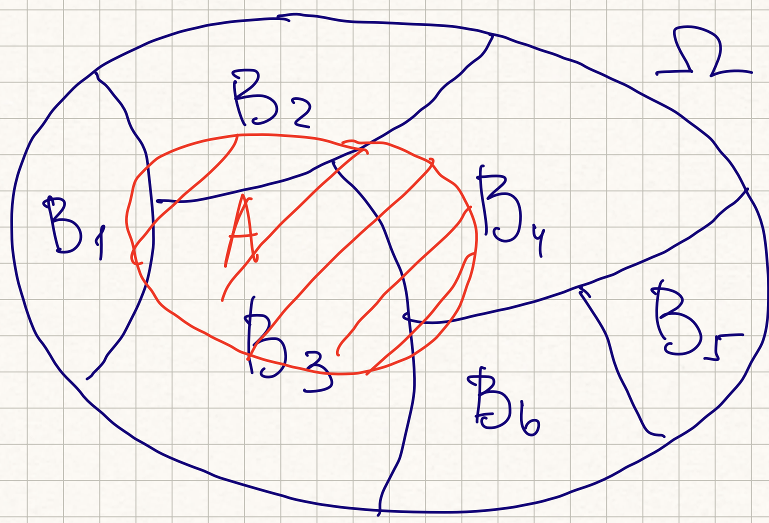
Теор. вер.

$B_1 \dots B_n$



$P(A) =$

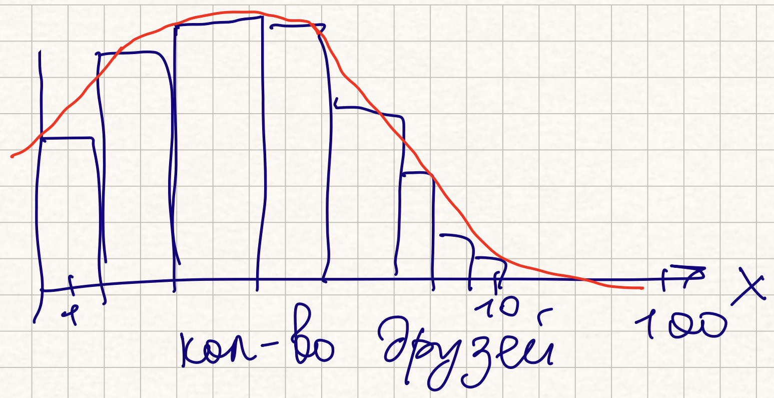
$\mathcal{B} = \bigcup [a, b]$

ξ – с. в. $\quad \xi(\mathcal{B}) \longrightarrow R$ +

$\mathbb{P}(\xi \in B)$, $B \subset \mathcal{B}$ −

$\mathbb{F}_\xi(x) = \mathbb{P}(\xi < x)$ +

$f_\xi(x)$: $\mathbb{F}_\xi(x) = \mathbb{P}(\xi < x) = \int_{-\infty}^{x} f_\xi(y)\,dy$



$$\int_{-\infty}^{+\infty} f_\xi(x)\,dx = 1 \qquad f_\xi(x) \geqslant 0$$

ЦПТ

$\xi_1 \ldots \xi_n$ – i.i.d, $\exists\, \mathbb{E}\xi_i = a$, $\exists\, \mathbb{D}\xi_i = \sigma^2$

$S_n = \xi_1 + \ldots + \xi_n$

$$\frac{S_n - n \cdot a}{\sqrt{n\sigma^2}} \xrightarrow[n \to +\infty]{} \mathcal{N}(0, 1)$$

$$\bar{\xi} \longrightarrow \mathcal{N}\left(a, \frac{\sigma^2}{\sqrt{n}}\right) \quad +$$

$$\frac{\lambda^k}{k!} \cdot e^{-\lambda} \qquad X_1 \dots X_n \overset{i.i.d.}{\sim} \text{Pois}(\underline{\underline{\lambda}})$$

$$\mathcal{L}(X|\lambda) = \prod_{i=1}^{n} \frac{\lambda^{X_i}}{X_i!} e^{-\lambda} =$$

$$= \underline{e^{-n\lambda} \cdot \lambda^{\sum_{i=1}^{n} X_i} \prod_{i=1}^{n} \frac{1}{X_i!}} \longrightarrow \max_{\lambda}$$

$$\ell = \ln \mathcal{L}(X|\lambda) = \ln e^{-n\lambda} + \ln \lambda^{\sum_{i=1}^{n} X_i} + \ln\left(\prod_{i=1}^{n} \frac{1}{X_i!}\right)$$

$$= -n\lambda + \ln \lambda \cdot \sum_{i=1}^{n} X_i + \underline{\underline{\sum_{i=1}^{n} \ln \frac{1}{X_i!}}}$$

$$\frac{\partial \ell}{\partial \lambda} = -n + \frac{1}{\lambda} \cdot \sum_{i=1}^{n} X_i = 0 \quad | \cdot \lambda$$

$$\underline{\underline{\lambda = \bar{X}}} \qquad \sum_{i=1}^{n} X_i = \lambda n$$

$$\underline{\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i = \lambda}$$

Задачи ML

регрессия

$\mathbb{R}^{n \times d} \to \underline{\underline{\mathbb{R}}}$

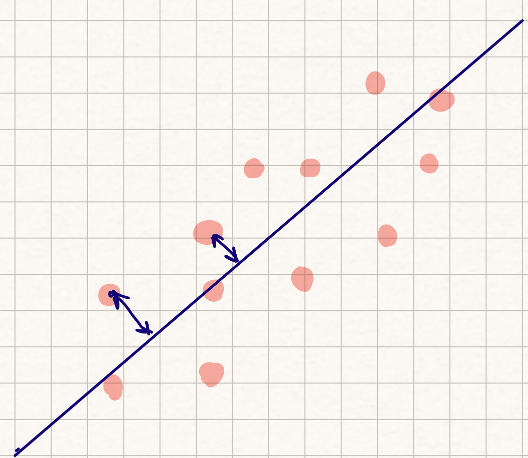
Классиф.

$\mathbb{R}^{n \times d} \to \{0, 1 \dots k\}$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - a(x_i) \right)^2$$

y_i – реальность

$a(x_i)$ – наша модель



Поиск параметров

1. Несмещённость

$\xi \sim Pois(\lambda) \qquad \mathbb{E}\xi = \lambda$

$X_1 \dots X_n \sim Pois(\lambda)$

$\overline{X} = \lambda = \mathbb{E}\xi$ – несмещённая

$X_1 \dots X_n \sim R(\Theta)$ $\quad \overline{\Theta}$ – теорет. оценка

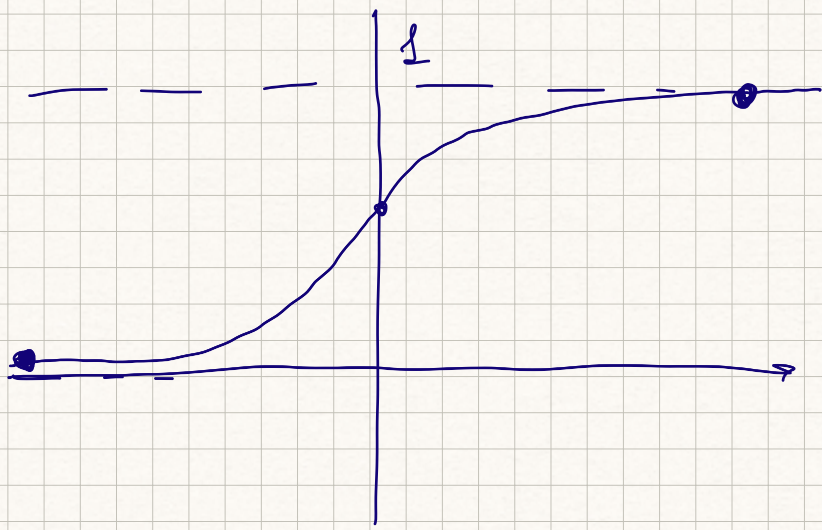
$g(X) = C(n)\overline{\Theta}$

Статистика – любая ф-ция от выборки

$\overline{X} = \frac{n-1}{n}\lambda \neq \lambda \xrightarrow[n\to\infty]{} \lambda$

$\frac{n-1}{n} \to 1$

s^2 – смещённая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