MATH 2574 (Calculus III)
Spring 2017

Name: ______________________

Fri 10 Feb 2017

Exam 1: Intro to Multidimensional Calculus (§11.1-11.7, 12.1-12.2)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems. No electronic devices (phones, iDevices, computers, etc) except for a **basic scientific calculator**. On story problems, round to one decimal place. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

In addition, please provide the following data:

Drill Instructor: ______________________

Drill Time: ______________________

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: **(1 pt)** ______________________

Good luck!

1. Determine whether the following statements are true or false. You must justify your answer.

   (a) **(5 pts)** The domain of the function $u = f(w, x, y, z)$ is a region in $\mathbb{R}^4$.

   (b) **(5 pts)** $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \mathbf{0}$

   (c) **(5 pts)** The domain of the function $f(x, y) = 1 - |x - y|$ is $\{(x, y) \mid x \geq y\}$.

   (d) **(5 pts)** All level curves of the plane $z = 2x - 3y$ are lines, except for when $z = 0$.

2. **(18 pts)** Determine an equation of the line that is perpendicular to the lines

$$\mathbf{r}(t) = \langle -1 + 3t, 3t, 2t \rangle$$
$$\mathbf{R}(s) = \langle -6 + 3s, -8 + 2s, -12 + s \rangle$$

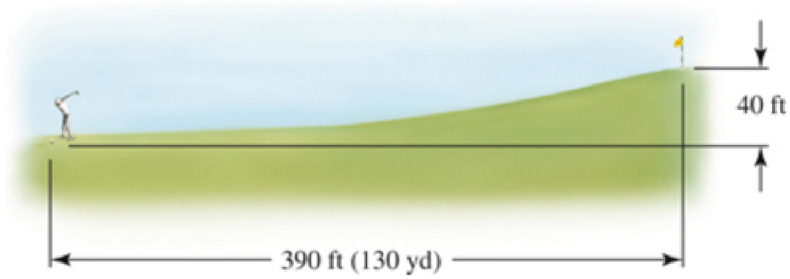
and passes through the point of intersection of the lines $\mathbf{r}$ and $\mathbf{R}$.

3. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable functions at $t = 0$ with $\mathbf{u}(0) = \langle 1, 0, 1\rangle$, $\mathbf{u}'(0) = \langle 7, 0, 1\rangle$, $\mathbf{v}(0) = \langle 0, 1, 1\rangle$, $\mathbf{v}'(0) = \langle 1, 3, 2\rangle$. Evaluate the following expressions:

   (a) **(6 pts)** $\left.\dfrac{d}{dt}(\sin(t)\mathbf{u}(t))\right|_{t=0}$

   (b) **(6 pts)** $\left.\dfrac{d}{dt}(\mathbf{u} \cdot \mathbf{v})\right|_{t=0}$

4. A golfer stands 400 ft horizontally from the hole and 40 ft below the hole (see figure).

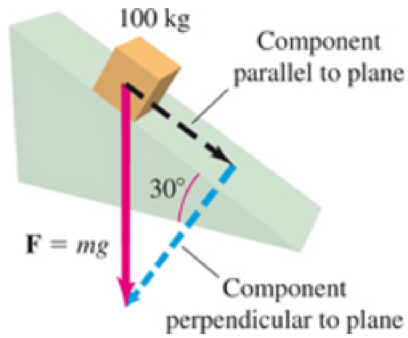


Suppose the ball is hit with an initial speed of 150 ft/s, at an angle of θ from the ground.

(a) **(12 pts)** Find the acceleration $\mathbf{a}(t)$, velocity $\mathbf{v}(t)$, and position $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ vectors for the trajectory of the ball. The gravitational constant is $g = 32$ ft/s^2.

(b) **(6 pts)** Write down a system of two equations to find the two unknowns: (1) time of flight and (2) θ. **Do not** solve the system.

5. **(16 pts)** A 100 kg box rests on a ramp with an incline of 30° to the floor (see figure). Find the components of the force perpendicular to and parallel to the ramp. (The vertical component of the force exerted by an object of mass m is its weight, which is mg, where $g = 9.8$ m/s^2 is the acceleration due to gravity.)



6. **(15 pts)** Match equations (a)-(f) with the surfaces (A)=(F).

   (a) $y = |x|$

   (b) $3x - 4y - z = 5$

   (c) $y - z^2 = 0$

   (d) $4x^2 + \dfrac{y^2}{4} + z^2 = 1$

   (e) $x^2 + \dfrac{y^2}{9} = z^2$

   (f) $x^2 + \dfrac{y^2}{9} - z^2 = 1$