Predator-Prey Model Report

Michael Whelan

May 7, 2023

Contents

1 Introduction to the Predator-Prey Model

When studying nature you will find it is always changing in many different ways. One way it changes is when animals eat other animals in order to survive. The animals that eat the are called the 'Predator' and the animals that are eaten are called the 'Prey'. It is a very interesting part of nature because it can have very significant consequences to the population of both groups. The predator consumes the prey decreasing their population and the prey can increase the predator's population as they survive longer and are able to reproduce. This can lead to cyclical patterns of predator and prey abundance, where prey increase in number and then, with abundant food, predator number increases until the predators begin to suppress prey numbers and then decrease as well. As long as predator and prey numbers don't drop to zero, this cycle can repeat indefinitely (MacPhee 2020). There are many examples of these relationships in nature such as

- Lions and Gazelles
- Birds and Insects
- Pandas and Eucalyptus trees
- Venus fly traps and Flies

(Kapolka 2001). This can be modelled mathematically and visualised this is known as a 'Predator-Prey Model'. What is very interesting about this model is it started off used for animals eating other animals but can be adapted and tweaked to other aspects of nature for example

- Epidemics (COVID-19, Influenza etc)
- Zombie Apocalypse
- Economics
- Human Relationships

In this report we will explain where it came from and will look at a specific model in detail.

2 Lotka-Volterra Differential Equations

The Lotka-Volterra Differential Equations also known as the 'predator-prey equations' are a system of first-order nonlinear differential equations, used to describe the interaction of two species one being a predator and the other being prey. The populations change with respect to time.

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{1}$$

$$\frac{dy}{dt} = \delta xy - \gamma y \tag{2}$$

where

- x is the number of prey
- y is the number of a predator
- $\frac{dx}{dt}$ and $\frac{dy}{dt}$ is the growth rates of the two populations
- t represents time
- α is the growth rate of prey,
- β is the rate at which predators eliminate prey
- γ is the death rate of predators
- δ is the rate at which predators increase by consuming prey

The Lotka–Volterra predator–prey model was initially proposed by Alfred J. Lotka he looked at plant species and a herbivorous animal species as an example. The same solutions were then published in 1926 by Vito Volterra studied the fish catches in the Adriatic Sea and had noticed that the percentage of predatory fish caught had increased during the years of World War I (Kingsland 1988).

A prey population x increases at a rate $dx = \alpha x dt$ (proportional to the number of prey) but is simultaneously destroyed by predators at a rate $dx = -\beta xy dt$ (proportional to the product of the numbers of prey and predators).

The equations have periodic solutions taht look like the following figure.

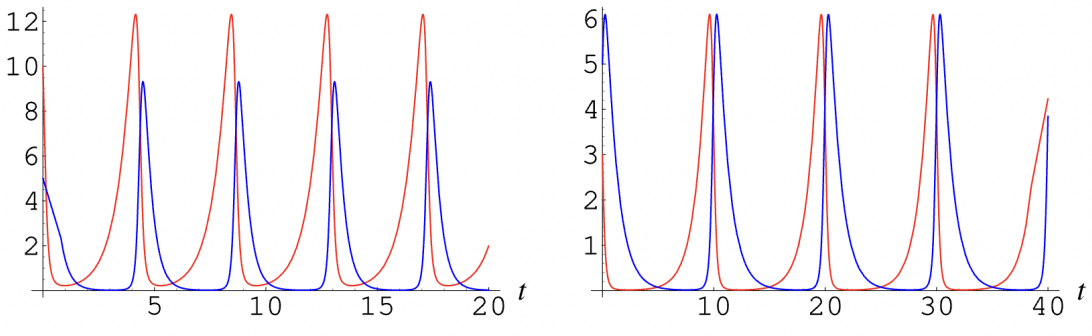

Figure 2.1:

The solutions of which are plotted above, where prey are shown in red, and predators in blue. In this sort of model, the prey curve always lead the predator curve Weisstein (2021).

3 Dublin's Success from 2015-2022

The predator-prey model we will look at in this report is the rise and fall of Dublin's success at winning the all Ireland Football Championship. Dublin dominated the All Ireland championship from 2015 up until 2020 winning 6 All Ireland titles in a row. They began their success in 2014 winning their first all Ireland since 2011 and before that since 1995. We will look at 2015 to 2022 for this model. We will begin by showing Dublin aganst the Rest of Ireland however unlike the Lotka-Volterra examples we had above we will have a third factor that comes in to change the relationships.

$$\frac{dR}{dt} = R - 0.1FR - 0.1R(M-1) \tag{3}$$

$$\frac{dF}{dt} = -1.5F + 0.18FR \tag{4}$$

$$\frac{dM}{dt} = -M + 0.1MR \tag{5}$$

Where

- R is the Rest of teams in Ireland
- B is Dublin
- F is Dessie Farrell

With our initial conditions as

$$R(2015) = 25 \tag{6}$$

$$F(1950) = 4 \tag{7}$$

$$M(1950) = 0 \tag{8}$$

We will start the model by just looking at Dublin's success against the Rest of Ireland's success and then we will introduce Dessie Farrell.

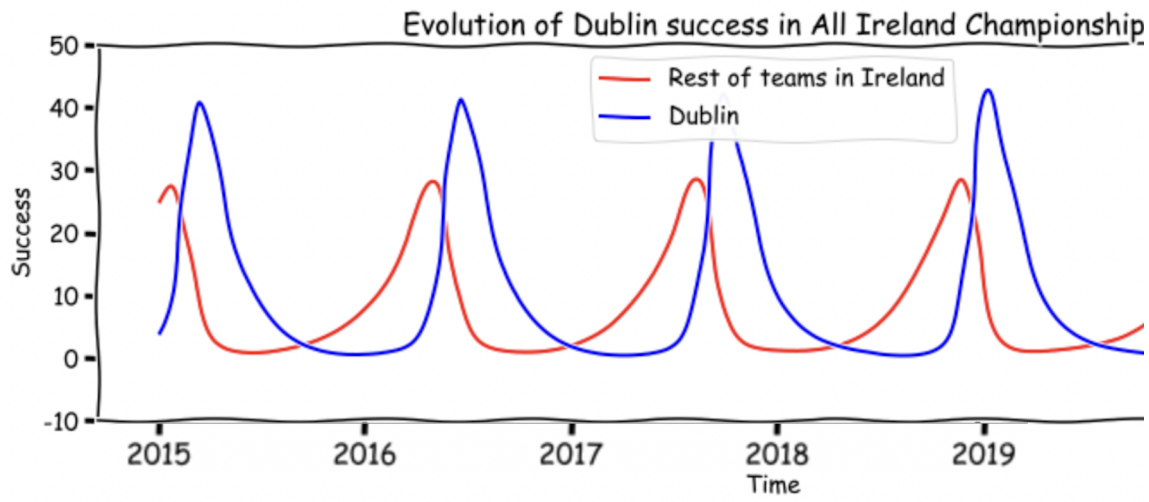

Figure 3.1:

The model is not really that interesting we can see Dublin dominating the whole championship where it looks like it will never end. The people from Dublin never thought their reign would end either. The success goes up and down as everyone starts at the same level at the beginning of the championship. The other teams have success playing amongst each other but Dublin overtake on the success combined by beating teams by a lot. The manager of Dublin at the time was Jim Gavin and he managed the team in a very strict and structured manor. The team's defense were nearly impossible to get by and the defenders would attack along with the midfield and forwards. In the model we can see the relationship that looks like a predator prey model. The main difference is in our examples above the predator population doesn't really exceed the population of the prey. This is because we are measuring against success and the more matches Dublin win and also by how much they win, the more successful they become. We can see the cyclic relationship they have year in and year out. When the season begins Dublin's success shoots up way over the Rest of Ireland in this case the prey.

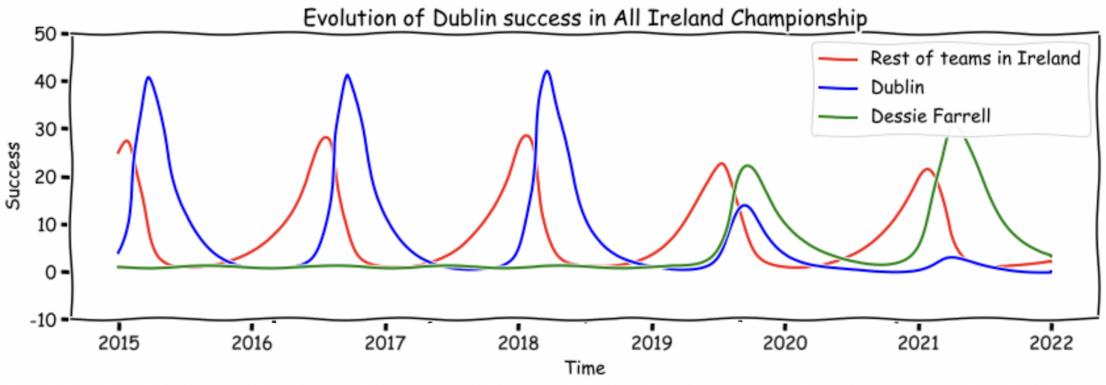

Figure 3.2:

Dublin's major change in the last few years was their manager. Jim Gavin stepped down and Dessie Farrell took over. Since Dessie Farrell took over Dublin's success has declined big time. From six championships in a row to not even making the final by 2021. Farrell's managing structure is not as strict as Gavin's was. Dublin play out of position and their blanket defence is no where to be seen.

So how does this affect the model and the inner relationships? Farrell takes control in 2019. This was a turning point according to the figure 3.2 because Dublin's success begins to decline. The Rest of Ireland becomes more successful because once someone bet Dublin it would mean it isn't impossible and taht someone else could win. In fact Tyrone won it in 2021. We can see the relationship of Farrell coming in. As Farrell comes in the success of Dublin lowers and the Rest of Ireland's success is a lot bigger. The success of Dessie Farrell is high which is something to note. We can argue this is high because he is taking on a successful team at the beginning and he has an All Ireland title to his name in 2020. Even though Dublin have that title too their success still declines because they start to lose games dropping them lower to eventually not even reaching the final in 2021.

4 Conclusion

We can say fairly the Predator-Prey model can be applied to anything with something dominant taking over something else. Whether it is animals in nature, bigger business taking over smaller business or sports teams winning leagues. In our particular model it worked although it had some flaws. Farrell may not have been the only reason Dublin declined. Perhaps they lost key players throughout the years or another reason that could be is politics within the team. It is seen in many sports where certain individuals in the dressing room have a say in tactics and other aspects. Some individuals even disrespect managers and other staff in doing so which can have an affect on success. We are now in 2022 and although it is not over yet Dublin's success is even lower again a long with Farrell's which would be interesting to model in the future.

References

Kapolka, D. (2001), 'Predator-Prey Models '.

URL: *https://services.math.duke.edu/education/webfeats/Word2HTML/Predator.html*

Kingsland, S. E. (1988), 'Modeling Nature: Episodes in the History of Population Ecology. '.

URL: *https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equation#cite_note-12*

MacPhee, L. (2020), 'Predator-Prey Interaction '.

URL: *https://www2.nau.edu/lrm22/lessons/predator_prey/predator_prey.html*

Weisstein, E. W. (2021), 'Lotka-Volterra Equations '.

URL: *https://mathworld.wolfram.com/Lotka-VolterraEquations.html*