# Mon 8 Feb

- EXAM 1 on Friday.
  - Covers up to §3.1 (see the semester schedule of material on the course webpage).
  - You must attend your own lecture on exam day.
  - CEA: Register with the CEA office for a time on 12 Feb, as close to your normal lecture time as possible.
  - Look at old Wheeler exams to study. `comp.uark.edu/~ashleykw`
  - Also look at Quiz and Drill solutions posted in MLP.
  - Do the book problems. Go to office hours or Calculus Corner to get feedback.

# Mon 8 Feb (cont.)

- Quizzes:
  - Include drill instructor and time.
  - Don't turn in the Quiz sheet with your work.
  - No quiz again until next week.
  - Drill Exercise Tues 16 Feb and Quiz 4 Thurs 18 Feb.

# Mon 8 Feb (cont.)

- Announcement:
  A student in this class requires a note-taker. If you are willing to upload your notes and plan to attend class on a REGULAR basis, please sign up via the CEA Online Services on the Center for Educational Access (CEA) website `http://cea.uark.edu`. On the CEA Online Services login screen, click on "Sign Up as a Note-taker". At the end of the semester you will receive verification of 48 community service hours OR a $50 gift card for providing class notes. All interested students are encouraged to sign up; preference may be given to volunteers seeking community service in an effort engage U of A students in community service opportunities. Please contact the Center for Educational Access at ceanotes@uark.edu if you have any questions.

### Exercise

Let $f(x) = x^2 - 4$. For $\epsilon = 1$, find a value for $\delta > 0$ so that

$$|f(x) - 12| < \epsilon \quad \text{whenever} \quad 0 < |x - 4| < \delta.$$

In this example, $\lim_{x \to 4} f(x) = 12$.

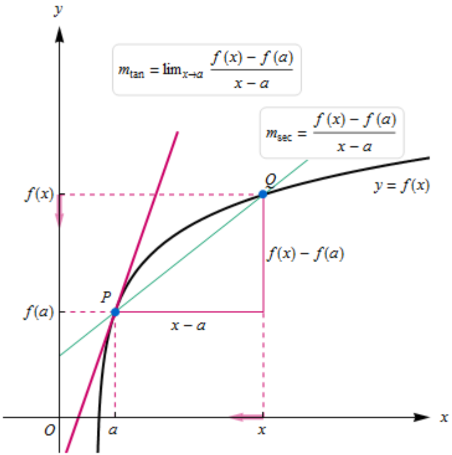
## 2.7 Book Problems

1-7, 9-18

# §3.1 Introducing the Derivative

**Recall from Ch 2:** We said that the slope of the tangent line at a point is the limit of the slopes of the secant lines as the points get closer and closer.

- slope of secant line: $\dfrac{f(x)-f(a)}{x-a}$ (average rate of change)
- slope of tangent line: $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ (instantaneous rate of change)


y
$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
$m_{\sec} = \frac{f(x) - f(a)}{x - a}$
f(x)
Q
y = f(x)
f(x) − f(a)
P
f(a)
x − a
O
a
x
x

### Exercise

Use the relationship between secant lines and tangent lines, specifically the slope of the tangent line, to find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point $P = (1, 5)$.

In the preceding exercise, we considered two points

$$P = (a, f(a)) \quad \text{and} \quad Q = (x, f(x))$$

that were getting closer and closer together.

Instead of looking at the points approaching one another, we can also view this as the distance $h$ between the points approaching 0. For

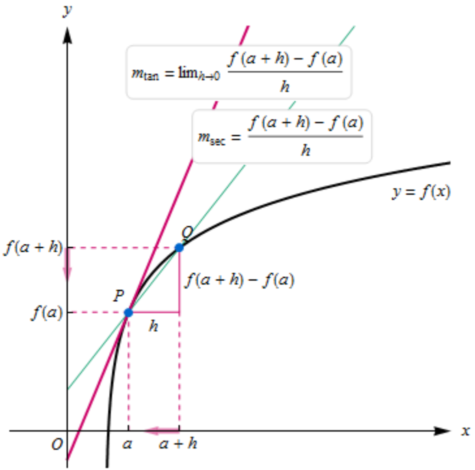
$$P = (a, f(a)) \quad \text{and} \quad Q = (a+h, f(a+h)),$$

- slope of secant line:

$$\frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

- slope of tangent line:

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$



y
$m_{\tan} = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$
$m_{\sec} = \dfrac{f(a+h)-f(a)}{h}$
$y = f(x)$
$Q$
$f(a+h)$
$f(a+h) - f(a)$
$P$
$f(a)$
$h$
$O$
$a$
$a+h$
x

### Exercise

Find the equation of a line tangent to the curve $f(x) = x^2 + 2x + 2$ at the point $P = (2, 10)$.

## Derivative Defined as a Function

The slope of the tangent line for the function $f$ is itself a function of $x$ (in other words, there is an expression where we can plug in any value $x = a$ and get the derivative at that point), called the derivative of $f$.

### Definition

The **derivative** of $f$ is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. If $f'(x)$ exists, we say $f$ is **differentiable** at $x$. If $f$ is differentiable at every point of an open interval $I$, we say that $f$ is differentiable on $I$.

## Exercise

Use the definition of the derivative to find the derivative of the function $f(x) = x^2 + 2x + 2$.

A standard notation for change involves the Greek letter $\Delta$.

$$\frac{f(x+h)-f(x)}{h} = \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\Delta y}{\Delta x}.$$

Apply the limit:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

## Other Notation

The following are alternative ways of writing $f'(x)$ (i.e., the derivative as a function of $x$):

$$\frac{dy}{dx} \qquad \frac{df}{dx} \qquad \frac{d}{dx}(f(x)) \qquad D_x(f(x)) \qquad y'(x)$$

The following are ways to notate the derivative of $f$ evaluated at $x = a$:

$$f'(a) \qquad y'(a) \qquad \left.\frac{df}{dx}\right|_{x=a} \qquad \left.\frac{dy}{dx}\right|_{x=a}$$

### Question

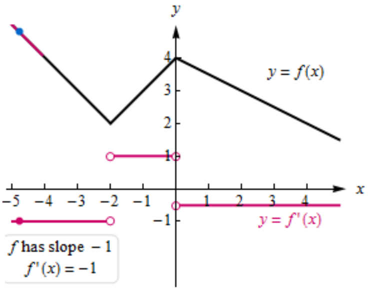
Do the words "derive" and "differentiate" mean the same thing?

The graph of the derivative is the graph of the collection of slopes of tangent lines of a graph. If you just have a graph (without an equation for the graph), the best you can do is approximate the graph of the derivative.

## Example

Simple checklist:

1. Note where $f'(x) = 0$.
2. Note where $f'(x) > 0$.
   (What does this look like?)
3. Note where $f'(x) < 0$.
   (What does this look like?)



Key points about the relationship between differentiability and continuity:

- If $f$ is differentiable at $a$, then $f$ is continuous at $a$.
- If $f$ is not continuous at $a$, then $f$ is not differentiable at $a$.
- $f$ can be continuous at $a$, but not differentiable at $a$.

A function $f$ is not differentiable at $a$ if at least one of the following conditions holds:

1. $f$ is not continuous at $a$.
2. $f$ has a corner at $a$.

**Question**

Why does this make $f$ not differentiable?

3. $f$ has a vertical tangent at $a$.

**Question**

Why does this make $f$ not differentiable?

## 3.1 Book Problems

9-45 (odds), 49-53 (odds)

- **NOTE:** You do not know any rules for differentiation yet (e.g., Power Rule, Chain Rule, etc.) In this section, you are strictly using the definition of the derivative and the definition of slope of tangent lines we have derived.