Math 115 Quiz 8: § 4.3, 4.4 **Mon 15 November 2010**
Optimization **Name:** _____

You have 30 minutes to complete this quiz. Make your variables clear and consistent (so if you want to say, for example, $\frac{dy}{dx}$, you should also mention $y = f(x)$, or "y is a function of x"). Calculators are OK.

1. **Definitions/Concepts.** (1 pt each)

   (a) Give an example of a family of functions, $f(x)$, depending on a parameter a, such that each member of the family has exactly one critical point.

   (b) TRUE or FALSE: If the radius of a circle is increasing at a constant rate, then so is the area.

2. **Questions/Problems.** Verify your extrema using either a well-labeled graph or the 2nd derivative.

   (a) (2pts) What point(s) on the graph of $y = \frac{1}{x^2}$ is closest to $(0, 0)$?

   (b) (3 pts) I have a square piece of sheet metal, 1 meter on a side. I plan to cut equal squares from the four corners and fold up the sides to make a box (with no top). How big should the cut-off squares be in order to maximize the volume of the box?

turn over →

3. **Computations/Algebra.** (1 pt each) Find the critical points.

(a) $f(x) = (x-a)^2 + b$

(b) $g(y) = y^3 - ay^2 + b$

(c) $h(z) = az(z-b)^2$

ChAlLeNgE PrObLeM: Give an example of a function f which satisfies the following. If no such function exists, say why. Assume f'' exists everywhere.

$$f(x)f'(x)f''(x)f'''(x) < 0 \text{ for all } x$$