Take-Home Quiz 2: Integration techniques (§5.3-5.5)

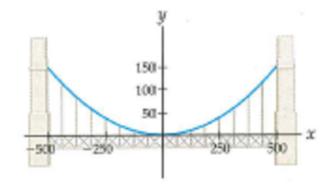
Directions: This quiz is due on September 18, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

- 1. (a) §5.3 #12 Try to perform a partial-fraction decomposition of the improper rational function $\frac{x^3}{(x-1)(x-2)}$, without using long division. What goes wrong?
 - (b) §5.3 #14 What goes wrong if you try to decompose $\frac{1}{(x^2+1)^2}$ into the form $\frac{A}{x^2+1} + \frac{B}{(x^2+1)^2}$?
- 2. Compute the integrals.

(a) §5.3 #54
$$\int \frac{e^x}{e^{3x} - 2e^{2x}} dx$$

(b) §5.3 #56
$$\int \frac{\ln x + 1}{x((\ln x)^2 - 4)} dx$$

- 3. Describe strategies for solving the following types of integrals (see the table on p. 445).
 - (a) §5.4 #10 $\int \cot^k x \, dx$, k odd
 - (b) §5.4 #12 $\int \csc^k x \ dx, k = 2$
 - (c) §5.4 #14 $\int \sin^m x \cos^n x \, dx$, one of m and n odd
 - (d) §5.4 #16 $\int \sec^m x \tan^n x \, dx$, m even
 - (e) §5.4 #18 $\int \sec^m x \tan^n x \ dx$, m odd and n even
- 4. §5.5 #92 The main cable on a certain suspension bridge follows a parabolic curve with equation $f(x) = (0.025x)^2$, measured in feet:



(a) In general, the length of a curve f(x) from x = a to x = b can be calculated from the formula

$$\int_{b}^{a} \sqrt{1 + (f'(x))^2} \ dx.$$

Write down a specific definite integral that represents the length of the main cable of the suspension bridge.

- (b) Use trigonometric substitution to solve the definite integral and determine the length of the cable.
- 5. §5.5 #94 Using part (a) as a guide, prove part (b) of Theorem 5.18:

For $x \in (-\infty, \infty)$ and $u \in (-\frac{\pi}{2}, \frac{\pi}{2})$, the substitution $x = a \tan u$ gives $x^2 + a^2 = a \sec u$.

Your proof should include a discussion of domains and a consideration of absolute values.