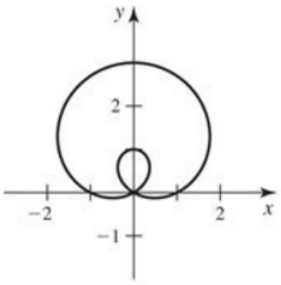
(B)



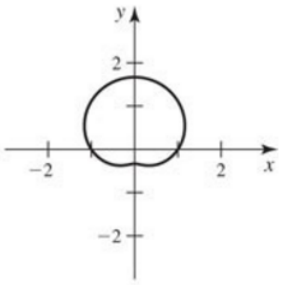
(C)



(D)



(E)



(F)

6. **12.6 #48** Consider the upper half of the ellipsoid

$$f(x,y) = \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{16}}$$

and the point $P = (0, \sqrt{8})$ given on the level curve $f(x,y) = \frac{1}{\sqrt{2}}$. Compute the slope of the line tangent to the level curve at P and verify that the tangent line is orthogonal to the gradient at that poitnt.

7. **12.7 #18** Find an equation of the plane tangent to the surface

$$z = 2 + 2x^2 + \frac{y^2}{2}$$

at the point $(-\frac{1}{2}, 1, 3)$. Sketch the surface along with the tangent plane.

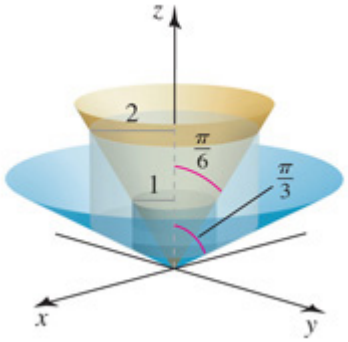
8. **12.7 #36** The volume of a right circular cone with radius r and height h is $V = \dfrac{\pi r^2 h}{3}$. Use linear approximation to:

   (a) approximate the change in the volume of the cone when the radius changes from $r = 6.5$ to $r = 6.6$ and the height changes from $h = 4.20$ to $h = 4.15$.

   (b) approximate the change in volume of the cone when the radius changes from $r = 5.40$ to $r = 5.37$ and the height changes from $h = 12.0$ to $h = 11.96$.

9. **13.5 #50, 52** Find the volume of the following solids:

   (a) the solid bounded by the cylinders $r = 1$ and $r = 2$ and the cones $\varphi = \frac{\pi}{6}$ and $\varphi = \frac{\pi}{3}$.



   (b) the solid inside the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.