Math 2554 Exam 3: Sections 4.6-5.4 **Name:** SOLUTIONS
Fri 5 Dec 2014

Calculus I
Exam 3

Please provide the following data:

Drill Instructor: ______________________________

Drill Time: ______________________________

Student ID or clicker #: ______________________________

Exam Instructions: Sit in every other chair. You have 50 minutes to complete this exam. One 3×5 inch notecard, 2-sided, is allowed. No graphing calculators. No programmable calculators. No electronic devices except for the approved calculators (so no phones, iDevices, computers, etc). If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: (1 pt) ____

Good luck!

(5pts) 1. State Rolle's Theorem.

Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b) with $f(a)=f(b)$. Then there is at least one point c in (a,b) such that $f'(c)=0$.

(12 pts) 2. Determine the point(s) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[-4, 5]$.

Slope of secant line:

$$\frac{f(5)-f(-4)}{5-(-4)} = \frac{5^3-(-4)^3}{9} = \frac{189}{9} = 21$$

$$f'(x) = 3x^2 = 21$$

$$\boxed{x = \pm\sqrt{7} \approx \pm 2.646}$$ (both are in the interval)

(12 pts)

3. Evaluate $\lim_{n\to\infty}\left(1+\frac{7}{n^2}\right)^{n^3}$. (1^∞)

$$\lim_{n\to\infty} n^3 \ln\left(1+\frac{7}{n^2}\right) = \lim_{n\to\infty} \frac{\ln\left(1+\frac{7}{n^2}\right)}{\frac{1}{n^3}} \quad \left(\frac{0}{0}\right)$$

L'Hôpital's Rule:

$$= \lim_{n\to\infty} \frac{\frac{1}{1+\frac{7}{n^2}}\left((-2)\frac{7}{n^3}\right)}{\frac{-3}{n^4}}$$

$$= \lim_{n\to\infty} \frac{(-2)(7)n^{4}}{\left(1+\frac{7}{n^2}\right)(-3)\cancel{n^3}} = \infty$$

Undo ln:

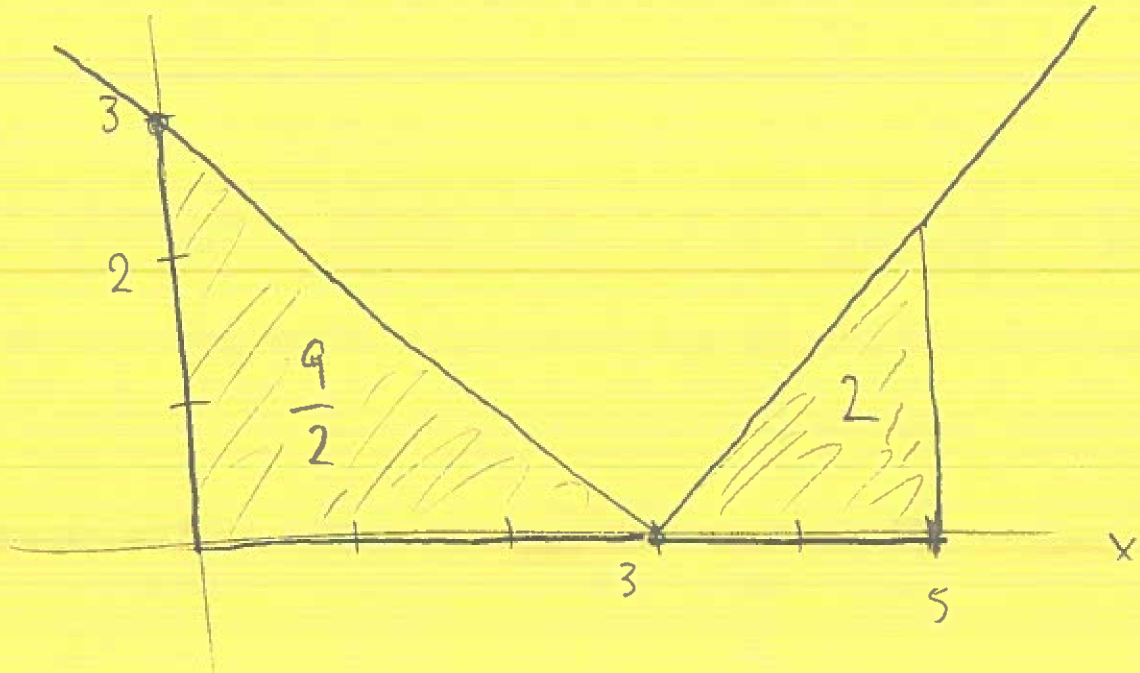
$$\boxed{e^{\infty} = \infty}$$

(15 pts)

4. Use geometry to evaluate

$$\int_0^5 |3-x|\,dx.$$

(Show your picture – it does not need to be to scale.)



The area is the sum of the shaded regions' areas:

$$\frac{9}{2} + 2 = \boxed{\frac{13}{2}}$$

5. Use the Fundamental Theorem of Calculus

(a) ... Part I to find the derivative of

(10 pts)

$$\int_{-3x}^{5} \sqrt{2t+3}\,dt. = -\int_{5}^{-3x} \sqrt{2t+3}\,dt$$

$$\frac{d}{dx}\left[-\int_{5}^{-3x}\sqrt{2t+3}\,dt\right] = -1(-3)\sqrt{2(-3x)+3}$$

$$= \boxed{3\sqrt{-6x+3}}$$

(5 pts)

(b) ... Part II to find the area between the curve $f(x) = 4x^3 + 16x + 13$ and the x-axis over the interval $[-1, 2]$.

$$\int_{-1}^{2} 4x^3 + 16x + 13\,dx = x^4 + 8x^2 + 13x \Big|_{-1}^{2}$$

$$= 2^4 + 8(2^2) + 13(2) - \left[(-1)^4 + 8(-1)^2 + 13(-1)\right]$$

$$= 74 + 4 = \boxed{78}$$

(11 pts)

6. (a) Determine the average height over the time interval [2, 5] of a particle whose position at any time t is $s(t) = t^2 + 6t$ m/sec.

$$\bar{s} = \frac{1}{5-2}\int_2^5 t^2 + 6t \, dt$$

$$= \frac{1}{3}\left(\frac{t^3}{3} - 3t^2\right)\Big|_2^5 = \frac{1}{3}\left[\frac{5^3}{3} - 3(5^2) - \left[\frac{2^3}{3} - 3(2^2)\right]\right]$$

$$\boxed{= 34 \text{ m}}$$

(9 pts)

(b) At what time t is the particle's height equal to its average height?

$$t^2 + 6t = 34$$

$$t^2 + 6t - 34 = 0$$

Use Q-formula:

$$t = \frac{-6 \pm \sqrt{36 - 4(1)(-34)}}{2(1)}$$

$$= -3 \pm \frac{\sqrt{172}}{2} \approx \boxed{3.556 \text{ sec}}, \cancel{-9.557}$$

(11 pts)

7. (a) Compute the midpoint Riemann sum for the function $f(x) = x^2 - 3x + 5$ on the interval $[-1, 5]$ with $n = 3$.

$$\Delta x = \frac{5-(-1)}{3} = 2 \qquad \bar{x}_k = -1 + \left(k - \frac{1}{2}\right)\cdot 2 = 2(k-1)$$

$$\sum_{k=1}^{3} f(2(k-1))\cdot 2 = 2\sum_{k=1}^{3}\left[(2(k-1))^2 - 3(2(k-1)) + 5\right]$$

$$= 2\left[(5) + (2^2 - 3(2) + 5) + (4^2 - 3(4) + 5)\right] = \boxed{34}$$

(9 pts)

(b) What is the **exact** area under the curve for $f(x) = x^2 - 3x + 5$ on the interval $[-1, 5]$?

$$\int_{-1}^{5} x^2 - 3x + 5\, dx = \frac{x^3}{3} - \frac{3}{2}x^2 + 5x \Big|_{-1}^{5}$$

$$= \frac{5^3}{3} - \frac{3}{2}(5^2) + 5(5)$$

$$- \left[\frac{(-1)^3}{3} - \frac{3}{2}(-1)^2 + 5(-1)\right]$$

$$= \boxed{36}$$

(+10 pts)

EXTRA CREDIT Suppose you are standing on the shore of a circular pond with radius 1 mile and you want to get to a point on the shore directly opposite your position (on the other end of a diameter). You plan to swim at 2 miles per hour from your current position to another point P on the shore and then walk at 3 miles per hour along the shore to the terminal point. How should you choose P to minimize the total time for the trip?

(MathFact: For a circle of radius r and a chord on the circle with central angle θ, the length of the chord is given by $2r\sin\frac{\theta}{2}$. Given an arc with central angle ϕ, the arc length is $r\phi$.)

* See Exam 2 Key