Take Home Quiz #2

SOLUTIONS

Math 2574 (Cal III)

Spring 2017

1. (a) i. $s = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$ $(z = 0 \Rightarrow z' = 0)$

$= \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dt$

$= \int_0^{2\pi} dt = \boxed{2\pi}$

ii. $s = \int_0^{\pi} \sqrt{(-2\sin t)^2 + (2\cos t)^2}\, dt$

$= \int_0^{\pi} \sqrt{4\sin^2 t + 4\cos^2 t}\, dt$

$= \int_0^{\pi} 2\sqrt{\sin^2 t + \cos^2 t}\, dt$

$= \int_0^{\pi} 2\, dt = \boxed{2\pi}$

arc length of a circle is 2π

$\longrightarrow$

(b) i. $s(t) = \int_0^t \sqrt{(-\sin u)^2 + (\cos u)^2}\, du$

$= \int_0^t du = \boxed{t}$

ii. $s(t) = \int_0^t \sqrt{(-2\sin u)^2 + (2\cos u)^2}\, du$

$= \int_0^t \sqrt{4(\sin^2 u + \cos^2 u)}\, du$

$= \int_0^t 2\, du = \boxed{2t}$

(c) Since $s(t) = \int_0^t \sqrt{x'(u)^2 + y'(u)^2 + z'(u)^2}\, du$, by FTOC, $\frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} \boxed{= |\vec{r}\,'(t)|}$

(d) If $\vec{r}(t)$ is parametrized by arc length, then $s(t) = t$ (in other words, arc length s equals time t). That means $s'(t) = 1$ and so from part (c) $\boxed{|\vec{r}\,'(t)| = 1}$

2. Substitute the x- and y- coordinates of $\vec{r}$ into the equation of the plane:

$$y = 2x + 1$$

$$\Rightarrow 2\sin t = 2(10\cos t) + 1$$

To solve for t, look for the points of intersection in the graphs (desmos.com/calculator)

$$y = 2\sin t - 20\cos t$$

$$y = 1$$

$\Rightarrow t \approx 1.521, 4.563$ (there are infinitely many points, but only these two are in the domain $0 \le t \le 2\pi$).

Use $\vec{r}(t)$ to find the coordinates in $\mathbb{R}^3$:

$$\boxed{\begin{aligned} \vec{r}(1.521) &\approx \langle 1.00, 2.00, 1 \rangle \\ \vec{r}(4.563) &\approx \langle -1.50, -1.99, 1 \rangle \end{aligned}}$$

3. The given planes have normal vectors

$\vec{n}_1 = \langle 2, 5, -3 \rangle$

$\vec{n}_2 = \langle -1, 5, 2 \rangle$

The orthogonal plane will have normal vector

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \langle \underbrace{5(2) - (-3)(5)}_{10\ +15}, \underbrace{(-3)(-1) - (2)(2)}_{3\ -4}, \underbrace{(2)(5) - 5(-1)}_{10\ +5} \rangle$$

$$= \langle 25, -1, 15 \rangle$$

Since $(0, -2, 4)$ is on the plane, the equation is

$$\boxed{25x - \overset{+2}{(y-2)} + \overset{-60}{15(z+4)} = 0}$$

or

$$25x - y + 15z = 58$$

4.(a) • Elliptic cone

• x-axis $\Rightarrow y = z = 0$

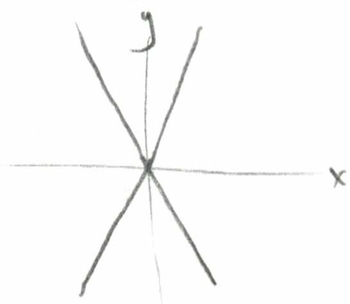
$$4(0)^2 + (0)^2 = x^2 = 0$$
$$\Rightarrow x = 0$$

and similarly for the other axes, the intercept is at the origin

• xy-trace $\Rightarrow z = 0$

$$4y^2 + 0^2 = x^2$$

intersecting lines



zx-trace $\Rightarrow y = 0$

$$z^2 = x^2$$

intersecting lines

yz-trace $\Rightarrow x = 0$

$$4y^2 + z^2 = 0$$

point

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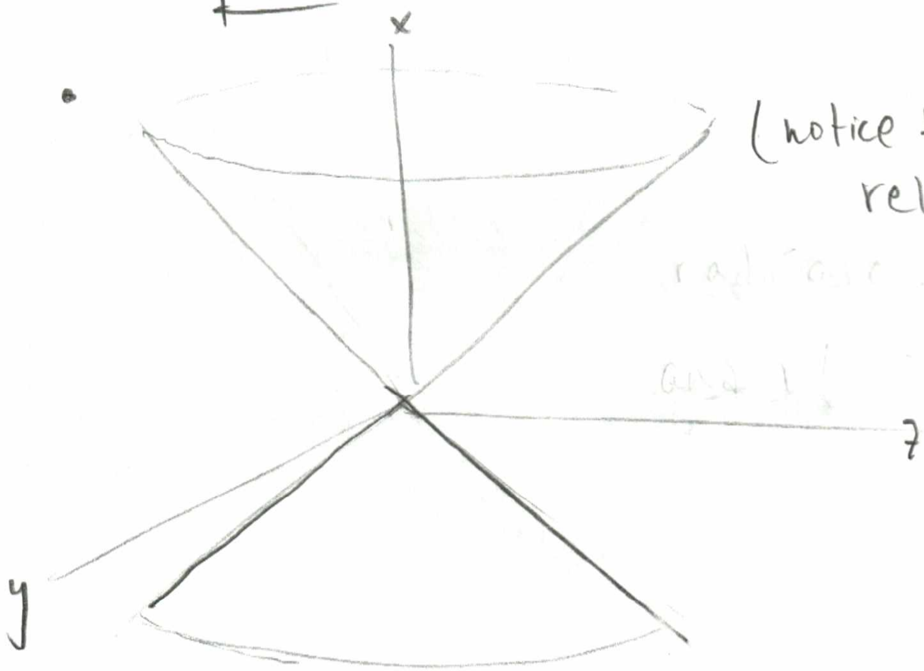


(notice the axes are relabelled)

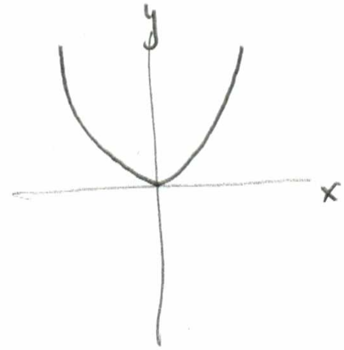
(b) • Hyperbolic paraboloid

• x-axis: $0 = \frac{x^2}{16} \Rightarrow \underline{x = 0}$

y-axis: $\underline{y = 0}$

z-axis: $0 = -4z^2 \Rightarrow \underline{z = 0}$

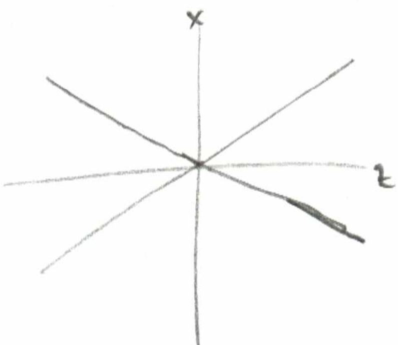
• xy-trace: $y = \frac{x^2}{16}$ parabola



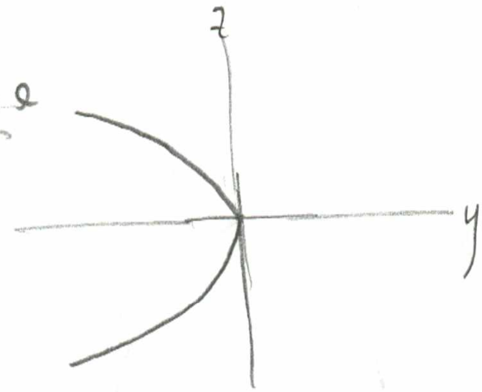
zx-trace: $0 = \frac{x^2}{16} - 4z^2$

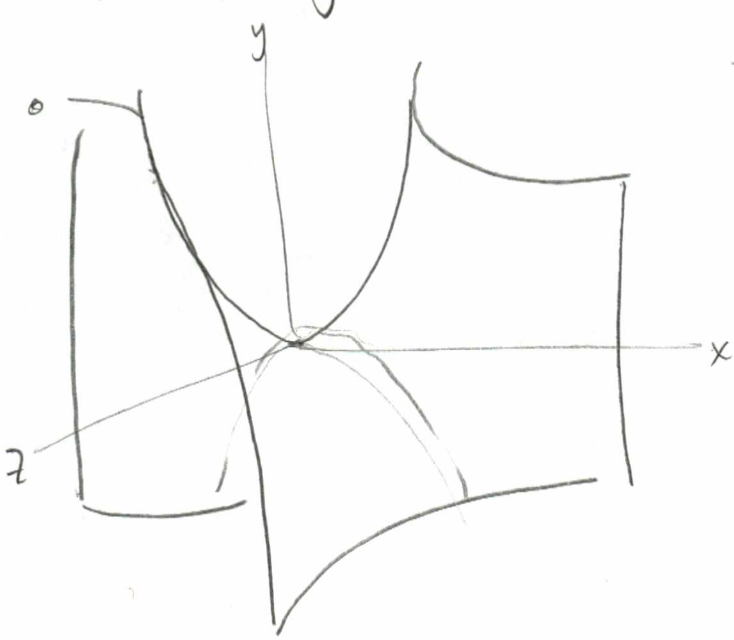
$\Rightarrow x^2 = 64z^2$

intersecting lines



yz-trace: $y = -4z^2$ parabola





(c) • <u>Elliptic paraboloid</u>

• x-intercept: $0 = \frac{x^2}{4} \Rightarrow \underline{x = 0}$

y-intercept: $0 = \frac{y^2}{9} \rightarrow \underline{y = 0}$

z-intercept: $\underline{z = 0}$

• xy-trace: $0 = \frac{x^2}{4} + \frac{y^2}{9}$ <u>point</u>

zx-trace: $z = \frac{x^2}{4}$ <u>parabola</u>

yz-trace: $z = \frac{y^2}{9}$ parabola

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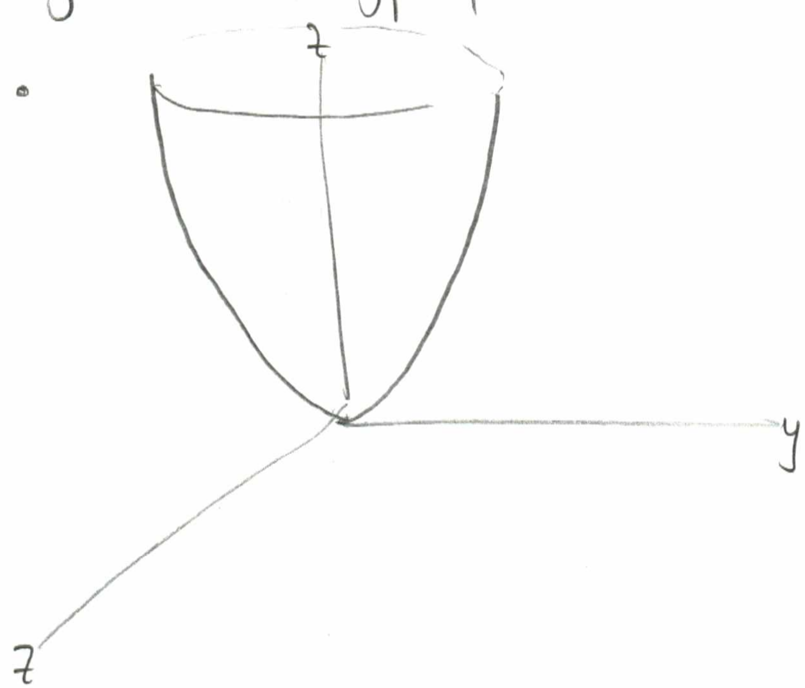


5. (a) $B = 2000 = Pe^{0.04t}$

Solve for $P \Rightarrow \frac{2000}{e^{0.04t}} = P$

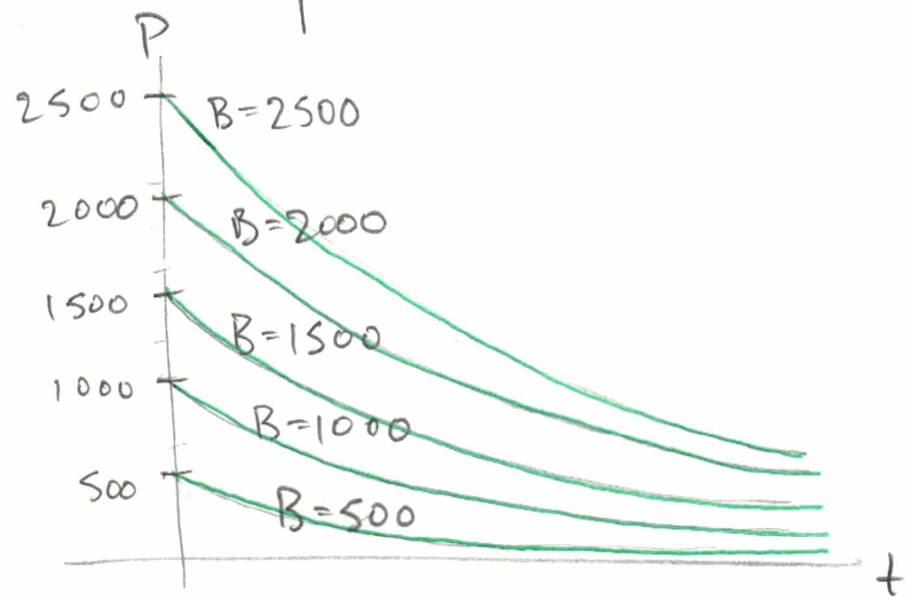
or $\boxed{P = 2000e^{-0.04t}}$

Can also solve for t as a function of P:

$\ln\left(\frac{2000}{P}\right) = 0.04t$

$\Rightarrow \boxed{t = \frac{1}{0.04}\left(\ln(2000) - \ln P\right)}$

(b) Plot P as a function of t — the curves are exponentials



(c) If t increases, then the principal P can be lower and yield the same balance.