Homework Assignment 4 due by Thursday 11:45 PM, November 7, 2019.
Test your answers on MATLAB Grader or Live Editor on MATLAB

1. Write a Matlab function called `euler_method` that solve the IVP
$$\frac{dy}{dt} = f(t, y), \quad a \le t \le b, \quad y(0) = \alpha \tag{1}$$
using Euler's method. The header should look like
```
function [y,t] = euler_method(f,a,b,alpha,N)
```
where N is the number of intervals used, so that $\Delta t = \frac{b-a}{N}$. Note that the output should be an array that contains the evaluation of the solution at all time steps. Use this method to solve the IVP
$$\frac{dy}{dt} = \frac{\sin(2t) - 2ty}{t^2}, \quad y(1) = 2, \quad t \in [1, 5]$$
with $N = 10, 10^2, 10^3$. Call `y0, y1, y2` the three results. Plot the three solutions with respect to t. On another figure, make a `loglog` plot of absolute error at $t = 5$ versus the number of intervals for all three methods on the same plot. To compute the exact solution use the matlab function `dsolve`. *MATLAB Grader might not recognize the function dsolve, compute it on MATLAB and copy the expression on MATLAB Grader.* Comment on your results (using `%`)

Now consider the IVP
$$\frac{dy}{dt} = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha. \tag{2}$$
$$\frac{dy}{dt} = -12y, \quad 0 \le t \le 1, \quad y(0) = 1. \tag{3}$$

2. Write a Matlab function called `RK2` that solve (2) using Order 2 Runge-Kutta method. Your function header should look like `[y,t] = RK2(f,a,b,alpha,N)`. Use this method to solve the IVP (3) with $N = 20, 50, 100$. Call `y0,y1,y2`, the three results and plot the three solutions on the same graph. Make a `loglog` plot of absolute error at $t = 1$ versus the number of intervals for all three methods on the same plot. Compare the solution at $t = 1$ with the exact solution at $t = 1$. Create `Err` a vector that stores the absolute error of the three solutions at $t = 1$. Comment on the result (is the method converging or not, etc.) using `%`.

3. Write a Matlab function called `RK4` that solve (2) using Order 4 Runge-Kutta method. Your function header should look like `[y,t] = RK4(f,a,b,alpha,N)`. Use this method to solve the IVP (3) with $N = 20, 50, 100$. Call `y3,y4,y5`, the three results and plot the three solutions on the same graph. Make a `loglog` plot of absolute error at $t = 1$ versus the number of intervals for all three methods on the same plot. Compare the solution at $t = 1$ with the exact solution at $t = 1$. Create `Err1` a vector that stores the absolute error of the three solutions at $t = 1$. Comment on the result (is the method converging or not, etc.) using `%`.

4. Write a Matlab function called `AB4` that solve (2) using four-step Adams-Bashforth method. You may use your textbook to get the coefficients. Your function header should look like `[y,t] = AB4(f,a,b,alpha,N)`. Use this method to solve the IVP (3) with $N = 20, 50, 100$. Call `y6,y7,y8`, the three results and plot the three solutions on the same graph. *Notice: be careful on how to evaluate the first steps !* Make a `loglog` plot of absolute error at $t = 1$ versus the number of intervals for all three methods on the same plot. Compare the solution at $t = 1$ with the exact solution at $t = 1$. Create `Err2` a vector that stores the absolute error of the three solutions at $t = 1$. Comment on the result (is the method converging or not, etc.) using `%`.