Debugging with the Power Equilibrium Check in Simulations

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Abstract

This paper aims to help beginners of research students debug and analyse their models and simulations by checking the power equilibrium. Electric motor simulations are usually complicated for the beginners. Applying the continuous time models in discrete-time increase the confusion how to validate the simulations and how to debug. The power equilibrium is a good method for these purposes; however, checking it is not straightforward. This paper explains the details on a doubly-fed induction machine model, which is one of the most complicated electric motor models.

Key words: Power equilibrium, electric motor modelling, doubly-fed induction machine.

1. Introduction

Research studies frequently meet failures and disruptions due to wrong interpretations or applications of basic equations such as modelling errors in practical studies or bugs in software applications and simulations. In order to surmount these failures, research students are encouraged to measure or observe each quantity in equations and then to check if the equations are really satisfied as assumed theoretically. If not, then the researcher should go one step back or sometimes go back to the mid-stage between the last tested correct and wrong stages and check the equations for that stage similarly. This approach usually helps the researcher identify and solve the failure. A basic component might not work properly or the researcher might have thought its function or usage in a wrong way when designing or coding. However, sometimes there is no back stage practically, an equation is simply not satisfied. In such a case, which probably includes a modelling error, the researcher should go back to former equations yielding that equation checking if they really yield it under the operating conditions considered properly. The equation

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revision process may continue until reaching the very basic equations. We can call this well-known approach "*going back to childhood of the equations*", which requires and/or develops a strong educational background both theoretically and practically.

An example of this approach in computer simulations is to check the power equilibrium because more than physical, it is a mathematical rule. It is applicable in most engineering subjects and it must be satisfied exactly at every time step of the simulation; however, it is usually confusing for most research students because instantaneously considering, the energy goes into or come from various forms. In addition, even the simple Ri^2 law on a resistor may not be applied directly if the equilibrium is checked exactly in discrete-time solutions.

Electric machinery simulations are good examples to clarify such confusions because their dynamics include both electrical and mechanical power, both generating and motoring modes, both electromagnetic and kinetic energy storage, both fast and slow modes, and both active and reactive power definitions are available instantaneously. In addition, stationary or rotating reference frames and pulse-width modulated inputs increase the complexity in a research student's mind. Among the electric machinery, one of the most complicated models belongs to **d**oubly-**f**ed **i**nduction **m**achines (DFIM), which can be fed both from the stator and the rotor. In order to clarify these aspects, the application of the power equilibrium check is shown on a DFIM in this paper.

2. Model of Doubly-Fed Induction Machine (DFIM)

With the definitions of

$$\begin{bmatrix} \phi_s \\ \phi_r \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} \tag{1}$$

$$T_e = \frac{3}{2} n_p \left(\phi_{sd} i_{sq} - \phi_{sq} i_{sd} \right) \tag{2}$$

the basic equations of the DFIM dynamics [1] are

$$v_s = R_s i_s + \frac{d\phi_s}{dt} + j\omega_g \phi_s \tag{3}$$

$$v_r = R_r i_r + \frac{d\phi_r}{dt} + j(\omega_g - \omega_r)\phi_r \tag{4}$$

$$J_i \frac{d(\omega_r/n_p)}{dt} = T_e - B_f(\omega_r/n_p) - T_L \tag{5}$$

where among the parameters n_p is the number of the pole pairs, B_f is the friction constant, J_i is the inertia, R_s and R_r are the stator and rotor resistances, L_s , L_r and M are the stator, rotor and mutual inductances, respectively; among the variables ω_g is the angular speed of the dq axes with respect to stator, T_e and T_L are the electromechanical and load torques, $\phi_s = \phi_{sd} + j\phi_{sq}$ and $\phi_r = \phi_{rd} + j\phi_{rq}$ are the stator and rotor flux linkages respectively; the inputs $v_s = v_{sd} + jv_{sq}$ and $v_r = v_{rd} + jv_{rq}$ are the stator and rotor voltages; and the state variables, $i_s = i_{sd} + ji_{sq}$ and $i_r = i_{rd} + ji_{rq}$ are the stator and rotor currents respectively and ω_r is the electrical angular speed of the rotor; hence ω_r/n_p is the mechanical angular speed of the rotor.

Defining the leakage constant $\sigma = \frac{L_r L_s - M^2}{L_r L_s} = 1 - \frac{M^2}{L_r L_s}$, stator and rotor time constants $\tau_s = L_s/R_s$ and $\tau_r = L_r/R_r$ respectively, and

$$A(\omega_r,\omega_g) = \begin{bmatrix} -\frac{1}{\sigma\tau_s} & \left(\omega_g + \frac{M^2}{\sigma L_r L_s}\omega_r\right) & \frac{M}{\sigma L_s \tau_r} & \frac{M}{\sigma L_s}\omega_r \\ -\left(\omega_g + \frac{M^2}{\sigma L_r L_s}\omega_r\right) & -\frac{1}{\sigma\tau_s} & -\frac{M}{\sigma L_s}\omega_r & \frac{M}{\sigma L_s \tau_r} \\ \frac{M}{\sigma L_r \tau_s} & -\frac{M}{\sigma L_r}\omega_r & -\frac{1}{\sigma\tau_r} & \left(\omega_g - \frac{1}{\sigma}\omega_r\right) \\ \frac{M}{\sigma L_r}\omega_r & \frac{M}{\sigma L_r \tau_s} & -\left(\omega_g - \frac{1}{\sigma}\omega_r\right) & -\frac{1}{\sigma\tau_r} \end{bmatrix} \tag{6}$$

state equations (3)-(5) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} = A(\omega_r,\omega_g)\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & -\frac{M}{\sigma L_r L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & -\frac{M}{\sigma L_r L_s} \\ -\frac{M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_r} & 0 \\ 0 & -\frac{M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_r} \end{bmatrix}\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} \tag{7}$$

$$\frac{d\omega_r}{dt} = \frac{3}{2}\frac{n_p^2 M}{J_i}\left(i_{rd}i_{sq} - i_{rq}i_{sd}\right) - \frac{B_f}{J_i}\omega_r - \frac{n_p}{J_i}T_L \tag{8}$$

When simulating the model in discrete-time with a time step of dt, denoting the next time step values with superscript " $^+$ ", (3)-(5) become

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = R_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \frac{1}{dt}\left(\begin{bmatrix} \phi_{sd}^+ - \phi_{sd} \\ \phi_{sq}^+ - \phi_{sq} \end{bmatrix} \right) + \begin{bmatrix} -\omega_g \phi_{sq} \\ \omega_g \phi_{sd} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix} = R_r \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} + \frac{1}{dt}\left(\begin{bmatrix} \phi_{rd}^+ - \phi_{rd} \\ \phi_{rq}^+ - \phi_{rq} \end{bmatrix} \right) + \begin{bmatrix} -(\omega_g - \omega_r)\phi_{rq} \\ (\omega_g - \omega_r)\phi_{rd} \end{bmatrix} \tag{10}$$

$$J_i \frac{(\omega_r^+ - \omega_r)}{n_p dt} = T_e - B_f \frac{\omega_r}{n_p} - T_L \tag{11}$$

3. Power Equilibrium

Since the effects of applied stator and rotor voltage are seen at the next time step, the instantenous stator and rotor input powers p_i^s and p_i^r should be calculated as multiplying by $\begin{bmatrix} i_{sd}^+ & i_{sq}^+ \end{bmatrix}$ and $\begin{bmatrix} i_{rd}^+ & i_{rq}^+ \end{bmatrix}$ from left respectively:

$$p_i^s = \frac{3}{2}\left(v_{sd} i_{sd}^+ + v_{sq} i_{sq}^+\right) = p_{Rs} + \frac{3}{2dt}\left(i_{sd}^+ (\phi_{sd}^+ - \phi_{sd}) + i_{sq}^+ (\phi_{sq}^+ - \phi_{sq})\right) + \frac{3}{2}\omega_g \left(-i_{sd}^+ \phi_{sq} + i_{sq}^+ \phi_{sd}\right) \tag{12}$$

$$p_i^r = \frac{3}{2}\left(v_{rd} i_{rd}^+ + v_{rq} i_{rq}^+\right) = p_{Rr} + \frac{3}{2dt}\left(i_{rd}^+ (\phi_{rd}^+ - \phi_{rd}) + i_{rq}^+ (\phi_{rq}^+ - \phi_{rq})\right) + \frac{3}{2}(\omega_g - \omega_r)\left(-i_{rd}^+ \phi_{rq} + i_{rq}^+ \phi_{rd}\right) \tag{13}$$

where p_{Rs} and p_{Rs} are the stator and rotor cupper losses respectively:

$$p_{Rs} = \frac{3}{2} R_s \left(i_{sd} i_{sd}^+ + i_{sq} i_{sq}^+\right) \tag{14}$$

$$p_{Rr} = \frac{3}{2} R_r \left(i_{rd} i_{rd}^+ + i_{rq} i_{rq}^+\right) \tag{15}$$

(12) and (13) are true if the multiplications of unbalance voltage and current components on both stator and rotor sides, $v_{so} i_{so}$ and $v_{ro} i_{ro}$ respectively, are zero, which is satisfied if a neutral line is not used. The multiplier 3/2 comes because the model used here assumes the equivalence of voltage and current amplitudes between three phase and two phase (*dq*) models. The sum of the middle terms in (12) and (13) is the power corresponding to the energy stored in the magnetic field:

$$p_{mag} = \frac{3}{2dt}\Big[L_s i_{sd}^+ (i_{sd}^+ - i_{sd}) + L_s i_{sq}^+ (i_{sq}^+ - i_{sq}) + L_r i_{rd}^+ (i_{rd}^+ - i_{rd}) + L_r i_{rq}^+ (i_{rq}^+ - i_{rq})$$
$$+ M\Big(i_{sd}^+ (i_{rd}^+ - i_{rd}) + i_{sq}^+ (i_{rq}^+ - i_{rq}) + i_{rd}^+ (i_{sd}^+ - i_{sd}) + i_{rq}^+ (i_{sq}^+ - i_{sq})\Big)\Big] \tag{16}$$

The sum of the last terms in (12) and (13) equals $p_m + p_{err}$, where p_m is the electromechanical power and p_{err} is the discrete-time modelling error:

$$p_m + p_{err} = \frac{3}{2}\omega_g\left(-i_{sd}^{+}\phi_{sq} + i_{sq}^{+}\phi_{sd}\right) + \frac{3}{2}(\omega_g - \omega_r)\left(-i_{rd}^{+}\phi_{rq} + i_{rq}^{+}\phi_{rd}\right) \tag{17}$$

In fact, if the present time values are used instead of the next time values in (17), the continuous-time p_m formula $\frac{\omega_g T_e}{n_p} + \frac{(\omega_g - \omega_r)(-T_e)}{n_p} = T_e\frac{\omega_r}{n_p}$ is obtained. However, since the effects of T_e and T_L are seen at the next time step, the electromechanical power in discrete-time is

$$p_m = T_e\frac{\omega_r^{+}}{n_p} \tag{18}$$

which is composed of three components:

$$p_f = B_f\frac{\omega_r\omega_r^{+}}{n_p^2} \tag{19}$$

$$p_{kin} = J_i\frac{\omega_r^{+}(\omega_r^{+} - \omega_r)}{n_p^2\, dt} \tag{20}$$

$$p_o = T_L\frac{\omega_r^{+}}{n_p} \tag{21}$$

which are the friction loss, the power corresponding to kinetically stored energy and the output power respectively.

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