MATH 2554 (Calculus I)
Summer 2015

Name: SOLUTIONS

Fri 5 June 2015

Exam 1: Limits ($\oint$2.1-3.1)

Version A

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt) ______________________

Good luck!

1. **(14 pts)** Given $f(x) = 2x^3 + x$, use the Intermediate Value Theorem to show there exists a solution to the equation $f(x) = 2$ on the interval $(-1, 1)$.

f is a polynomial so is continuous on $[-1, 1]$.

$f(-1) = 2(-1)^3 + (-1) = -2 - 1 = -3$

$f(1) = 2(1)^3 + (1) = 2 + 1 = 3$

Since $-3 < 2 < 3$, by IVT there exists c, between -1 and 1, so that $f(c) = 2$.

2. **(24 pts)** Determine the end behavior of $f(x) = \dfrac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}}$.

$$\lim_{x \to \infty} \frac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 1}} \left(\frac{\frac{1}{\sqrt{x^6}}}{\frac{1}{\sqrt{x^6}}} \right)$$

$$= \lim_{x \to \infty} \frac{4 + \cancel{\frac{1}{x^3}} \to 0}{2 + \sqrt{16 + \cancel{\frac{1}{x^6}} \to 0}} = \frac{4}{2 + 4} = \frac{4}{6} = \frac{2}{3}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{4 - \cancel{\frac{1}{x^3}} \to 0}{2 - \sqrt{16 + \cancel{\frac{1}{x^6}} \to 0}} = \frac{4}{2 - 4} = \frac{4}{-2} = -2$$

$(\sqrt{x^6} = -x^3)$

3. **(5 pts ea)** Evaluate the following limits analytically:

(a) $\lim_{t\to 3} \sqrt[3]{t^2 - 10}$

$= \sqrt[3]{3^2 - 10} = \sqrt[3]{-1} \boxed{= -1}$

(b) $\lim_{\theta\to -\infty} \frac{\cos(\theta^5)}{\theta} = 0$, by the Squeeze Theorem;

$-1 \le \cos\theta^5 \le 1$

implies $\frac{1}{\theta} \le \frac{\cos\theta^5}{\theta} \le -\frac{1}{\theta}$ for all $\theta < 0$.

$\frac{1}{\theta} \to 0$, $-\frac{1}{\theta} \to 0$

as $\theta \to -\infty$.

(c) $\lim_{x\to -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)}$

$= \lim_{x\to -b} \frac{(x+b)^6 + (x+b)^9}{4} = \frac{0}{4} \boxed{= 0}$

4. (a) **(7 pts)** Using the graph below, find the δ that satisfies $|f(x) - 5| < 1$ whenever $0 < |x - 4| < \delta$.

$\delta = 3$

(b) **(7 pts)** Use the same graph to find the δ that satisfies $|f(x) - 5| < \frac{1}{2}$ whenever $0 < |x - 4| < \delta$.

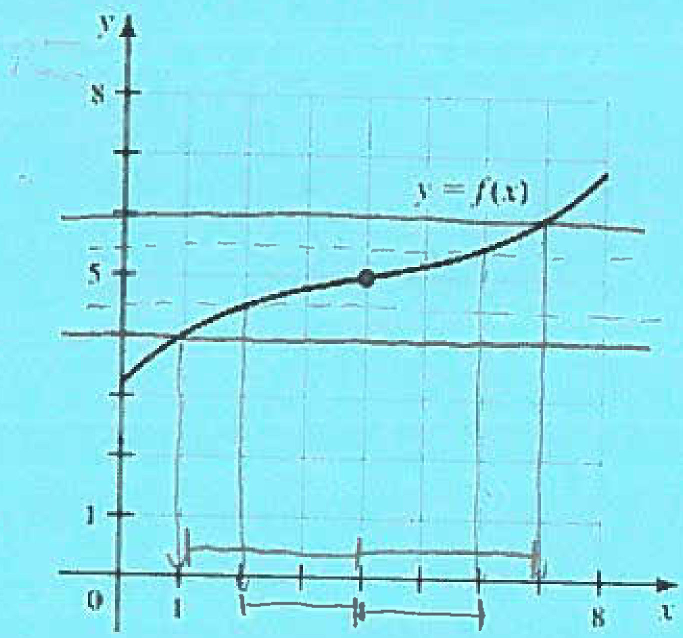
$\delta = 2$

(c) **Extra Credit (4 pts)** Using smaller and smaller ϵs and finding the corresponding δs, as in (a) and (b), will show

$$\lim_{x \to ?} f(x) = ?.$$

(rewrite the limit, with the ?s filled in).

$\lim_{x \to 4} f(x) = 5$



5. **(5 pts ea)** When computing derivatives in this problem you must use the limit definitions. Given the function,

$$s(t) = \frac{1}{t^2},$$

(a) write the formula for the slope of the secant line joining the points $(a, s(a))$ and $(b, s(b))$;

$$\frac{s(b)-s(a)}{b-a} = \frac{\frac{1}{b^2} - \frac{1}{a^2}}{b-a} = \frac{a^2-b^2}{(ab)^2(b-a)}$$

(b) find $s'(1)$;

$$s'(1) = \lim_{t\to 1} \frac{s(t)-s(1)}{t-1} = \lim_{t\to 1} \frac{1-t^2}{t^2(t-1)}$$

$$= \lim_{t\to 1} \frac{\cancel{(1-t)}(1+t)}{-t^2\cancel{(1-t)}}$$

$$= \lim_{t\to 1} \frac{1+t}{-t^2} = \frac{2}{-1} = -2$$

(c) write the equation of the line tangent to $s(t)$ at $t = 1$.

$$y - s(1) = s'(1)(t-1)$$

$$\boxed{y - 1 = -2(t-1)}$$

6. **(11 pts ea)** For each function, identify any vertical asymptotes; if there are none, then say so. Then match the function to its corresponding picture from among the graphs (A)-(C) (see the next page).

(a) $f(x) = \dfrac{x}{x^2+1}$

$\boxed{\text{no VA's ; (B)}}$

(b) $f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x+1)(x-1)}$

$$\lim_{x\to -1^-} \frac{\overset{+\to 1}{1}}{\underbrace{(x+1)}_{0,\text{neg}}\underbrace{(x-1)}_{-2}} = \infty \qquad \lim_{x\to 1^-} \frac{\overset{+\to 1}{1}}{\underbrace{(x+1)}_{2}\underbrace{(x-1)}_{0,\text{neg}}} = -\infty$$

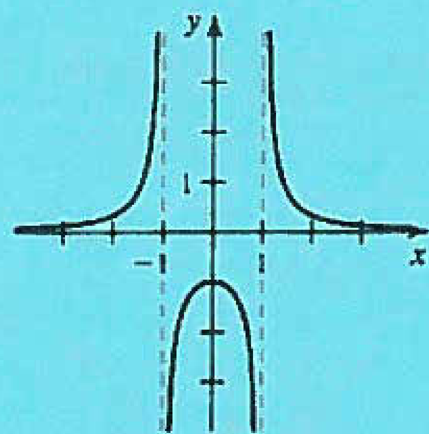
$$\lim_{x\to -1^+} \frac{\overset{+\to 1}{1}}{\underbrace{(x+1)}_{0,\text{pos}}\underbrace{(x-1)}_{-2}} = -\infty \qquad \lim_{x\to 1^+} \frac{\overset{+\to 1}{1}}{\underbrace{(x+1)}_{2}\underbrace{(x-1)}_{0,\text{pos}}} = \infty$$

$\boxed{\text{VA @ } x = \pm 1 \text{ ; (A)}}$

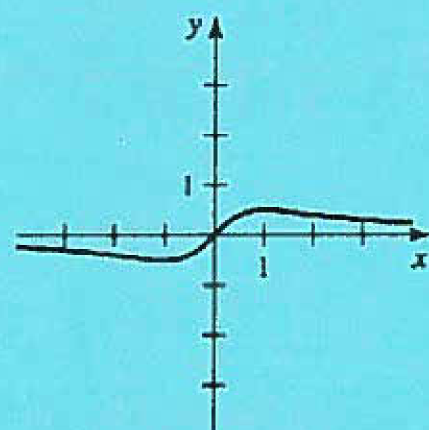
(c) $f(x) = \dfrac{x}{(x-1)^2}$

$$\lim_{x\to 1} \frac{\overset{+\to 1}{x}}{\underbrace{(x-1)^2}_{0,\text{pos}}} = \infty$$

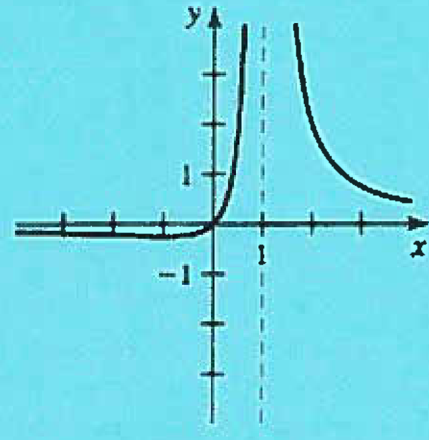
$\boxed{\text{VA @ } x = 1 \text{ ; (C)}}$



(A)

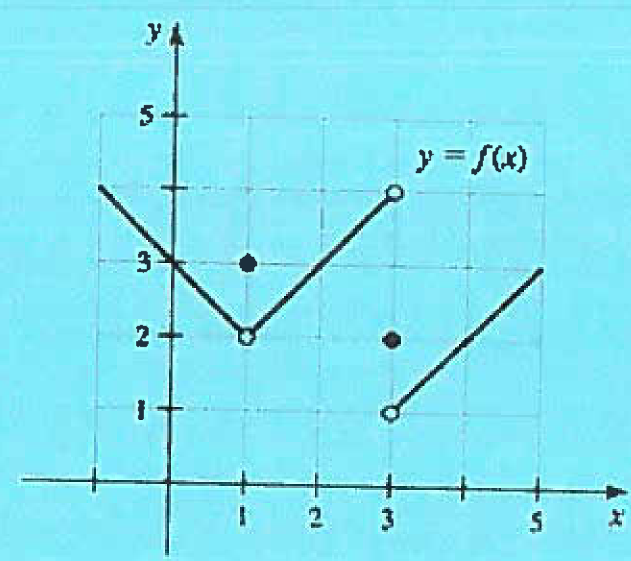


(B)



(C)

7. **(1 pt ea)** Use the graph of f in the figure to find the following values, if they exist. If a limit does not exist, write "DNE".



(a) $\lim_{x\to 2^-} f(x) = 3$

(b) $\lim_{x\to 3} f(x)$ DNE

(c) $\lim_{x\to 1^+} f(x) = 2$

(d) $f(3) = 2$

(e) $\lim_{x\to 3^-} f(x) = 4$

(f) $f(1) = 3$

(g) $\lim_{x\to 2} f(x) = 3$

(h) $f(2) = 3$

(i) $\lim_{x\to 1^-} f(x) = 2$