External memory model

I/O s

N – data size
B – minimal read/write block size
M – RAM capacity

$t(x) = \frac{x}{B} \approx \frac{x}{t_{ph}(x)}$

$t_{ph}(x) \approx 10ms + \frac{x}{100MB/s}$

$B \geq 1 MB$

Join = Sort(N)

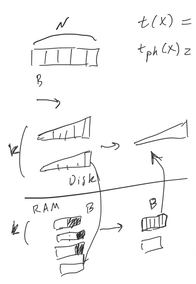
$(k, \ldots) \to (k, [\ \ ])$

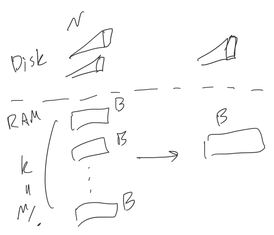
$(k, \ldots)$

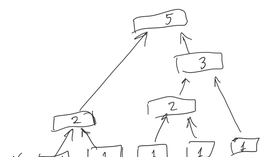
$Scan(N) = O(\frac{N}{B})$

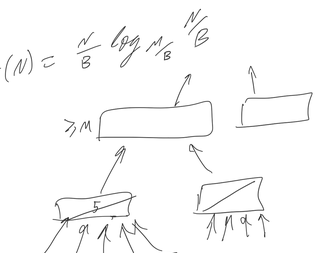
$Merge(N) = O(\frac{N}{B})$

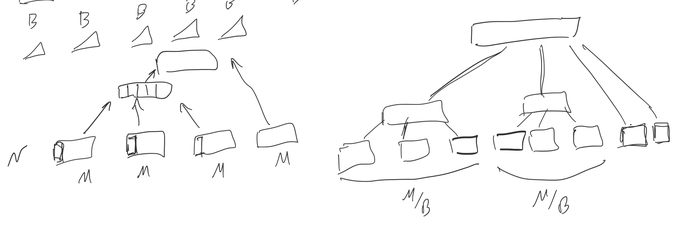
$Sort(N) = \frac{N}{B} \log_{M/B} \frac{N}{B}$









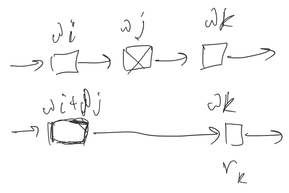


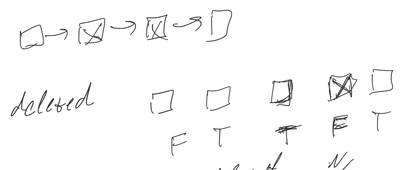
List ranking

next (1, 4, 3, ⊥, 2)
4 3 1 0 2
0 1 2 3 4

~~O(N) I/O~~

O(sort(N))





deleted

E #deleted nodes = N/4

$T(N) = O(Sort(N)) + T(\frac{3}{4}N)$

$T(N) = O(Sort(N))$

deleted
i, false
j, true
k, false

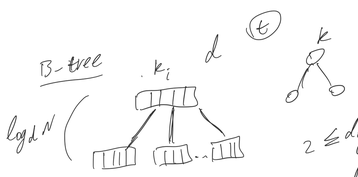
weights
i, w_i

ranks
i, r_i
k, r_k

ranks'
i, r_i

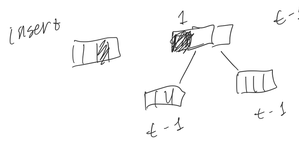
j k = next(j)
w_j w_k
deleted(j)
$r_j = w_j + r_k$

B-tree



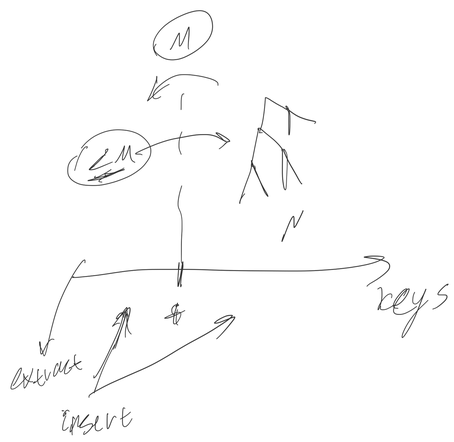
$2 \leq deg_{root} \leq 2t-1$

$t-1 \leq deg \leq 2t-1$



Buffered Tree

	insert	get		extract
B-tree	$\log_B N$	$\log_B N$		
Buffered Tree	$\frac{1}{B} \log_{M/B} \frac{N}{B}$	—	Heap	$\frac{1}{B} \log_{M/B} \frac{N}{B}$

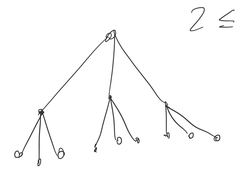


(d, b) - tree

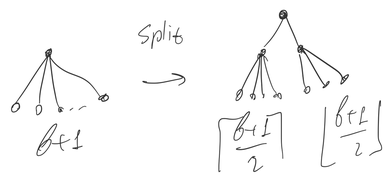
$2 \leq deg \leq b$

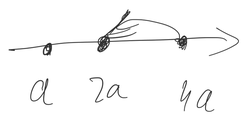
$a \leq deg \leq b$

$b = 4a$

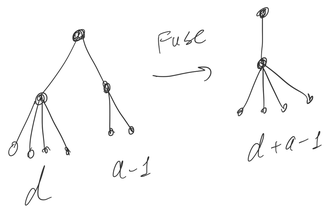


insert



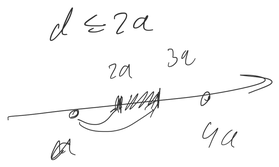


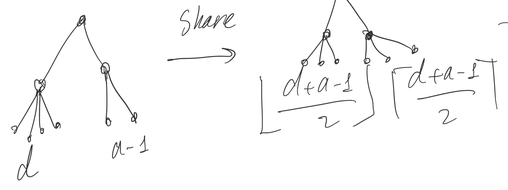
delete



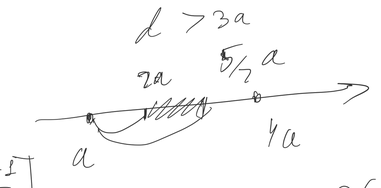
$d \leq 3a$

$d \leq 2a$

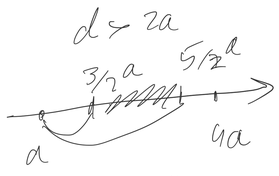




$d > 3a$



$d > 2a$



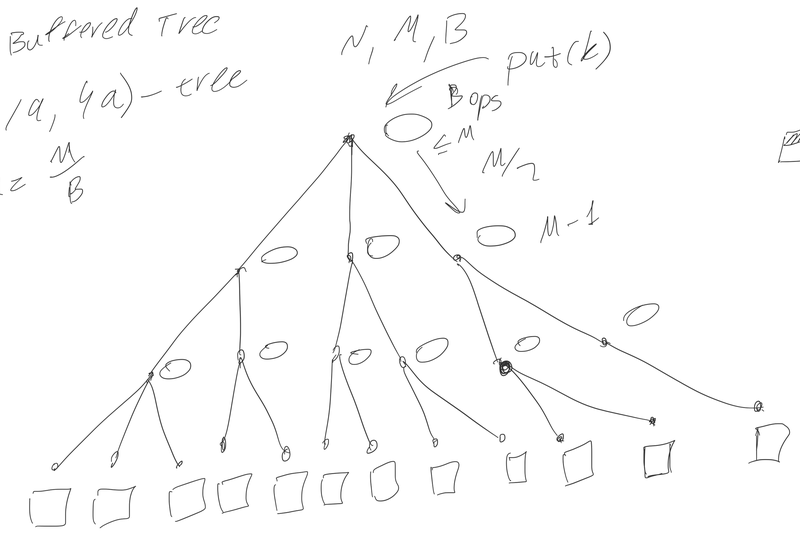
rebalance = $O(\frac{\#\text{operations}}{d})$

x levels

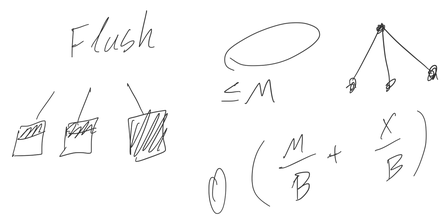
Buffered Tree

$(a, 4a)$ - tree

$a = \frac{M}{B}$



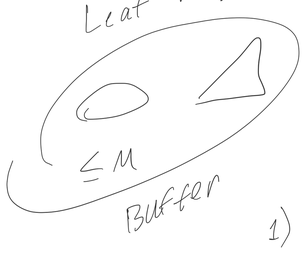
Flush

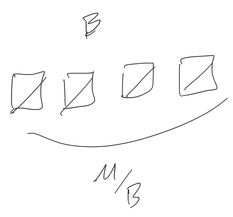


$O(\frac{M}{B} + \frac{X}{B})$

$\leq \log_{M/B} N \cdot M$

Leaf flush





$O(\frac{M}{B} + \frac{X}{B})$

1) Leaf split
2) Other splits

operations per key

$O(\frac{1}{B} \cdot \log_{M/B} \frac{N}{B})$

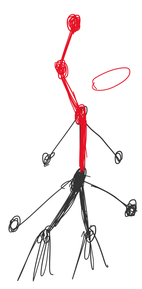
Flush N keys

#flushes

$O(\frac{M}{B} \cdot \frac{N}{M}) = O(\frac{N}{B})$

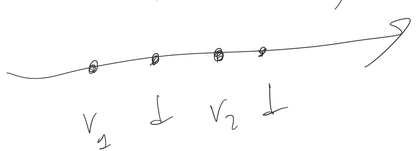
balance ops

$O(\frac{\#ops}{M/B}) = O(\frac{\frac{M}{B} \cdot N/B}{M/B}) = O(\frac{N}{B})$



delete

$(k, \perp)$

$(k, \perp, ts)$

tombstone
1) timestamp
2) empty blocks