Math 116: Notes on Chapter 6 **Last Updated: September 9, 2012**

1. $\oint$ 6.1 #7: Estimate $f(x)$ for $x = 2, 4, 6$, using the given values of $f'(x)$ and the fact that $f(0) = 50$.

x	0	2	4	6
$f'(x)$	17	15	10	2

Solution: Use the Fundamental Theorem of Calculus to get each of the values $f(2), f(4), f(6)$.

$$\int_0^2 f'(x)dx = f(2) - f(0)$$
$$\int_0^2 f'(x)dx + f(0) = f(2)$$

We don't actually know the value of the integral, but with the information given we can estimate using one rectangle. In computing a Riemann sum, $n = 1$, $\Delta t = 2$, $t_0 = 0$, and $t_1 = 2$. The lefthand sum is

$$\begin{aligned}\sum_{i=0}^{n-1} f'(t_i) \cdot \Delta t &= \sum_{i=0}^{0} f'(t_i) \cdot 2 \\ &= f'(t_0) \cdot 2 \\ &= f'(0) \cdot 2 \\ &= 34.\end{aligned}$$

The righthand sum is

$$\begin{aligned}\sum_{i=1}^{n} f'(t_i) \cdot \Delta t &= \sum_{i=1}^{1} f'(t_i) \cdot 2 \\ &= f'(t_1) \cdot 2 \\ &= f'(2) \cdot 2 \\ &= 30.\end{aligned}$$

Averaging the two, we get an estimate $\int_0^2 f'(x)dx \approx 32$. Going back to the FTOC formula, we can use the fact that $f(0) = 50$ to get

$$f(2) \approx 32 + 50 = 82.$$

To compute the other values, it does not matter which bounds a, b we choose when we apply the FTOC.

$$f(4) = \int_2^4 f'(x)dx + f(2)$$

We use Riemann sums again over one rectangle, so $n = 1$, $\Delta t = 2$, $t_0 = 2$, and $t_1 = 4$. For the lefthand sum we get $f'(2) \cdot 2 = 30$ and for the righthand sum we get $f'(4) \cdot 2 = 20$. Taking the average, we get $25 \approx \int_2^4 f'(x)dx$. Then we get the estimate

$$f(4) \approx 25 + 82 = 107.$$

Finally, computing $f(6)$ in the same way, we get

$$f(6) \approx 119.$$

2. Definite Integrals vs. Indefinite Integrals.
   In Chapter 5 we learned about the definite integral $\int_a^b f(x)dx$ and used techniques like Riemann sums and the FTOC to compute it. In practice, we almost never know a formula for an antiderivative F of f. Chapter 5 is full of examples where this happens and all we can compute are the specific values $F(a)$ and $F(b)$.

   In Chapter 6 we learn that if we know a formula for an antiderivative F then any vertical shift by a number C will also give an antiderivative. We express this by introducing a *different* symbol, the indefinite integral:

$$\int f(x)dx = F(x) + C$$

   The indefinite integral is not a number; notice how there are no bounds a and b and we don't have anything to compute. Rather, the indefinite integral is a generalized function of x. We say generalized because for every number we use for C we get a different function. Writing the antiderivative in this form shows that we are talking about them all. The indefinite integral gives another technique for computing definite integrals.

   To summarize, the symbol

$$\int_a^b f(x)dx$$

   is a value, usually a number (in some cases we'll have problems where we get a function of another variable but the point is there should be no xs in the answer). Here is where you plug in things for a and b and compute $F(b) - F(a)$. On the other hand, the symbol

$$\int f(x)dx$$

   is just an expression for the family of functions $F(x) + C$. It is not a number and there is nothing to compute.