MATH 2554 (Calculus I)

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Wednesday 21 January (Week 2)

- For old Calculus materials, see `comp.uark.edu/~ashleykw` and look for links under "Courses I've taught". Last semester's in-class exam solutions are posted in there, too.
- Thurs Jan 22 Quiz #2 (in drill).
- Sunday Jan 26: Computer HWs #1 and #2 Due

Squeeze Theorem

A final method for evaluating limits involves the relationship of functions with each other.

Theorem (Squeeze Theorem)

Assume the functions f, g, and h are functions and

$$f(x) \leq g(x) \leq h(x)$$

for all values of x near a, except possibly at a. If

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

Example:

(a) Draw a graph of the inequality

$$-|x| \le x^2 \ln x^2 \le |x|.$$

(b) Compute $\lim_{x \to 0} x^2 \ln x^2$.

HW from Section 2.3

Do problems 12–30 (every 3rd problem), 31, 33, 37–47 odds, 51, 53, 61–65 odds (pp. 73–75 in textbook).

∮ 2.4 Infinite Limits

In the next two sections, we examine limit scenarios involving infinity. The two situations are:

- **Infinite limits:** as x (i.e., the independent variable) approaches a finite number, y (i.e., the dependent variable) becomes arbitrarily large or small

  $$\text{looks like: } \lim_{x\to\text{number}} f(x) = \pm\infty$$

- **Limits at infinity:** as x approaches an arbitrarily large or small number, y approaches a finite number

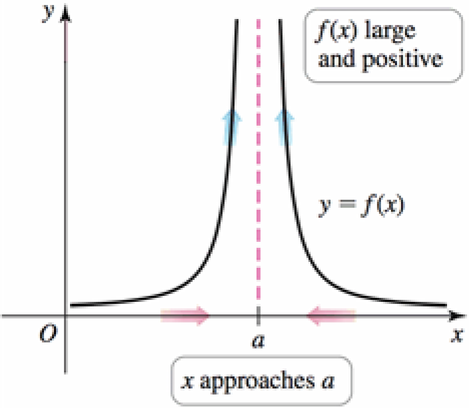
  $$\text{looks like: } \lim_{x\to\pm\infty} f(x) = \text{number}$$

Definition of Infinite Limits

Suppose f is defined for all x near a. If $f(x)$ grows arbitrarily large for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = \infty$$

and say that "the limit of $f(x)$ as x approaches a is infinity."

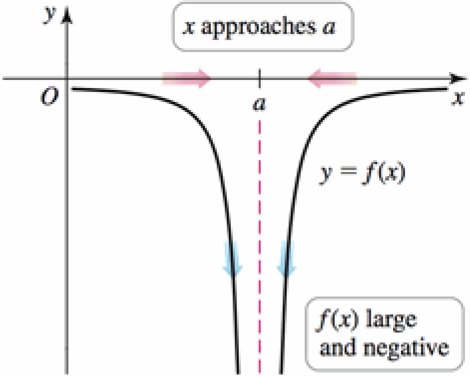

y
$f(x)$ large
and positive
$y = f(x)$
O
a
x
x approaches a

Definition of Infinite Limits (cont.)

Suppose f is defined for all x near a. If $f(x)$ is negative and grows arbitrarily large in magnitude for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = -\infty$$

and say that "the limit of $f(x)$ as x approaches a is negative infinity."


x approaches a
y
O
a
x
y = f(x)
f(x) large
and negative

The definitions work for one-sided limits, too.

Example: Using a graph and a table of values, given $f(x) = \dfrac{1}{x^2 - x}$, determine:

1. $\lim\limits_{x \to 0^+} f(x)$
2. $\lim\limits_{x \to 0^-} f(x)$
3. $\lim\limits_{x \to 1^+} f(x)$
4. $\lim\limits_{x \to 1^-} f(x)$

Definition of Vertical Asymptote

If any of the following are true:

- $\lim_{x \to a} f(x) = \pm\infty$,
- $\lim_{x \to a^+} f(x) = \pm\infty$
- $\lim_{x \to a^-} f(x) = \pm\infty$

then the line $x = a$ is called a **vertical asymptote** of f.

Determining Infinite Limits Analytically:

Given $f(x) = \dfrac{3x-4}{x+1}$, determine, without using a table or a graph,

- $\displaystyle\lim_{x\to -1^+} f(x)$
- $\displaystyle\lim_{x\to -1^-} f(x)$

Remember to check for factoring – what is/are the vertical asymptotes of

$$f(x) = \frac{3x^2 - 48}{x + 4}?$$

What is $\lim_{x \to -4} f(x)$?

HW from Section 2.4

Do problems 7–10, 15, 17–26, 36–37 (pp. 81–84 in textbook)

Friday 23 January (Week 2)

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$\oint$ 2.5 Limits at Infinity

Limits at infinity determine what is called the **end behavior** of a function.

Horizontal Asymptotes

If $f(x)$ becomes arbitrarily close to a finite number L for all sufficiently large and positive x, then we write

$$\lim_{x\to\infty} f(x) = L.$$

The line $y = L$ is a **horizontal asymptote** of f.

The limit at negative infinity, $\lim\limits_{x\to-\infty} f(x) = M$, is defined analogously and in this case, the horizontal asymptote is $y = M$.

Infinite Limits at Infinity

Is it possible for a limit to be both an infinite limit and a limit at infinity? (Yes.)

If $f(x)$ becomes arbitrarily large as x becomes arbitrarily large, then we write

$$\lim_{x\to\infty} f(x) = \infty.$$

(The limits $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$, and $\lim_{x\to-\infty} f(x) = -\infty$ are defined similarly.)

Infinite Limits at Infinity, cont.

Powers and Polynomials: Let n be a positive integer and let $p(x)$ be a polynomial.

- $n =$ even number: $\displaystyle\lim_{x\to\pm\infty} x^n = \infty$
- $n =$ odd number: $\displaystyle\lim_{x\to\infty} x^n = \infty$ and $\displaystyle\lim_{x\to-\infty} x^n = -\infty$

- (again, assuming n is positive)

$$\lim_{x\to\pm\infty} \frac{1}{x^n} = \lim_{x\to\pm\infty} x^{-n} = 0$$

- For a polynomial, only look at the term with the highest exponent:

$$\lim_{x\to\pm\infty} p(x) = \lim_{x\to\pm\infty} (\text{constant}) \cdot x^n$$

The constant is called the leading coefficient, $\text{lc}(p)$. The highest exponent that appears in the polynomial is called the degree, $\deg(p)$.

Rational Functions: Suppose $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function.

1. If $\deg(p) < \deg(q)$, i.e., the numerator has the smaller degree, then

$$\lim_{x\to\pm\infty} f(x) = 0$$

(also, $y = 0$ is a horizontal asymptote of f).

2. If $\deg(p) = \deg(q)$, i.e., numerator and denominator have the same degree, then

$$\lim_{x\to\pm\infty} f(x) = \frac{\mathsf{lc}(p)}{\mathsf{lc}(q)},$$

and $y = \dfrac{\mathsf{lc}(p)}{\mathsf{lc}(q)}$ is a horizontal asymptote of f.

3. If $\deg(p) > \deg(q)$, (numerator has the bigger degree) then

$$\lim_{x\to\pm\infty} f(x) = \infty \quad \text{or} \quad -\infty$$

and f has no horizontal asymptote.

4. Assuming that $f(x)$ is in reduced form (p and q share no common factors), vertical asymptotes occur at the zeroes of q.

(This is why it is a good idea to check for factoring and cancelling first!)

Exercises

Determine the end behavior of the following functions (in other words, compute both limits, as $x \to \pm\infty$, for each of the functions):

- $f(x) = \dfrac{x+1}{2x^2-3}$
- $g(x) = \dfrac{4x^3-3x}{2x^3+5x^2+x+2}$
- $h(x) = \dfrac{6x^4-1}{4x^3+3x^2+2x+1}$

Algebraic and Transcendental Functions:
Determine the end behavior of the following functions.

- $f(x) = \dfrac{4x^3}{2x^3 + \sqrt{9x^6 + 15x^4}}$ (radical signs appear)
- $g(x) = \cos x$ (trig)
- $h(x) = e^x$ (exponential)

HW from Section 2.5

Do problems 9–10, 13–35 odds, 39, 43, 45, 53 (pp. 92–93 in textbook)