FINAL EXAMINATION
Networked Life (NETS 112)
December 15, 2016
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so. You may also make annotations directly on any diagrams given.

Name:

Problem 1: ________/10

Problem 2: ________/10

Problem 3: ________/15

Problem 4: ________/10

Problem 5: ________/15

Problem 6: ________/10

Problem 7: ________/10

Problem 8: ________/10

Problem 9: ________/10

TOTAL: _________/100

Problem 1 (10 points). Answer "true" or "false" to each of the following assertions.

(a) Adding new roads can only reduce the average driving time, as long as drivers choose their routes in a self-interested fashion.

(b) The average number of retweets of an initial tweet is approximately 10.

(c) Erdos-Renyi networks will generally have less wealth inequality than preferential attachment networks in the Milk-Wheat trading model.

(d) It is not possible to have arbitrarily large networks in which the diameter is 6 or less unless every vertex has at least N/6 neighbors.

(e) Biased voting subjects generally did better on the minority power graphs than on the others.

(f) The virality of Facebook photo sharing is largely determined by features of the photo itself.

(g) Internet routers are responsible for making sure all packets have arrived at their destination, and in the correct order.

(h) A network without any cycles must always have clustering coefficient 0.

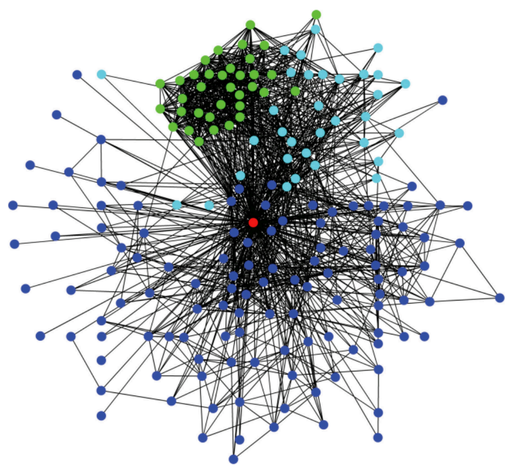
(i) Allowing subjects to create the network themselves made biased voting performance worse.

(j) Our class experiment on the Beauty Contest Game showed slightly worse collective performance than usual.

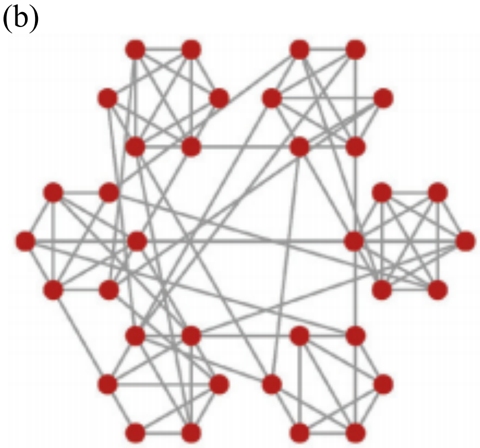
Problem 2 (10 points). Consider the following sentence from one of the assigned readings: *"We can thus sum up all of the actual payments made across all sessions and experiments, and divide it by the sum of all the maximum social welfare payments to arrive at a measure of the overall efficiency of the subject pools over the years."* What reading is this quote from? Discuss the context of the proposed calculation as precisely as you can, and give the value the author(s) obtained from it. Was this value viewed as being high or low?

Problem 3 (15 points). For each of the networks below, describe the generative or real-world process that created it. A precisely as you can, describe the context in which each network was discussed in class.

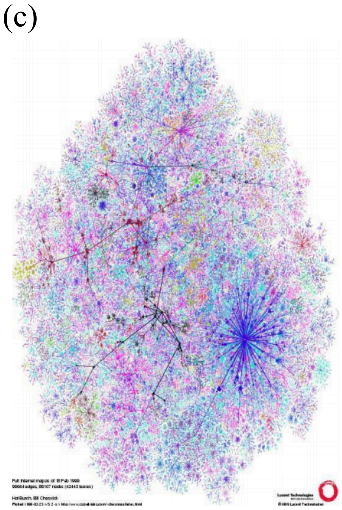
(a)



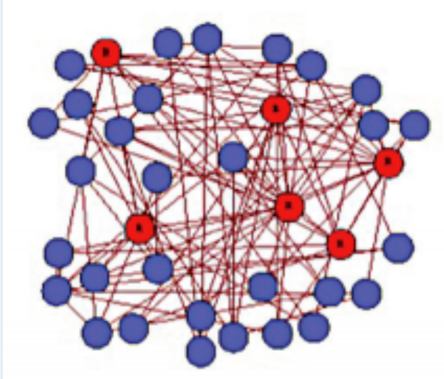
(b)



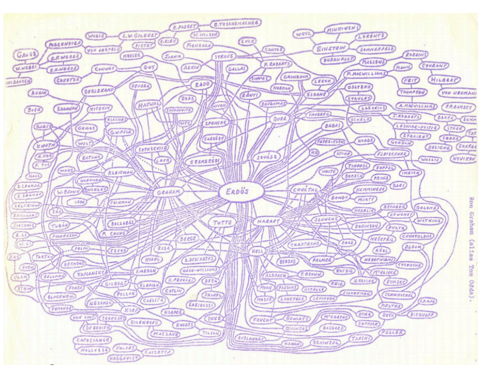
(c)



(d)



(e)



Problem 4 (10 points). For each of the items below, give its precise definition.

(a) vertex

(b) edge

(c) clustering coefficient of a vertex

(d) perfect matching

(e) network diameter

(f) connected component

(g) bipartite network

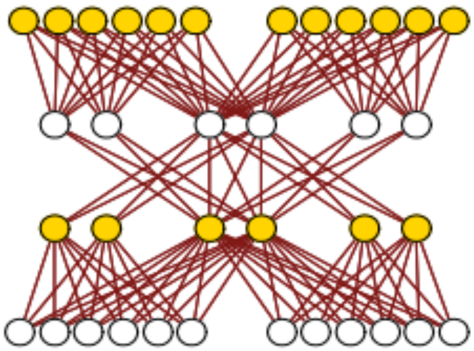
(h) degree distribution

(i) proper coloring

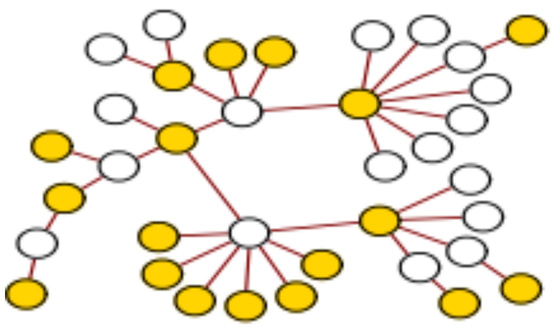
(j) giant component

Problem 5 (15 points). For each of the networks shown below, compute the equilibrium wealths in the Milk-Wheat trading model discussed in class.

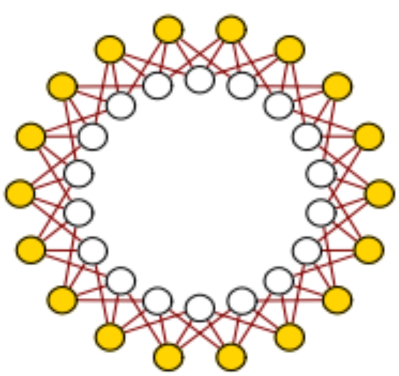
(a)



(b)



(c)



Problem 6 (10 points). Consider the two-player, two action game given by the payoff table below.

2, 2	0, 1
1, 0	1, 1

(a) Describe a simple scenario or story that motivates or justifies the payoffs of this game. Give names to the actions suggested by your motivation.

(b) Does this game have any Nash equilibria? If not, explain why. If so, precisely describe them.

Problem 7 (10 points). List as many differences as you can between the old AT&T telephony network and the modern Internet. Be as precise as possible.

Problem 8 (10 points). As precisely as you can, describe Schelling's housing segregation model and the associated demo we examined in class. What was the main point or lesson of this model and demo?

Problem 9 (10 points). Write a brief essay in which you summarize the topics in the entire course, ideally in the approximate order in which we proceeded.

FINAL EXAMINATION
Networked Life (NETS 112)
December 15, 2015
Prof. Michael Kearns

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If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: ________/10

Problem 2: ________/20

Problem 3: ________/15

Problem 4: ________/10

Problem 5: ________/20

Problem 6: ________/15

Problem 7: ________/10

TOTAL: _________/100

Problem 1 (10 points) For each of the following statements, simply write "TRUE" or "FALSE".

a. Social welfare is always maximized at Nash equilibrium.

b. Preferential attachment is an example of a "rich get richer" process.

c. It is possible for a network to have a high clustering coefficient, yet not contain distinct communities of vertices.

d. If the players in a game repeatedly best-respond to the actions of the other players, they will eventually converge to a Nash equilibrium.

e. If an Internet router is too busy to process an arriving packet, it will notify the sender to try again later.

f. A cycle in which all players purchase immunization is an equilibrium for our network formation game experiment.

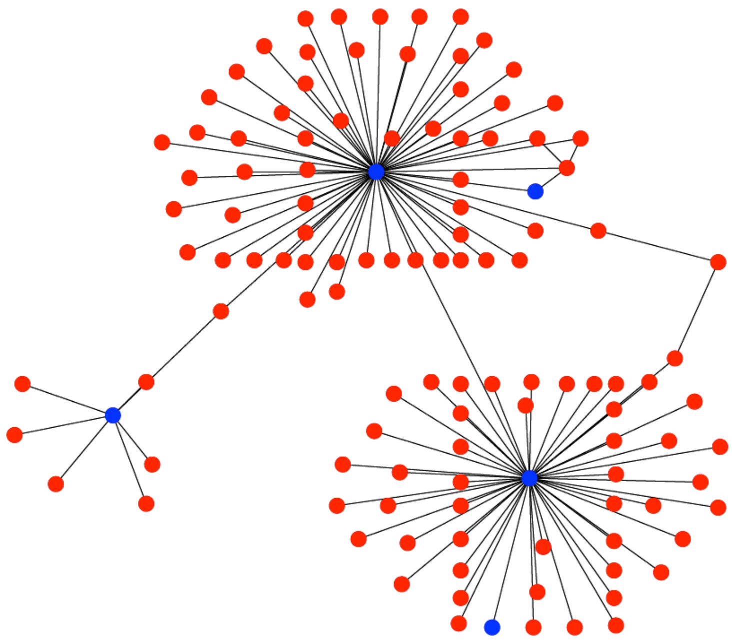
g. The technology underlying Skype is vastly different than that for the traditional land-line telephone network.

h. If we add roads to a network of roads, the average commute time at equilibrium can only improve.

i. Human subjects playing the Ultimatum Game tend to deviate significantly from the theoretical equilibrium.

j. Playing 17 is a better strategy than playing 0 when playing the Beauty Contest Game against a population of human opponents.

Problem 2 (20 points) The image below visualizes the final network collectively built in our behavioral experiment on a network formation game with attack and immunization. The blue vertices have purchased immunization, and the red vertices have not. Which vertex purchased each edge is not indicated, but in general the spokes bought the edges to the hubs.



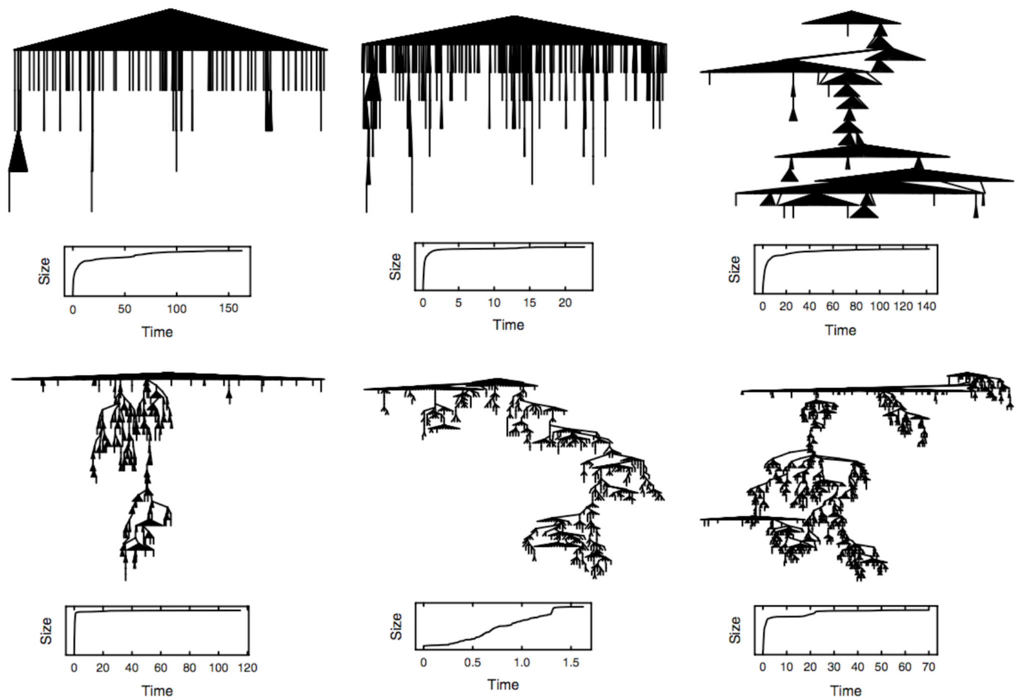
a. Is this network a Nash equilibrium? If not, clearly indicate on the image a player/vertex who is not best-responding to the actions of the other players, and describe what a better choice for this player would be and why.

b. Clearly label on the image all of the players who are receiving the lowest payoff; below do your best to calculate and explain that lowest payoff.

c. Exactly how many of the immunized players are best-responding to the actions of all the other players? Explain your answer below.

d. Suppose we change the behavior of the adversary, so that he now simply picks *any* non-immunized vertex v at random to attack; as before, this attack then spreads and kills all vertices in v's connected component after the removal of the immunized vertices. Clearly identify a vertex in the figure who was best-responding in the original game, but would not be under this new attack model. Explain your answer below.

Problem 3 (15 points) The following image is from one of the required readings.



a. As clearly and precisely as you can, describe what the tree-like structures are visualizing. What data source lies behind them? Exactly what process are they visualizing?

b. What do the authors suggest might account for the structural differences between the first two shallow trees and the deeper remaining four?

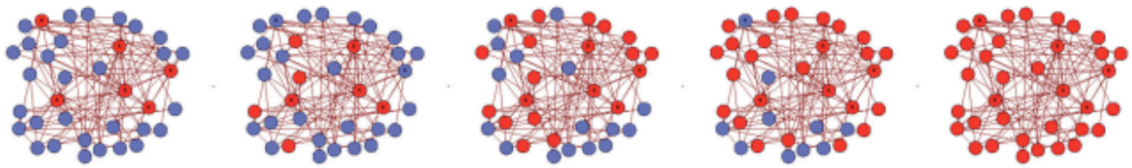
c. Do the authors conclude that the shallow or deep structures are more typical in their data set? What explanations do they provide for their conclusion?

Problem 4 (10 points) For a 2-player game in which all payoffs are positive, we define the Price of Anarchy to be the ratio x/y, where x is the maximum possible value for the sum of the payoffs to the two players, and y is the value of the sum of the payoffs for the two players in a Nash equilibrium.

a. Write down the numerical payoff matrix for a 2-player, 2-action game in which the Price of Anarchy is exactly equal to 1. Explain your reasoning, and clearly indicate the Nash equilibrium of your game.

b. Write down the numerical payoff matrix for a 2-player, 2-action game in which the Price of Anarchy is as large as you can make it. Explain your reasoning, and clearly indicate the Nash equilibrium of your game

Problem 5 (20 points) The following image was discussed in lecture, and is taken from the required reading “Experiments in Social Computation”.



a. Precisely describe the actions and payoffs to the players in this game. What is the main strategic tension that these payoffs create?

b. Name the generative model that produced the network shown, and describe how the different types of players were arranged in the network.

c. Describe what phenomenon the sequence of images is illustrating, and give at least one reason why it perhaps surprising.

d. Can the outcome shown be explained by players simply choosing to agree with the majority of their neighbors? Why or why not?

Problem 6 (15 points) This question is about the graph coloring problem, which we discussed at several points during the semester.

a. Precisely describe what the graph coloring problem is.

b. Precisely describe three different real-world settings which require the solution of an instance of the graph coloring problem.

c. Briefly describe a setting in which we can view graph coloring as a problem of social differentiation.

Problem 7 (10 points) Consider the Erdos-Renyi model of network formation. Suppose that at some point during the process of adding random edges, there exist two connected components, each of size approximately N/2. As precisely as you can, explain why it is extremely unlikely these two components will remain disconnected from each other as we add further random edges.

FINAL EXAMINATION
Networked Life (NETS 112)
December 18, 2014
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: _________ **/10**

Problem 2: _________ **/10**

Problem 3: _________ **/12**

Problem 4: _________ **/12**

Problem 5: _________ **/10**

Problem 6: _________ **/10**

Problem 7: _________ **/12**

Problem 8: _________ **/12**

Problem 9: _________ **/12**

Total: ______________ **/100**

Problem 1 (10 points) Indicate whether the following statements are *True* or *False*.

(a) In the bipartite networked trading model, making the network larger by adding more vertices and edges will always reduce wealth inequality.

(b) Every game with a finite number of players and actions has a mixed strategy Nash equilibrium which assigns non-zero probability to all the actions of all the players.

(c) In the Dodd's et al. paper "An Experimental Study of Search in Global Social Networks", the most frequent reason cited for sending messages to people later in completed chains was similarity in occupation to the target.

(d) The "medals" awarded in our own graph coloring experimental assignment were spread fairly evenly throughout the class.

(e) In the laboratory experiments on biased voting in networks, allowing the players to build the network themselves dramatically degraded collective performance.

(f) It is possible to create a network with 1 billion vertices in which every vertex has degree at most 3 and every pair of vertices has distance at most 3.

(g) The Ultimatum Game demonstrates what behavioral economists refer to as Inequality Amplification.

(h) For the majority of graphs in our experimental graph clustering assignment, the maximum score achieved was above 0.2.

(i) The geographic distances covered by successive router hops keeps increasing as a packet travels from its source to its destination.

(j) Networks in which there is a strong community structure, in the sense of our online clustering assignment, should generally also tend to have high clustering coefficient.

Problem 2 (10 points) Consider the recent online experiments in graph clustering or finding communities. You will have noticed that for some graphs, a much higher score was possible than for others. Clearly describe a probabilistic (randomized) model for generating networks that has a small number of parameters or “knobs”, and which is capable of generating networks in which the maximum score possible is essentially any desired value, depending on how the parameters are chosen.

Problem 3 (12 points) Consider a house with N housemates and a shared Wi-Fi network. Each housemate would like to download their favorite content over the network. If housemate i attempts to download b_i bits of content during a specified time period, then the common speed or rate r at which downloads occur for everyone is $r = 1/(b_1 + b_2 + \ldots + b_N)$, and the payoff or utility to housemate i is then $r \times b_i$. There is no limit on how large b_i may be.

(a) (3 points) Clearly describe the equilibrium of this game. Explain or justify your answer.

(b) (3 points) Clearly describe the maximum social welfare solution – that is, ignoring equilibrium considerations, the choices of actions for the housemates that would maximize their total utility.

(c) (3 points) Based on your answers to (a) and (b), what is the Price of Anarchy for this game?

(d) (3 points) Describe a simple economic or technological mechanism which could cause the housemates to choose the maximum social welfare solution over the equilibrium solution.

Problem 4 (12 points). Consider networks in which there are two types of individuals/vertices. Red vertices are happy if at least half of their neighbors are also Red, and otherwise are unhappy. Blue vertices are happy if at least half of their neighbors are also Blue, and otherwise are unhappy.

(a) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is maximized.

(b) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is minimized.

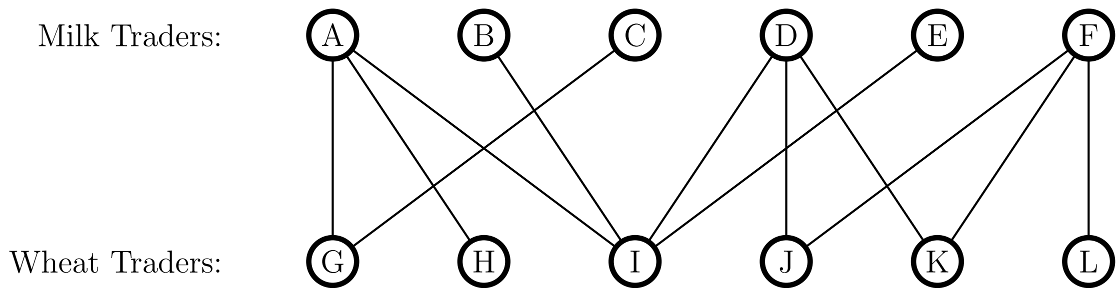
(c) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and only the Red vertices are happy.

(d) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and there are exactly 3 happy Red vertices, and exactly 3 happy Blue vertices.

Problem 5 (10 points) Consider the online experimental assignments on graph coloring, competitive contagion, and network clustering. Each of these assignments had very particular instructions and scoring/grading rules. Carefully compare and contrast these instructions and rules across the three assignments, and discuss what types of behavior or outcomes they were designed to incentivize. Where appropriate, you may want to discuss notions of equilibrium and competition between subjects. Your answer should demonstrate both knowledge of the rules themselves, as well as similarities and distinctions between the different assignments.

Problem 6 (10 points) This problem considers the bipartite networked trading model.

(a) (5 points) What are the equilibrium prices and trades for the network shown below?



(b) (5 points) What is the fewest number of edges you need to add in order for the equilibrium in this network to have no variation in wealth? Clearly list or indicate the added edges.

Problem 7 (12 points) Imagine a driving app that allows a user to specify their desired origin and destination, and then suggests a route to the user in a way that minimizes the collective driving time for all uses. Is it necessarily in a particular driver's best interests to:

(a) (4 points) Use the app at all? Justify your answer.

(b) (4 points) Truthfully report their desired origin and destination? Justify your answer.

(c) (4 points) Follow the route recommended by the app? Justify your answer.

Problem 8 (12 points) Consider the following two-player game. There are two separate islands – Island 1, whose size or area is S_1, and Island 2, whose size is S_2. The two players – call them Red and Blue – can invade only one island, and must decide which one to invade. If one player invades Island i, and the other does not, the payoff to the invader is S_i. If both players choose to invade Island i, they split the territory and each receive payoff $S_i/2$. For each setting of S_1 and S_2 below, carefully describe the Nash equilibria of this game, and the payoffs to the players at equilibrium.

(a) (4 points) $S_1 = 15, S_2 = 15$

(b) (4 points) $S_1 = 21, S_2 = 9$

(c) (4 points) $S_1 = 19, S_2 = 11$

Problem 9 (12 points) Consider our online competitive contagion assignment. Suppose that for some graph G, the empirical distribution of seed pairs chosen by the participants is P. We say that P is an equilibrium if every seed pair (i, j) appearing in P (i.e. with non-zero probability) receives the same average payoff (call it x) against P, and there is no seed pair (i, j) not appearing in P that receives a payoff greater than x against P.

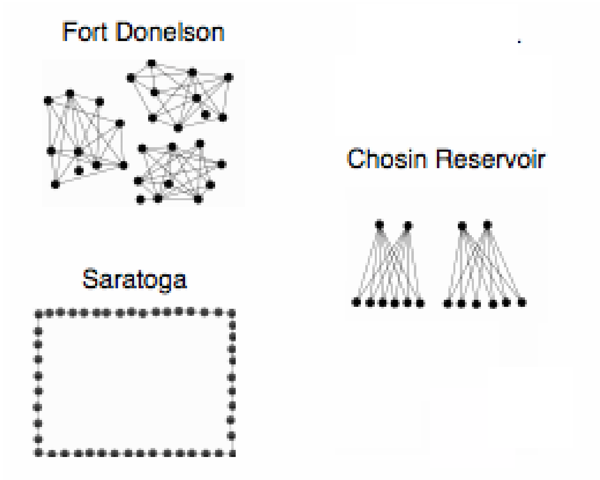


Figure 1: Competitive Contagion Networks

Consider the networks in Figure 1. For each property, mention all the networks that satisfy that property.

(a) (4 points) There is more than one equilibrium distribution.

(b) (4 points) There is an equilibrium distribution where more than one player would play the same seed pair.

(c) (4 points) Any equilibrium distribution requires all the players play the same seed pair.

FINAL EXAMINATION
Networked Life (NETS 112)
December 13, 2013
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: ________/10

Problem 2: ________/15

Problem 3: ________/15

Problem 4: ________/15

Problem 5: ________/20

Problem 6: ________/10

Problem 7: ________/15

TOTAL: __________/100

Problem 1 (10 points) For each lettered item on the left below, write the number of the item on the right that matches it best.

a) Penny matching game ______	1. Equilibrium welfare vs. maximum welfare
b) Magic number 150 ______	2. Inequality aversion
c) Beauty contest game ______	3. Efficient navigation
d) Volleyball ______	4. Quality of service
e) Braess's Paradox ______	5. Multiple pure strategy equilibria
f) Paris Metro Pricing ______	6. Social channel capacity
g) r = 2 in Kleinberg's model ______	7. Attendance dynamics
h) Ultimatum game ______	8. Bounded rationality
i) "Chicken" game ______	9. No pure strategy equilibrium
j) Price of Anarchy ______	10. Selfish routing

Problem 2 (15 points)

a. Describe two key findings of the Travers and Milgram study.

b. Describe two key findings of the Columbia Small Worlds Project.

c. Describe a key finding of the Watts et. al. paper on “Identity and Search in Social Networks.”

d. Describe a key finding of the “Where's George?” study from the Brockmann et. al. paper.

Problem 3 (15 points)

We have discussed several network formation models in which the choice of a parameter controls the structure and dynamics of the resulting network. For each of the following, describe the key properties of the network formed by the given model and parameter choice.

a) Kleinberg's model, r = 0. Recall that the probability of adding an edge to a vertex at grid distance d is proportional to $\left(\frac{1}{d}\right)^r$.

b) Erdos-Renyi model, p > $\frac{1}{N}$, where p is the edge density and N is the network size.

c) Alpha model, α → 0. Recall that the probability of connecting two vertices u and v is proportional to $p+\left(\frac{x}{N}\right)^{\alpha}$, where p is the background edge density, x is the number of current common neighbors of u and v, and N is the network size.

d) Ring-rewiring model, q = 1, where q is the probability of rewiring an edge.

Problem 4 (15 points)

In each part of this problem, you should draw a bipartite network between 4 "Wheat" traders and 4 "Milk" traders who each start with an endowment of 1.0 units of their initial good. The network you draw should have the properties described.

(a) Each vertex has degree 2, and at equilibrium all traders' wealths are equal to 1.0

(b) The deletion of any single edge leaves equilibrium wealths unchanged.

(c) All equilibrium wealths fall in the set $\{1/2, 2/3, 3/2, 2\}$.

(d) The network is connected, and all equilibrium wealths fall in the set $\{1/2, 1, 2\}$.

Problem 5 (20 points)

Imagine that you are the marketing director at a large advertising agency, and are responsible for developing a viral marketing campaign for a new product on Facebook. Your plan is to give the product away to a limited number of carefully chosen "seed" users, in the hopes of gathering "likes" for the product. While Facebook declined your request to see a complete description of their social network, fortunately you were able to download it from the National Security Agency website. Suppose that a typical Facebook user will "like" a product as long as $f(x) > y$, where x is the fraction of the user's neighbors who currently "like" the product, and y is a random number between 0 and 1 that measures the "immunity" of the user.

(a) Suppose the Facebook network structure is highly clustered, and that $f(x) = \sqrt{x}$. Describe how you would choose your seed users, and justify your strategy.

(b) Suppose the Facebook network structure is highly clustered, and that $f(x) = x^2$. Describe how you would choose your seed users, and justify your strategy.

(c) Suppose the Facebook network structure consists of two separate dense and highly clustered groups, with little connectivity between them, and that $f(x) = \sqrt{x}$. Describe how you would choose your seed users, and justify your strategy.

(d) Suppose the Facebook network structure consists of two separate dense and highly clustered groups, with little connectivity between them, and that $f(x)=x^2$. Describe how you would choose your seed users, and justify your strategy.

(e) Suppose the Facebook network structure was generated by preferential attachment, and that $f(x)=x$. Describe how you would choose your seed users, and justify your strategy.

Problem 6 (10 points)

Suppose you make a Skype call from your laptop to a friend in Europe. In as much detail as possible, describe both the technical process by which the call is conducted over the Internet, and the potential strategic, economic or game-theoretic issues that such a process entails. Then briefly describe any aspects of your answer that would change if instead of a Skype call, you send a short email to your friend.

Some topics you might want to touch upon in your answers: packets, routing, the IP protocol, IP addresses, TCP/IP, autonomous systems, the border gateway protocol, Quality of Service guarantees, bandwidth, latency, congestion, the Price of Anarchy, customer-provider, peer networks.

Problem 7 (15 points)

Throughout the course, we considered a variety of models for network formation in which the decisions to connect vertices are made probabilistically or randomly (as in Erdos-Renyi and Preferential Attachment). In this problem, we consider network formation from a game-theoretic perspective.

Consider a one-shot, simultaneous-move game in which each of N players must choose which other players they will purchase an edge to. The collection of all edges purchased by the players results in some overall network G. The payoff to a particular play i is then defined to be $B(G,i)-K\times C$, where K is the number of edges purchased by player i, C is some fixed cost per edge, and B(G,i) measures the "network benefit" enjoyed by player i in the overall network G. For example, B(G,i) might be the PageRank of i in G (so each player would like to balance being "important" with their edge expenditures), or B(G,i) might be the clustering coefficient of i in G (so each player wants to be in a tight-knit circle of friends without spending too much on edges). Note that in such games, a player may benefit from the edges purchased by other players, not just their own edge purchases.

(a) Must it be that for any choice of the function B(G,i) and any value for C, the resulting game has a Nash equilibrium? Why or why not?

(b) Suppose B(G,i) is the size of the connected component of player i in G, and that the edge cost C is very small (say, just slightly larger than 0). What can you say about the networks that can be formed at equilibrium? For instance, will they be connected? Why or why not? Can they contain cycles? Why or why not? Can the equilibrium networks have large diameter? Why or why not?

(c) Briefly describe any strengths or weaknesses you perceive regarding such game-theoretic network formation models, relative to the probabilistic ones we studied in the course. You may want to take into consideration the various properties of the probabilistic models we considered, and the motivation behind them.

FINAL EXAMINATION
Networked Life (MKSE 112)
December 14, 2012
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: ________/10

Problem 2: ________/15

Problem 3: ________/15

Problem 4: ________/15

Problem 5: ________/15

Problem 6: ________/10

Problem 7: ________/20

TOTAL: _________/100

Problem 1 (10 points) For each of the following statements, simply write "TRUE" or "FALSE".

a. Wealth variation at equilibrium in the trading model we studied is characterized by whether the network is connected or not.

b. The distance between two vertices is defined as the length of the shortest path between them.

c. The assigned reading "The Scaling Laws of Human Travel" relies on data about the geographic distances between Facebook friends.

d. Different types of Internet services (e.g. Skype, Facebook, web browsing) have specialized types of packets associated with each of them.

e. The behavioral experiments on biased voting established that a small but well-connected minority could systematically control the collective outcome.

f. The Columbia Small Worlds experiment reverses almost all of the findings of Travers and Milgram.

g. Schelling's segregation model was designed to demonstrate that segregation only arises when individuals have extreme preferences regarding their neighbors.

h. The "traceroute" program we demonstrated in class outputs the sequence of routers encountered as a packet travels through the Internet.

i. Even at low edge density, a bipartite version of the Erdos-Renyi model is likely to generate networks containing a perfect matching.

j. A pure Nash equilibrium always exists if the players have only two actions available.

Problem 2 (15 points) Define a vertex u in a network to be *crucial* if there exist two other vertices v and w such that there is a path from v to w that goes through u, but if u is deleted from the network, there is no path from v to w.

a. For any given number of vertices N, describe what network structure results in the greatest number of crucial vertices.

b. Draw an example of a network in which there are no crucial vertices.

c. Is it possible to have networks in which every vertex is crucial? Why or why not?

Problem 3 (15 points) Suppose you use a high-level application like Skype to initiate a call, or an email client to send an email. In as much detail as you are able, describe the process by which packets are generated and travel through the Internet in response. Topics to include are: what the packets contain, how they travel and are forwarded through the Internet, the series of devices and types of organizations the packets encounter, and how the Skype call or email is rendered on the receiving end. Greater detail will earn greater credit.

Problem 4 (15 points) Write a brief essay in which you compare and contrast the following three problems:

- navigation in social networks (Travers&Milgram, Columbia Small Worlds, Kleinberg's model)
- packet routing in the Internet (both routing table-based, and "selfish"/source routing)
- driving or commuting over a network of physical roadways

You should discuss the ways in which these problems are similar, and different, and support your claims with course readings and lectures.

Problem 5 (15 points) In lecture we briefly discussed the example of email spam as a problem with both technological approaches (spam filters, email blacklists and whitelists) and economic approaches (micropayments for email; that is, very tiny payments for sending email that would be negligible for ordinary users but prohibitive for professional spammers). Give another example, *different from any given in class*, of a problem with both technological and economic solutions. Identify what the proposed solution(s) are in each case.

Problem 6 (10 points)

a. Draw the simplest network you can of 15 vertices in which the diameter is at least 5, the clustering coefficient is much higher than the edge density, and the degree distribution is *not* heavy-tailed.

b. On the same network diagram above, using dashed instead of solid lines, clearly add a couple of edges so that the diameter will now be less than 4, while the clustering coefficient and degree distribution do not change much.

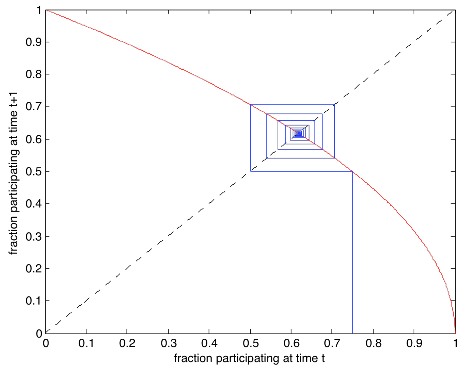


Figure 1

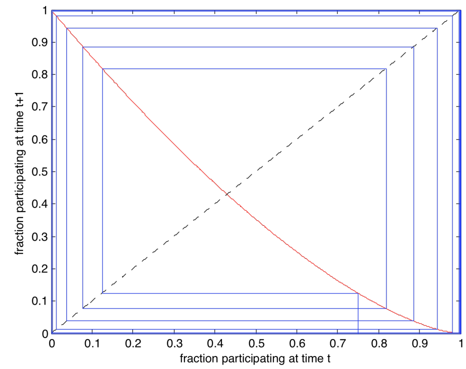


Figure 2

Problem 7 (20 points) The two figures above show, in the red curves, models for the amount of collective participation in some activity at time t+1 (y axis) as a function of the amount of participation at time t (x axis). Answer the following questions *for each of the two figures.* You may clearly annotate the diagrams if it will help clarify your answers.

a. How many equilibrium points are there, and are they stable or unstable?

b. What will be the limiting behavior of the population if we repeat the process indefinitely (i.e. let t go to infinity?)

c. If in part b. you answered differently for the two figures, discuss why you think the limiting behavior is different; and if you answered the same for the two figures, discuss why you think the limiting behavior is the same.

FINAL EXAMINATION
Networked Life (MKSE 112)
December 8, 2011
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: ________/10

Problem 2: ________/10

Problem 3: ________/10

Problem 4: ________/15

Problem 5: ________/10

Problem 6: ________/20

Problem 7: ________/10

Problem 8: ________/15

TOTAL: _________/100

Problem 1 (10 points) For each of the following statements, simply write "TRUE" or "FALSE".

a. The Price of Anarchy measures the extent to which a Nash equilibrium is better than the maximum social welfare solution.

b. In the behavioral experiments on biased voting, the well-connected minority tended to lose.

c. An edge always connects two vertices of equal degree.

d. The Marvel Comic Universe in some ways resembles a real social network.

e. A completely connected bipartite graph between Milks and Wheats may sometimes exhibit wealth variation at equilibrium.

f. If you know the Pageranks of all the pages that point to page P, and the Pageranks of all the pages that P points to, you can determine the Pagerank of P.

g. Segregation within a population can always be explained by strong discriminatory preferences of one of the types.

h. Low clustering coefficient and a heavy-tailed degree distribution are incompatible properties.

i. A monotone property of networks is one that can't go away by adding more edges to the network.

j. In the Erdos-Renyi model with N vertices, most interesting structural properties arise when p is approximately 1/sqrt(N).

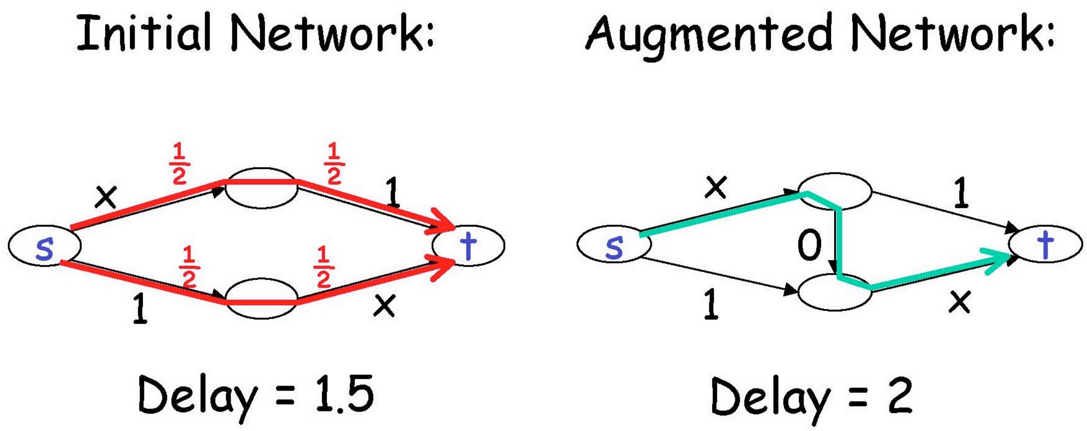
Problem 2 (10 points) In lecture we discussed many real-world quantities whose empirical distributions exhibit long tails. Pick and discuss any such quantity *other* than degree distributions of networks. Be as precise as possible about exactly what quantity you are discussing, and give a clear description of the process by which you think it came to have long tails.

Problem 3 (10 points) The following output is the result of running the "traceroute" command demonstrated in class. Briefly but precisely describe what this command is doing, and what its output is showing. Discuss any organizational, business and geographical inferences you can make from this trace.

```
bash-3.2$ traceroute www.cs.stanford.edu
traceroute to cs.stanford.edu (171.64.64.64), 64 hops max, 52 byte packets
 1  seas-apn-gw.router.upenn.edu (158.130.104.1)  1.669 ms  1.028 ms  1.024 ms
 2  external3-core2.dccs.upenn.edu (128.91.10.2)  1.067 ms  5.897 ms  4.505 ms
 3  external-core1.dccs.upenn.edu (128.91.9.1)  1.367 ms  1.095 ms  1.136 ms
 4  local.upenn.magpi.net (216.27.100.73)  1.951 ms  2.613 ms  9.942 ms
 5  remote.internet2.magpi.net (216.27.100.54)  21.424 ms  11.438 ms  7.331 ms
 6  64.57.28.193 (64.57.28.193)  8.745 ms  11.784 ms  8.848 ms
 7  ae-8.10.rtr.atla.net.internet2.edu (64.57.28.6)  22.381 ms  24.723 ms  22.859 ms
 8  xe-1-0-0.0.rtr.hous.net.internet2.edu (64.57.28.112)  45.974 ms  59.457 ms  98.308 ms
 9  * * ge-6-1-0.0.rtr.losa.net.internet2.edu (64.57.28.96)  475.088 ms
10  hpr-lax-hpr--i2-newnet.cenic.net (137.164.26.133)  77.909 ms  77.845 ms  77.838 ms
11  svl-hpr2--lax-hpr2-10g.cenic.net (137.164.25.38)  86.509 ms  86.313 ms  91.700 ms
12  hpr-stanford--svl-hpr2-10ge.cenic.net (137.164.27.62)  86.844 ms  102.996 ms  104.788 ms
13  boundarya-rtr.stanford.edu (171.66.0.34)  172.854 ms  86.867 ms  86.742 ms
14  bbra-rtr.stanford.edu (171.64.255.129)  120.103 ms  89.006 ms  87.124 ms
15  yoza-rtr-a.stanford.edu (171.64.255.144)  87.040 ms  86.913 ms  87.120 ms
16  cs.stanford.edu (171.64.64.64)  87.844 ms  87.193 ms *
```

Problem 4 (15 points) In the first half of the class, we were primarily concerned with the dynamics of contagion in networks; in the second half, with "rational" dynamics. Write a brief essay in which you discuss the differences and similarities between the two. Be sure to give examples of each, and also to discuss the differing notions of outcome we employed in the analysis of each type of dynamics. You may want to refer to some of the course readings in your essay.

Problem 5 (10 points) The following diagram illustrates Braess' Paradox. Briefly but precisely describe what the figure is showing, and why there is a paradox. What implications might Braess' Paradox have for traffic management in large cities?



Problem 6 (20 points) Write 10 single sentences, each articulating a distinct principle, phenomenon or fact that you learned in this class. In addition to accuracy and clarity, your answers will be graded for their generality and their diversity.

Problem 7 (10 points) Clearly draw a connected, bipartite network between 4 Milk players and 4 Wheat players in which (a) there is wealth variation at equilibrium, but (b) there is a single edge whose addition eradicates wealth variation at equilibrium. Clearly annotate your diagram with the numerical wealths of all players for part (a), and with the added edge for part (b).

Problem 8 (15 points) The table below represents the payoffs for a two-player, one-shot, simultaneous move game of the type we considered in lecture.

	A	B
One	10, 10	1, 11
Two	11, 1	2, 2

(a) Is there a Nash equilibrium for this game? If so, what is it?

(b) What is the Price of Anarchy for this game?

(c) Changing only two numbers in the table, alter the payoffs so that the Price of Anarchy becomes as small as possible. Indicate the changes clearly on the diagram.

FINAL EXAMINATION

Networked Life
CIS 112
Prof Michael Kearns

May 5, 2009

This is a closed-book examination. You should have no materials on your desk other than this exam and a pen or pencil. If you need more space to answer a problem, use the reverse side of the same page and clearly indicate you have done so.

YOUR NAME: ________________________________

Problem 1 _______/10

Problem 2 _______/10

Problem 3 _______/10

Problem 4 _______/10

Problem 5 _______/10

Problem 6 _______/10

Problem 7 _______/10

Problem 8 _______/20

Problem 9 _______/10

TOTAL: _______/100

Problem 1 (10 Points) For each item on the left, write the index of the item on the right which matches best.

a. minority power ____

b. frequency of English words ____

c. rubber necking ____

d. generates high clustering & small diameter ____

e. Kleinberg-esque network formation game____

f. an abstract commodity ____

g. generates low clustering & small diameter ____

h. behavioral game theory ____

i. Nash equilibrium ____

j. navigation in a small world ____

k. Paris Metro Pricing ____

l. inequality aversion ____

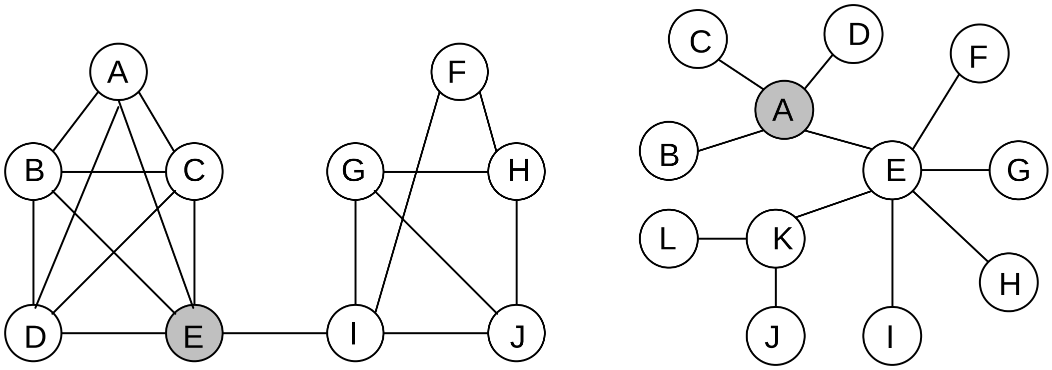
m. theory of efficient markets ____

n. consensus ____

o. magic number 150 ____

1. alpha model
2. Arrow & Debreu
3. biased voting experiments
4. no one can unilaterally improve his payoff
5. efficient navigation when r = 2
6. the ultimatum game (nearly all cultures)
7. preferential attachment
8. money
9. a.k.a. the social channel capacity
10. networks of constant diameter for a <= 2
11. Zipf's law
12. bounded rationality
13. yet another unfortunate equilibrium
14. decongestion through differential pricing
15. a game that's the opposite of coloring

Problem 2 (10 points)



(Left) (Right)

For the “left” and “right” networks above:

(a) Write the letter of the vertex with the highest degree and give its degree.

Left: _________ Right: _________

(b) State the size of the worst case diameter.

Left: _________ Right: _________

(c) State whether the network is bipartite.

Left: _________ Right: _________

(d) Give the clustering coefficient of the gray colored vertex.

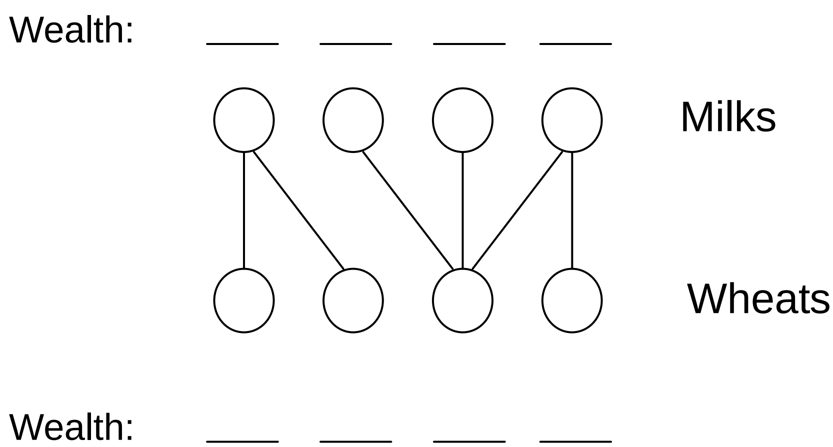
Left: _________ Right: _________

(e) State which model --- Erdos-Renyi, preferential attachment, or the alpha model --- was most likely to have generated the network.

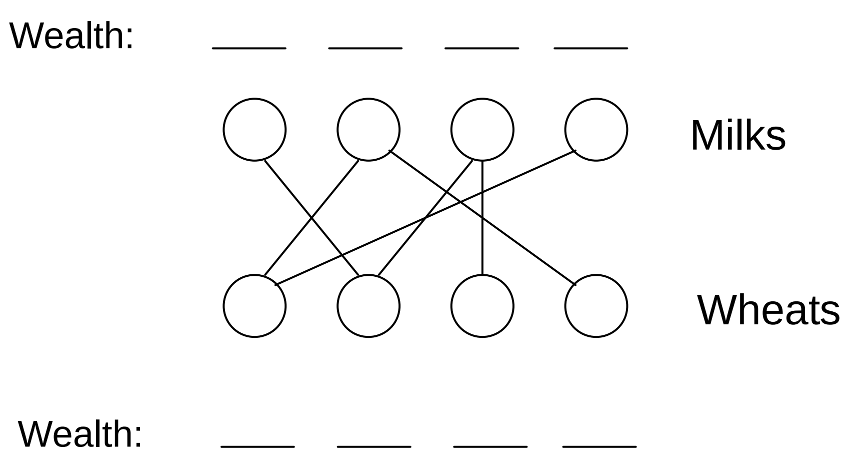
Left: _________ Right: _________

Problem 3 (10 points) For the networks below, write the equilibrium in the space beside each vertex, assuming all vertices are initially endowed with one unit of milk or wheat and have preferences only for the other good.

(a)

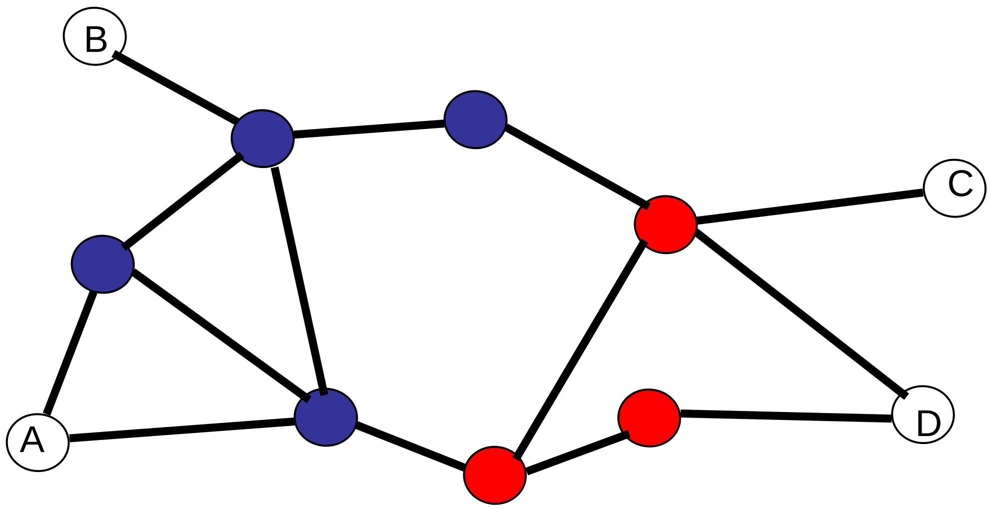


(b)



Problem 4 (10 points) Briefly discuss two problems or protocols arising on the Internet whose formulation or solution involves game-theoretic or economic considerations. For each, clearly identify the parties involved, their individual incentives, and how those incentives interact to create strategic tension. Describe any underlying technological facts (e.g. how a protocol is designed to work) needed to explain these strategic tensions.

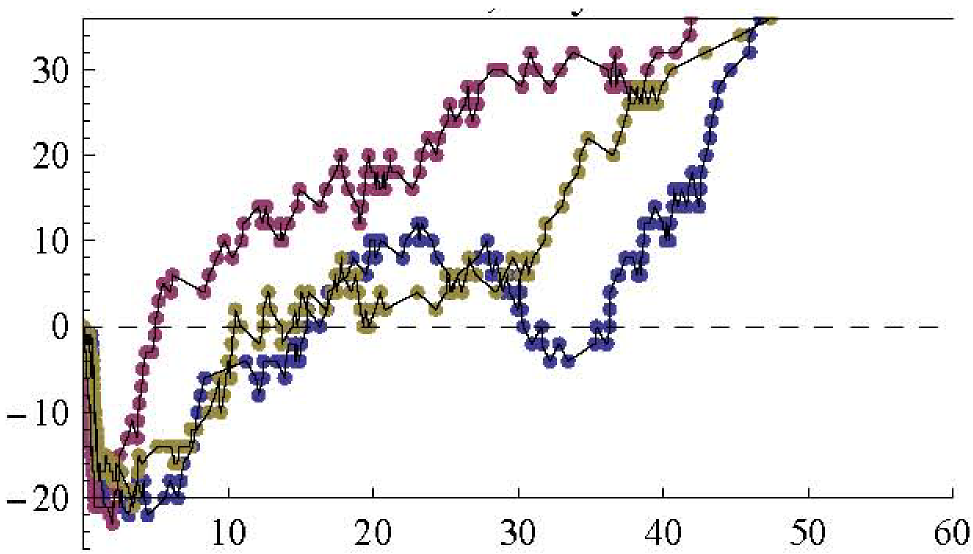
Problem 5 (10 points) Describe and discuss two games (in the formal sense of game theory) in which it is known that human subject behavior deviates from equilibrium predictions, and discuss the nature of this deviation.



Problem 6 (10 points) The network diagram above shows 4 Internet end-users: A, B, C and D. Users A and B have their Internet service provided by Provider Blue, who operates the routers shown in blue, while users C and D have their Internet service provided by Provider Red, who operates the red routers. Providers Red and Blue operate independently, and each has the incentive to get traffic destined for parties on the other network off of their own network as quickly as possible (that is, with the fewest router hops --- “early-exit” routing).

Will the combined early-exit routing behavior of Providers Red and Blue guarantee that all traffic between all pairs of end users will always travel on a *globally* shortest path? Answer yes or no. If your answer is no, give a specific counterexample to global optimality, clearly annotating the diagram with both the path actually taken and the globally shortest path.

Problem 7 (10 points) The image below is reproduced from the required reading "Behavioral Experiments on Biased Voting in Networks", which was also discussed in class.



(a) Clearly describe what is being plotted in this diagram --- i.e. what is measured by the x-axis and the y-axis. Be as precise and complete as you can --- for instance, your answer should make it clear why there can be negative y values.

(b) What network structure was underlying the experiments represented in the diagram? What interesting behavioral phenomenon do the plots show?

Problem 8 (20 points). For this problem you are asked to reconsider the network formation game introduced in the last problem of Homework 3. To review: Consider a network formation game in which each of N players may purchase edges from their own vertex to other players for a fixed cost of c per edge. Let G denote the network formed by the collective edge purchases of all players. The overall payoff to a given player X is then equal to the *number of players X is connected to in G, minus the total edge expenditures of player X*. By "connected to", we mean reachable by any finite-length path. Note that we view this as a one-shot game, in which all players simultaneously decide which edges to purchase.

(a) Consider a network that is a simple cycle over the N players. Are there values for the edge cost c such that this network is an equilibrium of the formation game? If not, why not? If so, which value(s) of c?

(b) Consider a network that is a line or chain over the N players:

1 --- 2 --- 3 --- 4 --- 5 --- … --- N-1 --- N

Suppose that c = N/4. Is it possible for this network to be an equilibrium? If not, why not? If so, describe who would purchase which edges at equilibrium.

(c) Repeat part (b) but for edge cost $c = 3N/4$.

(d) Suppose that $c = 6$ and that N is very large. Consider the “universal” structural properties of social networks we discussed in the first half of the course: small diameter, heavy-tailed degree distributions and high clustering coefficient. For each of these properties, briefly discuss whether the equilibrium networks of the formation game *must always* have the property, *may sometimes* have the property, or *will never* have the property.

Problem 9 (10 points) Briefly describe Braess' Paradox, illustrating it in a simple network diagram with whatever annotations you need. Why is it called a paradox?

FINAL EXAMINATION

Networked Life
CIS 112
Prof Michael Kearns

May 7, 2008

This is a closed-book examination. You should have no materials on your desk other than this exam and a pen or pencil.

YOUR NAME: ________________________

Problem 1 _______/10

Problem 2 _______/10

Problem 3 _______/10

Problem 4 _______/15

Problem 5 _______/10

Problem 6 _______/10

Problem 7 _______/10

Problem 8 _______/15

Problem 9 _______/10

TOTAL: _______/100

Problem 1 (10 Points) For each item on the left, write the index of the item on the right which matches best.

a. kings and pawns ____	1. market for lemons
b. Gladwell ____	2. a model for network clustering
c. maximum social welfare ____	3. money
d. IDS ____	4. mixed strategy equilibrium

e. Poisson distribution ____

f. Caveman & Solaria ____

g. 1/3 for selfish routing ____

h. they always “tip” ____

i. IP ____

j. iterated dominance ____

k. Travers & Milgram ____

l. every game has one ____

m. result of top-down design ____

n. information asymmetry ____

o. alpha = 2 ____

p. encodes all pair-wise exchange rates ____

q. trivial for centralized computation ____

r. early exit routing ____

s. rock paper scissors ____

t. frequency of English words ____

5. “knife’s edge” result

6. full / no investment at equilibrium

7. exponential decay from the mean

8. independent set

9. best effort packet delivery

10. telephony network

11. price of anarchy

12. the most M.K. pays out

13. peering agreements

14. six degrees of separation

15. has no pure strategy equilibrium

16. monotone graph properties

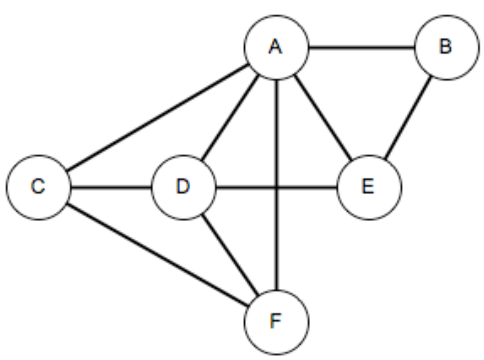
17. power law distributed

18. fads as epidemics

19. beauty contest game

20. consensus

Problem 2 (10 points)



For the network above:

a) What is the value of the worst case diameter?

b) What is the maximum degree?

c) What is the minimum degree?

d) Which node has the smallest clustering coefficient and what is its value?

e) Determine whether there exists a perfect matching and if so, list the corresponding pairs of vertices.

Problem 3 (10 points) Give a real-world example, other than those discussed in class or the readings, of a “rich get richer” phenomenon, in which parties already possessing a larger amount of some resource are differentially advantaged in obtaining more of it. Discuss what you think might be the resulting distribution of this resource across the population.

Any reasonable example here OTHER than preferential attachment for network formation and degree distribution. Their example should clearly have the property that those with more “stuff” are arguably more likely to get even more; they should ideally say something to support this, or it should be self-evident. Presumably these processes should lead to heavy-tailed distribution (not necessarily power law).

Problem 4 (15 points) During the course we examined both stochastic (or randomized) models of network formation, and game-theoretic ones. Briefly discuss the main commonalities and differences between these two broad classes of models, illustrating your discussion with at least one example of both a stochastic and a game-theoretic formation model. What do we mean for each of these models when we say that it generates networks with a particular property (for instance, small diameter)?

Commonalities: both describe distributed, decentralized NW formation w/o any master plan or centralized authority; edge "decisions" are made by individual vertices (in the sense that we can view the vertices as choosing edges, either randomly or game-theoretically). This is a partial list, any reasonable common property is acceptable.

Differences: the obvious one (random vs rational edge choice); in stochastic models the edges chosen are unconstrained by global behavior, while in GT models there is the global equilibrium constraint; again a partial list

Example: be sure they use legitimate examples of mathematical NW formation models discussed in class

Generating properties: for stochastic models, we mean that a NW chosen at random according to the model will have the property with high probability; for GT models, we mean that all Nash equilibria of the formation game have the property.

Problem 5 (10 points) Give a real-world example, other than those discussed in class or the readings, of an "unhappy equilibrium": a situation in which most or all of a large population is unhappy, but no individual can unilaterally improve things for themselves.

Any plausible example here that is well-argued is acceptable OTHER than those discussed in class or Schelling (you'll have to check his examples).

Problem 6 (10 points) Describe and discuss two games (in the formal sense of game theory) in which it is known that human subject behavior deviates from equilibrium predictions, and discuss the nature of this deviation.

I am expecting Ultimatum Game, Beauty Contest Game (the one we did in class where the target is 2/3 the average), and possibly the behavioral experiments in networked trade. All are acceptable. In ultimatum and NW trade, inequality aversion is a main deviation, but there are others described in the slides/reading (you will have to check) that are acceptable. For Beauty Contest, the main finding is the limited number of rounds of iterated reasoning people perform.

Problem 7 (10 points) The image on the following page is reproduced from the lecture on behavioral network games in which players are incentivized to agree with the color of their neighbors (consensus).

(a) Describe the underlying network structure that accompanies this image.

Chain of 6 6-cliques. It's OK if they don't mention the possibility of some random rewiring (or if they do). See class slides.

(b) As precisely as you can, describe what the image is showing (i.e. what are the x and y coordinates and the meaning of the colors).

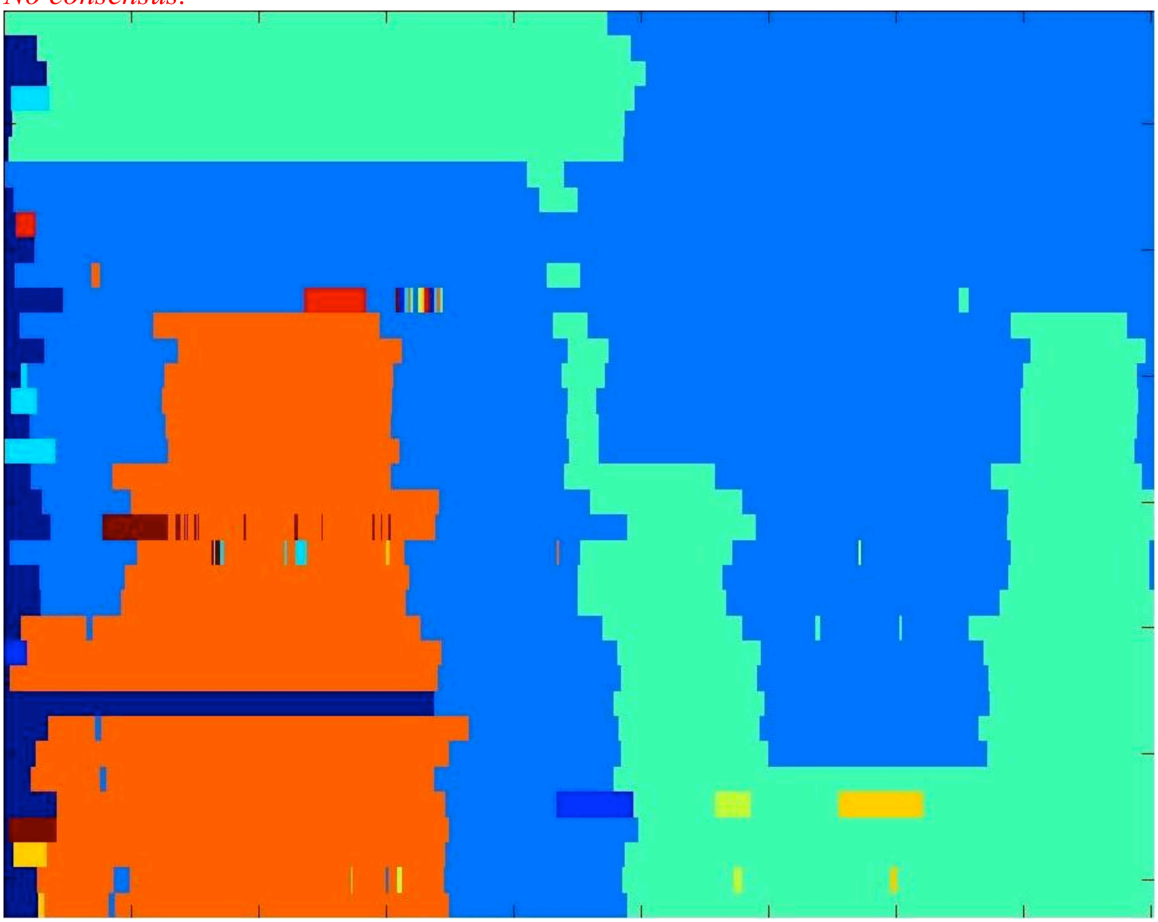
For a single experiment: x-axis is time in the experiment, y-axis has one row for each of 36 players, value shown is color of that player at that time. See class slides.

(c) Discuss interesting instances of both collective and individual dynamics represented in the image.

There are many and they were discussed in class; obvious individual signaling and stubbornness; collective block structure induced by NW structure; and the obvious oscillation between two colors that results in no consensus.

(d) What was the final outcome of this experiment?

No consensus.



Problem 8 (15 points) The two plots labeled (1) and (2) on the following page are reproduced from the lectures and paper on behavioral experiments in networked trade.

(a) Precisely describe what each circle represents, and the meaning of the x and y coordinates in each diagram. Be sure to clearly describe both plots (1) and (2).

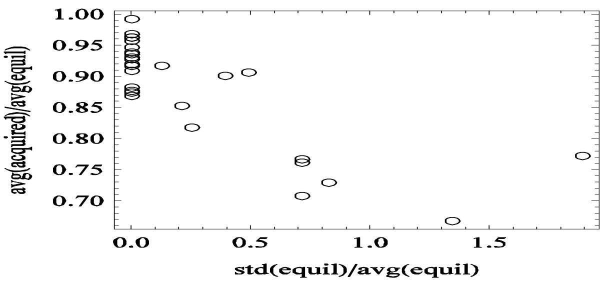
See detailed discussion in accompanying paper.

(b) Describe the overall phenomenon or finding being illustrated by each diagram.

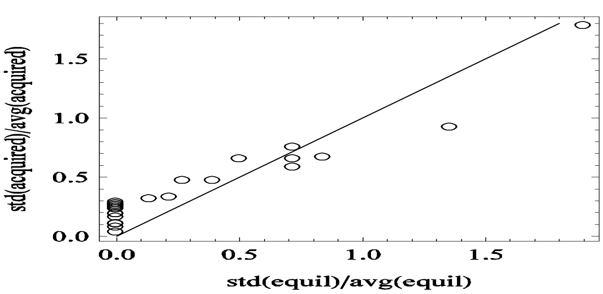
See paper.

(c) Discuss how these findings agree and/or disagree with equilibrium theory.

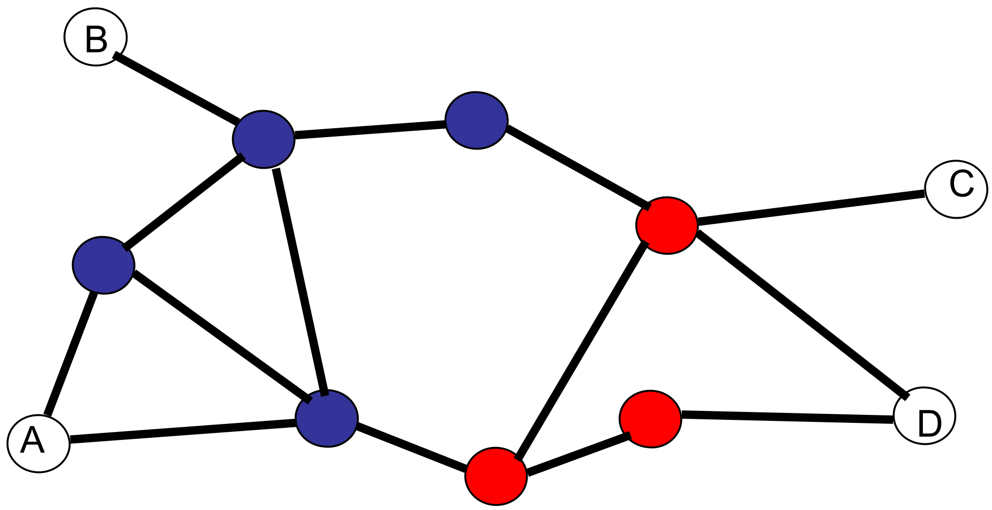
See paper.


avg(acquired)/avg(equil)
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.0
0.5
1.0
1.5
std(equil)/avg(equil)

(1)


std(acquired)/avg(acquired)
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
std(equil)/avg(equil)

(2)



Problem 9 (10 points) The network diagram above shows 4 Internet end-users: A, B, C and D. Users A and B have their Internet service provided by Provider Blue, who operates the routers shown in blue, while users C and D have their Internet service provided by Provider Red, who operates the red routers. Providers Red and Blue operate independently, and each has the incentive to get traffic destined for parties on the other network off of their own network as quickly as possible (that is, with the fewest hops).

Will the combined routing behavior of Providers A and B guarantee that all traffic between all pairs of end users will always travel on a *globally* shortest path? Answer yes or no. If your answer is no, give a specific counterexample to global optimality.

No. The counterexample I had in mind was traffic sent from D to A: the globally shortest route just follows along the bottom (two red hops, one blue hop, length 4), while under early-exit routing, it first goes to the upper red router, resulting in a path length of 5. There may be other examples; you'll have to check them carefully. Also note that the counterexamples may NOT be symmetric --- e.g. traffic from A to D above DOES achieve the global optimal under early-exit routing. They should be docked points if they exhibit confusion about this (though it is fine as long as their example is unidirectional and they don't mention it).

Networked Life
CSE 112
Prof. Michael Kearns
Final Examination
May 1, 2007

NAME: ___________________________________

PENN ID: __________________________________

Exam Score:

Problem 1:__________/10
Problem 2:__________/10
Problem 3:__________/10
Problem 4:__________/10
Problem 5:__________/10
Problem 6:__________/10
Problem 7:__________/10
Problem 8:__________/10
Problem 9:__________/10
Problem 10:_________/10

TOTAL:____________/100

This is a closed-book exam; you should have no materials other than this exam and a pencil or pen.

If you need more space to answer a problem, use the reverse side of the page, but clearly indicate where your answers are.

1. (10 points) For each item on the left, write the index of the item on the right which is the best match.

a. connectors __7__	1. fewest colors required
b. market for lemons __3__	2. neural network
c. dollar bill migration _8___	3. cascading
d. C. Elegans _2___	4. Nash equilibrium
e. baggage screening _10___	5. no wealth variation
f. perfect matching _5___	6. independent set
g. forest fire __9__	7. the heavy tail
h. complement of clique __6__	8. scaling laws of human travel
i. chromatic number _1___	9. viral spread
j. no unilateral gain _4___	10. interdependent security

2. (10 points)

(a) (5 points) Draw a network with 10 vertices in which the clustering coefficient and the overall rate of connectivity (i.e. the fraction of all possible edges in the network that are present) are both low. What network formation model studied in class would give rise to networks with this property?

One (not the only) possibility is something like a cycle --- very regular structure, very sparse.

Formation model: Erdos-Renyi with small value of p (points off if they do not specify small p). Note that the phrasing of the problem does NOT require the specific graph they draw to match the formation model they name.

(b) (5 points) Draw a network with 10 vertices in which the clustering coefficient is relatively high, but the overall rate of connectivity is relatively low. What network formation model studied in class would give rise to networks with this property?

One possibility is something like a chain of cliques.

Possible formation models: alpha model, rewired chain of cliques

3. (10 points) Consider the behavioral network science experiments in which the games being played were Coloring, Consensus, and Kings and Pawns (without tips). In each of these games, there was a precise specification of how individuals would be paid in response to their own actions and those of their neighbors in the network, so we can discuss both the maximum social welfare states (the global configurations in which the total payoff to the population is highest), and the Nash equilibria. For each of the three games, answer the following questions:

    (i) Is a maximum social welfare state always a Nash equilibrium? Explain.

    (ii) Is a Nash equilibrium always a maximum social welfare state? Explain.

Coloring:
(i)
Yes. Max social welfare corresponds to a proper coloring, in which everyone is paid the maximum possible amount, so this is also Nash.
(ii)
No. It is possible for a player to have the colors used in its neighborhood in a Nash equilibrium (think of a red-blue-blue-red chain) and thus not be able to improve its payoff from 0.

Consensus:
(i)
Yes. Max welfare corresponds to consensus, in which everyone receives the max possible amount, so this is also Nash.
(ii)
No --- think of a cycle with three colorings alternating around it --- nobody is getting paid anything, but also cannot do anything about it since both their neighbors have different colors, so it is Nash

Kings and Pawns without tips:
(i)
Yes. Max welfare is a max independent set, and the pawns cannot improve their payoff.

(ii)
No --- any maximal independent set is Nash but may not be maximum.

4. (10 points) Consider the assigned reading “The Scaling Laws of Human Travel”.

(a) (3 points) Briefly summarize the source and nature of the data analyzed in the paper.

Data source is from wheresgeorge.com, a web-based dollar bill tracking service. The data consists of the sequence of distances traveled by individual dollar bills in consecutive “sightings”. The main interest of the paper is in studying the empirical distribution or histogram of distances traveled.

(b) (4 points) Briefly summarize the main empirical findings of the paper.

The main empirical claim of the paper is that the empirical distribution of distances traveled has a heavy tail, specifically with a decay ~ 1/r^(1.59) where r is the distance traveled. (Full credit if they at least state this finding clearly.) They also point out that the initial hop after the first sighting is often a very short one, leading the distribution to be slightly more peaked near small r and thus causing some departure from a line on a log-log plot.

(c) (3 points) Discuss the implications of the empirical findings for the network formation model and theoretical results of Kleinberg’s paper “Navigation in a Small World”.

They should first point out that the exponent found in the Scaling Laws paper is different than the specific exponent of 2 required in Kleinberg’s work for efficient navigation. Then they should either point out that a) these two exponents (1.59 and 2) are different enough that there is inconsistency between the two papers, and that if we believe that humans can in fact solve efficient navigation then Kleinberg’s theory needs amending; or b) that these exponents are close enough to be broadly consistent and thus render Kleinberg’s explanation plausible. Either argument is fine as long as they make it clearly enough.

5. (10 points)

(a) (6 points) Briefly discuss what is being illustrated in the figure below, and what points are demonstrated by the diagram.

The figure is from the IDS baggage security case study. There are 36 tiny plots corresponding to the 36 largest carriers by volume. x-axis of each plot is simulation time, y-axis is fraction made of possible security investment. Dynamics of simulation is that at each step, each carrier incrementally adjusts their investment upwards if doing so improves their payoff (reduces their overall cost). The diagram shows that the Price of Anarchy is large --- while smaller carriers invest fully at equilibrium the largest ones do not.

(b) (2 points) Briefly discuss how the figure below differs from that in (a), and what point is being demonstrated.

In this figure the two largest carriers have been clamped at full investment, representing (e.g.) government subsidy for their investment. Now all of the carriers eventually converge to full investment, showing that the two largest carriers form a "tipping set" for the entire population. The figure also demonstrates cascading behavior, as there is a distinct order in which the larger carriers begin investing, in response to the investments of their neighbors.

(c) (2 points) Briefly discuss how the figure below differs from that in (a) and (b), and what point is being demonstrated.

In this figure only the largest carrier is subsidized, and there is again a high Price of Anarchy --- many of the large carriers still do not invest. So subsidizing both of the largest really was required to get the result in part (b).

6. (10 points) Consider the network Milk-Wheat economic exchange model considered extensively in class. Draw the smallest bipartite network you can in which the number of Milk players and Wheat players is the same, each player begins with an endowment of 1.0 of their respective good, the network is connected (i.e. there is a path between any pair of players), and the equilibrium wealths are *not* all equal. Annotate your diagram with the wealth of each player at equilibrium.

Any example that is simple and meets all the criteria of the statement is acceptable. The connectivity requirement excludes trivial solution like A connected to 1 and 2 on the other side, and 3 connected to B and C, giving two disjoint components with wealth variation.

7. (10 points). The following assertions all refer to the behavioral network science experiments from this semester and/or the ones from last year described in the paper "An Experimental Study of the Coloring Problem on Human Subject Networks". For each assertion, circle True or False.

(a) All networks used in the Kings and Pawns experiments were bipartite.
TRUE FALSE

(b) In the network formation model used for this semester's coloring experiments, larger values of p tended to increase the time to solution
TRUE FALSE

(c) In the network formation model used for this semester's consensus experiments, larger values of p tended to increase the time to solution
TRUE FALSE

(d) Allowing the exchange of tips tended to reduce social welfare in Kings and Pawns
TRUE FALSE

(e) In last year's coloring experiments, showing participants the entire network increased the time to solution, regardless of the network structure
TRUE FALSE

(f) In Kings and Pawns with tips allowed, the instantaneous social welfare approached its maximum possible value at some point during most experiments
TRUE FALSE

(g) In last year's coloring experiments, the Leader Cycle networks yielded the smallest average time to solution
TRUE FALSE

(h) Across all of the experiments from both last year and this year, smaller network diameter tended to lead to faster solution, regardless of the game type
TRUE FALSE

(i) Across all of the experiments from both last year and this year, higher network clustering coefficient tended to lead to faster solution, regardless of the game type
TRUE FALSE

(j) Allowing the exchange of tips dramatically improved the social welfare in Kings and Pawns when play occurred in isolated pairs
TRUE FALSE

8. (10 points) Consider a bipartite network in which one set of vertices represents movies, the other set of vertices represents actors, and there is an edge between a movie and an actor if and only if that actor appeared in that movie.

(a) (3 points) Suppose there is an actor whose degree is very large. Does this imply that there is a movie with very large degree? Explain your answer.

No --- as long as the number of movies is large compared to the max actor degree (e.g. if the number of movies is at least as large as the number of actors, a reasonable assumption), the high degree of this actor could be "spread out" over many movies, each of small degree.

(b) (3 points) Suppose the degree distribution of the actors is heavy-tailed. Does this imply that the degree distribution of the movies is heavy-tailed? Explain your answer.

No --- imagine first specifying the degrees of the actors in a way that gives them a heavy-tailed distribution. But now choose the destination movies of each of the edges uniformly at random. Then the degree distribution of the movies will be sharply peaked, not heavy-tailed.

(c) (4 points) Suppose that the number of movies and actors is the same, and that every actor has appeared in at least d movies. What can you say about the degrees of the movies? Explain your answer.

The average degree of the movies must be at least d by a simple counting argument --- if there are N actors and N movies, and each actor has degree at least d, then there are at least N*d edges total, and thus the average movie must have at least d edges or actors.

9. (10 points) Consider a population of N people. Suppose there is a group activity for this population that obeys the following dynamics:
   - If k people participated in the activity last time, then N-k will participate this time.

To be fair to all students, I want to make this problem worth only 5 extra credit points total. In order to get the 5 points, they need to simply observe that if you start with 0 you will oscillate forever between 0 and N, and that more generally if you start with k you will oscillate forever between k and N-k. There is no need for them to draw a diagram.

(a) (3 points) Draw a diagram below of the type found in Schelling's book to represent these dynamics. Be sure to label your axes precisely.

(b) (4 points) Re-draw your diagram from part (a) below and use it to compute the eventual number of people participating if the initial number of participants is 0.

(c) (3 points) Is there any number of initial participants that will cause the eventual number of people participating to be different from your answer to part (b)?

10. (10 points)

(a) (5 points) Give an example of a model of economic exchange taking place over networks in which the wealth of individuals is entirely determined by their degrees. Be precise in your description.

The example I had in mind was the “brain-dead” model from early in the course --- each vertex starts with $1, and at each time step gives away all of its money to its neighbors, equally divided. Thus at each step every vertex gives all its money away, but also receives new money from its neighbors. This process converges to a player’s average wealth being exactly proportional to its degree.

Other examples are acceptable (including original ones) as long as they are precise and correct.

(b) (5 points) Give an example of a model of economic exchange taking place over networks in which the wealth of individuals is *not* entirely determined by their degrees, and explain why it is not. Be precise in your description.

The most obvious example would be the bipartite Milk-Wheat exchange model from class, where it was pointed out at some length that the degree distribution does NOT determine the equilibrium wealth in all cases --- e.g. if you have two neighbors that have no neighbors besides you, you will make $2 at equilibrium, but if you have degree two and the network has a perfect matching you will make only $1.

Networked Life ***(CSE 112)***

Prof. Michael Kearns

Final Examination

May 3, 2006

The final exam is closed-book; you should have no materials present other than the exam and a pen or pencil.

NAME: ___________________________________

PENN ID: __________________________________

Exam Score:

Problem 1:__________/15
Problem 2:__________/10
Problem 3:__________/10
Problem 4:__________/10
Problem 5:__________/10
Problem 6:__________/10
Problem 7:__________/10
Problem 8:__________/10
Problem 9:__________/7
Problem 10:_________/8

TOTAL:____________/100

1. (15 points) For each item on the left, write down the number of the item on the right that most closely corresponds to it.

(a) Differential Pricing ______	(1) Hubs & authorities
(b) Quality of Service ______	(2) No incentive for unilateral change
(c) Routers ______	(3) Multi-dimensional search
(d) Nash Equilibrium ______	(4) Price equilibrium
(e) Prisoner's Dilemma ______	(5) An online social network
(f) Arrow & Debreu ______	(6) Preferential attachment
(g) Kleinberg ______	(7) Personal valuation of goods
(h) Utility function ______	(8) Switches that receive & forward packets
(i) ping ______	(9) Guarantee of service on the Internet
(j) Watts, Dodds & Newman ______	(10) A non zero-sum game
(k) Power law ______	(11) Gladwell's Law of the Few
(l) Rich gets richer ______	(12) Tool to check reachability over IP network
(m) Orkut ______	(13) Linear log-log plot
(n) Market equilibrium ______	(14) Clearance of goods
(o) Connectors, Mavens & Salesmen ______	(15) Premium/first class service in airlines

2. (10 points) Schelling discusses the phenomenon in which individual people choose whether or not to send holiday cards to each of their friends. In this setting there is a cost associated with sending each card, but a higher cost associated with the embarrassment of not sending a card to someone who has sent a card to you.

a) One equilibrium in this setting is the situation in which everybody sends a card to everybody else. Explain why this scenario is an equilibrium.

b) The equilibrium described above is bad in the sense that everyone must pay the costs associated with sending cards to everyone else. Are there other equilibria? If not, explain why others can't exist. If so, describe the equilibrium that is socially optimal, i.e. the equilibrium with the lowest total cost to all participants.

3. (10 points) Consider the hypothetical spread of disease on a network. Here, each node represents an individual person who may or may not decide to receive a vaccination. Each edge represents potential contact between people. A person can contract the disease from any of his neighbors on the graph who are infected, but cannot contract the disease if none of his neighbors are infected.

If an individual chooses to receive the vaccination, he pays a fixed cost V and runs no risk of obtaining the disease. If a person chooses not to receive the vaccination, he pays a cost equal to N*X where N is the number of his neighbors who are not vaccinated and X is a fixed cost associated with the chance of contracting the disease. Assume that X is much larger than V.

a) Suppose everyone chooses to receive the vaccination. Is this an equilibrium? Why or why not?

b) Suppose the government decides to subsidize half of the population by providing them with the vaccination for free (i.e. this half of the population pays 0 and runs no risk of obtaining the disease). Will the other half of the population be more likely to purchase the vaccination now? Briefly explain why or why not.

c) In class we discussed the interdependent security game in which each node represents an airline and an edge between airlines A and B implies that customers may transfer between flights on A and B without having their baggage rescreened. In this game, each airline must decide whether or not to invest in a new security device for screening baggage. Briefly compare the effects of subsidization in the vaccination game with the effects of subsidization in the airline interdependent security game.

4. (10 points) Consider the undirected graph shown in Figure 4.1. Answer following questions with respect to Figure 4.1:

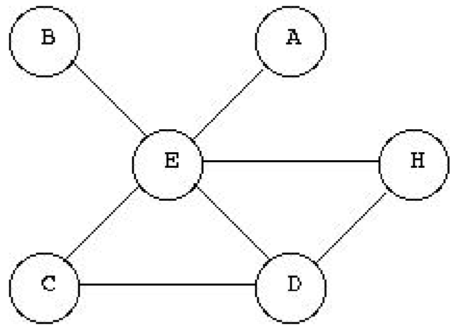


Figure 4.1

a) What is the value of worst-case diameter in this graph? Identify the vertices that are farthest apart (if more than one such pair is possible, then list all of them).

b) What is the size of the largest clique in the graph? List the vertices in each clique of this size.

c) What is the minimum number of edges that could be removed to make the graph unconnected? List these edges. (If more than one answer is possible, then list all of them.)

d) What is the clustering coefficient of node E?

e) What is the minimum number of edges whose addition to the existing graph would increase the size of the largest clique by 1 (compared to the size of the largest clique in the existing graph)? List these edges.

5. (10 points) Consider a network of exchange in which each node represents either a buyer or a seller. Buyers start with one dollar and value only wheat; sellers start with one unit of wheat and value only dollars. All edges on the network connect a buyer with a seller and represent the fact that the buyer and the seller are allowed to trade with each other. Assume that each seller must sell all of her wheat at a single price, and each buyer will purchase wheat only from the seller(s) in his neighborhood with the lowest price.

a) Suppose the network contains ten buyers and six sellers and is connected. Will the average price of wheat be higher or lower than one dollar? (No explanation necessary.)

b) Suppose Seller A is connected to three buyers and that two of these buyers have no other neighbors. What is the minimum price that Seller A will be able to demand at equilibrium? (Again, no explanation necessary.)

c) Is it possible in a buyer-seller network for two sellers of the same degree to charge different prices at equilibrium? If so, draw a network in which this is the case, clearly labeling the buyers and sellers and marking the two sellers of the same degree. If not, explain why sellers of the same degree must charge the same price.

6. (10 points) Consider a repeated game of Prisoner's Dilemma. The game matrix for a single round of this game is given below.

	Cooperate	Defect
Cooperate	-1, -1	-10, -0.25
Defect	-0.25, -10	-8, -8

a) If the game will be played for a known number of rounds R, there is only one equilibrium. State what this equilibrium is and describe why no other can exist.

b) If the game will be played for R rounds where R is *unknown* it is possible for both players to receive higher payoffs at equilibrium. Describe a strategy that can be followed by both players to lead to these higher payoffs. Why does this strategy result in an equilibrium only when R is unknown?

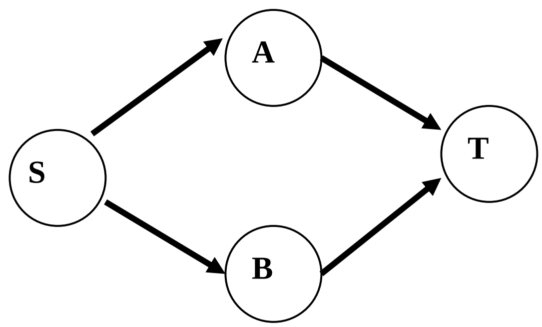
7. (10 points) Some internet services, such as email, can tolerate a network lag of 1 or 2 seconds without any noticeable degradation of service. Others, such as internet telephony, would be basically unusable with such a lag. In class we discussed a possible economic-based approach to handling such differences in demand for quality of service that was based on an analogy to the Paris subway system.

a) Briefly describe this approach and its appeal, including properties such as self-regulation.

b) Give an example of a real-world domain in which a similar technique has worked.

8. (10 points) Consider the following network where *S* is the source node and *T* is the termination node. Traffic (*e.g.* Internet packets, drivers on a freeway etc.) flows from *S* to *T*. The latency function $L_e(x)$ of an edge *e* specifies the latency or delay that traffic on that edge will suffer when a fraction *x* of the total traffic is traveling on that edge. Assume that each packet or driver is selfish and tries to minimize its own total latency.

In class, we defined cost of a flow as the mean of all latencies incurred by traffic in that flow. We also defined optimal flow as the flow that minimizes this cost. On the other hand, a flow is at Nash equilibrium if no traffic (*e.g.* packet or driver) can improve its latency by unilaterally taking an alternative route.



Latency functions for each edge in the above network are as follows:

$$L_{SA}(x) = 0.5x + 0.4, L_{SB}(x) = 1, L_{AT}(x) = 1, L_{BT}(x) = 0.6x + 0.3$$

a) In the above network, one unit of traffic needs to travel from *S* to *T*. The traffic can be divided fractionally along the two possible paths, *S-A-T* and *S-B-T*. What is the cost of Nash equilibrium flow of this one unit of traffic from *S* to *T*? Show your work.

b) Without explicitly computing the value, what can you say about the cost of optimal (minimum cost) flow of this one unit of traffic from *S* to *T* in this network?

9. (7 points)

a) In Figure 9.1, Watts' α parameter is plotted along the horizontal axis. Values of path length (L) and clustering coefficient (C) are plotted along the vertical axis.

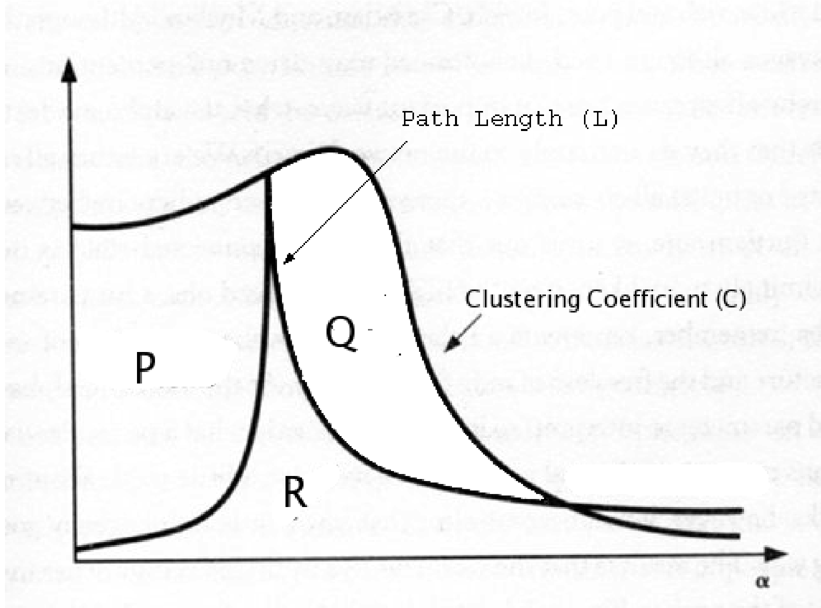


Figure: 9.1

(i) In Figure 9.1, identify the region corresponding to small world networks. Indicate your choice by selecting one of the three regions below:

P **Q** **R**

(ii) State two characteristic features (which can be inferred from Figure 9.1) of such small world networks.

b) In one or two words, answer the following:

(i) State the network formation model with the following properties: small diameter, with heavy tailed degree distribution, but without high clustering.

(ii) State the network formation model with the following properties: small diameter without high clustering and without heavy tailed degree distribution.

10. (8 points) Kleinberg proposed the HITS algorithm for ranking web pages as either hubs or authorities. The intuition behind this algorithm is that a good hub should point to many good authorities, and a good authority should be pointed to by many good hubs.

a) How might one manipulate the HITS algorithm in order to artificially increase the authority ranking of a page? Is this same manipulation technique possible under the PageRank algorithm? Briefly explain your answer.

b) The HITS algorithm makes strong assumptions about the nature of the internet. Aside from manipulability, what are some problems we might run into if we tried to use HITS as a ranking tool for search over the full internet?

Networked Life
CSE 112
Prof. Michael Kearns
Final Examination
May 6, 2005

The final exam is closed-book; you should have no materials present other than the exam and a pen or pencil.

NAME: ___________________________________

PENN ID: __________________________________

Exam Score:

Problem 1:__________/10
Problem 2:__________/10
Problem 3:__________/12
Problem 4:__________/10
Problem 5:__________/12
Problem 6:__________/10
Problem 7:__________/12
Problem 8:__________/12
Problem 9:__________/12

TOTAL:____________/100

1. (10 points, 1 point each) For each item on the left, write down the number of the item on the right that it is most closely associated with.

a) Stanford Prison Experiment: _____	1. central limit theorem
b) alpha model: _____	2. friends of friends
c) contagion: _____	3. dynamics of transmission
d) social capacity: _____	4. Zipf's Law
e) convergence to expectation: _____	5. power of context
f) North American city sizes: _____	6. Caveman and Solaria
g) threshold for monotone graph properties: _____	7. the magic number 150
h) clustering coefficient: _____	8. threshold for participation
i) mixed strategy: _____	9. randomization
j) volleyball: _____	10. formalization of tipping

2. (10 points) As precisely and as generally as you can, define the notion of the "Price of Anarchy" in economic and game-theoretic settings. Give an example from class materials in which the Price of Anarchy was relatively high, and another example in which it was relatively low. Briefly discuss the Price of Anarchy as applied to the Prisoner's Dilemma.

3. (12 points) On the following page is the output of an invocation of the traceroute program discussed in class.

a) (2 points) What is the IP address of the destination?

b) (10 points) In two or three paragraphs in the space below, discuss the technological, organizational, economic and geographic inferences one can draw from this output. You should refer to specific parties appearing in the output to illustrate your discussion.

traceroute output for Problem 3:

```
C:\Documents and Settings\Administrator>tracert cs.berkeley.edu

Tracing route to cs.berkeley.edu [169.229.60.161]
over a maximum of 30 hops:

  1    <1 ms    <1 ms    <1 ms  subnet-50-router.seas.upenn.edu [158.130.50.1]
  2     1 ms     1 ms     1 ms  INTERNAL-BORDER-ROUTER.CIS.upenn.edu [158.130.21
.2]
  3     2 ms     1 ms     1 ms  EXTERNAL-BORDER-ROUTER2.CIS.upenn.edu [158.130.2
1.14]
  4     2 ms     2 ms     2 ms  ISC-GW-FE.CIS.upenn.edu [158.130.20.4]
  5     3 ms     3 ms     2 ms  EXTERNAL-GE2.ROUTER.UPENN.EDU [165.123.217.1]
  6     2 ms     2 ms     3 ms  local.upenn.magpi.net [198.32.42.249]
  7     3 ms     2 ms     2 ms  phl-02-01.backbone.magpi.net [216.27.100.221]
  8     5 ms     5 ms     5 ms  remote.oc48.abilene.magpi.net [216.27.100.22]
  9    31 ms    25 ms    25 ms  chinng-nycmng.abilene.ucaid.edu [198.32.8.82]
 10    29 ms    29 ms    48 ms  iplsng-chinng.abilene.ucaid.edu [198.32.8.77]
 11    43 ms    40 ms    38 ms  kscyng-iplsng.abilene.ucaid.edu [198.32.8.81]
 12    55 ms    53 ms    48 ms  dnvrng-kscyng.abilene.ucaid.edu [198.32.8.13]
 13    73 ms    73 ms    73 ms  snvang-dnvrng.abilene.ucaid.edu [198.32.8.1]
 14    82 ms    81 ms   104 ms  losang-snvang.abilene.ucaid.edu [198.32.8.94]
 15    81 ms    81 ms    81 ms  hpr-lax-gsr1--abilene-LA-10ge.cenic.net [137.164
.25.2]
 16    91 ms    90 ms    90 ms  sac-hpr--lax-hpr-10ge.cenic.net [137.164.25.11]

 17    92 ms    92 ms    92 ms  oak-hpr--sac-hpr-10ge.cenic.net [137.164.25.16]

 18    93 ms    92 ms    96 ms  hpr-ucb-ge--oak-hpr.cenic.net [137.164.27.130]
 19   115 ms   114 ms    94 ms  vlan190.inr-202-doecev.Berkeley.EDU [128.32.0.39
]
 20    93 ms    92 ms    93 ms  doecev-soda-br-6-4.EECS.Berkeley.EDU [128.32.255
.170]
 21    95 ms    94 ms    94 ms  sbd2a.EECS.Berkeley.EDU [169.229.59.226]
 22   116 ms    94 ms    94 ms  soda-cr-1-2-cory-cr-1-1.EECS.Berkeley.EDU [169.2
29.59.234]
 23    94 ms    95 ms    94 ms  mx1.EECS.Berkeley.EDU [169.229.60.161]

Trace complete.
```

4. (10 points) The 1999 AltaVista web crawl study discussed in class and the readings identified five components of the web graph according to their different connectivity properties. For each of these five components, give its name, briefly describe what constitutes a web page being a member of that component, and give an example of either a specific web page or a type of web page that you might expect to find in that component.

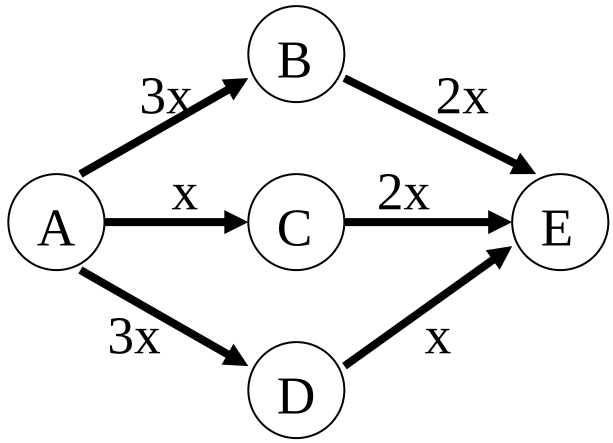
5. (12 points) For each of the current problems or challenges regarding the use or management of the Internet, briefly describe a potential solution that can be viewed as having an economic or financial flavor, and that can be administered in a relatively distributed, decentralized fashion.

a) (4 points) e-mail spam

b) (4 points) free-riding in peer-to-peer file sharing

c) (4 points) lack of quality-of-service guarantees for high-bandwidth, real-time applications

6. (10 points) Consider the network diagram below, and suppose that a large volume of traffic (say, Internet packets or drivers on freeways) would like to travel from vertex A to vertex E. Each directed edge in the network is annotated by the latency or delay that traffic will suffer on that edge if a fraction x of the total volume is traveling that edge. Assume that each packet or driver is selfish, meaning that it will always travel a route from A to E that minimizes its own total latency, given the behavior of the rest of the traffic population.



a) (5 points) For the three possible paths A-B-E, A-C-E, A-D-E, write down the order of these paths according to how much of the total volume of traffic they carry at equilibrium, from the most to the least traffic. For the path carrying the least traffic in this ordering, will it carry *any* traffic at equilibrium?

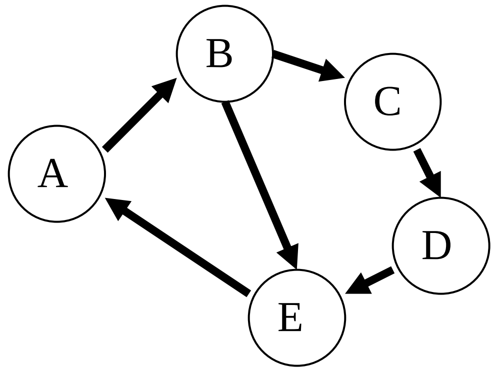
b) (5 points) Suppose that we add a directed edge from C to D, thus creating a new path A-C-D-E. Suppose that this new edge from C to D is a "teleport" edge, meaning it has zero latency no matter how much traffic it carries. As in part a), order the four paths according to how much traffic they carry at equilibrium, from the most to least traffic. For the path carrying the least traffic in this ordering, will it carry *any* traffic at equilibrium?

7. (12 points) One of the major themes of the course was the distinction between the "dynamics of transmission" and the "dynamics of rationality".

a) (6 points) Briefly describe what is meant by the dynamics of *transmission*, and give at least two examples (either from course materials or from the real world). You should make sure your answer is clearly distinguished from your answer to part b).

b) (6 points) Briefly describe what is meant by the dynamics of *rationality*, and give at least two examples (either from course materials or from the real world). You should make sure your answer is clearly distinguished from your answer to part a).

8. (12 points) Consider the following directed network diagram:



a) (3 points) Which vertex do you think has the highest PageRank in this network? Briefly justify your answer in terms of the properties of the algorithm. (You do not need to numerically calculate anything.)

b) (3 points) Which vertex has the highest hub weight in this network? Briefly justify your answer in terms of the properties of the algorithm. (You do not need to numerically calculate anything.)

c) (3 points) If your answers to a) and b) above were different, discuss what it is about the PageRank and Hubs and Authorities algorithms that causes them to choose different vertices.

9. (12 points) This question examines concepts in behavioral game theory.

a) (3 points) Briefly describe the Ultimatum Game, and describe its Nash equilibrium

b) (3 points) Briefly describe the behavioral findings on the Ultimatum Game, and discuss the systematic ways in which they differ from Nash equilibrium.

c) (3 points) Briefly describe some known factors or settings in which the systematic results of part b) are violated.

d) (3 points) Briefly describe the theory of inequality aversion, and how it modifies the classic concepts of game theory.

1. Write the number of the left-hand item next to the item on the right that corresponds to it.

1. Stanford prison experiment	model of how the rich get richer
2. Friendster	models random pairwise conflicts
3. neuron	describes North American city sizes
4. router	obtained by selfish routing
5. tipping	component of the web graph
6. small worlds	improved by the power of weak ties
7. job-hunting	phenomenon found by Travers and Milgram
8. social capacity	clearance of goods
9. independent set	model of random graphs
10. planar graphs	based on shared randomization
11. preferential attachment	Arrow's impossibility theorem
12. Erdos-Renyi	complement of clique
13. Zipf's law	vertex in a biological network
14. tendrils	model of a random web surfer
15. PageRank algorithm	sudden amplification of the incremental
16. bag of words	model used in information retrieval
17. correlated equilibrium	subject of the four-color theorem
18. social choice theory	explored power of context
19. evolutionary game theory	online social network
20. market equilibrium	the magic number 150
21. Price of Anarchy	vertex in a technological network
22. differential pricing	measure in a mathematical collaboration network

---

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

2. Consider the diagram above, whose x-axis value indicates the number of individuals currently attending some activity (like a seminar or volleyball game), and whose y-axis value indicates how many will attend the activity the next time, given the x-axis value.

A. If 55 people attend the first time, what will be the eventual (or limiting) number attending?

B. If 25 people attend the first time, what will be the eventual number?

C. Are there any other points of equilibrium? If yes, where are they?

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3. Consider what we referred to in class and the readings as the “market for lemons”.

A. [One paragraph.] Briefly describe the dynamics of the market for lemons. Indicate whether the price and volume of lemons is expected to increase or decrease with time, and explain why. Be sure to highlight any aspects of the market for lemons that distinguish it from markets with different behavior.

B. [Short phrase.] Suggest at least one real-world market for a product or service (other than lemons and insurance) that might exhibit the same behavior, and indicate why.

	Median choice													
Choice	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	45	49	52	55	56	55	46	−59	−88	−105	−117	−127	−135	−142
2	48	53	58	62	65	66	61	−27	−52	−67	−77	−86	−92	−98
3	48	54	60	66	70	74	72	1	−20	−32	−41	−48	−53	−58
4	43	51	58	65	71	77	80	26	8	−2	−9	−14	−19	−22
5	35	44	52	60	69	77	83	46	32	25	19	15	12	10
6	23	33	42	52	62	72	82	62	53	47	43	41	39	38
7	7	18	28	40	51	64	78	75	69	66	64	63	62	62
8	−13	−1	11	23	37	51	69	83	81	80	80	80	81	82
9	−37	−24	−11	3	18	35	57	88	89	91	92	94	96	98
10	−65	−51	−37	−21	−4	15	40	89	94	98	101	104	107	110
11	−97	−82	−66	−49	−31	−9	20	85	94	100	105	110	114	119
12	−133	−117	−100	−82	−61	−37	−5	78	91	99	106	*112*	118	123
13	−173	−156	−137	−118	−96	−69	−33	67	83	94	103	110	117	123
14	−217	−198	−179	−158	−134	−105	−65	52	72	85	95	104	112	120

4. Consider Table 1.1., which is reproduced from the class readings on behavioral game theory, and describes the payoffs to a player for each of their possible choices as a function of the median choice of the population. The group of players each simultaneously chooses a “choice” 1-14 (the rows) and gets a payoff based on which number, 1-14, ended up being closest to the median choice of the whole group.

A. List all the Nash equilibria of this game.

B. [Short Phrase.] Suppose that a large fraction of the players in the game come from a culture in which the number 5 is considered lucky, and choose to play it in the first round of repeated play. What is the outcome likely to be?

C. [Short Phrase.] Suppose we were to draw a network in which each vertex was a player in this game, and there is an edge between players i and j if there is some joint action in which the action of player i can influence the payoff to player j, and vice versa. Briefly describe the structure of this network.

5. In this problem we will examine networks that embody a simplified version of the Interdependent Security (IDS) games that we studied in class.

Consider an undirected network. We will refer to the degree of player i as d(i). Each player in the network must decide whether to invest in some security mechanism (e.g. airline baggage screening, computer users updating their anti-virus software, etc.). We assume that there is some k, either 0 or a positive integer, with the following property: for any player i, player i will invest if and only if *at least* d(i) – k neighbors *are* investing.

A. [Short Phrase.] Let k=1. what happens to a node of degree 1?

B. [At most 5 sentences.] Assume that all players are initially not investing, but may update their behavior in response to changes in their neighbor's decisions. Suppose the network is a tree, and that k = 1. What will the final result (equilibrium) be? Why?

C. Consider the following computational problem: given as input such a network and the value of k, find the smallest set of vertices whose subsidization causes "tipping" to full investment — that is, the smallest set S of players such that if we somehow force those players to invest, all the remaining players will have the financial incentive to invest. For the network given below, find this smallest set S when k = 2.

   [NETWORK NEEDED!]

D. When k = 0, the problem of finding the minimum number who could be subsidized/forced to invest is equivalent to one of the standard optimization problems on graphs studied in class. Name this problem, and explain briefly why they are equivalent.

6. Consider economic networks like those we studied in class. Each vertex is either a buyer or a seller. Buyers begin with $1 and sellers with 1 unit of wheat. Buyers have utility only for wheat, and thus want to exchange their cash for wheat; sellers have the opposite incentive. All edges in the network connect a buyer to a seller, so the network is bipartite.

At equilibrium in such networks, every seller sells exactly one unit of wheat (and thus has neither excess supply or demand); every buyer spends exactly $1 (and thus neither exceeds nor underutilizes their budget); and every buyer purchases wheat only from sellers offering the lowest price available to that buyer. Note that we are *not* allowing price discrimination in this definition; **a seller must sell only at a single price**, not different prices to different buyers.

A. Give the smallest connected network possible in which at equilibrium, there are at least two sellers charging different prices for wheat. Draw the network clearly, indicating buyers with a B and sellers with an S. Annotate each seller with the price they charge at equilibrium.

B. Suppose that such a buyer-seller network has the property that every vertex has the exact same number of neighbors. What is the ratio of buyers to sellers?

C. Is the scenario mentioned in b sufficient to guarantee that all sellers will charge the same price at equilibrium? If you answer yes, briefly argue why this is so, and say what the common equilibrium price is. If you answer no, provide a counterexample.

D. True or false: In a buyer-seller network, the seller with the highest degree will always be the one who charges the highest price at equilibrium.

7. Draw one line from each item on the left to each item in the middle, and draw one line from each item in the middle to each item on the right, to connect the concepts that correspond.

(E,V) such that V can be split into subsets V1 and V2 and for each {a,b} in E, a in V1 and b in V2
/
Bipartite graph
/
Describes a networked economy of buyers and sellers, each of whom only can trade with a certain number of those in the other class

(E,V) such that for each v in V, v=a or v=b for some {a,b} in E
/
Connected graph
/
A Friendster-like network in which people are not added until the accept friendship, people cannot unfriend one another, and people cannot close their accounts or otherwise leave

(E,V) such that for each v in V, v appears choose(N,2) times in the elements of E
/
Complete graph
/
Describes a Web site where each page is only one click away from each other page

X and Y axes are scaled logarithmically
/
Log-log plot
/
Provides a way to detect heavy-tailed distributions in the underlying data

U1 = P1 - alpha1(max(P2-P1,0)) - beta1(max(P1-P2,0))

/
Inequality aversion
/
Models the guilt and envy of participants, can explain some "irrational" economic behavior

n by n grid of veritces, all connected within distance p, q additional connections with probability that goes as 1/d^r
/
Kleinberg's model
/
Explains one way that it may be possible to find short paths using only local navigation

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8. The following question refers to Malcolm Gladwell's book *The Tipping Point* and his discussion of how the structure of certain networks matters.

A. [Three words.] Write down the three types of influential people who are described in "The Law of the Few."

B. [One word.] If you were given just an undirected, unweighted graph of an acquaintance network, which of these three types of people would be easiest to identify?

C. [Two short phrases.] What are *two* features you might look at, or look for, to determine which people are in this category?

D. [Four words.] Consider making each undirected link into a pair of directed links, and then allowing these directed links to have weights. So, for instance, person A might have a link with weight 10 to person B, while person B might have a link leading back to A that only has weight 10. There are two categories of people left. First, write down the two remaining categories. For each one, imagine you are assigning a meaning to the weights of the graph. What should these weights should represent, in each case, if we want to capture how the influence of these types of people works? One word is sufficent for the meaning of the weights in each case.

9.

[One sentence.] Define "clustering." Be precise enough so that someone who knows the basics of graph theory could tell, based on your definition, which of two networks is more highly clustered.

[One sentence.] Define "small diameter." Be precise enough so that someone who knows the basics of graph theory could tell, based on your definition, which of two networks has smaller diameter.