

3.3. The given signal is

$$\begin{aligned}x(t) &= 2 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} - 2je^{j(5\pi/3)t} + 2je^{-j(5\pi/3)t} \\&= 2 + \frac{1}{2}e^{j2(2\pi/6)t} + \frac{1}{2}e^{-j2(2\pi/6)t} - 2je^{j5(2\pi/6)t} + 2je^{-j5(2\pi/6)t}\end{aligned}$$

From this, we may conclude that the fundamental frequency of  $x(t)$  is  $2\pi/6 = \pi/3$ . The non-zero Fourier series coefficients of  $x(t)$  are:

$$a_0 = 2, \quad a_2 = a_{-2} = \frac{1}{2}, \quad a_5 = a_{-5} = -2j$$

3.21. Using the Fourier series synthesis eq. (3.38),

$$\begin{aligned}x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_5 e^{j5(2\pi/T)t} + a_{-5} e^{-j5(2\pi/T)t} \\&= je^{j(2\pi/8)t} - je^{-j(2\pi/8)t} + 2e^{j5(2\pi/8)t} + 2e^{-j5(2\pi/8)t} \\&= -2\sin\left(\frac{\pi}{4}t\right) + 4\cos\left(\frac{5\pi}{4}t\right) \\&= -2\cos\left(\frac{\pi}{4}t - \pi/2\right) + 4\cos\left(\frac{5\pi}{4}t\right).\end{aligned}$$

3.22. (a) (i)  $T = 1$ ,  $a_0 = 0$ ,  $a_k = \frac{d(-1)^k}{k\pi}$ ,  $k \neq 0$ .

(ii) Here,

$$x(t) = \begin{cases} t+2, & -2 < t < -1 \\ 1, & -1 < t < 1 \\ 2-t, & 1 < t < 2 \end{cases}$$

$T = 6$ ,  $a_0 = 1/2$ , and

$$a_k = \begin{cases} 0, & k \text{ even} \\ \frac{6}{\pi^2 k^2} \sin\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi k}{6}\right), & k \text{ odd} \end{cases}$$

(iii)  $T = 3$ ,  $a_0 = 1$ , and

$$a_k = \frac{3j}{2\pi^2 k^2} [e^{jk2\pi/3} \sin(k2\pi/3) + 2e^{jk\pi/3} \sin(k\pi/3)], \quad k \neq 0.$$

(iv)  $T = 2$ ,  $a_0 = -1/2$ ,  $a_k = \frac{1}{2}(-1)^k$ ,  $k \neq 0$ .

(v)  $T = 6$ ,  $\omega_0 = \pi/3$ , and

$$a_k = \frac{\cos(2k\pi/3) - \cos(k\pi/3)}{jk\pi/3}.$$

Note that  $a_0 = 0$  and  $a_k \text{ even} = 0$ .

(vi)  $T = 4$ ,  $\omega_0 = \pi/2$ ,  $a_0 = 3/4$  and

$$a_k = \frac{e^{-jk\pi/2} \sin(k\pi/2) + e^{-jk\pi/4} \sin(k\pi/4)}{k\pi}, \quad \forall k.$$

此处 (f) 答案明显错误, 积分结果形式多样, 请自行积分。

3.24. (a) We have

$$a_0 = \frac{1}{2} \int_0^1 t dt + \frac{1}{2} \int_1^2 (2-t) dt = 1/2.$$

(b) The signal  $g(t) = dx(t)/dt$  is as shown in Figure S3.24.

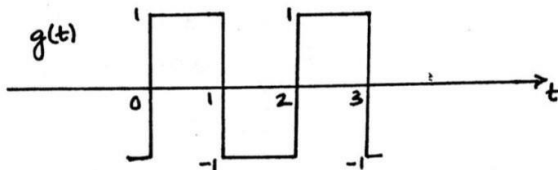


Figure S3.24

The FS coefficients  $b_k$  of  $g(t)$  may be found as follows:

$$b_0 = \frac{1}{2} \int_0^1 dt - \frac{1}{2} \int_1^2 dt = 0$$

and

$$\begin{aligned}b_k &= \frac{1}{2} \int_0^1 e^{-jk\pi t} dt - \frac{1}{2} \int_1^2 e^{-jk\pi t} dt \\&= \frac{1}{j\pi k} [1 - e^{-jk\pi}].\end{aligned}$$

(c) Note that

$$g(t) = \frac{dx(t)}{dt} \xrightarrow{FS} b_k = jk\pi a_k.$$

Therefore,

$$a_k = \frac{1}{jk\pi} b_k = -\frac{1}{\pi^2 k^2} \{1 - e^{-jk\pi}\}.$$

3.25. (a) The nonzero FS coefficients of  $x(t)$  are  $a_1 = a_{-1} = 1/2$ .

(b) The nonzero FS coefficients of  $x(t)$  are  $b_1 = b_{-1}^* = 1/2j$ .

(c) Using the multiplication property, we know that

$$z(t) = x(t)y(t) \xrightarrow{FS} c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}.$$

Therefore,

$$c_k = a_k * b_k = \frac{1}{4j} \delta[k-2] - \frac{1}{4j} \delta[k+2].$$

This implies that the nonzero Fourier series coefficients of  $z(t)$  are  $c_2 = c_{-2}^* = (1/4j)$ .

(d) We have

$$z(t) = \sin(4t) \cos(4t) = \frac{1}{2} \sin(8t).$$

答案 (d) 有问题, 这里应和 (c) 结论一样

Therefore, the nonzero Fourier series coefficients of  $z(t)$  are  $c_2 = c_{-2} = (1/4j)$ .