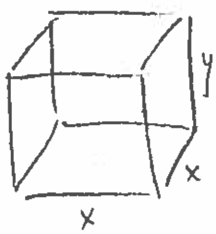
Math 2554 Pop Quiz Make-up
Mon 17 Nov 2014

Name: SOLUTIONS

You have 15 minutes to answer the question. Graphing calculators are allowed.

Find the dimensions of a closed box with a square base that has the following characteristics:

1. The maximum volume, given that the surface area is 16 square meters.



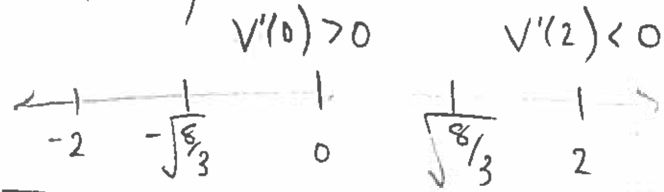
Surface Area $= S = 2x^2 + 4xy = 16 m^2$

$\Rightarrow y = \dfrac{16-2x^2}{4x}$ m

Volume $= V = x^2 y = x^2\left(\dfrac{16-2x^2}{4x}\right) = 4x - \dfrac{1}{2}x^3$ m^3

maximize:

$\dfrac{dV}{dx} = 4 - \dfrac{3}{2}x^2 = 0 \Rightarrow x = \pm\sqrt{\dfrac{8}{3}}$



Since x cannot be negative, $x = \sqrt{8/3} \approx 1.633$ gives a max by the 1^{st} Deriv. Test. $\Rightarrow y = \dfrac{16 - 2(\sqrt{8/3})^2}{4\sqrt{8/3}} \approx 1.633$

$\boxed{\text{Dimensions: } \approx 1.633\text{ m} \times 1.633\text{ m} \times 1.633\text{ m}}$

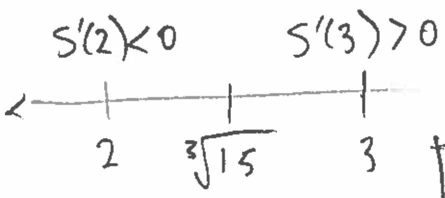
2. The ~~maximum~~ minimum surface area, given that the volume is 15 cubic meters.

$V = x^2 y = 15 m^3 \Rightarrow y = \dfrac{15}{x^2}$ m

$S = 2x^2 + 4xy = 2x^2 + 4x\left(\dfrac{15}{x^2}\right) = 2x^2 + \dfrac{60}{x}$ m^2

optimize:

$\dfrac{dS}{dx} = 4x - \dfrac{60}{x^2} = 0 \Rightarrow 4x^3 - 60 = 0 \Rightarrow x = \sqrt[3]{15}$



By the 1^{st} Deriv Test, $x = \sqrt[3]{15} \approx 2.466$ gives a min $\Rightarrow y = \dfrac{15}{(\sqrt[3]{15})^2} = \sqrt[3]{15}$

$\boxed{\text{Dimensions: } \approx 2.466\text{ m} \times 2.466\text{ m} \times 2.466\text{ m}}$