5.2 12011124 冯柏钧 13班

(a) let $x[n] = \delta[n-1] + \delta[n+1]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= e^{-j\omega} + e^{j\omega} = 2\cos\omega$$

(b) let $x[n] = \delta[n+2] - \delta[n-2]$.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = e^{2j\omega} - e^{-2j\omega} = 2j\sin(2\omega)$$

5.5.

$$x[n] = \frac{\pi}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \cancel{\frac{\pi}{2}\int_{-\pi}^{\pi} |X(e^{j\omega})| e} \ \frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-\frac{3}{2}\omega} e^{j\omega n} d\omega$$

$$= \frac{\sin(\frac{\pi}{4}(n-\frac{3}{2}))}{\pi(n-\frac{3}{2})}$$

$x[n]$ is zero when $\frac{\pi}{4}(n-\frac{3}{2})$ is ~~zero~~ a nonzero integer multiple of π or when $|n| \to \infty$.

$\therefore x[n] = 0$ only for $n = \pm\infty$

5.15.

$y[n] = x[n]x[n]$.

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\theta}) X(e^{j(\omega-\theta)}) d\theta$$

define $\overset{*}{X}(e^{j\omega}) = \begin{cases} X(e^{j\omega}) & -\pi < \omega \le \pi \\ 0 & \text{otherwise} \end{cases}$

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overset{*}{X}(e^{j\theta}) X(e^{j(\omega-\theta)}) d\theta.$$

$\omega_c = \frac{3}{4}\pi$.

5.21

(a) $z[n] = \overset{u[n-2]}{\cancel{u}} - u[n-6] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$.

$$X(e^{j\omega}) = e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega}.$$

(b) $X(e^{j\omega}) = \sum_{n=-\infty}^{-1} (\frac{1}{2})^{-n} e^{-j\omega n}$

$$= \sum_{n=1}^{\infty} (\tfrac{1}{2}e^{j\omega})^n.$$

$$= \frac{e^{j\omega}}{2}\frac{1}{1-\frac{1}{2}e^{j\omega}}$$

(c) $X(e^{j\omega}) = \sum_{n=-\infty}^{-2} (\frac{1}{3})^{-n} e^{-j\omega n}$

$$= \sum_{n=2}^{\infty} (\tfrac{1}{3}e^{j\omega})^n.$$

$$= \frac{e^{2j\omega}}{9}\frac{1}{1-\frac{1}{3}e^{j\omega}}$$

(d) $X(e^{j\omega}) = \sum_{n=-\infty}^{0} 2^n \sin(\frac{n\pi}{4}) e^{-j\omega n}$

$$= -\sum_{n=0}^{\infty} 2^{-n}\sin(\tfrac{n\pi}{4}) e^{j\omega n}.$$

$$= -\frac{1}{2j}[(\tfrac{1}{2})^n e^{j\frac{\pi}{4}n} e^{j\omega n} - (\tfrac{1}{2})^n e^{-\frac{\pi}{4}j\pi} e^{j\omega n}].$$

$$= -\frac{1}{2j}\left[\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{4}}e^{j\omega}} - \frac{1}{1-\frac{1}{2}e^{-\frac{\pi}{4}j}e^{j\omega}}\right].$$

(e) $X[e^{j\omega}] = \sum_{n=-\infty}^{\infty} (\frac{1}{2})^{|n|} \cos[\pi\frac{(n-1)}{8}] e^{-j\omega n}$

$$= \frac{1}{2}\left[\frac{e^{-j\frac{\pi}{8}}}{1-\frac{1}{2}e^{\frac{\pi}{8}j}e^{j\omega}} + \frac{e^{\frac{\pi}{8}j}}{1-\frac{1}{2}e^{-\frac{\pi}{8}j}e^{-j\omega}}\right]$$

$$+ \frac{1}{4}\left[\frac{e^{\frac{\pi}{4}j}}{1-\frac{1}{2}e^{\frac{\pi}{8}j}e^{j\omega}} + \frac{e^{-\frac{\pi}{4}j}e^{j\omega}}{1-\frac{1}{2}e^{-\frac{\pi}{8}j}e^{j\omega}}\right].$$

(f) $x[n] = -3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$.

$$X(e^{j\omega}) = -3e^{3j\omega} - 2e^{2j\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-2j\omega} + 3e^{-3j\omega}$$

(h) $x[n] = \sin(\frac{5}{3}\pi n) + \cos(\frac{7}{3}\pi n) = -\sin(\frac{\pi}{3}n) + \cos(\frac{\pi}{3}n)$

$$= -\frac{1}{2j}[e^{\frac{\pi}{3}j\omega} - e^{-\frac{\pi}{3}j\omega}] + \frac{1}{2}[e^{\frac{\pi}{3}j\omega} + e^{-\frac{\pi}{3}j\omega}].$$

$$X(e^{j\omega}) = \cancel{-\frac{\pi}{j}}\, j\pi[\delta(\omega-\tfrac{\pi}{3}) - \delta(\omega+\tfrac{\pi}{3})] + \pi[\delta(\omega-\tfrac{\pi}{3}) + \delta(\omega+\tfrac{\pi}{3})]$$

in $0 \le |\omega| < \pi$