MATH 236 (Calculus II)
due Thurs 12 Oct 2017

Take-Home Quiz 4: Sequences (§7.1-7.2)

Directions: This quiz is due on October 12, 2017 at the beginning of lecture. You may use whatever resources you like – e.g., other textbooks, websites, collaboration with classmates – to complete it **but YOU MUST DOCUMENT YOUR SOURCES**. Acceptable documentation is enough information for me to find the source myself. Rote copying another's work is unacceptable, regardless of whether you document it.

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1. **§7.1 #8** Give the first five terms of the following recursively defined sequence:

$$a_1 = 1, \quad a_k = a_{k-1} + 2 \text{ for } k \geq 2.$$

   Also, give a closed formula for the sequence.

2. **§7.1 #24** Let $\{a_k\}$ be the sequence $a_1 = 3$, $a_2 = 3.1$, $a_3 = 3.14$, $a_4 = 3.141$, etc. That is, each term a_k contains the first k digits of π.

   (a) Explain why a_k is a rational number for each positive integer k.

   (b) Explain why the sequence a_k is increasing.

   (c) Provide an upper bound for $\{a_k\}$. What is the least upper bound?

   (d) Use this sequence to explain why the Least Upper Bound Axiom does not work for the set of rational numbers.

3. (a) i. **§7.1 #79** Use Newton's method to derive the recursion formula

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$$

   for approximating $\sqrt{a}$. *Hint: Let* $f(x) = x^2 - a$.

   ii. **§7.1 #80** Starting with $x_0 = 1$, use this formula to approximate $\sqrt{2}$ to within ten decimal places. How many terms did you use?

   iii. Plot the points $(0, x_0)$, $(1, x_1)$, $(2, x_2)$, $(3, x_3)$, $(4, x_4)$. *Tip: In* `desmos.com/calculator`*, use the window* $-0.1 \leq x \leq 4.1$*,* $1.4 \leq y \leq 1.55$.

   (b) **§7.1 #84** Explain why Newton's method will fail if you choose a value of x_0 for which $f'(x_0)$.

4. **§7.2 #62** We may use a recursively defined sequence to approximate the current amount of a radioactive element. For example, radioactive radium changes into lead over time. The rate of decay is proportional to the amount of radium present. Experimental data suggests that a gram of radium decays into lead at a rate of $\frac{1}{2337}$ grams per year. Let a_k be the amount of radium at the end of year k. Since the decay rate is constant, if we use a linear model to approximate the amount that remains after one year has passed, we have

$$a_1 = a_0 - \frac{1}{2337}a_0 = \frac{2336}{2337}a_0.$$

   More generally, we obtain the recursion formula

$$a_{k+1} = \frac{2336}{2337}a_k.$$

   Use this formula to estimate how much radium remains after 100 years if we start off with $a_0 = 10$ grams of radium.