

Rapid Prototyping: A 3D Visualization Tool Takes on Sculpture and Mathematical Forms

Rendering intricate geometries in tangible 3D form provides a more immediate experience and deeper insight than is possible through pictures and words alone.

Many technical and scientific objects are far too complex to be properly understood through pictures. 3D representations that can be touched and physically manipulated by the observer convey information not obtainable from 2D projections. Thanks to some emerging affordable rapid prototyping (RP) technologies, such models are beginning to find a role in design, science, and manufacturing. RP is already firmly established in the automotive industry and among designers of consumer products, including household appliances, toys, and electronics. With this personal case



VOLUTION_O, A
MINIMAL SURFACE OF
GENUS ZERO, DESIGNED
BY CARLO H. SÉQUIN OF
THE UNIVERSITY OF
CALIFORNIA, BERKELEY,
AND FABRICATED ON A
3D PRINTER FROM Z
CORPORATION, SHOWING
THE SHAPE A SOAP FILM
MIGHT TAKE ON WHEN
SPANNING A FRAME
COMPOSED OF 12
QUARTER CIRCLES LYING
ON THE SURFACE OF A
FIVE-INCH CUBE.

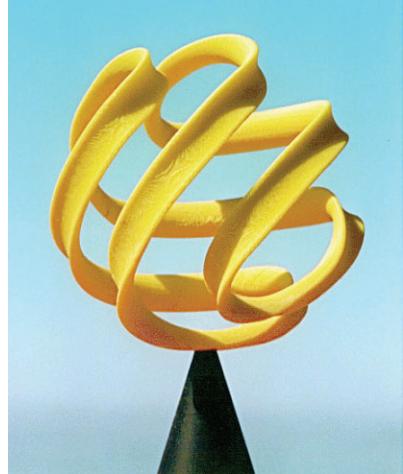


study from the arts and mathematics, I hope to encourage designers from other application domains to use RP technologies as a truly 3D physical visualization tool.

Two decades ago, few sculptors used computers in the creative phases of their work. Most notable among them was Helaman Ferguson [3], an artist and mathematics professor at Brigham Young University, who has combined mathematics and sculpting for most of his life. His creativity and analytical skills came together in a



a



b



c

custom-built, computer-controlled carving tool that allows him to transfer shapes described by mathematical expressions with high precision into large-scale stone sculptures.

My own introduction to the field of computer-aided design of artistic geometry dates to the early 1980s, when I heard a talk by Frank Smullin [11], a creator of many large tubular metal sculptures. He had developed a simple program for the Apple II computer that could plot the projected outlines of his planned sculptures and calculate the exact elliptical intersections at the joints; however, he had to apply shading by hand in order to visualize the resulting shape. The talk inspired me to create the Berkeley UniGrafix System [9] to allow me to visualize complex geometrical shapes with all hidden features eliminated. About 15 years later, my laboratory acquired a couple of RP machines, and I have used this visualization tool ever since to create complex mathematical models, as well as small maquettes of geometrical sculptures.

Figure 1. Chains of holes and saddles: (a) Brent Collins' Hyperbolic Hexagon; (b) four-story Scherk tower; and (c) Hyperbolic Hexagon II.

SCULPTURE GENERATOR I

A 1992 article [4] made me aware of the fascinating geometrical saddle shapes Brent Collins, a sculptor in Gower, MO, was then carving from wood. My collaboration with Collins began in 1994 after seeing a picture of his Hyperbolic Hexagon (see Figure 1a). This shape can be understood as a toroidal ring formed by a chain of six holes and six saddles, a constellation found in its purest form in Scherk's Second Minimal Surface, a famous mathematical surface of zero mean curvature (formed by soap films spanning warped wire loops). In a phone conversation, Collins and I discussed conceptual extensions of this paradigm. We wondered what might happen if we used an odd number of saddles in such

a toroidal loop. Clearly, the Scherk tower (see Figure 1b) would have to be given a 90-degree longitudinal twist so its two ends could be joined smoothly into a ring structure. We wondered what the twist would do to its original edges. Would they now form a complex knot rather than four individual simple loops, as in the Hyperbolic Hexagon? And would the surface become single-sided, as in a Moebius band? In the days following our conversation, we each built small mock-up models from paper and tape (Séquin) and from pipe segments and wire meshing (Collins) to assist our visualizations. These models made clear that something mathematically interesting would happen. Less clear was whether the new structure had enough aesthetic merit to warrant a three-month effort by Collins to carve it from solid maple or mahogany.

In subsequent discussions, we expanded the basic paradigm of these Scherk-Collins toroids, realizing that one could also twist hole-saddle chains with an even number of elements, tangling the edges but keeping the resulting surface two-sided. We then replaced the ordinary second-order biped saddle in



a



b



c

Figure 2. Scherk-Collins Toroids, or hole-saddle chains closed into a loop: (a) Family of 12 Trefoils; (b) the Whirled White Web, a 12-foot snow sculpture; and (c) Totem_3, a bronze sculpture cast from a fused deposition modeling maquette.

code, the rendering module used OpenGL, and the user interface was built on Mosaic. The user could manipulate a dozen sliders to specify the topology and geometry of the object—the order of the saddles used, their number in the chain, the amount of twist and total bending being applied, and the width and thickness of the surface itself, as well as the detailed shape of the edges being formed. New virtual shapes were formed in real time as the user moved the sliders, making it possible to explore dozens of new ideas in a few minutes. For the most promising design constellations, the geometrical parameters could then be fine-tuned to obtain the most aesthetically pleasing configuration.

Additional computer help is required to physically construct these sculptures. I enhanced the program to slice a chosen shape at specified intervals, typically 7/8 of an inch, to produce construction drawings for the individual boards from which Collins could then assemble the gross shape of the sculpture. Subsequently, he would fine-tune the detailed shape and sand the surface to aesthetic perfection. Our first joint sculpture, Hyperbolic Hexagon II (see Figure 1c), featuring monkey saddles instead of the original biped saddles, was constructed this way.

ITCHING FOR REALIZATIONS

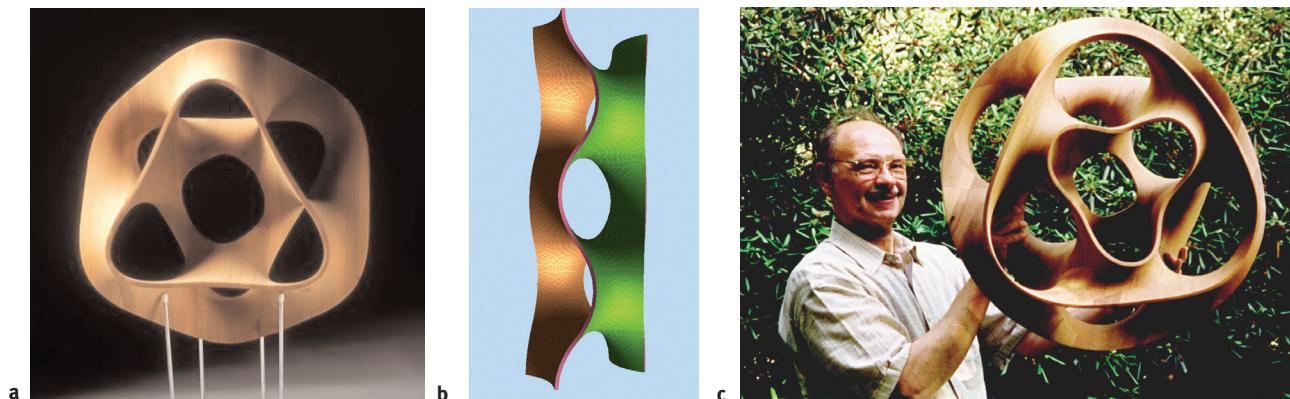
Even with the help of blueprints from Sculpture Generator I, it still takes Collins months to create a complex wood sculpture. But I was itching to quickly see realizations of many more of these geometries. In the late 1990s, RP machines became

Collins' original sculpture with saddles of higher order (such as the third-order monkey-saddle we eventually used in Hyperbolic Hexagon II (see Figure 1c).

Our ideas poured forth at a rate that made it clear we could not keep making physical models of all these potentially intriguing structures. I thus started to build special-purpose visualization tools to explore (relatively effortlessly) the vast universe of Scherk-Collins toroids. The first program was Sculpture Generator I in 1996 [8]. Its geometry kernel consisted of about 5,000 lines of C

more affordable due to their general proliferation and the pressure of competition, so I added an output module to Sculpture Generator I that would send a triangulated description of the surface in the de facto standard STL stereolithography format to any of the machines. This networking capability allowed me to fabricate six-inch models in hours or days.

I mostly used a fused deposition modeling



(FDM) machine from Stratasys, Inc. (www.stratasys.com). In this model-fabricating process, the geometry of the given shape is geometrically sliced into thin (0.01-inch-thick) layers. These layers are deposited individually, one on top of another, by a computer-controlled nozzle that dispenses the acrylonitrile butadiene styrene (ABS) thermoplastic modeling material (the kind used in Lego bricks) in a semi-liquid state at 270°C, until the precise 3D shape is recreated (see the article by Sara McMains in this section). RP models play a crucial role in the final check of whether the chosen parameters in Sculpture Generator I really lead to an aesthetically pleasing sculpture. They also are a great help to Collins when carving complex sculptures from the blueprints I generate for him.

The FDM maquettes are attractive enough that they can be enjoyed as miniature sculptures in their own right. Given how easy it is to produce them, it

now becomes practical to build whole arrays of them. Thus, if one is willing to accept the plastic prototypes as the real thing, the technology makes it affordable to build a whole family of little sculptures with systematic variations of one or two key parameters, thereby creating a special type of hypersculpture that reveals more clearly the underlying constructive logic of the geometrical shapes. An example is Family of 12 Trefoils (see Figure 2a); an

Figure 3. A sweep of a crescent-shaped cross section forms a ribbon then artfully looped around a sphere: (a) Pax Mundi by Collins; (b) Altamont by Séquin; and (c) Via Globi 3 by Séquin.

individual trefoil is obtained—if one starts with a Scherk tower with three stories and applies an overall twist of 270 degrees, so the resulting toroid has three-fold symmetry. Moreover, for two values of the azimuthal orientation of the saddles on the toroidal ring, the sculpture exhibits front-to-back symmetry. Figure 2a shows a collection of such symmetrical trefoils with saddles ranging from order one to order four (left to right), forming a single (foreground) or a double (background) toroidal loop. Each model is about six inches tall and took about 20 hours to build.

In January 2003, we used one such symmetrical trefoil with monkey saddles to design a 12-foot snow sculpture we then created at the International Snowsculpting Championships in Breckenridge,

An interactive virtual environment is conducive to experimentation and to the exploration of many fleeting ideas.

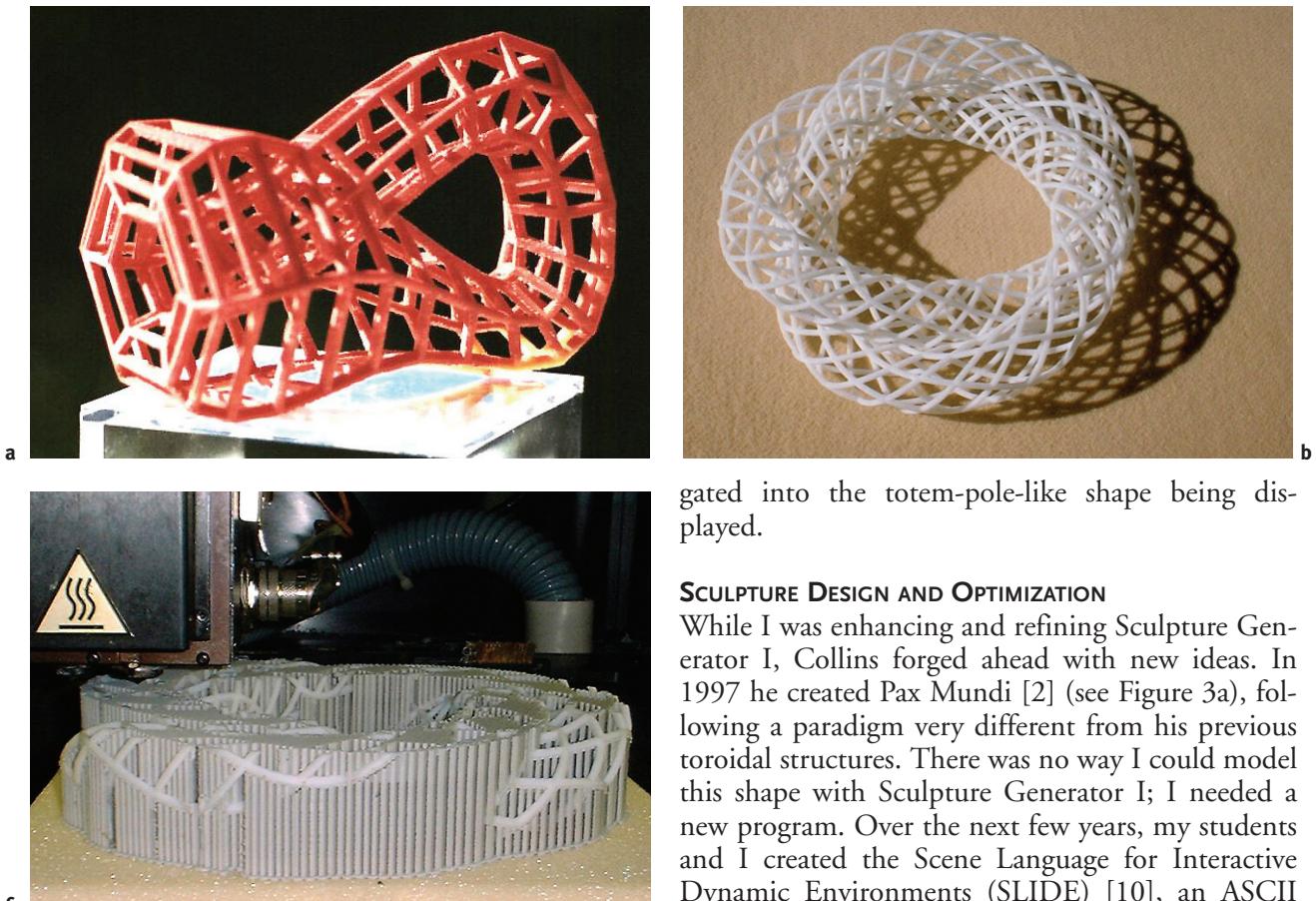


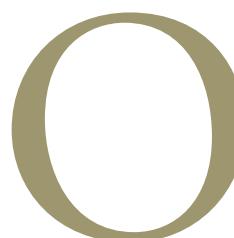
Figure 4. Gridded “transparent” RP models of Klein bottles: (a) generic Klein bottle; (b) figure-eight Klein bottle; and (c) model under construction in a fused deposition modeling machine.

CO. It would have been impossible to carve this piece from a solid block of snow weighing 20 metric tons without constant reference to a six-inch-tall FDM maquette. Called the Whirled White Web (see Figure 2b), it was awarded the silver medal [1].

With some experimentation, Steve Reinmuth [6], currently bronze casting most of Collins’ works in his studio in Eugene, OR, found that ABS maquettes could be used directly as the disposable originals in an investment-casting process (see the article by Paul K. Wright in this section). A plaster shell is formed around them by repeatedly dipping them into colloidal silica slurry and fused silica stucco. This shell is then heated with great care to about 1,600°F. The ABS plastic liquifies and is drained from the plaster shell, whereupon the hollow is refilled with liquid bronze. Reinmuth has cast several of my models (created through Sculpture Generator I) in bronze and added a variety of patinas to them. A recent example is the Totem 3 sculpture (see Figure 2c), demonstrating a further extension of the Scherk-Collins paradigm. Using nonuniform affine scaling, the original toroidal structure can be elongated into the totem-pole-like shape being displayed.

SCULPTURE DESIGN AND OPTIMIZATION

While I was enhancing and refining Sculpture Generator I, Collins forged ahead with new ideas. In 1997 he created Pax Mundi [2] (see Figure 3a), following a paradigm very different from his previous toroidal structures. There was no way I could model this shape with Sculpture Generator I; I needed a new program. Over the next few years, my students and I created the Scene Language for Interactive Dynamic Environments (SLIDE) [10], an ASCII text file language for describing and interacting with hierarchical, dynamic environments. SLIDE is more modular than Sculpture Generator I and contains a powerful sweep generator module and many types of subdivision surfaces. A sculpture like Pax Mundi would typically call on three different modules: one to create a smooth, looping sweep path on the surface of a sphere; one to define the desired crescent-like cross section; and one to specify the exact way the cross section is rotated, tilted, and/or scaled as it travels along the sweep path.



nce I understood Pax Mundi in terms of these modules, it was easy to extend Collins’ key idea of sweeping a crescent-like cross section along an undulating path around a sphere into a much broader paradigm. Collins’ original sculpture exhibited four full waves of a line circling a globe while oscillating about its equator. I could now readily change the path to include three, five, or six waves, and apply different amounts of amplitude modulation to obtain pleasing results. In a later enhancement I allowed each lobe to be modulated laterally, too, obtaining even more intriguing

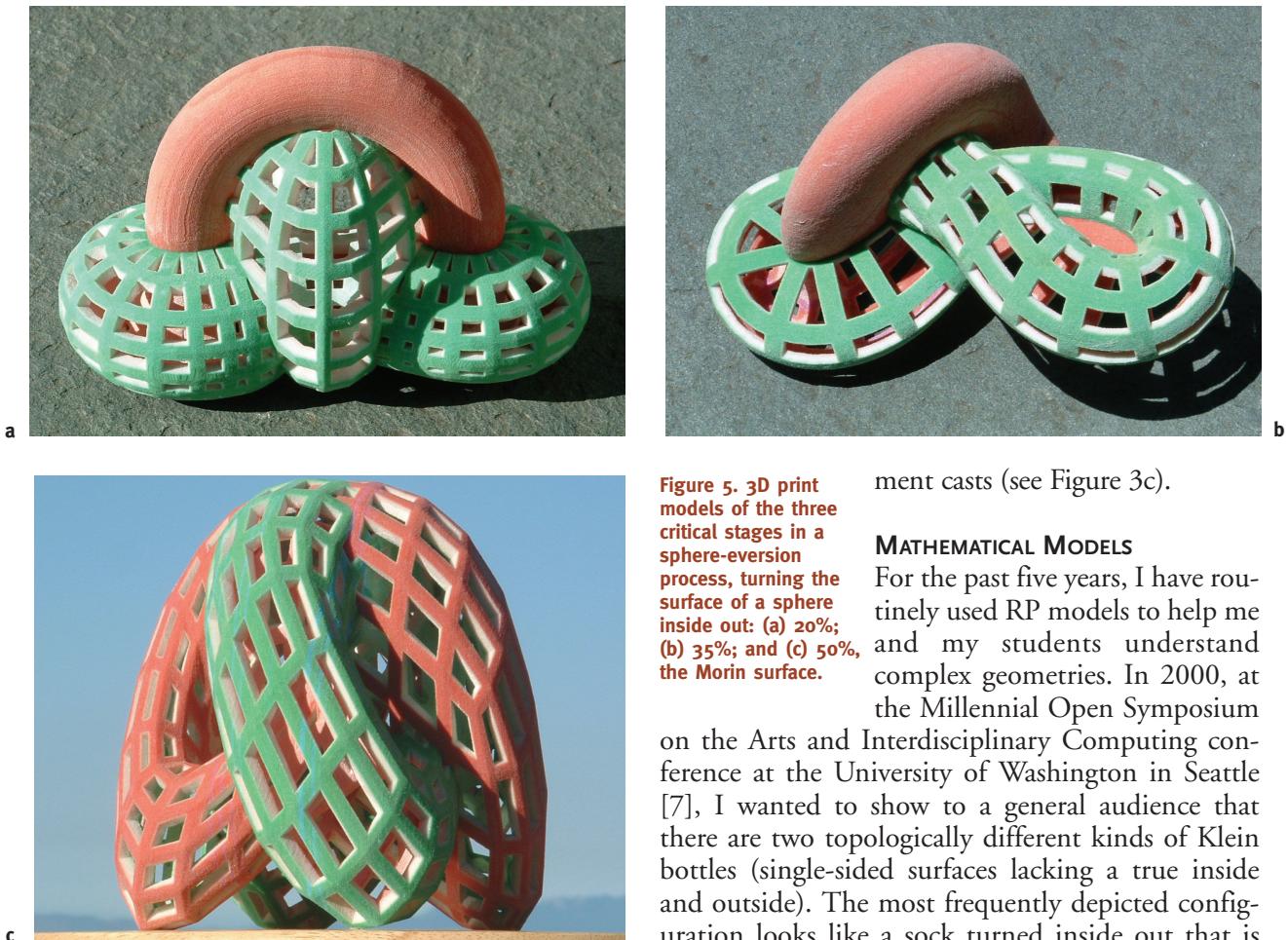


Figure 5. 3D print models of the three critical stages in a sphere-eversion process, turning the surface of a sphere inside out: (a) 20%; (b) 35%; and (c) 50%, the Morin surface.

ment casts (see Figure 3c).

MATHEMATICAL MODELS

For the past five years, I have routinely used RP models to help me and my students understand complex geometries. In 2000, at the Millennial Open Symposium on the Arts and Interdisciplinary Computing conference at the University of Washington in Seattle [7], I wanted to show to a general audience that there are two topologically different kinds of Klein bottles (single-sided surfaces lacking a true inside and outside). The most frequently depicted configuration looks like a sock turned inside out that is then stuffed through its own side to close the surface (see Figure 4a). A topologically different Klein bottle results if one sweeps a figure eight cross section around a loop while it undergoes an odd number of half-twists before closing on itself. It turns out that this Klein bottle (see Figure 4b) can be formed by a single contiguous filament woven around the shape so it avoids self-intersection at the junction where the surface must pass through itself for topological reasons.

Mathematicians ignore surface self-intersections; specifically, one is not allowed to step from one branch of the surface to the other at the self-intersec-

results (see Figure 3b).

A virtual environment is conducive to such experimentation and to the exploration of many fleeting ideas. When artists or designers discover promising constellations, they can then seek out their optimal realization. It turned out that in almost all cases the first physical maquette of the design revealed aesthetic flaws I had not observed on the computer screen. RP thus provides a powerful visualization tool one should not do without, if one is truly concerned with finding the optimal design. The final, improved maquettes can then also be used as originals for making invest-

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tion lines. Conveying these concepts with pictures alone is difficult. RP has allowed me to create grid-like—and thus transparent—physical models students can handle and inspect closely. Figure 4c shows the model being built in the FDM machine and the extraordinary amount of gray support material it required. Fortunately, some RP machines from Stratasys now use a water-soluble model-support material that can be dissolved and washed away once the desired part is built.



Another object demanding a 3D model is the Morin surface (see Figure 5c), the halfway-point in a process of turning a sphere inside out. Beautiful computer animations have depicted

this process, the earliest in 1976 by Nelson Max, now a scientist at Lawrence Livermore National Laboratory and a professor at the University of California, Davis [5]. However, one needs to watch them many times before one truly understands the process and the geometry of the various stages. Physical models can give people the necessary insight much more quickly. In the 1960s Charles Pugh at the University of California, Berkeley, painstakingly created a dozen beautiful, large-scale chicken-wire models (later stolen and never returned).

RP now offers an easier way to recreate such models; the 3D printing process developed by Z Corporation (www.zcorporation.com) can even color the two sides of the sphere surface differently (see Figure 5). This fabrication process is derived from inkjet printing, but rather than squirting colored ink onto paper, the RP machines squirt a colored binder into the surface of a bed of plaster powder. After printing a few hundred layers on top of each other, the model is then defined by the regions in which the plaster is glued together. The part can be freed from its bed of plaster powder by simply brushing and blowing away the loose powder particles. This process is particularly attractive for geometries where it would be too tedious or even impossible to remove the internal scaffolding needed in the traditional FDM process.

CONCLUSION

Representing geometrical shapes in the form of a computer model offers several advantages in almost any kind of design. A parameterized, procedural description provides an easy way to explore a number of possible solutions, then fine-tune the chosen configurations. With the help of the computer, more complex designs can be generated than could be cre-

ated through traditional hand-crafted means. The model can then be scaled to any size and used to produce small models through layered free-form fabrication, as well as large sections of the sculpture for some casting process.

RP already plays a major role in automotive and consumer-product design where the tactile feedback from a shape under development is as important as its visual appearance. As RP processes become better known and more affordable, artists, mathematicians, scientists, and many other potential users will also come to rely on the technology as a powerful visualization tool. ■

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