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Problem: 1:

The given matrix $M = \begin{bmatrix} -1 & 2 & 0 \\ a-b & 3 & 0 \\ a & -b & 1 \end{bmatrix}$

My roll number last two digit are 55.

So, $a = 5$, $b = 5$

$$\therefore M = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 0 \\ 5 & -5 & 1 \end{bmatrix}$$

Let, λ be an eigenvalue of M.

Here,

$$M - \lambda I = \begin{bmatrix} -1-\lambda & 2 & 0 \\ 0 & 3-\lambda & 0 \\ 5 & -5 & 1-\lambda \end{bmatrix}$$

$$|M - \lambda I| = \begin{vmatrix} -1-\lambda & 2 & 0 \\ 0 & 3-\lambda & 0 \\ 5 & -5 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - \begin{pmatrix}\text{Sum of diagonal} \\ \text{of } M\end{pmatrix} \lambda^2 + \begin{pmatrix}\text{Sum of Anti-diagonal} \\ \text{of } M\end{pmatrix} \lambda - |M| = 0$$

$$\Rightarrow \lambda^3 - (3)\lambda^2 + (-1)\lambda + 3 = 0$$

$$\Rightarrow \lambda^3 - 3\lambda^2 - \lambda + 3 = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 - 2\lambda^2 + 2\lambda - 3\lambda + 3 = 0$$

$$\Rightarrow \lambda^2(\lambda - 1) - 2\lambda(\lambda - 1) - 3(\lambda - 1) = 0$$

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$\Rightarrow (\lambda-1)(\lambda^2-2\lambda-3)=0 \Rightarrow (\lambda-1)\{(\lambda^2-3\lambda+\lambda-3)\}=0$

$\Rightarrow (\lambda-1)(\lambda+1)(\lambda-3)=0$

$\therefore \lambda = 1, -1, 3$

Therefore, spectrum of M is $\{1, -1, 3\}$

Let,
$V_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigenvector associated to the eigenvalue.

$\lambda_1 = 1$,

Then, $(M-\lambda_1 I)\vec{V_1} = \vec{0}$

$$\Rightarrow \begin{bmatrix} -1+1 & 2 & 0 \\ 0 & 3+1 & 0 \\ 5 & -5 & 1+1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 4 & 0 \\ 5 & -5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2y = 0$, $\quad 5x - 5y + 2z = 0$
$4y = 0$,

$$\left\{ \begin{array}{l} 5x - 5y + 2z = 0 \\ \Rightarrow y = 0 \end{array} \right.$$

Let, z is a free variable and assign $z = a$,

then, $y = 0$ and $x = -\frac{2a}{5}$

Therefore, $V_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{2a}{5} \\ 0 \\ a \end{bmatrix}$. where a is a non-zero real number.

Let, $V_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigenvector associated to the eigenvalue.

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$\lambda_2 = -1$

Then,

$(M - \lambda_2 I)V_2 = 0$

$$\Rightarrow \begin{bmatrix} -1-1 & 2 & 0 \\ 0 & 3-1 & 0 \\ 5 & -5 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 2 & 0 \\ 0 & 2 & 0 \\ 5 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow -2x + 2y = 0, \quad 2y = 0, \quad 5x - 5y = 0$

let, z in the free variable,

then, $z = b, \; y = 0, \; x = 0$

Then, $V_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$

let, $V_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigenvector associated to the eigenvalue,

$\lambda_3 = 3$

Then, $(M - \lambda_3 I)V_3 = 0$

$$\Rightarrow \begin{bmatrix} -1-3 & 2 & 0 \\ 0 & 3-3 & 0 \\ 5 & -5 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 2 & 0 \\ 0 & 0 & 0 \\ 5 & -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow 4x + 2y = 0, \quad 5x - 5y - 2z = 0$

Let y is the free variable and assign, $y = c$

Then $x = \frac{2c}{4} = \frac{c}{2}, \quad z = -\frac{5c}{4}$

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Therefore. $V_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a/2 \\ c \\ -\frac{5c}{4} \end{bmatrix}$

for, $a = b = c = 1$, we have,

$$V_1 = \begin{bmatrix} -5/2 \\ 0 \\ 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1/2 \\ 1 \\ -5/4 \end{bmatrix}$$

Observation,

$$P = \begin{bmatrix} -5/2 & 0 & 1/2 \\ 0 & 0 & 1 \\ 1 & 1 & -5/4 \end{bmatrix}$$ (modal matrix) (V_1, V_2, V_3)

$$P^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 0 \\ 5 & -5 & 1 \end{bmatrix}$$

$$\therefore P^{-1}MP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$(P^{-1}MP)^{199} = P^{-1}M^{199}P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{199} = \begin{bmatrix} -1^{199} & 0 & 0 \\ 0 & 1^{199} & 0 \\ 0 & 0 & 3^{199} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3^{199} \end{bmatrix}$$

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$$\Rightarrow M^{199} = PM^{199}P^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 0 \\ 5 & -5 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3^{199} \end{bmatrix} \begin{bmatrix} -5/2 & 0 & 1/2 \\ 0 & 0 & 1 \\ 1 & 1 & -5/4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ -5 & -5 & 3^{199} \end{bmatrix} \begin{bmatrix} -5/2 & 0 & 1/2 \\ 0 & 0 & 1 \\ 1 & 1 & -5/2 \end{bmatrix}$$

$$= \begin{bmatrix} -5/2 & 0 & 5/2 \\ 0 & 0 & 3 \\ 25/2 + 3^{199} & 3^{199} & -5/2 - 5 - 5/2 \times 3^{199} \end{bmatrix}$$

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Problem 2:

Let $V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $V_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

Let, $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$

$$AV_1 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \\ -1 \\ 0 \end{bmatrix}$$

So, $2a - 2b + c = -5$ ..... (i)

$$AV_2 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -1 \\ -1 \end{bmatrix}$$

So, $b - c = 5$ .... (ii)

$$AV_3 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \\ -3 \end{bmatrix}$$

So, $a - b - c = 2$ ... (iii)

forming matrix from (i), (ii), (iii)

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$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 1 & -5 \\ 0 & 1 & -1 & 5 \\ 1 & -1 & -1 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 1 & -5 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & -3 & 9 \end{array}\right] R_3 \leftarrow 2R_3 - R$$

from row-echelon form:

$-3c = 9$, $\Rightarrow c = -3$

$b + 3 = 5 \Rightarrow b = 2$

$2a - 4 - 3 = -5 \Rightarrow a = 1$

from AV_1, AV_2, AV_3,

$2d - 2e + f = 8$ .... (i)

$e - f = -2$ .... (ii)

$d - e - f = 4$ .... (iii)

forming matrix from (i), (ii), (iii).

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 1 & 8 \\ 0 & 1 & -1 & -2 \\ 1 & -1 & -1 & 4 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 1 & 8 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -3 & 0 \end{array}\right] R_3 \leftarrow 2R_3 - R_1$$

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So,

$f = 0, \quad e = -2, \quad d = \frac{-4+8}{2} = -2$

Again from, Av_1, Av_2, Av_3,

$2g - 2h + i = -1 \quad \cdots \text{(i)}$

$h - i = -1 \quad \cdots \text{(ii)}$

$g - h - i = -2 \quad \cdots \text{(iii)}$

forming matrix from (i), (ii) and (iii)

$$\left[\begin{array}{ccc|c} 2 & -2 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

$i = \frac{-3}{-3} = 1, \quad h = 0, \quad g = -1$

again from $Av_1, Av_2 \; Av_3$.

$2j - 2k + l = 0$

$k - l = -1$

$j - k - l = -3$

$$\left[\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$l = -\frac{6}{-3} = 2, \quad k = 1, \quad j = 0$

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So, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

for any vector $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in R^3$

$$Av = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} x+2y-3z \\ 2x-2y \\ -x+z \\ y+2z \end{bmatrix}$$