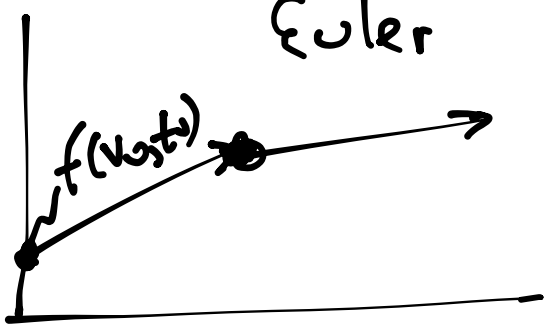
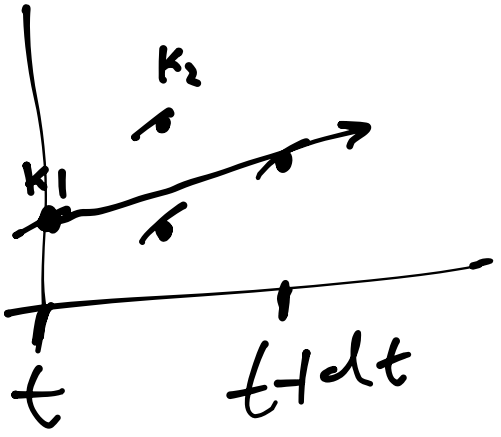
$O(h)$ $h \sim 0.1$ $O \sim 0.1$

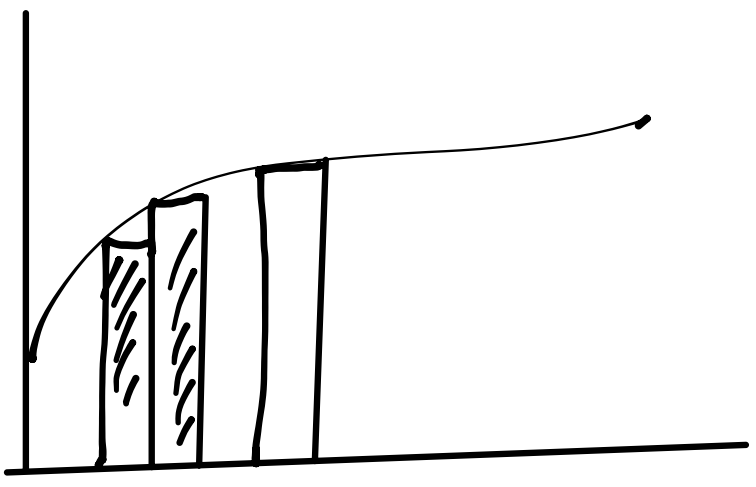
$O(h^2)$ $h \sim 0.1$ $O \sim 0.01$

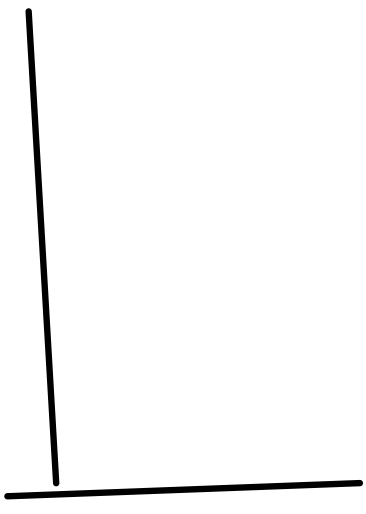
$O(h^3)$ $h \sim 0.1$ $O \sim 0.001$

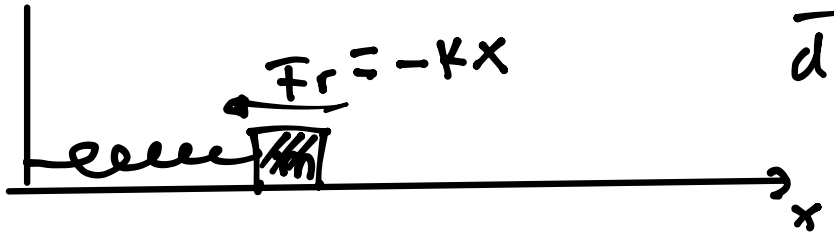
$$\frac{dy}{dt} = f(y,t)$$








$$m\frac{d^2x}{dt^2} = -kx$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\begin{cases} t=0 \quad x(0)=x_i \\ \qquad v(0)=v_i \end{cases}$$

$$v = \frac{dx}{dt}$$

$$m\frac{dv}{dt} + kx = 0$$

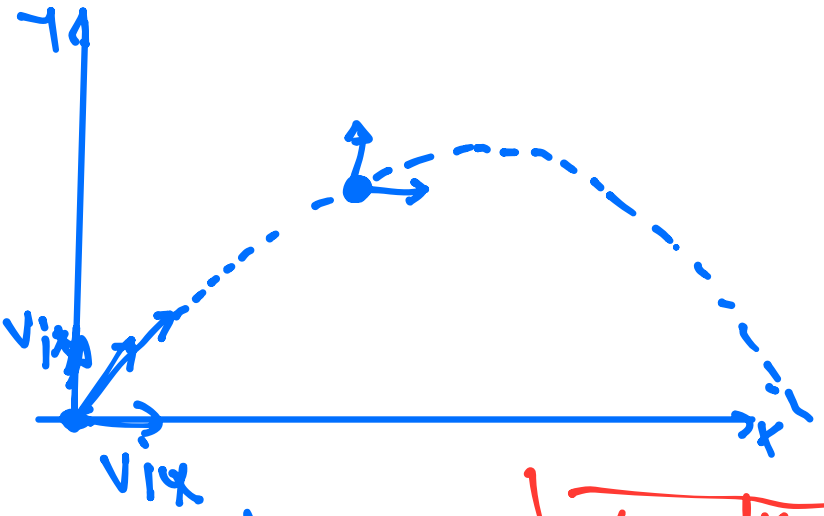
$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = -\frac{k}{m}x \end{cases}$$

$$Y = \begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} Y_0 \\ Y_1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dv}{dt} \end{pmatrix} = \begin{pmatrix} v \\ -\frac{k}{m}x \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ -\frac{k}{m}x \end{pmatrix}$$

$$\begin{cases} \frac{dY}{dt} = \begin{bmatrix} Y_1 \\ -\frac{k}{m}Y_0 \end{bmatrix} = \vec{F}(\vec{Y}) \\ \vec{Y}_0 = \begin{pmatrix} x_i \\ v_i \end{pmatrix} \end{cases}$$




$$F_x \rightarrow \quad m\frac{d^2x}{dt^2} = 0$$

$$F_x \rightarrow \quad m\frac{d^2y}{dt^2} = -my$$

$$V_x = \frac{dx}{dt} \qquad m\frac{dV_x}{dt} = 0$$

$$V_y = \frac{dy}{dt} \qquad m\frac{dV_y}{dt} = -mg$$

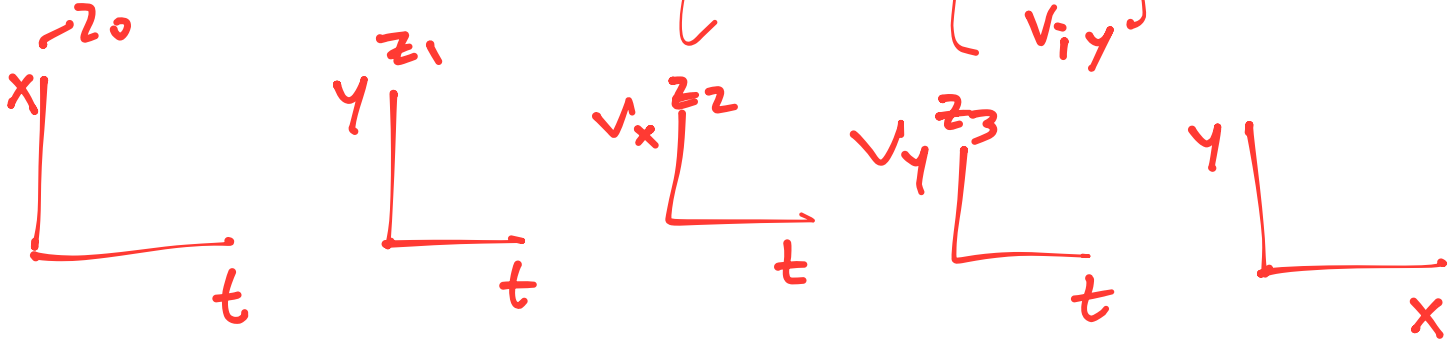
$$X_0 = 0$$
$$Y_0 = 0$$
$$V_{x0} = V_{ix}$$
$$V_{y0} = V_{iy}$$

$$\begin{cases} \frac{dx}{dt} = V_x & \frac{dz_0}{dt} = z_2 \\ \frac{dy}{dt} = V_y & \frac{dz_1}{dt} = z_3 \\ \frac{dV_x}{dt} = 0 & \frac{dz_2}{dt} = 0 \\ \frac{dV_y}{dt} = -g & \frac{dz_3}{dt} = -g \end{cases}$$

$$\vec{z} = \begin{Bmatrix} x \\ y \\ V_x \\ V_y \end{Bmatrix} = \begin{Bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{Bmatrix}$$

$$\frac{d}{dt}\{z\} = \begin{Bmatrix} z_2 \\ z_3 \\ 0 \\ -g \end{Bmatrix} = \vec{F}(\vec{z})$$

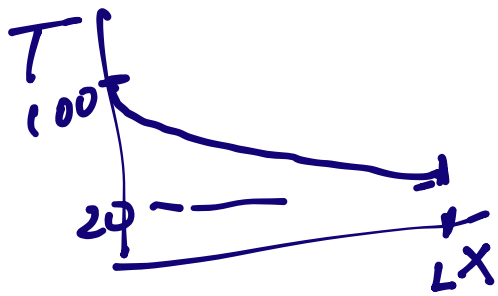
$$\vec{z}_0 = \begin{Bmatrix} 0 \\ 0 \\ V_{ix} \\ V_{iy} \end{Bmatrix}$$




$$\frac{d^2T}{dx^2} - 0.1(T - T_m) = 0 \qquad T_m = 20$$

$x = 0 \quad T(0) = 100^\circ C$

$x = L = 0.20 \quad T(L) = 20^\circ C$




$$q = \frac{dT}{dx} \qquad \frac{dq}{dx} - 0.1(T - 20) = 0$$

$$\frac{dT}{dx} = q$$

$$\frac{dq}{dx} = 0.1(T - 20)$$

$x = 0 \quad T(0) = 100$

$q(0) = ?$

shooting

$$\theta = \begin{bmatrix} T \\ q \end{bmatrix} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \quad \begin{bmatrix} \frac{d\theta_0}{dx} = \theta_1 \\ \frac{d\theta_1}{dx} = 0.1(\theta_0 - 20) \end{bmatrix} = \dot{F}$$

$$\frac{d}{dx}\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ 0.1(\theta_0 - 20) \end{bmatrix} = \vec{F}(\vec{\theta})$$

$$X=0 \quad \theta_{ini} = \begin{bmatrix} 100 \\ q_{ini} \end{bmatrix}$$

$q_i = 1 \qquad T(L) = 100.8$

$q = -10 \qquad T(L) = 98$