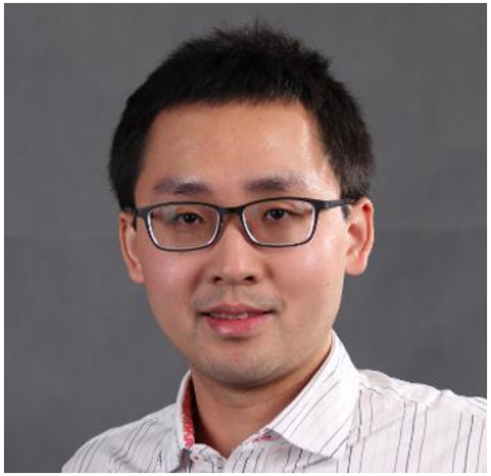
# Signals and Systems

**Department of Electrical & Electronic Engineering**

**Southern University of Science and Technology**

**2022 Spring**



## WANG Rui

- USTC - CSE – BEng (2000-2004)
- HKUST - ECE – PhD (2004-2009)
- Huawei - Senior Research Engineer (2009-2012)
- SUSTech – EEE - Associate Professor

## Research Interests:

- Wireless communications: 5G/B5G/6G, VLC, mmWave and etc.
- Integrated sensing, computing and communications
- Stochastic optimization, Reinforcement learning, convex optimization and etc.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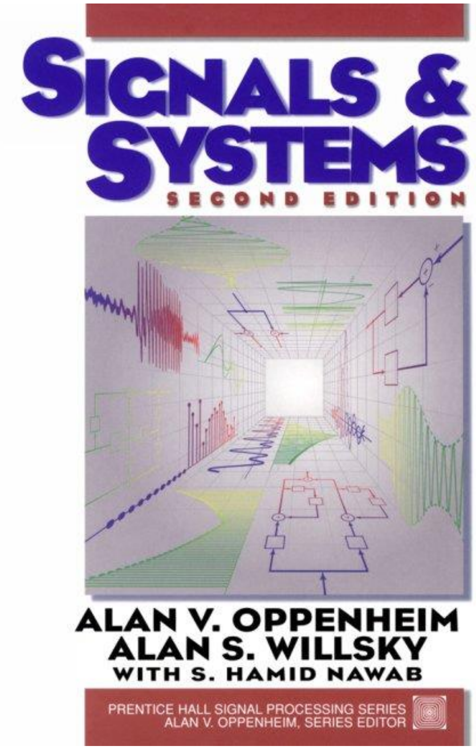
- **Email: wang.r@sustech.edu.cn**
- **Website: http://eee.sustc.edu.cn/p/wangrui/**


# Scope of Lecture

- "Signals and Systems", Oppenheim, Willsky and Nawab, 2nd Edition, 1997, Prentice-Hall.




- This course teaches Chapters 1 to 8.
  - Roughly two weeks for one chapter
  - Middle-term exam for Chapters 1 to 4
  - Final exam for all

**Textbook reading is crucial, as I cannot cover every detail in slides**

# Three Pillars

**Lectures (Tutorial)**

**Matlab Labs**

**YOU**

**100%**

***Assignment/Quiz 20%***

***Mid-term Exam 25%***

***Final Exam 25%***

***Lab Reports***

***Project Report & Presentation***

***30%***

# Class Schedules

- Lab Session – Starts from the first week
- Instructor: WU Guang (吴光) & WANG Xiaojin (王小静)

- Tutorials – Please negotiate with TAs
- Every week (except week 1&2)

- Assignment: Every week (except week 1&2)
- Submit assignment in softcopy to Blackborad system
- Deadline: Next Tuesday, 12pm.

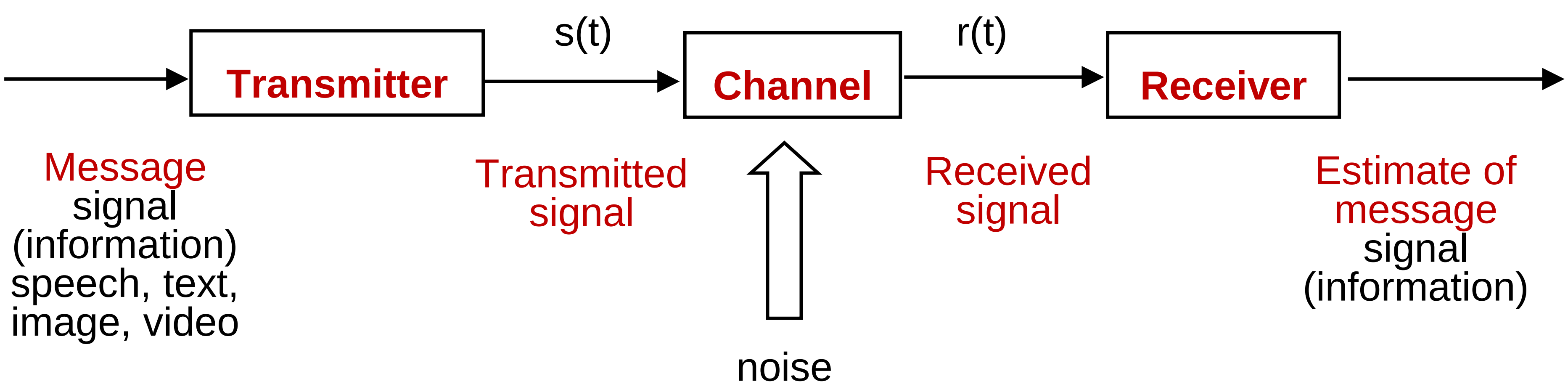
# Signals and Systems

- **Signals:** everything which carries information
- **Systems:** everything which processes input signal and generate output signal

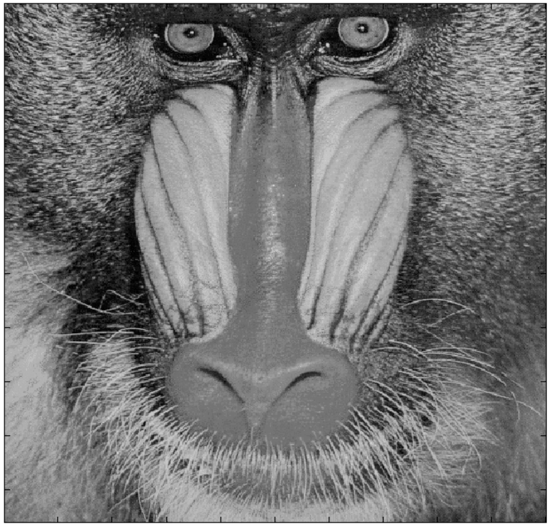
# Communication Signals & Systems

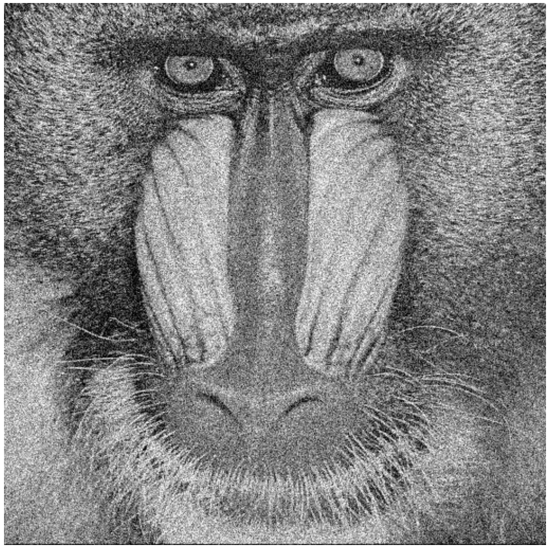
Can you find any example of signals and systems when making a phone call?

- Transmitter, channel and receiver are all systems.
- Each system has one input signal and one output signal.



# Image Processing





# More examples of signals

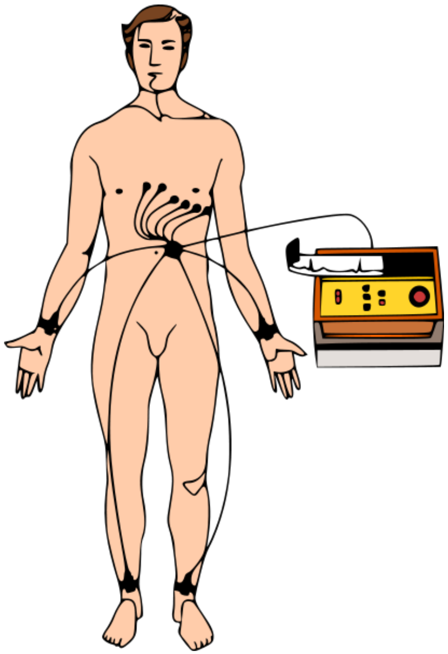
- Electrical signals – voltages and currents in a circuit
- Acoustic signals – audio or speech signals
- Video signals – movie
- Biological signals – sequence of bases in a gene
- We will treat noise as unwanted signals.

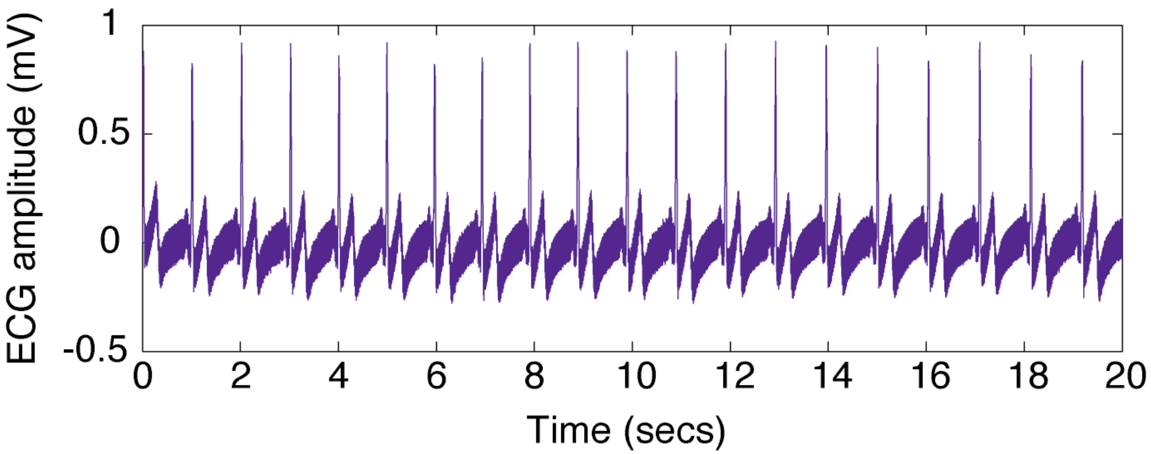
# Signals and Systems from Our Point of View

- Signals are variables that carry information, like function.
- Systems process input signals to produce output signals.
- The course is about using mathematical techniques to analyze and synthesize systems which process signals.

# Independent Variable of Signals

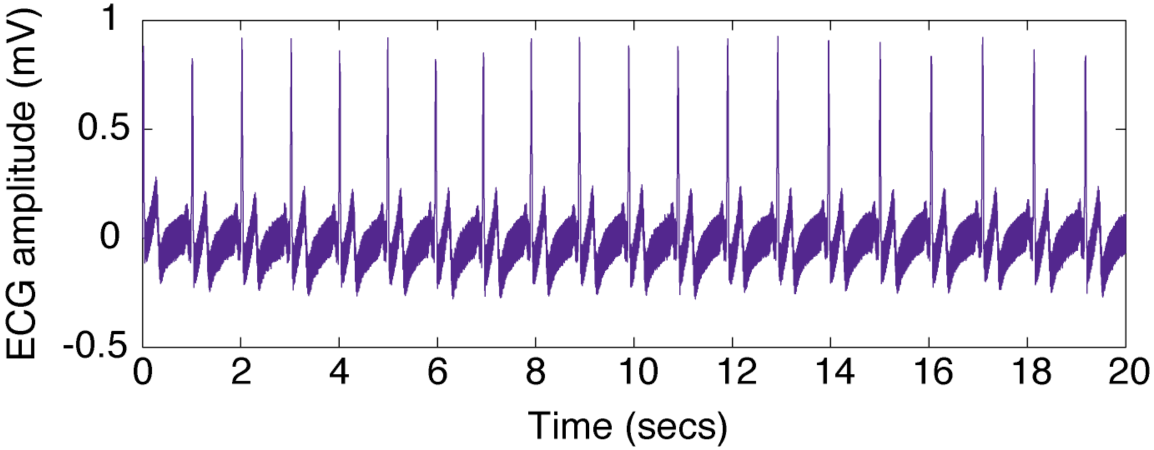
- Time is often the independent variable.
- Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG).

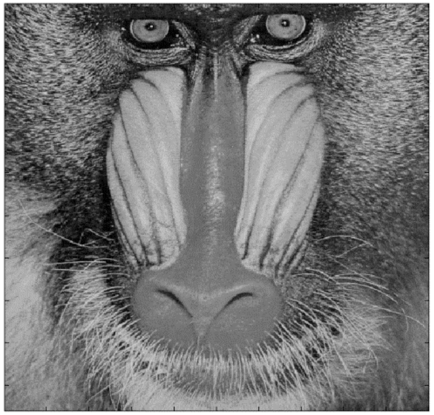




# Signal Classification 1: Dimension of Independent Variable

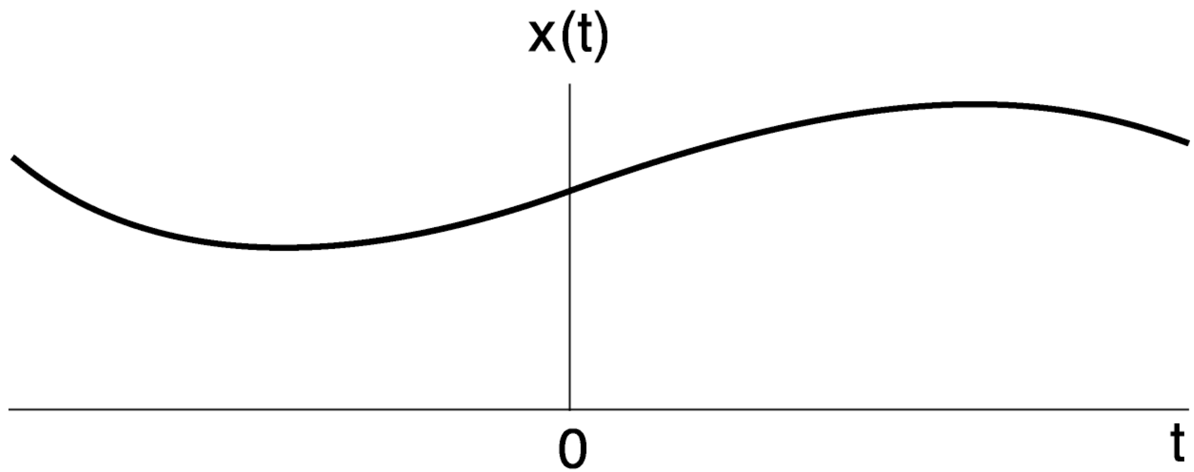
- An independent variable can be 1-D (*t* in the ECG), 2-D (*x*, *y* in an image), or 3-D (*x*, *y*, *t* in an video).



- We focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions.

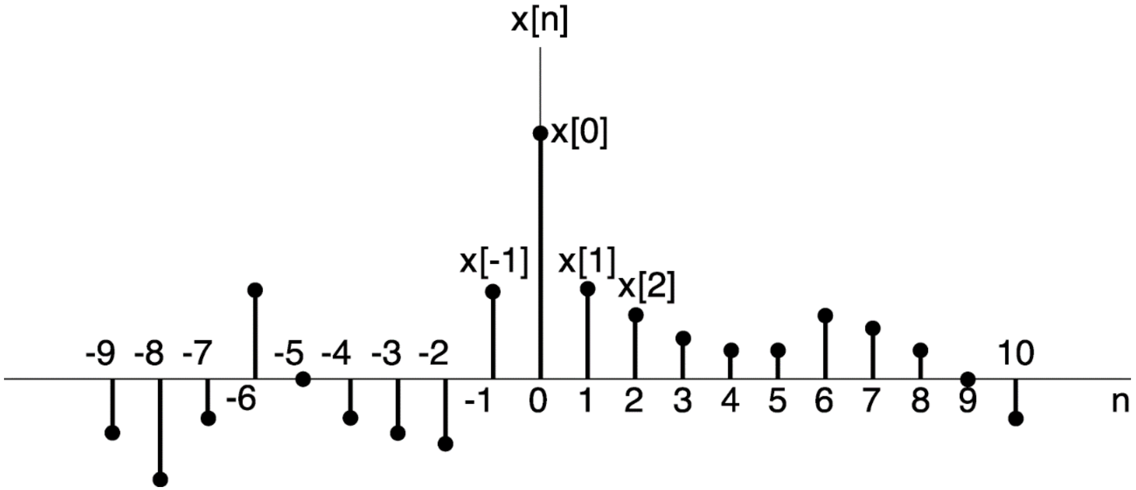
# Signal Classification 2: Continuous-time (CT) Signals




- Independent variable is continuous
- Most of the signals in the physical world are CT signals.
- E.g. voltage & current, pressure, temperature, velocity, etc.

Notation: x(t)

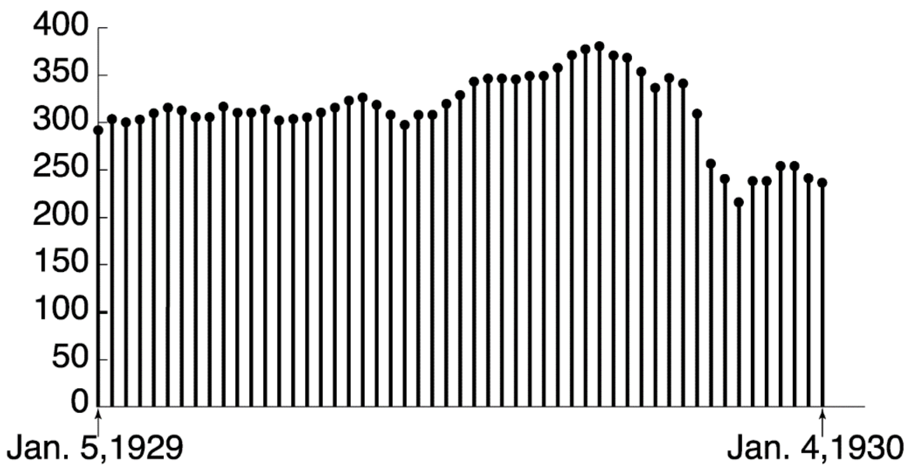
# Discrete-time (DT) Signals




- Independent variable is integer
- Examples of DT signals: DNA sequence, population of the *n*-th generation of certain species

> Notation: $x[n]$

# Many Human-made Signals are DT

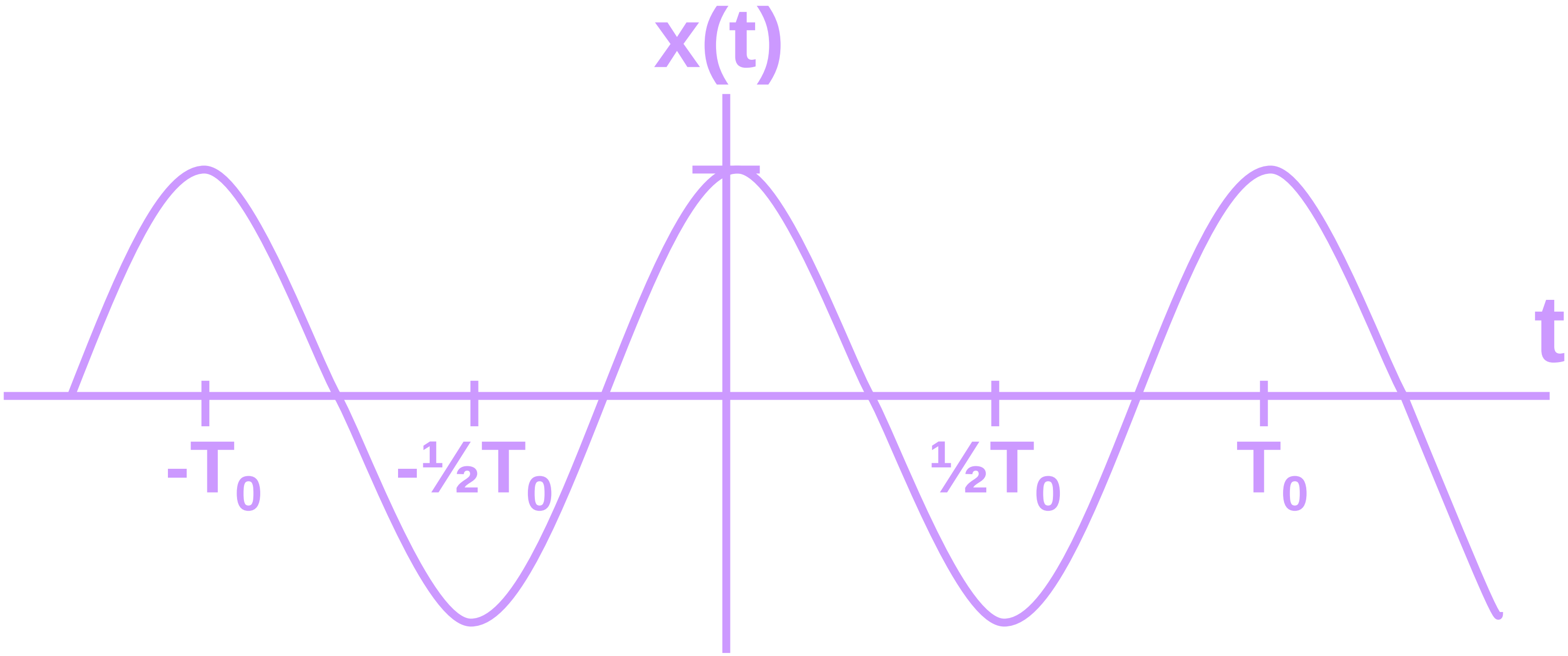



*Weekly Dow-Jones industrial average*

*Digital image*

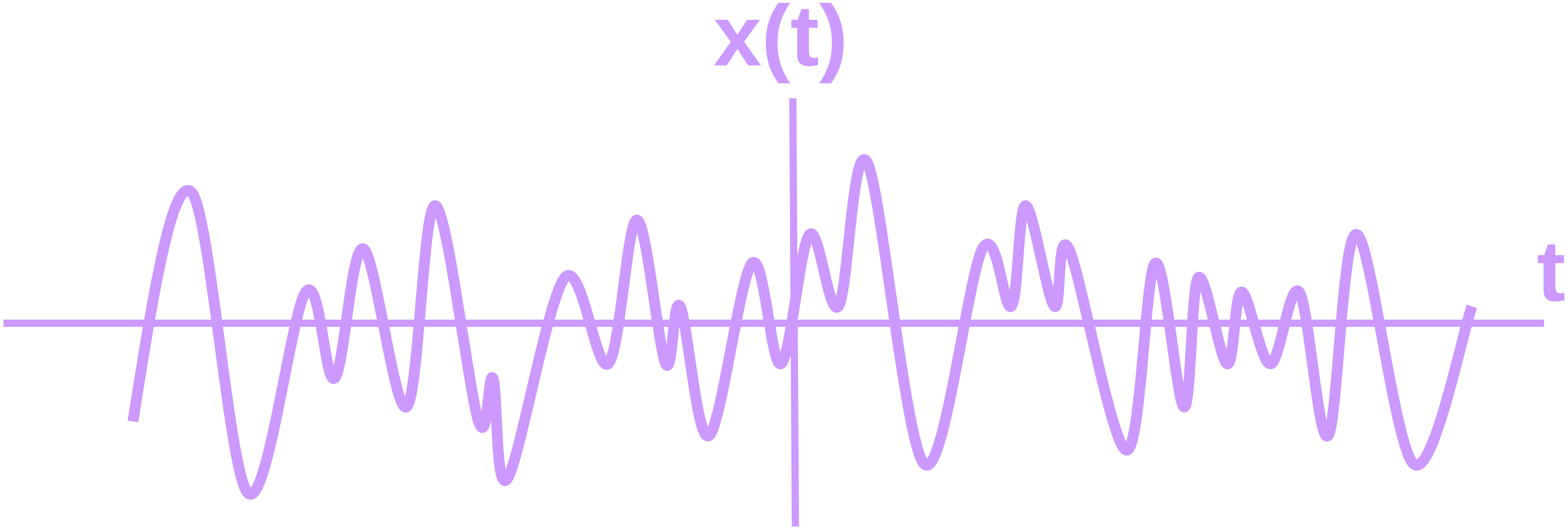
- Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

# Signal Classification 3: Deterministic Signal




- Each value of the signal is fixed, and can be determined by a mathematical expression, rule, or table.

# Signal Classification 3: Random Signal




- Signal value at any time instance is a random variable.

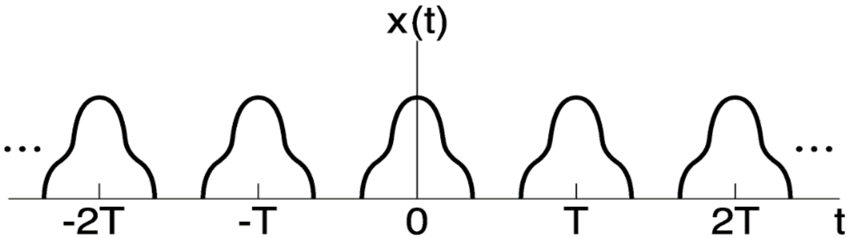
# Signal Classification 4: Periodic / Aperiodic

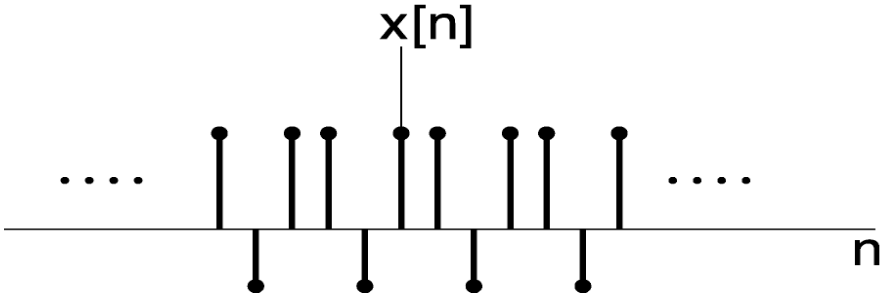
- Periodic Signals

  CT: $x(t) = x(t + T)$, $T$ : *period*

  $x(t) = x(t + mT)$, *m: integer*

  DT: $x[n] = x[n + N] = x[n + mN]$, N: period



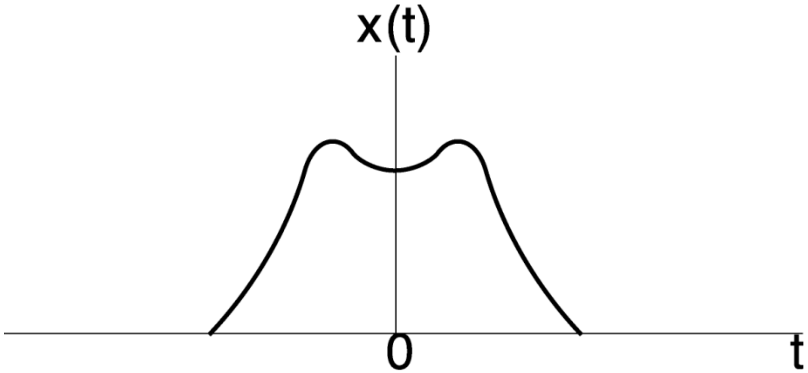

- Fundamental period: the smallest positive period

- Aperiodic: NOT periodic

# Signal Classification 5: Even / Odd

- Even and Odd Signals

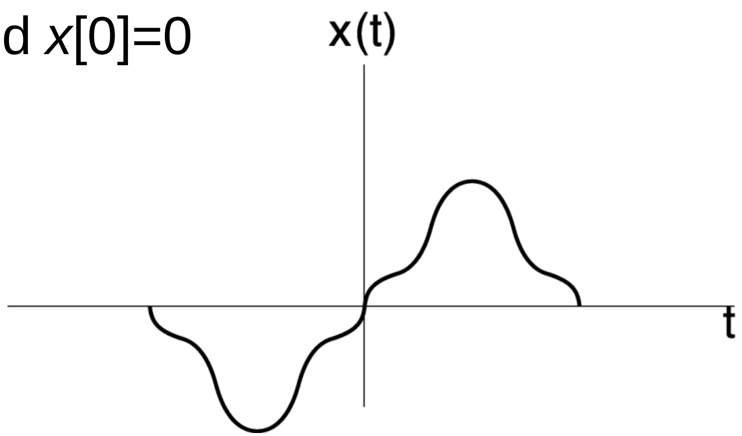
Even $x(t) = x(-t)$ *or* $x[n] = x[-n]$




Example: cos(t)

Odd $x(t) = -\,x(-t)$ *or* $x[n] = -\,x[-n]$

$x(0)=0$, and $x[0]=0$




Example: sin(t)

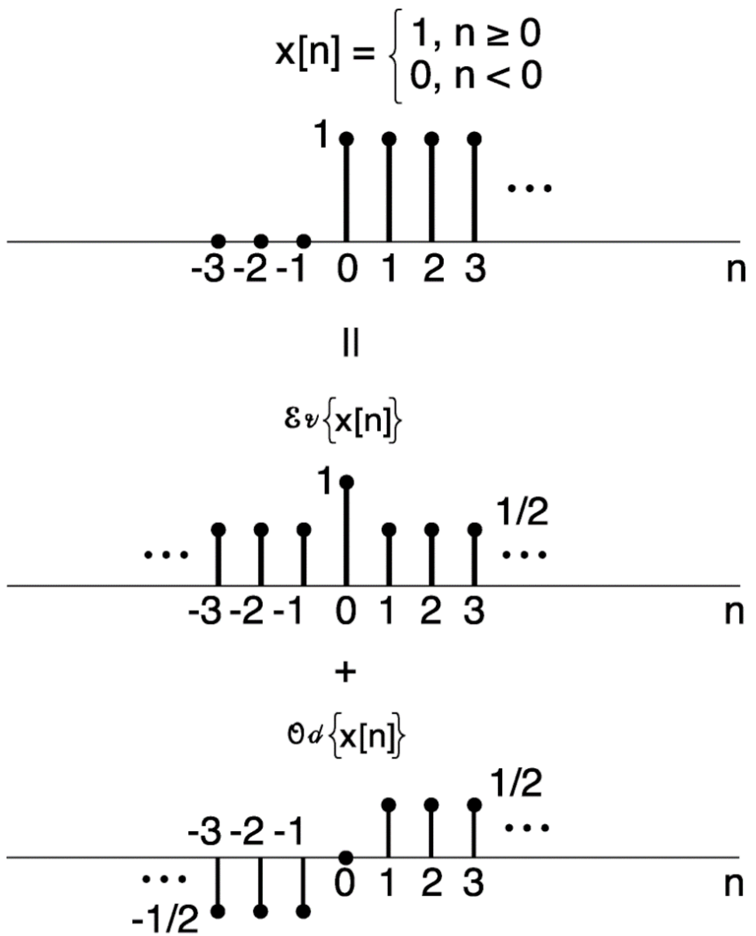
- Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

$$x(t) = x_{even}(t) + x_{odd}(t),$$

where:

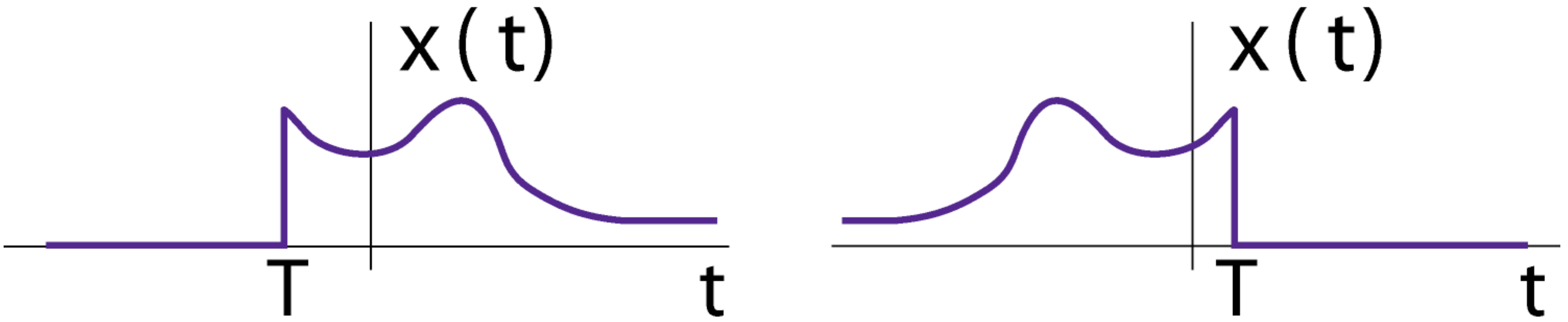
$$x_{even}(t) = [x(t) + x(-t)]/2,$$

$$x_{odd}(t) = [x(t) - x(-t)]/2.$$

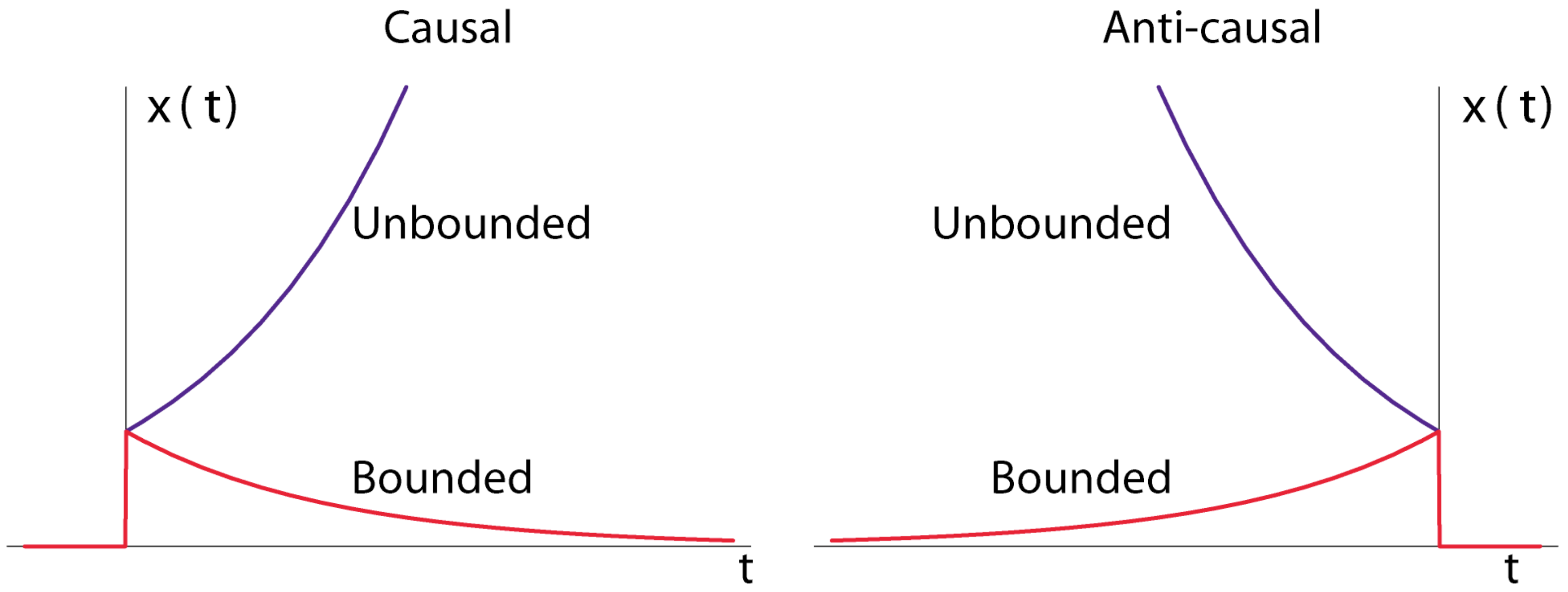


# Signal Classification 6: Right- and Left-Sided

- A right-sided signal is zero for $t < T$, and
- A left-sided signal is zero for $t > T$, where $T$ can be positive or negative.



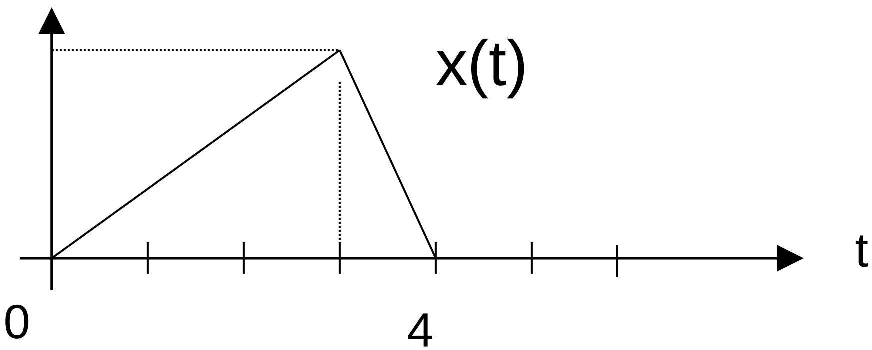
# Classification 7: Bounded and Unbounded

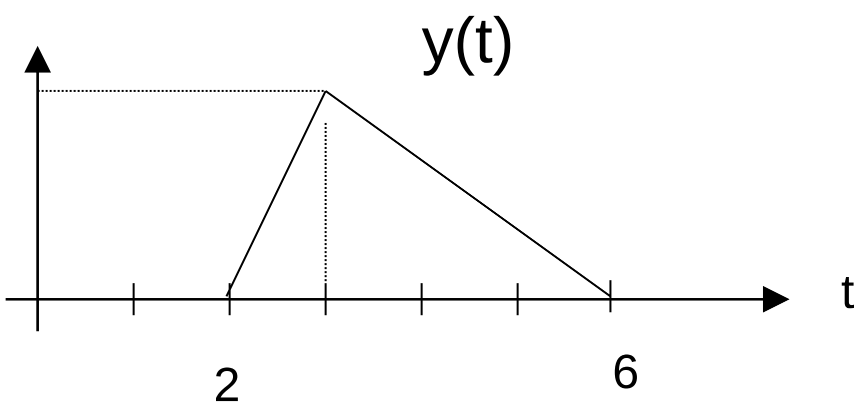


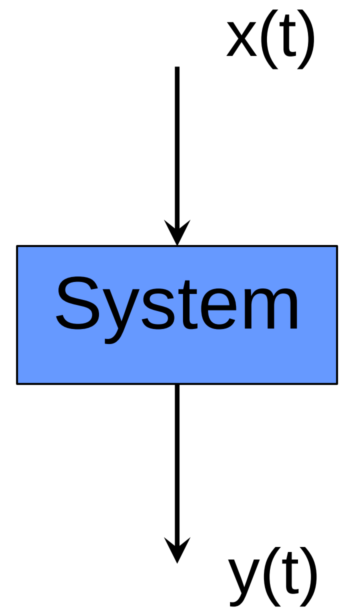

- Bounded signal: the absolute value of signal is bounded.
- Unbounded signal: otherwise

$$\exists C, |x(t)| \leq C \ \forall t$$

# Transformation of a Signal



# Transformation of a Signal

- Time Shift

$$x(t) \rightarrow x(t - t_0) \quad , \quad x[n] \rightarrow x[n - n_0]$$

- Time Reversal

$$x(t) \rightarrow x(-t) \quad , \quad x[n] \rightarrow x[-n]$$

- Time Scaling

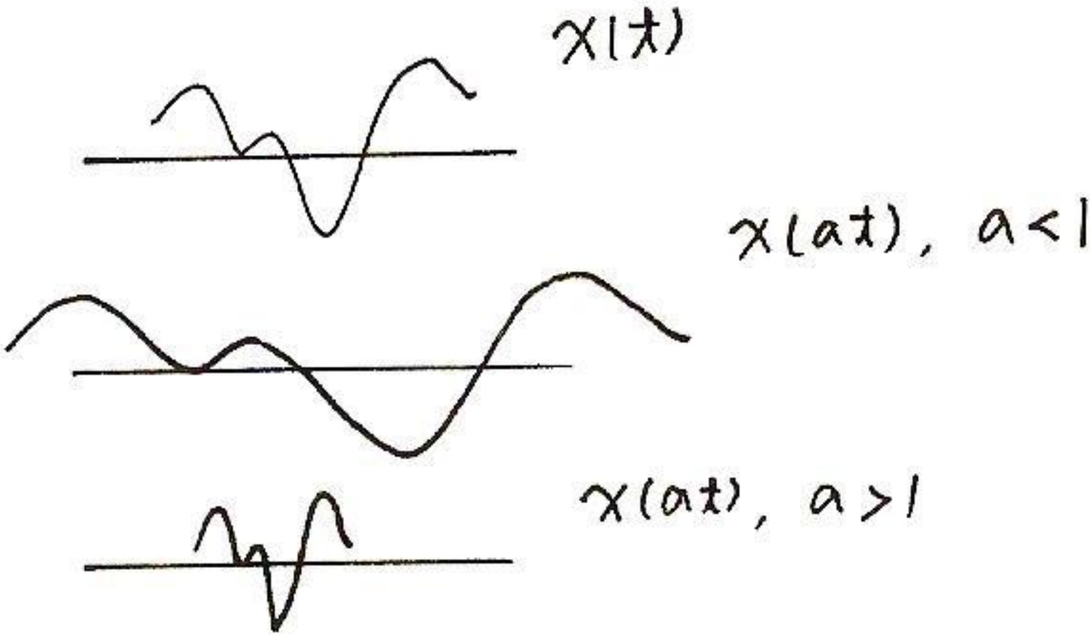
$$x(t) \rightarrow x(at) \quad , \quad x[n] \rightarrow ?$$

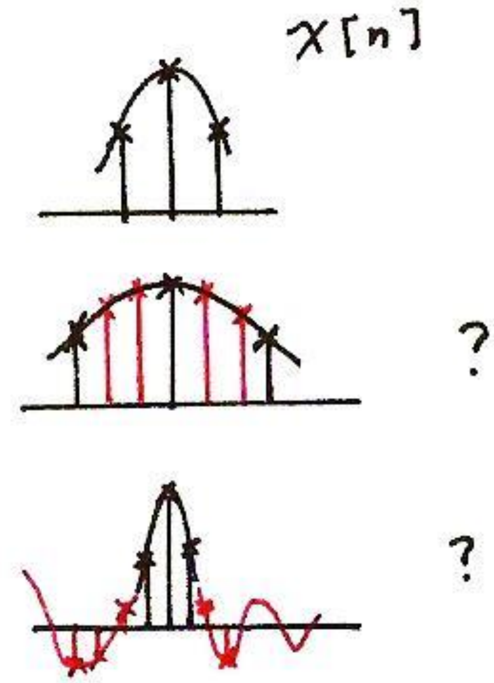
- Combination

$$x(t) \rightarrow x(at + b) \quad , \quad x[n] \rightarrow ?$$

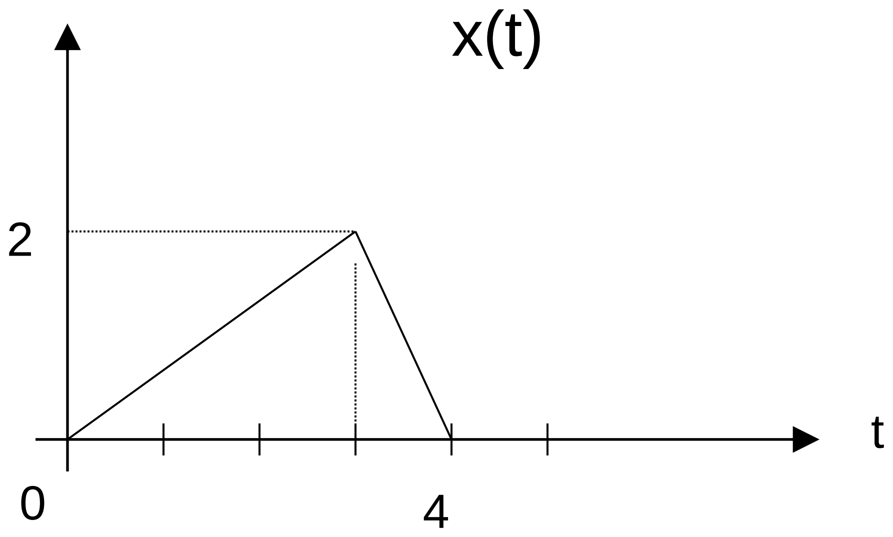
# Transformation of a Signal

## Time Scaling



# Class problem

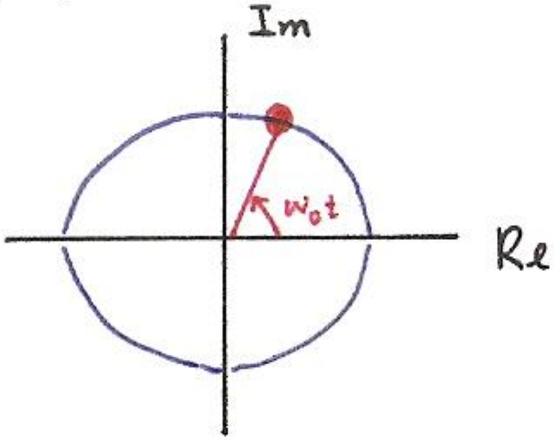



x(-2t+2) ?

# Exponential Signals

- A very important class of signals is presented as:
  - CT signals of the form $x(t) = e^{j\omega_0 t}$
  - DT signals of the form $x[n] = e^{j\omega_0 n}$
- For both *exponential* CT and DT signals, *x* is a complex quantity and has:
  - a real and imaginary part [i.e., *Cartesian form*], or equivalently
  - a magnitude and a phase angle [i.e., *polar form*].
- We will use whichever form that is convenient.

$$x(t) = e^{j\omega_0 t}$$

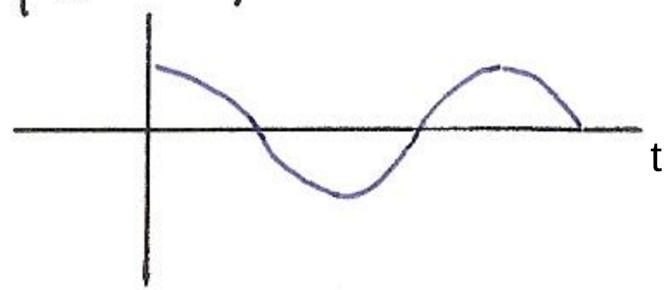



**Euler's relation**

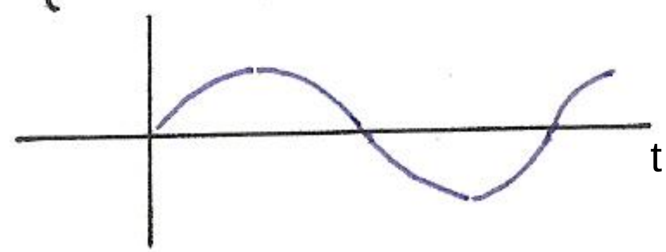
$$e^{jx} = \cos x + j \sin x$$

$\omega_0 t$ is defined as phase

$$\mathrm{Re}\{e^{j\omega_0 t}\} = \cos \omega_0 t$$

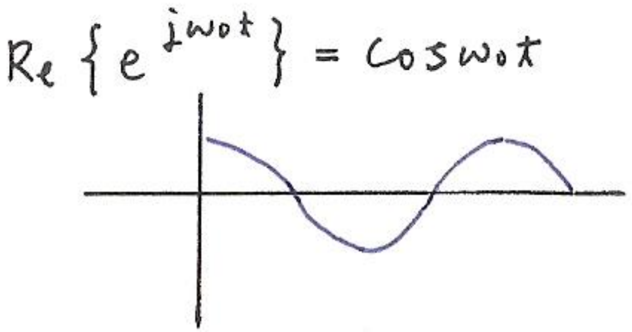


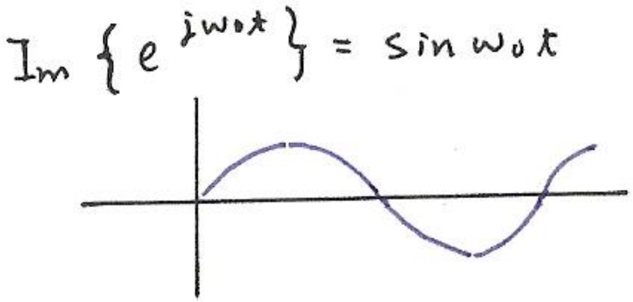

$$\mathrm{Im}\{e^{j\omega_0 t}\} = \sin \omega_0 t$$




Real and imaginary parts are periodic signals with the same period, but out of phase (90° phase difference)

$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$




**-Fundamental (angular) frequency:** $|\omega_0|$

**-Fundamental period:** $T_0 = \dfrac{2\pi}{|\omega_0|}$

**-In CT,** $e^{j\omega_0 t}$ **always periodic**

**-larger** $\omega_0$ **=> higher frequency**

$$x[n] = e^{j\omega_0 n} = cos\omega_0 n + \mathrm{j}\ \mathrm{sin}\omega_0 n$$

Two special cases:

$$e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Question 1: Is it periodic?

Question 2: Larger $\omega_0$ => higher oscillation frequency?

# Periodicity of DT Complex Exponentials

Important difference between $e^{j\omega_0 n}$ and $e^{j\omega_0 t}$:

- $e^{j\omega_0 n}$ is a periodic signal only when $\frac{\omega_0}{2\pi}$ is a rational number

$$e^{j\omega_0 n} = e^{j\omega_0 (n+N)} \rightarrow e^{j\omega_0 N} = 1 \rightarrow \omega_0 N = 2\pi m$$

Hence, $\dfrac{\omega_0}{2\pi} = \dfrac{m}{N}$

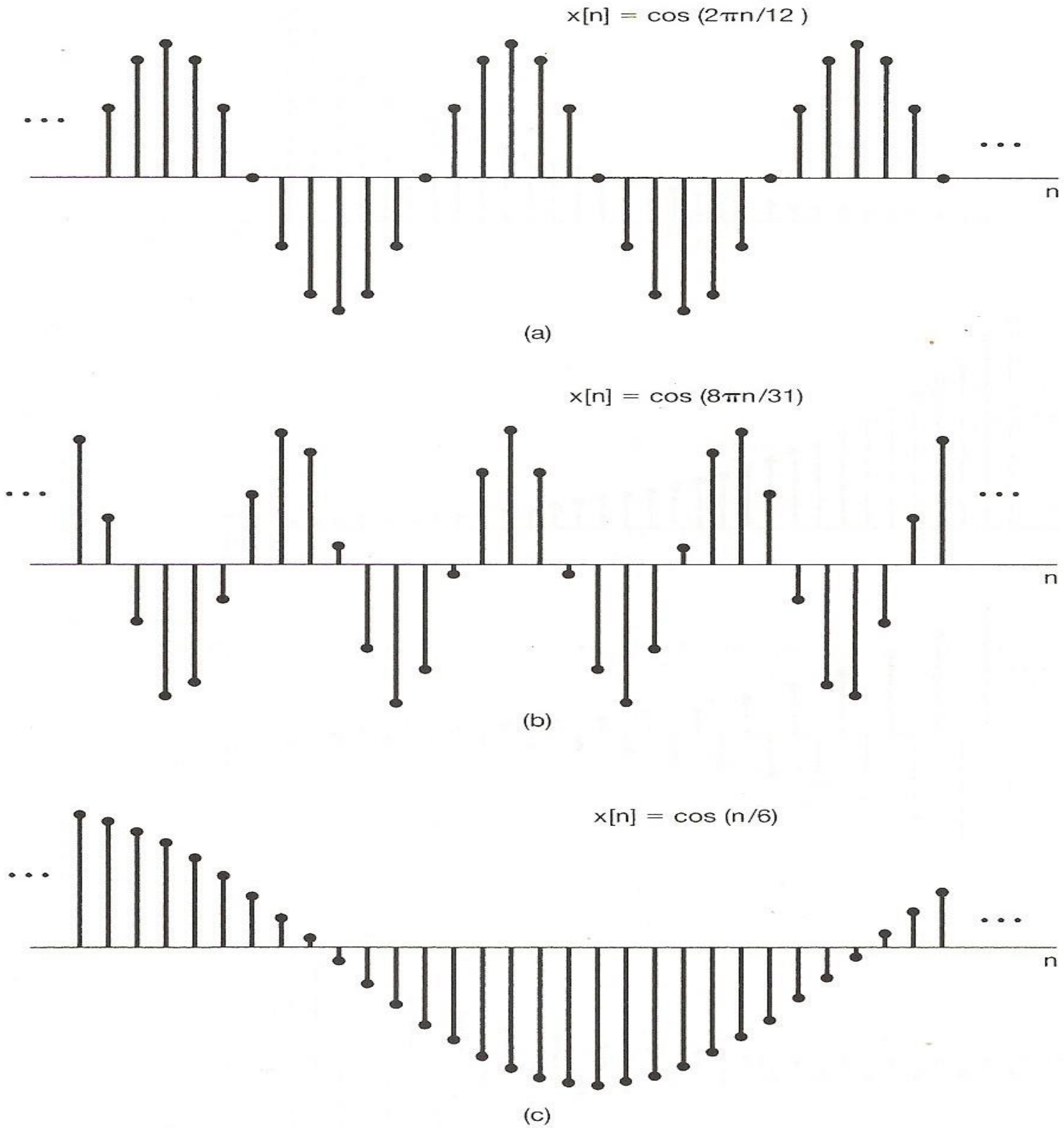



Figure 1.25 Discrete-time sinusoidal signals.

$$\frac{\omega_0}{2\pi} = \frac{2\pi/12}{2\pi} = \frac{1}{12}$$

$$\frac{\omega_0}{2\pi} = \frac{8\pi/31}{2\pi} = \frac{4}{31}$$

$$\frac{\omega_0}{2\pi} = \frac{1/6}{2\pi} = \frac{1}{12\pi}$$

# How to determine the fundamental period of $e^{j\omega_0 n}$?

**Solution:**

- Let $N$ be the fundamental period, then
$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \to e^{j\omega_0 N} = 1.$$
- $\exists\ integer\ m,\ \ \omega_0 N = 2\pi m.$
- Therefore,
$$N = \frac{2\pi}{\omega_0} m.$$
- Hence, N is the minimum positive integer in the set $\{\frac{2\pi}{\omega_0} m | \forall\ integer\ m\}$.

# Example

- **What is the fundamental period of $e^{j\frac{6}{5}\pi n}$?**

$$\left\{\frac{2\pi}{\omega_0}m \middle| \forall\ integer\ m\right\} = \left\{\frac{5}{3}m \middle| \forall\ integer\ m\right\}$$
$$= \left\{\dots, 0, \frac{5}{3}, \frac{10}{3}, 5, \frac{20}{3}, \dots\right\}$$

Hence, the fundamental period is 5 and fundamental frequency is $\frac{2\pi}{5}$.

# Oscillations of DT Complex Exponentials

Important difference between $e^{j\omega_0 n}$ and $e^{j\omega_0 t}$:

- $e^{j\omega_0 n}$ is periodic w.r.t. $\omega_0$

$$\boxed{e^{j(\omega_0+m\cdot 2\pi)n} = e^{j\omega_0 n} \cdot e^{jm\cdot 2\pi n} = e^{j\omega_0 n}}$$

- However, $e^{j\omega_0 t}$ *is aperiodic w.r.t.* $\omega_0$

$$\boxed{\forall x \neq 0, e^{j(\omega_0+x)t} = e^{j\omega_0 t} e^{jxt} \neq e^{j\omega_0 t}}$$

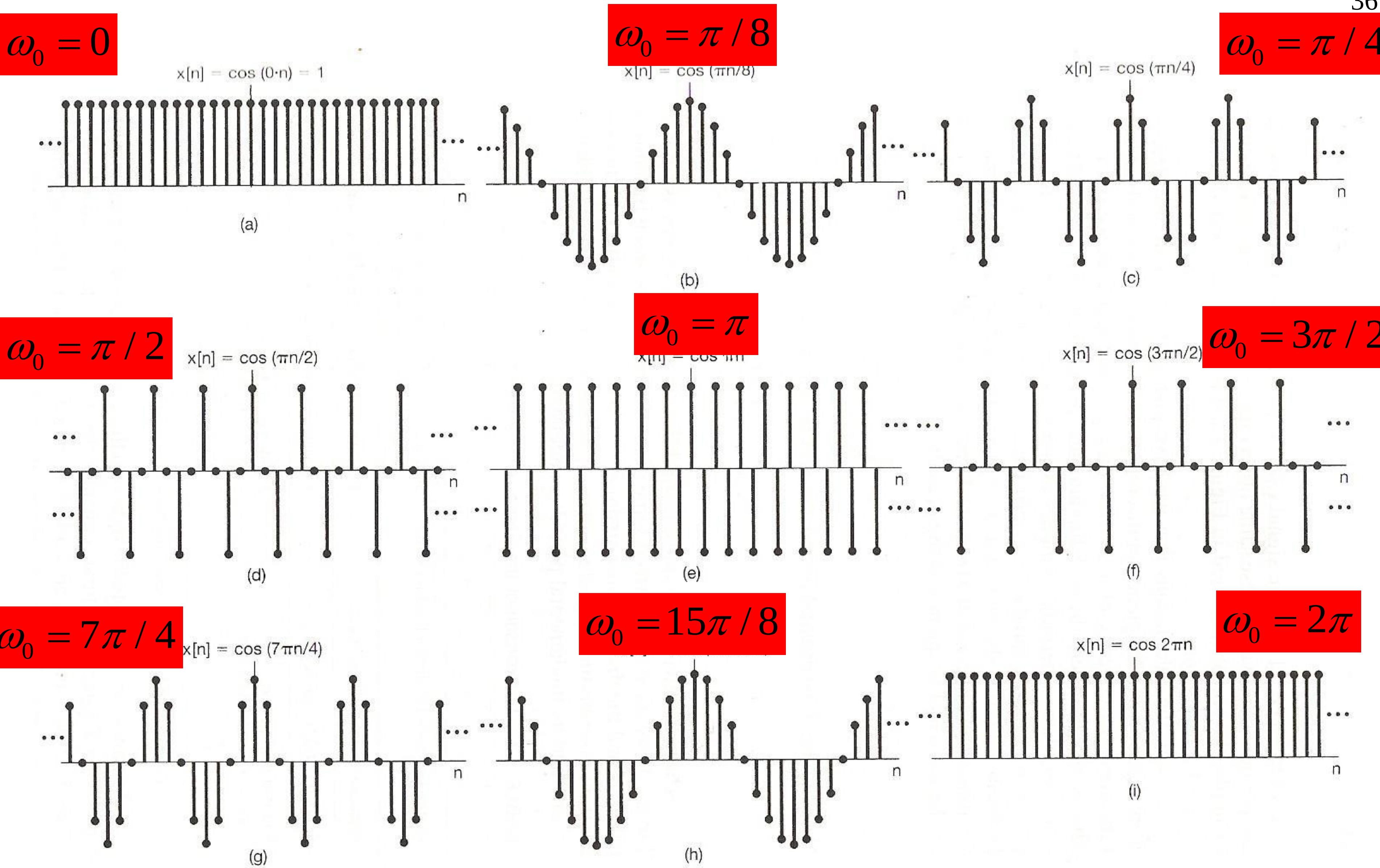



**Figure 1.27** Discrete-time sinusoidal sequences for several different frequencies.

- We need only consider a frequency interval of length 2π, and on most cases, we use the interval: $0 \le \omega_0 < 2\pi$, or $-\pi \le \omega_0 < \pi$

- $e^{j\omega_0 n}$ does **not** have a continually increasing rate of oscillation as $\omega_0$ is increased.

  lowest-frequency (slowly varying): $\omega_0$ near 0, 2π, …, or 2k • π

  highest-frequency (rapid variation): $\omega_0$ near ± π, …, or (2k+1) • π

$$e^{j(2k+1)\pi n} = e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$