

Quiz 12: Divergence, Curl, and Green's Theorem (§14.4-14.5)

1. (4 pts) $\mathbf{F} = \langle e^{-x+y}, e^{-y+z}, e^{-z+x} \rangle$ is a vector field in \mathbb{R}^3 .

(a) $\text{curl } \mathbf{F} =$

(b) $\text{div } \mathbf{F} =$

2. (2 pts) **Green's Theorem:** Let C be a simple closed smooth curve, oriented counterclockwise, that encloses a connected and simply connected region R in the plane. Let $\mathbf{F} = \langle f(x, y), g(x, y) \rangle$ denote a vector field, where f and g have continuous first partial derivatives on R . Then

(a) Green's Theorem says the circulation of \mathbf{F} on R is (write the equation):

(b) and that the flux of \mathbf{F} across the boundary of R is (write the equation):

3. (4 pts) Let $\mathbf{F} = \langle x - y, -x + 2y \rangle$ denote a vector field on the parallelogram

$$R = \{(x, y) \mid 1 - x \leq y \leq 3 - x, 0 \leq x \leq 1\}.$$

Compute (a) the circulation of \mathbf{F} on R and (b) the outward flux of \mathbf{F} across the boundary of R .