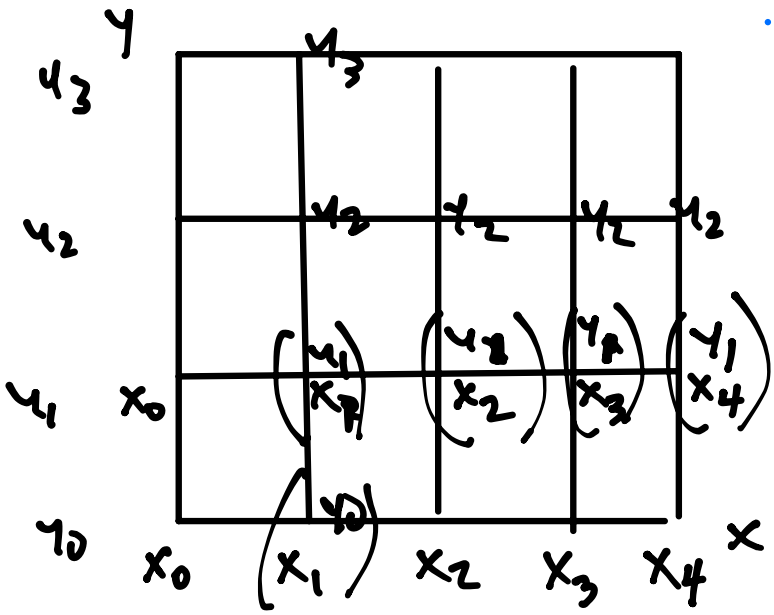
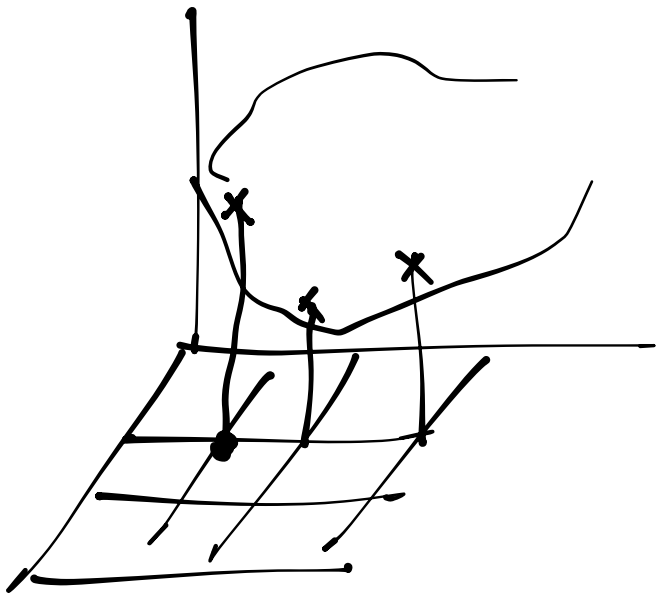
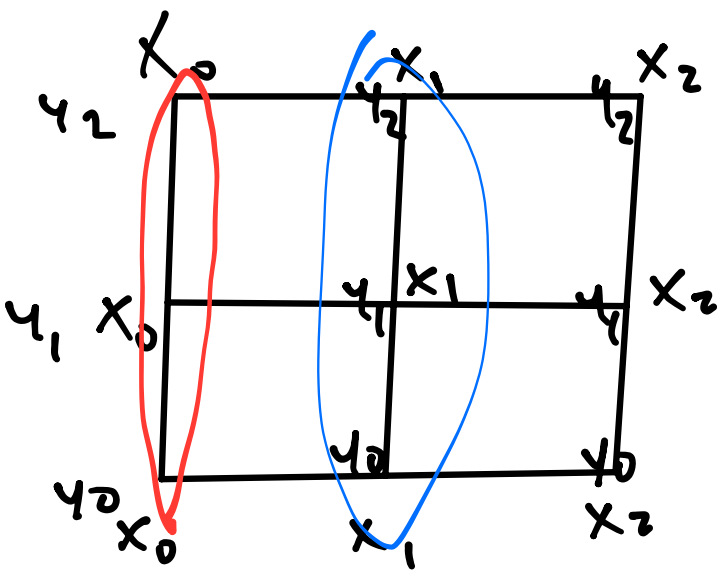






$$X = \begin{bmatrix} x_0 & x_1 & x_2 \\ x_0 & x_1 & x_2 \\ x_0 & x_1 & x_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_0 & y_0 & y_0 \\ y_1 & y_1 & y_1 \\ y_2 & y_2 & y_2 \end{bmatrix}$$

(x_0, y_0) (x_0, y_1) (x_0, y_2)

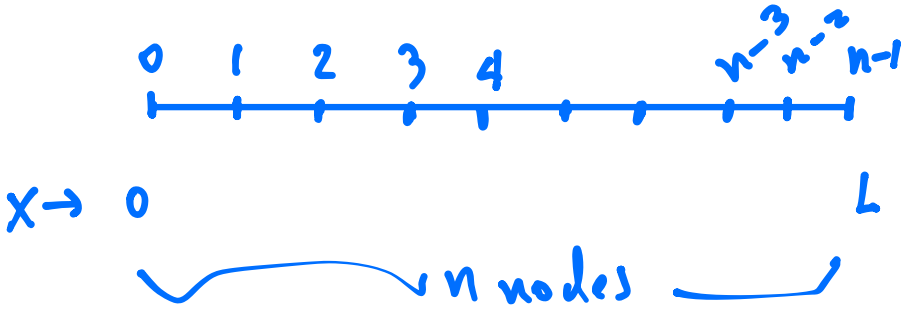
(x_1, y_0) (x_1, y_1) (x_1, y_2)

$$\frac{d^2T}{dx^2} = -f(x)$$

Finite differences
F-D

$x = 0 \quad T(0) = T_a$

$x = L \quad T(L) = T_b$



$$\Delta x = \frac{L}{n-1}$$

$$\left.\frac{dT}{dx}\right|_x \approx \frac{T(x+\Delta x) - T(x)}{\Delta x} = \frac{\Delta T}{\Delta x} \lim_{\Delta x \to 0} \to f'$$

(T_{i+1}, T_i)

$$\left.\frac{dT}{dx}\right|_x \quad \frac{T(x) - T(x-\Delta x)}{\Delta x}$$

(T_i, T_{i-1})

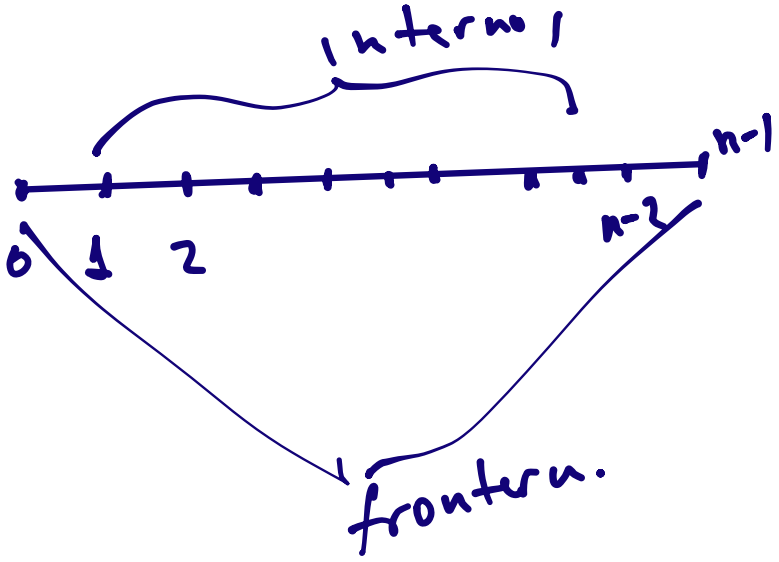
$$\left.\frac{dT}{dx}\right|_x \approx \frac{T(x+\Delta x) - T(x-\Delta x)}{2\Delta x}$$

(T_{i+1}, T_{i-1})

$$\left.\frac{d^2T}{dx^2}\right|_x \approx \frac{T(x+\Delta x) - 2T(x) + T(x-\Delta x)}{\Delta x^2}$$

($T(x+\Delta x) = T_{i+1}$, $T(x) = T_i$, $T(x-\Delta x) = T_{i-1}$)

$$\left.\frac{d^2T}{dx^2}\right|_{x_i} = -f(x_i)$$



$$\begin{cases} \dfrac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} = -f(x_i) & i = 1, 2, 3 \dots, n-2 \\ 1\,T_0 = T_a & \\ 1\,T_{n-1} = T_b & \end{cases}$$

$i = 1 \quad \dfrac{T_2 - 2T_1 + T_0}{\Delta x^2} = -f(x_1)$

$i = 2 \quad \dfrac{T_3 - 2T_2 + T_1}{\Delta x^2} = -f(x_2)$

$i = 3 \quad \dfrac{T_4 - 2T_3 + T_2}{\Delta x^2} = -f(x_3)$

$\vdots$

$$T_0 = T_a$$

$$T_{n-1} = T_b$$

$$\begin{matrix} & 0 \\ 0 & \\ 1 & \\ 2 & \\ & \\ n-1 & \end{matrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\Delta x^2} & \frac{-2}{\Delta x^2} & \frac{1}{\Delta x^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\Delta x^2} & \frac{-2}{\Delta x^2} & \frac{1}{\Delta x^2} & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ \\ T_{n-1} \end{bmatrix} = \begin{bmatrix} T_a \\ -f(x_1) \\ -f(x_2) \\ \vdots \\ T_b \end{bmatrix}$$

$$\underset{n \times n}{[A]}\{T\} = \{B\}$$