自伴線運 $T = T^*$

自伴方陣

自伴小觀察 Let β is an orthonormal basis of a finite-dimensional inner-product space V, then T is self-adjoint iff $[T]_\beta^\beta$ is self-adjoint.

自伴線運基本性質 If T is a self-adjoint linear operator on an inner-product space V over $F \in \{\mathbb{R}, \mathbb{C}\}$ with $\dim(V) < \infty$, then 1. each eigenvalue of T is real (even if $F = \mathbb{C}$, and) 2. the characteristic polynomial $f_T(t)$ of T splits (even if $F = \mathbb{R}$)

6.24 投影：正交 $\Leftrightarrow$ 自伴 If T is a projection of inner-product space W, then T is an orthogonal projection of W iff T is self-adjoint

自伴線運推論 Let $T \in \mathbb{L}(V)$ for inner-product space W over $F = \mathbb{C}$ wth $\dim(V) < \infty$. If T is self-adjoint iff every eigenvalue of T is real.

6.17 自伴定理

Def 么正方陣、正交方陣 Let $Q \in F^{n\times n}$ with $F \in \{\mathbb{C}, \mathbb{R}\}$, • Q is unitary if $Q^*Q = I_n$ (i.e. $Q^* = Q^{-1}$) • Q is orthogonal if $Q^tQ = I_n$ (i.e. $Q^t = Q^{-1}$)

Def 么正、正交等價 Let $A, B \in F^{n\times n}$ with $F \in \{\mathbb{C}, \mathbb{R}\}$, • A is unitarily equivalent to B if there is a unitary matrix Q with $A = Q^*BQ$ • A is orthogonally equivalent to B if there is an orthogonal matrix Q with $A = Q^tBQ$

6.19

6.20 If $A \in \mathbb{R}^{n\times n}$, then A is self-adjoint (i.e. symmetric) iff A is orthogonally (i.e. unitarily) equivalent to a diagonal matrix in $\mathbb{R}^{n\times n}$