§5.5 Slides
Cal I Spring 2016

1. $\int \sin^{10}x \cos x\,dx \longrightarrow u = \sin x$

$du = \cos x\,dx$

$$\int u^{10}\,du = \frac{u^{11}}{11} + C = \boxed{\frac{\sin^{11}x}{11} + C}$$

Check:

$$\frac{d}{dx}\left(\frac{\sin^{11}x}{11} + C\right) = \frac{1}{\cancel{11}}\left(\cancel{11}\sin^{10}x\right)\cos x + 0 \quad \checkmark$$

2. $-\int \frac{\csc x \cot x}{1+\csc x}\,dx \longrightarrow u = 1 + \csc x$

$du = -\csc x \cot x\,dx$

$$= \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|1+\csc x| + C}$$

Check:

$$\frac{d}{dx}\left(\ln|1+\csc x| + C\right) = \frac{1}{1+\csc x}\cdot\left(-\csc x \cot x\right) \quad \checkmark$$

3. $\int \frac{1}{(10x-3)^2}\,dx \longrightarrow u = 10x - 3$

$du = 10\,dx \Rightarrow \frac{1}{10}\,du = dx$

$$= \int \frac{1}{10}\cdot\frac{1}{u^2}\,du = \frac{1}{10}\left(\frac{u^{-1}}{-1}\right) + C = \boxed{\frac{-1}{10}\left(\frac{1}{10x-3}\right) + C}$$

→

Check: $\frac{d}{dx}\left(\frac{-1}{10}\left(\frac{1}{10x-3}\right)+C\right) = \frac{-1}{10}(-1)(10x-3)^{-2}(10)+0$

$= \frac{1}{(10x-3)^2}$ ✓

4. $\int (3x^2+8x+5)^8(3x+4)\,dx \longrightarrow u = 3x^2+8x+5$

$du = (6x+8)\,dx$

$\Rightarrow \frac{1}{2}du = (3x+4)\,dx$

$= \int \frac{1}{2}u^8\,du = \frac{1}{2}\cdot\frac{u^9}{9}+C = \boxed{\frac{(3x^2+8x+5)^9}{18}+C}$

Check: $\frac{d}{dx}\left(\frac{(3x^2+8x+5)^9}{18}+C\right)$

$= \frac{9(3x^2+8x+5)^8}{18_2}(6x+8)$

$= (3x^2+8x+5)^8(3x+4)$ ✓

Check: $\frac{d}{dx}\left(\frac{-1}{x+1}+\frac{1}{(x+1)^2}-\frac{1}{3(x+1)^3}+C\right)$

$= -1(-1)(x+1)^{-2} - 2(x+1)^{-3} - \frac{1}{3}(-3)(x+1)^{-4}$

→

$$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} + \frac{1}{(x+1)^4}$$

$$= \frac{(x+1)^2 - 2(x+1) + 1}{(x+1)^4} = \frac{((x+1)-1)^2}{(x+1)^4} = \frac{x^2}{(x+1)^4} \checkmark$$

$$\int \frac{x}{\sqrt{x+1}}\,dx \longrightarrow u = x+1 \Rightarrow x = u-1$$
$$du = dx$$

$$= \int \frac{u-1}{\sqrt{u}}\,du = \int \left(u^{1-\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + C$$

$$= \frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + C$$

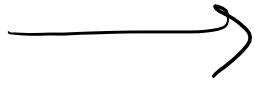
Check:

$$\frac{d}{dx}\left(\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + C\right)$$

$$= \frac{2}{3}\left(\frac{3}{2}\right)(x+1)^{1/2}(1) - 2\left(\frac{1}{2}\right)(x+1)^{-1/2} + 0$$

$$= \sqrt{x+1} - \frac{1}{\sqrt{x+1}} = \frac{(\sqrt{x+1})^2 - 1}{\sqrt{x+1}}$$

$$= \frac{x+1-1}{\sqrt{x+1}} = \frac{x}{\sqrt{x+1}} \checkmark$$



$$\int_0^2 \frac{2x}{(x^2+1)^2}\,dx \longrightarrow u = x^2+1 \qquad x=0 \Rightarrow u = 0^2+1 = 1$$

$$du = 2x\,dx \qquad x=2 \Rightarrow u = 2^2+1 = 5$$

$$= \int_1^5 \frac{du}{u^2} = \frac{u^{-1}}{-1}\Bigg|_1^5 = -\frac{1}{5} - \left(-\frac{1}{1}\right)$$

$$\boxed{= \frac{4}{5}}$$