

**Directions:** No calculators, phones or other electronic aids are allowed. Show all your work. If you use a formula from memory, write it down. *Clearly indicate your final answer.* You will be graded not only on your final answer, but on the clarity of your solutions.

Name \_\_\_\_\_

TA Name: \_\_\_\_\_

Drill Time: \_\_\_\_\_

GRADE	
Problem 1	/ 20
Problem 2	/ 10
Problem 3	/ 10
Problem 4	/ 25
Problem 5	/ 20
Problem 6	/ 15
Total	/100

1. **(20 pts)** Evaluate the following integral exactly as written.

$$\int_0^{\ln 8} \int_0^4 \int_0^{\ln 2} y e^{-x-z} dx dy dz$$

2. **(10 pts)** Compute the average value of  $f(x, y) = \sin x \sin y$  over the region

$$R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq \pi\}.$$

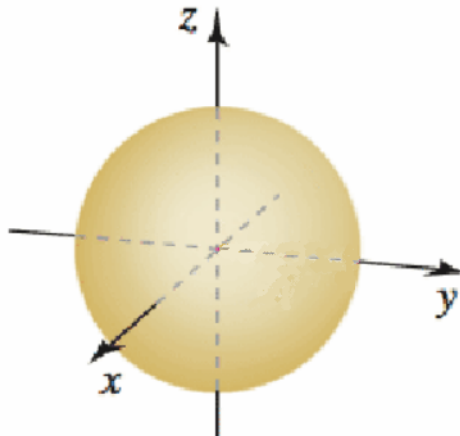
3. **(10 pts)** Consider the integral

$$\int_0^1 \int_1^{e^y} f(x, y) \, dx \, dy.$$

Sketch the region of integration and then rewrite the integral in the order  $dy \, dx$ .

4. A spherical fish tank of radius 1 ft is filled with water to a level 6 in from the top.

- (a) **(4 pts)** On the sphere below, draw and label the tank's radius and water level, with units included.



- (b) **(2 pts)** Write the equation for your sphere, in spherical coordinates.

- (c) **(9 pts)** Write down a triple integral that will give the volume of the *empty space* in the fish tank.

- (d) **(7 pts)** Evaluate the integral from (c).

- (e) **(3 pts)** What is the volume of the water in the tank?

5. **(20 pts)** Evaluate the following integral using a change of variables of your choice. Sketch the original and new regions of integration,  $R$  and  $S$ .

$$\iint_R (z - w)\sqrt{2z - w} \, dA$$

$R$  is bounded by the lines  $w = 2z - 2$ ,  $w = 2z$ ,  $w = z - 3$ , and  $w = z - 1$ .

6. **(15 pts)** For the integral below, sketch the region of integration and evaluate the integral using polar coordinates.

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} (4 - x^2 - y^2) dx dy$$

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Problem 6	/ 15
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1. **(20 pts)** Evaluate the following integral exactly as written.

$$\int_0^8 \int_0^{\ln 4} \int_0^{\ln 2} 2ze^{-x-y} dx dy dz$$



2. **(10 pts)** Compute the average value of  $g(x, y) = \cos x \sin y$  over the region

$$R = \{(x, y) : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \pi\}.$$

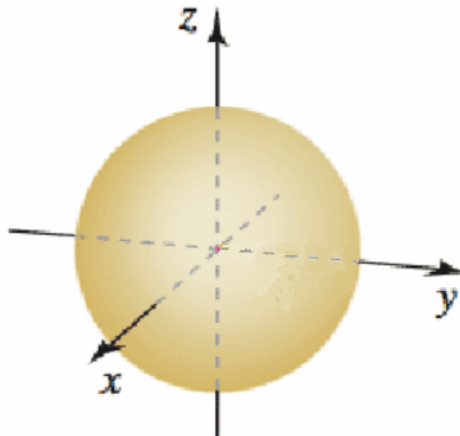
3. **(10 pts)** Consider the integral

$$\int_1^e \int_0^{\ln x} f(x, y) \, dy \, dx.$$

Sketch the region of integration and then rewrite the integral in the order  $dx \, dy$ .

4. A spherical fish tank of radius 2 ft is filled with water to a level 1 ft from the top.

- (a) **(4 pts)** On the sphere below, draw and label the tank's radius and water level, with units included.



- (b) **(2 pts)** Write the equation for your sphere, in spherical coordinates.

- (c) **(9 pts)** Write down a triple integral that will give the volume of the *empty space* in the fish tank.

- (d) **(7 pts)** Evaluate the integral from (c).

- (e) **(3 pts)** What is the volume of the water in the tank?

5. **(20 pts)** Evaluate the following integral using a change of variables of your choice. Sketch the original and new regions of integration,  $R$  and  $S$ .

$$\iint_R (x - y)\sqrt{y - 2x} \, dA$$

$R$  is bounded by the lines  $y = 2x - 2$ ,  $y = 2x$ ,  $y = x - 3$ , and  $y = x - 1$ .

6. **(15 pts)** For the integral below, sketch the region of integration and evaluate the integral using polar coordinates.

$$\int_{-3}^3 \int_0^{\sqrt{9-y^2}} (9 - x^2 - y^2) dx dy$$

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1. **(20 pts)** Evaluate the following integral exactly as written.

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2. **(10 pts)** Compute the average value of  $f(x, y) = \sin x \cos y$  over the region

$$R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq \frac{\pi}{2}\}.$$

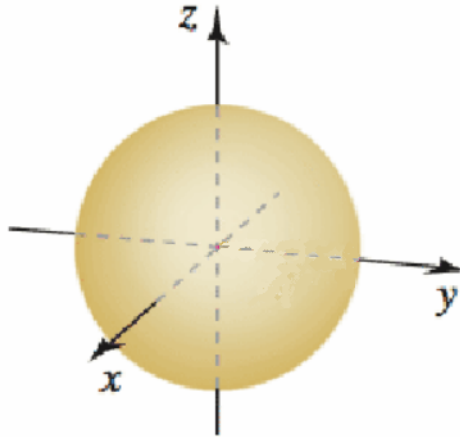
3. **(10 pts)** Consider the integral

$$\int_0^1 \int_1^{e^x} f(x, y) dy dx.$$

Sketch the region of integration and then rewrite the integral in the order  $dx dy$ .

4. A spherical fish tank of radius 10 in is filled with water to a level 5 in from the top.

- (a) **(4 pts)** On the sphere below, draw and label the tank's radius and water level, with units included.



- (b) **(2 pts)** Write the equation for your sphere, in spherical coordinates.

- (c) **(9 pts)** Write down a triple integral that will give the volume of the *empty space* in the fish tank.

- (d) **(7 pts)** Evaluate the integral from (c).

- (e) **(3 pts)** What is the volume of the water in the tank?



5. **(20 pts)** Evaluate the following integral using a change of variables of your choice. Sketch the original and new regions of integration,  $R$  and  $S$ .

$$\iint_R (z - w)\sqrt{z - 2w} \, dA$$

$R$  is bounded by the lines  $w = \frac{z}{2} - 2$ ,  $w = \frac{z}{2}$ ,  $w = z - 3$ , and  $w = z - 1$ .

6. **(15 pts)** For the integral below, sketch the region of integration and evaluate the integral using polar coordinates.

$$\int_{-5}^5 \int_0^{\sqrt{25-y^2}} (25 - x^2 - y^2) \, dx \, dy$$

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$$R = \{(x, y) : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}\}.$$

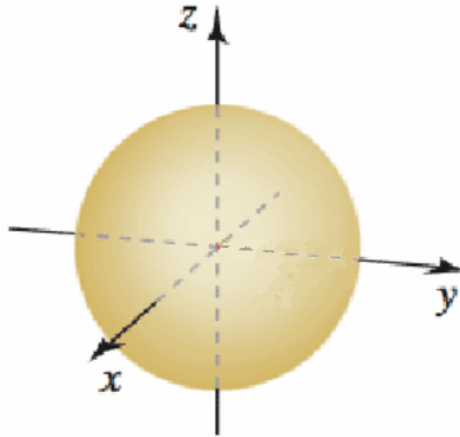
3. **(10 pts)** Consider the integral

$$\int_1^e \int_0^{\ln y} f(x, y) \, dx \, dy.$$

Sketch the region of integration and then rewrite the integral in the order  $dy \, dx$ .

4. A spherical fish tank of radius 20 in is filled with water to a level 10 in from the top.

- (a) **(4 pts)** On the sphere below, draw and label the tank's radius and water level, with units included.



- (b) **(2 pts)** Write the equation for your sphere, in spherical coordinates.

- (c) **(9 pts)** Write down a triple integral that will give the volume of the *empty space* in the fish tank.

- (d) **(7 pts)** Evaluate the integral from (c).

- (e) **(3 pts)** What is the volume of the water in the tank?

5. **(20 pts)** Evaluate the following integral using a change of variables of your choice. Sketch the original and new regions of integration,  $R$  and  $S$ .

$$\iint_R (x - y)\sqrt{2x - y} \, dA$$

$R$  is bounded by the lines  $y = 2x + 2$ ,  $2x - y = 0$ ,  $y = x - 3$ , and  $y = x - 1$ .

6. **(15 pts)** For the integral below, sketch the region of integration and evaluate the integral using polar coordinates.

$$\int_{-4}^4 \int_0^{\sqrt{16-y^2}} (16 - x^2 - y^2) \, dx \, dy$$