Shahjalal University of Science and Technology

Institute of Information and Communication Technology

Software Engineering

Final Examination, 1st Year 2nd Semester, 2018

Course No: MAT-107W Course Title: Linear and Abstract Algebra

Credits: 3 Full Marks: 70 Time: 3 Hours

GROUP-A

(Answer any seven questions)

1.	Define idempotent matrix and Involutory matrix. Give example for each. For any two idempotent matrix A and B, AB is idempotent – prove or disprove it.	5
2.	Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & -2 \end{bmatrix}$, (if it exists) using elementary row operation.	5
3.	Define linear equation and system of linear equations in n variables. When is a system said to be consistent? Picture the type of solutions of a system of linear equation.	5
4.	Solve the following system with the help of matrix: $x + 2y + z = 1$, $3x + 7y + 6z = 5$, $-2x - y + 7z = 4$	5
5.	Define rank of a matrix. Find the rank of the matrix A, where $A = \begin{pmatrix} 1 & 3 & -2 & -1 \\ 2 & 6 & -4 & -2 \\ 1 & 3 & -2 & 1 \\ 2 & 6 & 1 & -1 \end{pmatrix}$	5
6.	Define linear dependence and linear independence. Test the dependency of the sets $\{ (2, 0, -1), (1, 1, 0), (0, -1, 1) \}$	5
7.	Let S′ be any subset of a vector space V. Under what conditions S′ will be a subspace of V. Determine whether or not ω is a subspace of $\mathbb{R}^3$ where ω consists of all vectors (a, b, c) in $\mathbb{R}^3$ such that (i) $a \leq b \leq c$; (ii) $b = a^2$	5
8.	Define basis and dimension of a vector space V. Suppose W be the subspace of $\mathbb{R}^4$ generated by the vectors $\omega_1 = (1, -2, 5, -3)$, $\omega_2 = (2, 3, 1, -4)$, $\omega_3 = (3, 8, -3, -5)$. i. Find basis and dimension of W. ii. Extend this basis to the basis of $\mathbb{R}^4$.	5
9.	Define linear transformations. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$, find the rank and the nullity of T.	5

10. Consider the basis $S = \{(1,2,0),(1,3,2),(0,1,3)\}$ of $\mathbb{R}^3$. Find i. The change-of-basis matrix P from the usual basis E to S. ii. The change-of-basis Q from the basis S to E.	5

GROUP-B

(Answer any seven questions)

11. Define permutation of a set. When is it said to be even or odd? Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 2 & 5 & 3 & 1 & 6 \end{pmatrix}$ as a product of disjoint cycles. Hence determine the parity of σ.	5
12. Define a group. If in a group G, $xy^2 = y^3x$ and $yx^2 = x^3y$, then show that $x = y = e$, where e is the identity of G.	5
13. Define residue class modulo n. Prove that Z_n the residue class modulo n forms a finite group under residue addition.	5
14. Define subgroup of a group. Prove that HCG is a subgroup of a group G if and only if H is closed under the binary operation on G and also closed under the formation of invers.	5
15. Define normal subgroup and centre of a group. Prove that centre of a group is a normal subgroup.	5
16. Define right cosets. Find all the right cosets of A_3. Find all the right cosets of A_3 in S_3, where S_3, the set of all permutations of degree 3 and A_3, the set of all even permutation of S_3.	5
17. Define group homomorphism and its kernel. Let $G = (\mathbb{Z}\,;+)$ and $k = (\mathbb{Z}_n\,, +)$. Define $f: G \to k$ by $f(r) = \bar{r}$. Show that f is a group homomorphism. Also find its kernel.	5
18. Define automorphism. For a fixed $a \in G$, define $f_a: G \to G$ by $f_a(x) = axa^{-1}$ for all $x \in G$. Show that f_a is an automorphism.	5
19. Define a principal ideal ring. Let R be a commutative ring with 1. Let $a \in R$, then prove that the set $S = \{ ra \mid r \in R \}$ is a principal ideal.	5
20. Define maximal ideal and prime ideal of a ring. Let R be a commutative ring with 1 and let I be any ideal in R. Then prove that R/I is an integral domain if and only if I is a prime ideal of R.	5