Signed Number Addition (and Subtraction)

Q: How do I know whether it is an addition or subtraction problem?

A: The operation we are performing immediately follows the first number.

Examples:

-6 + (-3) this is addition because the + immediately follows the first number

9 – 30 this is a subtraction problem because the – immediately follows the first number

Assessment 1

Q: How do I know the signs of the first and second number?

A: The sign of the first number precedes the first number and the sign of the second number follows the operation symbol. If there is no sign the number is positive.

Examples:

-6 + (-3) The first number is negative because a – sign precedes the first number, and the second number is negative because a – follows the operation symbol.

9 – (+30) The first number is positive because there is no sign preceding the first number, and the second number is positive because a + follows the operation symbol.

+17 – 8 The first number is positive because a + precedes the first number, and the second number is positive because there is no sign following the operation symbol.

Assessment 2

Q: What is the absolute value of a number?

A: The absolute value of a number is its distance from zero. It is the number without it's sign and is always taken to be positive. It is denoted by vertical bars surrounding a number.

Examples:

|-25| = 25 because it is 25 units from zero

|12| = 12 because it is 12 units from zero

Q: What is meant by greater in absolute value (GAV) and less in absolute value?

A: When comparing two numbers, the GAV, or greater in absolute value is the number that is further from zero. The number that is less in absolute value is the number that is closer to zero.

Examples:

Which number is GAV and which number is less in absolute value?

-25 and 17 -25 is GAV because it is 25 units from zero. 17 is less in absolute value because it is closer to zero

Assessment 3

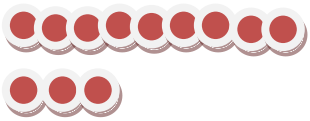
In your own words, explain how to determine whether a problem is an addition problem or a subtraction problem, and explain how to determine the sign of the second number.

Q: What do I do if it is an addition problem?

A: Apply the rules for adding integers. In an addition problem, we are putting two groups together. This is something that we ALL can visualize in our heads using integer chips.

If my signs are the same, then I am putting together two groups of chips that are the same color so when I get done I will have more chips than either of the numbers I am adding. Specifically, I will have the sum of the two numbers and the sign will be the same.

Example: -9 + (-3)



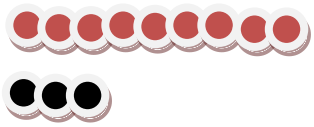
I have a total of 9+3 negative chips and so my sum is -12

Example: 1 + 9

I have a total of 1+9 positive chips and so my sum is +10

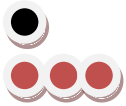
If the signs of my numbers are different, I am going to have some chips of each color and so I am going to have zero sum pairs. Specifically I am going to have the same number of zero sum pairs as the number that is less in absolute value (closer to zero). Left over, I will have fewer chips than either of my two numbers. Specifically I will have the difference of the greater in absolute value and the smaller in absolute value (or, the difference between the number that is further from zero and the number that is closer to zero). And, my left over chips will all be the color of the GAV (the number further from zero).

Example: -9 + 3



I have three zero sum pairs which, when removed leaves me with 9-3=6 red chips and so my sum is -6.

Example: 1 + (-3)



I have one zero sum pair which, when removed, leave me with 3-1=2 red chips and so my sum is -2

Assessment 4a and 4b

In your own words, explain how adding signed numbers works.

Q: What do I do if it is a subtraction problem, and why?

A: Find an equivalent addition problem and proceed as we did above, because we can always visualize an addition problem, but it is often very difficult to visualize a subtraction problem.

In the simplest case, I would be subtracting a smaller number of chips from a larger number of the same colored chips. These problems can take on two forms.

Form 1: A smaller number of positive chips from a larger number of positive chips.

Example: 9 – 4

There are 9 black (positive) chips.

Now remove 4 black chips, and we are left with 5 black chips and the difference is +5

Alternatively we could have added -4 to positive 9 and ended up with +5.

Keeping the first number the same,

We can add the opposite of the second number and get the same answer as the difference.

Form 2: A smaller number of negative chips from a larger number of negative chips. This example would look identical to the previous problem but with red and black chips interchanged. Again, by keeping the first number the same and adding the opposite of the second number, we could get the same answer as the original difference.

The real reason why we change subtraction to addition by the opposite comes from the difficulty in modeling the following differences.

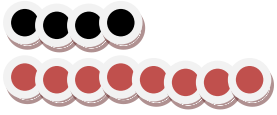
1. Taking a larger number of chips from a smaller number of the same sign. Ex. 4 – 8

There are not 8 positive chips to take away because we only have four.

Now, we can overcome this problem by merely adding in zero sum pairs until we have enough positives to remove eight of them specifically we will need four zero sum pairs.

Now we can remove eight positives and we are left with four negatives and the difference is -4.

Then again, from the beginning, instead of subtracting 8 positives, I could add 8 negatives (the opposite) and I would have a problem that is easier to visualize than putting in zero sum pairs. 4 – 8 = 4 + (-8)

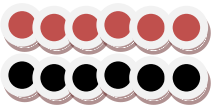


And removing our zero sum pairs we have -4 just as we did in the more complicated situation above.

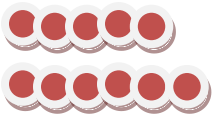
2. Removing chips of one color from an initial chip set of another color. Ex. (-5) – 6

Here I am trying to remove six positive chips from five negative chips.

Here are 5 negatives, but I can't remove any positives because there are none.



Again I can overcome this by adding in zero pairs until I have enough positives to remove, but alternatively, right from the start I could have added the opposite of the chips I was trying to remove and get the same solution. In this case, -11.



Instead of worrying about zero sum pairs, from the beginning I changed (-5) – 6 to (-5) + (-6), an easier problem to visualize. My answer was still -11 but I didn't have to add any zero sum pairs because I changed to an addition problem.

Assessment 5

Bottom Line:

1. Identify whether the problem is addition or subtraction by looking at the operation sign immediately following the first number.
2. Identify the sign of the second number which follows the operation sign we found above, and remember if there is no sign following the operation sign, the sign is positive.
3. If the operation is addition and;
    a. The signs are the same, add the absolute values and keep the sign.
    b. The signs are different, subtract the smaller absolute value from the larger and keep the sign of the GAV (number that is greater in absolute value).
4. If the operation is subtraction, keep the first number, change the operation to addition and change the second number to its opposite.

Assessment 1: Are the following problems addition or subtraction? Explain.

1. 7 – (-30) _____
2. -7 + (-30) _____
3. -15 – 90 _____
4. -9 + (-16) _____

Assessment 2: What is the sign of the second number? Explain.

1. 7 – (-30) _____
2. -7 + (-30) _____
3. -15 – 90 _____
4. -9 + (-16) _____

Assessment 3: Which of the two numbers is the GAV? Explain.

1. 7 – (-30) _____
2. -7 + (-30) _____
3. -15 – 90 _____
4. -9 + (-16) _____

Assessment 4a: What will be the sign of the following addition problems? Explain.

1. 7 + (-30) _____
2. -7 + 30 _____
3. -15 + 90 _____
4. -9 + (-16) _____

Assessment 4b: Find the following sums. Explain in words or pictures.

1. 7 + (-30) _____
2. -7 + 30 _____
3. -15 + 90 _____
4. -9 + (-16) _____

Assessment 5: Find the following difference by finding an equivalent sum. Be sure to include your new addition equation and explain your work.

1. 7 - (-30)_____
2. -7 - 30 _____
3. -15 - 90 _____
4. -9 - (-16) _____