10.7.2011 Constructing GCF and LCM

Do Now:

Factorization

- A number
- Written as a product
- Of two or more of its factors

Ex. Factorizations of 48

48 = 6•8 48 = 12•4 48 = 4•3•4

Prime Factorization

- A number
- Written as a product
- Of all of its prime factors

Ex. Prime Factorizations of 48 and 60

48 = 2•2•2•2•3 60 = 2•2•3•5

The advantage of a prime factorization is that it is unique (always the same). This is compared to factorizations that are not necessarily so.

Prime Factorizations and Divisibility

The question of whether one number evenly divides another number, can be restated as: "Can I cancel all of the prime factors of my divisor with corresponding prime factors from my dividend?" If the answer is yes, the divisor divides the dividend evenly. If the answer is no, the divisor does not.

Ex. Does 16 divide 96 evenly?

$$\frac{96}{16} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2}$$

Since I can cancel all prime factors of 16 with prime factors of 96, the answer is yes. Furthermore, the quotient is what we have left in the numerator after canceling.

$$\frac{96}{16} = \frac{2 \cdot 3}{1} = 6$$

Ex. Does 36 divide 96 evenly?

$$\frac{96}{36} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3}$$

I can cancel both 2s and one 3, but there remains one three in my denominator and therefore, 36 does not divide 96 evenly.