PS 12 NOTES: WEEK 1

TA: WADE HANN-CARUTHERS

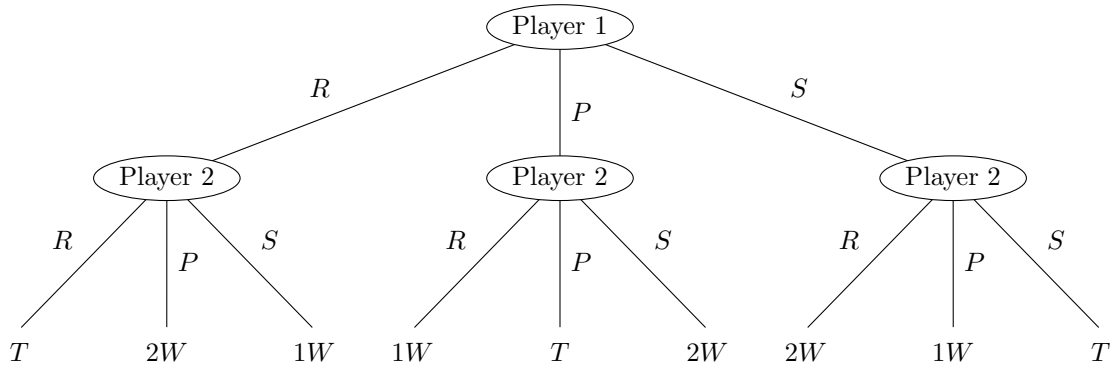
1. Extensive form games

An extensive form game has:

- Players
- Outcomes
- Nodes (decision/chance)
- Branches
- Terminal nodes

The interpretation of the extensive form is that each player chooses one of the available actions ("branches") at each node[1] where it is that player's turn. The game ends at terminal nodes, and each terminal node has an associated outcome. Each player i has a preference, usually denoted $\succ_i$, over the outcomes, and her goal is to get the best outcome she can.

One simple example of an extensive form game is "Silly Rock-Paper-Scissors". The extensive form for this game looks like this:



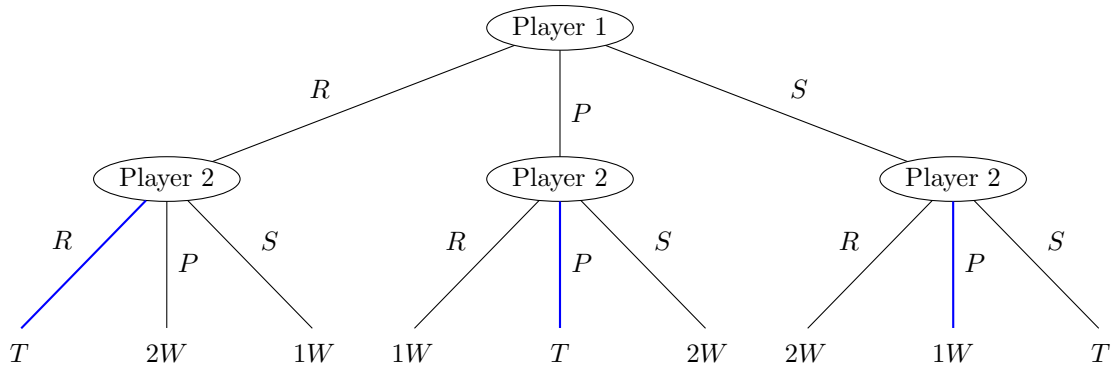
where the player's preferences are $1W \succ_1 T \succ_1 2W$ (read: player 1 prefers winning to tying, and prefers tying to losing) and $2W \succ_2 T \succ_2 1W$.

A *strategy* is a specification of what decision a particular player takes at every decision node associated with her. For example, one (not very good) strategy for player 2 is:

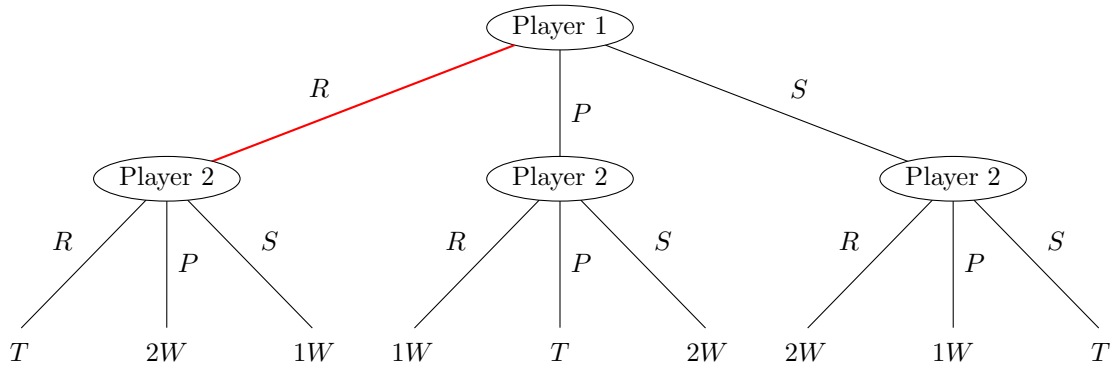
- play R if player 1 plays R
- play P if player 1 plays P
- play P if player 1 plays S

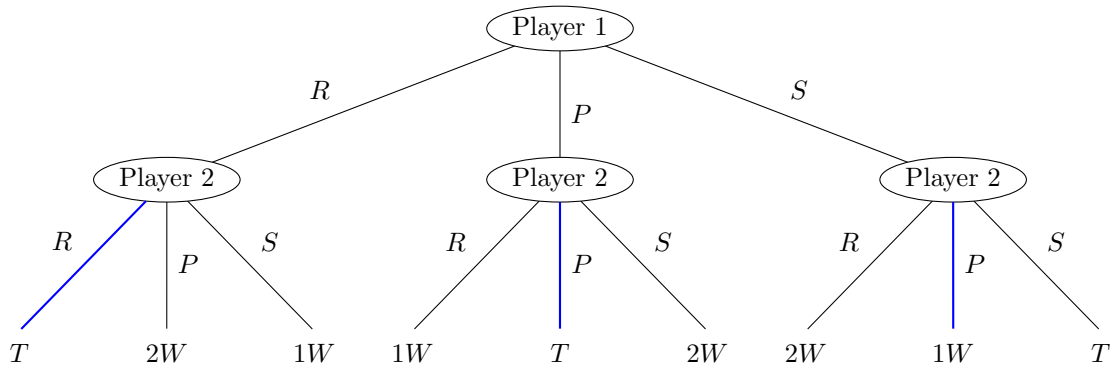
We can make a diagram for this strategy by coloring each of the corresponding branches in the extensive form:

[1]All of the nodes we will see for now are decision nodes; later, we will introduce chance nodes, where the branch followed is not the result of any player's choices but is instead determined randomly.

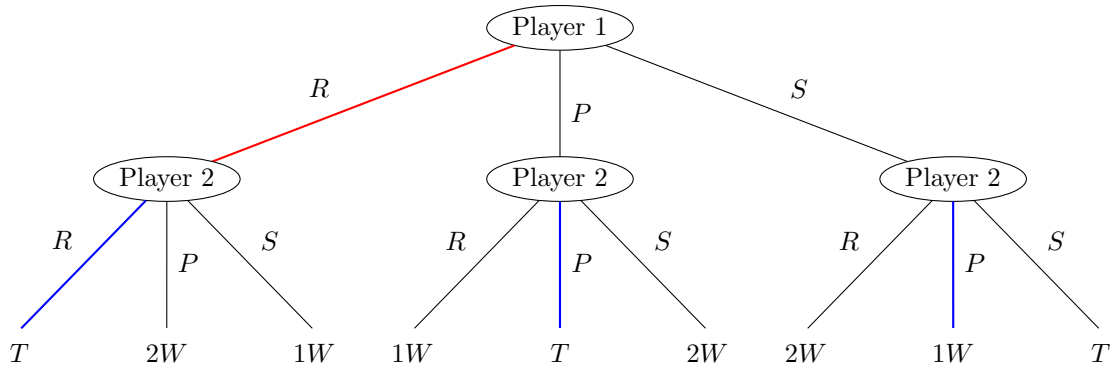


A *strategy profile* is a specification of a strategy for every player. Consider the strategy profile where player 1 plays R and player 2 plays the strategy above. One way we can draw a diagram for this strategy profile is to make a separate drawing for each player:





Another way we could draw a diagram for this strategy profile is to use color the branches for each of the players all on the same extensive form:

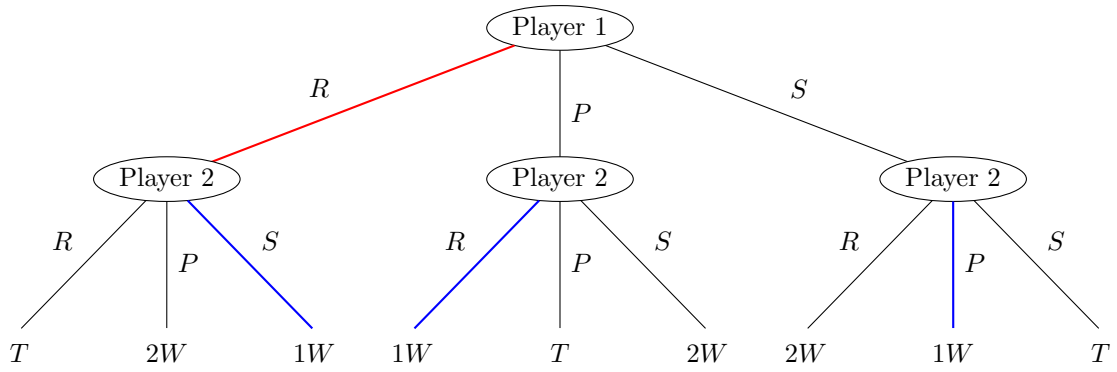


This is the way we will usually draw strategy profiles, since a single diagram is easier to analyze.

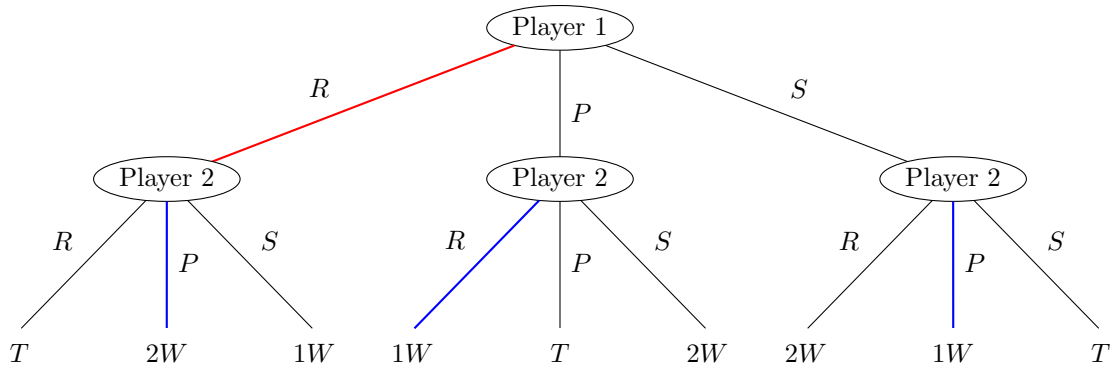
Given a strategy profile, a *unilateral deviation* is a strategy profile where all players except for one use the same strategy. For example, a unilateral deviation from the strategy profile above would be for player 1 to use the same strategy and player 2 to use the strategy:

- play S if player 1 plays R
- play R if player 1 plays P
- play P if player 1 plays S

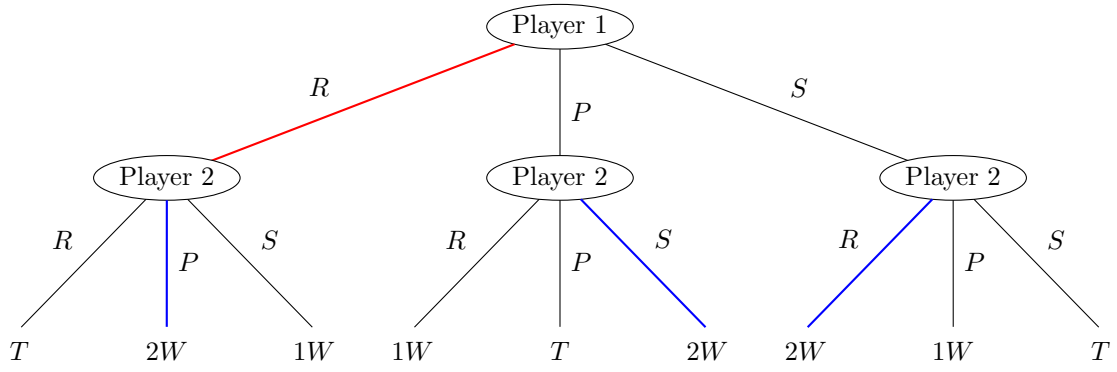
The diagram for this new strategy profile looks like this:



A unilateral deviation is *strictly profitable* if the player whose strategy changes strictly prefers (read: likes more, not the same amount) the outcome under the new strategy profile to the outcome under the old strategy profile. Notice that the unilateral deviation above is not strictly profitable for player 2 (in fact, it makes player 2 strictly worse off.) Here's an example of a unilateral deviation for player 2 that is strictly profitable:

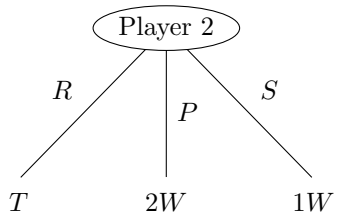


A strategy profile is an *equilibrium* if there is no player that has a strictly profitable unilateral deviation. Notice that this strategy profile is not an equilibrium, because player 1 could change to playing S and be strictly better off. Here is an example of a strategy profile that is an equilibrium:

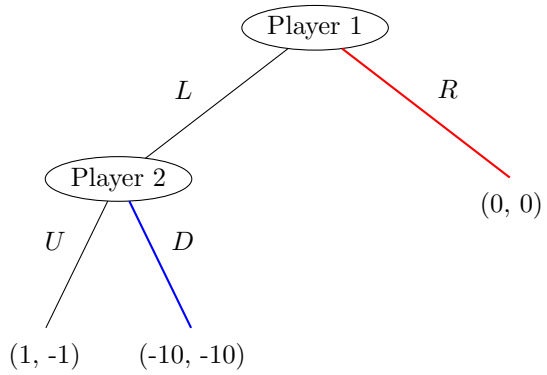


2. Subgame perfect equilibrium

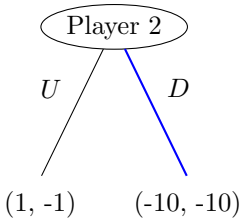
Notice that if you choose any node in the tree of an extensive form game, the subtree rooted at that node also satisfies the definition of an extensive form game. This is called a *subgame*. Here is an example of a subgame of Silly Rock-Paper-Scissors:



The concept of equilibrium is intended to provide predictions about what behavior we should observe when economic or political agents play a game. However, there are obvious instances of equilibria which we would not expect to see. Here is an example of a game and an equilibrium that we would not expect to see:[2]

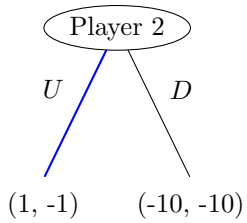


What is strange about this equilibrium is that it is hard to believe that player 2 would actually take the action D if player 1 took the action L. Another way to put this is that restricting the strategy profile to that subgame, like this:

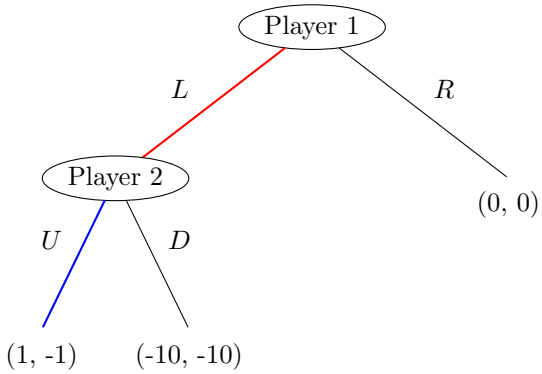


does not result in an equilibrium of the subgame. Because player 2 does strictly better by taking action U than by taking action D in this subgame, the only equilibrium of this subgame looks like this:

[2]Instead of labeling terminal nodes without outcomes, I have labeled them with the "payoffs" the players get, read (player 1 payoff, player 2 payoff).



To capture this idea, that even restricting to subgames we should see reasonable behavior, we use the notion of subgame perfect equilibrium. We say that a strategy profile is a *subgame perfect equilibrium* if for every subgame, the strategy profile that results from restricting to that subgame is an equilibrium of that subgame. Now observe that the only subgame perfect equilibrium of the game from above is:



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