A Variant of the TSP: Prize Collecting Around National Parks

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Outline

A Reminder: The TSP

The classic Traveling Salesman Problem (TSP), an important problem in operations research, can be stated as the following:

> *"Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" [4]*

A Reminder: The TSP

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> *"Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" [4]*

- How can we make this more *interesting*?

The Modification: Prize Collecting

- Prize-Collecting TSP
- Given a set of cities, each city gives positive utility to visit
- Constraint on maximum distance traveled
- Goal: Find the path that maximizes utility
- Setting: US National Parks

US National Parks

Figure 1: Yellowstone, taken from [3].

US National Parks

- 59 US National Parks: 27 states & various territories
- National Park Service $\rightarrow$ *nice data*
- Important historically
- First national park: Yellowstone in 1872.
- Beauty, diverse ecosystems, geological features
- Fun, interesting setting (but also arbitrary)

Additional Modifications

- Relax the constraint that each site must be visited
- Utility: function of the annual number of visitors to each park
- Return to the city of origin (Boston, MA)
- Sub-tour elimination constraints

Algebraic Model

Table 1: List of all notations (decision variables, parameters, and sets/subsets)

Notation	Object Type	Definition
v_j	Decision Variable	Parks
x_{ij}	Decision Variable	Roads
c_j	Parameter	Utility coefficients
y_{ij}	Parameter	Distance between parks
d	Parameter	Maximum travel distance
V	Set	Set of all parks
V_1	Subset	Set of visited parks
S	Subset	Subset of V_1

Algebraic Model

$$max \sum_{j=1}^{n} c_j v_j$$

$$s.t. \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} y_{ij} \leq d$$

$$v_j = \sum_{i=1}^{n} x_{ij}$$

$$\sum_{i \in S} \sum_{j \in V_1 \setminus S} x_{ij} \geq 1, \forall_S \subset V_1$$

$$x_{ii} = 0, \forall_i \in V_1$$

$$v_j \in \{0,1\}, \forall_j \in V$$

$$x_{ij} \in \{0,1\}, \forall_i, \forall_j \in V$$

Algebraic Model

Table 2: Description of Objective Function and Constraints

Objective Function	$max \sum_{j=1}^{n} c_j v_j$	Maximize utility by selecting the parks
Constraint 1	$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} y_{ij} \leq d$	Sum of y_{ij} for $x_{ij} \leq d$.
Constraint 2	$v_j = \sum_{i=1}^{n} x_{ij}$	We go to a park v_j if and only if we take a road leaving that park
Constraint 3	$\sum_{i \in S} \sum_{j \in V_1 \setminus S} x_{ij} \geq 1, \forall_S \subset V_1$	Sub-tour elimination constraint

Algebraic Model

Constraint 4	$x_{ii} = 0, \forall_i \in V_1$	No self path constraint
Constraint 5	$v_j \in \{0,1\}, \forall_j \in V$	v_j binary variable $\forall$ elements in V: $v_j = \begin{cases} 1 & \text{if park j is chosen} \\ 0 & \text{if park j is not chosen} \end{cases}$
Constraint 6	$x_{ij} \in \{0,1\}, \forall_i, \forall_j \in V$	x_{ij} binary variable $\forall$ elements in V: $x_{ij} = \begin{cases} 1 & \text{if road } ij \text{ is chosen} \\ 0 & \text{if road } ij \text{ is not chosen} \end{cases}$

Data

- Data on 59 US National Parks
  - GPS Coordinates
  - Number of annual visitors (ζ_j)
- GPS coordinates used to compute distances
- ζ_j used to establish the utility values for each national park
- Utility value $c_j = \zeta_j^{1/5}$ used for appropriate scale
- True distances computed in the Julia program
- Utilities computed separately and read into the Julia file

Results

max distance (km)	value
18000	693.07
16000	653.50
15000	624.47
14000	589.02
13000	543.58
12000	494.60
10000	386.79
9000	325.12
8000	260.99
7000	152.19
6000	131.95
5000	108.62
4000	102.23
3000	59.44
2500	37.04
2100	34.64
TSP Distance (km)	**TSP Value**
35805.21	832.34

Results

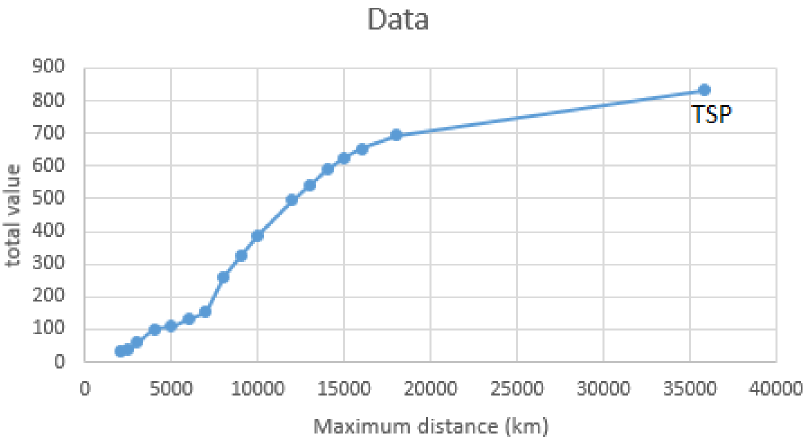

Figure 2: This graph shows the optimal objective value for each maximum distance d in kilometers.

Maps

Figure 3: $d = 2,100$ km.

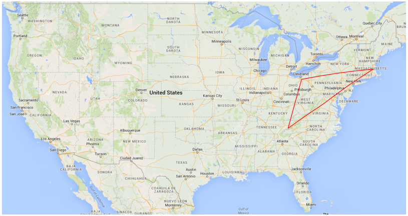

Figure 4: $d = 3,000$ km.

Figure 5: $d = 2,500$ km.

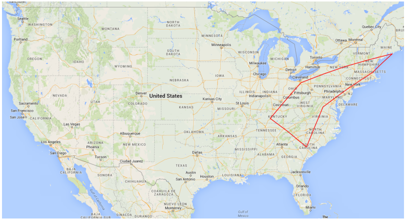

Figure 6: $d = 4,000$ km.

Maps

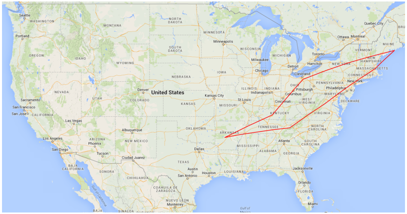

Figure 7: $d = 5,000$ km.

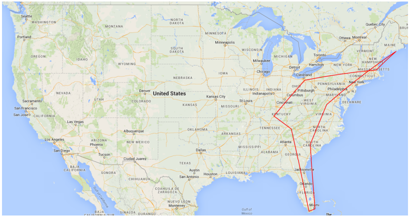

Figure 9: $d = 6,000$ km.

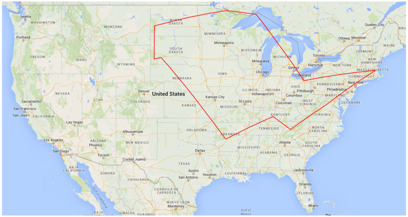

Figure 8: $d = 7,000$ km.

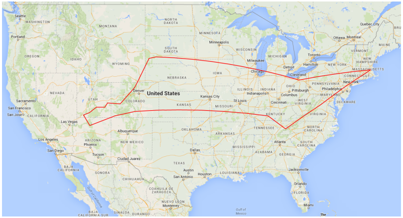

Figure 10: $d = 8,000$ km.

Maps

Figure 11: $d = 9,000$ km.

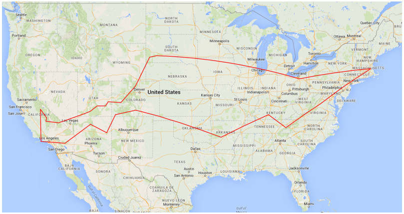

Figure 13: $d = 10,000$ km.

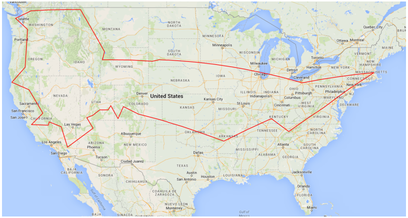

Figure 12: $d = 12,000$ km.

Figure 14: $d = 13,000$ km.

Maps

Figure 15: $d = 14,000$ km.

Figure 16: $d = 16,000$ km.

Figure 17: $d = 18,000$ km.

Maps

Figure 18: The optimal solution for the Traveling Salesman Problem that goes through all the National Parks in the United States, including Alaska, Hawaii, and other US territories.

Summary

- Variant of the TSP: Prize Collecting
- Setting: 59 US National Parks
- Utility, Maximal Distance
- Implemented in Julia/JuMP to obtain IP solutions
- Variational analysis

Further Considerations

Some easy choices:

- Change the set of locations (i.e. famous landmarks, different starting points)
- Change how utility is calculated
- Add a “budget" rather than maximal distance

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Some more interesting considerations:

- Desires
  - Visit as many states as possible
  - Specify certain “necessary" destinations
  - Group prize collecting problem
- Seasonality: Spring, Summer, Fall, Winter
  - Cheaper to visit parks in the north when it’s cold, etc...
- Time constraint to go from one park to the other: *time-consuming* to implement

References

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Volcano Café. Picture of Yellowstone. https://volcanocafe.files.wordpress.com/2013/10/yellowstone-fountain.jpg, 2013.

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