CAPACITORS AND INDUCTORS

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Outline

- *Introduction to Storage Elements*
- *Capacitors and it's Working Principle*
- *Characteristics of Capacitors*
- *Inductors and it's Working Principle*
- *Characteristics of Inductors*

Introduction to Storage Elements

- These elements store energy, which can be retrieved at a later time; hence the name storage element.
- Storage elements store energy in the form of either ***Electric Field*** or ***Magnetic Field***.
- Every storage element has ***two*** phases.

a) ***Charging Phase:*** Takes energy from another source and stores the acquired energy.
b) ***Discharging phase:*** Delivers the stored energy to the other elements of the circuit or system.

- According to the sign convention of power, an element delivers power if the power is negative ($p=vi<0$) and consumes power if it is positive ($p=vi>0$).
- Therefore, a storage element has positive power ($v,i>0$ or $v,i<0$) in the charging phase and negative power in the discharging phase ($v<0,i>0$ or $v>0,i<0$).
- Capacitors and Inductors are two such storage elements.

Capacitor and it's Working Principle

- Stores energy as electric field.

- A capacitor consists of two conducting plates separated by an insulator (or dielectric). A typical capacitor is illustrated in Figure: 1.

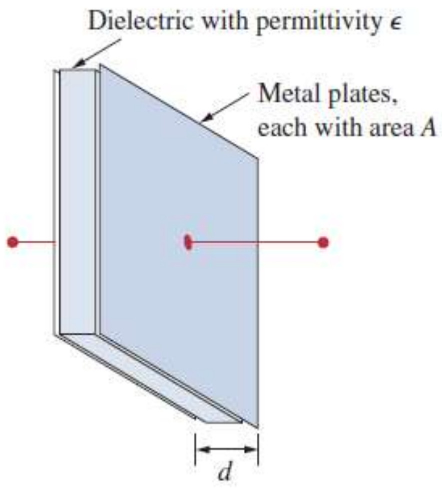


Figure: 1

- When a voltage source is connected to the capacitor, as in Figure: 2, the source deposits a positive charge *q* on one plate and a negative charge on the other.

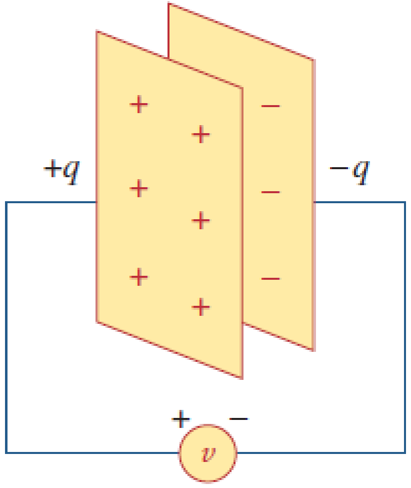


Figure: 2

- The capacitor is said to store the electric charge.

- The amount of cha $q = Cv$, represented by *q*, is directly proportio applied voltage *v* so that

Capacitor and it's Working Principle (cont.)

$$\boxed{q = Cv}$$

- In the above equation, C is the constant of proportionality and is known as the ***capacitance*** of the capacitor. The unit of capacitance is the farad (F).
- Capacitance depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Figure: 1, the capacitance is given by:

$$C = \frac{\epsilon A}{d}$$

From the above equation, we may infer that three factors determine the value of the capacitance:

1. The surface area (A) of the plates—the larger the area, the greater the capacitance.
2. The spacing (d) between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity (ε) of the material—the higher the permittivity, the greater the capacitance.

Characteristics of a Capacitor

- Let's first determine the current-voltage (I-V) relationship of a capacitor.
- To obtain the current-voltage relationship of the capacitor, we take the derivative of the equation below with respect to time on both sides.

$$\boxed{q = Cv}$$

Or, $$\frac{dq}{dt} = C\frac{dv}{dt}$$

Or, $$\boxed{i_c = C\frac{dv_c}{dt}}$$

[*Since,* $i = \frac{dq}{dt}$]

Here, i_c = *Current through the capacitor*

v_c = *Voltage across the capacitor*

The above equation describes the current-voltage relationship of a capacitor.

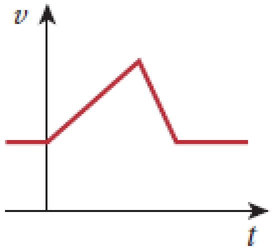
Characteristics of a Capacitor (cont.)

$$\boxed{i_c = C\frac{dv_c}{dt}}$$

- From the equation above, we can see that the current through the dc conditions is zero as the voltage across the capacitor is constant with respect to time.
- Therefore, we can conclude on the following facts.

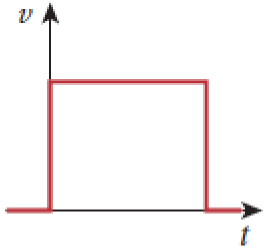
1. The capacitor works as an open circuit under dc conditions.
2. The capacitor resists a sudden change of voltage across it.

To illustrate this point, let’s take a look at two waveforms.



(a)

- The voltage across a capacitor may take the form shown in figure:(a) as the voltage changes ***gradually*** (the rate of change of *v is a non-zero value*)w.r.t time.



(b)

- The voltage across a capacitor cannot take the form shown in figure:(b) as the voltage changes ***suddenly*** **(**the rate of change of *v is infinite at the transition***).** It makes the current i_c = *infinity*, which is practically impossible.

Characteristics of a Capacitor (cont.)

$$\boxed{i_c = C\frac{dv_c}{dt}}$$

- From the equation above, we can obtain the equations of voltage across the capacitor , the power and the energy stored in the capacitor.

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

or

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)}$$

- Where $v\ (t_0)$ is the voltage across the capacitor at time t_0(initial time or the time when the analysis is initiated). This voltage is due to the initially stored charge on the capacitor plates. If the capacitor is assumed initially uncharged than $v\ (t_0)$=0.
- The equation shows that capacitor voltage depends on its initial charging condition which implies it has a memory.

Characteristics of a Capacitor (cont.)

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$\boxed{w = \frac{1}{2}Cv^2}$$

Or, $$w = \frac{q^2}{2C}$$

Inductor and it’s Working Principle

- Stores energy as magnetic field.

- Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in **Figure:1**.

- The working principle of an inductor based on the fact that whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. The induced emf in a coil is equal to the rate of change of flux linkage.

$$\boxed{e = N\frac{d\phi}{dt}}$$

- Current through a coil of wire, with or without a core, establishes a magnetic field through and surrounding the unit.

- If the current through the coil increases in magnitude, the flux linking the coil also increases.

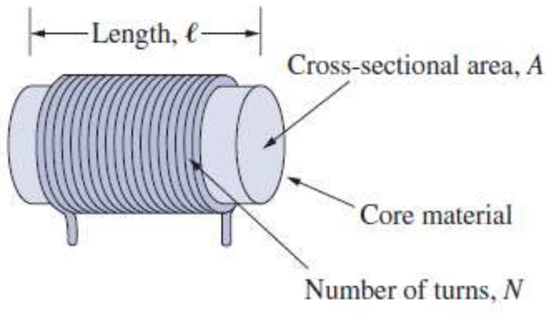


Figure 1: A typical form of an Inductor

Inductor and it's Working Principle (cont.)

- If the current through the coil increases in magnitude, the flux linking the coil also increases. This change in magnetic field induces a voltage.
- The polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current in the first place.
- The quicker the change in current through the coil, the greater the opposing induced voltage to squelch the attempt of the current to increase in magnitude.

$$v = L\frac{di}{dt}$$

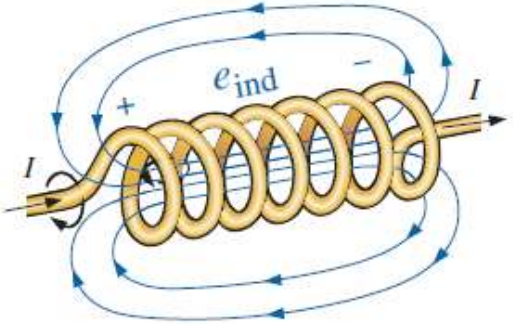


Figure 2: Demonstrating the effect of Lenz's law.

- This property of a conducting coil or wire whereby it exhibits opposition to the change of current flowing through it is called inductance (L, measured in henrys, H) and the resulting circuit element it called an inductor.

$$\boxed{L = N\frac{d\phi}{di_L}} \qquad \text{(henries, H)}$$

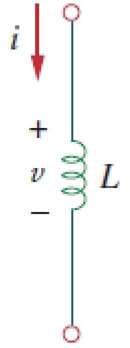


Figure 3: Circuit symbol of an Inductor

Characteristics of an Inductor

- The following equation describes the current-voltage relationship of an inductor.

$$\boxed{v_L = L\frac{di_L}{dt}}$$

- The current through an inductor,

$$i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

or

$$\boxed{i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)}$$

- The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i$$

- The energy stored in an inductor is

$$w = \frac{1}{2}Li^2$$

Characteristics of an Inductor (cont.)

$$\boxed{v_L = L\frac{di_L}{dt}}$$

- From the equation above, we can see that the change in current through the dc conditions is zero as the current through the inductor is constant with respect to time, which make the voltage across the inductor zero.
- Therefore, we can conclude on the following facts.

1. The inductor works as an short circuit under dc conditions.
2. The inductor resists a sudden change of current through it.

To illustrate this point, let's take a look at two waveforms.

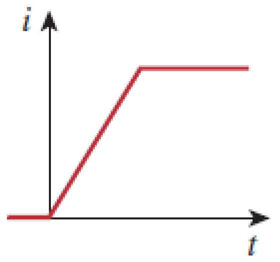


Figure (a)

- The current through the inductor may take the form shown in figure:(a) as the current changes ***gradually*** (the rate of change of *i is a non-zero value*) w.r.t time.

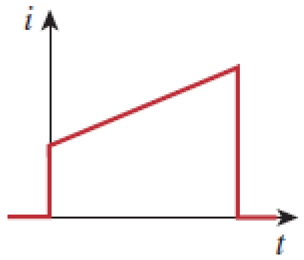


Figure (b)

- The current through the inductor cannot take the form shown in figure:(b) as the current changes ***suddenly*** **(**the rate of change of *i is infinite at the transition***).** It makes the current v_L = *infinity*, which is practically impossible.

Summary

We can summarize the lecture with the following conclusions:

- Inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.
- Both capacitor and inductor exhibits true characteristics only when there is a gradual change in either voltage and current.
- In many ways, capacitor and inductor act as the dual of each other, i.e. we can obtain the I-V characteristics of capacitor by replacing the voltage with current and the inductance with capacitance in the following equation:

$$v_L = L\frac{di_L}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

Numerical Problems/ Home Work

- Practice all the related exercises and examples.

Reference: Chapter 6 (Capacitors and Inductors)
Fundamentals of Electric Circuits
Charles K. Alexander & Matthew N.O. Sadiku

Any Questions?