

Math 2554 Exam 1: Limits
date

Name: _____

You have 50 minutes to complete this exam. Eyes on your own paper and good luck!

1. Definitions/Concepts.

- (a) (2.1) Suppose $s(t)$ is the position of an object moving along a line at a time $t \geq 0$. What is the average velocity between the times $t = a$ and $t = b$? What is the instantaneous velocity at $t = a$? Which one is the slope of the secant line between the points $(a, s(a))$ and $(b, s(b))$ on the graph of s ? Which one gives the slope of the line tangent to the graph of s at $(a, s(a))$?
- (b) When $\lim_{x \rightarrow a} f(x)$ exists, it always equals $f(a)$. (True or False?)
- (c) (2.5) The end behavior for e^x and e^{-x} (on the entire real line) and $\ln x$ (on the interval $(0, \infty)$) is given by
- i. $\lim_{x \rightarrow \infty} e^x =$
 - ii. $\lim_{x \rightarrow -\infty} e^x =$
 - iii. $\lim_{x \rightarrow \infty} e^{-x} =$
 - iv. $\lim_{x \rightarrow -\infty} e^{-x} =$
 - v. $\lim_{x \rightarrow \infty} \ln x =$
 - vi. $\lim_{x \rightarrow 0^+} \ln x =$
2. (2.4) The function $f(x)$ has a vertical asymptote at the line $x = a$ means at least one of the following conditions holds:
- (a)
 - (b)
 - (c)
3. (2.6) Suppose a function $f(x)$ is continuous at the point a . Why is it OK to plug in the value $x = a$ when computing $\lim_{x \rightarrow a} f(x)$ (Hint: What are the three conditions on the Continuity Checklist?)?

Questions/Problems.

1. (2.1/2.2) Sketch the graph of a function with the given properties. You do not need to find a formula for the function.

$$f(1) = 0, f(2) = 4, f(3) = 6, \lim_{x \rightarrow 2^-} f(x) = -3, \lim_{x \rightarrow 2^+} f(x) = 5$$

2. The value of $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ does not exist. (t/f)
3. The value of $\lim_{x \rightarrow a} f(x)$ does not exist if $f(a)$ is undefined. (t/f)

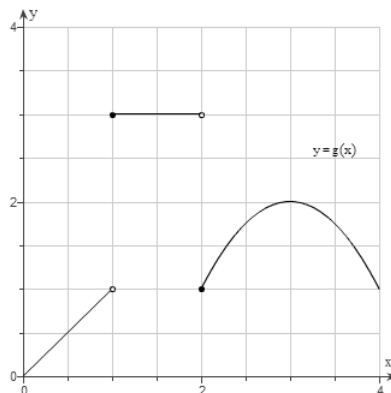


Figure 1: g (Briggs, W. and Cochran, L. *Calculus: Early ranscententials*)

4. (2.1/2.2) Use the graph of g in the figure to find the following values, if they exist. If a limit does not exist, explain why.
 - (a) $g(1)$
 - (b) $\lim_{x \rightarrow 1} g(x)$
 - (c) $\lim_{x \rightarrow 2^+} g(x)$
 - (d) $\lim_{x \rightarrow 1^-} g(x)$
 - (e) $g(2)$
 - (f) $g(3)$
 - (g) $\lim_{x \rightarrow 1^+} g(x)$
 - (h) $\lim_{x \rightarrow 2^-} g(x)$
 - (i) $\lim_{x \rightarrow 3} g(x)$
5. Using the figure as a guide, explain how the Squeeze Theorem can be used to compute $\lim_{x \rightarrow 0} x^2 \sin(\frac{1}{x})$, and then say what the limit is.
6. (CHALLENGE) Suppose a spaceship is traveling at velocity v , relative to an observer. Say the length of the spaceship is L_0 . To the observer, the ship appears to have a smaller length, given by the Lorentz contraction formula:

$$\text{length to the observer} = L_0 \sqrt{1 - \frac{v^2}{c^2}},$$

where c is the speed of light.

- (a) If $v = 0.5c$, i.e., the ship is traveling at half the speed of light, then what is the length of the ship to the observer?
- (b) If the ship is traveling 75% of the speed of light then how long does the ship look to the observer?
- (c) Compute $\lim_{v \rightarrow c^-} L_0 \sqrt{1 - \frac{v^2}{c^2}}$. What is physically interesting about this limit?

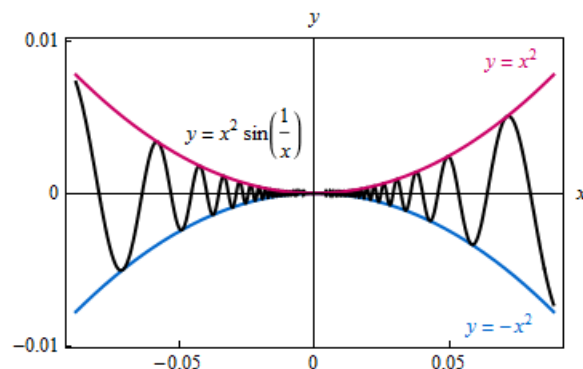


Figure 2: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

7. (2.3) Suppose $\lim_{x \rightarrow 1} f(x) = 4$. What is $\lim_{x \rightarrow -1} f(x^2)$? (Hint: There is another limit law that says how to compute this limit.)
8. (2.3/2.4) Suppose $g(x) = \begin{cases} \frac{x-5}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Evaluate
 - (a) $\lim_{x \rightarrow 0^+} g(x)$. Make sure your justification involves determining the sign of the numerator and denominator for x -values slightly larger than 0.
 - (b) $\lim_{x \rightarrow 0^-} g(x)$. Make sure your justification involves determining the sign of the numerator and denominator for x -values slightly smaller than 0.
 - (c) $\lim_{x \rightarrow 0} g(x)$.
 - (d) $g(0)$.
9. (2.3/2.4) Suppose $h(x) = \begin{cases} \frac{x-5}{x} & x < 0 \\ 0 & x \geq 0 \end{cases}$. Evaluate
 - (a) $\lim_{x \rightarrow 0^+} h(x)$.
 - (b) $\lim_{x \rightarrow 0^-} h(x)$. Make sure your justification involves determining the sign of the numerator and denominator for x -values slightly smaller than 0.
 - (c) $\lim_{x \rightarrow 0} h(x)$.
 - (d) $h(0)$.
 - (e) Does h have a vertical asymptote at the line $x = 0$?
10. (2.4) The graph of f in the figure has vertical asymptotes at $x = 1$ and $x = 2$. Find the following limits, if possible. If not possible, then say so and why.
 - (a) $\lim_{x \rightarrow 1^-} f(x)$
 - (b) $\lim_{x \rightarrow 1^+} f(x)$
 - (c) $\lim_{x \rightarrow 1} f(x)$
 - (d) $\lim_{x \rightarrow 2^-} f(x)$

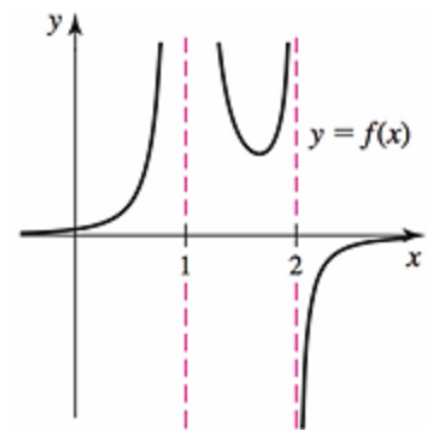


Figure 3: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

- (e) $\lim_{x \rightarrow 2^+} f(x)$
 - (f) $\lim_{x \rightarrow 2} f(x)$
11. (2.4) Sketch a possible graph of a function f , together with vertical asymptotes, satisfying all of the following conditions.
- (a) $f(1) = 0$
 - (b) $f(3)$ is undefined
 - (c) $\lim_{x \rightarrow 3} f(x) = 1$
 - (d) $\lim_{x \rightarrow 0^+} = -\infty$
 - (e) $\lim_{x \rightarrow 2} f(x) = \infty$
 - (f) $\lim_{x \rightarrow 4^-} f(x) = \infty$
12. (2.1/3.1) Given the graph of f in the following figures, find the slope of the secant line that passes through $(0, 0)$ and $(h, f(h))$, in terms of h , for $h > 0$ and $h < 0$. Then calculate the limit of that slope as $h \rightarrow 0^+$ and as $h \rightarrow 0^-$. What does this tell you about the tangent line to the curve at $(0, 0)$? Why doesn't f itself have a vertical asymptote in either of these cases?
- (a) $f(x) = x^{\frac{1}{9}}$
 - (b) $f(x) = x^{\frac{1}{3}}$
13. (up to 2.5) Sketch a possible graph of a function f that satisfies all of the given conditions. Be sure to identify all the vertical and horizontal asymptotes.
- (a) $f(-1) = -2$
 - (b) $f(1) = 2$
 - (c) $f(0) = 0$
 - (d) $\lim_{x \rightarrow \infty} f(x) = 1$
 - (e) $\lim_{x \rightarrow -\infty} f(x) = -1$

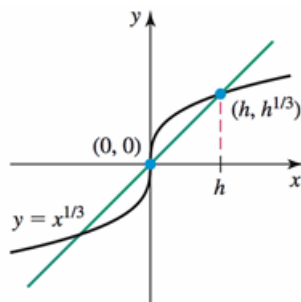


Figure 4: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

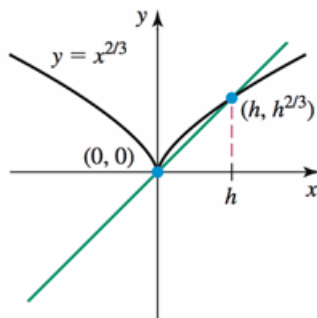


Figure 5: (Briggs, W. and Cochran, L. *Calculus: Early Transcendentals*)

14. (2.4/2.5) For each function $f(x)$, evaluate $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$, and then identify any horizontal asymptotes. Next, find the vertical asymptotes. For each vertical asymptote $x = a$, evaluate $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$.

(a) $f(x) = \frac{x^2 - 4x + 3}{x - 1}$

(b) $f(x) = \frac{\sqrt{16x^4 + 64x^2 + x^2}}{2x^2 - 4}$

(c) $f(x) = \frac{x^2 - 4}{x(x - 2)}$

Computations/Algebra.

1. (2.3) $\lim_{x \rightarrow 4} (3x - 7)$
2. (2.3) $\lim_{x \rightarrow -5} \pi$
3. (2.3) $\lim_{x \rightarrow -5} 2$
4. (2.3) Suppose $\lim_{x \rightarrow 1} f(x) = 8$, $\lim_{x \rightarrow 1} g(x) = 3$, and $\lim_{x \rightarrow 1} h(x) = 2$. Compute the following limits and state the limit laws used (and why you are allowed to use them in that instance, if there is a caveat) to justify your computations. If the limit does not exist then say so.

- (a) $\lim_{x \rightarrow 1} 4f(x)$
- (b) $\lim_{x \rightarrow 1} \frac{f(x)g(x)}{h(x)}$
- (c) $\lim_{x \rightarrow 1} \sqrt[3]{f(x)g(x) + 3}$
- (d) $\lim_{x \rightarrow 1} (2x^3 - 3x^2 + 4x + 5)$
- (e) $\lim_{x \rightarrow 1} (x^2 - x)^5$
- (f) $\lim_{x \rightarrow 3^-} \sqrt{\frac{x-3}{2-x}}$
- (g) $\lim_{x \rightarrow 3} \sqrt{\frac{x-3}{2-x}}$
- (h) $\lim_{x \rightarrow 3^+} \sqrt{\frac{x-3}{2-x}}$
- (i) $\lim_{x \rightarrow -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)}$
- (j) $\lim_{t \rightarrow a} \frac{\sqrt{3t+1} - \sqrt{3a-1}}{t-a}$
- (k) $\lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2}}{x^2}$
- (l) $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{2}{x^2-2x} \right)$
- (m) $\lim_{x \rightarrow c} \frac{x^2 - 2cx + c^2}{x-c}$
- (n) $\lim_{x \rightarrow \infty} \frac{2x}{x+1}$
- (o) $\lim_{x \rightarrow -\infty} \left(5 + \frac{100}{x} + \frac{\sin^4 x^3}{x^2} \right)$
- (p) $\lim_{x \rightarrow -\infty} (3x^7 + x^2)$
- (q) $\lim_{x \rightarrow \infty} -12x^{-5}$