$y = \cos 7x$

Find $y^{(101)}(0)$.

$y' = -7\sin 7x$

$y'' = -7^2 \cos 7x$

$y''' = 7^3 \sin 7x$

$y^{(4)} = 7^4 \underline{\cos 7x}$

$y^{(10)} = -7^{10} \cos 7x$

$y^{(101)} = -7^{101} \sin 7x$

$y^{(101)}(0) = 0$

Find a, b so that (-1, 2) is an inflection point for

$$y = ax^3 + bx^2 - 8x + 2 \leftarrow$$

$$y' = 3ax^2 + 2bx - 8$$

$$y'' = 6ax + 2b = 0$$

$$6a(-1) + 2b = 0$$

$$-6a + 2b = 0$$

$$b = \frac{6a}{2} = 3a$$

$$y'' = 6ax + 2(3a) = 0$$

$$6a(-1) + 6a = 0$$

$$6a = 6a$$

$$\cancel{2} = a(-1)^3 + b(-1)^2 - 8(-1) + \cancel{2}$$

$$0 = -a + b + 8$$

$$0 = -a + 3a + 8 \longrightarrow a = -4, \quad b = -12$$

$\longrightarrow$

Check it is an inflection point.

$$y''(-2) = 6(-4)(-2) + 2(-12) > 0$$

$$y''(0) = 6(-4)(0) + 2(-12) < 0$$

$$\boxed{\text{Infl. pt at } (-1, 2) \text{ when } a = -4 \text{ and } b = -12.}$$

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a) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$ $\left(\frac{0}{0}\right)$ Evaluate analytically.

$$\lim_{x \to 2} \frac{(x+3)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \boxed{\frac{5}{4}}$$

$$\ln\left(\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x\right) = L \qquad \text{ans: } e^L$$

$$= \lim_{x\to\infty} x \ln\left(\frac{x+1}{x}\right) \quad (\infty \cdot 0)$$

$$= \lim_{x\to\infty} \frac{\ln\left(\frac{x+1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \quad \left(\frac{0}{0}\right)$$

l'Hôpital:

$$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{1}{1+\cancel{\frac{1}{x}}_{\to 0}}$$

$$= 1$$

$$\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x = e^L = e^1 = e$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2+\cancel{x^2}}\right) \quad (\#36 \text{ in } \S 4.7)$$
$$$$

$$=\lim_{x\to\infty}\left(x - x\sqrt{1+\frac{1}{x^2}}\right)$$

$$=\lim_{x\to\infty} x\left(1-\sqrt{1+\frac{1}{x^2}}\right) \quad (\infty\cdot 0)$$

$$=\lim_{x\to\infty} \frac{1-\sqrt{1+\frac{1}{x^2}}}{\frac{1}{x}} \quad \left(\frac{0}{0}\right)$$

Let $y=\frac{1}{x}$ As $x\to\infty$,
$y\to 0^+$

$$=\lim_{y\to 0^+}\frac{1-\sqrt{1+y^2}}{y}$$

L'Hôp!

$$= \lim_{y \to 0^+} \frac{-\cancel{\frac{1}{2}}(1+y^2)^{-1/2} \cdot \cancel{2}y}{1}$$

$$= \lim_{y \to 0^+} \frac{-y}{\sqrt{1+y^2}} = 0$$

$f(x) = \frac{\pi}{4} \sin x$; $[0, \pi]$

Find the point(s) at which f equals its average on the given interval.

$$\bar{f} = \frac{1}{\pi - 0} \int_0^{\pi} \frac{\pi}{4} \sin x \, dx$$

$$= \frac{1}{4} \int_0^{\pi} \sin x \, dx$$

$$= \frac{1}{4}\left(-\cos x \Big|_0^{\pi}\right) = \frac{1}{4}\left(-\overset{-1}{\cos \pi} - (-\overset{1}{\cos 0})\right)$$

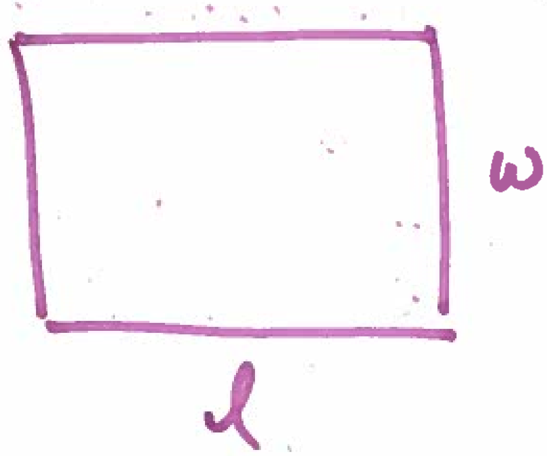
$1 + 1$

$$= \frac{1}{2}$$

$$f(x) = \frac{1}{2} = \frac{\pi}{4} \sin x$$

$$\frac{2}{\pi} = \sin x$$

$$\boxed{\arcsin\left(\frac{2}{\pi}\right) = x \approx \cancel{0.0111}\ 0.69}$$



$10m = 2l + 2w$

<u>Maximize</u>:

$A = \text{Area} = lw$

$$w = \frac{10 - 2l}{2} = 5 - l$$

$$A = l(5 - l) = 5l - l^2$$

$$\frac{dA}{dl} = 5 - 2l = 0 \implies l = \frac{5}{2}\, m$$

Know: $0 \le l \le 5$

$A(0) = 0(5-0) = 0$

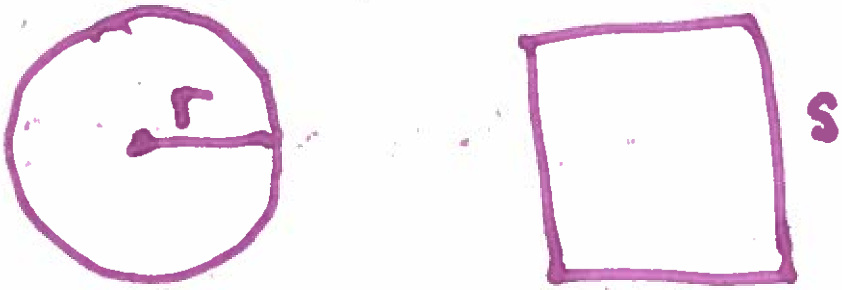
$A(5) = 5(5-5) = 0$

$A\left(\frac{5}{2}\right) = \frac{5}{2}\left(5-\frac{5}{2}\right) = +\#$ so >0

so is a max.

$$\boxed{\begin{array}{l}\text{dimensions:} \\ l = 2.5\text{m} \\ w = 2.5\text{m}\end{array}}$$

#20 §4.4



Minimize

$A = \text{Area} = \pi r^2 + s^2$

$60\text{ cm} = 2\pi r + 4s$

$60 - 4s = 2\pi r$

$\Rightarrow r = \dfrac{60-4s}{2\pi} = \dfrac{30 - 2s}{\pi}$

$A = \pi\left(\left(\dfrac{1}{\pi}\right)^2 \underbrace{(30-2s)^2}\right) + s^2$

$\dfrac{dA}{ds} = \dfrac{1}{\pi}\left(2(30-2s)\cdot(-2)\right) + 2s = 0$

$= \dfrac{1}{\pi}(-120 + 8s) + 2s = 0$

$\left(\dfrac{8}{\pi} + 2\right)s = \dfrac{120}{\pi}$

Area is minimized when

$s=0$.

$$A\left(\frac{120}{8+2\pi}\right)=\frac{1}{\pi}\left(30-\frac{2(120)}{8+2\pi}\right)^2+\left(\frac{120}{8+2\pi}\right)^2$$

$$=\frac{1}{\pi}\left(30^2-120\left(\frac{120}{8+2\pi}\right)+\left(\frac{120}{8+2\pi}\right)^2\right)+\left(\frac{120}{8+2\pi}\right)^2$$

Know:

$0 \leq s \leq 15$

$$s = \frac{120}{\pi\left(\frac{8}{\pi}+2\right)} = \frac{120}{8+2\pi}$$

$$A(0) = \frac{1}{\pi}\left(30-2(0)\right)^2 + 0^2$$

$$= \frac{30^2}{\pi}$$

$$A\left(\frac{120}{8+2\pi}\right) = \frac{1}{\pi}\left(30 - \frac{2(120)}{(8+2\pi)}\right)^2 + \left(\frac{120}{8+2\pi}\right)^2$$

$$\cancel{= \frac{30}{\pi} - \frac{2}{\pi}\left(\frac{120}{8+2\pi}\right)^2 + \left(\frac{120}{8+2\pi}\right)^2}$$

$$\cancel{= \frac{30}{\pi}\left(\frac{120}{8+2\pi}\right)^2\left(1-\frac{2}{\pi}\right)} \rightarrow \frac{30}{\pi}$$

$$A(15) = \frac{1}{\pi}\left(30-2(15)\right)^2 + 15^2 = 15^2$$

$$\cancel{= \frac{30}{\pi} - \frac{2\cdot 15^2}{\pi} + 15^2}$$

$$\cancel{= \frac{30}{\pi} - 15^2\left(1-\frac{2}{\pi}\right) < 0}$$

$$s = \frac{60 - 2\pi r}{4}$$

$$= 15 - \frac{\pi}{2} r$$

$$A = \pi r^2 + \left(15 - \frac{\pi}{2} r\right)^2$$

$$= \pi r^2 + 15^2 - 15\pi r + \frac{\pi^2}{4} r^2$$

$$\frac{dA}{dr} = 2\pi r - 15\pi + \frac{\pi^2}{2} r = 0$$

$$\left(2\pi + \frac{\pi^2}{2}\right) r = 15\pi$$

$$r = \frac{15\pi}{2\pi + \frac{\pi^2}{2}}$$

$$= \frac{30\pi}{4\cancel{\pi} + \pi^{\cancel{2}}}$$

$$= \frac{30}{4 + \pi}$$

$$0 \le r \le \frac{30}{\pi}$$

$$A(0) = \pi(0^2) + 15^2 - 15\pi(0) + \frac{\pi^2}{4}(0^2)$$

$$= 15^2$$

$$A\left(\frac{30}{\pi}\right) = \pi\left(\frac{30}{\pi}\right)^2 + 15^2 - 15\pi\left(\frac{30}{\pi}\right) + \frac{\pi^2}{4}\left(\frac{30}{\pi}\right)^2$$

$$= \frac{900}{\pi}$$

$$A\left(\frac{30}{4+\pi}\right) =$$

$h(x) = \sqrt{4x-9}$

$h'(x) = \frac{1}{2}(4x-9)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4x-9}}$

$y - b = \frac{2}{\sqrt{4(a)-9}}(x-a)$

Solution: Dr. Paulk

$\boxed{y = 4x - 9}$

slope $= 4$

$(y+9) = 4(x-0)$

$y = x^2 - kx$

$y' = 2x - k = 4$

$2(0) - k = 4$

$\Rightarrow k = -4$

$$f(x) = \ln\left(\frac{1}{3}\cdot(1-x^2)^{3/2}\right)$$

$$= \ln\left(\frac{1}{3}\right) + \ln\left((1-x^2)^{3/2}\right)$$

$$= \ln\left(\frac{1}{3}\right) + \frac{3}{2}\ln(1-x^2)$$

$$f'(x) = 0 + \frac{3}{\cancel{2}}\left(\frac{-\cancel{2}x}{1-x^2}\right) = \frac{-3x}{1-x^2}$$

<u>OR</u>

$$f'(x) = \frac{1}{\frac{1}{3}(1-x^2)^{3/2}}\cdot\frac{1}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{2}}\right)(1-x^2)^{1/2}\cdot(-\cancel{2}x)$$

$$= \frac{-3x}{1-x^2}\ ✓$$

$$2r = 3h \Rightarrow r = \frac{3}{2}h$$

$$\frac{dV}{dt} = 10 \frac{ft^3}{min}$$

$$\left.\frac{dh}{dt}\right|_{h=15ft} = ?$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h = \frac{3}{4}\pi h^3$$

$$\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} = \underbrace{\frac{9}{4}\pi h^2}\cdot\frac{dh}{dt}$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=15ft} = \frac{10\, ft^3/min}{\frac{9}{4}\pi(15\, ft)^2} \approx 0.006 \frac{ft}{min}$$