Math 116 Quiz 7: $\oint$ 9.1-9.3 **Tue 20 Nov 2012**
(Sequences and Series) **Name:** SOLUTIONS

You have 35 minutes to complete this quiz. Eyes on your own paper and good luck!

1. **Definitions/Concepts.** (1 pt ea) Decide whether each of the statements below is *True* or *False*. Write the entire word *True* or *False*. If the statement is false, briefly explain why.

   (a) A convergent sequence is bounded.
   True

   (b) A bounded sequence converges.
   False; the sequence $1, -1, 1, -1, \ldots$ is bounded but does not converge.

   (c) Changing a finite number of terms in a series does not change whether or not it converges, although it may change the value of its sum if it does converge.
   True

   (d) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
   True

2. **Questions/Problems.** *(from April 2011 Final Exam)* You are trapped on an island, and decide to build a signal fire to alert passing ships. You start the fire with 200 pounds of wood. During the course of a day, 40% of the wood pile burns away (so 60% remains). At the end of each day, you add another 200 pounds of wood to the pile. Let W_n denote the weight of the wood pile immediately after adding the nth load of wood (the inital 200-pound pile counts as the first load).

   (a) (3 pts) Find expressions for W_1, W_2, W_3.
   -see the solution posted on the course website -

   (b) (3 pts) Find a closed form expression for W_n (a *closed form* expression means your answer should not contain a large summation).
   -see the solution posted on the course website -

(c) (2 pts) Instead of starting with 200 pounds of wood and adding 200 pounds every day, you decide to start with P pounds of wood and add P pounds every day. If you plan to continue the fire indefinitely, determine the largest value of P for which the weight of the wood pile will never exceed 1000 pounds.

-see the solution posted on the course website -

3. **Computations/Algebra.**

(a) (1 pt) Find a formula for the general term of the sequence $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \ldots$.

$$s_n = \frac{n}{2n+1}$$

(b) (2 pts) Does the sequence given by $s_n = \frac{2n+(-1)^n 5}{4n-(-1)^n 3}$ converge or diverge? If it converges then find its limit.

To see if the limit exists, write

$$\begin{aligned}\lim_{n\to\infty} s_n &= \lim_{n\to\infty} \frac{2n+(-1)^n 4}{4n-(-1)^n 3} \\ &= \frac{\lim_{n\to\infty} 2n+(-1)^n 4}{\lim_{n\to\infty} 4n-(-1)^n 3} \\ &= \frac{\lim_{n\to\infty} 2n}{\lim_{n\to\infty} 4n},\end{aligned}$$

because whether n is even or odd, we are still adding or subtracting the same constant, no matter how large n gets. Therefore the end behavior will be dominated by the linear terms $2n$ and $4n$.

$$\begin{aligned}&= \lim_{n\to\infty} \frac{2n}{4n} \\ &= \lim_{n\to\infty} \frac{2}{4} = \frac{1}{2}.\end{aligned}$$

So the sequence converges, to $\frac{1}{2}$.

(c) (3 pts) Find the first three terms of the sequence of partial sums for the series

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

$$\begin{aligned}
S_1 &= \frac{1}{1(1+1)} \\
&= \frac{1}{2} \\
S_2 &= S_1 + \frac{1}{2(2+1)} \\
&= \frac{1}{2} + \frac{1}{6} = \frac{4}{6} \\
&= \frac{2}{3} \\
S_3 &= S_2 + \frac{1}{3(3+1)} \\
&= \frac{2}{3} + \frac{1}{12} = \frac{9}{12} \\
&= \frac{3}{4}
\end{aligned}$$

(d) (2 pts) Does the series $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$ converge or diverge?
We can try the integral test to check for convergence. The series is positive. To see if it is decreasing beyond a certain point, check the accuracy of the following statement:

$$\begin{aligned}
\frac{n+1}{2n+3} &> \frac{(n+1)+1}{2(n+1)+3} = \frac{n+2}{2n+5} \\
(n+1)(2n+5) &> (n+2)(2n+3) \\
2n^2+8n+5 &> 2n^2+7n+6 \\
8n+5 &> 7n+6 \\
n &> 1.
\end{aligned}$$

From this we can conclude the function in the series is decreasing for all $n > 1$.
Evaluate the corresponding integral:

$$\begin{aligned}
\int_1^{\infty} \frac{x+1}{2x+3} dx &= \frac{1}{2} \int_5^{\infty} \frac{u-3}{2u} du \\
&\text{(by setting } u = 2x+3) \\
&= \frac{1}{2} \left(\int_5^{\infty} \frac{1}{2} du - \int_5^{\infty} \frac{3}{2u} du \right),
\end{aligned}$$

but both terms in the parentheses diverge by the p-test ($p = 0$ and then $p = 1$) so we don't need to evaluate any further, and the sequences diverges.