# Wed 17 Feb

- Expect Exam back on Thursday. Feedback on Friday. Scores $\to$ MLP?
- Instructions for when you get your exam back:
    - Look over your test, but don't write on it.
    - If you find discrepancies on points or grading, write your grievances on a separate sheet of paper.
    - Return that paper with your exam to your drill instructor by the end of drill.
    - Once you leave the room with your exam you lose this opportunity.
    - This is the only way you can get points back on the exam.

# Wed 17 Feb (cont.)

- MIDTERM in less than three weeks.
  - Tuesday 8 March 6-7:30p
  - If you have legitimate conflict, i.e., anything that is also scheduled in ISIS, I need to know now. If you are not sure if it conflicts with a course, please have that instructor contact me ASAP.
  - Morning Section: Walker rm 124
    Afternoon Section: Walker rm 218
- Later this month: Sub on Friday 26 Feb and Monday 29 Feb.

### Exercise

(a) Find the slope of the line tangent to the curve $f(x) = x^3 - 4x - 4$ at the point $(2, -4)$.

(b) Where does this curve have a horizontal tangent?

If we can write the derivative of $f$ as a function of $x$, then we can take *its* derivative, too. The derivative of the derivative is called the **second derivative** of $f$, and is denoted $f''$.

In general, we can differentiate $f$ as often as needed. If we do it $n$ times, the $n$th derivative of $f$ is

$$f^{(n)}(x) = \frac{d^n f}{dx^n} = \frac{d}{dx}[f^{(n-1)}(x)].$$

## 3.3 Book Problems

9-48 (every 3rd problem), 51-53, 58-60

- For these problems, use only the rules we have derived so far.

# §3.4 The Product and Quotient Rules

Issue: Derivatives of products and quotients do NOT behave like they do for limits.

As an example, consider $f(x) = x^2$ and $g(x) = x^3$. We can try to differentiate their product in two ways:

- $$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}\left(x^5\right) = 5x^4$$
- $$f'(x)g'(x) = (2x)(3x^2) = 6x^3$$

**Question**

Which answer is the correct one?

# Product Rule

If $f$ and $g$ are any two functions that are differentiable at $x$, then

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x).$$

In the example from the previous slide, we have

$$\begin{aligned}
\frac{d}{dx}[x^2 \cdot x^3] &= \frac{d}{dx}(x^2) \cdot (x^3) + x^2 \cdot \frac{d}{dx}(x^3) \\
&= (2x) \cdot (x^3) + x^2 \cdot (3x^2) \\
&= 2x^4 + 3x^4 \\
&= 5x^4
\end{aligned}$$

## Derivation of the Product Rule

$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\left(\frac{f(x+h)g(x+h) + [-f(x)g(x+h) + f(x)g(x+h)] - f(x)g(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{f(x+h)g(x+h) - f(x)g(x+h)}{h}\right) + \left(\lim_{h\to 0} \frac{f(x)g(x+h) - f(x)g(x)}{h}\right)$$

## Derivation of the Product Rule (cont.)

$$= \lim_{h\to 0}\left(g(x+h)\frac{f(x+h)-f(x)}{h}\right)+\left(\lim_{h\to 0} f(x)\frac{g(x+h)-g(x)}{h}\right)$$

$$= g(x)f'(x)+f(x)g'(x)$$

## Exercise

Use the product rule to find the derivative of the function $(x^2 + 3x)(2x - 1)$.

A. $2(2x + 3)$

B. $6x^2 + 10x - 3$

C. $2x^3 + 5x^2 - 3x$

D. $2x(x + 3) + x(2x - 1)$

# Derivation of Quotient Rule

### Question

Let $q(x) = \dfrac{f(x)}{g(x)}$. What is $\dfrac{d}{dx} q(x)$?

**Answer:** We can write $f(x) = q(x)g(x)$ and then use the Product Rule:

$$f'(x) = q'(x)g(x) + g'(x)q(x)$$

and now solve for $q'(x)$:

$$q'(x) = \frac{f'(x) - q(x)g'(x)}{g(x)}.$$

Then, to get rid of $q(x)$, plug in $\dfrac{f(x)}{g(x)}$:

$$q'(x) = \frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}$$

$$= \frac{g(x)\left(f'(x) - g'(x)\frac{f(x)}{g(x)}\right)}{g(x) \cdot g(x)}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

"LO-D-HI minus HI-D-LO over LO squared"

Just as with the product rule, the derivative of a quotient is not a quotient of derivatives, i.e.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \neq \frac{f'(x)}{g'(x)}.$$

Here is the correct rule, the Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}.$$

## Exercise

Use the Quotient Rule to find the derivative of

$$\frac{4x^3 + 2x - 3}{x + 1}.$$

## Exercise

Find the slope of the tangent line to the curve

$$f(x) = \frac{2x - 3}{x + 1} \text{ at the point } (4, 1).$$

The Quotient Rule also allows us to extend the Power Rule to negative numbers – if $n$ is any integer, then

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}.$$

**Question**

How?

### Exercise

If $f(x) = \dfrac{x(3-x)}{2x^2}$, find $f'(x)$.

## Derivative of $e^{kx}$

For any real number $k$,

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}.$$

### Exercise

What is the derivative of $x^2e^{3x}$?

The derivative provides information about the instantaneous rate of change of the function being differentiated (compare to the limit of the slopes of the secant lines from §2.1).

For example, suppose that the population of a culture can be modeled by the function $p(t)$. We can find the instantaneous growth rate of the population at any time $t \geq 0$ by computing $p'(t)$ as well as the **steady-state population** (also called the **carrying capacity** of the population). The steady-state population equals

$$\lim_{t \to \infty} p(t).$$

## 3.4 Book Problems

9-49 (every 3rd problem), 57, 59, 63, 75-79 (odds)