MATH 2554 Quiz 5: §3.2-3.3

$\boxed{\text{Solutions}}$

1. $f'(x) = 5 \frac{d}{dx} x^3$ (Constant Multiple Rule)

$= 5(3x^2)$ (Power Rule)

$= 15x^2$

2. $g'(w) = 2 \frac{d}{dw} w^3 + 3 \frac{d}{dw} w + \frac{d}{dw} e^w$ ((Constant Multiple Rule)

$= 2(3w^2) + 3(1) + e^w$ (Power Rule, then Exponential Rule)

$= 6w^2 + 3 + e^w$

3. Quotient Rule:

$$f'(x) = \frac{(x+1)\frac{d}{dx}(x^2-7x-8) - (x^2-7x-8)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(2x-7) - (x^2-7x-8)(1)}{(x+1)^2}$$

$$= \frac{(2x^2+2x-7x-7) - x^2+7x+8}{(x+1)^2}$$

$$= \frac{x^2+2x+1}{(x+1)^2} = \frac{(x+1)^2}{(x+1)^2} = 1.$$

$\longrightarrow$

Alternate Method: Factoring

$$f(x) = \frac{x^2-7x-8}{x+1} = \frac{\cancel{(x+1)}(x-8)}{\cancel{x+1}}$$

$$= x-8$$

$$f'(x) = \frac{d}{dx}(x-8)$$

$$= 1.$$

$$f''(x) = \frac{d}{dx}f'(x) = \frac{d}{dx}(1) = 0.$$

$$f'''(x) = \frac{d}{dx}(0) = 0.$$

4. (A) $f'(t) = 3t^2 - 27 = 0$

$$3t^2 = 27$$

$$t^2 = 9$$

$$t = \pm 3$$

(b) $f'(t) = 3t^2 - 27 = 21$

$$3t^2 = 48$$

$$t^2 = 16$$

$$t = \pm 4$$

5. (a) $\frac{d}{dx} e^x = e^x$, so for $x = \ln 3$,

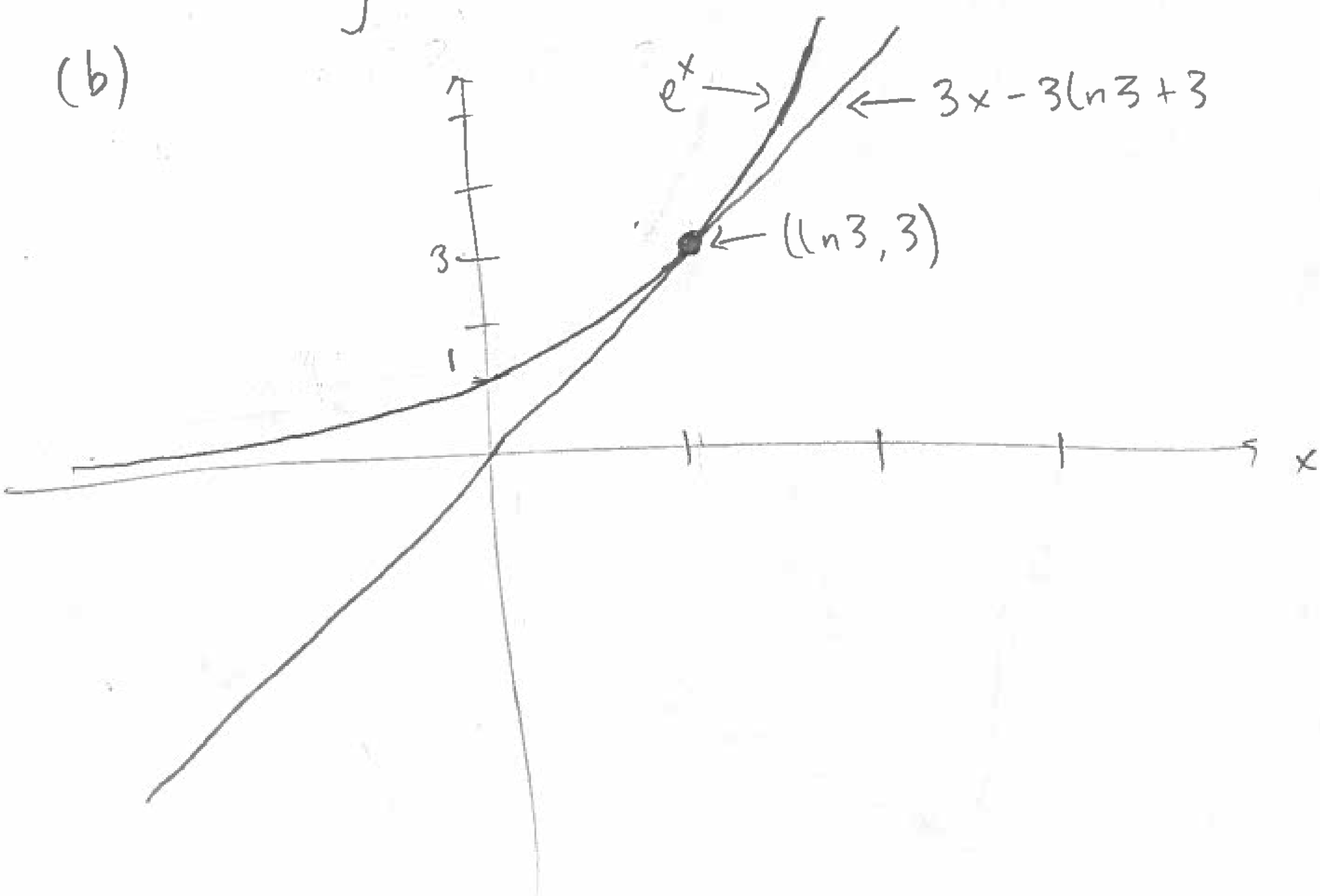
$$\left.\frac{d}{dx}\right|_{x=\ln 3} e^x = e^{\ln 3} = 3.$$

Tangent line:

$$y - 3 = 3(x - \ln 3)$$

$$y = 3x - 3\ln 3 + 3$$

(b)



6. (a) Quotient Rule:

$$f'(x) = \frac{(3x-1)\frac{d}{dx}(2x^2) - 2x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}$$

$$= \frac{(3x-1)(4x) - 2x^2(3)}{(3x-1)^2} = \frac{12x^2-4x-6x^2}{(3x-1)^2}$$

$$= \frac{6x^2-4x}{(3x-1)^2}$$

$$f'(1) = \frac{6(1^2)-4(1)}{(3(1)-1)^2} = \frac{6-4}{2^2} = \frac{1}{2}$$

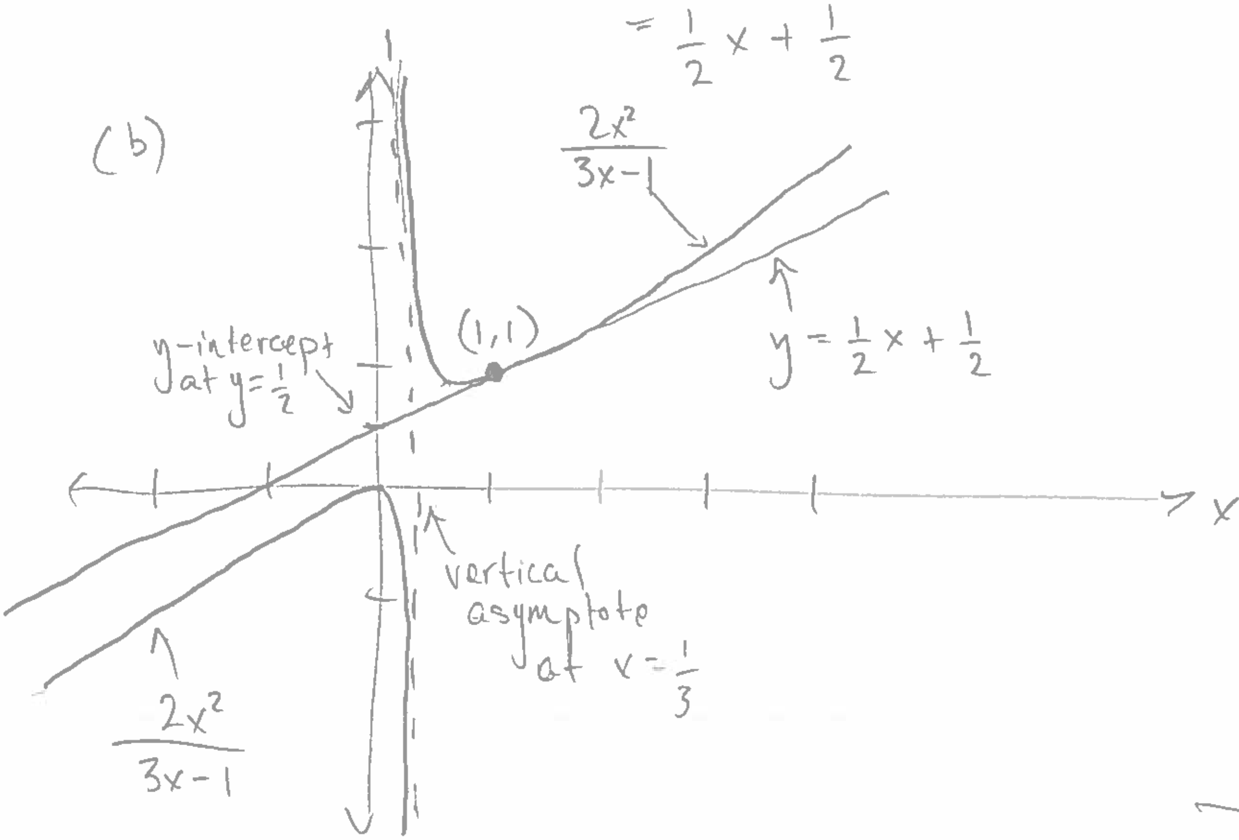
$$f(1) = \frac{2(1^2)}{3(1)-1} = \frac{2}{2} = 1$$

Tangent line: $y - 1 = \frac{1}{2}(x-1)$

$$y = \frac{1}{2}x - \frac{1}{2} + 1$$

$$= \frac{1}{2}x + \frac{1}{2}$$

(b)



7. Method 1: Product Rule

$$f'(x) = \left(\frac{d}{dx}(x-3)\right)(x^2+4) + (x-3)\frac{d}{dx}(x^2+4)$$

$$= (1)(x^2+4) + (x-3)(2x)$$

$$= x^2+4+2x^2-6x$$

$$= 3x^2-6x+4$$

Method 2: "FOIL" (Simplify), then differentiate

$$f'(x) = \frac{d}{dx}(x^3+4x-3x^2-12)$$

$$= 3x^2+4-6x$$

8. (a) $A(10) = 200e^{0.0398(10)}$

$\approx \$297.77$ ← must be rounded correctly

(b) $A'(t) = 200(0.0398e^{0.0398t})$

$A'(10) \approx \$11.85 / \text{year}$

(c) $y - A(10) = A'(10)(t-10)$

$y - 200e^{0.398} = (7.96e^{0.398})(t-10)$

(should have exact answers here)

9. Factor first: $p(x) = \dfrac{4x^3}{2x^5} + \dfrac{3x}{2x^5} + \dfrac{1}{2x^5}$

$$= \frac{2}{x^2} + \frac{3}{2x^4} + \frac{1}{2x^5}$$

→

$$p'(x) = \frac{d}{dx}\left(\frac{2}{x^2}\right) + \frac{d}{dx}\left(\frac{3}{4x^4}\right) + \frac{d}{dx}\left(\frac{1}{2x^5}\right)$$

$$= 2\frac{d}{dx}\left(\frac{1}{x^2}\right) + \frac{3}{2}\frac{d}{dx}\left(\frac{1}{x^4}\right) + \frac{1}{2}\frac{d}{dx}\left(\frac{1}{x^5}\right)$$

$$= 2\left(\frac{x^2\frac{d}{dx}1 - 1\frac{d}{dx}x^2}{(x^2)^2}\right) + \frac{3}{2}\left(\frac{x^4\frac{d}{dx}(1) - 1\cdot\frac{d}{dx}x^4}{(x^4)^2}\right) + \frac{1}{2}\left(\frac{x^5\frac{d}{dx}(1) - 1\cdot\frac{d}{dx}x^5}{(x^5)^2}\right)$$

$$= 2\left(\frac{0-2x}{x^4}\right) + \frac{3}{2}\left(\frac{0-4x^3}{x^8}\right) + \frac{1}{2}\left(\frac{0-5x^4}{x^{10}}\right)$$

$$= \frac{-4}{x^3} - \frac{6}{x^5} - \frac{5}{2x^6} = \frac{-8x^3 - 12x - 5}{2x^6}$$

Alternate Method: Quotient Rule

$$p'(x) = \frac{2x^5\frac{d}{dx}(4x^3+3x+1) - (4x^3+3x+1)\frac{d}{dx}(2x^5)}{(2x^5)^2}$$

$$= \frac{2x^5(12x^2+3) - (4x^3+3x+1)(10x^4)}{(2x^5)^2}$$

$$= \frac{24x^7 + 6x^5 - 40x^7 - 30x^5 - 10x^4}{4x^{10}}$$

$\longrightarrow$

$$= -\frac{16x^7}{4x^{10}} - \frac{24x^5}{4x^{10}} - \frac{10x^4}{4x^{10}}$$

$$= \frac{-4}{x^3} - \frac{6}{x^5} - \frac{5}{2x^6} = \frac{-8x^3 - 12x - 5}{2x^6}$$

10 (a) (Product Rule)

$$\frac{d}{dx}\left(xe^{2x}\right) = x\frac{d}{dx}e^{2x} + \left(\frac{d}{dx}x\right)e^{2x}$$

$$= x(2e^{2x}) + (1)e^{2x}$$

can never equal zero

set the derivative equal to zero $\rightarrow = (2x+1)e^{2x} = 0$

$$2x+1 = \frac{0}{e^{2x}} = 0$$

$$x = \frac{-1}{2}$$

Tangent line:

$$y - \left(\tfrac{-1}{2}e^{2(\frac{-1}{2})}\right) = (0)\left(x - \left(\tfrac{-1}{2}\right)\right)$$

$$y + \frac{1}{2}e^{-1} = 0$$

$$y = \frac{-1}{2}e^{-1} = \frac{-1}{2e}$$

(b) The tangent line is horizontal when $x = \frac{-1}{2}$ and the function "flattens out".