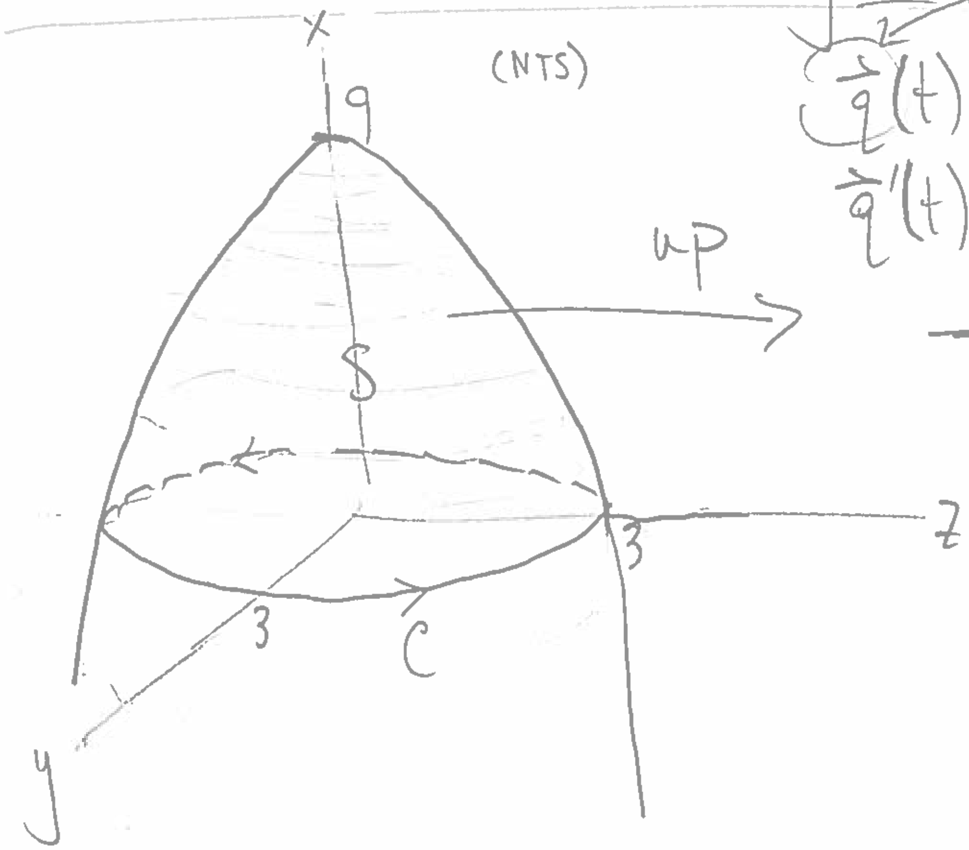
§14.7 | #18: "Evaluate the line integral in Stokes' Theorem to determine the value of the surface integral $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} \, dS$. Assume that $\hat{n}$ points in an upward direction"

$\vec{F} = \dfrac{\vec{r}}{|\vec{F}|}$ where $\vec{r} = \langle x, y, z \rangle$

S = paraboloid $x = 9 - y^2 - z^2$ for $0 \le x \le 9$ (excluding its base)



($\vec{r}$ was already used)

$\vec{q}(t) = \langle 0, 3\cos t, 3\sin t \rangle$

$\vec{q}\,'(t) = \langle 0, -3\sin t, 3\cos t \rangle$

$\longrightarrow \vec{F} = \dfrac{\langle 0, 3\cos t, 3\sin t \rangle}{3}$

$= \langle 0, \cos t, \sin t \rangle$

Stokes' Theorem says

($\vec{r}$ was already used)

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS = \oint_C \vec{F} \cdot d\vec{q}$$

$$= \int_0^{2\pi} \Big((0)(0) + (\cos t)(-3\sin t) + (\sin t)(3\cos t) \Big)\, d$$

$$\boxed{= 0}$$

* Just for fun ($\vec{r}$ was already used)

Parametrize S: $\vec{q}(u,v) = \langle 9v, 3v\cos u, 3v\sin u \rangle$

$$\vec{q}_u = \langle 0, -3v\sin u, 3v\cos u \rangle$$

$$\vec{q}_v = \langle 9, 3\cos u, 3\sin u \rangle$$

$$\vec{q}_u \times \vec{q}_v = \langle \underbrace{-9v\sin^2 u - 9v\cos^2 u}_{-9v}, 27v\cos u - 0,$$

$$0 - (-27v\sin u) \rangle$$

$\longrightarrow$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix}$$

$$= \Big\langle z\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}y)\right] - y\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)\right],$$

$$x\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)\right] - z\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}x)\right],$$

$$y\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}x)\right] - x\left[-\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}y)\right]\Big\rangle$$

$$= \langle 0, 0, 0 \rangle$$

$\longrightarrow$

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS = \int_0^1 \int_0^{2\pi} \langle 0,0,0 \rangle \cdot \langle -9v, 27v\cos u, 27v\sin u \rangle \, du\, dv$$

$$\boxed{= 0}$$