Math 2603 Exam 2 **Name:** ______________________________
Wed 24 Sep 2014

Discrete Math
Exam 2 (Ch. 3-6 as we have covered)

Please provide the following data:

Drill Time: __________________________________

Student ID: __________________________________

Exam Instructions: You have 50 minutes to complete this exam. One 3×5 inch notecard is allowed. No graphing calculators. No programmable calculators. No phones, iDevices, computers, etc. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature:
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Good luck!

1. Determine which of the following are functions. If they are functions decide if they are one-to-one, onto, or both.

   (a) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = 4x - 5$.

   (b) $R = \{(a, y), (b, z), (c, x), (d, x)\} \subset \{a, b, c, d\} \times \{x, y, z\}$

2. Find an explicit formula for the nth term of each of the following sequences. You must state where your indices n start.

   (a) $s = \left\{ \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots \right\}$

   (b) $t = \left\{ 0, \frac{-1}{2}, \frac{2}{3}, \frac{-3}{4}, \frac{4}{5}, \frac{-5}{6}, \frac{6}{7}, \ldots \right\}$

3. Suppose R is a relation on a set X. Write down the following definitions:

(a) R is *reflexive* means

(b) R is *symmetric* means

(c) R is *antisymmetric* means

(d) R is *transitive* means

(e) R is an *equivalence relation* means

(f) R is a *partial order* means

4. Define the relation R on $\mathbb{Z}$ by mRn if and only if $3 \mid (m^2 - n^2)$. Which of the properties from Problem 3 does R satisfy? Give justification (proof or counterexample) for each of the items (a)-(f) from Problem 3.

5. Prove that for the Fibonacci sequence $(f_1 = 1,\ f_2 = 1, \ldots)$

$$\sum_{i=1}^{n} f_i^2 = f_n f_{n+1}.$$

6. A student council consists of 16 students, 5 of whom are men and 11 of whom are women.

   (a) In how many ways can a committee of 5 be selected from the membership of the council?

   (b) How many committees of 5 contain at least one man?

   (c) Suppose the council has 2 freshmen, 4 sophomores, 6 juniors, and 4 seniors. How many committees of 8 contain exactly 2 representatives from each class?

7. **cHaLlEnGe PrObLeM** The sequence of *Catalan numbers* is defined as

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

for each $n \in \mathbb{Z}_{\geq 1}$.

(a) Prove that $C_n = \binom{2n}{n} - \binom{2n}{n+1}$. Hint: Use the following Lemma (and prove it).

Lemma. *For all nonnegative integers n and r with $r + 1 \leq n$,*

$$\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}.$$

(b) Find the first 6 Catalan numbers.

8. **EXTRA CREDIT** Find integers s, t such that

$$\gcd(825, 315) = 825s + 315t.$$