Assignment 4 Solution

3.2 Using the Fourier series synthesis eq. (3.95)

$$x[n] = a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2} e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4} e^{-j4(2\pi/N)n}$$
$$= 1 + e^{j(\pi/4)} e^{j2(2\pi/5)n} + e^{-j(\pi/4)} e^{-j2(2\pi/5)n}$$
$$= 1 + 2\cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4\cos(\frac{8\pi}{5}n + \frac{\pi}{3})$$
$$= 1 + 2\sin(\frac{4\pi}{5}n + \frac{3\pi}{4}) + 4\sin(\frac{8\pi}{5}n + \frac{5\pi}{6})$$

3.27.Using the Fourier series synthesis eq.(3.38),

$$x[n] = a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2} e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4} e^{-j4(2\pi/N)n}$$
$$= 2 + 2e^{j\pi/6} e^{j(4\pi/5)n} + 2e^{-j\pi/6} e^{-j(4\pi/5)n} + e^{j\pi/3} e^{j(8\pi/5)n} + e^{-j\pi/3} e^{-j(8\pi/5)n}$$
$$= 2 + 4\cos[(4\pi n/5) + \pi/6] + 2\cos[(8\pi n/5) + \pi/3]$$
$$= 2 + 4\sin[(4\pi n/5) + 2\pi/3] + 2\sin[(8\pi n/5) + 5\pi/6]$$

3.36. We will first evaluate the frequency response of the system. Consider an input x[n] of the form e^{jwn} .From the discussion in Section 3.9 we know that the response to this input will be y[n]=H(e^{jw}) e^{jwn} .Therefore , substituting these in the given difference equation. We get

$$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}H(e^{j\omega}) = e^{j\omega n}.$$

Therefore,

$$H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}.$$

Form eq. (3.131), we know that coefficients

$$y[n] = \sum_{k=<N>} a_k H(e^{j2\pi k/N}) e^{jk(2\pi/N)n}$$

when the input is x[n]. x[n] has the Fourier series coefficients a_k and fundamental frequency 2/N. therefore, the Fourier series coefficients of y[n] are a_k $H(e^{j2\pi k/N})$

(a) Here, N=$\frac{8}{3}$ and the nonzero FS coefficients of x[n] are $a_3 = a^*_3 = 1/2j$.Therefore,the nonzero FS coefficients of y[n] are

$$b_3 = a_1 H(e^{3j\pi/4}) = \frac{1}{2j(1-(1/4)e^{-j3\pi/4})}, \quad b_{-3} = a_{-1}H(e^{-3j\pi/4}) = \frac{-1}{2j(1-(1/4)e^{j3\pi/4})}.$$

Then the FS representation of y[n] is

$$y[n] = \frac{1}{2j(1-\frac{1}{4}e^{-j\frac{3\pi}{4}})} e^{j\frac{3\pi}{4}n} - \frac{1}{2j(1-\frac{1}{4}e^{j\frac{3\pi}{4}})} e^{-j\frac{3\pi}{4}n}$$

(b) Here, N=8 and the nonzero FS coefficients of x[n] are $a_1 = a_{-1}$ =1\2 and $a_2 = a_{-2} = 1$. Therefore, the nonzero FS coefficients of y(t) are

$$b_1 = a_1 H(e^{j\pi/4}) = \frac{1}{2(1-(1/4)e^{-j\pi/4})}, \quad b_{-1} = a_{-1}H(e^{-j\pi/4}) = \frac{1}{2(1-(1/4)e^{j\pi/4})},$$
$$b_2 = a_2 H(e^{j\pi/2}) = \frac{1}{(1-(1/4)e^{-j\pi/2})}, \quad b_{-2} = a_{-2}H(e^{-j\pi/2}) = \frac{1}{(1-(1/4)e^{j\pi/2})}.$$

Then the FS representation of y[n] is

$$y[n] = \frac{1}{2-\frac{1}{2}e^{-j\frac{\pi}{4}}} e^{j\frac{\pi}{4}n} + \frac{1}{2-\frac{1}{2}e^{j\frac{\pi}{4}}} e^{-j\frac{\pi}{4}n}$$
$$+ \frac{1}{1-\frac{1}{4}e^{-j\frac{\pi}{2}n}} e^{j\frac{\pi}{2}n} + \frac{1}{1-\frac{1}{4}e^{+j\frac{\pi}{2}n}} e^{-j\frac{\pi}{2}n}$$

3.38 The frequency response of the system may be evaluated as

$$H(e^{jw})=\sum_{n=-\infty}^{+\infty}h[n]e^{-jwn}=1+e^{-jw}+e^{-2jw}-e^{jw}-e^{2jw}=1-2j\sin(w)-2j\sin(2w)$$

For x[n],N=4 and $w_0=\pi/2$. the FS coefficients of input x[n] are

$a_k=1/4$, for all k

Therefore, the FS coefficients of output are

$$b_k=\frac{1}{4}[1-2j\sin(k\tfrac{\pi}{2})-2j\sin(k\pi)]=\frac{1}{4}[1-2j\sin(k\tfrac{\pi}{2})]$$

3.50 from Table 3.2,we know that if

$$x[n]\xleftrightarrow{FS}a_k,$$

then ,

$$(-1)^n x[n]=e^{(2\pi/N)(N/2)n}x[n]\xleftrightarrow{FS}a_{k-N/2}$$

In this case, N =8. Therefore,

$$(-1)^n x[n]\xleftrightarrow{FS}a_{k-4}$$

This implies that $x[0]=x[\pm 2]x[\pm 4]=\cdots=0$.

We are also given that $x[1]=x[5]=\cdots=and=x[3]x[7]=-1$.therefore , one period of x[n] is as shown in Figure S3.50

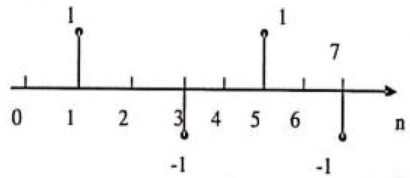

Figure S3.50