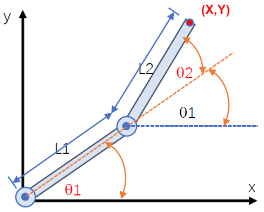
Kinematics of 2D Manipulator



- Forward (Simple geometry & trigo.)
  - $x = l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2)$
  - $y = l_1 sin(\theta_1) + l_1 sin(\theta_1 + \theta_2)$
- Inverse
  - $\theta_1 = arctan\left(\frac{y}{x}\right) - \arcsin\left(\frac{l_2 \sin(\theta_2)}{\sqrt{x^2+y^2}}\right)$
  - $\theta_2 = \arccos\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1 l_2}\right)$

Vector

- Inner product (Dot product)
- Outer product
  - $A \otimes B = AB^T$
  - $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} [b_1 \quad b_2] = \begin{bmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{bmatrix}$

- Eigenvalues & Eigenvectors
  - $Ax = \lambda x;\ A\ is\ a\ matrix, \lambda\ is\ a\ number$
  - $x\ is\ eigenvector\ of\ A,$
    $\lambda\ is\ eigenvalue\ of\ A$
  - **TRUE** when $|A - \lambda I| = 0$

$\dot{x}_1 = x_1 + x_2$ (1)
$\dot{x}_2 = 4x_1 + x_2$

$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Try $x(t) = Ve^{\lambda t}$
$AV = \lambda V$

$\mathbf{det}(\mathbf{A} - \lambda I) = \lambda^2 - 2\lambda - 3 = 0$
$(\lambda - \mathbf{3})(\lambda + \mathbf{1}) = 0$
$\lambda_1 = -\mathbf{1}$
$\lambda_2 = \mathbf{3}$

$\lambda_1 = -\mathbf{1}$;
$(\mathbf{A} - \lambda_1 I)V_1 = 0$
$\left(\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right)\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$2V_{11} + V_{21} = 0$
$2V_{11} = -V_{21}$ if $V_{11} = 1, V_{21} = -2$
$V_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda_2 = \mathbf{3}$;
$(\mathbf{A} - \lambda_2 I)V_2 = 0$
$\left(\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$-2V_{12} + V_{22} = 0$
$2V_{12} = V_{22}$ if $V_{12} = 1, V_{22} = 2$
$V_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\boxed{x(t) = C_1\begin{bmatrix} 1 \\ -2 \end{bmatrix}e^{-t} + C_2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{3t} \qquad x_1(t) = C_1 e^{-t} + C_2 e^{3t}\ ,\quad x_2(t) = -2C_1 e^{-t} + 2C_2 e^{3t}}$

Transformation Matrix (Rotation)

- Notation (T_{AB}/ ${}^A T_B$)
  - $P_A = {}^A T_B \cdot P_B$
- ${}^B R_A = \left({}^A R_B\right)^{-1} = \left({}^A R_B\right)^T$
- counter-clockwise (+), clockwise (-)

2D
- $R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

3D
- $R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$

  $R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$

  $R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- P1 is rotated … by $\theta \rightarrow {}^0 R_1(\theta)$

Composition of matrix

- Just multiply directly
- About current frame ($\times\ back, POST$)
  - $R_x(\theta) \rightarrow R_x(\theta)R_y(\emptyset)$
- About fixed frame ($\times\ front, PRE$)
  - $R_x(\theta) \rightarrow R_y(\emptyset)R_x(\theta)$

$$\boldsymbol{atan2} = \begin{cases} atan\left(\frac{y}{x}\right) ; x > 0 \\ \text{atan}\left(\frac{y}{x}\right) + \pi; x < 0, y \geq 0 \\ \text{atan}\left(\frac{y}{x}\right) - \pi; x < 0, y < 0 \end{cases}$$

$$\left.\begin{matrix} {}^*y \geq 0 \rightarrow (+) \\ y < 0 \rightarrow (-) \end{matrix}\right\} x = 0 \rightarrow \pm 90$$

*Take the sign in $\frac{y}{x} \rightarrow \frac{-0.766}{0.684}$

Minimal representation (Euler angle)

- Represents as $ZX'Z''$ about current axis.
  * *Post Product*
  - $(\phi,\ \theta,\ \psi) \rightarrow R_z(\phi)R_x(\theta)R_z(\psi)$
- Inverse
  - $\theta = \begin{cases} atan2\left(\sqrt{{r_{13}}^2 + {r_{23}}^2}, r_{33}\right); & Q1, Q2 \\ atan2\left(-\sqrt{{r_{13}}^2 + {r_{23}}^2}, r_{33}\right); & Q3, Q4 \end{cases}$

    $\psi = atan2(r_{31}/\mathrm{S}\theta, r_{32}/\mathrm{S}\theta)$

    $\phi = atan2(r_{13}/\mathrm{S}\theta, -r_{23}/\mathrm{S}\theta)$

  $*\,\mathrm{S}\theta = \sin\theta \neq 0$

Minimal representation (Roll-Pitch-Yaw)

- Represents as $Roll \cdot Pitch \cdot Yaw$ about fixed axis.
  Pre product
  - $(\ \psi,\ \theta,\ \phi) \rightarrow R_z(\phi)R_y(\theta)R_x(\psi)$
- Inverse
  - $\theta = \begin{cases} atan2\left(-r_{31}, \sqrt{{r_{32}}^2 + {r_{33}}^2}\right); & Q1, Q4 \\ atan2\left(-r_{31}, -\sqrt{{r_{32}}^2 + {r_{33}}^2}\right); & Q2, Q3 \end{cases}$

    $\psi = atan2(r_{32}/\mathrm{C}\theta, r_{33}/\mathrm{C}\theta)$

    $\phi = atan2(r_{21}/\mathrm{C}\theta, r_{11}/\mathrm{C}\theta)$

  $*\,\mathrm{C}\theta = \cos\theta \neq 0$

Quaternion

- Consists of scalar (w) and imaginary parts (x, y, z)
  - $Q = [w, x, y, z] = [w, \epsilon] = \left[\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) i, \sin\left(\frac{\theta}{2}\right) j, \sin\left(\frac{\theta}{2}\right) k\right]$
- Properties
  - $Q_1 + Q_2 = [w_1 + w_2, \epsilon_1 + \epsilon_2]$
  - $Q^* = [w, -\epsilon]$
  - $||Q|| = \sqrt{w^2 + \epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2} \rightarrow is\ 1\ for\ unit\ quarnion$
  - $Q^{-1} = \frac{Q^*}{||Q||^2}$
  - $Q_1 \circ Q_2 = [w_1 w_2 - \epsilon_1 \cdot \epsilon_2, w_1\epsilon_2 + w_2\epsilon_1 + (\epsilon_1 \times \epsilon_2)]$
- Application on vector v
  - Convert v to $(0, v)$ – quaternion.
  - Apply rotation quaternion $\vec{v}_{rot} = Q \circ \vec{v} \circ Q^*$
  - Answer $\vec{v}_{rot} = (0, v_{rot})$
- Convert to rotation matrix.
  - $R(w, \epsilon) = \begin{bmatrix} 2(w^2 + x^2) - 1 & 2(xy - wz) & 2(xz + wy) \\ 2(xy + wz) & 2(w^2 + y^2) - 1 & 2(yz - wx) \\ 2(xz - wy) & 2(yz + wx) & 2(w^2 + z^2) - 1 \end{bmatrix}$
- Convert back to quaternion.
  - $w = \frac{1}{2}\sqrt{r_{11} + r_{22} + r_{33} + 1}$
  - $\epsilon = \frac{1}{2}\begin{bmatrix} sgn(r_{32} - r_{23})\sqrt{r_{11} - r_{22} - r_{33} + 1} \\ sgn(r_{13} - r_{31})\sqrt{r_{22} - r_{33} - r_{11} + 1} \\ sgn(r_{21} - r_{12})\sqrt{r_{33} - r_{11} - r_{22} + 1} \end{bmatrix} \rightarrow sgn(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$= (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_z)\hat{k}$

- **Product of Quaternions**

$Q_1 \circ Q2 = [w_1 \quad (x_1 \quad y_1 \quad z_1)][w_2 \quad (x_2 \quad y_2 \quad z_2)]$

$= [w_1 \quad v_1][w_2 \quad v_2]$

$= [w_1 w_2 - v_1 \cdot v_2 \quad w_1 v_2 + w_2 v_1 + v_1 \times v_2]$

$= \begin{bmatrix} w_1 w_2 - x_1 x_2 - y_1 y_2 - z_1 z_2 \\ \begin{pmatrix} w_1 x_2 + x_1 w_2 + y_1 z_2 - z_1 y_2 \\ w_1 y_2 - x_1 z_2 + y_1 w_2 + z_1 x_2 \\ w_1 z_2 + x_1 y_2 - y_1 x_2 + z_1 w_2 \end{pmatrix} \end{bmatrix}$

Homogenous Transformation Matix

- ${}^A T_B = \begin{bmatrix} {}^A R_B & {}^A t_B \\ 0 & 1 \end{bmatrix}$, $transformation\ from\ B\ to\ A.$
- ${}^A t_B$ is translation matrix $\rightarrow \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$
  - $is\ a\ vector\ pointing\ from\ A\ to\ B$
- ${}^B T_A = \left({}^A T_B\right)^{-1} = \begin{bmatrix} {}^A R_B & {}^A t_B \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix}$
- ${}^C T_A = {}^C T_B\, {}^B T_A;\ pre\ multiplication\ A \rightarrow B\ then\ B \rightarrow C.$
- Keyword:
  - With respect to A
  - Related to A $\Big\}\ {}^A T_{any\ axis}$
- Position $p \rightarrow [p, 1]^T$

DH Matrix

- θ: $rotation\ about\ z\ axis$
  d: $translation\ along\ z\ axis$
  α: $rotation\ about\ x\ axis$
  a: $translation\ along\ x\ axis$
- Reference all movement to the previous axis.
  - Find θ_1, so find the rotation of frame 1 ref to frame 0 about z axis.
- Each row of the table gives you the translation matrix from i to $i-1$

Joint	θ_i	d_i	a_i	α_i
1	θ_1	d_1	a_1	α_1
2	θ_2	d_2	a_2	α_2
⋮	⋮	⋮	⋮	⋮

$\rightarrow {}^0 T_1$

- ${}^{i-1} T_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

NOTES

- p_0 is rotated in counterclockwise by θ about y axis. $\rightarrow p_1 = {}^1 R_0(\theta) \cdot p_0$
- p_0 is the result from rotating in clockwise by θ about x axis. $\rightarrow p_0 = {}^0 R_1(\theta) \cdot p_1$

From the given rotation matrix equivalent to the Roll-Pitch-Yaw angles, Find rotated angle in $X(\psi)\ Y(\theta)\ Z(\phi)$ when θ's in Quadrant 3.

$$R = R_z(\phi)R_y(\theta)R_x(\psi) = \begin{bmatrix} 0 & -0.866 & 0.5 \\ -0.766 & -0.3214 & -0.5567 \\ 0.6428 & -0.383 & -0.6634 \end{bmatrix}$$

a) $X(\psi) = 30\ deg,\ Y(\theta) = -140\ deg,\ Z(\phi) = 90$ deg.

b) $X(\psi) = 30\ deg,\ Y(\theta) = -140\ deg,\ Z(\phi) = $ -90 deg.

c) $X(\psi) = 30\ deg,\ Y(\theta) = 220\ deg,\ Z(\phi) = 90$ deg.

d) a) and c) are correct.

$\theta = atan2\left(-0.6428,\ -\left[0.383^2 + 0.6634^2\right]^{1/2}\right)$

$= atan2(-0.6428,\ -0.766)$

$= atan\left(\frac{-0.6428}{-0.766}\right) - \pi$

$= -140$

$\psi = atan2\left(\frac{-0.383}{\cos(-140)},\ \frac{-0.6634}{\cos(-140)}\right)$

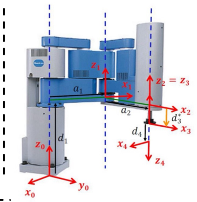
$= atan2(0.5,\ 0.866)$

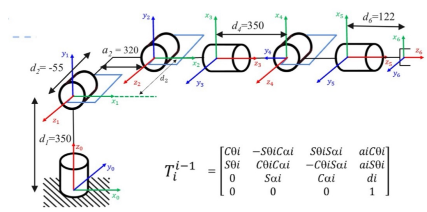
$= 30$

$\phi = atan2\left(\frac{-0.766}{\cos(-140)},\ 0\right)$

$= 90$

Kinematics	θ_i Theta [rad]	d_i [m]	a_i [m]	α_i alpha [rad]
Joint 1	θ_1^*	d_1	a_1	0
Joint 2	θ_2^*	0	a_2	0
Joint 3	0	$-d_3^*$	0	0
Joint 4	θ_4^*	$-d_4$	0	π



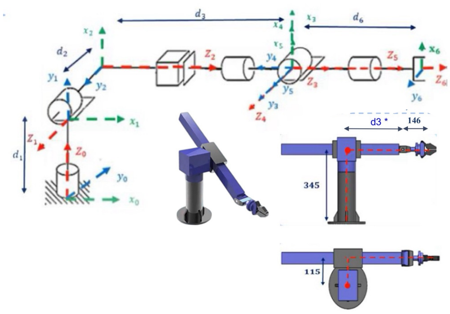


$$T_i^{i-1} = \begin{bmatrix} C\theta i & -S\theta iC\alpha i & S\theta iS\alpha i & aiC\theta i \\ S\theta i & C\theta iC\alpha i & -C\theta iS\alpha i & aiS\theta i \\ 0 & S\alpha i & C\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH parameters

Frame {0}, {1}, {2}, {3}, {4}, {5}, {6}, Joint1, Joint2, Joint3, Joint4, Joint5, Joint6

Kinematics	θ_i Theta [deg]	d_i [mm]	a_i [mm]	α_i alpha [deg]
Joint 1	θ1 *	350	0	90
Joint 2	θ2 *	-55	320	0
Joint 3	θ3* + 90	0	0	90
Joint 4	θ4 *	350	0	-90
Joint 5	θ5 *	0	0	90
Joint 6	θ6 *	122	0	0



Frame {0}, {1}, {2}, {3}, {4}, {5}, {6}, Joint1, Joint2, Joint3, Joint4, Joint5, Joint6

	Kinematics	θ_i Theta [deg]	d_i [mm]	a_i [mm]	α_i alpha [deg]
{0}, {1}	Joint 1	θ1 *	d1 = 345	0	90
{1}, {2}	Joint 2	θ2 * or θ2 * + 90	d2 = -115	0	90
{2}, {3}	Joint 3	0 (Prismatic)	d3 *	0	0
{3}, {4}	Joint 4	θ4 *	0	0	-90
{4}, {5}	Joint 5	θ5 *	0	0	90
{5}, {6}	Joint 6	θ6 *	146	0	0

Find the unit quaternion that is equivalent to the following rotation matrix

$$R = \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$w = \frac{1}{2}\left(-\frac{1}{2} + 1 - \frac{1}{2} + 1\right)^{1/2}$

$= 0.5$

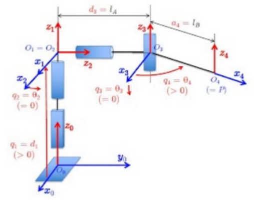
$x = \frac{1}{2}\ sgn(0-0)(1.5 - 1 + 0.5 + 1)^{1/2} = 0$

$y = \frac{1}{2}\ sgn\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)(1 + 0.5 + 0.5 + 1)^{1/2}$

$= -0.866$

$z = \frac{1}{2}\ sgn(1-0)(0.5 - 0.5 - 1 + 1)^{1/2} = 0$

$Q = (0.5,\ 0,\ -0.866,\ 0)$



Kinematics	θ_i Theta [rad]	d_i [m]	a_i [m]	α_i alpha [rad]
Joint 1	0	q_1	0	0
Joint 2	θ_2	0	0	$-\pi/2$
Joint 3	θ_3	l_A	0	$\pi/2$
Joint 4	θ_4	0	l_B	0

Find the rotation of point p= (3, 5, 2) by an angle of 60° about (i, 0, 0)

a) using a rotation matrix,

b) using quaternions.

a) $R_x(60^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix} \rightarrow P = R_x \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0.768 \\ 5.33 \end{bmatrix}$

b) $Q = (\cos 30^\circ,\ \sin 30^\circ,\ 0, 0)$

$= \left(\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 0,\ 0\right)$

$Q^* = \left(\frac{\sqrt{3}}{2},\ \frac{-1}{2},\ 0,\ 0\right)$

$P_o = Q \circ (0, 3, 5, 2) \circ Q^*$

$= \left(\frac{\sqrt{3}}{2} \cdot 0 - \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix},\ \frac{\sqrt{3}}{2}\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} + 0 \cdot \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}\right) \circ Q^*$

$= \left(-1.5,\ \begin{bmatrix} 3\sqrt{3}/2 \\ 5\sqrt{3}/2 \\ \sqrt{3} \end{bmatrix} + 0 + \begin{bmatrix} 0 \\ -1 \\ 2.5 \end{bmatrix}\right) \circ Q^*$

$= \left(-1.5,\ 3\sqrt{3}/2,\ \frac{-2+5\sqrt{3}}{2},\ 2.5+\sqrt{3}\right) \circ Q^*$

$= \left(-1.5 \times \frac{\sqrt{3}}{2} - \begin{bmatrix} 3\sqrt{3}/2 \\ (-2+5\sqrt{3})/2 \\ 2.5+\sqrt{3} \end{bmatrix} \cdot \begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix},\right.$

$\left. -1.5\begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix} + \frac{\sqrt{3}}{2}\begin{bmatrix} 3\sqrt{3}/2 \\ (-2+5\sqrt{3})/2 \\ 2.5+\sqrt{3} \end{bmatrix} + \begin{bmatrix} 3\sqrt{3}/2 \\ (-2+5\sqrt{3})/2 \\ 2.5+\sqrt{3} \end{bmatrix} \times \begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix}\right)$

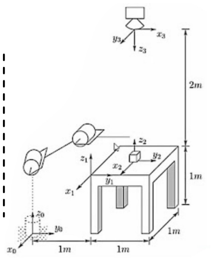
$= (0,\ 3,\ 0.768,\ 5.33)$

What's the composition of rotation matrices?

1. Rotation of 10 deg about the **current** Z axis
2. Rotation of 20 deg about the **fixed** Y axis
3. Rotation of -30 deg about the **fixed** X axis
4. Rotation of 40 deg about the **current** Z axis
5. Rotation of -50 deg about the **current** Y axis

a) $\begin{bmatrix} 0.6503 & -0.7198 & -0.2429 \\ 0.7157 & 0.6877 & -0.122 \\ 0.2548 & -0.0945 & 0.9624 \end{bmatrix}$

$R_x(-30)R_y(20)R_z(10)R_z(40)R_y(-50)$



The figure shows a robot whose base is 1m away from the base of the table. The table is 1m height and its surface is a square. Frame {1} is fixed on a corner of the table. A 20 cm cube is located on the middle of the table, and it has frame {2} attached to its center. A camera is located 2m above the table, just over the cube, and it has frame {3} attached to it.

a) Find the homogeneous transformations that relate each of these frames with the base system {0}

b) Find the homogeneous transformation that relates the cube frame {2} with respect to the camera frame {3}

$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$^0T_2 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

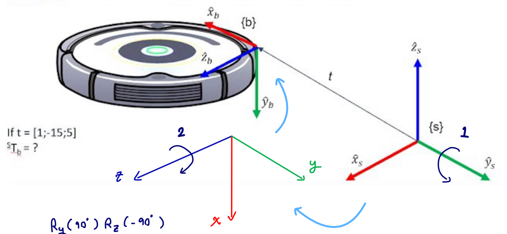
$^0T_3 = R_y(180)R_z(90)\begin{bmatrix} -0.5 \\ 1.5 \\ 3 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) $^3T_2 = {}^3T_0\ {}^0T_2$

$= {}^0T_3^{-1}\ {}^0T_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Example Expressing the configuration of a frame relative to a space frame



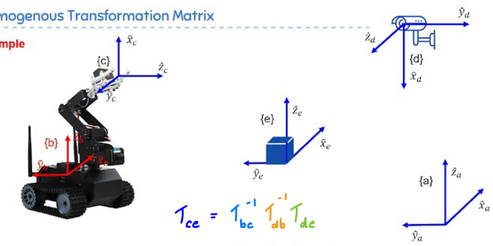
If t = [1;-15;5]
^sT_b = ?

$R_y(90^\circ)R_z(-90^\circ)$

$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \longrightarrow {}^sT_b = \begin{bmatrix} 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & -15 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Homogenous Transformation Matrix

Example



$T_{ce} = T_{bc}^{-1}\ T_{db}^{-1}\ T_{de}$

$T_{db} = \begin{bmatrix} 0 & 0 & -1 & 250 \\ 0 & -1 & 0 & -150 \\ -1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T_{de} = \begin{bmatrix} 0 & 0 & -1 & 300 \\ 0 & -1 & 0 & 100 \\ -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T_{bc} = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 30 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -40 \\ 1 & 0 & 0 & 25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T_{ad} = \begin{bmatrix} 0 & 0 & -1 & 400 \\ 0 & -1 & 0 & 50 \\ -1 & 0 & 0 & 300 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, T_{ce} =?

Find the rotation matrix that is equivalent to the following unit quaternion

The quaternion is $Q = (\frac{1}{2}, 0, -\frac{\sqrt{3}}{2}, 0)$

$2(w^2 + \cancel{x^2}) - 1 = -0.5$	$2(\cancel{xy} - \cancel{wz}) = 0$	$2(\cancel{xz} + wy) = -0.866$
$2(xy + wz) = 0$	$2(w^2 + y^2) - 1 = 1$	$2(\cancel{yz} - \cancel{wx}) = 0$
$2(\cancel{xz} - wy) = 0.866$	$2(\cancel{yz} + \cancel{wx}) = 0$	$2(w^2 + z^2) - 1 = -0.5$

$M = \begin{bmatrix} -0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & -0.5 \end{bmatrix}$

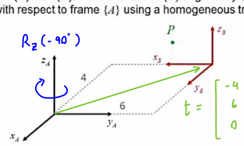
4. If Vector ρ1(3, 7, 4) is rotated in counter-clockwise direction by 180 deg about x-axis, Find the rotation matrix and the coordinate value ρ0(?, ?, ?).

$$^0R_{x1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(180) & -\sin(180) \\ 0 & \sin(180) & \cos(180) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$^0P = {}^0R_{x1}{}^1P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \\ -4 \end{bmatrix}$$

Consider frame $\{A\}$ and $\{B\}$. Point P in frame $\{B\}$ is given by (5, 6, 7) find its coordinates with respect to frame $\{A\}$ using a homogeneous transformation matrix

$R_z(-90^\circ)$



$^AT_B = \begin{bmatrix} 0 & 1 & 0 & -4 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$t = \begin{bmatrix} -4 \\ 6 \\ 0 \end{bmatrix}$ $\quad {}^AT_B P_o = \begin{bmatrix} 2 \\ 1 \\ 7 \\ 0 \end{bmatrix}$

Define values of
θ1 = 0 deg
θ2 = 10 deg
θ4 = 0 deg
θ5 = 90 deg
θ6 = 0 deg
d3 = 100 mm

$^0T_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 345 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T01 = 4x4
     1      0      0      0
     0      0     -1      0
     0      1      0    345
     0      0      0      1
```

$^1T_2 = \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ 0 & 1 & 0 & -115 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T12 = 4x4
   -0.1736       0    0.9848         0
    0.9848       0    0.1736         0
         0  1.0000         0 -115.0000
         0       0         0    1.0000
```

$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T23 = 4x4
     1      0      0      0
     0      1      0      0
     0      0      1    100
     0      0      0      1
```

$^3T_4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T34 = 4x4
     1      0      0      0
     0      0      1      0
     0     -1      0      0
     0      0      0      1
```

$^4T_5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T45 = 4x4
     0      0      1      0
     1      0      0      0
     0      1      0      0
     0      0      0      1
```

$^5T_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 146 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

```
T56 = 4x4
     1      0      0      0
     0      1      0      0
     0      0      1    146
     0      0      0      1
```

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$^0T_6 = {}^0T_1{}^1T_2{}^2T_3{}^3T_4{}^4T_5{}^5T_6 = ?$

```
T06 = 4x4
   -0.9848         0   -0.1736   73.1281
         0   -1.0000         0  115.0000
   -0.1736         0    0.9848  506.1467
         0         0         0    1.0000
```

A frame $\{A\}$ is rotated 90° about x, and then it is translated a vector (6, -2, 10) with respect to the fixed (initial) frame. Consider a point **p** = (-5, 2, -12) with respect to the new frame $\{B\}$. Determine the coordinates of that point with respect to the initial frame

$R_x(90^\circ)\ \&\ \begin{bmatrix} 6 \\ -2 \\ 10 \end{bmatrix} \Rightarrow {}^AT_B = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow {}^AT_B \begin{bmatrix} -5 \\ 2 \\ -12 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ 12 \\ 1 \end{bmatrix}$

1. If Vector ρ0(4, 5, 6) is the results from rotation in counter-clockwise direction by 30 deg about x- axis, Find the rotation matrix and the coordinate value ρ1(?, ?, ?).

$$^0P = {}^0R_{1x}{}^1P$$

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \end{bmatrix}\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \end{bmatrix}$$

$$^1P = {}^0R_{1x}^{-1}{}^0P$$

$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 7.3301 \\ 2.6962 \end{bmatrix}$$

If Vector ρ'''(5, 6, -7) is rotated by rotation matrix equivalent to the Roll-Pitch-Yaw angles ($R_x(\psi)$ 90 deg, $R_y(\theta)$ 200 deg, $R_z(\phi)$ 30 deg). Find the coordinate value ρ0(?, ?, ?).

$R = R_z(30)R_y(200)R_x(90)$

$P_o = R\begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix} = \begin{bmatrix} -4.35 \\ 2.69 \\ -5.93 \end{bmatrix}$