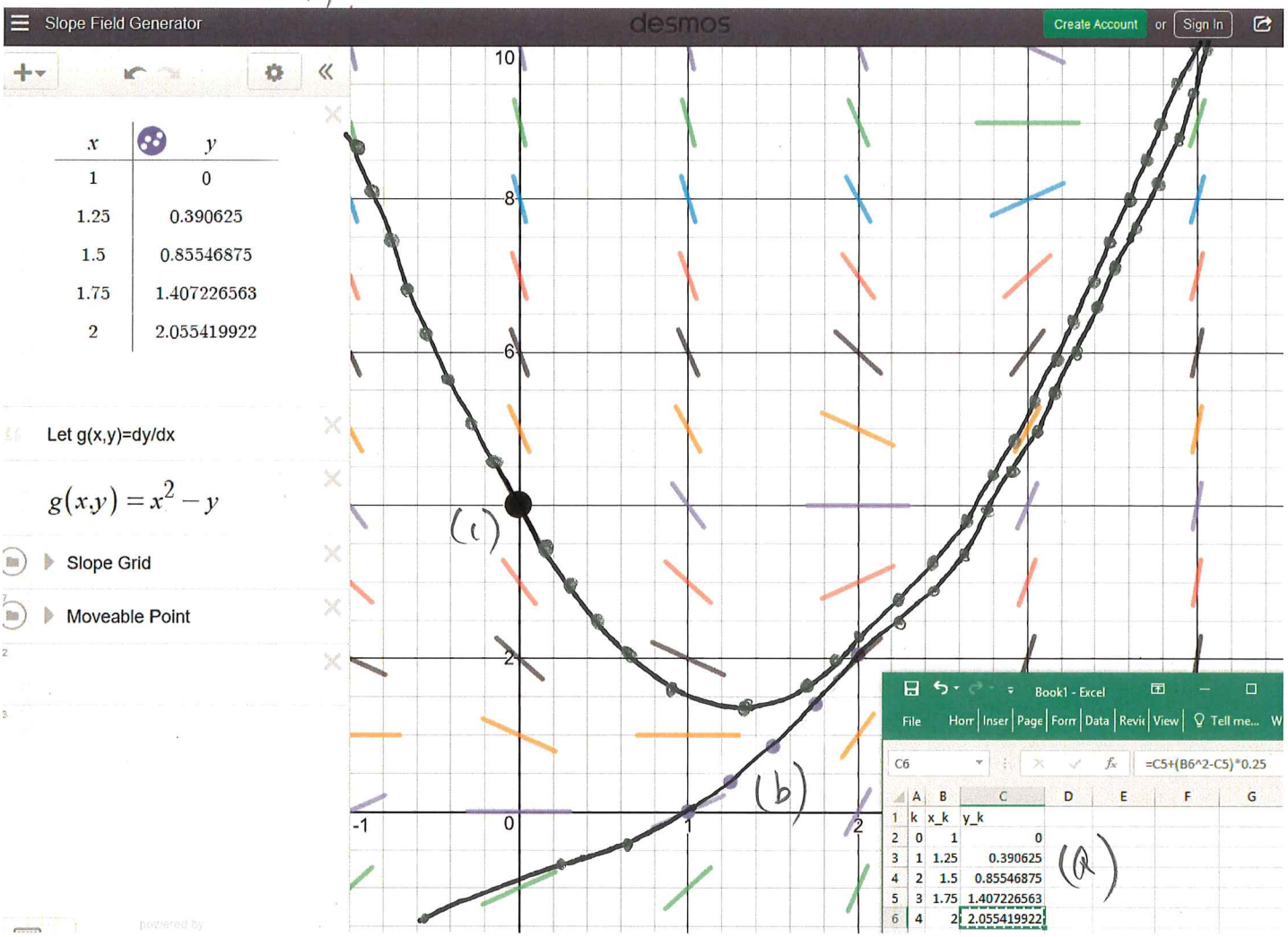
Take-Home Quiz 8

SOLUTIONS

Math 236 (Calc II)
Fall 2017

1. (a) (see Excel)



(d) If the two curves cross each other at a point (x_0, y_0), then the slope $\left.\frac{dy}{dx}\right|_{(x_0,y_0)}$ will be two different numbers, which is impossible.

2(a) $T'(t) = 0 - (A - T_0)e^{-kt}(-k)$

$= k\underbrace{(A - T_0)e^{-kt}}_{A - T(t)}$

(b) $\frac{dT}{dt} = k(A - T)$

$\Rightarrow \int \frac{1}{A-T}\, dT = \int k\, dt$

$\ln|A - T| = kt + C$

$A - T = e^{kt+C}$

$\Rightarrow T = A - e^{kt+C}$

Use the initial value:

$T(0) = A - e^{k(0)+C} = T_0$

$A - T_0 = e^C \Rightarrow C = \ln(A - T_0)$

Then $T(t) = A - e^{kt + \ln(A-T_0)}$

$= A - e^{kt} \cdot e^{\ln(A-T_0)}$

$= A - (A - T_0)e^{kt}$

3. The given parametric equations are for a circle centered at the origin. Shift each coordinate to shift the circle:

$$x(t) = r\cos t + a$$
$$y(t) = r\sin t + b$$

4(a) $\frac{dx}{dt} = 2 \Rightarrow \int dx = \int 2\,dt$

$$x = 2t + C$$

At time $t=0$, $x=0$:

$$x(0) = 2(0) + C = 0 \Rightarrow C = 0.$$

$$\frac{dy}{dt} = 1.778t^2 - 2.667t$$

$$\Rightarrow y = 1.778\frac{t^3}{3} - 2.667\frac{t^2}{2} + C$$

At time $t=0$, $y=0$:

$$y(0) = \frac{1.778}{3}(0)^3 - \frac{2.667}{2}(0)^2 + C = 0$$

$$\Rightarrow C = 0$$

$$x(t) = 2t$$

$$t \geq 0$$

$$y(t) = \frac{1.779}{3}t^3 - \frac{2.667}{2}t^2$$

(b) Annie must paddle 3 miles east:

$$x(t) = 2t = 3$$

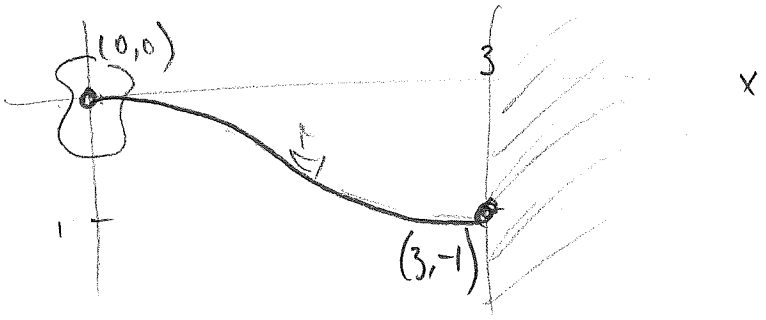
$$\Rightarrow t = \frac{3}{2}$$

means it takes an hour and a half for Annie to make landfall. Her y-position at that time is

$$y\left(\frac{3}{2}\right) = \frac{1.778}{3}\left(\frac{3}{2}\right)^3 - \frac{2.667}{2}\left(\frac{3}{2}\right)^2 \approx -1.000$$

$\Rightarrow$ about 1 mile South

(c)



$$\text{arc length} = \int_0^{3/2} \sqrt{(2)^2 + (1.778t^2 - 2.667t)^2}\, dt$$

$$\approx 3.191 \text{ miles}$$

(desmos)