1. a.

$$\varepsilon_1 = k_x \sin\left(\frac{\theta}{2}\right)$$

$$\varepsilon_2 = k_y \sin\left(\frac{\theta}{2}\right)$$

$$\varepsilon_3 = k_z \sin\left(\frac{\theta}{2}\right)$$

$$\varepsilon_4 = \cos\left(\frac{\theta}{2}\right)$$

(1, 1) element $= k_x^2 v\theta + c\theta$

$1 - 2\left(k_y \sin\left(\frac{\theta}{2}\right)\right)^2 - 2\left(k_z \sin\left(\frac{\theta}{2}\right)\right)^2$

$= 1 - 2k_y^2 \cdot \frac{(1-\cos\theta)}{2} - 2k_z^2 \cdot \frac{(1-\cos\theta)}{2}$

$= 1 - k_y^2 \cdot (1-\cos\theta) - k_z^2 \cdot (1-\cos\theta)$

$= 1 - (k_y^2 + k_z^2)(1-\cos\theta)$

$\because k_x^2 + k_y^2 + k_z^2 = 1, \quad v\theta = 1 - c\theta$

$\therefore = 1 - (1 - k_x^2)(1-\cos\theta) = 1 - (1-k_x^2)v\theta$

$= k_x^2 v\theta + \cos\theta$

b. (2, 1) element $= k_x k_y v\theta + k_z s\theta$

$\cancel{2\varepsilon_1 \; k_x k_y (1-c\theta) + k_z s\theta}$

$2(\varepsilon_1\varepsilon_2 + \varepsilon_3\varepsilon_4) = 2\left(k_x \sin\left(\frac{\theta}{2}\right) \cdot k_y \sin\left(\frac{\theta}{2}\right) + k_z \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)$

$= 2\left(k_x k_y \frac{(1-\cos\theta)}{2} + k_z \cdot \frac{1}{2}\sin\theta\right)$

$= k_x k_y v\theta + k_z s\theta$

$\therefore$ they are equal

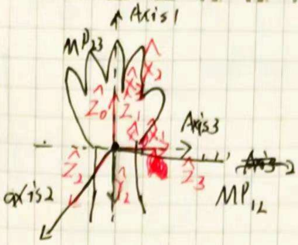
c. $c\theta = \frac{r_{11} + r_{22} + r_{33} - 1}{2}$ $\quad \theta = 180$

$\therefore -1 = k_x^2(1-c\theta) + c\theta + k_y^2(1-c\theta) + c\theta + k_z^2(1-c\theta) + c\theta$

$-1 = 2k_x^2 - 1 + 2k_y^2 - 1 + 2k_z^2 - 1$

$k_x^2 + k_y^2 + k_z^2 = 1$

2. a. Step 1 : draw axes



Step 2 : draw mp lites (insect in same part)

Step 3 : Attach frames {1} to {n-1}
(red lines)

Step 4 : Attach frame {0}

Step 5 : Attach frame {n}

Step 6 : a_0 to a_{n-1} : a_0 = distance from $\hat{Z}_0$ to $\hat{Z}_1$ about $\hat{X}_0 = 0$
a_1 = distance from $\hat{Z}_1$ to $\hat{Z}_2$ about $\hat{X}_1 = 0$
a_2 = distance from $\hat{Z}_2$ to $\hat{Z}_3$ about $\hat{X}_2 = 0$

Step 7 : α_0 to α_{i-1} : α_0 = angle between $\hat{Z}_0$ to $\hat{Z}_1$ about $\hat{X}_0 = 0$
α_1 = angle between $\hat{Z}_1$ to $\hat{Z}_2$ about $\hat{X}_1 = 90$
α_2 = angle between $\hat{Z}_2$ to $\hat{Z}_3$ about $\hat{X}_2 = 90$

Step 8 : d_i : $d_1 = 0$ $d_2 = 0$ $d_3 = 0$

Step 9 : θ_i : θ_1 = angle between $\hat{X}_0$ to $\hat{X}_1$ about $\hat{Z}_1$ = variable
$\theta_2 = \hat{X}_1 \rightarrow \hat{X}_2$ about $\hat{Z}_2$ = variable
$\theta_3 = \hat{X}_2 \rightarrow \hat{X}_3$ about $\hat{Z}_3$ = variable

Step 10 DH table :

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	0	0	θ_2
3	90	0	0	θ_3

$$\cancel{{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\cancel{{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\cancel{{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\cancel{{}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T}$$

$$\cancel{= \begin{bmatrix} c\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -s\theta_1 & 0 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & c\theta_1 & 0 \\ -s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot {}^2_3T}$$

→ next page

2 . a continue $\rightarrow$ =

$$\begin{bmatrix} c\theta_1 c\theta_2 c\theta_3 + s\theta_1 s\theta_3 & -c\theta_1 c\theta_2 s\theta_3 + s\theta_1 c\theta_3 & c\theta_1 s\theta_2 & 0 \\ s\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_3 & -s\theta_1 c\theta_2 s\theta_3 - c\theta_1 c\theta_3 & s\theta_1 s\theta_2 & 0 \\ s\theta_2 c\theta_3 & -s\theta_2 s\theta_3 & -c\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 . b I'm not sure about 2a so !

$${}^{A_0}_{123}R = {}^0_1R \cdot {}^1_2R \cdot {}^2_3R$$

$${}^0_1T = \left[\begin{array}{c|c} {}^0_1R & {}^{A}P_{B_{org}} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$\therefore\ {}^0_1R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_1 & -s\theta_1 \\ 0 & s\theta_1 & c\theta_1 \end{bmatrix} \qquad {}^1_2R = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 \\ 0 & 1 & 0 \\ -s\theta_2 & 0 & c\theta_2 \end{bmatrix}$$

$${}^2_3R = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3R = \left[\begin{array}{c|c|c} c\theta_2 \cdot c\theta_3 & c\theta_3 \cdot s\theta_2 \cdot s\theta_1 - s\theta_3 \cdot c\theta & c\theta_3 \cdot s\theta_2 \cdot c\theta_1 + s\theta_3 \cdot s\theta_2 \\ s\theta_3 \cdot c\theta_2 & s\theta_3 \cdot s\theta_2 \cdot s\theta_1 + c\theta_3 \cdot c\theta_1 & s\theta_3 \cdot s\theta_2 \cdot c\theta_1 - c\theta_3 \cdot s\theta_1 \\ -s\theta_2 & c\theta_2 \cdot c\theta_1 & c\theta_2 \cdot c\theta_1 \end{array}\right]$$

I don't think there is rotational singularity.

$$\begin{bmatrix} 0 & 0 & 0 & \theta_1 \\ 90 & 0 & 0 & \theta_2 \\ 90 & 0 & 0 & \theta_3 \end{bmatrix}$$

2 . a . continue :

$${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \longrightarrow$$ top page

4.

$$u(t) = \begin{cases} \cancel{\frac{u_f - u_0}{t_f} \cdot t + u_0} & \cancel{t < t_f} \\ \cancel{u_f} & \cancel{t \geq t_f} \end{cases}$$

$$u(t) = \begin{cases} a_0 + a_1 t + a_2 t^2 + a_3 t^3 & t < t_f \\ u_f & t \geq t_f \end{cases}$$

$$a_0 = u_0 \qquad a_1 = 0 \qquad a_2 = \frac{3}{t_f^2}(u_f - u_0)$$

$$a_3 = -\frac{3}{t_f^3}(u_f - u_0)$$

$$u(t) = \begin{cases} u_0 + \frac{3}{t_f^2}(u_f - u_0)t^2 - \frac{3}{t_f^3}(u_f - u_0) & t < t_f \\ u_f & t \geq t_f \end{cases}$$

~~velocity~~ $\dot{\theta} = 2a_2 t + 3a_3 t^2$

$$\ddot{\theta} = 2a_2 + 6a_3 t$$

$$\dot{\theta} = \frac{6}{t_f^2}(u_f - u_0) + \frac{9}{t_f^3}(u_f - u_0)$$

$\because \dot{\theta} \leq \dot{\theta}_{max}$ $\therefore$ $\dot{\theta}_{max} \geq \frac{6}{t_f^2}(u_f - u_0) - \frac{9}{t_f^3}(u_f - u_0)$

$\because \ddot{\theta} \leq \ddot{\theta}_{max}$ $\therefore$ $\ddot{\theta}_{max} \geq \frac{6}{t_f^2}(u_f - u_0) - \frac{18}{t_f^3}(u_f - u_0)$

6.

$${}^{A}_{B}R = R_z(\gamma) \cdot R_Y(\alpha) \cdot R_z(\beta)$$

$$= \begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix} \cdot \begin{bmatrix} c\beta & -s\beta & 0 \\ s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\gamma c\alpha & -s\gamma & c\gamma s\alpha \\ s\gamma c\alpha & c\gamma & s\gamma s\alpha \\ -s\alpha & 0 & c\alpha \end{bmatrix} \cdot \begin{bmatrix} c\beta & -s\beta & 0 \\ s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\gamma \cdot c\alpha \cdot c\beta - s\gamma \cdot s\beta & -c\gamma c\alpha s\beta - s\gamma c\beta & c\gamma s\alpha \\ s\gamma c\alpha c\beta + c\gamma s\beta & -s\gamma c\alpha s\beta + c\gamma c\beta & s\gamma s\alpha \\ -s\alpha c\beta & s\alpha s\beta & c\alpha \end{bmatrix}$$

$\because r_{13}^2 + r_{23}^2 = s^2\alpha$

$\therefore s\alpha = \sqrt{r_{13}^2 + r_{23}^2}$

$c\alpha = r_{33}$

$\therefore \alpha = \cancel{\arctan}\ \arctan 2\left(\sqrt{r_{13}^2 + r_{23}^2},\ r_{33}\right)$

$= \arctan 2\left(\sqrt{0.25^2 + 0.4330^2},\ 0.8660\right)$

$= 30^\circ$

$\beta = \arctan_2(s\beta,\ c\beta)$

$= \arctan_2\left(\frac{r_{32}}{s\alpha},\ \frac{r_{31}}{-s\alpha}\right)$

$= \arctan_2\left(\frac{0.3586}{s\alpha},\ \frac{0.3586}{s\alpha}\right) = 45^\circ$

$\gamma = \arctan 2(s\gamma,\ c\gamma)$

$= \arctan 2\left(\frac{r_{23}}{s\alpha},\ \frac{r_{13}}{s\alpha}\right)$

$= \arctan 2\left(0.\overset{4330}{\cancel{25}},\ 0.\underset{25}{\cancel{4330}}\right) = 60^\circ$