Constructing GCF

Prime Factorizations and Divisibility

***remember** – we find the prime factorization through a factor tree

The question of whether one number evenly divides another number can be restated as: "Can I cancel all of the prime factors of my divisor with corresponding prime factors from my dividend?" If the answer is yes, the divisor divides the dividend evenly. If the answer is no, the divisor does not.

Ex. Does 16 divide 96 evenly?

$$\frac{96}{16} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2}$$

Since I can cancel all prime factors of 16 with prime factors of 96, the answer is yes. Furthermore, the quotient is what we have left in the numerator after canceling.

$$\frac{96}{16} = \frac{2 \cdot 3}{1} = 6$$

Ex. Does 36 divide 96 evenly?

$$\frac{96}{36} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3}$$

I can cancel both 2s and one 3, but there remains one 3 in my denominator and therefore, 36 does not divide 96 evenly.

Definition of Greatest Common Factor(GCF) – Given two or more numbers, the GCF is the "largest" number that evenly divides all the given numbers.

Ex. Find GCF (12,30)

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 30: 1, 2, 3, 5, 10, 15, 30

GCF = 3 because it is the largest number in both factor lists

Constructing GCF from Prime Factorization

Most of us can already see the GCF of relatively small numbers, but when the numbers get bigger, it gets harder to list all the factors. Fortunately, the prime factorization shows us how to construct a GCF using our definition and our relationship between prime factorization and divisibility.

Ex. Construct the GCF of 28 and 70.

First, find the Prime Factorization of both.

28 = 2•2•7

70 = 2•5•7

Second, GCF = product of all common prime factors, so 28 and 70 share the prime factors of 2 and 7.

GCF = 2•7 = 14

Ex. Construct GCF (1092, 1638, 1365)

1092 = 2•2•3•7•13

1638 = 2•3•3•7•13

1365 = 3•5•7•13

GCF (1092, 1638, 1365) = 3•7•13