

**Axioms** A1. The speed of any light beam, measured in  $S$  or  $S'$ , is 1.

A2.  $T_v : \mathbb{R}^4 \Rightarrow \mathbb{R}^4$  is an isomorphism.

A3. If  $T_v \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$ , then  $y' = y$  and  $z' = z$ .

A4. If  $T_v \begin{pmatrix} x \\ y_1 \\ z_1 \\ t \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$  &  $T_v \begin{pmatrix} x \\ y_2 \\ z_2 \\ t \end{pmatrix} = \begin{pmatrix} x'' \\ y'' \\ z'' \\ t'' \end{pmatrix}$ , then  $x'' = x'$  and  $t'' = t'$ .

A5. The origin of  $S'$  moves in the positive direction of the  $x$ -axis of  $S$  at the constant velocity  $v > 0$  as measured from  $S$ .

**Thm Lorenz** If  $\alpha$  is the standard basis of  $\mathbb{R}^4$ , then  $[T_v]_\alpha^\alpha = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} & 0 & 0 & \frac{-v}{\sqrt{1-v^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-v}{\sqrt{1-v^2}} & 0 & 0 & \frac{1}{\sqrt{1-v^2}} \end{pmatrix}$

**Notation** For any linear operator  $T$  on  $V$  over  $F$  and any scalar  $a \in F$ , let  $t_a \stackrel{def}{=} T - a \cdot I_V$

**Def** Let  $T \in \mathbb{L}(V)$  for vector space  $V$  over  $F$ . For any  $a \in F$ , define  $E_T(a) = N(T_a)$

**Def 純量重數** The *multiplicity* of a scalar  $a$  w.r.t.  $T \in \mathbb{L}(V)$  is the largest integer  $m$  with  $(t - a)^m | f_T(t)$

**Def 推廣特徵組** Let  $T \in \mathbb{L}(V)$  for  $V$  over  $F$ .  $(\lambda, x)$  with  $\lambda \in F$  and  $x \in V \setminus \{0_V\}$  is a *generalized eigenpair* of  $T$  if  $T_\lambda^l(x) = 0_V$  holds for some positive integer  $l$ .

**Obs 暖身觀察** If  $(\lambda, x)$  is a generalized eigenpair of  $T$ , then  $\lambda$  is an eigenvalue of  $T$ .

**Def**  $E_T(a) \rightarrow G_T(a)$  Let  $T \in \mathbb{L}(V)$  for  $V$  over  $F$ . For any  $a \in F$ , let  $G_T(a) = \{x \in V | T_a^l(x) = 0_V \text{ holds for a positive integer } l\}$