Math 116 Quiz 1: $\oint$ 5.1-5.4, 6.1-6.2 **Name:** ____________________
Tue 11 Sep 2012

You have 15 minutes to complete this quiz. Eyes on your own paper and good luck!

1. **Definitions/Concepts.** (3 pts) Write down the Fundamental Theorem of Calculus.
   If f is continuous on the interval $[a, b]$ and $f(t) = F'(t)$, then
   $$\int_a^b f(t)dt = F(b) - F(a).$$

2. **Questions/Problems.**

   (a) (2 pts) Recall that when we want to estimate area under a curve for a function $f(t)$ over the interval $t \in [a, b]$ we can use a left-hand or right-hand approximation. Let n denote the number of equally-sized subdivsions we use to divde the interval $[a, b]$. Then
   $$\Delta t = \frac{b-a}{n}$$
   and we can let $t_0 = a,\ t_1 = t_0 + \Delta t,\ t_2 = t_1 + \Delta t$, etc.
   Suppose you have the data:

t	0	4	8	12	16
f(t)	25	23	22	20	17

Table 1: number of students awake after t minutes into a boring lecture

Use this data to fill in the missing information:

$n = 4$				
$\Delta t = 4$				
$a = 0$	$b = 16$			
$t_0 = 0$	$t_1 = 4$	$t_2 = 8$	$t_3 = 12$	$t_4 = 16$
$f(t_0) = 25$	$f(t_1) = 23$	$f(t_2) = 22$	$f(t_3) = 20$	$f(t_4) = 17$
$n = 2$				
$\Delta t = 8$				
$a = 0$	$b = 16$			
$t_0 = 0$	$t_1 = 8$	$t_2 = 16$		
$f(t_0) = 25$	$f(t_1) = 22$	$f(t_2) = 17$		

MORE QUIZ ON THE BACK –>

(b) (3 pts each) Write out the entire word, either True or False. No justification is needed.

i. If $\int_0^2 (3f(x)+1)dx = 8$, then $\int_0^2 f(x)dx = 2$.

True. Use the linearity property of integrals (Theorem 5.3 from the book) and algebraic manipulation to solve for $\int_0^2 f(x)dx$:

$$\begin{aligned} \int_0^2 (3f(x)+1)dx &= 8 \\ 3\int_0^2 f(x)dx + \int_0^2 1dx &= 8 \\ 3\int_0^2 f(x)dx + x\big|_0^2 &= 8 \\ 3\int_0^2 f(x)dx &= 6 \\ \int_0^2 f(x)dx &= 2 \end{aligned}$$

ii. If $f(x) = \int_{-2x}^0 (1+t^4)dt$, then $f(x)$ is decreasing.

False. The bounds of integration are allowed to have a different variable from the one we're integrating over. Any symbols other than t we just treat like a constant while integrating:

$$\begin{aligned} f(x) = \int_{-2x}^0 (1+t^4)dt &= \left(t + \frac{t^5}{5}\right)\Bigg|_{-2x}^0 \\ &= 0 - \left((-2x) + \frac{(-2x)^5}{5}\right) \\ &= \frac{42}{5}x, \end{aligned}$$

a line with positive slope everywhere.

iii. If $f(x) \le g(x)$ for $x \in [0,1]$, then $\int_0^1 f(x)dx \le \int_0^1 g(x)dx$.

True. Theorem 5.4 in the text says so if we put $a = 0$ and $b = 1$.

iv. If $g(x)$ is odd and $\int_1^3 g(x)dx = 2$, then $\int_{-3}^{-1} g(x)dx = 2$.

False. An odd function is symmetric about the origin, meaning the left of the vertical axis is a mirror image of the right, turned upside down. The value of g for $x \in [1,3]$ is negated for $-x \in [-3,-1]$. That means $\int_{-3}^{-1} g(x)dx = -2$.

v. If $f(t)$ is measured in dollars per year, and t in measured in years, then $\int_a^b f(t)dt$ is measured in dollars per years squared.

False. Think of the integral symbol as a sum, and dt as a tiny change in t. Then to see the units we can write:

$$\int_a^b f(t)dt = \text{“sum over } t \in [a,b] \text{ of } \left(f(t)\,\frac{\text{dollars}}{\text{year}}\right) \cdot (dt \text{ years})\text{”}$$

The measurements in years cancel, so the integral is just measured in dollars.

3. **Computations/Algebra.**

   -none this week-