STUDENT NAME

UNIT 4 MATHEMATICAL METHODS

Unit 4 SAC: Integration

Monday 29th July 2024

Reading time: 11:00am to 11:10am (10 minutes)
Writing time: 11:10am to 1:00pm (110 minutes)

QUESTION AND ANSWER BOOK

Number of questions	*Number of marks*
2	40

- Students are permitted to bring into the SAC room: pens, pencils, highlighters, erasers, sharpeners, and rulers
- Students are permitted to have one bound reference/log book, CAS calculator, and scientific calculator
- Students are permitted to have a translation dictionary between English and their chosen language

Materials supplied

- Question and answer book of 10 pages
- Formula sheet
- Working space is provided throughout pages AND extra working out paper can be requested

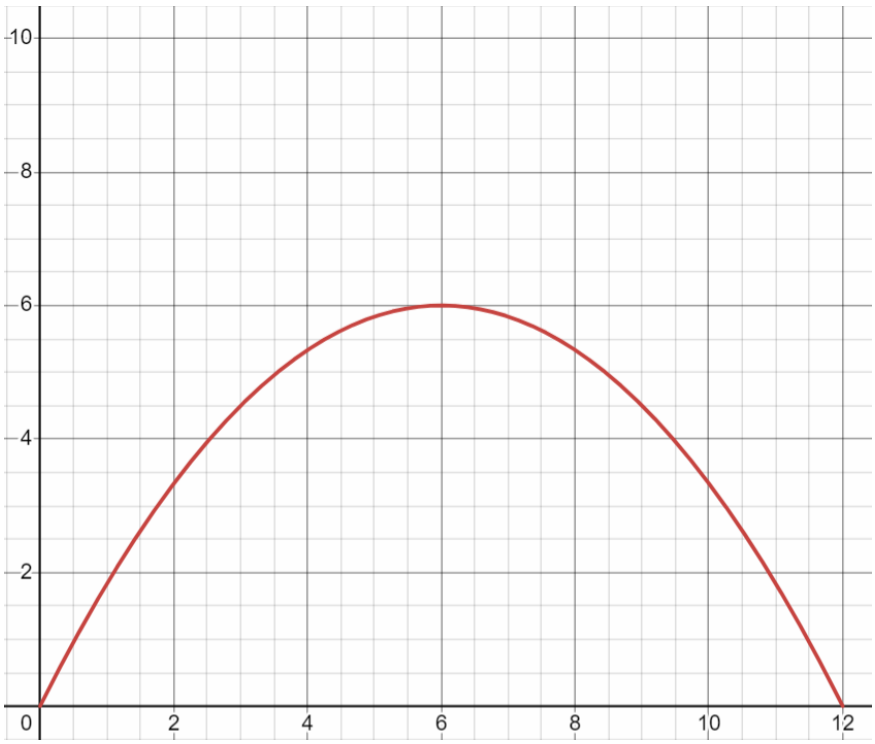
Instructions

- All written responses must be in English.
- Marks will not be deducted for incorrect answers.
- No marks will be given if a question is not attempted, answer provided is unclear/ambiguous, or more than one answer is written.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

Engineers would like to dig a tunnel through a hill for a new train line to go through. The tunnel would have a constant cross-sectional area shown in the diagram. The tunnel area has the shape of a parabola. Units are in metres.



a) The rule for this parabola is . Show that and find the value of . Also write the domain and range of this function. (4 marks)

b) Workers want to roughly estimate how much ground will need to be removed to make the tunnel. Use the trapezium rule, with column widths of 2m, to find the exact estimate of the cross-

sectional area of the tunnel. If the tunnel is 100m long and it costs $1000 per removed, how much is the estimate for the tunnel removal cost, rounded to the nearest whole number?

(3 marks)

c) Use calculus to calculate the exact area. What is the difference between the estimated cost and the actual cost, to the nearest whole number? (2 marks)

d) A new project manager suggests that money could be saved by modelling the cross-sectional area of the tunnel with a **sine** function, with the same x-intercepts and maximum point as the original parabola. They say it would save money because less dirt will need to be removed for the tunnel area.

Find the sine equation that would model the tunnel and sketch it on the same axis above. State whether the new project manager is correct, including how much difference there is to the cost with this new model. (6 marks)

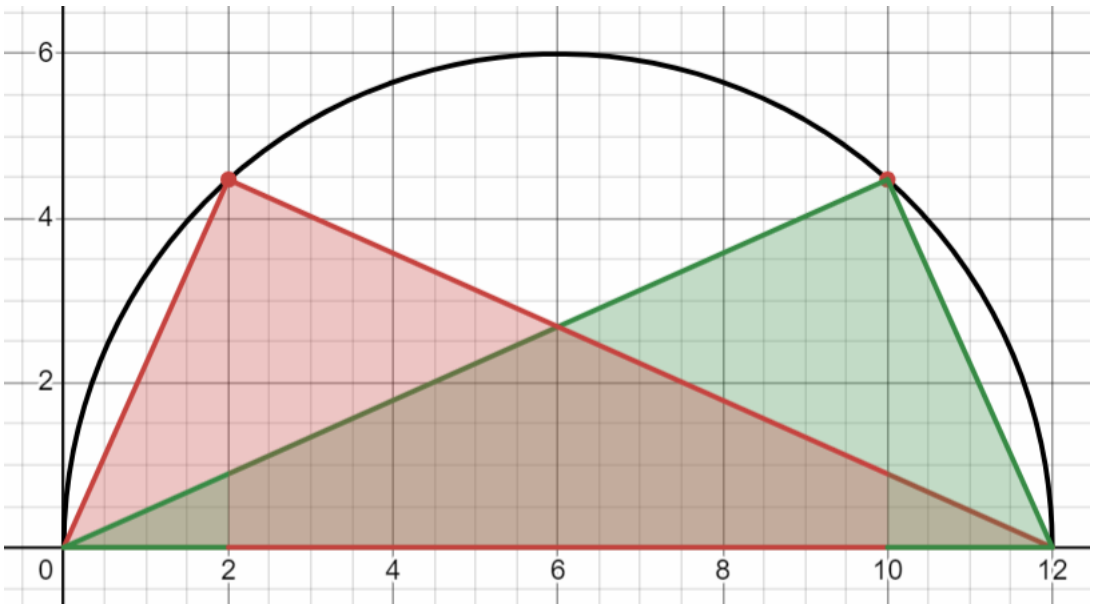
e) The engineers ignore the project manager and use the original parabolic model for the tunnel area. The surface of the hill above the tunnel can be modelled by the function .

Sketch this function on the same axis above, labelling key features, correct to two decimal places.

Engineers know that a maximum of 45 of dirt can be directly above the cross-sectional area of the tunnel before it collapses. Write an equation that would calculate the area of dirt directly above the tunnel. Show whether the tunnel will collapse. An exact value of dirt is not needed. (5 marks)

f) A work experience student, who has been studying hard at Mathematical Methods, has been told to investigate an alternative model for the cross-sectional area of the tunnel. This model uses a semicircle with function .

Two lights will be attached to the tunnel roof at and . The area lit by the lights is shown below.



What percentage of the tunnel cross-sectional area will be lit by the lights? Round your answer to the nearest integer. (4 marks)

g) The largest trucks that might go through the tunnel are 3.9m high and have a width of 2.2m. For two such trucks to pass each other in the tunnel safely (going in opposite directions), they must

have a minimum of 2m between them and a minimum of 1m between the truck and any part of the tunnel wall. Is this possible using the semicircular model? Justify your answer. (4 marks)

Question 2

One of the engineers working on the building site loves to drink tea. She has calculated that the rate of temperature loss from her cup of tea (in per minute) is given by:

for

Where is the total accumulated temperature loss (in) at time minutes after she first makes the cup of tea. **Round all answers to two decimal places.**

a) For what values of time is the rate of temperature loss greater than per minute? (1 mark)

b) When is her tea losing temperature fastest? Justify your answer. (2 marks)

c) What is the average rate of temperature loss over the first 10 minutes after making the cup of tea?

(2 marks)

d) How much total temperature has been lost if she left the tea for an hour after making it? (2 marks)

e) If the cup of tea was initially , write the equation that models the temperature of her tea (not temperature loss). (2 marks)

f) She finally drinks the tea 100 minutes after making it. How much longer did the tea take to lose the last (before it was drunk) than the first (after it was first made)? (3 marks)

END OF TASK