Notes of Promi & Notes of Shanto

"Notes of Shanto" & "Notes of Promi" Presents

(MAT-105W)

ALL QUESTIONS

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Limit and Continuity

(1) If $f(x) = \begin{cases} x & ; \ 0 \le x \le \frac{1}{2} \\ 1 - x & ; \ \frac{1}{2} \le x \le 1 \end{cases}$

Show that $f(x)$ is continuous at $x = \frac{1}{2}$ but $f(x)$ is not differentiable at that point.

(2) Discuss the continuity and differentiability at $x = 0$ and $x = \frac{\pi}{2}$ of the function

$$f(x) = \begin{cases} 1 & ; \ x < 0 \\ 1 + \sin(x) & ; \ 0 \le x \le \dfrac{\pi}{2} \\ 2 + \left(x - \dfrac{\pi}{2}\right)^2 & ; \ x > \dfrac{\pi}{2} \end{cases}$$

(3) Discuss the continuity and differentiability at $x = 0$ and $x = 1$ of the function

$$f(x) = \begin{cases} x^2 + 1 & ; \ x \le 0 \\ x & ; \ 0 < x < 1 \\ \dfrac{1}{x} & ; \ x \ge 1 \end{cases}$$

Graph of a function

(1) Sketch the graphs of the following functions:

(i) $$f(x) = \begin{cases} x & ; \ x < 1 \\ e^x & ; \ 1 \le x \le 2 \\ 2 & ; \ x > 2 \end{cases}$$

(ii) $$f(x) = \begin{cases} \sin(x) & ; \ x < \frac{\pi}{2} \\ 0 & ; \ x = \frac{\pi}{2} \\ x & ; \ x > \frac{\pi}{2} \end{cases}$$

Derivative

(1) $\sin y = x \cdot \sin(a+y)$; $\frac{dy}{dx} = ?$

Formula
$\frac{d}{dx}(u^v) = u^v \cdot \frac{d}{dx}\{v \cdot \ln(u)\}$

(2) $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$; $\frac{dy}{dx} = ?$

(3) $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$; $\frac{dy}{dx} = ?$

(4) $e^x + e^y = xy$; $\frac{dy}{dx} = ?$

(5) If $y = e^{-x} \cdot \sin x$, show that $y_4 + 4y = 0$.

(6) If $y = a^{bx+c}$, show that

$$y_n = a^{bx+c} \cdot \{\ln(a)\}^n \cdot b^n$$

(7) If $y = \ln(ax+b)$, show that

$$y_n = (-1)^{n-1} \cdot (n-1)! \cdot (ax+b)^{-n} \cdot a^n$$

Leibnitz's Theorem
$(uv)_n = u_n v + n_{c_1} \cdot u_{n-1} v_1 + \cdots + n_{c_r} \cdot u_{n-r} v_r + \cdots + u v_n$

(8) If $y = \tan^{-1}(x)$, show that

$$(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$$

(9) If $\ln y = \tan^{-1} x$, show that

$$(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$$

(10) If $y = \sin(m \sin^{-1} x)$, prove that

$$(1-x^2)y_{n+2} - (2n+1)x \cdot y_{n+1} + (m^2 - n^2)y_n = 0$$

Maxima – Minima

(1) Investigate for what values of x,

$$f(x) = 5x^6 - 18x^5 + 15x^4 - 10$$

is minimum or maximum.

(2) Find for what values of x, the following expression is maximum or minimum respectively

$$2x^3 - 21x^2 + 36x - 20$$

Find out the maximum and minimum values of the expression.

Rolle's Theorem

(1) Verify Rolle's theorem for $f(x) = x^3 - 12x$ in the interval $0 \leq x \leq 2\sqrt{3}$.

(2) Does Rolle's theorem apply to the function

$$f(x) = 1 - (x-3)^{\frac{2}{3}}$$

Mean Value Theorem

(No Math Found)

Euler's Theorem

$$x \cdot \frac{\delta F}{\delta x} + y \cdot \frac{\delta F}{\delta y} + \cdots = nF$$

(1) If $u = \tan^{-1}\frac{x^3+y^3}{x+y}$, show that

$$x \cdot \frac{\delta u}{\delta x} + y \cdot \frac{\delta u}{\delta y} = \sin(2u)$$

(2) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x \cdot \frac{\delta u}{\delta x} + y \cdot \frac{\delta u}{\delta y} + \frac{1}{2}(\cot u) = 0$$

(3) If $u = \tan^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x \cdot u_x + y \cdot u_y = \frac{1}{4}\sin 2u$$

Taylor & Maclaurin's Polynomial

(1) Find Maclaurin's series for e^x.

(2) Find Maclaurin's series for

(i) e^{mx}

(ii) $\sin 2x$

(3) Find Taylor series for $f(x) = \ln x$ at $x = 2$.

Integration : As An Inverse Process Of Differentiation

- Find the anti-derivative of –

(1) $\sin 2x + e^{5x}$

(2) $\frac{8^{1+x} + 4^{1-x}}{2^x}$

- Find the Indefinite Integrals –

(1) $\int \frac{sin^8 x - cos^8 x}{1 - 2sin^2 x cos^2 x}\ dx$

(2) $\int \frac{dx}{x\{10 + 7\ln(x) + (lnx)^2\}}$

(3) $\int \frac{e^x\ dx}{e^{2x}+2e^x+5}$

(4) $\int \frac{dx}{(x^2-16)\sqrt{x+1}}$

(5) $\int \frac{dx}{3+2cosx}$

- Fundamental theorem of integral calculus and its application to definite integrals –

(1) $\int \frac{dx}{(a^2+x^2)^{\frac{3}{2}}}$

(2) $\int \frac{x\sqrt{2-x^2}}{\sqrt{1+x^2}}\ dx$

Definite Integral As The Limit Of A Sum

(1) $\lim_{n\to\infty}[\frac{n}{n^2} + \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+(n-1)^2}]$

(2) $\lim_{n\to\infty}[\frac{1}{n} + \frac{\sqrt{n^2-1^2}}{n^2} + \cdots \frac{\sqrt{n^2-(n-1)^2}}{n^2}]$

(3) $\lim_{n\to\infty}[\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \cdots \frac{1}{n}]$

Reduction Formula

(1) Apply reduction formula for $\int cos^n x\, dx$ and then evaluate $\int cos^8 x\, dx$.

Improper Integral

(1) Is the area under the curve $y = \frac{1}{\sqrt{x}}$ from $x = 0$ to $x = 1$ finite? If so, what is it?

(2) Evaluate $\int_2^{\alpha} \frac{x+3}{(x-1)(x^2+1)} dx$

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