$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & -1 \\ 4 & 2 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Ax = B \qquad x = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$$

$$\begin{array}{c} F_0 \rightarrow \\ F_1 \\ F_2 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 5 & -1 & 1 \\ 4 & 2 & 7 & 1 \end{array}\right)$$

$$\begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix} = U$$

$$\begin{pmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{pmatrix} = L$$

$$F_1 + (-2) \cdot F_0 \rightarrow F_1$$

$$\cancel{F_2 + (-4) F_0} \rightarrow F_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & -6 & -5 & -3 \end{array}\right)$$

$$F_2 + 6 \cdot F_1 \rightarrow F_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & -47 & -9 \end{array}\right) \uparrow$$

$$x_0 + 2x_1 + 3x_2 = 1$$
$$x_1 - 7x_2 = -1$$
$$-47x_2 = -9$$

Backsubstitution $x_2 = \frac{-9}{-47}$

$$x_1 = -1 + 7x_2$$

$$x_0 = 1 - 2x_1 - 3x_2$$

Factorización LU

$$A = \begin{pmatrix} \alpha_{00} & 0 & 0 \\ \alpha_{10} & \alpha_{11} & 0 \\ \alpha_{20} & \alpha_{21} & \alpha_{22} \end{pmatrix} \cdot \begin{pmatrix} \beta_{00} & \beta_{01} & \beta_{02} \\ 0 & \beta_{11} & \beta_{12} \\ 0 & 0 & \beta_{22} \end{pmatrix}$$

$A = L \cdot U$ $\qquad \alpha_{00} = \alpha_{11} = \alpha_{22} = 1$

$A_{00} = \overset{=1}{\alpha_{00}} \beta_{00}$ $\qquad A_{01} = \overset{=1}{\alpha_{00}} \beta_{01}$ $\qquad A_{02} = \overset{=1}{\alpha_{00}} \beta_{02}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & -1 \\ 4 & 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \overset{=2}{\alpha_{10}} & 1 & 0 \\ \alpha_{20} & \alpha_{21} & 1 \end{pmatrix} \begin{pmatrix} \overset{=1}{\beta_{00}} & \overset{=2}{\beta_{01}} & \overset{=3}{\beta_{02}} \\ 0 & \overset{=1}{\beta_{11}} & \overset{=-7}{\beta_{12}} \\ 0 & 0 & \beta_{22} \end{pmatrix}$$

$1 = \beta_{00}$ $\qquad 2 = \beta_{01}$ $\qquad 3 = \beta_{02}$

$2 = \alpha_{10} \overset{=1}{\beta_{00}}$

$5 = \alpha_{10}\beta_{01} + \beta_{11}$

$5 = 2 \cdot 2 + \beta_{11}$

$\beta_{11} = 1$

$-1 = \alpha_{10}\beta_{02} + \beta_{12}$

$-1 = 2 \cdot 3 + \beta_{12}$

Algoritmo de Crout

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 2 & -1 & 7 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 7 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$PA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 2 & -1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 7 \end{pmatrix}$$

$Ax = B$ $\qquad L(Ux) = B$

$Ly = B$

$Ux = y$

$$\begin{array}{c} L \\ \begin{pmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{pmatrix} \end{array} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$$

$$\begin{array}{c} U \\ \begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix} \end{array} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}$$

$Ax = B$

$X = A^{-1}B \quad \equiv \quad \underline{\underline{Ax = B}}$ LU

$A X_{exacta} - B = \vec{0}$

$A x_{num} - B = \vec{e}$

$$\|\vec{e}\| = \|AX_{min} - B\|$$