Assignment 1 Solution

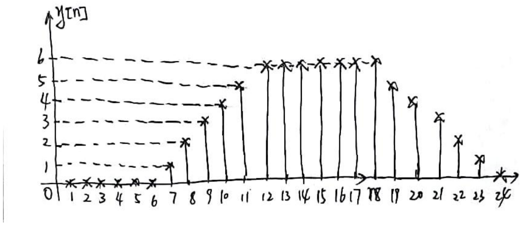
2.4 We know that

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

The signals x[n] and y[n] are as shown in Figure S2.4.From this figure,we see that the above summation reduces to

$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$ This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & otherwise \end{cases}$$



2.6. From the given information, we have :

$$\text{y[n]= x[n]} * \text{h[n]} = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

$$= \sum_{k=-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1]u[n-k-1]\cdot$$

$$= \sum_{k=-\infty}^{-1} (\frac{1}{3})^{-k} u[n-k-1]\cdot$$

$$= \sum_{k=1}^{\infty} (\frac{1}{3})^{-k} u[n+k-1]\cdot$$

Replacing k by p-1,

$$\text{y[n]=} = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} u[n+p] \qquad \text{(S2.6-1)}$$

For $n \ge 0$ the above equation reduces to,

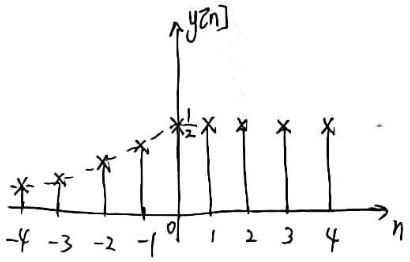
$$\text{y[n]=} \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} = \frac{1}{3} \frac{1}{1-\frac{1}{3}} = \frac{1}{2}$$

For n<0 eq.(S2.6-1) reduces to,

$$\text{y[n]=} \sum_{p=-n}^{\infty} (\frac{1}{3})^{p+1} = (\frac{1}{3})^{-n+1} \sum_{p=0}^{\infty} (\frac{1}{3})^{p} = (\frac{1}{3})^{-n+1} \frac{1}{1-\frac{1}{3}} = (\frac{1}{3})^{-n} \frac{1}{2} = \frac{3^n}{2}$$

Therefore,

$$\text{y[n]=} \begin{cases} (3^n/2), n < 0 \\ (1/2), n \ge 0 \end{cases}$$



2.19. (a) Consider the difference equation relating $y[n]$ and $w[n]$ for s_2:

$$y[n] = \alpha y[n-] + \beta w[n]$$

From this we may write

$$w[n] = \frac{1}{\beta} y[n] + \frac{\alpha}{\beta} y[n-1]$$

and

$$w[n-1] = \frac{1}{\beta} y[n-1] + \frac{\alpha}{\beta} w[n-2]$$

Weighting the previous equation by 1/2 and subtracting from the one before ,we obtain

$$w[n] - \frac{1}{2} w[n-1] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{\beta} y[n-2]$$

Substituting this in the difference equation relating $w[n]$ and $x[n]$ for S_1.

$$\frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2] = x[n]$$

Comparing with the given equation relating $y[n]$ and $x[n]$, we obtain

$$\alpha = \frac{1}{4}, \quad \beta = 1$$

(b) The difference equation relating the input and output of the system S_1 and S_2 are

$$w[n] = \frac{1}{2} w[n-1] + x[n] \qquad \text{and} \qquad y[n] = \frac{1}{4} y[n-1] + w[n]$$

From these, we can use the method specified in Example 2.15 to show that the impulse response of S_1 and S_2 are

$$h_1[n] = (\frac{1}{2})^n u[n]$$

and

$$h_2[n] = (\frac{1}{4})^n u[n] \cdot$$

Respectively. The overall impulse response of the system made up of a cascade of S_1 and S_2 will be

$$\begin{aligned} h[n] = h_1[n] * h_2[n] &= \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k] \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} n[n-k] \\ &= \sum_{k=0}^{n} (\frac{1}{2})^k (\frac{1}{4})^{n-k} n[n-k] \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^{2(n-k)} = [2(\frac{1}{2})^n - (\frac{1}{4})^n] u[n] \end{aligned}$$

2.21

(c)

$$y[n]=x[n]*h[n]$$

$$=\sum_{k=-\infty}^{+\infty}\left(-\frac{1}{2}\right)^{k}u[k-4]\,4^{n-k}\,u[2-n+k]$$

$$=\sum_{k=-\infty}^{+\infty}4^{n}\left(-\frac{1}{8}\right)^{k}u[k-4]\,u[2-n+k]$$

$$=\begin{cases}4^{n}\sum_{k=4}^{+\infty}\left(-\frac{1}{8}\right)^{k}=\frac{4^{n}\left(-\frac{1}{8}\right)^{4}}{\frac{9}{8}}=\frac{4^{n}}{9\times 8^{3}}=\frac{4^{n}}{4608}, & n\leq 6\\ 4^{n}\sum_{k=n-2}^{+\infty}\left(-\frac{1}{8}\right)^{k}=\frac{4^{n}\left(-\frac{1}{8}\right)^{n-2}}{\frac{9}{8}}=\frac{8^{3}\times\left(-\frac{1}{2}\right)^{n}}{9}=\frac{512}{9}\left(-\frac{1}{2}\right)^{n}, & n>6\end{cases}$$

(d) the desired convolution is

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

$$=x[0]h[n]+x[1]h[n-1]+x[2]h[n-2]+x[3]h[n-3]+x[4]h[n-4]$$

$$=h[n]+h[n-1]+h[n-2]+h[n-3]+h[n-4]$$

This is shown in figure s2.21