Wed 3 Feb

- GET YOUR CLICKER NOW. If you haven't gotten any email from me, then your clicker should be working fine.
- EXAM 1 is one week from Friday. Covers up to §3.1 (see the semester schedule of material on the course webpage). You must attend your own lecture on exam day. CEA: Register with the CEA office for a time on 12 Feb, as close to your normal lecture time as possible.
- Look at old Wheeler exams to study.

Example

Let $f(x) = \begin{cases} x^3 + 4x + 1 & \text{if } x \leq 0 \\ 2x^3 & \text{if } x > 0. \end{cases}$

1. Use the continuity checklist to show that f is not continuous at 0.
2. Is f continuous from the left or right at 0?
3. State the interval(s) of continuity.

(assuming m and n are positive integers and $\frac{n}{m}$ is in lowest terms)

- If m is odd, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points at which f is continuous.
- If m is even, then $[f(x)]^{\frac{n}{m}}$ is continuous at all points a at which f is continuous and $f(a) \geq 0$.

Question

Where is $f(x) = \sqrt[4]{4 - x^2}$ continuous?

Continuity of Transcendental Functions

Trig Functions: The basic trig functions are all continuous at all points IN THEIR DOMAIN. Note there are points of discontinuity where the functions are not defined – for example, $\tan x$ has asymptotes everywhere that $\cos x = 0$.

Exponential Functions: The exponential functions b^x and e^x are continuous on all points of their domains.

Inverse Functions: If a continuous function f has an inverse on an interval I (meaning if $x \in I$ then $f^{-1}(y)$ passes the vertical line test), then its inverse f^{-1} is continuous on the interval J, which is defined as all the numbers $f(x)$, given x is in I.

Theorem (Intermediate Value Theorem)

Suppose f is continuous on the interval $[a, b]$ and L is a number satisfying

$$f(a) < L < f(b) \quad \text{or} \quad f(b) < L < f(a).$$

Then there is at least one number $c \in (a, b)$, i.e., $a < c < b$, satisfying

$$f(c) = L.$$

Example

Let $f(x) = -x^5 - 4x^2 + 2\sqrt{x} + 5$. Use IVT to show that $f(x) = 0$ has a solution in the interval $(0, 3)$.

Exercise

Which of the following functions is continuous for all real values of x?

(A) $f(x) = \dfrac{x^2}{2x+1}$

(B) $g(x) = \sqrt{3x^2 - 2}$

(C) $h(x) = \dfrac{5x}{|x^8 - 1|}$

(D) $j(x) = \dfrac{5x}{x^8 + 1}$

2.6 Book Problems

9-25 (odds), 35-45 (odds), 59, 61, 63, 83, 85

§2.7 Precise Definitions of Limits

So far in our dealings with limits, we have used informal terms such as "sufficiently close" and "arbitrarily large". Now we will formalize what these terms mean mathematically.

Recall: $|f(x) - L|$ and $|x - a|$ refer to the distances between $f(x)$ and L and between x and a.

Also, recall that when we worked informally with limits, we wanted x to approach a, but not necessarily equal a. Likewise, we wanted f to get arbitrarily close to L, but not necessarily equal L.

Definition

Assume that $f(x)$ exists for all x in some open interval (open means: neither of the endpoints not included) containing a, except possibly at a. **"The limit of $f(x)$ as x approaches a is L"**, i.e.,

$$\lim_{x \to a} f(x) = L,$$

means **for any $\epsilon > 0$ there exists $\delta > 0$ such that**

$$|f(x) - L| < \epsilon \quad \textbf{whenever} \quad 0 < |x - a| < \delta.$$

Question

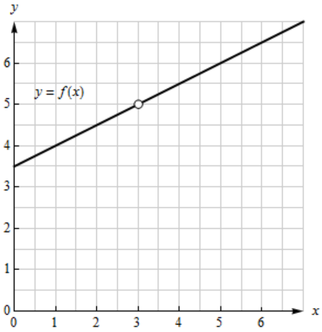
Why $0 < |x - a|$ but not for $|f(x) - L|$?

When we worked informally with limits, we saw $f(x)$ get closer and closer to L as x got closer and closer to a.

Question

If we want the distance between $f(x)$ and L to be less than 1, how close does x have to be to a? What if we want $|f(x) - L| < 0.5$? 0.5? 0.1? 0.01?

Example



Using the graph, for each $\epsilon > 0$, determine a value of $\delta > 0$ to satisfy the statement

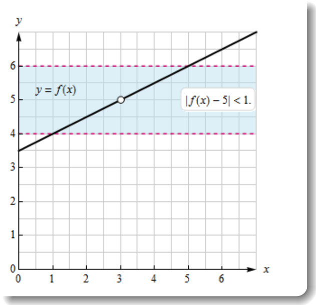
$$|f(x) - 5| < \epsilon \quad \text{whenever} \quad 0 < |x - 3| < \delta.$$

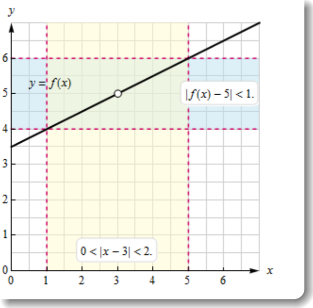
(a) $\epsilon = 1$

(b) $\epsilon = 0.5$.

Seeing ϵs and δs on a Graph, cont.

When $\epsilon = 1$:

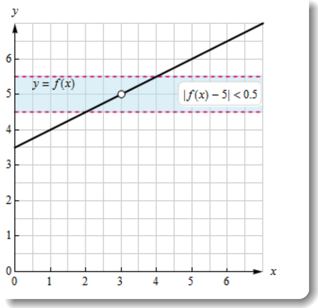


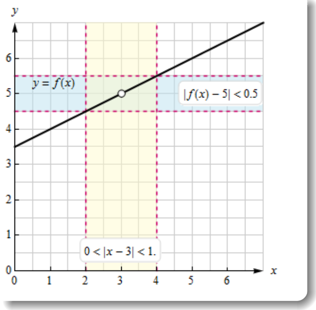


$\ldots \; \delta = 2$

Seeing ϵs and δs on a Graph, cont.

When $\epsilon = 0.5$:



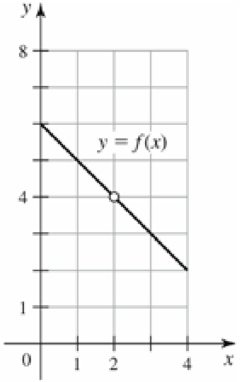


$\ldots\ \delta = 1$

The ϵs and δs give a way to visualize computing the limit, and prove it exists. As the ϵs get smaller and smaller, we want there to always be a δ. In this example,

$$\lim_{x\to 3} f(x) = 5.$$

Exercise



Using the graph, for each $\epsilon > 0$, determine a value of $\delta > 0$ to satisfy the statement

$$|f(x) - 4| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta.$$

(a) $\epsilon = 1$

(b) $\epsilon = 0.5$.

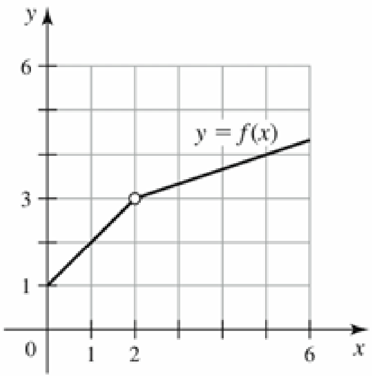
Finding a Symmetric Interval

Question

When finding an interval $(a - \delta, a + \delta)$ around the point a, what happens if you compute two different δs?

Answer: To obtain a symmetric interval around a, use the smaller of the two δs as your distance around a.

Exercise



The graph of $f(x)$ shows

$$\lim_{x \to 2} f(x) = 3.$$

For $\epsilon = 1$, find the corresponding value of $\delta > 0$ so that

$$|f(x) - 3| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta.$$

Exercise

Let $f(x) = x^2 - 4$. For $\epsilon = 1$, find a value for $\delta > 0$ so that

$$|f(x) - 12| < \epsilon \quad \text{whenever} \quad 0 < |x - 4| < \delta.$$

In this example, $\lim_{x \to 4} f(x) = 12$.

2.7 Book Problems

1-7, 9-18