Take-Home Quiz #4

SOLUTIONS

Math 235 (Calc I)

Fall 2017

1. (a)

- $\dfrac{2530-2948}{36-42} = \dfrac{-418}{-6} \approx$ 70 beats/min
- $\dfrac{2661-2948}{38-42} = \dfrac{-287}{-4} \approx$ 72 beats/min
- $\dfrac{2806-2948}{40-42} = \dfrac{-142}{-2} =$ 71 beats/min
- $\dfrac{2948-3080}{42-44} = \dfrac{-132}{-2} =$ 66 beats/min

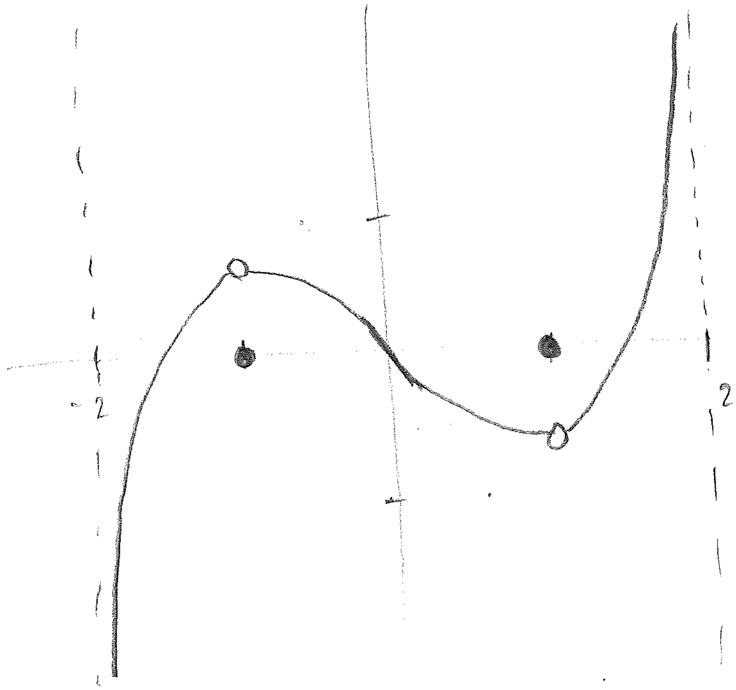
(b) The patient's heart rate seems constant for a few minutes, ~71 bpm. Then, at 42 minutes it starts decreasing.

2. Since the tangent line is at the point (4,3),

$\boxed{f(4)=3.}$

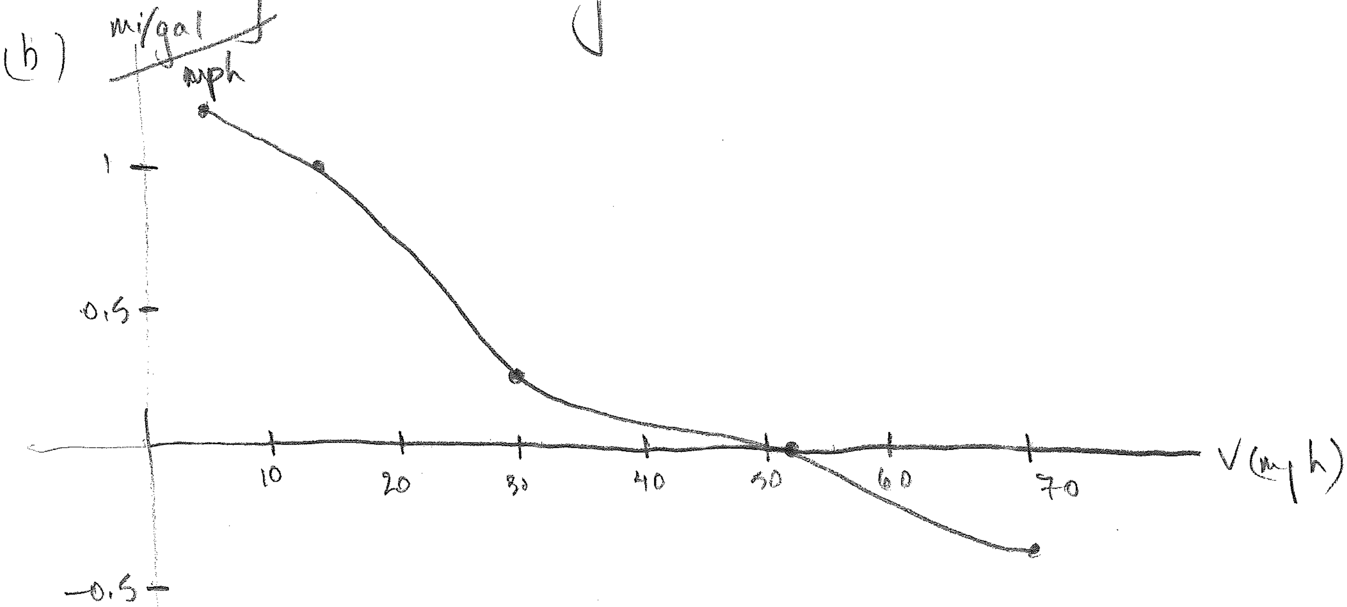
The slope of the tangent line, since it also passes through (0,2), is $\dfrac{3-2}{0-4} = \boxed{\dfrac{-1}{4} = f'(4)}$

3.



4. (a) $F'(v)$ is the change in fuel economy, according to velocity.

(b)



(c) Fuel economy is maximized ~ $\boxed{51 \text{ mph.}}$

5. (a) $\lim_{x\to 0^-} \frac{g(x)-g(0)}{x-0} = \lim_{x\to 0^-} \frac{2x-0}{x-0} = 2$

$\lim_{x\to 0^+} \frac{g(x)-g(0)}{x-0} = \lim_{x\to 0^+} \frac{2x-x^2-0}{x-0} = \lim_{x\to 0^+} 2-x = 2$

$\Rightarrow g(x)$ is differentiable at $x=0$.

$\lim_{x\to 2^-} \frac{g(x)-g(2)}{x-2} = \lim_{x\to 2^-} \frac{2x-x^2-0}{x-2} = \lim_{x\to 2^-} \frac{x(2-x)}{x-2}$

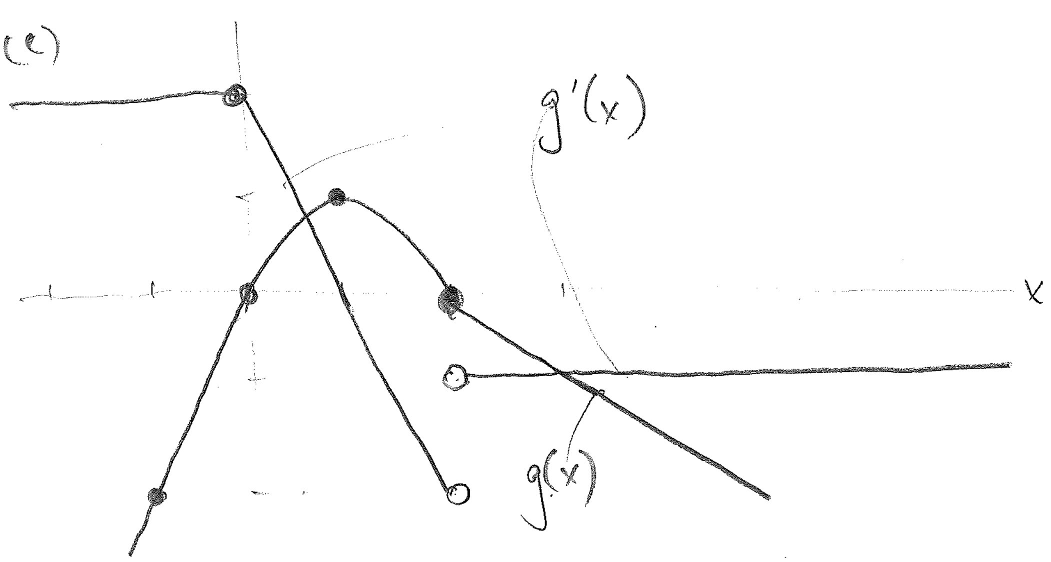
$= \lim_{x\to 2^-} -x = -2$

$\lim_{x\to 2^+} \frac{g(x)-g(2)}{x-2} = \lim_{x\to 2^+} \frac{2-x-0}{x-2} = -1$

$\Rightarrow g(x)$ is not differentiable at $x=2$.

(b) $g'(x) = \begin{cases} 2 & \text{if } x \le 0 \\ 2-2x & \text{if } 0 \le x < 2 \\ -1 & \text{if } 2 < x \end{cases}$

(c)



6. At any point x on the parabola, the slope of the tangent line is $\underline{f'(x)=2ax+b}$. So the average of the slopes of the tangent lines to the endpoints of an interval $[p,q]$ is

$$\frac{f'(p)+f'(q)}{2} = \frac{(2ap+b)+(2aq+b)}{2}$$

$$= \frac{\cancel{2}a(p+q)+\cancel{2}b}{\cancel{2}} = \underline{a(p+q)+b}.$$

The midpoint of $[p,q]$ is $\frac{p+q}{2}$. The slope of the tangent line there is

$$f'\left(\frac{p+q}{2}\right) = \cancel{2}a\left(\frac{p+q}{\cancel{2}}\right)+b = \underline{a(p+q)+b}.$$

$\blacksquare$

7. Write $Q(x) = \frac{f(x)}{g(x)} = \frac{1+x+x^2+xe^x}{1-x-x^2-xe^x}$.

Then $f'(x) = 1+2x+e^x+xe^x \Rightarrow f'(0)=1$

$g'(x) = -1-2x-e^x-xe^x = -f'(x) \Rightarrow g'(0) = -f'(0) = -1$

$$Q'(x) = \frac{g(x)f'(x)-f(x)g'(x)}{g(x)^2} = \frac{-f'(x)^2+f(x)f'(x)}{g(x)^2}$$

$\longrightarrow$

$$\Rightarrow Q'(0) = \frac{-f'(0)^2 + f(0)f'(0)}{g(0)^2}$$

$$= \frac{-(1)^2 + (1)(1)}{(1)^2} = \boxed{0}$$

8. (a) (typo in the problem). Saying $f(20) = 10{,}000$ means at \$20/yard, the manufacturer can sell 10,000 yd of fabric. Saying $f'(20) = -350$ means if they increase the price from \$20 to \$21/yd, then they will sell 350 fewer yards of fabric.

(b) $R'(p) = f(p) + pf'(p)$

$$\Rightarrow R'(20) = f(20) + 20f'(20)$$

$$= 10{,}000 + 20(-350) = 10{,}000 - 7000$$

$$\boxed{= 3{,}000.}$$

If the manufacturer increases the price from \$20 to \$21/yd, then they will still make about \$3000 more (despite selling less fabric).