MATH 2574 (Calculus III)
Spring 2017

Name: KEY

Fri 10 Feb 2017

Exam 1: Intro to Multidimensional Calculus (§11.1-11.7, 12.1-12.2)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems. No electronic devices (phones, iDevices, computers, etc) except for a **basic scientific calculator**. On story problems, round to one decimal place. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

In addition, please provide the following data:

Drill Instructor: ______________

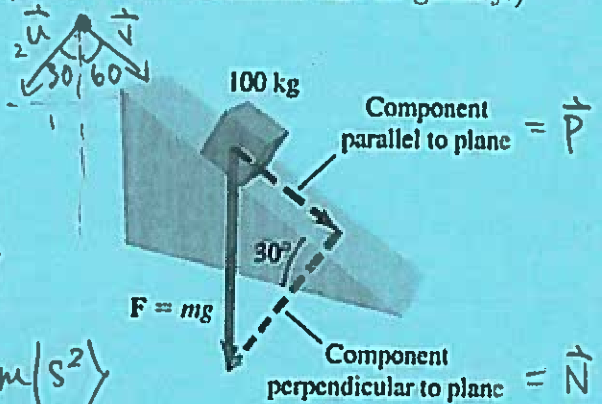
Drill Time: ______________

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt) ______________

Good luck!

1. **(16 pts)** A 100 kg box rests on a ramp with an incline of 30° to the floor (see figure). Find the components of the force perpendicular to and parallel to the ramp. (The vertical component of the force exerted by an object of mass m is its weight, which is mg, where $g = 9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity.)



$\vec{u} = \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

$\vec{v} = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

$\vec{F} = \langle 0, -980\ \mathrm{kgm/s^2} \rangle$

$$\vec{N} = \mathrm{proj}_{\vec{u}} \vec{F} = \frac{\vec{u}\cdot\vec{F}}{\vec{u}\cdot\vec{u}}\vec{u}$$

$$= \frac{\left(-\frac{1}{2}\right)(0) + \left(-\frac{\sqrt{3}}{2}\right)(-980)}{\underbrace{\left(\frac{-1}{2}\right)^2}_{\frac{1}{4}} + \underbrace{\left(\frac{-\sqrt{3}}{2}\right)^2}_{\frac{3}{4}}} \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$$

$$= \left\langle -\frac{490\sqrt{3}}{2}, -\frac{490(3)}{2} \right\rangle \mathrm{kgm/s^2}$$

$$\vec{P} = \mathrm{proj}_{\vec{v}} \vec{F} = \frac{\vec{v}\cdot\vec{F}}{\vec{v}\cdot\vec{v}}\vec{v}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)(0) + \left(\frac{-1}{2}\right)(-980)}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$$

$$= \left\langle \frac{490\sqrt{3}}{2}, -\frac{490}{2} \right\rangle =$$

2. Determine whether the following statements are true or false. You must justify your answer.

   (a) **(5 pts)** The domain of the function $f(x, y) = 1 - |x - y|$ is $\{(x, y) \mid x \geq y\}$.

False. The absolute value function is defined for all real numbers and so the domain is

$$\mathbb{R}^2 = \{(x,y) \mid x, y \in \mathbb{R}\}.$$

   (b) **(5 pts)** $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 0$

True. $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$.

   (c) **(5 pts)** The domain of the function $u = f(w, x, y, z)$ is a region in $\mathbb{R}^3$.

False. u is a function of four variables so its domain is in $\mathbb{R}^4$.

   (d) **(5 pts)** All level curves of the plane $z = 2x - 3y$ are lines.

True. For $z = z_0$,

$$z_0 = 2x - 3y$$

$$\Rightarrow y = \frac{2x - z_0}{3}$$

$$\Rightarrow y = \frac{2}{3}x - \frac{z_0}{3}$$ is the equation of a line.

3. **(18 pts)** Determine an equation of the line l that is perpendicular to the lines

$$\mathbf{r}(t) = \langle -2+3t, 2t, 3t\rangle = \langle -2,0,0\rangle + t\langle 3,2,3\rangle$$
$$\mathbf{R}(s) = \langle -6+s, -8+2s, -12+3s\rangle = \langle -6,-8,-12\rangle + s\langle 1,2,3\rangle$$

and passes through the point of intersection of the lines $\mathbf{r}$ and $\mathbf{R}$.

l is parallel to the vector

$$\langle 3,2,3\rangle \times \langle 1,2,3\rangle = \langle 2(3)-2(3), 3(1)-3(3), 3(2)-2(1)\rangle$$

1 2 3   3 - 6   6 - 2

$$= \langle 0,-3,4\rangle$$

P = point of intersection:

$$-2+3t = -6+s \Rightarrow s = 4+3t$$
$$2t = -8+2(4+3t)$$
$$= -8+\cancel{8}+6t \Rightarrow t=0,\ s=4+3(0)=4$$

so $P=(-2,0,0)$

and l is given by

$$\langle -2,0,0\rangle + t\langle 0,-3,4\rangle$$

$$\boxed{= \langle -2,-3t,4t\rangle}$$

4. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable functions at $t = 0$ with $\mathbf{u}(0) = \langle 0, 1, 1 \rangle$, $\mathbf{u}'(0) = \langle 0, 7, 1 \rangle$, $\mathbf{v}(0) = \langle 0, 1, 1 \rangle$, $\mathbf{v}'(0) = \langle 1, 1, 2 \rangle$. Evaluate the following expressions:

(a) **(6 pts)** $\left.\frac{d}{dt}(\mathbf{u} \cdot \mathbf{v})\right|_{t=0} = \left(\vec{u}' \cdot \vec{v} + \vec{u} \cdot \vec{v}'\right)\Big|_{t=0}$

$= \vec{u}'(0) \cdot \vec{v}(0) + \vec{u}(0) \cdot \vec{v}'(0)$

$= \langle 0,7,1 \rangle \cdot \langle 0,1,1 \rangle + \langle 0,1,1 \rangle \cdot \langle 1,1,2 \rangle$

$= 0 + 7 + 1 + 0 + 1 + 2$

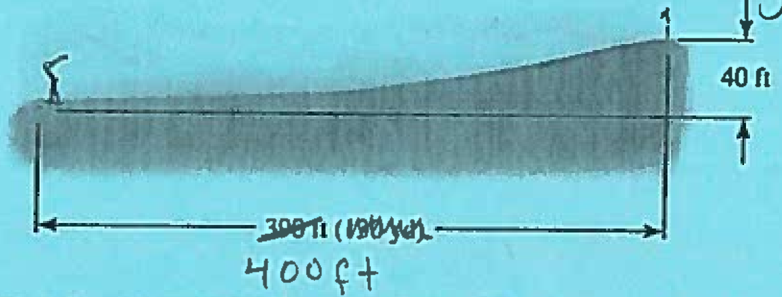
$\boxed{= 11}$

(b) **(6 pts)** $\left.\frac{d}{dt}(\cos(t)\mathbf{u}(t))\right|_{t=0} = \cancel{-\sin(0)\vec{u}(0)} + \cos(0)\vec{u}(0)$

$\boxed{= \langle 0,7,1 \rangle}$

5. A golfer stands 400 ft horizontally from the hole and 40 ft below the hole (see figure).

$g = 32\ ft/s^2$



Suppose the ball is hit with an initial speed of 150 ft/s, at an angle of θ from the ground.

(a) **(12 pts)** Find the acceleration $\mathbf{a}(t)$, velocity $\mathbf{v}(t)$, and position $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ vectors for the trajectory of the ball.

$$\boxed{\vec{a}(t) = \langle 0, -32\rangle\ ft/s^2}$$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0, -32t\rangle + \vec{C}$$

$$\vec{v}(0) = \langle 150\cos\theta, 150\sin\theta\rangle = \langle 0, 0\rangle + \vec{C}$$

$$\Rightarrow \boxed{\vec{v}(t) = \langle 150\cos\theta, 150\sin\theta - 32t\rangle\ ft/s}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 150\cos\theta\, t, 150\sin\theta\, t - 16t^2\rangle + \vec{C}$$

$$\vec{r}(0) = \langle 0, 0\rangle \Rightarrow \vec{C} = \langle 0, 0\rangle$$

$$\Rightarrow \boxed{\vec{r}(t) = \langle 150\cos\theta t, 150\sin\theta t - 16t^2\rangle\ ft}$$

(b) **(6 pts)** Write down a system of two equations to find the two unknowns: (1) time of flight and (2) θ. **Do not** solve the system.

$$x(t) = 150\cos\theta t = 400$$

$$y(t) = 150\sin\theta t - 16t^2 = 40$$

6. **(15 pts)** Match equations (a)-(f) with the surfaces (A)-(F).

(D) (a) $y - z^2 = 0$ parabola cylinder

(E) (b) $4x^2 + \frac{y^2}{9} + z^2 = 1$ ellipsoid

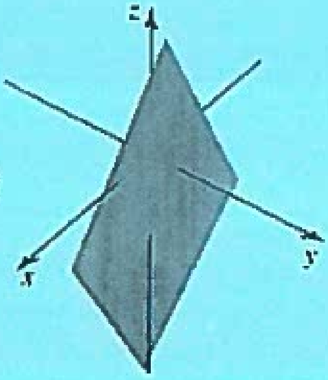
(B) (c) $x^2 + \frac{y^2}{9} = z^2$ traces are intersecting lines

(A) (d) $2x - 3y - z = 5$ plane

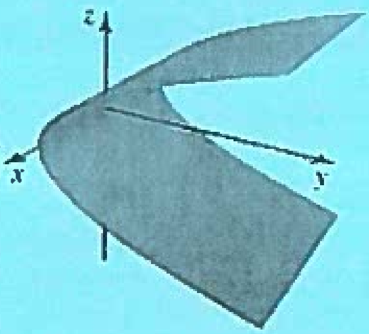
(F) (e) $x^2 + \frac{y^2}{9} - z^2 = 1$ hyperbolic

(C) (f) $y = |x|$ absolute value cylinder

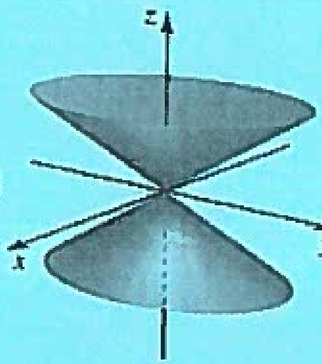
(A)



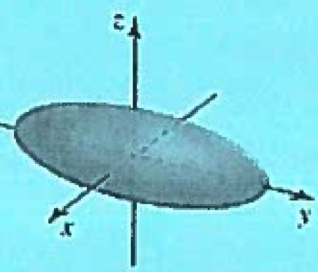
(D)



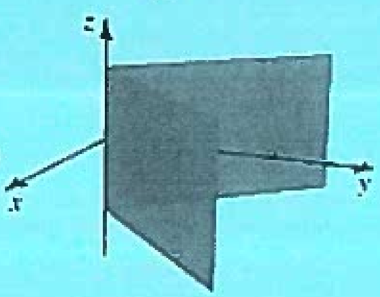
(B)



(E)



(C)



(F)