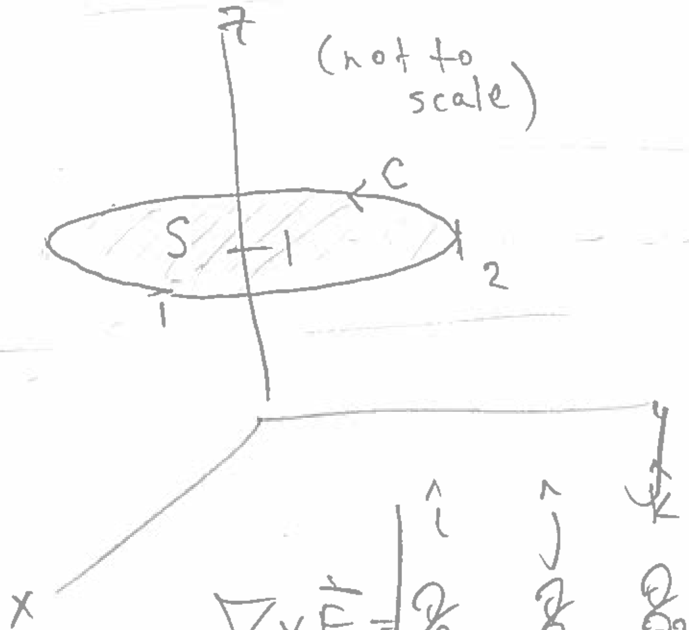
§14.7 #12 "Evaluate the line integral $\oint_C \vec{F}\cdot d\vec{r}$ by evaluating the surface integral in Stokes' Theorem with an appropriate choice of S. Assume that C has a counterclockwise orientation."

$$\vec{F} = \langle y, xz, -y \rangle$$

C = ellipse $x^2 + \frac{y^2}{4} = 1$ in the plane $z = 1$



$$\vec{r}(u,v) = \langle v\cos u, 2v\sin u, 1 \rangle$$

$$\vec{r}_u = \langle -v\sin u, 2v\cos u, 0 \rangle$$

$$\vec{r}_v = \langle \cos u, 2\sin u, 0 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle 0, 0, \underbrace{-2v\sin^2 u - 2v\cos^2 u}_{-2v} \rangle$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & xz & -y \end{vmatrix} = \langle -1 - x, 0 - 0, z - 1 \rangle$$

$$= \langle -1 - v\cos u, 0, \underbrace{1 - 1}_{0} \rangle$$

$\longrightarrow$

Stokes' Theorem says:

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$$

$$= \int_0^1 \int_0^{2\pi} \left((-1 - v\cos u)(0) + (0)(0) + 0(-2v) \right) du\, dv$$

$$\boxed{= 0}$$

* Just for fun:

Parametrize C: $\vec{r}(t) = \langle \cos t, 2\sin t, 1 \rangle$

$\vec{r}\,'(t) = \langle -\sin t, 2\cos t, 0 \rangle$

$\Rightarrow \vec{F} = \langle 2\sin t, \cos t(1), -2\sin t \rangle$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} (-2\sin^2 t + 2\cos^2 t)\, dt$$

. . .

$\longrightarrow$

$$= \cancel{2}\int_0^{2\pi}\left(-\left(\frac{\cancel{1}-\cos 2t}{\cancel{2}}\right)+\left(\frac{\cancel{1}+\cos 2t}{\cancel{2}}\right)\right)dt$$

$$= \int_0^{2\pi} 2\cos 2t\,dt = \sin 2t\Big|_0^{2\pi} \boxed{= 0}$$