Calc II Exam 2 Review

§7.1 Sequences

- notation $\{a_k\}$, $\{a_k\}_{k=1}^{\infty}$, $a_k : \mathbb{Z}_{\geq r} \to \mathbb{R}$
- closed form vs. recursive
- upper/lower bounds, g.l.b., l.u.b.
- LUBA: only works for $\mathbb{R}$ numbers
- geometric vs. arithmetic
- monotonicity
- modelling, Newton's Method

§7.2 Limits of Sequences

- Definition 7.8: $\lim_{k\to\infty} a_k = L$ means for any $\varepsilon > 0$ (tiny #) there exists $N > 0$ (large #) such that for all $k > N$,

  $a_k \in (L-\varepsilon, L+\varepsilon)$

- limit rules
  - linearity
  - limits of functions of sequences

- uniqueness of limits
- Squeeze Theorem

- subsequences: A subsequence of a convergent sequence converges (the converse is not true).
- dominance: $(a>0, b>1)$

  $\ln k \ll k^a \ll b^k \ll k!$
- given: $\{k^{1/k}\} \to 1$
- bounded & monotone $\Rightarrow$ convergence
- convergence $\Rightarrow$ boundedness

§5.6 Improper Integrals

- unbounded intervals

$$\int_a^{\infty} f(x)\,dx = \lim_{B\to\infty} \int_a^B f(x)\,dx$$

or

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{A\to-\infty} \int_A^b f(x)\,dx$$

- Asymptotes

$$\int_a^b f(x)\,dx = \lim_{A\to a^-} \int_A^b f(x)\,dx$$ (if $f(x)$ has an asymptote at $x=a$)

or

$= \lim_{B \to b^+} \int_a^B f(x)\,dx$ (if $f(x)$ has an asymptote at $x = b$)

- power functions

$\int_1^\infty \frac{1}{x^p}\,dx$ converges when $p > 1$

$\int_0^1 \frac{1}{x^p}\,dx$ converges when $p < 1$

Q: What do they converge to?

- comparisons

Q: Does $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ converge?

ans: Yes, b/c $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx \le \int_1^\infty \frac{1}{x^2}\,dx$.

(b/c $0 \le \sin^2 x \le 1$)

§7.3 Series

- $\sum_{k=1}^\infty a_k = \lim_{n \to \infty} \underbrace{\sum_{k=1}^n a_k}_{S_n}$

- geometric series
  - $|r| < 1 \Rightarrow \sum_{k=0}^\infty r^k = \frac{1}{1-r}$

  Q: How to modify if k doesn't start at 0?

  - For any r, $\sum_{k=0}^n r^k = S_n = \frac{1-r^{n+1}}{1-r}$

$\longrightarrow$

- telescoping series

§7.4 Intro to Convergence Tests

- divergence test
- integral test
  - $\sum_{k=1}^{\infty} a_k \geq \int_1^{\infty} a(x)\,dx \geq \sum_{k=2}^{\infty} a_k \geq \int_2^{\infty} a(x)\,dx \geq \sum_{k=3}^{\infty} a_k \geq \ldots$
  - approximating series: Don't have to memorize the notation in the book, but understand what it means.
  - p-series, harmonic series
- $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$

§7.5 Comparison Tests

Usually compare to a p-series or a geometric series.

- comparison test
- Limit comparison test

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§7.6 Ratio and Root Tests

factorials, exponentials

exponentials, but not factorials

§7.7 Alternating Series

- alternating series test
- conditional vs. absolute convergence