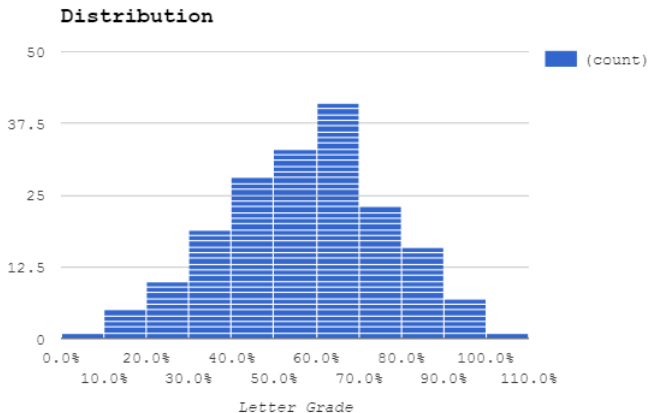


π Day 2016

- Exam 2: Curve, etc. is posted.



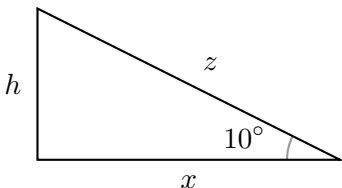
π Day 2016 (cont.)

- Midterm: expect it back Thursday in drill. Don't expect a curve. :(
- "Fast Track Calculus": Dr. Kathleen Morris will be teaching a second 8 weeks Calculus One class. "If you have a student who is maybe doing poorly because of illness or a tragic event in their life during the beginning of the semester, this might be an opportunity for a new start for them." The class requires departmental consent so the student will need to contact Kathleen to get permission to enroll.

Steps for Solving Related Rates Problems

1. Read the problem carefully, making a sketch to organize the given information. Identify the rates that are given and the rate that is to be determined.
2. Write one or more equations that express the basic relationships among the variables.
3. Introduce rates of change by differentiating the appropriate equation(s) with respect to time t .
4. Substitute known values and solve for the desired quantity.
5. Check that the units are consistent and the answer is reasonable.

Step 1: There are three variables: the distance the shadow has traveled (x), the altitude of the jet (h), and the distance the jet has actually traveled on its line of flight (z). We know that $\frac{dz}{dt} = 550$ miles/hr and we want to find $\frac{dx}{dt}$ and $\frac{dh}{dt}$. We also see that these variables are related through a right triangle:



Step 2: To answer how fast the altitude is increasing, we need an equation involving only h and z . Using trigonometry,

$$\sin(10^\circ) = \frac{h}{z} \implies h = \sin(10^\circ) \cdot z.$$

To answer how fast the shadow is moving, we need an equation involving only x and z . Using trigonometry,

$$\cos(10^\circ) = \frac{x}{z} \implies x = \cos(10^\circ) \cdot z.$$

Step 3: We can now differentiate each equation to answer each question:

$$h = \sin(10^\circ) \cdot z \implies \frac{dh}{dt} = \sin(10^\circ) \frac{dz}{dt}$$
$$x = \cos(10^\circ) \cdot z \implies \frac{dx}{dt} = \cos(10^\circ) \frac{dz}{dt}$$

Step 4: We know that $\frac{dz}{dt} = 550$ miles/hr. So

$$\frac{dh}{dt} = \sin(10^\circ) \cdot 550 \approx 95.5 \text{ miles/hr}$$
$$\frac{dx}{dt} = \cos(10^\circ) \cdot 550 \approx 541.6 \text{ miles/hr}$$

Step 5: Because both answers are in terms of miles/hr and both answers seem reasonable within the context of the problem, we conclude that the jet is gaining altitude at a rate of 95.5 miles/hr, while the shadow on the ground is moving at about 541.6 miles/hr.

Example

The sides of a cube increase at a rate of R cm/sec. When the sides have a length of 2 cm, what is the rate of change of the volume?

Exercise

A 13 foot ladder is leaning against a vertical wall when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.5 ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

Exercise

Sand falls from an overhead bin and accumulates in a conical pile with a radius that is always three times its height.

Suppose the height of the pile increases at a rate of 2 cm/sec.

When the pile is 12 cm high, at what rate is the sand leaving the bin? *Recall the volume of a cone: $V = \frac{1}{3}\pi r^2 h$.*

3.11 Book Problems

5-14, 16-19, 21-24, 37-38

§4.1 Maxima and Minima

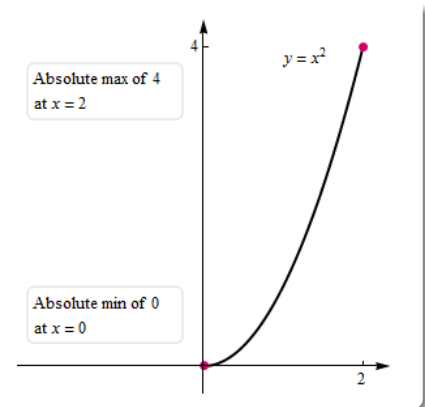
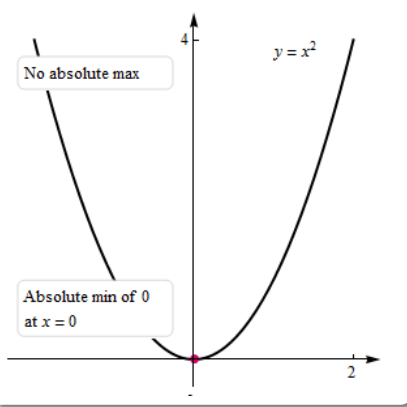
Chapter 4 is all about applications of the derivative. In the first couple of sections we examine the graphs of functions and what the derivative can tell us about the graph's behavior and characteristics.

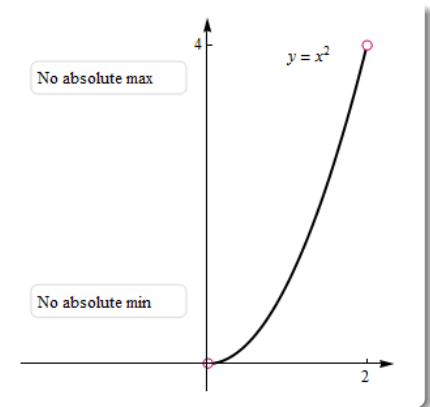
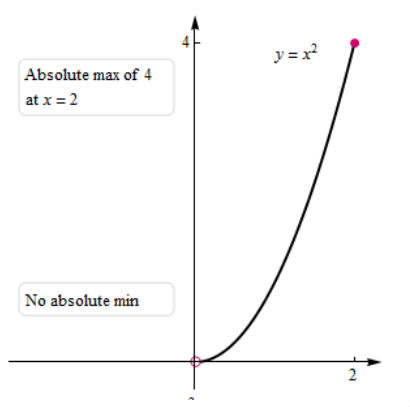
Definition

Let f be defined on an interval I containing c .

- f has an **absolute maximum** value on I at c means $f(c) \geq f(x)$ for every x in I .
- f has an **absolute minimum** value on I at c means $f(c) \leq f(x)$ for every x in I .

The existence and location of absolute extreme values depend on the function and the interval of interest:





Extreme Value Theorem

Theorem (Extreme Value Theorem)

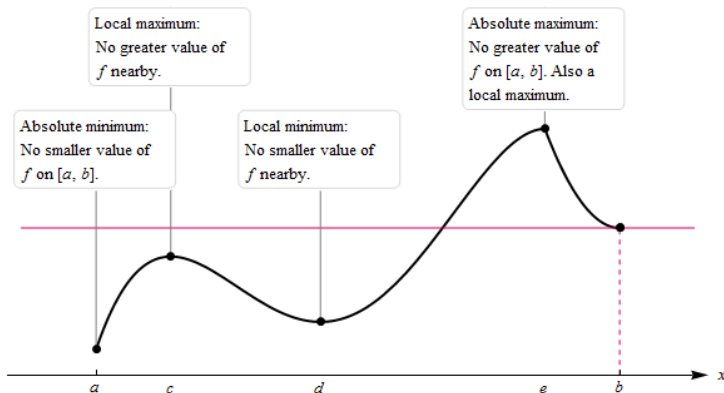
A function that is continuous on a closed interval $[a, b]$ has an absolute maximum value and an absolute minimum value on that interval.

The EVT provides the criteria that ensures absolute extrema:

- the function must be continuous on the interval of interest;
- the interval of interest must be closed and bounded.

Local Maxima and Minima

Beyond absolute extrema, a graph may have a number of peaks and dips throughout its interval of interest:



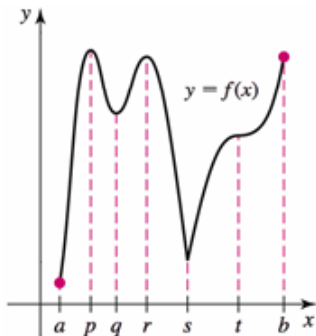
Definition

Suppose I is an interval on which f is defined and c is an interior point of I .

- If $f(c) \geq f(x)$ for all x in some open interval containing c , then $f(c)$ is a **local maximum** value of f .
- If $f(c) \leq f(x)$ for all x in some open interval containing c , then $f(c)$ is a **local minimum** value of f .

Exercise

Use the graph below to identify the points on the interval $[a, b]$ at which local and absolute extreme values occur.



Critical Points

Based on the previous graph, how is the derivative related to where the local extrema occur?

Local extrema occur where the derivative either does not exist or is equal to 0.

Definition

An interior point c of the domain of f at which $f'(c) = 0$ or $f'(c)$ fails to exist is called a **critical point** of f .

Local Extreme Point Theorem

Theorem (Local Extreme Point Theorem)

If f has a local minimum or maximum value at c and $f'(c)$ exists, then $f'(c) = 0$. (Converse is not true!)

It is possible for $f'(c) = 0$ or $f'(c)$ not to exist at a point, yet the point not be a local min or max. Therefore, critical points provide **candidates** for local extrema, but do not guarantee that the points are local extrema (see p. 227 immediately before Figure 4.9 for examples).

Locating Absolute Min and Max

Two facts help us in the search for absolute extrema:

- Absolute extrema in the interior of an interval are also local extrema, which occur at critical points of f .
- Absolute extrema may occur at the endpoints of f .

Procedure: Assume that the function f is continuous on $[a, b]$.

1. Locate the critical points c in (a, b) , where $f'(c) = 0$ or $f'(c)$ does not exist. These points are **candidates** for absolute extrema.
2. Evaluate f at the critical points and at the endpoints of $[a, b]$.
3. Choose the largest and smallest values of f from Step 2 for the absolute max and min values, respectively.

NOTE: In this section, given an equation, we can identify critical points and absolute extrema, **BUT NOT LOCAL EXTREMA**. Techniques for locating local extrema come in later sections.

Example

On the interval $[-2, 2]$, the function $f(x) = x^4$

- A. has no local or absolute extrema.
- B. has a local minimum but no absolute minimum.
- C. has an absolute maximum but no local maxima.
- D. has an absolute maximum at an interior point of the interval.

Exercise

Given $f(x) = (x + 1)^{4/3}$ on $[-8, 8]$, determine the critical points and the absolute extreme values of f .

4.1 Book Problems

11–35 (odds), 37–49 (odds)