

$$P_5(x) = \underline{a_0} + \underline{a_1}x + \underline{a_2}x^2 + \underline{a_3}x^3 + \underline{a_4}x^4 + \underline{a_5}x^5 \rightarrow \{1, x, x^2, \dots, x^5\}$$

$$P_2(x) = a_0 + a_1x + a_2x^2 \rightarrow \{1, x, x^2\}$$

~~$$\{1, x, 2x, 3x\}$$~~ 
$$\{1, \sin x, e^{-x}\}$$

x	y
$x_0$	$y_0$
$x_1$	$y_1$
$x_2$	$y_2$

$$p(x) = a_0 + a_1x + a_2x^2$$

$$y_0 = a_0 + a_1x_0 + a_2x_0^2$$

$$y_1 = a_0 + a_1x_1 + a_2x_1^2$$

$$y_2 = a_0 + a_1x_2 + a_2x_2^2$$

$${}^3\begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} \quad f(g(x))$$

Diferenciación numérica

$$\frac{df}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x) - f(x-h)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2} + \frac{f'''(x)h^3}{6} + \dots$$

$$f(x+h) = f(x) + f'(x)h + \overset{\text{Landau}}{O(h^2)}$$

$$f(x+h) \approx f(x) + f'(x)h$$

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \quad \text{Differenz finit. F.D.}$$

$$f'(5) \approx \frac{f(5+0.1) - f(5)}{0.1} \quad \text{OK!}$$

$$\approx \frac{f(5+4) - f(5)}{4} \quad \text{XXX}$$

$$h \propto \epsilon^{\frac{1}{L}} \quad h \approx 1 \cdot 10^{-3} |x|$$



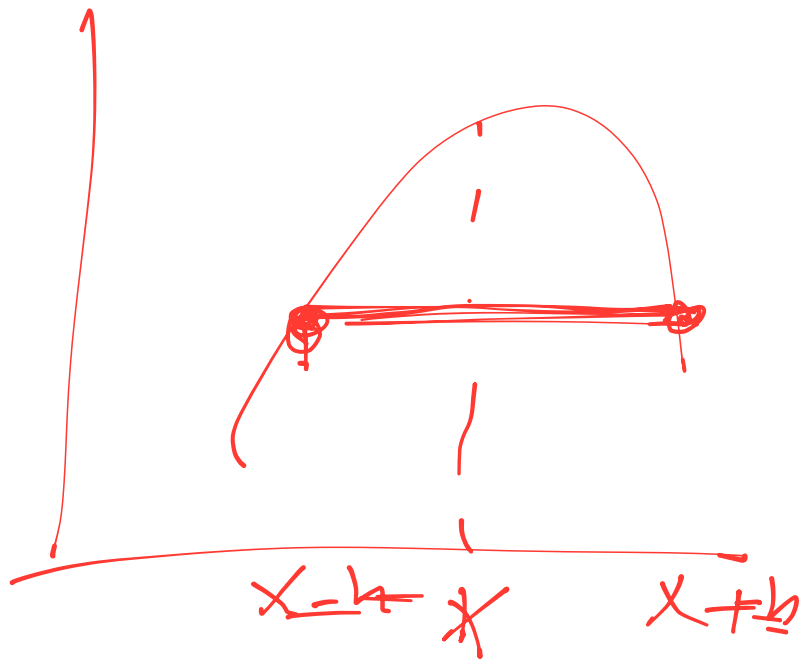
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

$$h = \epsilon \times |x|$$

$$x = 5$$

$$h = 4$$

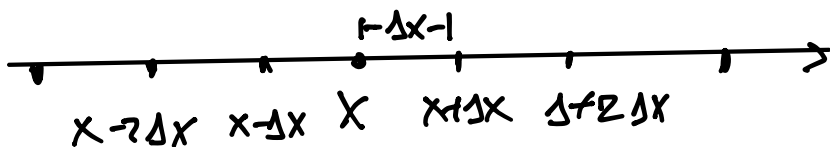
$$h = 1, 0.1$$



$$\frac{dP}{dT} = \frac{P(T+h) - P(T)}{h} \quad h = 0.1$$

$$P'(T=50) \quad P'(50) = \frac{P(\textcircled{50.1}) - P(50)}{0.1}$$

$$f(x), f(x+\Delta x), f(x-\Delta x), f(x+2\Delta x), \dots$$



$$i-2 \quad i-1 \quad i \quad i+1 \quad i+2$$

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \frac{f(i+1) - f(i-1)}{2\Delta x}$$

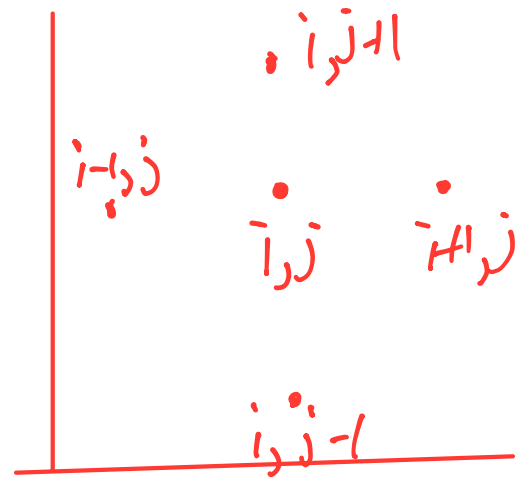
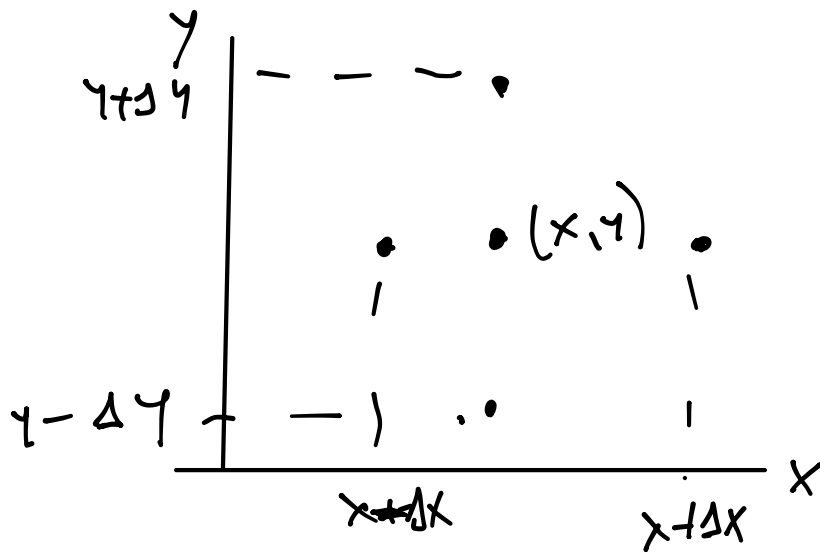
The diagram shows red arrows pointing from the  $f(x+\Delta x)$  and  $f(x-\Delta x)$  terms in the first fraction to the  $f(i+1)$  and  $f(i-1)$  terms in the second fraction. The terms  $f(i+1)$  and  $f(i-1)$  are circled in red.

$$f(x, y) = x^2 + y^2 \quad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial f}{\partial x} = \frac{f(x+\Delta x, y) - f(x-\Delta x, y)}{2\Delta x}$$

$$\frac{\partial f}{\partial y} = \frac{f(x, y+\Delta y) - f(x, y-\Delta y)}{2\Delta y}$$

$$f(x, y) = xy^2 \quad \frac{\partial f}{\partial x} = y^2 \quad \frac{\partial f}{\partial y} = 2xy$$



$$h = x^2 + y^2$$

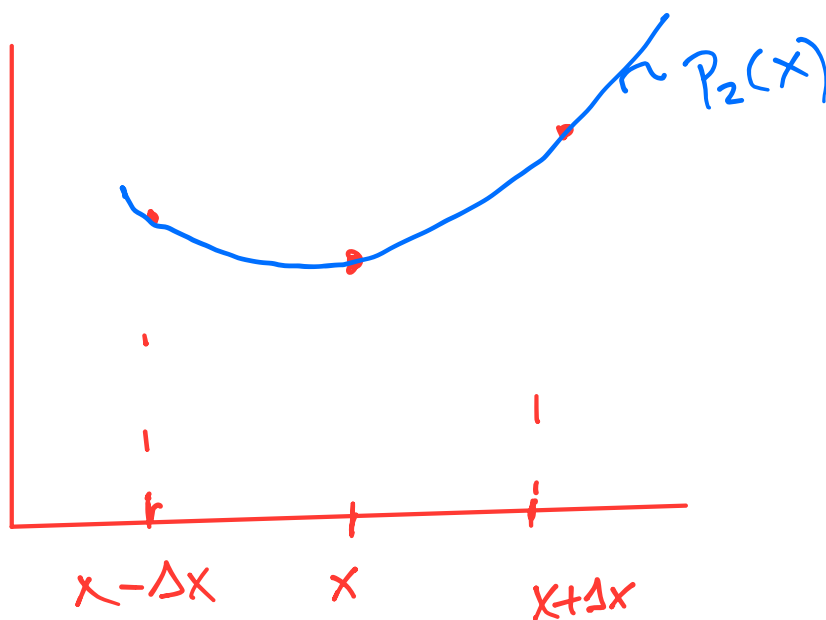
$$x = 1 \quad y = 3$$

$$\frac{\partial h}{\partial x} = 2x$$

$$\frac{\partial h}{\partial y} = 2y$$

$$\frac{\partial h}{\partial x} = 2$$

$$\frac{\partial h}{\partial y} = 6$$

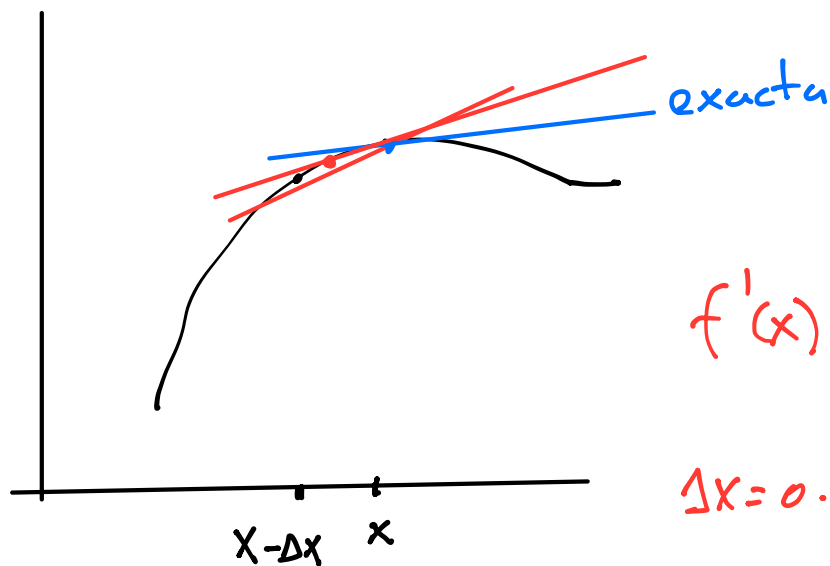
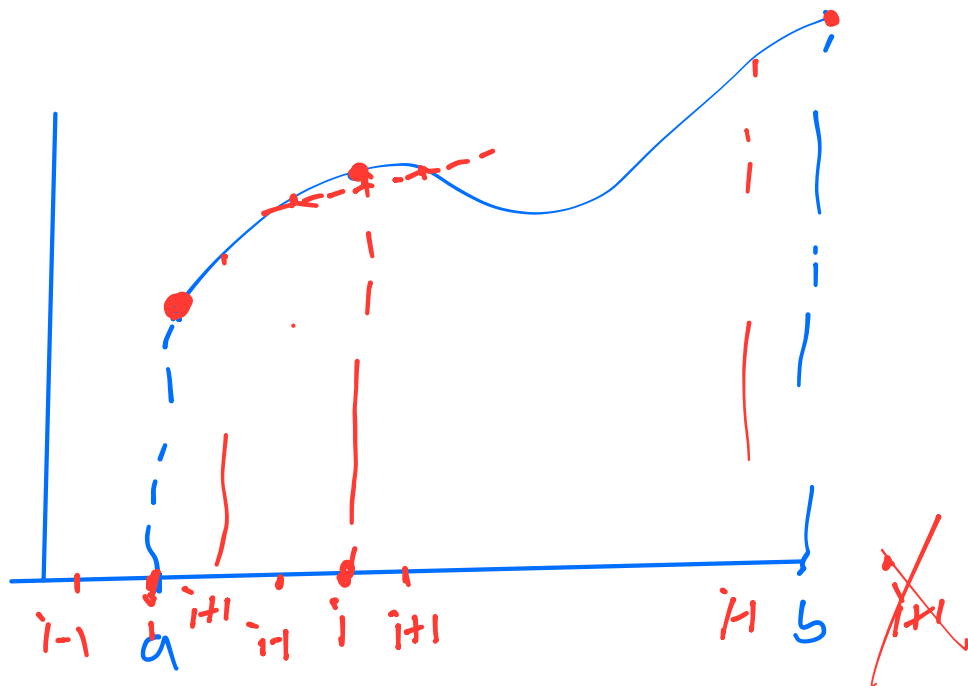


x	y
x - Δx	y <sub>i-1</sub>
x	y <sub>i</sub>
x + Δx	y <sub>i+1</sub>

$$P_2(x) = a_0 + a_1x + a_2x^2$$

1 2 3

$$p_2'(x) = a_1 + 2a_2x$$



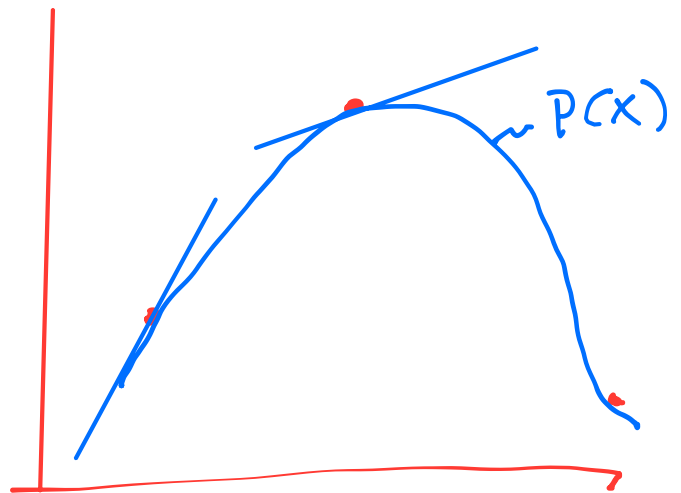
$$f'(x) \approx \frac{f(x) - f(x-\Delta x)}{\Delta x} + O(\Delta x)$$

$\Delta x = 0.1 \quad O(\Delta x) \approx 0.1$

$$f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2)$$

$\Delta x = 0.1 \quad O(\Delta x^2) = \underline{0.01}$

$x$	$f(x)$
$x - \Delta x$	$f(x - \Delta x)$
$x$	$f(x)$
$x + \Delta x$	$f(x + \Delta x)$



$$P_2(x) = a_0 + a_1 x + a_2 x^2$$

$$f'(x) \approx P_2'(x)$$

$$P_2'(x) = a_1 + 2a_2 x$$

$$P_2'(x - \Delta x) = a_1 + 2a_2(x - \Delta x)$$

$$f(x, y) = x^2 + y^2$$

$$f_x = \frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\begin{aligned} x &= y \\ y &= 3 \end{aligned}$$

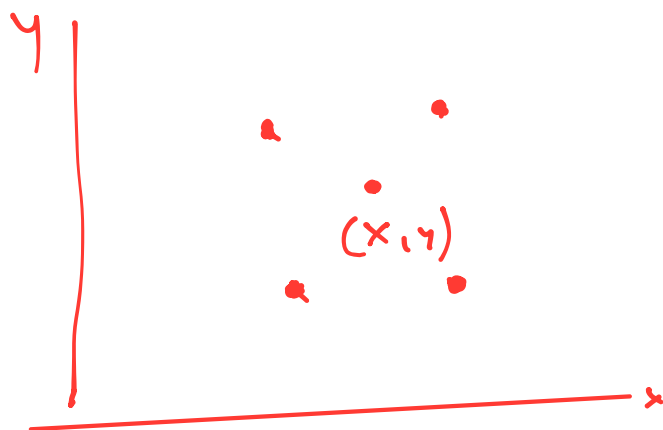
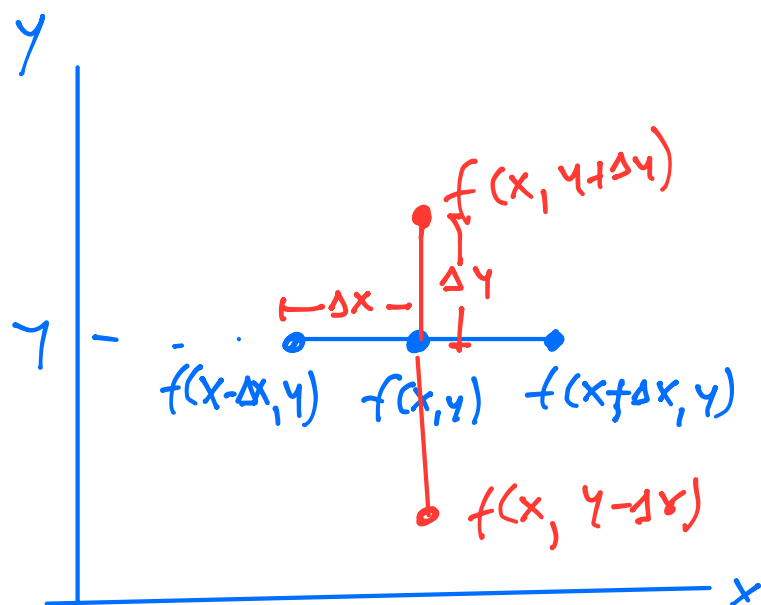
$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$f_{,xxxxy}$$





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$$P(x) = 0.0375x^5 - 0.525x^4 + 2.6625x^3 - 4.775x^2 + 4.5x + 0$$

$$P'(x) = 5 \times 0.0375x^4 - 4 \times 0.525x^3 + 3 \times 2.6625x^2 - 2 \times 4.775x + 4.5$$

$t$	$x(t)$
$t_1$	$x_1$
$t_2$	$x_2$
$\vdots$	
$t_n$	$x_n$

$$v = \frac{dx}{dt}$$

