MATH 2574 (Calculus III)
Spring 2017

Name: ____________________________

Wed 15 Mar 2017

Exam 2: Multivariate Derivatives and Multiple Integrals (§12.3-12.9, 10.1-10.3, 13.1-13.5)

Exam Instructions: You have 50 minutes to complete this exam. Justification is required for all problems. No electronic devices (phones, iDevices, computers, etc) except for a **basic scientific calculator**. On story problems, round to one decimal place. If you finish early then you may leave, UNLESS there are less than 5 minutes of class left. To prevent disruption, if you finish with less than 5 minutes of class remaining then please stay seated and quiet.

In addition, please provide the following data:

Drill Instructor: ____________________________

Drill Time: ____________________________

Your signature below indicates that you have read this page and agree to follow the Academic Honesty Policies of the University of Arkansas.

Signature: **(1 pt)** ____

Good luck!

1. **(6 pts)** The density of a thin circular plate of radius 2 is given by $\rho(x, y) = 4 + xy$. The edge of the plate is described by the parametric equations $x = \cos t$, $y = \sin t$, for $0 \le t \le 2\pi$. Find the rate of change of the density with respect to t on the edge of the plate.

2. Evaluate (or show non-existence of) the following limits:

   (a) **(5 pts)** $\displaystyle\lim_{(x,y,z)\to(\ln 2,3,1)} (1 + x) \ln e^{yz}$

   (b) **(5 pts)** $\displaystyle\lim_{(u,v)\to(0,0)} \frac{|uv|}{uv}$

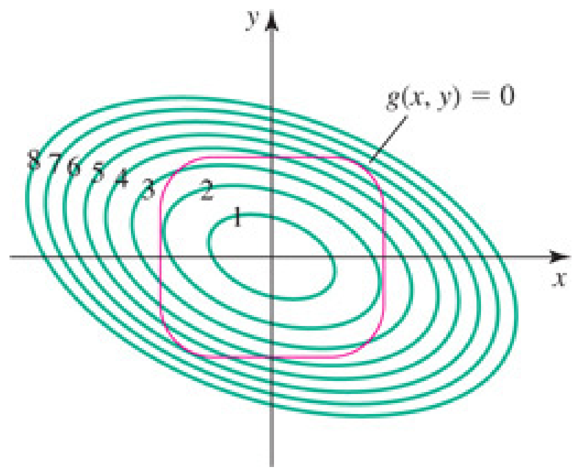
3. **(10 pts)** Find the area of the region inside the rose $r = 4\cos 2\theta$ and outside the circle $r = 2$. *(In case you need it, the half-angle formula is* $\cos^2 x = \dfrac{1+\cos 2x}{2}$*.)*

4. **(12 pts)** Find the absolute maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 2x - 2y$$

on the closed region R, bounded by the triangle with vertices $(0, 0)$, $(2, 0)$, $(0, 2)$.

5. **(8 pts)** The following figure shows the level curves for various $z = z_0$ of the function f, along with the constraint curve $g(x, y) = 0$. Estimate the maximum and minimum values of f subject to the constraint. At each point where an extreme value occurs, indicate the direction of ∇f and the direction of ∇g.



6. **(6 pts)** Compute the directional derivative of

$$g(x, y) = \sin(\pi(2x - y))$$

at the point $P = (-1, -1)$ in the direction of $\mathbf{u} = \langle \frac{12}{13}, -\frac{5}{13} \rangle$.

7. Determine whether the following statements are true or false. You must justify your answer.

   (a) **(4 pts)** The graphs of $r = 2$ and $\theta = \dfrac{\pi}{4}$ intersect exactly once.

   (b) **(4 pts)** The point $\left(3, \dfrac{\pi}{2}\right)$ lies on the graph of $r = 3\cos 2\theta$.

   (c) **(4 pts)** The graphs of $r = 2\sec\theta$ and $r = 3\csc\theta$ are lines.

8. **(10 pts)** Set up, but **do not evaluate**, the integral for the volume of material remaining in a hemisphere of radius 4 after a cylindrical hole of radius 2 is drilled through the center of the hemisphere perpendicular to its base.

   ExTrA cReDiT (5pts) Evaluate the integral you set up.