2.10. From the given information, we may sketch $x(t)$ and $h(t)$ as shown in Figure S2.10.

(a) With the aid of the plots in Figure S2.10, we can show that $y(t) = x(t) * h(t)$ is as shown in Figure S2.10.

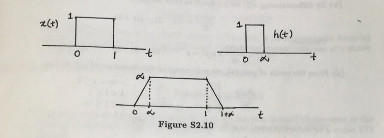

Figure S2.10

Therefore,

$$y(t) = \begin{cases} t, & 0 \le t \le \alpha \\ \alpha, & \alpha \le t \le 1 \\ 1+\alpha-t, & 1 \le t \le (1+\alpha) \\ 0, & \text{otherwise} \end{cases}$$

(b) From the plot of $y(t)$, it is clear that $\frac{dy(t)}{dt}$ has discontinuities at 0, α, 1, and $1+\alpha$. If we want $\frac{dy(t)}{dt}$ to have only three discontinuities, then we need to ensure that $\alpha = 1$.

2.11. (a) From the given information, we see that $h(t)$ is non zero only for $0 \le t \le \infty$. Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\ &= \int_{0}^{\infty} e^{-3\tau}(u(t-\tau-3) - u(t-\tau-5))d\tau \end{aligned}$$

We can easily show that $(u(t-\tau-3) - u(t-\tau-5))$ is non zero only in the range $(t-5) < \tau < (t-3)$. Therefore, for $t \le 3$, the above integral evaluates to zero. For $3 < t \le 5$, the above integral is

$$y(t) = \int_{0}^{t-3} e^{-3\tau}d\tau = \frac{1-e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau}d\tau = \frac{(1-e^{-6})e^{-3(t-5)}}{3}$$

Therefore, the result of this convolution may be expressed as

$$y(t) = \begin{cases} 0, & -\infty < t \le 3 \\ \frac{1-e^{-3(t-3)}}{3}, & 3 < t \le 5 \\ \frac{(1-e^{-6})e^{-3(t-5)}}{3}, & 5 < t \le \infty \end{cases}$$

(b) By differentiating $x(t)$ with respect to time we get

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

Therefore,

$$g(t) = \frac{dx(t)}{dt} * h(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5).$$

(c) From the result of part (a), we may compute the derivative of $y(t)$ to be

$$\frac{dy(t)}{dt} = \begin{cases} 0, & -\infty < t \le 3 \\ e^{-3(t-3)}, & 3 < t \le 5 \\ (e^{-6}-1)e^{-3(t-5)}, & 5 < t \le \infty \end{cases}$$

This is exactly equal to $g(t)$. Therefore, $g(t) = \frac{dy(t)}{dt}$.

2.22. (a) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{0}^{t} e^{-\alpha t}e^{-\beta(t-\tau)}d\tau, \ t \ge 0 \end{aligned}$$

Then

$$y(t) = \begin{cases} \frac{e^{-\beta t}\{e^{-(\alpha-\beta)t}-1\}}{\beta-\alpha}u(t) & \alpha \ne \beta \\ te^{-\beta t}u(t) & \alpha = \beta \end{cases}.$$

(b) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{0}^{2} h(t-\tau)d\tau - \int_{2}^{5} h(t-\tau)d\tau. \end{aligned}$$

This may be written as

$$y(t) = \begin{cases} \int_{0}^{2} e^{2(t-\tau)}d\tau - \int_{2}^{5} e^{2(t-\tau)}d\tau, & t \le 1 \\ \int_{t-1}^{2} e^{2(t-\tau)}d\tau - \int_{2}^{5} e^{2(t-\tau)}d\tau, & 1 \le t \le 3 \\ -\int_{t-1}^{5} e^{2(t-\tau)}d\tau, & 3 \le t \le 6 \\ 0, & 6 < t \end{cases}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \le 1 \\ (1/2)[e^{2} + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \le t \le 3 \\ (1/2)[e^{2(t-5)} - e^{2}], & 3 \le t \le 6 \\ 0, & 6 < t \end{cases}$$

(c) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{0}^{2} \sin(\pi\tau)h(t-\tau)d\tau. \end{aligned}$$

This gives us

$$y(t) = \begin{cases} 0, & t < 1 \\ (2/\pi)[1-\cos\{\pi(t-1)\}], & 1 < t < 3 \\ (2/\pi)[\cos\{\pi(t-3)\} - 1], & 3 < t < 5 \\ 0, & 5 < t \end{cases}$$

(d) Let

$$h(t) = h_1(t) - \frac{1}{3}\delta(t-2),$$

where

$$h_1(t) = \begin{cases} 4/3, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Now,

$$y(t) = h(t) * x(t) = [h_1(t) * x(t)] - \frac{1}{3}x(t-2).$$

We have

$$h_1(t) * x(t) = \int_{t-1}^{t} \frac{4}{3}(a\tau + b)d\tau = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)].$$

Therefore,

$$y(t) = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)] - \frac{1}{3}[a(t-2) + b] = at + b = x(t).$$

(e) $x(t)$ periodic implies $y(t)$ periodic. $\therefore$ determine 1 period only. We have

$$y(t) = \begin{cases} \int_{t-1}^{-\frac{1}{2}} (t-\tau-1)d\tau + \int_{-\frac{1}{2}}^{t} (1-t+\tau)d\tau = \frac{1}{4} + t - t^2, & -\frac{1}{2} < t < \frac{1}{2} \\ \int_{t-1}^{\frac{1}{2}} (1-t+\tau)d\tau + \int_{\frac{1}{2}}^{t} (t-1-\tau)d\tau = t^2 - 3t + 7/4, & \frac{1}{2} < t < \frac{3}{2} \end{cases}.$$

The period of $y(t)$ is 2.

2.25. (a) We may write $x[n]$ as

$$x[n] = \left(\frac{1}{3}\right)^{|n|}.$$

Now, the desired convolution is

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{-1} (1/3)^{-k}(1/4)^{n-k}u[n-k+3] + \sum_{k=0}^{\infty}(1/3)^{k}(1/4)^{n-k}u[n-k+3] \\ &= (1/12)\sum_{k=0}^{\infty}(1/3)^{k}(1/4)^{n+k}u[n+k+4] + \sum_{k=0}^{\infty}(1/3)^{k}(1/4)^{n-k}u[n-k+3] \end{aligned}$$

By consider each summation in the above equation separately, we may show that

$$y[n] = \begin{cases} (12^4/11)3^n, & n < -4 \\ (1/11)4^4, & n = -4 \\ (1/4)^n(1/11) + -3(1/4)^n + 3(256)(1/3)^n, & n \ge -3 \end{cases}.$$

(b) Now consider the convolution

$$y_1[n] = [(1/3)^n u[n]] * [(1/4)^n u[n+3]].$$

We may show that

$$y_1[n] = \begin{cases} 0, & n < -3 \\ -3(1/4)^n + 3(256)(1/3)^n, & n \ge -3 \end{cases}.$$

Also, consider the convolution

$$y_2[n] = [(3)^n u[-n-1]] * [(1/4)^n u[n+3]].$$

We may show that

$$y_2[n] = \begin{cases} (12^4/11)3^n, & n < -4 \\ (1/4)^n(1/11), & n \ge -3 \end{cases}.$$

Clearly, $y_1[n] + y_2[n] = y[n]$ obtained in the previous part.

2.28. (a) Causal because $h[n] = 0$ for $n < 0$. Stable because $\sum_{n=0}^{\infty}(\frac{1}{5})^n = 5/4 < \infty$.

(b) Not causal because $h[n] \ne 0$ for $n < 0$. Stable because $\sum_{n=-2}^{\infty}(0.8)^n = 5 < \infty$.

(c) Anti-causal because $h[n] = 0$ for $n > 0$. Unstable because $\sum_{n=-\infty}^{0}(1/2)^n = \infty$

(d) Not causal because $h[n] \ne 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^{3} 5^n = \frac{625}{4} < \infty$

(e) Causal because $h[n] = 0$ for $n < 0$. Unstable because the second term becomes infinite as $n \to \infty$.

(f) Not causal because $h[n] \ne 0$ for $n < 0$. Stable because $\sum_{n-\infty}^{\infty}|h[n]| = 305/3 < \infty$

(g) Causal because $h[n] = 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^{\infty}|h[n]| = 1 < \infty$.