6.16 常態定理 Let $T \in \mathbb{L}(V)$ for inner-product space V over $\mathbb{C}$ with $\dim(V) < \infty$. V has an orthonormal eigenbasis for T iff T is normal, i.e. $TT^* = T^*T$.

Def 自伴線運、方陣 T is self-adjoint if $T^* = T$. Square matrix A is Hermitian if $A^* = A$.

Obs 自伴小觀察 If β is an orthonormal basis of a inner-product space V with $\dim(V) < \infty$, then T is self-adjoint iff $[T]_\beta^\beta$ is self-adjoint.

Obs 自伴線運基本性質 If T is a self-adjoint linear operator on an inner-product space V over $F \in \{\mathbb{R}, \mathbb{C}\}$ with $\dim(V) < \infty$, then 1. each eigenvalue of T is real (even if $F = \mathbb{C}$, and) 2. the characteristic polynomial $f_T(t)$ of T splits (even if $F = \mathbb{R}$)

6.24 投影：正交 $\Leftrightarrow$ 自伴 If T is a projection of inner-product space W, then T is an orthogonal projection of W iff T is self-adjoint

Cor 自伴線運推論 Let $T \in \mathbb{L}(V)$ for inner-product space W over $F = \mathbb{C}$ wth $\dim(V) < \infty$. If T is normal, T is self-adjoint iff every eigenvalue of T is real.

6.17 自伴定理 Let $T \in \mathbb{L}(V)$ for V over $\mathbb{R}$ with $\dim(V) < \infty$. V has an orthonormal eigenbasis for T iff T is self-adjoint.

Cor 常態線運推論 If $T \in \mathbb{L}(V)$ for W over $\mathbb{C}$ with $\dim(W) < \infty$, then T is normal iff $T^* = g(T)$ for some polynomial $g \in \mathbb{P}(\mathbb{C})$

Def 么正方陣、正交方陣 Let $Q \in F^{n\times n}$ with $F \in \{\mathbb{C}, \mathbb{R}\}$, • Q is unitary if $Q^*Q = I_n$ (i.e. $Q^* = Q^{-1}$) • Q is orthogonal if $Q^tQ = I_n$ (i.e. $Q^t = Q^{-1}$)

Def 么正、正交等價 Let $A, B \in F^{n\times n}$ with $F \in \{\mathbb{C}, \mathbb{R}\}$, • A is unitarily equivalent to B if there is a unitary matrix Q with $A = Q^*BQ$ • A is orthogonally equivalent to B if there is an orthogonal matrix Q with $A = Q^tBQ$

6.19

6.20 If $A \in \mathbb{R}^{n\times n}$, then A is self-adjoint (i.e. symmetric) iff A is orthogonally (i.e. unitarily) equivalent to a diagonal matrix in $\mathbb{R}^{n\times n}$

6.21 方陣舒爾

6.18

么正定理

正交自伴定理

6.13 最短解 If $E : Ax = b$ with $A \in F^{m\times n}$ and $b \in F^m$ is a system of linear equations with $S(E) \neq \emptyset$, then there is exactly one vector x in

$$S(E) \cap L_{A^*}(F^m)$$

w.r.t. the standard inner product. Moreover, the vector x is the unique vector in $S(E)$ with minimum $||x||$

6.12 最佳近似解

Obs 標準內積觀察

Obs 矩陣位階觀察

Obs 伴隨矩陣觀察

6.27 SVD For any matrix $A = F^{m\times n}$ with rank r,

$$A = QSR*$$

holds for:

Def 正定 A self-adjoint $T \in \mathbb{L}(V)$ for inner-product space V over F is positive definite if

$$< T(x)|x >$$

6.26 奇異值定理 Let $T \in \mathbb{L}(V, W)$ for inner-product spaces V and W over $F \in \{\mathbb{C}, \mathbb{R}\}$ with $\mathrm{rank}(T) = r$, $\dim(V) = n$, and $\dim(W) = m$.

6.27 SVD For any matrix $A = F^{m\times n}$

Def Pseudoinverse $T^\dagger \in \mathbb{L}(W, V)$ defined by $T^\dagger = (T')^{-1}T''$

偽反線轉定理