$$\phi \frac{\partial s}{\partial t} = \nabla \cdot \left[D_0 D(s) \nabla s - K_s K(s) \hat{k} \right] - F_w$$

$$\phi \frac{\partial s}{\partial t} = D_0 D(s) \nabla \cdot \nabla s + D_0 D'(s) \nabla s \cdot \nabla s - K_s K'(s) \nabla s \cdot \hat{k} - F_w$$

$$\phi \frac{\partial s}{\partial t} = D_0 D(s) \nabla \cdot \nabla s + \left[D_0 D'(s) \nabla s - K_s K'(s) \hat{k} \right] \cdot \nabla s - F_w$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c\phi \frac{\partial s}{\partial t} = \nabla \cdot \left\{ D_f \phi^{d+1} s^{d+1} \nabla c + c \left[D_0 D(s) \nabla s - K_s K(s) \hat{k} \right] \right\} - F$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c\phi \frac{\partial s}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla \cdot \nabla c + D_f \phi^{d+1} (d+1) s^d \nabla s \cdot \nabla c +$$

$$+ c \left[D_0 D(s) \nabla \cdot \nabla s + D_0 D'(s) \nabla s \cdot \nabla s - K_s K'(s) \nabla (s) \cdot \hat{k} \right] +$$

$$+ \left[D_0 D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c - F$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c\phi \frac{\partial s}{\partial t} = \left[D_f \phi^{d+1} s^{d+1} \right] \nabla \cdot \nabla c +$$

$$+ \left[D_f \phi^{d+1} (d+1) s^d \nabla s + D_0 D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c +$$

$$c \left[D_0 D(s) \nabla \cdot \nabla s + \left[D_0 D'(s) \nabla s - K_s K'(s) \hat{k} \right] \cdot \nabla s \right] - F$$

$$(b + \phi s) \frac{\partial c}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla^2 c +$$

$$+ \left[D_f \phi^{d+1} (d+1) s^d \nabla s + D_0 D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c +$$

$$+ c F_w - F$$

$$(b+\phi s)\frac{\partial c}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla^2 c + D_f \phi^{d+1}(d+1) s^d \nabla s \cdot \nabla c - u \cdot \nabla c$$
$$+ c F_w - F$$

$$l = K_0\left(1 - e^{-\frac{r_0}{K_0}(t-t_0)}\right)$$

$$\frac{l}{K_0} = 1 - e^{-\frac{r_0}{K_0}(t-t_0)}$$

$$e^{-\frac{r_0}{K_0}(t-t_0)} = 1 - \frac{l}{K_0}$$

$$-\frac{r_0}{K_0}(t-t_0) = \ln\left(1 - \frac{l}{K_0}\right)$$

$$t - t_0 = -\frac{K_0}{r_0}\ln\left(1 - \frac{l}{K_0}\right)$$

$$(b+\phi s)\frac{\partial c}{\partial t} + c\phi\frac{\partial s}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla \cdot \nabla c + D_f \phi^{d+1}(d+1) s^d \nabla s \cdot \nabla c +$$
$$+ c\left[D_0 D(s) \nabla \cdot \nabla s + D_0 D'(s) \nabla s \cdot \nabla s - K_s K'(s) \nabla(s) \cdot \hat{k}\right] +$$
$$+ \left[D_0 D(s) \nabla s - K_s K(s)\hat{k}\right] \cdot \nabla c - F$$

$$\phi\frac{\partial s}{\partial t} = D_0 D(s) \nabla \cdot \nabla s + \left[D_0 D'(s) \nabla s - K_s K'(s)\hat{k}\right] \cdot \nabla s - F_w$$

$$\phi \frac{\partial s}{\partial t} = \nabla \cdot \left[D_o D(s) \nabla s - K_s K(s) \hat{k} \right] - F_w$$

$- K(s) \nabla K_s \cdot \hat{k}$

$$\phi \frac{\partial s}{\partial t} = D_o D(s) \nabla \cdot \nabla s + D_o D'(s) \nabla s \cdot \nabla s - K_s K'(s) \nabla s \cdot \hat{k} - F_w$$

$- K(s) \nabla K_s \cdot \hat{k}$

$$\phi \frac{\partial s}{\partial t} = D_o D(s) \nabla \cdot \nabla s + \left[D_o D'(s) \nabla s - K_s K'(s) \hat{k} \right] \cdot \nabla s - F_w$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c \phi \frac{\partial s}{\partial t} = \nabla \cdot \left\{ D_f \phi^{d+1} s^{d+1} \nabla c + c \left[D_o D(s) \nabla s - K_s K(s) \hat{k} \right] \right\} - F$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c \phi \frac{\partial s}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla \cdot \nabla c + D_f \phi^{d+1} (d+1) s^d \nabla s \cdot \nabla c +$$

$- K(s) \nabla K_s \cdot \hat{k}$

$$+ c \left[D_o D(s) \nabla \cdot \nabla s + D_o D'(s) \nabla s \cdot \nabla s - K_s K'(s) \nabla (s) \cdot \hat{k} \right] +$$

$$+ \left[D_o D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c - F$$

$$(b + \phi s) \frac{\partial c}{\partial t} + c \phi \frac{\partial s}{\partial t} = \left[D_f \phi^{d+1} s^{d+1} \right] \nabla \cdot \nabla c +$$

$$+ \left[D_f \phi^{d+1} (d+1) s^d \nabla s + D_o D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c +$$

$$c \left[D_o D(s) \nabla \cdot \nabla s + \left[D_o D'(s) \nabla s - K_s K'(s) \hat{k} \right] \cdot \nabla s \right] = c \left[\phi \frac{\partial s}{\partial t} + F_w \right] - F$$

$- K(s) \nabla K_s \cdot \hat{k}$

$$(b + \phi s) \frac{\partial c}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla^2 c +$$

$$\left[\ldots \phi^{d+1} (d+1) s^d \nabla s + D_o D(s) \nabla s - K_s K(s) \hat{k} \right] \cdot \nabla c +$$

$$+\big[D_f \phi^{\cdots} (d+1) s \cdots$$ [illegible]

$$+ c \cdot F_w \quad - F$$

$$(b+\phi s)\frac{\partial c}{\partial t} = D_f \phi^{d+1} s^{d+1} \nabla^2 c + D_f \phi^{d+1} (d+1) s^d \nabla s \cdot \nabla c - u \cdot \nabla c$$
$$+ c F_w - F$$