Math 2603 Exam 3 **Tues 2 Dec 2014**
(Take-Home Version) **Name:** ______________________________

Discrete Math
Exam 3 Take-Home Version (Ch. 6-8 as we have covered)

Please provide the following data:

Drill Time: __________________________________

Student ID: __________________________________

Exam Instructions: THIS EXAM IS OPTIONAL. If you do not submit on time, then your score from the original Exam 3 will be recorded. You have until 12a Monday 8 December, 2014 (i.e., midnight Sunday night) to submit this exam. Rules:

1. You are welcome to use whatever resources you wish. You *must* cite your source if it's not your brain. The more specific you are, the more lenience you get on partial credit/showing work, BUT your written answer must be coherent on its own (e.g., you can't just say "page blah blah of blerg blip textbook").

2. Your answers must be in your own hand-writing.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: ____

1. (a) How many strings can be formed using all of the letters

A S S I S T A N T S H I P

(b) How many such strings have all the S's consecutive?

(c) How many such strings from (a) have no consecutive S's?

2. (a) The Binomial Theorem (BT) states $(x+y)^n =$

(b) Using the BT, write down the coefficient for x^2yz in the expansion of $(2x+y-z)^4$.

(c) Prove that

$$\left(\frac{m}{m+n}\right)^m \left(\frac{n}{m+n}\right)^n \cdot \binom{m+n}{m} < 1$$

for all $m, n \in \mathbb{Z}_{>0}$. *Hint:* Consider the term for $k = m$ in the BT expansion of $(x+y)^{m+n}$ for appropriate x and y.

3. Professor Euclid is paid every other Friday. Using the Pigeonhole Principle, prove that in one year's time, there will be some month where she got paid three times. To get credit, you MUST state which version of the Principle Pigeonhole you used and how you applied it in solving the problem.

4. How many positive integer solutions are there of

$$x_1 + x_2 + x_3 = 20?$$

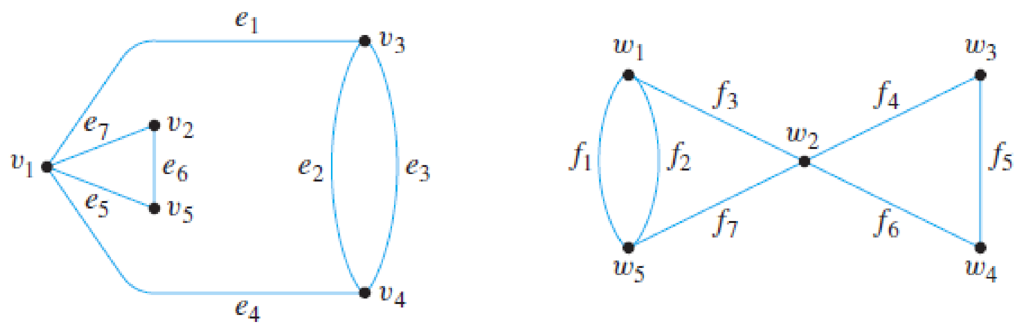
5. (a) Draw the complete bipartite graph $K_{2,4}$, with vertices and edges legibly labeled.

(b) Write the adjacency matrix for $K_{2,4}$.

(c) Write the incidence matrix for $K_{2,4}$.

(d) Does $K_{2,4}$ have an Euler cycle? If yes, then list the ordering of edges that give one. If no, then prove why not.

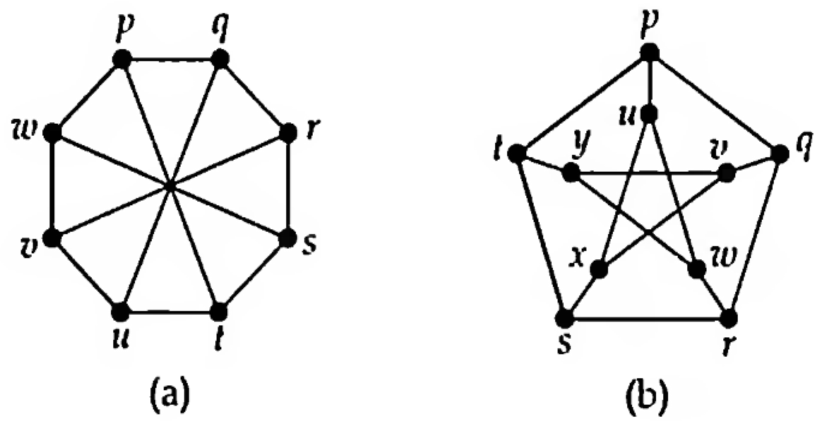
6. Are the following[1] graphs G, G' isomorphic? If so then, exhibit an isomorphism. If not, then state an invariant not shared by the two graphs. If the invariant you cite was not mentioned in class then you must prove it is actually an invariant.



G $\qquad$ G'

[1]Image Credit: Epp, Susanna. *Discrete Mathematics with Applications*. Cengage Learning, 2010. p. 677.

7. DO BOTH[2]:



Is the graph planar? Prove your answer: If it is planar then redraw it without any edges overlapping; if it is not planar then exhibit, by series reduction if necessary, a subgraph homeomorphic to $K_{3,3}$ or K_5.

[2]Image Credit: Aldous, Joan M. and Wilson, Robin J. *Graphs and Applications: An Introductory Approach.* Springer-Verlag London, 2000. p. 262.

8. The Fibonacci sequence is defined by the recurrence relation

$$f_n = f_{n-1} + f_{n-2}, \qquad \text{for } n \geq 3.$$

(a) How many initial conditions should there be and what are they?

(b) Solve the relation to get an explicit formula for f_n.

9. Find a formula for the probability that out of n millennials, at least two will have the same birthday, where "same birthday" means same month, date, AND year. Millennials are those born in the years 1980-1995. Assume no leap years in that time period.

10. Find such a formula, only this time, taking leap years into account.