MATH 131: Numerical Methods for scientists and engineers - Assignment 1

Homework Assignment 1 due by 11:45 PM, September 22, 2019. Please complete this assignment on MATLAB Grader.

1. Write a `MATLAB` function, called `bisection_method` that inputs a function f, two numbers a, b, an error tolerance, `tol`, and a maximum number of iterations, N, and finds a root c of f in the interval $[a, b]$ using the bisection method. Your function should compute a bound on the error, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first.

   (a) Hint #1: The function should start with the line

   ```
   function [c,n,err] = bisection_method(f,a,b,tol,N)
   ```

   (b) Hint #2: The function should contain a 'while' loop, looking something like:

   ```
   while err > tol && n < N
   ```

   (c) Use the code that you just developed to solve $f(x) = (2x^3 + 3x - 1)\cos(x) - x = 0$ in $[-1, 1]$, for an accuracy of 10^{-5}.

   i. What is the number, n of iterations used?

   ii. What is the error?

   iii. Plot on the same graph the function f and the axis $y = 0$.

2. Write a `MATLAB` function, called `fixed_point_iteration` that inputs a function, g, an initial guess x_0, an error tolerance, `tol`, and a maximum number of iterations, N, and outputs the fixed point of g, obtained using the fixed point iteration, starting with x_0. Your function should have an error defined by $E = |x_n - x_{n-1}|$, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first. Your function header should look something like:

   ```
   function [c,n,err] = fixed_point_iteration(g,x0,tol,N)
   ```

   Use the code you just developed to find the solution to the equations $x = e^{-x}$ and $x = \frac{1}{1+x}$, with an accuracy of 10^{-10} for $x \in [-1, 1]$. State your initial guess, and how many iterations it took. Plot on the same graph, $y = g(x)$ and $y = x$. Here c is the fixed-point solution and n the number of iterations the method takes. See MATLAB Grader Assignment 1 Fixed Point Problem for details on variable naming on the two functions.

3. Write a `MATLAB` function, called `newtons_method` that inputs a function, f, its derivative f', an initial guess x_0, an error tolerance, `tol`, and a maximum number of iterations, N, and outputs the root of f obtained using Newton's method (denoted by c), starting with x_0. Your function should have an error defined by `err`$= |x_n - x_{n-1}|$, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first. Your function header should look something like:

   ```
   function [c,n,err] = Newtons_method(f,fp,x0,tol,N)
   ```

   where `n` is the last iteration when you stop.

   Use the function you created to find the root of the equation $\arctan(x) = 1$ with initial guess $x_0 = 2$, to an accuracy of less than $\epsilon = 10^{-8}$. Did your method converge, and if so, how many iterations did it take? If not, why didn't it converge, and what happened–did it diverge, or end up in an infinite loop? Plot on the same graph the function and the axis $y = 0$. Test with $x_0 = -2$. What is happening ?