Hiding Leaders in Covert Networks: A Computational Complexity Perspective

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Introduction

Motivation

- Help to deepen the understanding of hiding in networks
- Help to make identifying key actors in criminal organizations easier

Problem Definition

Hiding Leaders (HL)

Instance: (G, L, b, c, d), where

- $G = (V, E)$ is a network
- $L \subseteq V$ are leaders, $F = V \setminus L$ are followers
- b is a budget
- $c : G \times V \to \mathbb{R}$ is a centrality measure
- d is a safety margin

Question: Is there $W \subseteq F \times F$ and $F' \subseteq F$ such that

- $|F'| \geq d$
- $|W| \leq b$
- $\forall_{f' \in F'} \forall_{\ell \in L} : c((V, E \cup W), f') \geq c((V, E \cup W), \ell)$

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Problem Definition

Hiding Leaders (HL_{deg})

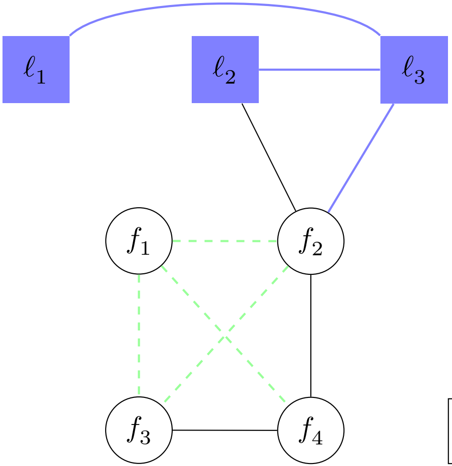
Instance: (G, L, b, c, d), where

- $G = (V, E)$ is a network
- $L \subseteq V$ are leaders, $F = V \setminus L$ are followers
- b is a budget
- $c = c_{deg}$, $c_{deg}(G, v) = \deg(v)$
- d is a safety margin

Question: Is there $W \subseteq F \times F$ and $F' \subseteq F$ such that

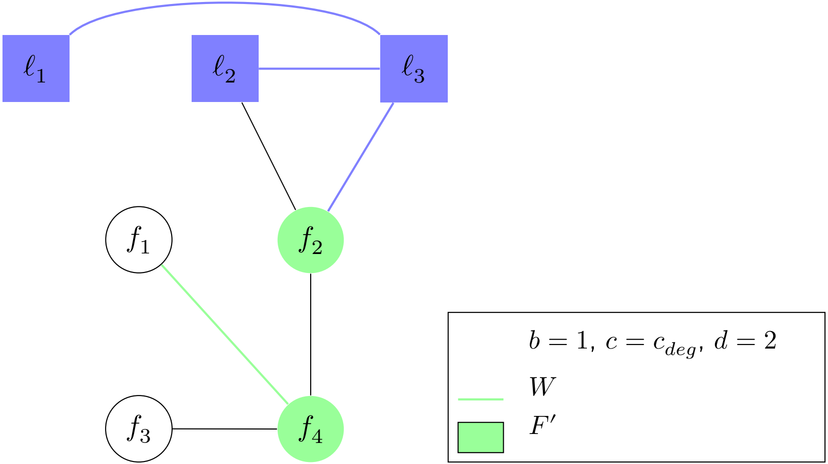
- $|F'| \geq d$
- $|W| \leq b$
- $\forall_{f' \in F'}: \deg(f') \geq \max\limits_{\forall_{\ell \in L}} \deg(\ell)$ in $(V, E \cup W)$

Sample Instance



$b = 1,\ c = c_{deg},\ d = 2$

Sample Instance



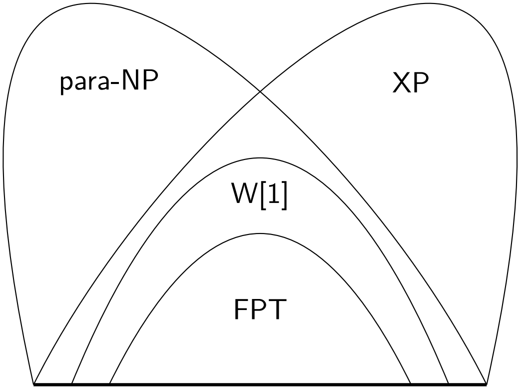
Goals

- Survey the computational complexity of the problem variants studied in the literature
- Derive new complexity and algorithmic results for various parameters with the use of the framework of parameterized complexity

Parameterized Complexity Classes

- FPT : $f(k) \cdot |(x,k)|^c$, $c \in \mathbb{N}$
- XP : $f(k) \cdot |(x,k)|^{g(k)}$

- W[1]-hard $\Rightarrow$ not in FPT
- para-NP-hard $\Rightarrow$ not in XP

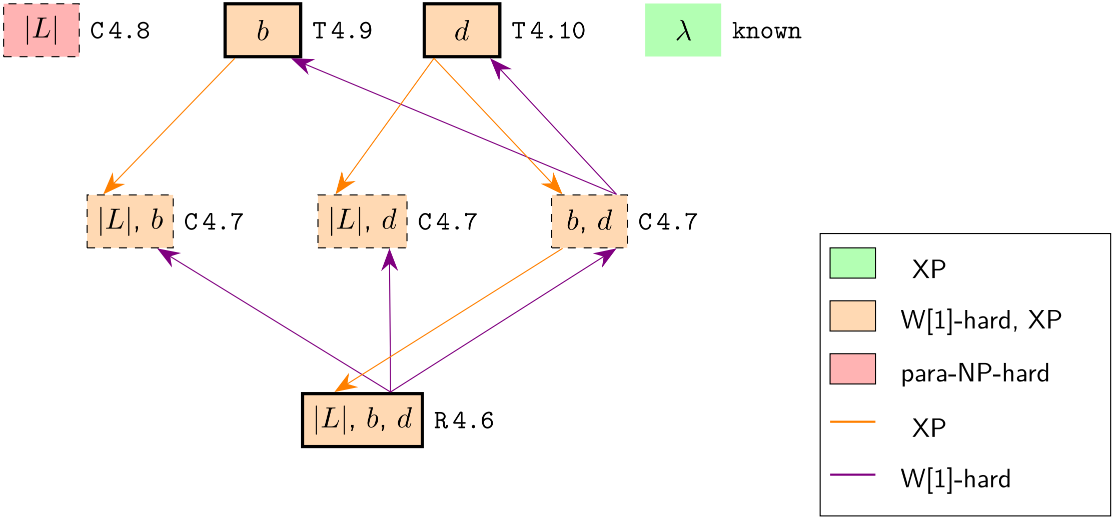


Contribution

Literature Review

- Literature results summarized and described in the language of parameterized complexity theory
- Pointed out that there are two definitions of the problem
  - Proof of NP-hardness of HL_{deg} reviewed for our definition

Own Results



$|L|$ C4.8
b T4.9
d T4.10
λ known
$|L|$, b C4.7
$|L|$, d C4.7
b, d C4.7
$|L|$, b, d R4.6
XP
W[1]-hard, XP
para-NP-hard
XP
W[1]-hard

Own Results

Theorem

Hiding Leaders parameterized by $b + d$ is W[1]-hard even if $|L| = 1$

Hiding Leaders parameterized by $|L| + b + d$ is W[1]-hard

- Proved by showing a parameterized reduction from k-Clique on regular graphs
  - k-Clique instance (G, k), where G is a r-regular graph
  - Hiding Leaders instance $(H, \{\ell\}, \frac{k\cdot(k-1)}{2}, c_{deg}, k)$, where H is constructed from G
  - Solving Hiding Leaders in H corresponds to solving k-Clique in G

Own Results

Corollary

Hiding Leaders is para-NP-hard with respect to $|L|$

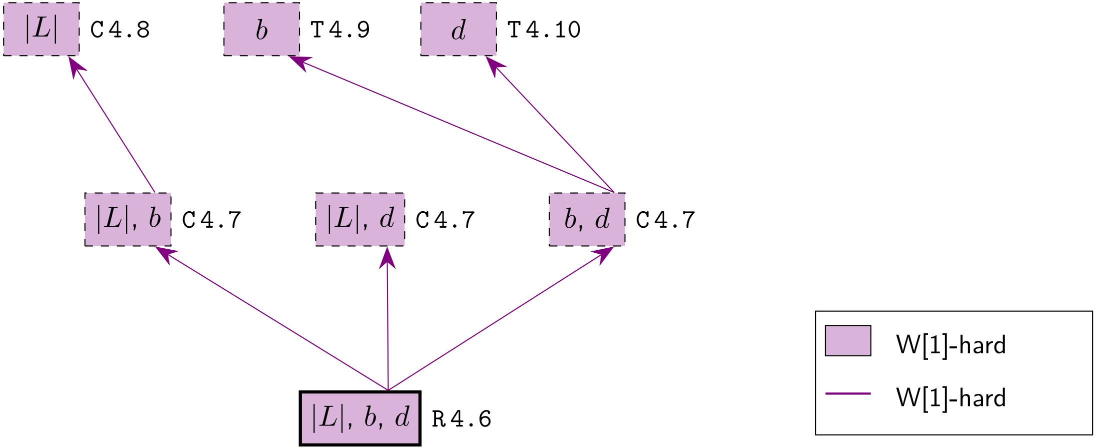
- The parameterized reduction is also a polynomial reduction
- k-Clique on regular graphs is NP-hard $\xrightarrow{\text{reduction}}$ HL with $|L| = 1$ is NP-hard

$\mathbf{HL}_{deg}$ Complexity Picture

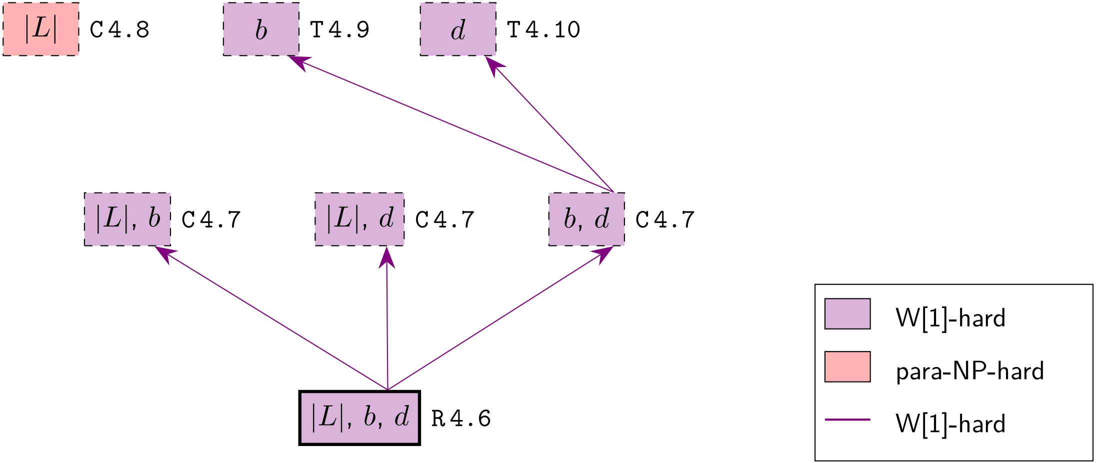
$\boxed{|L|,\ b,\ d}$ R4.6

W[1]-hard

HL_{deg} Complexity Picture



$\mathbf{HL}_{deg}$ Complexity Picture



Own Results

Theorem

Hiding Leaders parameterized by b is in XP

- Try to add every b-element subset of edges that can be added between followers
  - Check if it is a solution
- The algorithm runs in time $n^{\mathcal{O}(b)}$

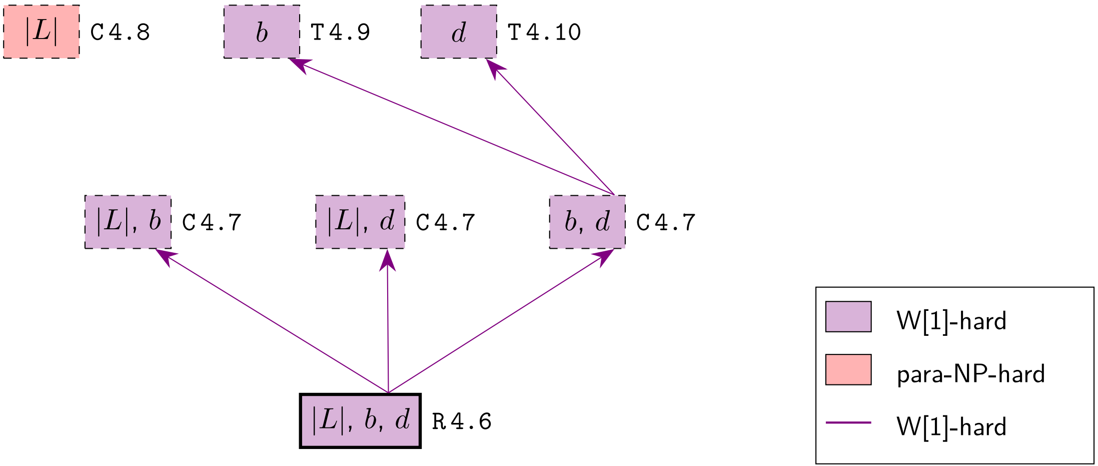
Own Results

Theorem

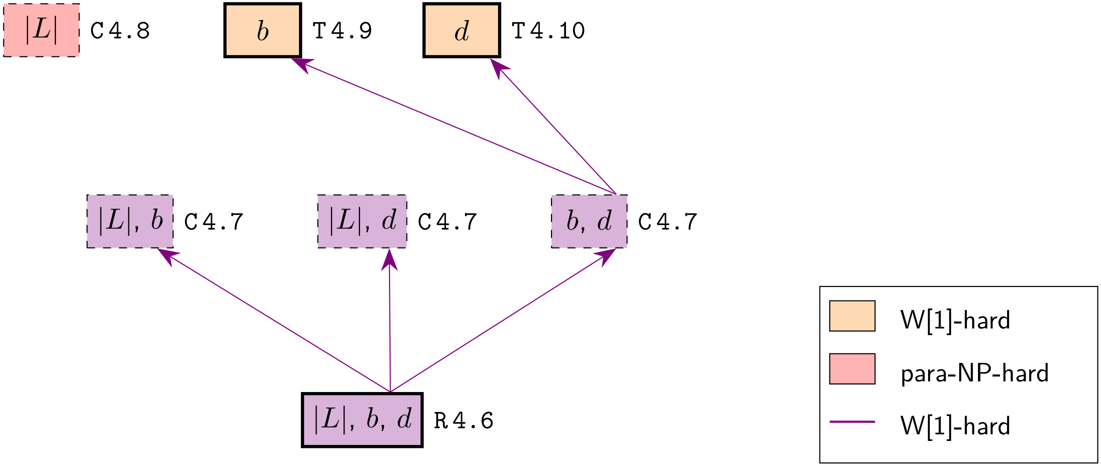
Hiding Leaders parameterized by d is in XP

- Try every d-element subset of followers
- Add edges between followers from the subset
  - Check if it is a solution
  - If no, check if a solution can be found by connecting followers outside the subset
- The algorithm runs in time $\mathcal{O}(2^{d^2}) \cdot n^{\mathcal{O}(d)}$

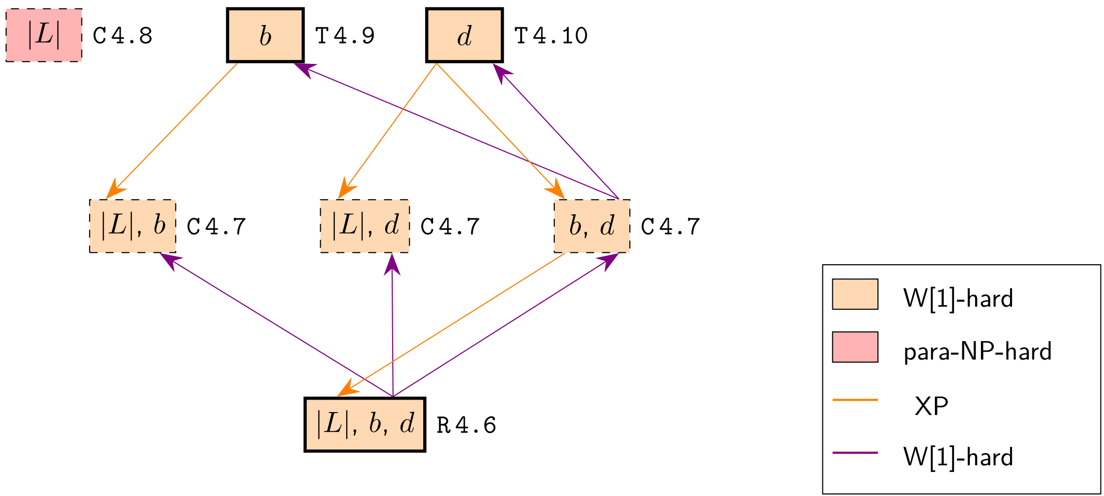
$\mathbf{HL}_{deg}$ Complexity Picture



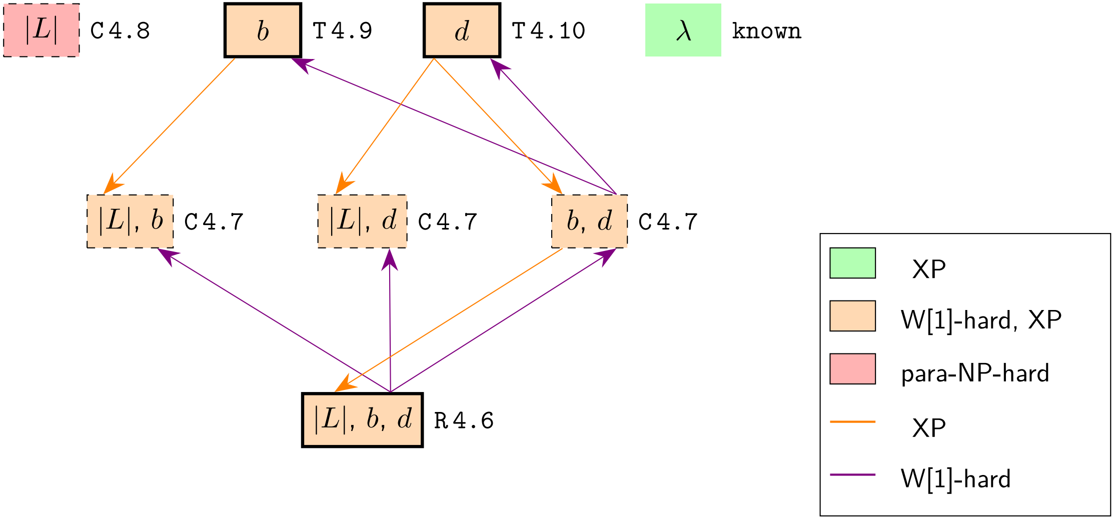
HL_{deg} Complexity Picture



$\mathbf{HL}_{deg}$ Complexity Picture



$\mathbf{HL}_{deg}$ Complexity Picture



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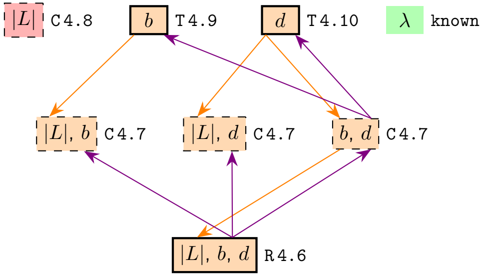
- New complexity and algorithmic results obtained



Outlook

- Different centralities or parameters
  - vertex cover number
- W[1]-hardness for λ

Thank you for your attention!

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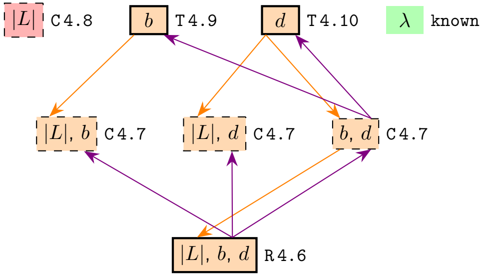

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Questions from the opponent

Question

V definici 2.11 definujete centralitu měřenou stupněm uzlu jako hodnotu danou stupněm daného vrcholu (1). Články, na které se odkazujete v konstrukcích a důkazech ovšem tuto míru definují jinak (2) (zdroje [2, 51]). Není tato odlišná definice problémem v důkazech, které v práci uvádíte? Prodiskutujte prosím odlišnosti a navrhněte řešení.

$$(1)\ c(G, v) = \deg v$$

$$(2)\ c(G, v) = \deg \frac{v}{|V|-1}$$

Questions from the opponent

$$(1)\ c(G,v) = \deg v \qquad (2)\ c(G,v) = \deg \frac{v}{|V|-1}$$

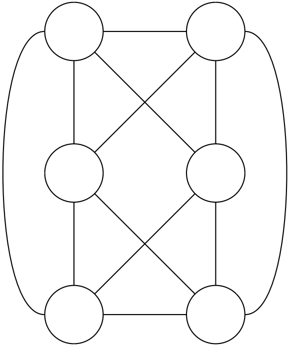
Answer

Odlišná definice není problémem v důkazech, které v práci uvádím.

Normalizační faktor $\frac{1}{|V|-1}$ ve vztahu (2) představuje pro daný graf konstantu a tedy nemění relativní pořadí vrcholů oproti pořadí určeného pomocí vztahu (1). Po přidání/odebrání vrcholů do/z grafu se sice tento faktor změní, zůstane však stejný pro všechny vrcholy v grafu a relativní pořadí oproti vzathu (1) se tak nezmění. Během zkoumání problému Hiding Leaders jde právě o relativní pořadí vrcholů mezi sebou, čili vztah (1) nepředstavuje v tomto ohledu vůči vztahu (2) žádný rozdíl, avšak umožňuje jednodušší argumentaci díky možné záměně centrality vrcholu a stupně vrcholu.

Vztah (1) je také použitý v definici ve zdroji [3].

Sample Construction

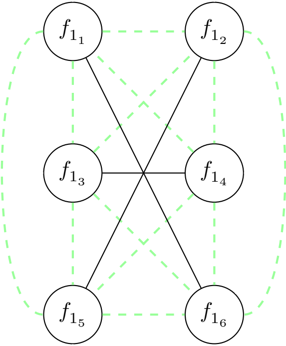


$$k = 3$$

$$b = \frac{k \cdot (k-1)}{2} = 3$$

$$d = k = 3$$

Sample Construction



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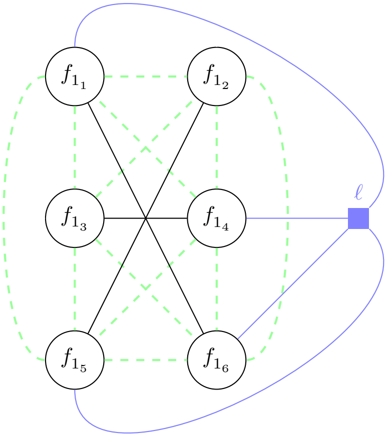

$k = 3$

$b = \frac{k \cdot (k-1)}{2} = 3$

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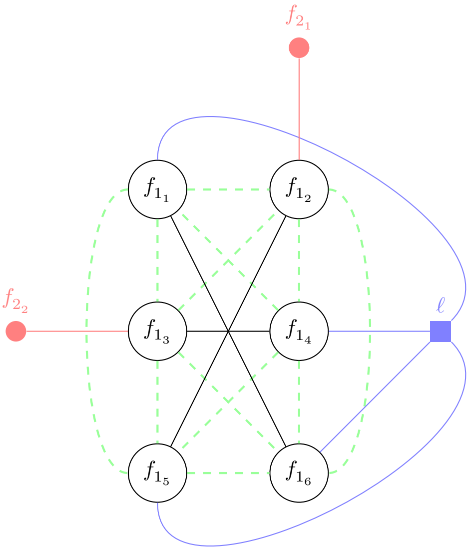
$\deg(\ell) = n - r + (k-1) = 4$

Sample Construction



$k = 3$

$b = \frac{k\cdot(k-1)}{2} = 3$

$d = k = 3$

$\deg(\ell) = n - r + (k-1) = 4$

$\deg(f_{2_i}) = 1$