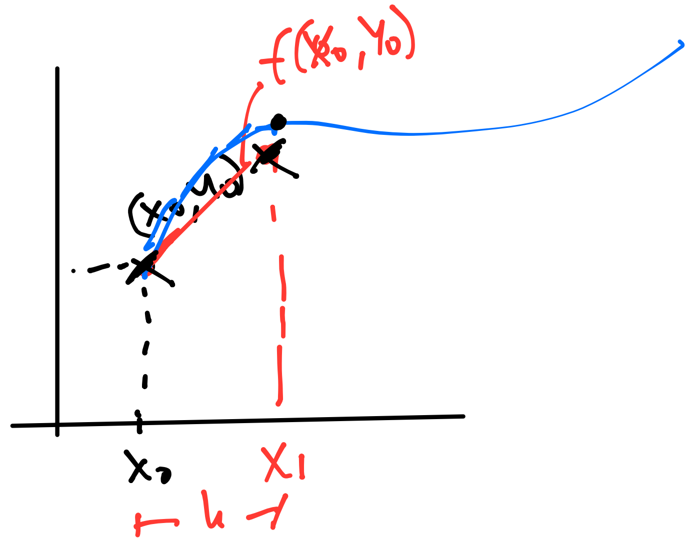
$$\frac{dy}{dx} = f(x,y)$$

$x = x_0 \qquad y(x_0) = y_0$



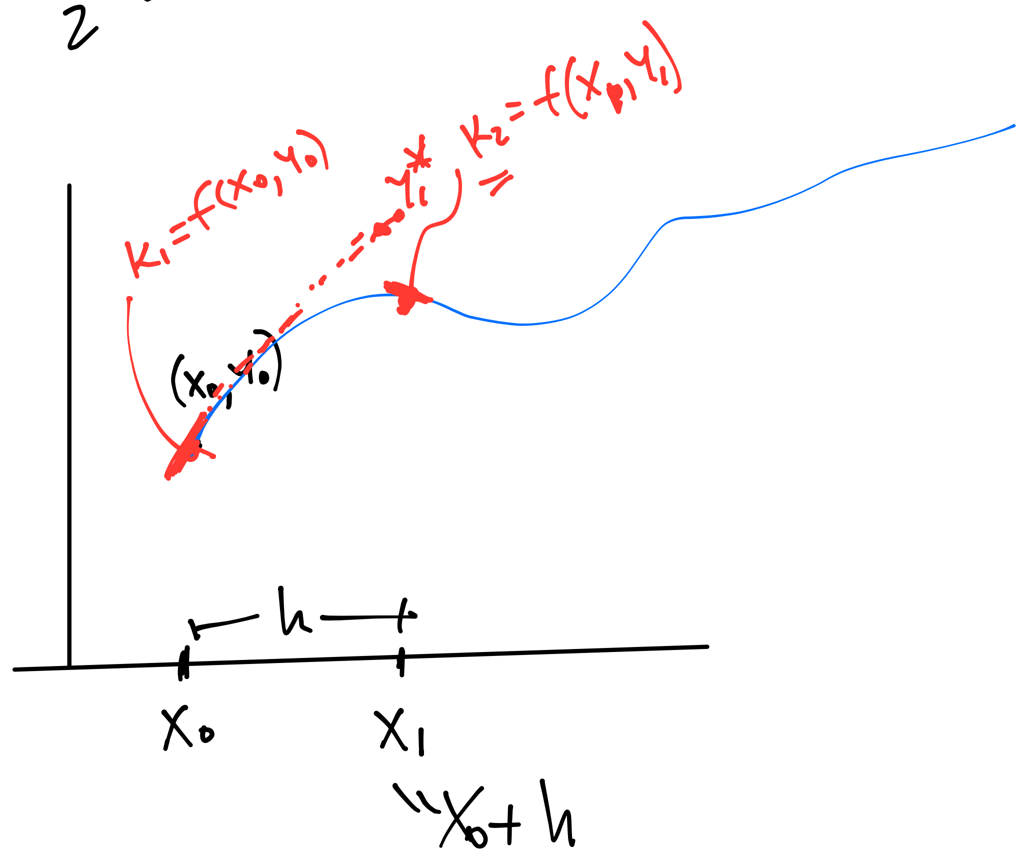
$$K_1 = f(x_0, y_0)$$

$$K_2 = f(x_0+h, y_1^*) = f(x_0+h, y_0+hK_1)$$

$$y_1^* \approx y_0 + hf(x_0,y_0)$$

$$y_1 = y_0 + h\frac{1}{2}(K_1+K_2)$$

$$y_1^* = y_0 + hK_1$$



$$\frac{dy}{dx} = -y$$

$$y(0) = 1$$

$$y_1 = y_0 + h\frac{1}{2}\left(f(x_0, y_0) + f(x_0+h, y_1)\right)$$

Runge Kutta

Runge-Kutta method (RK2) which is summarized as follows.

$$\begin{cases} k_1 = hf(y_n, t_n) \\ k_2 = hf(y_n + k_1, t_n + h) \end{cases}$$

$y_{n+1} = y_n + (k_1 + k_2)/2,$ Second Order Runge-Kutta Method (

$O(h^2)$

$h = 0.1 \quad \varepsilon \propto 0.1^2 = 0.01$

$h = 0.5 \quad \varepsilon \propto 0.5^2 = 0.25$

$k_1 = hf(y_n, t_n)$

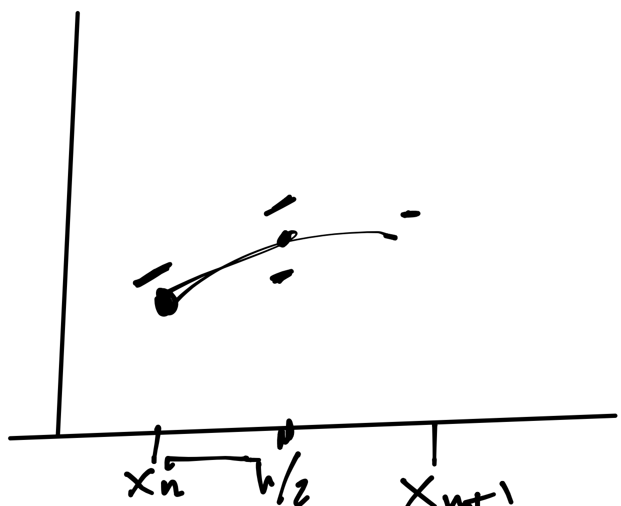
$k_2 = hf(y_n + k_1/2, t_n + h/2)$

$k_3 = hf(y_n + k_2/2, t_n + h/2)$ Fourth Order Runge-Kutta Method (RK4).

$k_4 = h(y_n + k_3, t_n + h)$

$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)/6.$

$h = 0.1 \quad \varepsilon \propto 0.1^4$



h

$$\frac{dy}{dx} = f(x, y)$$

$$y'' + 3y' - y = 10$$

$$y(0) = a$$

$$y'(0) = b$$

$$\begin{cases} u = y' \\ u' + 3u - y = 10 \end{cases}$$

$$y' = u$$

$$u' = 10 - 3u + y$$

$$y''' + 2y'' - y' + 5y = 1$$

$y(0) = a$ $\qquad$ $u = y'$

$y'(0) = b$ $\qquad$ $w = y'' = u'$

$y''(0) = c$ $\qquad$ $w' + 2w - u + 5y = 1$

$$\begin{cases} y' = u \\ u' = w \\ w' = 1 - 2w + u - 5y \end{cases}$$

$$\vec{X} = \begin{pmatrix} y \\ u \\ w \end{pmatrix} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$$

$y(0)=a$
$v(0)=b$
$w(0)=c$

$$\begin{pmatrix} y \\ v \\ w \end{pmatrix}' = \begin{pmatrix} v \\ w \\ 1-2w+v-5y \end{pmatrix}$$

$$\vec{X}' = \begin{pmatrix} x_1 \\ x_2 \\ 1-2x_2+x_1-5x_0 \end{pmatrix}$$

$$\vec{X}_0 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$x'' + 0.1x' + x = 0$$

$x(0)=a$

$x'(0)=b$

$x' = v$

$v' + 0.1v + x = 0$

$x' = v$

$v' = -0.1v - x$

$$x'' + x = 0$$

$$x'' = -x$$

$$mx'' = -kx$$

$$x'' = -\frac{k}{m}x$$

$$\vec{z} = \begin{pmatrix} x \\ v \end{pmatrix} \begin{matrix} z_0 \\ z_1 \end{matrix}$$

$$z' = \begin{pmatrix} z_1 \\ -0.1\, z_1 - z_0 \end{pmatrix}$$

$$z_0 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$m x'' = -kx$$

$t=0 \quad x(0) = x_{ini}$

$x'(0) = V_{ini}$

$m = 0.01$

$K = 0.025$

$x_{in} = 0.10$ ✓

$V_{in} = 0$

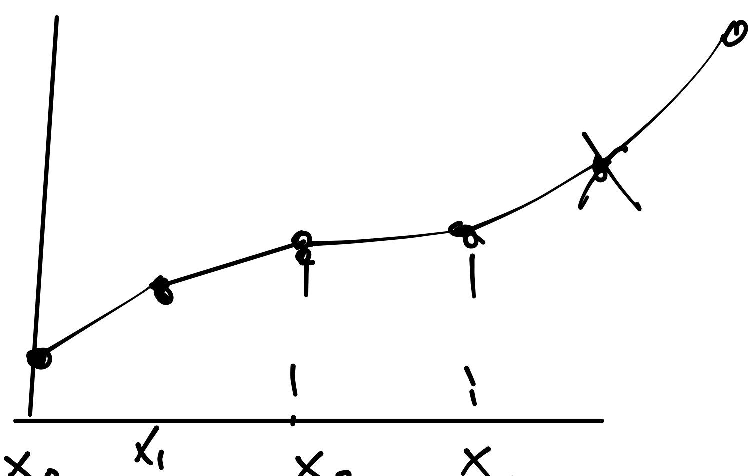
$$u = x'$$
$$m u' = -kx$$

$$x' = u$$
$$u' = -\frac{k}{m} x$$

$$x(0) = x_{ini}$$
$$u(0) = v_{ini}$$

$$z = \begin{pmatrix} x \\ u \end{pmatrix} = \begin{pmatrix} z_0 \\ z_1 \end{pmatrix}$$

$$\begin{cases} z' = \begin{pmatrix} u \\ -\frac{k}{m} x \end{pmatrix} = \begin{pmatrix} z_1 \\ -\frac{k}{m} z_0 \end{pmatrix} \\ z_0 = \begin{pmatrix} x_{ini} \\ v_{ini} \end{pmatrix} \end{cases}$$



x_3 x_2 x_1