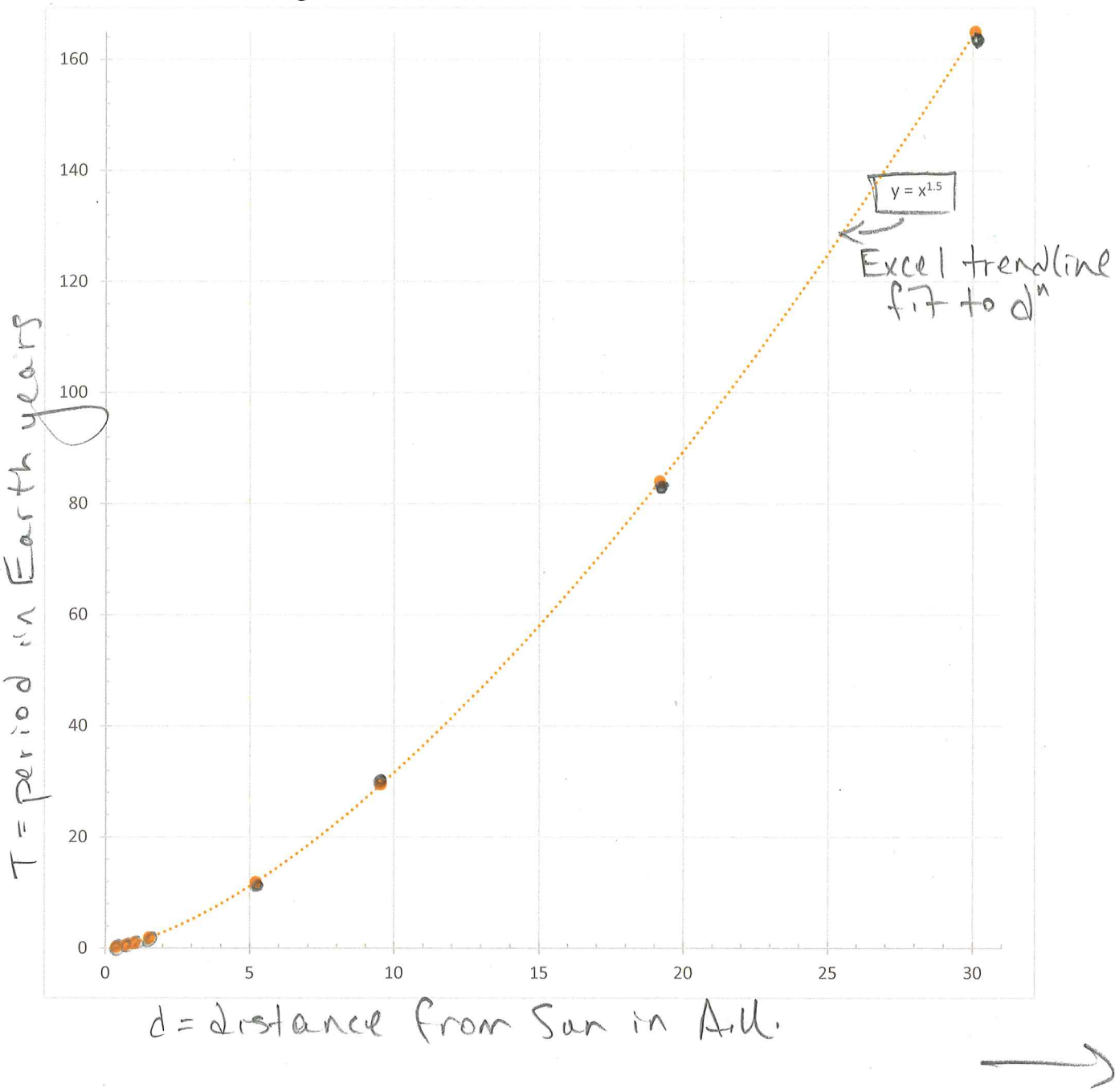
Take-Home Quiz #1
SOLUTIONS

Math 235 (Calc I)
Fall 2017

1. (a)

planet	d	T	$\log_d(T)$	d^n
Mercury	0.387	0.241	1.498907089	0.24076
Venus	0.723	0.615	1.498809685	0.614771
Earth	1	1		1
Mars	1.523	1.881	1.501855176	1.879499
Jupiter	5.203	11.861	1.499637792	11.86725
Saturn	9.541	29.547	1.501145932	29.46789
Uranus	19.19	84.008	1.499772565	84.05386
Neptune	30.086	164.784	1.499572747	164.9998
		average ->	1.499957284	<- n



(b) Yes. In the model, $T(d) \approx d^{1.5}$

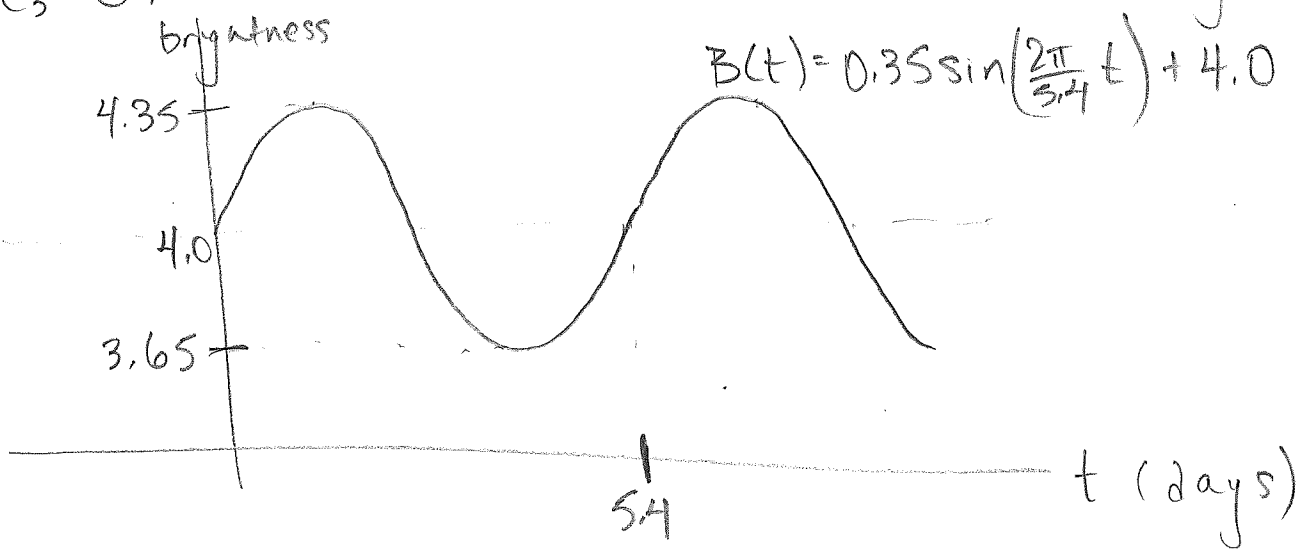
$$\Rightarrow T^2 \approx (d^{1.5})^2 = (d^{3/2})^2 = d^3.$$

2. The brightness is periodic, so should be of the form

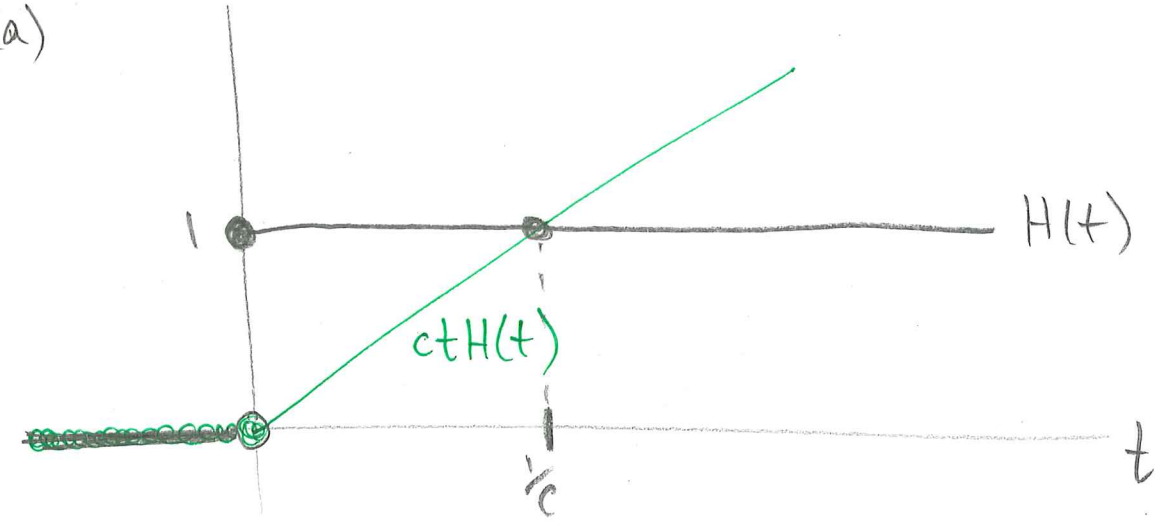
$$B(t) = C_1 \sin\left(C_2(t - C_3)\right) + C_4.$$

The average brightness is the midline of the sine function, so set $C_4 = 4$. For a ± 0.35 variation in magnitude we must scale the function, so set $C_1 = 0.35$. The period of Delta Cephei is 5.4 days instead of the usual period of 2π radians for a sine function, so set $C_2 = \frac{2\pi}{5.4}$.

Finally, assume Delta Cephei is at average brightness and increasing at time $t = 0$ days, so $C_3 = 0$.



3. (a)



(b) The constant c must be positive in the context of the problem, so that there is non-negative voltage.

The graphs intersect when

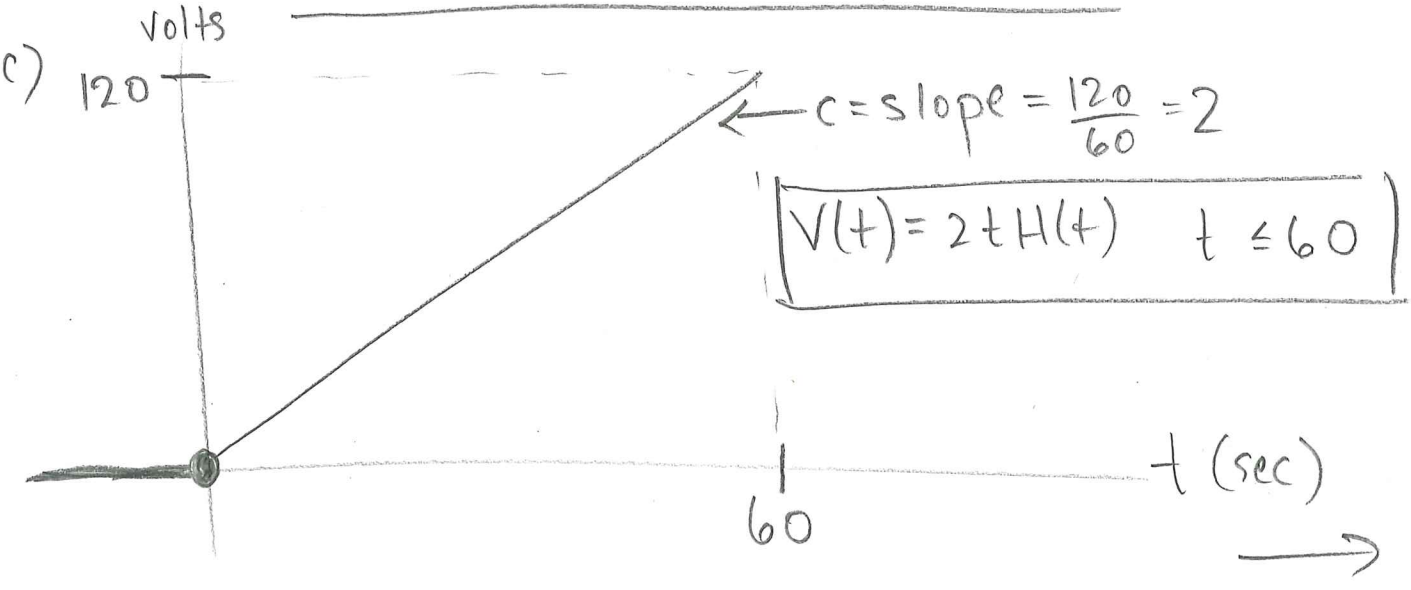
$$ctH(t) = H(t)$$

$$\Rightarrow ct = 1 \quad \text{when } t > 0$$

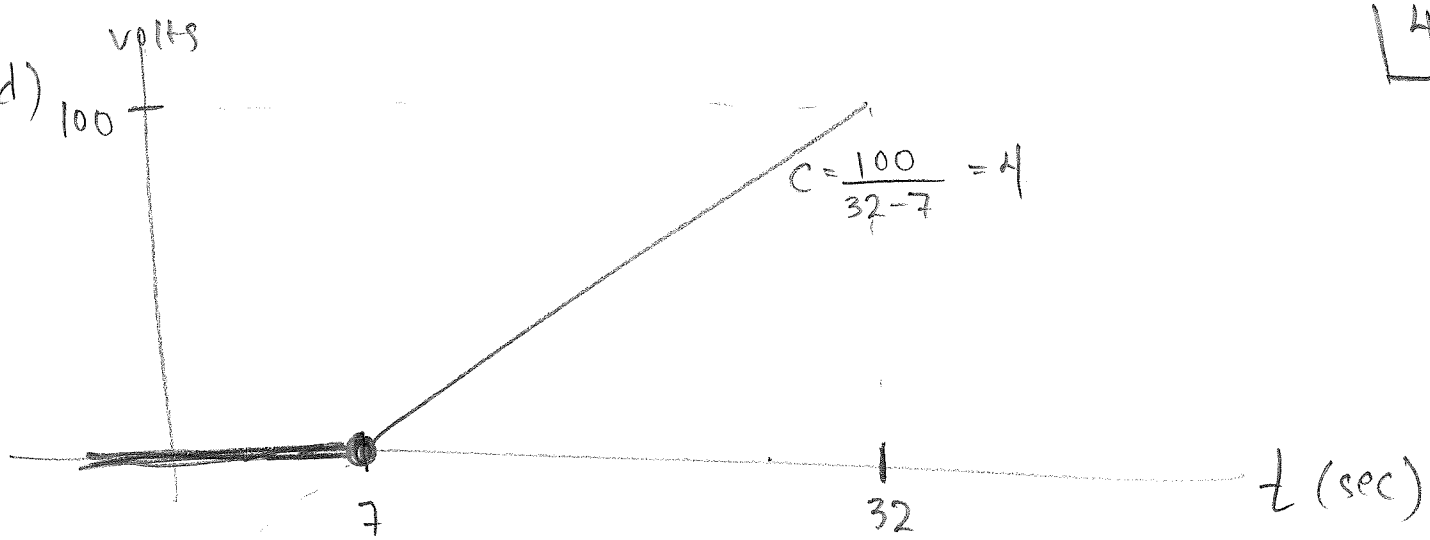
$$\boxed{t = \frac{1}{c}}$$

and they intersect everywhere when $t < 0$.

(c)



(d)



ramp function shifted to the right by 7:

$$V(t) = 4(t-7)H(t-7) \quad t \leq 32$$

$$= \begin{cases} 0 & t \leq 7 \\ 4(t-7) = 4t-28 & 7 < t \leq 32 \end{cases}$$

4. $A \circ A = 1.04(1.04x)$ ← investment after 2 years

$A \circ A \circ A = 1.04(1.04(1.04x))$ ← " " 3 years

$A \circ A \circ A \circ A = 1.04(1.04(1.04(1.04x)))$ ← " " 4 years

$\underbrace{A \circ A \circ \ldots \circ A}_{n} = 1.04^n x$ = investment after n years.

5. For $g \circ f = h$, let $g = h \circ f^{-1}$

$= h(x-4) = 4(x-4) - 1 = \boxed{4x-17}$

Check: $g \circ f = 4(x+4) - 17 = 4x + 16 - 17 = 4x - 1 = h$. ✓