MATH 2554 (Calculus I)

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Monday 20 April (Week 14)

- Computer HW this week: $\oint 5.3 - 5.4$; next week: $\oint 5.5$
- Quiz #14 tomorrow (Tues) in drill – in class
- Exam #4 Friday 24 April covers $\oint 4.6 - 5.4$ open in class
- Quiz #15 next Tues also in class
- FINAL! Monday 4 May 6-8p OZAR 026
- dead day review?

$\oint$ 5.4 Working with Integrals

Integrating Even and Odd Functions

Recall the definition of an even function,

$$f(-x) = f(x),$$

and of an odd function,

$$f(-x) = -f(x).$$

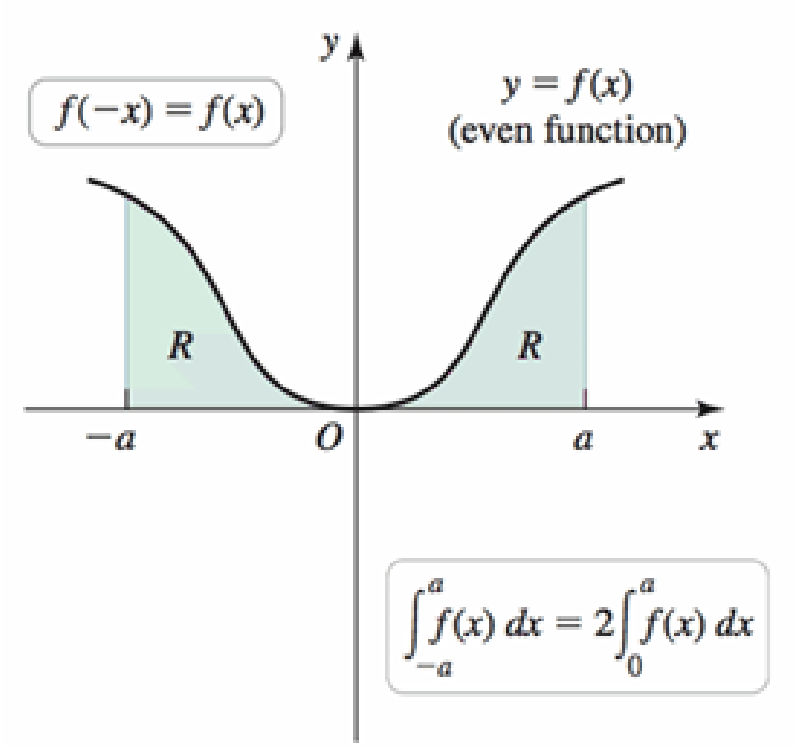
These properties simplify integrals when the interval in question is centered at the origin.

Integrating Even Functions

Even functions are symmetric about the y-axis. So

$$\int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(x)\,dx$$

i.e., the area under the curve to the left of the y-axis is equal to the area under the curve to the right

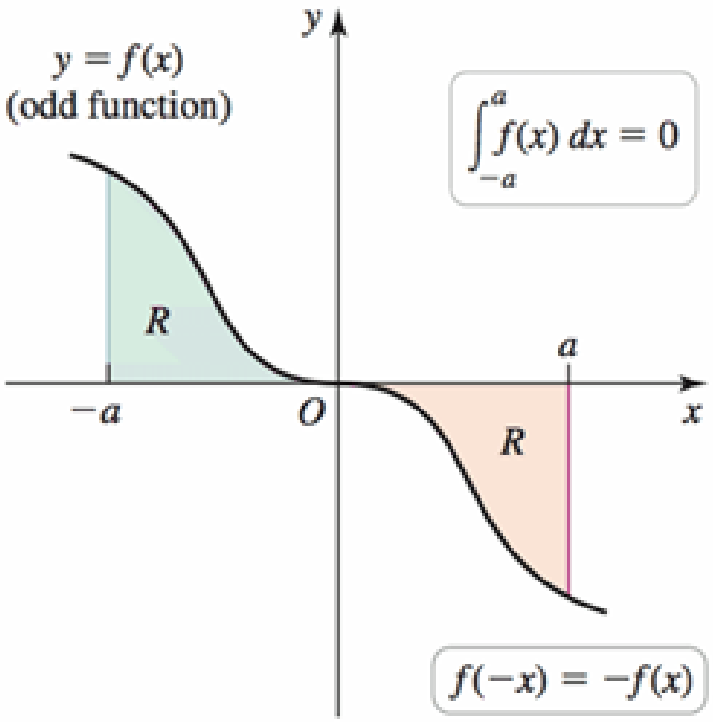


Integrating Odd Functions

On the other hand, odd functions have 180° rotation symmetry about the origin. So

$$\int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx$$

i.e., the area under the curve to the left of the origin is the negative of the area under the curve to the right of the origin.



Exercise

Evaluate the following integrals using the properties of even and odd functions:

1. $\displaystyle\int_{-4}^{4} (3x^2 - x)\, dx$
2. $\displaystyle\int_{-1}^{1} (1 - |x|)\, dx$
3. $\displaystyle\int_{-\pi}^{\pi} \sin x\, dx$

Average Value of a Function

To find the average of $f(x)$ between points a and b, we can estimate by choosing y-values $\overline{x}_k$. If we take n of them, then the average is

$$\frac{f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)}{n} = \frac{f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)}{\left(\frac{b-a}{\Delta x}\right)}$$

$$= \frac{1}{b-a}\left(f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)\right)\Delta x$$

$$= \frac{1}{b-a}\sum_{k=1}^{n} f(\overline{x}_k)\Delta x$$

The estimate gets more accurate, the more y-values we take; as $n \to \infty$, this is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

Average Value of a Function

The average value of an integrable function f on the interval $[a, b]$ is

$$\overline{f} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Exercise

Find the average value of the function $f(x) = x(1-x)$ on the interval $[0, 1]$.

Mean Value Theorem for Integrals

Theorem

If f is continuous on $[a, b]$, then there is at least one point c in $[a, b]$ such that

$$f(c) = \overline{f} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

In other words, the horizontal line $y = \overline{f} = f(c)$ intersects the graph of f for some point c in $[a, b]$. (See Figure 5.54)

Exercise

Find or approximate the point(s) at which $f(x) = x^2 - 2x + 1$ equals its average value on $[0, 2]$.

HW from Section 5.4

Do problems 7–27 odd, 31–35 odd (pp. 354–355 in textbook)

Wednesday 22 April (Week 14)

- Computer HW this week: $\oint 5.3 - 5.4$; next week: $\oint 5.5$
- Exam #4 Friday 24 April covers $\oint 4.6 - 5.4$ open in class
  - open resources but the exam MUST be completed by the end of lecture
  - The best way to prepare for this exam is to complete the assigned book problems. Use the same amount of effort and care you used on Exam 3.
  - CEA: Since the exam is collaborative you have the option to work in class and then finish in my office after. Please let me know ASAP if this is what you prefer.
- Quiz #15 ($\oint 5.5$) next Tues in class
- FINAL! Monday 4 May 6-8p OZAR 026
- dead day review details TBA

4.6 Mean Value Theorem

- Know and be able to state Rolle's Thm and the Mean Value Thm, including knowing the hypotheses and conclusions for both.
- Be able to apply Rolle's Thm to find a point in a given interval.
- Be able to apply the MVT to find a point in a given interval.
- Be able to use the MVT to find equations of secant and tangent lines.

You are still responsible for every derivative rule and every derivative formula we have covered this semester.

4.7 L'Hôpital's Rule

- Know how to use L'Hôpital's Rule, including knowing under what conditions the Rule works.
- Be able to apply L'Hôpital's Rule to a variety of limits that are in indeterminate forms (e.g., $0/0$, ∞/∞, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0).
- Be able to use L'Hôpital's Rule to determine the growth rates of two given functions.
- Be aware of the pitfalls in using L'Hôpital's Rule.
- PRACTICE THESE. Some of the book problems have non-obvious algebra tricks that simplify an otherwise crazy problem.

4.8 Antiderivatives

- Know the definition of an antiderivative and be able to find one or all antiderivatives of a function.
- Be able to evaluate indefinite integrals, including using known properties of indefinite integrals (i.e., Power Rule, Constant Multiple Rule, Sum Rule).
- Know how to find indefinite integrals of the six trig functions, of e^{ax}, of $\ln x$, and of the three inverse trig functions listed in the notes.
- Be able to solve initial value problems to find specific antiderivatives.
- Be able to use antiderivatives to work with motion problems.

5.1 Approximating Areas under Curves

- Be able to use rectangles to approximate area under the curve for a given function.
- Know how to calculate left Riemann sums, right Riemann sums, and midpoint Riemann sums for a function.
- Be able to sum a series of numbers written in sigma notation. You need to know these common sums: $\sum_{k=1}^{n} c = cn$ and $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.
- Be able to identify whether a given Riemann sum written in sigma notation is a left, right, or midpoint sum.

5.2 Definite Integrals

- Be able to compute left, right, or midpoint Riemann sums for curves that have negative components, and understand the concept of net area.
- Be able to evaluate a definite integral using geometry or a given graph.
- Know the properties of definite integrals and be able to use them to evaluate a definite integral.

5.3 Fundamental Theorem of Calculus

- Understand the concept of an area function, and be able to evaluate an area function as x changes.
- Know the two parts of the Fundamental Theorem of Calculus and its significance (i.e., the inverse relationship between differentiation and integration).
- Use the FTC to evaluate definite integrals or simplify given expressions.

5.4 Working with Integrals

- Be able to integrate even and odd functions knowing the "shortcuts" provided by these functions' characteristics.
- Be able to find the average value of a function.
- Know the Mean Value Theorem for Integrals and be able to use it to find points associated with the average value of a function.

Other Remarks on the Exam

Why you MUST study for this exam, even though it's open:

- Exam #3: How many hours did you put into completing it, given all the resources you had?
- Exams #1 and #2: The notecard alone wasn't enough to help you finish the exam in time.

Tips for studying efficiently and effectively:

- Given today's lists of materials you should know for the exam, if you see a topic you don't know then go back to the slides covering that topic first.
- Review slides for days you missed.
- Redo the quizzes until you can get a perfect score without looking at the key.
- Book problems. Do those problems with the same attention and care you put into Exam #3.
- If you spent 10 hours on Exam #3 then spend 9 hours studying for Exam #4, with one hour left for the exam itself.