Mon 1 Feb

- GET YOUR CLICKER NOW.
- EXAM 1 is one week from Friday. Covers up to §3.1 (see the semester schedule of material on the course webpage). You must attend your own lecture on exam day.

Exercise

What are the vertical and horizontal asymptotes of $f(x) = \dfrac{x^2}{2x+1}$?

2.5 Book Problems

9-14, 15-33 (odds), 41-49 (odds), 53-59 (odds), 67

§2.6 Continuity

Informally, a function f is "continuous at $x = a$" means for x-values anywhere close enough to a the graph can be drawn without lifting a pencil. In other words, no holes, breaks, asymptotes, etc.

Definition

A function f is **continuous** at a means

$$\lim_{x \to a} f(x) = f(a).$$

If f is not continuous at a, then a is a **point of discontinuity**.

In order to claim something is continuous, you must verify all three:

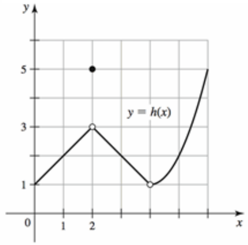
1. $f(a)$ is defined (i.e., a is in the domain of f – no holes, asymptotes).
2. $\lim_{x \to a} f(x)$ exists. You must check both sides and make sure they equal the same number.
3. $\lim_{x \to a} f(x) = f(a)$ (i.e., the value of f equals the limit of f at a).

Question

What is an example of a function that satisfies this condition?

Example

- Where are the points of discontinuity of the function below?
- Which aspects of the checklist fail?



recall (Continuity Checklist):

1. function is defined
2. the two-sided limit exists
3. 2. = 1.

If f and g are continuous at a, then the following functions are also continuous at a. Assume c is a constant and $n > 0$ is an integer.

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\dfrac{f}{g}$, provided $g(a) \neq 0$
6. $[f(x)]^n$

From the rules above, we can deduce:

1. Polynomials are continuous for all $x = a$.
2. Rational functions are continuous at all $x = a$ except for the points where the denominator is zero.
3. If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ is continuous at a.

Consider the cases where f is not defined past a certain point.

Definition

A function f is **continuous from the left** (or **left-continuous**) at a means

$$\lim_{x \to a^-} f(x) = f(a);$$

a function f is **continuous from the right** (or **right-continuous**) at a means

$$\lim_{x \to a^+} f(x) = f(a).$$

Definition

A function f is **continuous on an interval** I means it is continuous at all points of I.

Notation: Intervals are usually written

$$[a,b]\,,\ (a,b]\,,\ [a,b)\,,\ \text{or}\ (a,b)\,.$$

When I contains its endpoints, "continuity on I" means continuous from the right or left at the endpoints.

Example

Let $f(x) = \begin{cases} x^3 + 4x + 1 & \text{if } x \leq 0 \\ 2x^3 & \text{if } x > 0. \end{cases}$

1. Use the continuity checklist to show that f is not continuous at 0.
2. Is f continuous from the left or right at 0?
3. State the interval(s) of continuity.

(assuming m and n are positive integers and $\frac{n}{m}$ is in lowest terms)

- If m is odd, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points at which f is continuous.
- If m is even, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points a at which f is continuous and $f(a) \geq 0$.

Question

Where is $f(x) = \sqrt[4]{4 - x^2}$ continuous?

Trig Functions: The basic trig functions are all continuous at all points IN THEIR DOMAIN. Note there are points of discontinuity where the functions are not defined – for example, $\tan x$ has asymptotes everywhere that $\cos x = 0$.

Exponential Functions: The exponential functions b^x and e^x are continuous on all points of their domains.

Inverse Functions: If a continuous function f has an inverse on an interval I (meaning if $x \in I$ then $f^{-1}(y)$ passes the vertical line test), then its inverse f^{-1} is continuous on the interval J, which is defined as all the numbers $f(x)$, given x is in I.

Theorem (Intermediate Value Theorem)

Suppose f is continuous on the interval $[a,b]$ and L is a number satisfying

$$f(a) < L < f(b) \quad \text{or} \quad f(b) < L < f(a).$$

Then there is at least one number $c \in (a,b)$, i.e., $a < c < b$, satisfying

$$f(c) = L.$$

Example

Let $f(x) = -x^5 - 4x^2 + 2\sqrt{x} + 5$. Use IVT to show that $f(x) = 0$ has a solution in the interval $(0, 3)$.

Exercise

Which of the following functions is continuous for all real values of x?

(A) $f(x) = \dfrac{x^2}{2x+1}$

(B) $g(x) = \sqrt{3x^2 - 2}$

(C) $h(x) = \dfrac{5x}{|x^8 - 1|}$

(D) $j(x) = \dfrac{5x}{x^8 + 1}$

2.6 Book Problems

9-25 (odds), 35-45 (odds), 59, 61, 63, 83, 85