Math 116 Quiz 5: $\oint$ 11.1-11.6 **Tue 23 Oct 2012**
(Differential Equations) **Name:** SOLUTIONS

You have 30 minutes to complete this quiz. Eyes on your own paper and good luck!

1. **Definitions/Concepts.** (1 pt ea) Consider a differential equation of the form

$$\frac{dH}{dt} = k(H - C),$$

where k and C are constants.

(a) This is a __1st__ order differential equation.

(b) What is the general solution to this equation?

$$\begin{aligned}
\frac{dH}{dt} &= k(H - C) \\
\frac{dH}{H - C} &= kdt \\
\int \frac{dH}{H - C} &= \int kdt \\
\ln|H - C| &= kt + C' \text{ (note the constant } C' \text{ is not the same as the constant } C) \\
H - C &= e^{kt+C'} \\
&= Ae^{kt} \text{ (where } A = e^{C'} \text{ is a nonzero constant)} \\
H(t) &= Ae^{kt} + C
\end{aligned}$$

(c) What is the equilibrium solution to this equation?

$$\begin{aligned}
H - C &= 0 \\
H &= C
\end{aligned}$$

2. **Questions/Problems.**

A box is dropped from an airplane. The downward veleocity $v(t)$ of the box, once its parachute opens, satisfies the differential euqation

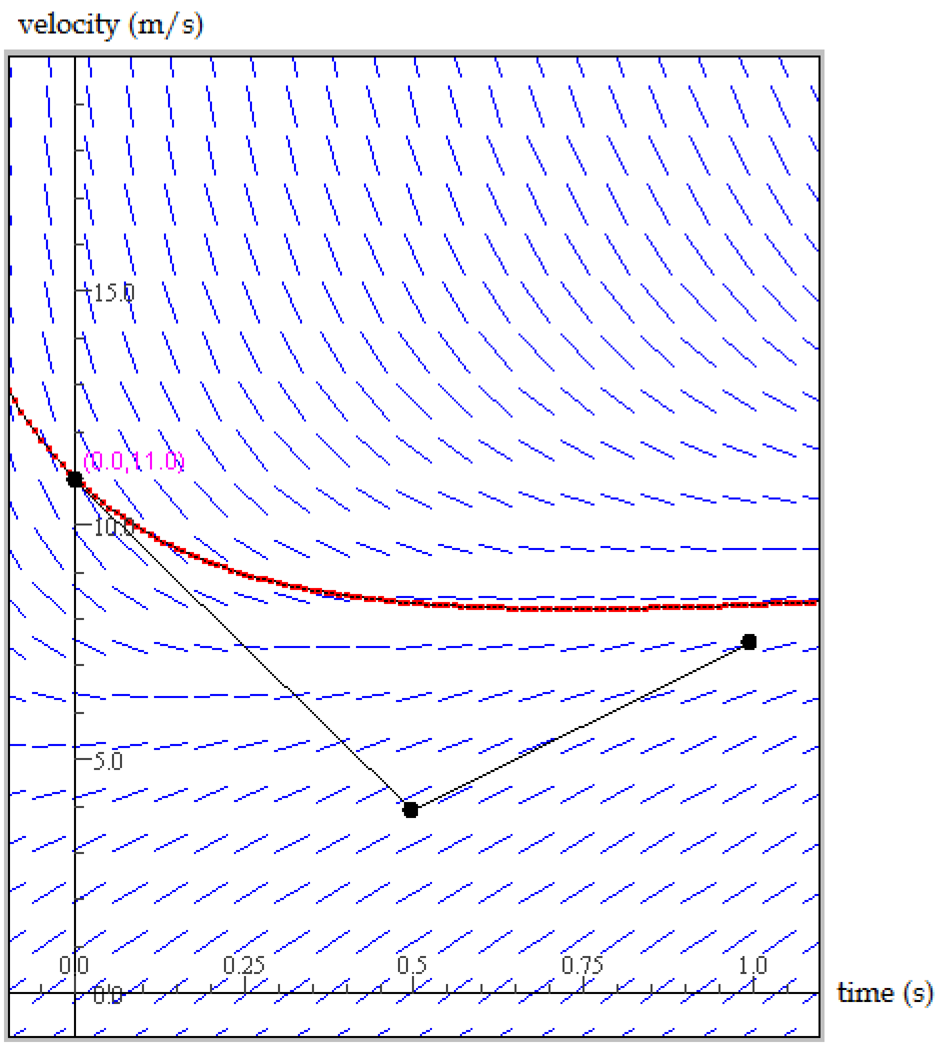
$$\frac{dv}{dt} = 10 - \frac{1}{10}(1 + e^{-t})v^2.$$

(a) (3 pts) Suppose the parachute opens when the velocity of the box is 11 m/s. Use Euler's method with three steps to approximate the velocity of the box one second after the parachute opens.

t	$v(t)$	dv/dt	Δv
0	11	-14.2	-7.1
0.5	3.9	7.556	3.778
1	7.678		

What does your estimate say the velocity is after 1 second?
approximately 7.678 m/s

(b) (2 pts) Draw your Euler approximation on the following slope field:



(c) (2 pts) Say something **about slope fields** to argue whether your approximation is an overestimate or an underestimate.

The approximation lies below the red solution curve, so it is an underestimate.

3. **Computations/Algebra.** (1 pt ea) Find the solutions to the following differential equations subject to their given initial conditions.

(a) $\frac{dy}{dx} + \frac{y}{3} = 0, \;\; y(0) = 10$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{y}{3} \\ \frac{dy}{y} &= -\frac{dx}{3} \\ \ln|y| &= -\frac{1}{3}x + C \\ y &= ke^{-\frac{1}{3}x} \\ 10 &= ke^{-\frac{1}{3}(0)}; \quad k = -10 \\ y &= 10e^{-\frac{1}{3}x} \end{aligned}$$

(b) $2\frac{du}{dt} = u^2, \;\; u(0) = 1$

$$\begin{aligned} 2\frac{du}{u^2} &= dt \\ -\frac{2}{u} &= t + C \\ -\frac{2}{t+C} &= u \\ 1 &= -\frac{2}{0+C}; \quad C = -2 \quad \text{(switching sides in the equation then using the intial value)} \\ u &= -\frac{2}{t-2} \end{aligned}$$

(c) $\frac{dz}{dy} = zy, \;\; z = 1$ when $y = 0$

$$\begin{aligned} \frac{dz}{z} &= y\,dy \\ \ln|z| &= \frac{y^2}{2} + C \\ z &= ke^{\frac{y^2}{2}} \\ 1 &= ke^{\frac{0^2}{2}}; \quad k = 1 \\ z &= e^{\frac{y^2}{2}} \end{aligned}$$

(d) $\frac{dz}{dt} = te^z$, through the origin

$$\begin{aligned}
e^{-z} dz &= t\, dt \\
-e^{-z} &= \frac{t^2}{2} + C \\
e^{-z} &= -\frac{t^2}{2} + C \text{ (constant is different now)} \\
-z &= \ln\left|-\frac{t^2}{2} + C\right| \\
z &= -\ln\left|-\frac{t^2}{2} + C\right| \\
0 &= -\ln\left|-\frac{0^2}{2} + C\right|; \quad C = 0 \\
z &= -\ln\left|-\frac{t^2}{2} + 1\right| \quad \text{(assuming } |t| < \sqrt{2}\text{)}
\end{aligned}$$

(e) $\frac{dw}{d\theta} = w + w\theta^2$, $w = 5$ when $\theta = 0$

$$\begin{aligned}
\frac{dw}{w} &= (1 + \theta^2) d\,\theta \\
\ln|w| &= \theta + \frac{\theta^3}{3} + C \\
w &= ke^{\theta + \frac{\theta^3}{3}} \\
5 &= ke^{0 + \frac{0^3}{3}}; \quad k = 5 \\
w &= 5e^{\theta + \frac{\theta^3}{3}}
\end{aligned}$$