homework_chapter7.pdf

Homework: 4.33, 4.37, 4.50 of the attachment

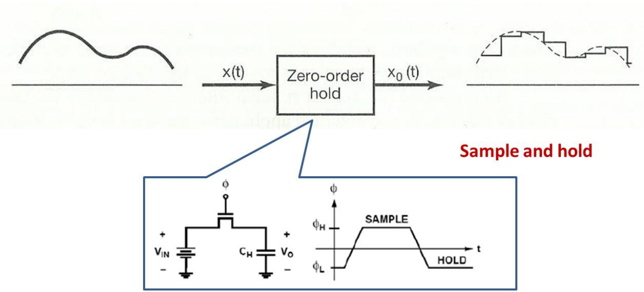
Tutorial Problems: 7.41, 7.44

Zero-Order Hold

- It's difficult to generate ideal impulse chain in practical implementation.

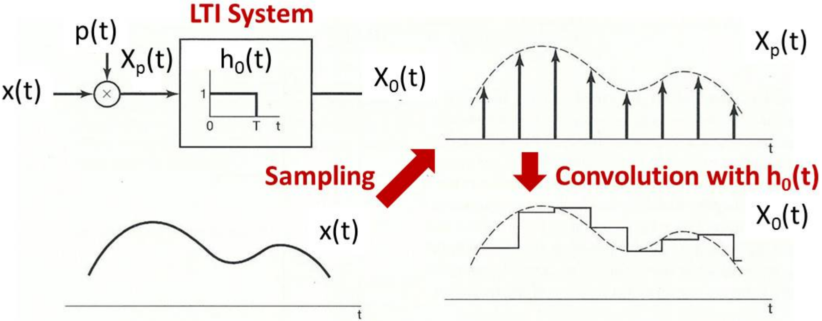
$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

- Alternative approach: zero-order hold



- How to interpret the system of "zero-order hold" mathematically?

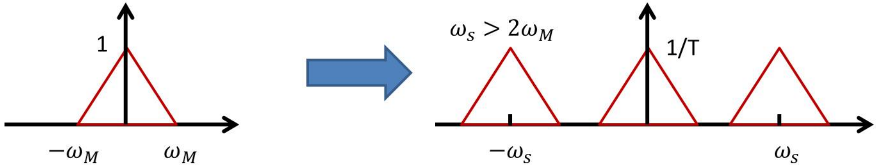
Interpretation of Zero-Order Hold



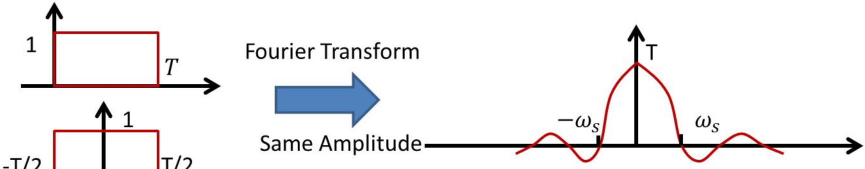
- Zero-order hold: sampling + interpolation with rectangular impulse response
- An approximation of the signal to be sampled.

Frequency Analysis (1/2)

- Step 1: Impulse-train sampling

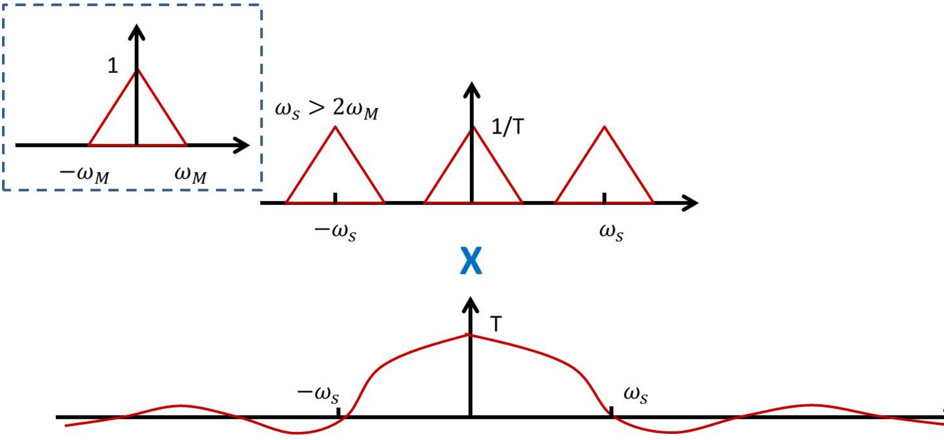


- Step 2: Frequency response of $h_0(t)$



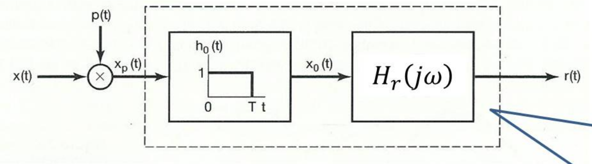
$$H(j\omega) = 2e^{-j\omega T/2}\frac{sin\omega T/2}{\omega}$$

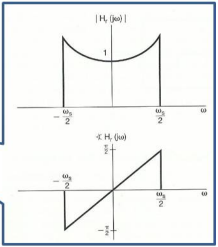
Frequency Analysis (2/2)



Reconstruction

$$H(j\omega) = H_0(j\omega)H_r(j\omega)$$



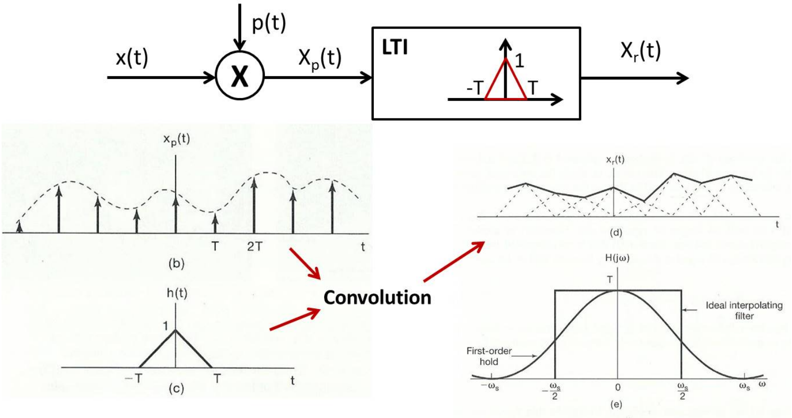


Compensate the frequency response of $h_0(t)$, such that H works as ideal lowpass filter

$$H_r(j\omega) = e^{j\omega T/2}\frac{\omega T}{2sin\omega T/2} \qquad -\frac{\omega_s}{2} \leq \omega \leq \frac{\omega_s}{2}$$

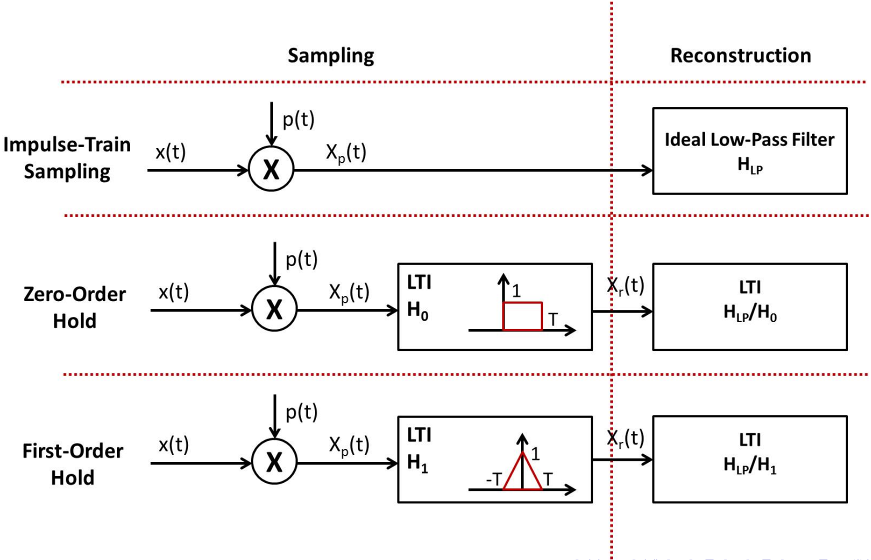
- $H(j\omega)$ should be a idea low-pass filter from $-\omega_s/2$ to $\omega_s/2$

First-Order Hold



- First-order hold: sampling + interpolation with triangular wave
- How to reconstruct?

Summary: Sampling Approaches



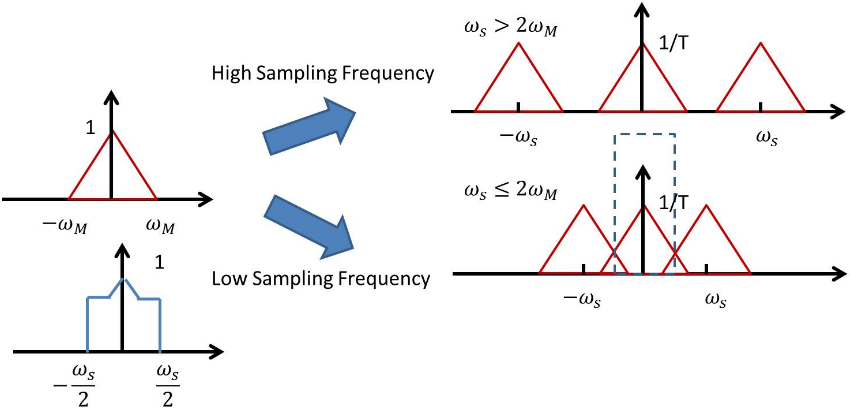
Problem 2

Problem (7.7)

A signal $x(t)$ undergoes a zero-order hold operation with an effective sampling period T to produce a signal $x_0(t)$. Let $x_1(t)$ denote the result of a first-order hold operation on $x(t)$. Specify the frequency response of a filter that produces $x_1(t)$ as its output when $x_0(t)$ is the input.

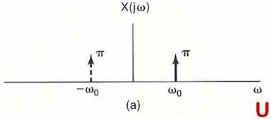
Undersampling & Aliasing

- Undersampling: insufficient sampling frequency $\omega_s < 2\omega_M$
- Perfect reconstruction is impossible with undersampling.
- Aliasing: distortion due to undersampling



Aliasing: Example

Signal before sampling: $\cos \omega_0 t$ **Sampling rate: ω_s**

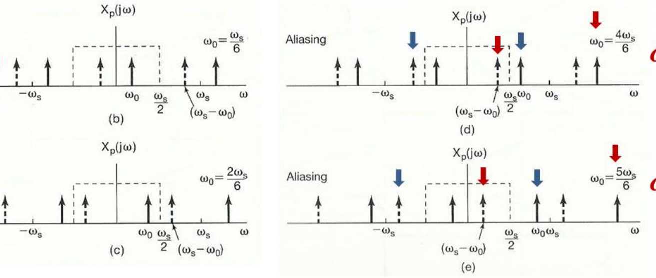


Lowpass Filter: $\dfrac{-\omega_s}{2} \sim \dfrac{\omega_s}{2}$

Undersampling

$\omega_s = 6\omega_0$

$\omega_s = \dfrac{3}{2}\omega_0$

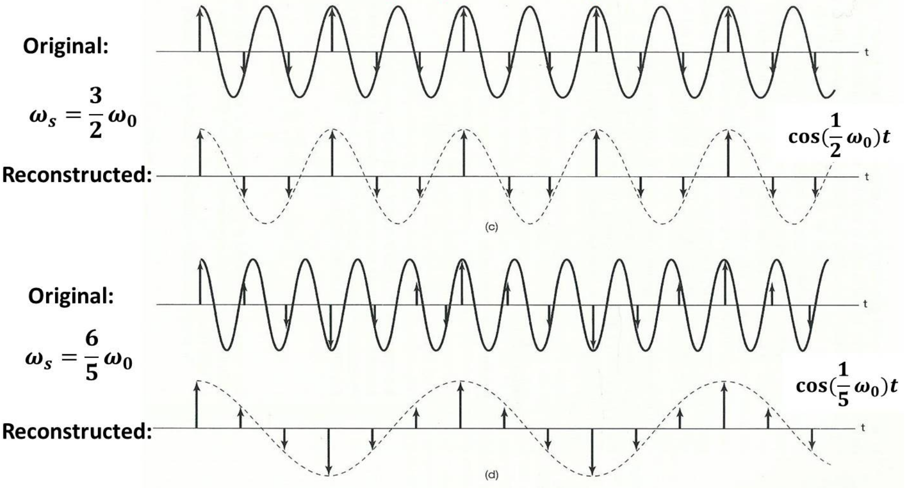


$\omega_s = 3\omega_0$

$\omega_s = \dfrac{6}{5}\omega_0$

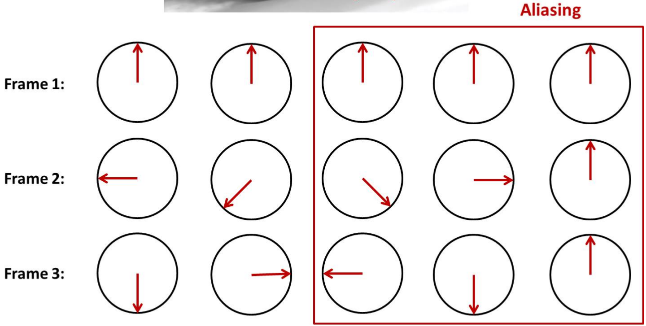
$\cos \omega_0 t$ **Aliasing: $\cos(\omega_s - \omega_0)t$**

Low-pass filtering: Interpret the samples by cosine function with frequency lower than $\omega_s/2$



Aliasing in Movies

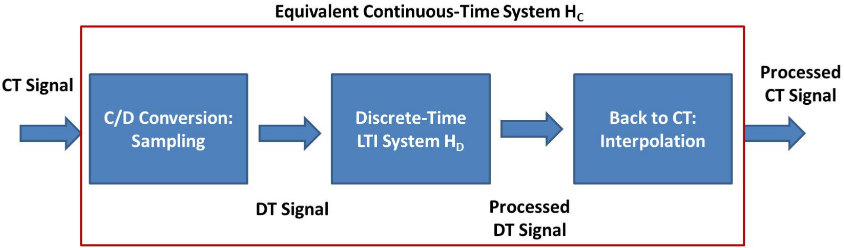
- Wheel's rotation in movies



Process Continuous-Time Signals Discretely

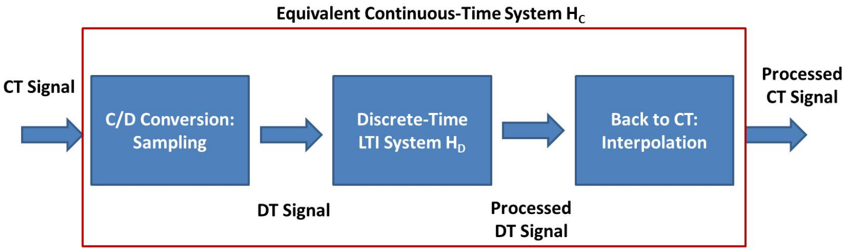
- People would like to process continuous-time signal in discrete-time (digital) domain

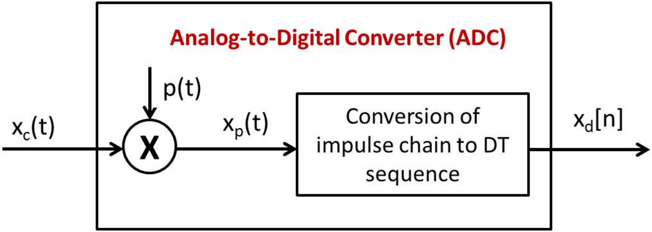
Block Diagram



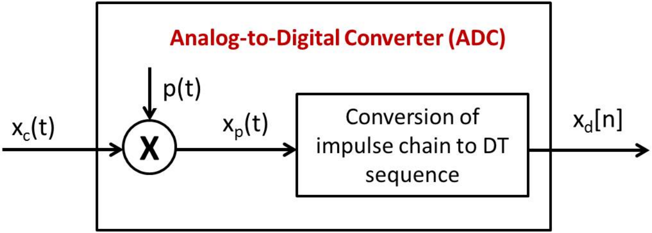
- It is much easier to design DT system.
- What's the relation between H_C and H_D?

Discretization: C/D Conversion





Discretization: C/D Conversion



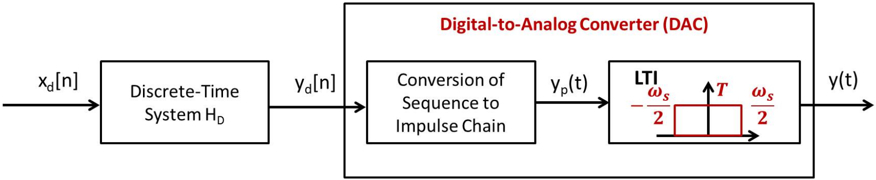
- Mathematical Interpretation (Fourier Transform)

$$x_c(t) \longleftrightarrow X_c(j\omega)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT)\delta(t-nT) \longleftrightarrow X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s))$$

$$x_d[n] = x_c(nT) \longleftrightarrow X_d(e^{j\omega}) = X_p(j\omega/T)$$

DT Processing and Conversion



- Mathematical Interpretation (Fourier Transform)

$$y_d[n] = x_d[n] * h_D[n] \quad \longleftrightarrow \quad Y_d(e^{j\omega}) = X_d(e^{j\omega})H_D(e^{j\omega})$$

$$y_p(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT) \quad \longleftrightarrow \quad Y_p(j\omega) = Y_d(e^{j\omega T})$$

$$y(t) = y_p(t) * h_{LP}(t) \quad \longleftrightarrow \quad Y(j\omega) = Y_p(j\omega)H_{LP}(j\omega)$$

Discussion

- From $x_c(t)$ to $x_d[n]$, from $y_d[n]$ to $y(t)$, what happen in time or frequency domain? Can you imagine it?
- What's the relation between $x_c(t)$ and $y(t)$?

Input vs. Output

$$
\begin{aligned}
Y(j\omega) &= Y_p(j\omega)H_{LP}(j\omega) = Y_d(e^{j\omega T})H_{LP}(j\omega) \\
&= X_d(e^{j\omega T})H_D(e^{j\omega T})H_{LP}(j\omega) \\
&= X_p(j\omega)H_D(e^{j\omega T})H_{LP}(j\omega) \\
&= \left[\frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s))\right] H_D(e^{j\omega T})H_{LP}(j\omega) \\
&= X_c(j\omega)H_D(e^{j\omega T}) \\
&= X_c(j\omega)\widetilde{H}_D(e^{j\omega T}) \qquad (1)
\end{aligned}
$$

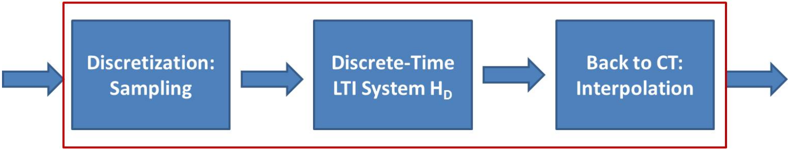
where

$$
\widetilde{H}_D(e^{j\omega T}) = \begin{cases} H_D(e^{j\omega T}) & |\omega| < \omega_s/2 \\ 0 & otherwise \end{cases} \qquad (2)
$$

- It is equivalent to a continuous-time LTI system $H_C(j\omega) = \widetilde{H}_D(e^{j\omega T})$
- $H_D(e^{j\omega T})$ is a periodic extension of $\widetilde{H}_D(e^{j\omega T})$ with period $\omega_s = 2\pi/T$

System Design

$$H_C(j\omega) = \widetilde{H}_D(e^{j\omega T})$$



- How can we design a CT LTI system with frequency response H_C via DT LTI system?
- Step 1: Sampling frequency ω_s or $2\pi/T$ should be larger than Nyquist rate
- Step 2: $\widetilde{H}_D(e^{j\omega T}) = H_C(j\omega)$
- Step 3: Frequency response of DT LTI system
  $H_D(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} \widetilde{H}_D(e^{j(\omega - k\omega_s)T})$ or
  $H_D(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \widetilde{H}_D(e^{j(\omega - k\omega_s T)}) = \sum_{k=-\infty}^{\infty} H_C(j\frac{\omega - 2k\pi}{T})$

System Design Example

- How to implement an ideal CT lowpass filter?