Math 115 Quiz 2: § 1.6-1.8, 2.1 **Name:** _____
Mon 27 September 2010

You have 15 minutes to complete this quiz. Calculators are OK. Eyes on your own paper and good luck!

1. **Definitions/Concepts.** (1 pt each) Suppose that $\lim_{x\to 3} f(x) = 7$. Are the following statements true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample.

   (a) $\lim_{x\to 3} (xf(x)) = 21$. **TRUE**

   $$\begin{aligned} \lim_{x\to 3} (xf(x)) &= \left(\lim_{x\to 3} x\right)\left(\lim_{x\to 3} f(x)\right) \\ &= 3 \cdot 7 \\ &= 21. \end{aligned}$$

   (b) If $g(3) = 4$, then $\lim_{x\to 3} (f(x) + g(x)) = 28$. **FALSE**

   Suppose

   $$g(x) = \begin{cases} 4 & \text{when } x = 3 \\ x & \text{when } x \neq 3. \end{cases}$$

   Then $\lim_{x\to 3} g(x) = 3$. Therefore

   $$\begin{aligned} \lim_{x\to 3} (f(x) + g(x)) &= \lim_{x\to 3} f(x) + \lim_{x\to 3} g(x) \\ &= 3 + 3 \\ &= 6 \\ &\neq 28. \end{aligned}$$

   This problem was taken out of the Chapter 1 Review section and was not meant to have the plus sign. However, the above solution illustrates why you should still not assume the limit of g is 4 as x approaches 3, even though $g(3) = 4$.

2. **Questions/Problems.** A ball is tossed into the air from a bridge, and its height, y (in feet), above the ground t seconds after it is thrown is given by

   $$y = f(t) = -16t^2 + 50t + 36.$$

   (a) (1 pt) How high above the ground is the bridge?

   The height of the bridge is given at the initial moment, when $t = 0$. So

   $$\begin{aligned} f(0) &= -16(0)^2 + 50(0) + 36 \\ &= 36 \text{ ft.} \end{aligned}$$

(b) (1 pt) What is the average velocity of the ball for the first second?

Compute the difference in height from the initial moment to the first second:

$$\begin{aligned} f(1) - f(0) &= 70 - 36 \\ &= 34 \text{ ft/sec.} \end{aligned}$$

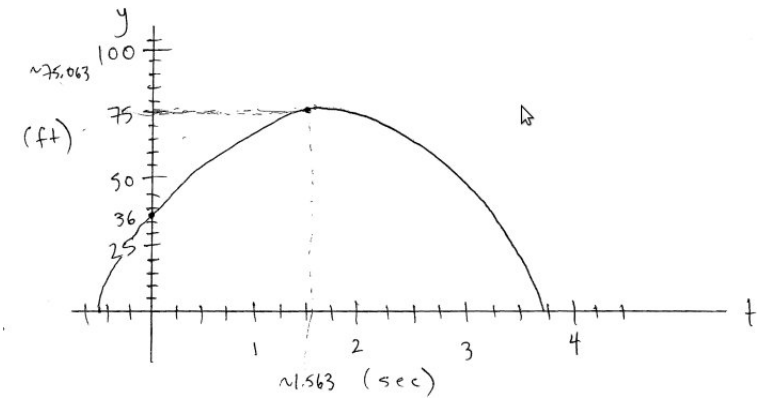
(c) (1 pt) Approximate the velocity of the ball at $t = 1$ second.

Look at the change in height at times close to $t = 1$ second.

$$\begin{aligned} \frac{f(1) - f(0.999)}{0.001} &= 18.016 \\ \frac{f(1) - f(0.9999)}{0.0001} &\approx 18.002 \\ \frac{f(1) - f(0.99999)}{0.00001} &\approx 18.000 \\ \frac{f(1.00001) - f(1)}{0.00001} &\approx 18.000 \\ \frac{f(1.0001) - f(1)}{0.0001} &\approx 17.998 \end{aligned}$$

Conclude the velocity at $t = 1$ second is approximately 18.000 ft/sec.

(d) (2 pts) Graph f, and determine the maximum height the ball reaches. What is the velocity at the time the ball is at its peak?



The maximum height is approximately 75.063 ft. The velocity at that height is 0 ft/sec.

(e) (1 pt) Use the graph to decide at what time, t, the ball reaches its maximum height.

The ball reaches its maximum height at approximately 1.563 sec.

3. **Computations/Algebra.** (1 pt each)

(a) Find k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} kx & x \leq 3 \\ 5 & 3 < x \end{cases}$$

The function is continuous on any interval that does not contain 3, unless k is assigned the appropriate value. To make the function continuous at $x = 3$, set $k \cdot (3) = 5$, which implies $k = \frac{5}{3}$.

(b) Find k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} kx & 0 \leq x < 2 \\ 3x^2 & 2 \leq x \end{cases}$$

As in part (a), set $k \cdot (2) = 3(2)^2$. Then $k = 6$.

The original quiz had a typo in this problem which has since been corrected.