Motion

Motion is the action of changing location or positive over time. Motion is mathematically described in terms of displacement, distance, velocity, acceleration, speed, and time.

A sprinter running on a straight race course……………………………………….……………1D

Dropping a ball from a building…………………………………………………………1D

Ant moving on a table……………………………………………………………………2D

Kicking a soccer ball at an angle from horizontal……………………………………….2D

A car moving in a circular path …………….………………………………………………2D

Swinging pendulum……………..……………………………………………………..2D

An outswing delivery in cricket…………………….…………………………………………3D

Motion in a Plane

Position vector & Displacement

In fig:a A particle moves along a curved path in xy plane. P and Q represent the positions of the particle at two different times. The position of the particle at P can be described by a displacement vector $\vec{r_1}$ from the origin O to P; $\vec{r_1}$ is called the position vector of the particle at this point. The components of $\vec{r_1}$ are the coordinates x and y and may write- $\vec{r_1} = x_1\hat{i} + y_1\hat{j}$

Similarly the position vector at point Q will be $\vec{r_2}$.

$\vec{r_2} = x_2\hat{i} + y_2\hat{j}$

The particle's displacement as it moves from P to Q is the change $\triangle\vec{r}$ in the position vector $\vec{r_1}$. So the displacement can be expressed as-

$\triangle\vec{r} = \vec{r_2} - \vec{r_1}$

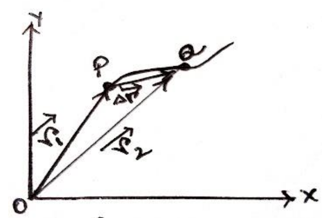


fig: a

Average velocity and Instantaneous Velocity

Let ∆t be the time interval during which the particle moves from P to Q (fig:a). The average velocity during this interval is defined to be a vector quantity equal to the displacement divided by the time interval.

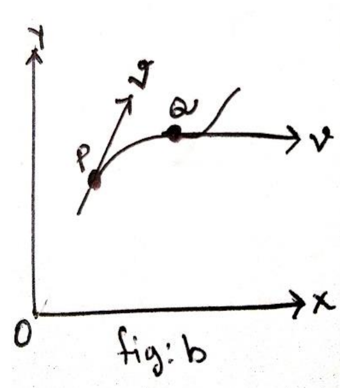
Average velocity, $\bar{v} = \frac{\Delta r}{\Delta t}$

The average velocity is a vector quantity having the same direction as $\triangle\vec{r}$ and a magnitude of $\triangle\vec{r}$ divided by ∆t.

The instantaneous velocity $\boldsymbol{v}$ at point P is defined in magnitude and direction as the limit approached by the average velocity when point Q is taken closer and closer to point P.

Instantaneous velocity $\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d\mathbf{r}}{dt}$

As point Q approaches point P, the direction of the vector $\triangle \vec{r}$ approaches that of the tangent to the path at P, so that the instantaneous velocity at any point is tangent to the path at that point. The instantaneous velocities at point P and Q are shown in fig:b.



The component $\bar{v}_x$ and $\bar{v}_y$ of average velocity are the corresponding components of displacement, divided by Δt

$\bar{v}_x = \frac{\Delta x}{\Delta t}$ and $\bar{v}_y = \frac{\Delta y}{\Delta t}$

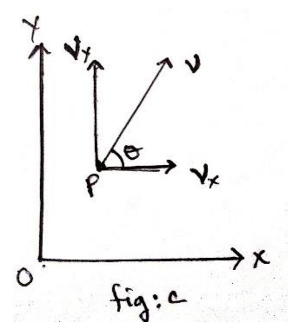
Similarly, the instantaneous velocity can be written as

$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ and $v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt}$

The magnitude of the instantaneous velocity can be written as

$|\, \boldsymbol{v} \,| = \sqrt{v_x^2 + v_y^2}$

And the angle Θ in fig:c by $tan\theta = \frac{v_y}{v_x}$



In terms of unit vectors,

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(\mathrm{x}\hat{\imath} + \mathrm{y}\hat{\jmath}) = \frac{\mathrm{dx}}{\mathrm{dt}}\hat{\imath} + \frac{\mathrm{dy}}{\mathrm{dt}}\hat{\jmath}$$

Average and Instantaneous acceleration

In fig:2a the vectors $\vec{v}_1$and $\vec{v}_2$ represent the instantaneous velocities at point P and Q of a particle moving in a curved path. The velocity $\vec{v}_2$ differs in direction from the velocity $\vec{v}_1$. The diagram has been constructed for a case in which it also differs in magnitude, although in some special cases the magnitude of the velocity may remain constant.

The average acceleration $\overline{\boldsymbol{a}}$ of the particle as it moves from P to Q is defined as the vector change in velocityΔv divided by Δt.

Average velocity, $\overline{\boldsymbol{a}} = \frac{\Delta v}{\Delta t}$

Average acceleration is a vector quantity, in the same direction as the Δv.

The vector change in velocity Δv, means the vector difference $\boldsymbol{v}_2 - \boldsymbol{v}_1$:

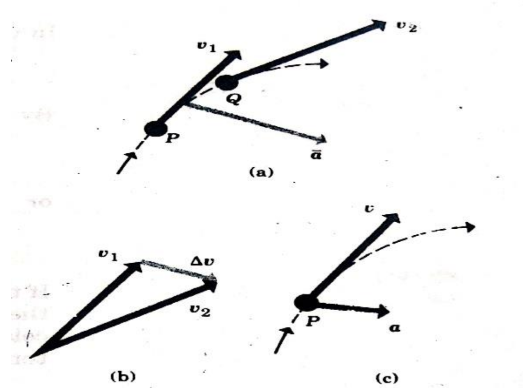


fig:2 (a) The vector $\bar{a} = \Delta v/\Delta t$ represents the average acceleration between P and Q. (b) Construction for obtaining $\Delta v = v_2 - v_1$. (c) Instantaneous acceleration a at point P. Vector v is tangent to the path; vector a points toward the concave side of the path.

$\Delta v = v_2 - v_1$

This relationship is shown in fig:2b.

The instantaneous acceleration $\boldsymbol{a}$ at point P is defined in magnitude and direction as the limit approached by the average acceleration when point Q approaches point P and Δv and Δt both approach zero.

Instantaneous acceleration, $\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

The instantaneous acceleration vector at point P is shown in fig:2c. It does not have the same direction as the velocity vector.

Mathematical Problems: (Solve the following problems)

1. The motion of a particle is described by the equation x= 20 cm+4(cm.s^{-2}) t^2

Find- a) the displacement of the particle in the time interval between t_1=2s and t_2=5s

b) the average velocity in this time interval

c) the instantaneous velocity at time 2s.

2. Suppose the velocity of a particle is given by the equation v=10 cm.s^{-1}+ 2(cm.s^{-3}) t^2

Find- a) the change in velocity of the particle in the time interval between t_1= 2s and t_2= 5s.

b) the average acceleration in this interval

c) the instantaneous acceleration at time 2s.

3. A body moves along the x-axis with constant acceleration 4m/s^2. At time t=0 it is at x=5m and has velocity v=3m/s

a) Find the position and velocity at time t=2s

b) Where is the body when its velocity is 5 m/s?

4. The motion of a particle along a straight line is described by the function x= 6 + 4t^2-t^4

a) Find the position, velocity and acceleration at time t=2s

b) During what time interval is the velocity positive?