MATH 2574 (Calculus III)
Spring 2017

Name: SOLUTIONS

Fri 10 Feb 2017

Exam 1: Intro to Multidimensional Calculus (§11.1-11.7, 12.1-12.2)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems. No electronic devices (phones, iDevices, computers, etc) except for a **basic scientific calculator**. On story problems, round to one decimal place. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

In addition, please provide the following data:

Drill Instructor: ______________________

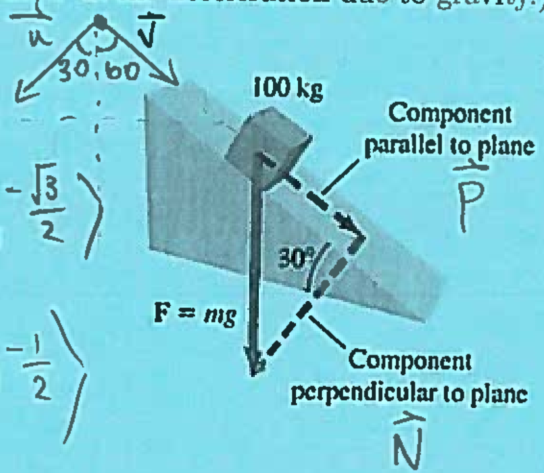
Drill Time: ______________________

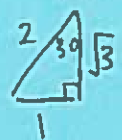
Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt) ______________________

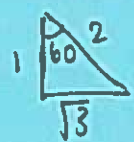
Good luck!

1. **(16 pts)** A 100 kg box rests on a ramp with an incline of 30° to the floor (see figure). Find the components of the force perpendicular to and parallel to the ramp. (The vertical component of the force exerted by an object of mass m is its weight, which is mg, where $g = 9.8\ \text{m/s}^2$ is the acceleration due to gravity.)





$\vec{u} = \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$



$\vec{v} = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

$\vec{F} = \langle 0, -100g \rangle = \langle 0, -980 \rangle\ \text{kgm/s}^2$

$$\vec{N} = \text{proj}_{\vec{u}} \vec{F} = \frac{\left(-\frac{1}{2}\right)(0) + \left(-\frac{\sqrt{3}}{2}\right)(-980)}{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$$

$$= \left\langle -\frac{980}{4}\sqrt{3}, -\frac{3}{4}(980) \right\rangle = \boxed{\langle -245\sqrt{3}, -735 \rangle\ \text{kgm/s}^2}$$

$$\approx \langle -424.4, -735 \rangle\ \text{kgm/s}^2$$

$$\vec{P} = \text{proj}_{\vec{v}} \vec{F} = \frac{\left(\frac{\sqrt{3}}{2}\right)(0) + \left(-\frac{1}{2}\right)(-980)}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$$

$$= \left\langle \frac{980}{4}\sqrt{3}, -\frac{980}{4} \right\rangle = \boxed{\langle 245\sqrt{3}, -245 \rangle\ \text{kgm/s}^2}$$

$$\approx \langle 424.4, -245 \rangle\ \text{kgm/s}^2$$

2. Determine whether the following statements are true or false. You must justify your answer.

(a) **(5 pts)** The domain of the function $f(x, y) = 1 - |x - y|$ is $\{(x, y) \mid x \geq y\}$.

False. The absolute value function does not have a restricted domain. The domain for $f(x,y)$ is $\mathbb{R}^2$.

(b) **(5 pts)** $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = \mathbf{0}$

True. $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$, by definition of the cross product. Orthogonal vectors have a dot product of 0.

(c) **(5 pts)** The domain of the function $u = f(w, x, y, z)$ is a region in $\mathbb{R}^3$.

False. $f(w,x,y,z)$ has four independent variables and so its domain is in $\mathbb{R}^4$.

(d) **(5 pts)** All level curves of the plane $z = 2x - 3y$ are lines.

True. If $z = z_0 = 2x - 3y$,

then $y = \frac{2x - z_0}{3} = \frac{2}{3}x - \frac{z_0}{3}$ is the equation of a line.

3. **(18 pts)** Determine an equation of the line $\vec{\ell}(t)$ that is perpendicular to the lines

$$\mathbf{r}(t) = \langle -2+3t, 2t, 3t\rangle = \langle -2,0,0\rangle + t\langle 3,2,3\rangle$$
$$\mathbf{R}(s) = \langle -6+s, -8+2s, -12+3s\rangle = \langle -6,-8,-12\rangle + s\langle 1,2,3\rangle$$

and passes through the point of intersection of the lines **r** and **R**.

Intersection point:

$$-2+3t = -6+s \Rightarrow s = 4+3t$$
$$2t = -8+2s$$
$$= -8+2(4+3t) = \cancel{-8}+\cancel{8}+6t$$
$$\Rightarrow 0 = 4t \Rightarrow t = 0,\ s = 4+3(0) = 4$$

Check z-component!

$$3t = -12+3s$$
$$3(0) = -12+3(4)\ \checkmark$$

$$\vec{r}(0) = \vec{R}(4) = \langle -2,0,0\rangle$$

Find a parallel vector:

$$\langle 3,2,3\rangle \times \langle 1,2,3\rangle = \langle \underbrace{(2)(3)-3(2)}_{0}, \underbrace{3(1)-3(3)}_{-6}, \underbrace{3(2)-2(1)}_{4}\rangle$$

1 2 3

$$\Rightarrow \boxed{\vec{\ell}(t) = \langle -2,0,0\rangle + t\langle 0,-6,4\rangle}$$

$$= \langle -2, -6t, 4t\rangle$$

4. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable functions at $t = 0$ with $\mathbf{u}(0) = \langle 0, 1, 1\rangle$, $\mathbf{u}'(0) = \langle 0, 7, 1\rangle$, $\mathbf{v}(0) = \langle 0, 1, 1\rangle$, $\mathbf{v}'(0) = \langle 1, 1, 2\rangle$. Evaluate the following expressions:

(a) **(6 pts)** $\left.\frac{d}{dt}(\mathbf{u}\cdot\mathbf{v})\right|_{t=0} = \left.\left(\vec{u}'\cdot\vec{v} + \vec{u}\cdot\vec{v}'\right)\right|_{t=0}$

$= \vec{u}'(0)\cdot\vec{v}(0) + \vec{u}(0)\cdot\vec{v}'(0)$

$= \langle 0,7,1\rangle\cdot\langle 0,1,1\rangle + \langle 0,1,1\rangle\cdot\langle 1,1,2\rangle$

$= 0 + 7 + 1 + 0 + 1 + 2$

$= \boxed{11}$

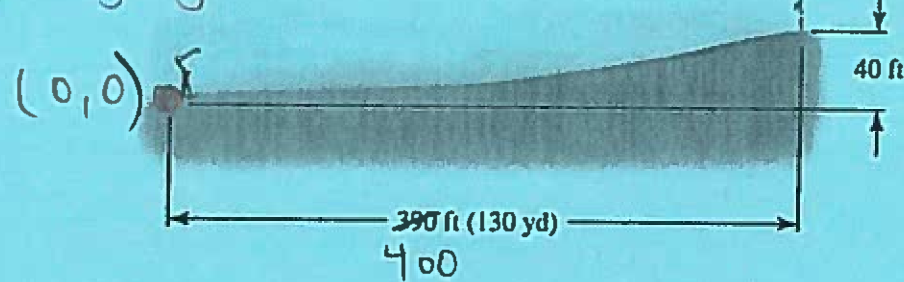
(b) **(6 pts)** $\left.\frac{d}{dt}(\cos(t)\mathbf{u}(t))\right|_{t=0}$

$= -\sin(0)\vec{u}(0) + \cos(0)\vec{u}'(0)$

$= \vec{u}'(0) = \boxed{\langle 0,7,1\rangle}$

5. A golfer stands 400 ft horizontally from the hole and 40 ft below the hole (see figure).

gravity $= g = 32$ ft/s^2



Suppose the ball is hit with an initial speed of 150 ft/s, at an angle of θ from the ground.

(a) **(12 pts)** Find the acceleration $\mathbf{a}(t)$, velocity $\mathbf{v}(t)$, and position $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ vectors for the trajectory of the ball.

$$\boxed{\vec{a}(t) = \langle 0, -32\rangle \text{ ft/s}^2}$$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0, -32t\rangle + \vec{c}$$

$$\vec{v}(0) = \langle 0, 0\rangle + \vec{c} = \langle 150\cos\theta, 150\sin\theta\rangle$$

$$\Rightarrow \boxed{\vec{v}(t) = \langle 150\cos\theta, -32t + 150\sin\theta\rangle \text{ ft/s}}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 150\cos\theta\, t, -16t^2 + 150\sin\theta\, t\rangle + \vec{c}$$

$$\vec{r}(0) = \langle 0, 0\rangle + \vec{c} = \langle 0, 0\rangle$$

$$\Rightarrow \boxed{\vec{r}(t) = \langle 150\cos\theta\, t, -16t^2 + 150\sin\theta\, t\rangle \text{ ft}}$$

(b) **(6 pts)** Write down a system of two equations to find the two unknowns: (1) time of flight and (2) θ. **Do not** solve the system.

$$150\cos\theta\, t = 400$$

$$-16t^2 + 150\sin\theta\, t = 40$$

6. **(15 pts)** Match equations (a)-(f) with the surfaces (A)=(F).

D (a) $y - z^2 = 0$ ← cylinder of a parabola

E (b) $4x^2 + \frac{y^2}{9} + z^2 = 1$ ← ellipsoid

B (c) $x^2 + \frac{y^2}{9} = z^2$ ← elliptic cone

A (d) $2x - 3y - z = 5$ ← linear

F (e) $x^2 + \frac{y^2}{9} - z^2 = 1$ ← hyperboloid

C (f) $y = |x|$ ← absolute value cylinder