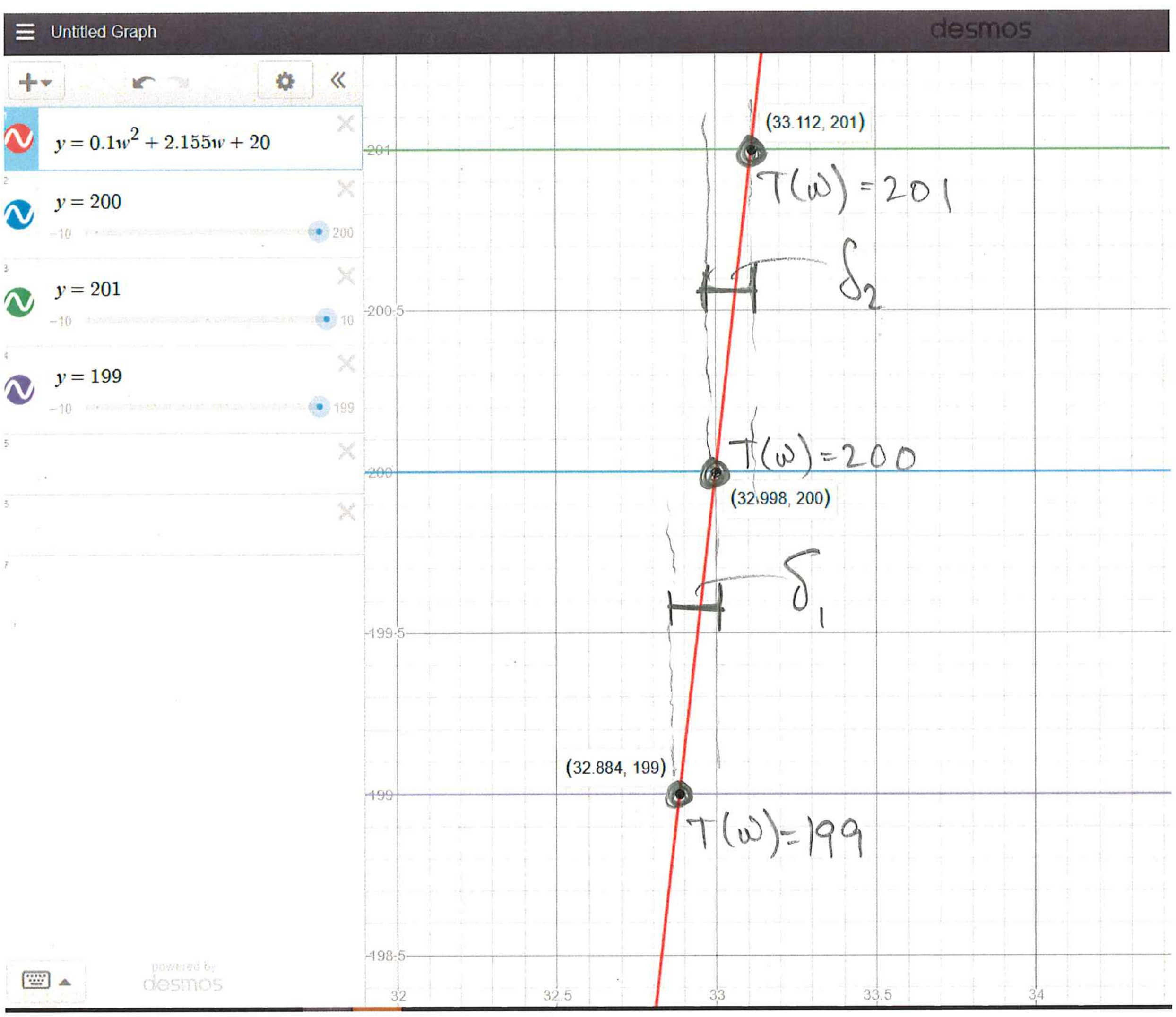
Take-Home Quiz No. 3

SOLUTIONS

Math 235 (Calc I)
Fall 2017

1. (a) Set $T(w)=200$ and solve for w; the context of the problem requires the positive solution. Using desmos.com, the amount of power needed is $w \approx \boxed{32.998 \text{ watts}}$

(b) Since $T(w)$ is continuous, use the solutions of $T(w)=200 \pm 1$ find bounds on the range. The wattage should be between $\boxed{\sim 32.884 \text{ and } \approx 33.112 \text{ watts.}}$



→

(c) $\lim_{x \to a} f(x) = L$ ← 200°F

w → x, a ≈ 32.998 watts, $f(x)$ ← $T(w)$

$\varepsilon = 1\,°F.$

To find δ, find the smaller of

$$\delta_1 = |32.884 - 32.998| = 0.114$$

$$\delta_2 = |33.112 - 32.998| = 0.114$$

$\Rightarrow \delta = 0.114$, i.e., if $|w - 32.988| < 0.114$

then $|T(w) - 200| < 1$.

2. Yes. On $(0,R)$ $F(r) = \frac{GM}{R^3}r$ is a line, and so is continuous.

On (R,∞), $F(r) = \frac{GM}{r^2}$ is well-defined and rational, so is continuous.

We must check continuity at $r = R$:

① $F(R) = \frac{GM}{R^2}$ exists. ✓ (R = radius of Earth is nonzero)

② $\lim_{r \to R^-} F(r) = \lim_{r \to R^-} \frac{GM}{R^3} r = \frac{GM}{R^3}(R) = \frac{GM}{R^2}$

(lines are continuous)

$\lim_{r \to R^+} F(r) = \lim_{r \to R^+} \frac{GM}{r^2} = \frac{GM}{R^2}$

(continuity for $r \neq 0$)

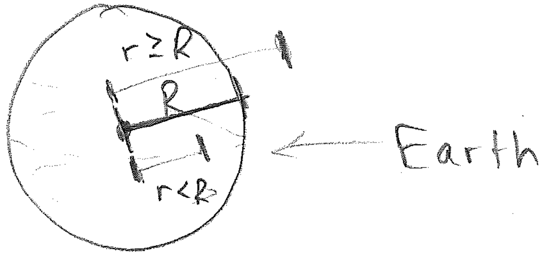
$\Rightarrow \lim_{r \to R} F(r) = \frac{GM}{R^2}$ exists. ✓

③ $\lim_{r \to R} F(r) = \frac{GM}{R^2} = F(R)$ ✓

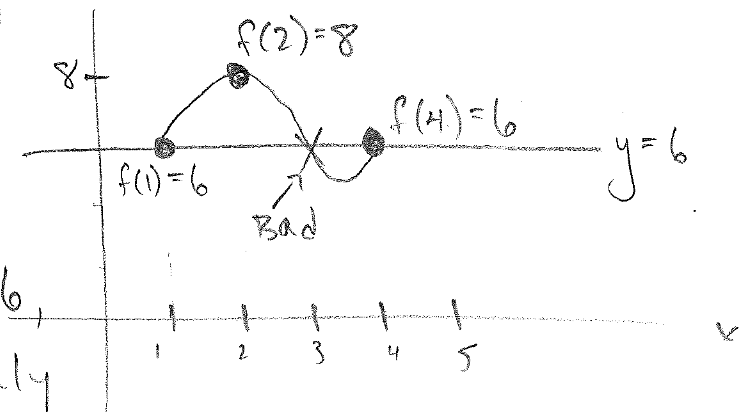
$\Rightarrow F(r)$ is continuous on all of $(0,\infty)$

→

(as one would expect)



3. Since f is continuous on $[1,5]$ it is continuous on $[2,3]$. Therefore by the Intermediate Value Theorem, it must take on every value between $f(2)$ and $f(3)$. That includes 6 if $f(3)<6$, since $f(2)>6$. But $f(x)$ only equals 6 at $x=1$ and $x=4$, not for any $x \in (2,3)$.



We also can't have $f(3)=6$, because only $f(1)$ and $f(4)$ equal 6.

$\Rightarrow f(3)>6$.

4. (a) Concentration is $\dfrac{\text{amount of salt (g)}}{\text{total amt of liquid (L)}}$

$$= \frac{(30\,g/L)(25\,L/min)(t \text{ minutes})}{5000L + (25\,L/min)(t\,min)}$$

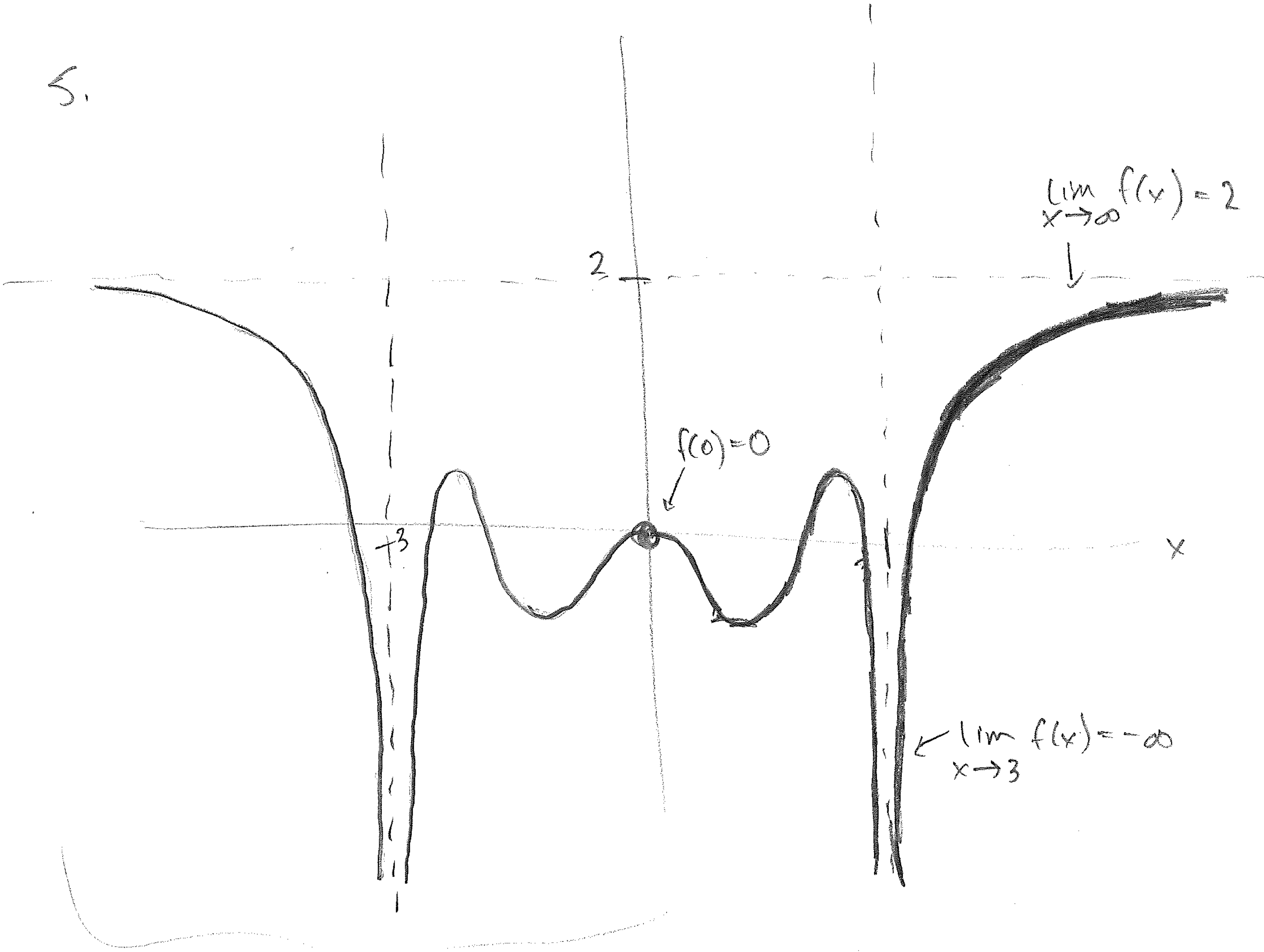
amount of brine after t minutes

$$\Rightarrow C(t) = \frac{(30)(25)t}{5000+25t} = \frac{30t}{200+t}$$

(b) $\lim_{t \to \infty} \frac{30t}{200+t}\left(\frac{\frac{1}{t}}{\frac{1}{t}}\right) = \lim_{t \to \infty} \frac{30}{\underbrace{\frac{200}{t}}_{0} + 1} = 30 \text{ g/L}.$

The solution becomes more like the 30 g/L brine that is added over time.

5.



even: symmetric about y-axis.