$\boxed{\S 14.7\ \#20}$

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot d\vec{r}$$

$$\vec{r}(t) = \langle \cos t, 2\sin t, \sqrt{3}\cos t \rangle$$

$$\Rightarrow \vec{r}\,'(t) = \langle -\sin t, 2\cos t, -\sqrt{3}\sin t \rangle$$

$$\vec{F} = \langle \cos t + 2\sin t, 2\sin t + \sqrt{3}\cos t, \sqrt{3}\cos t + \cos t \rangle$$

$$\int_0^{2\pi} \left(-\cos t \sin t - 2\sin^2 t + 4\cos t \sin t + 2\sqrt{3}\cos^2 t - 3\cos t \sin t - \sqrt{3}\cos t \sin t \right) dt$$

$$= \int_0^{2\pi} \left(-2\sin^2 t + 2\sqrt{3}\cos^2 t \right) dt + \int_0^{2\pi} \left(\cancel{-1+4-3} - \sqrt{3} \right) \cos t \sin t\, dt$$

Put $u = \sin t \Rightarrow du = \cos t\, dt$

$u(2\pi) = 0$

$u(0) = 0$

$\rightarrow \int = 0$

$$= -2\left(\frac{1}{\cancel{2}}\right)\int_0^{2\pi}(1-\cos 2t)\,dt + \cancel{2}\sqrt{3}\left(\frac{1}{\cancel{2}}\right)\int_0^{2\pi}(1+\cos 2t)\,dt$$

$$\left(t - \cancel{\tfrac{1}{2}\sin 2t}\right)\Big|_0^{2\pi} \qquad \left(t + \tfrac{1}{2}\sin 2t\right)\Big|_0^{2\pi}$$

$$= -2\pi + \sqrt{3}(2\pi)$$

$$\boxed{= (\sqrt{3} - 1)2\pi}$$