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2.4

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$\therefore y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$

$$\Rightarrow y[n] = \begin{cases} n-6 & 7 \le n \le 11, \\ 6 & 12 \le n \le 18, \\ 24-n & 19 \le n \le 23. \\ 0 & \text{else} \end{cases}$$

2.6

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$.

$= \sum_{k=-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1]u[n-k-1]$

$= \sum_{k=-\infty}^{-1} (\frac{1}{3})^{-k} u[n-k-1]$

$= \sum_{k=1}^{\infty} (\frac{1}{3})^{k} u[n+k-1]$

$k = p-1$.

$$\Rightarrow y[n] = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} u[n+p] = \begin{cases} \frac{1}{2} & n \ge 0. \\ \frac{3^n}{2} & n < 0 \end{cases}$$

2.19

(a) $y[n] = \alpha y[n-1] + \beta w[n]$

$w[n] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1]$.

$u[n-1] = \frac{1}{\beta} y[n-1] - \frac{\alpha}{\beta} y[n-2]$.

$w[n] - \frac{1}{2} w[n-1] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2]$

$\frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2] = x[n]$.

$y[n] = (\alpha + \frac{1}{2}) y[n-1] - \frac{\alpha}{2} y[n-2] + \beta x[x]$.

$\alpha = \frac{1}{4}$, $\beta = 1$.

(b)

S_1 : $w[n] = \frac{1}{2} w[n-1] + x[n]$

S_2 : $y[n] = \frac{1}{4} y[n-1] + w[n]$.

$h_1[n] = \frac{1}{2^n} u[n]$ $\quad h_2[n] = \frac{1}{4^n} u[n]$.

$h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k]$.

$= \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} u[n-k]$

$= \sum_{k=0}^{n} (\frac{1}{2})^k (\frac{1}{4})^{n-k}$

$= \sum_{k=0}^{n} (\frac{1}{2})^{2(n-k)}$

$= [2 \times (\frac{1}{2})^n - (\frac{1}{4})^n] u[n]$.

2.21

(c) for $n \le 6$.

$y[n] = 4^n \{ \sum_{k=0}^{n} (-\frac{1}{8})^k - \sum_{k=0}^{3} (-\frac{1}{8})^k \}$.

$n = 6$.

$y[n] = 4^n \{ \sum_{k=0}^{\infty} (-\frac{1}{8})^k - \sum_{k=0}^{n-1} (-\frac{1}{8})^k \}$

$$\therefore y[n] = \begin{cases} (\frac{8}{9}) \times (-\frac{1}{8})^4 \times 4^n & n \le 6 \\ (\frac{8}{9}) \times (-\frac{1}{2})^n & n > 6 \end{cases}$$

(d)

$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + \cdots + x[4]h[n$

$= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]$.

y[n].