Wed 20 Apr

- Exam 3: Issue with increasing/decreasing, number lines, etc. Point for signature.
- April 22: Last day to drop with a "W".
- Exam 4 next week, probably Friday. Covers §4.7-5.4

Without formally examining methods to evaluate definite integrals, we can use geometry.

Exercise

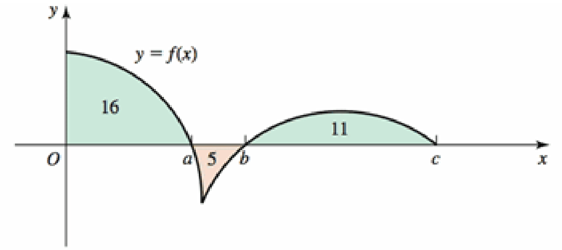
Using geometry, evaluate $\int_1^2 (4x-3)\ dx$.

(*Hint:* The area of a trapezoid is $A = \frac{h(l_1+l_2)}{2}$, where h is the height of the trapezoid and l_1 and l_2 are the lengths of the two parallel bases.)

Exercise

Using the picture below, evaluate the following definite integrals:

1. $\displaystyle\int_0^a f(x)\,dx$
2. $\displaystyle\int_0^b f(x)\,dx$
3. $\displaystyle\int_0^c f(x)\,dx$
4. $\displaystyle\int_a^c f(x)\,dx$



Properties of Integrals

1. (Reversing Limits) $\int_b^a f(x)\ dx = -\int_a^b f(x)\ dx$
2. (Identical Limits) $\int_a^a f(x)\ dx = 0$
3. (Integral of a Sum)
   $\int_a^b (f(x) + g(x))\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$
4. (Constants in Integrals) $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$

Properties of Integrals, cont.

5. (Integrals over Subintervals) If c lies between a and b, then

$$\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx.$$

6. (Integrals of Absolute Values) The function $|f|$ is integrable on $[a, b]$ and $\int_a^b |f(x)|\ dx$ is the sum of the areas of regions bounded by the graph of f and the x-axis on $[a, b]$. (See Figure 5.31 on p. 329)

   (This is the total area, no negative signs.)

Exercise

If $\int_2^4 f(x)\ dx = 3$ and $\int_4^6 f(x)\ dx = -2$, then compute $\int_2^6 f(x)\ dx$.

5.2 Book Problems

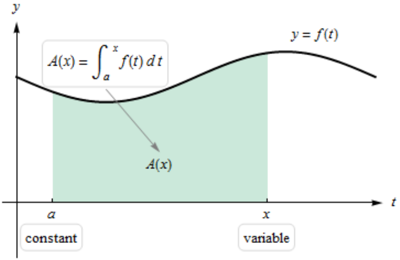
11-45 (odds), 67-74

§5.3 Fundamental Theorem of Calculus

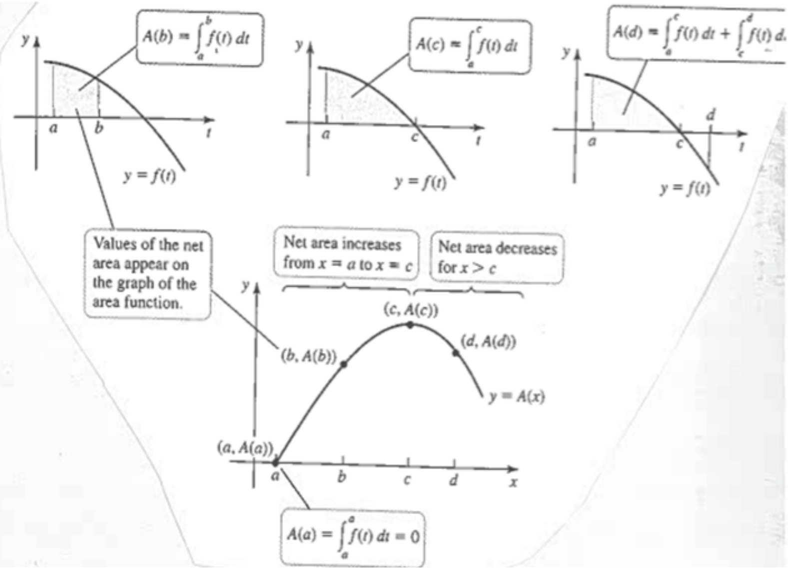
Using Riemann sums to evaluate definite integrals is usually neither efficient nor practical. We will develop methods to evaluate integrals and also tie together the concepts of differentiation and integration.

To connect the concepts of differention and integration, we first must define the concept of an area function.

Let $y = f(t)$ be a continuous function which is defined for all $t \geq a$, where a is a fixed number. The area function for f with left endpoint at a is given by $A(x) = \int_a^x f(t)\, dt$.



This gives the net area of the region between the graph of f and the t-axis between the points $t = a$ and $t = x$.



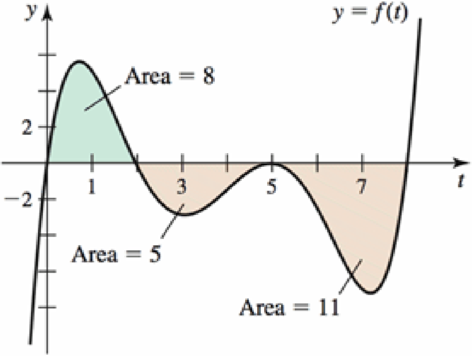
$A(b) = \int_a^b f(t)\,dt$
$A(c) = \int_a^c f(t)\,dt$
$A(d) = \int_a^c f(t)\,dt + \int_c^d f(t)\,dt$
$y = f(t)$
Values of the net area appear on the graph of the area function.
Net area increases from $x = a$ to $x = c$
Net area decreases for $x > c$
$(c, A(c))$
$(b, A(b))$
$(d, A(d))$
$y = A(x)$
$(a, A(a))$
$A(a) = \int_a^a f(t)\,dt = 0$

Example

The graph of f is shown below. Let

$$A(x) = \int_0^x f(t)\ dt \quad \text{and} \quad F(x) = \int_2^x f(t)\ dt$$

be two area functions for f. Compute $A(2), F(5), A(5), F(8)$.



Linear functions help to build the rationale behind the Fundamental Theorem of Calculus.

Example

Let $f(t) = 4t + 3$ and define $A(x) = \int_1^x f(t)\, dt$. What is $A(2)$? $A(4)$? $A(x)$? $A'(x)$?

In general, the property illustrated with this linear function works for all continuous functions and is one part of the FTOC (Fundamental Theorem of Calculus).

Theorem (FTOC I)

If f is continuous on $[a, b]$, then the area function $A(x) = \int_a^x f(t)\ dt$ for $a \leq x \leq b$ is continuous on $[a, b]$ and differentiable on (a, b). The area function satisfies $A'(x) = f(x)$; or equivalently,

$$A'(x) = \frac{d}{dx} \int_a^x f(t)\ dt = f(x)$$

which means that the area function of f is an antiderivative of f.

The Fundamental Theorem of Calculus (Part 2)

Since A is an antiderivative of f, we now have a way to evaluate definite integrals and find areas under curves.

Theorem (FTOC II)

If f is continuous on $[a, b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

We use the notation $F(x)|_a^b = F(b) - F(a)$.

Overview of FTOC

In essence, to evaluate an integral, we

- Find any antiderivative of f, and call if F.
- Compute $F(b) - F(a)$, the difference in the values of F between the upper and lower limits of integration.

The two parts of the FTOC illustrate the inverse relationship between differentiation and integration – the integral "undoes" the derivative.

Example

(1) Use Part 1 of the FTOC to simplify $\dfrac{d}{dx}\displaystyle\int_x^{10} \frac{dz}{z^2+1}$.

(2) Use Part 2 of the FTOC to evaluate $\displaystyle\int_0^{\pi} (1-\sin x)\ dx$.

(3) Compute $\displaystyle\int_1^{y} h'(p)\ dp$.

Exercise

(1) Simplify $\dfrac{d}{dx}\displaystyle\int_{3x^4}^{4} \frac{t-5}{t^2+1}\,dt$.

(2) Evaluate $\displaystyle\int_{1}^{5} (x^2-4)\,dx$.

5.3 Book Problems

11-17, 19-57 (odds), 61-67 (odds)