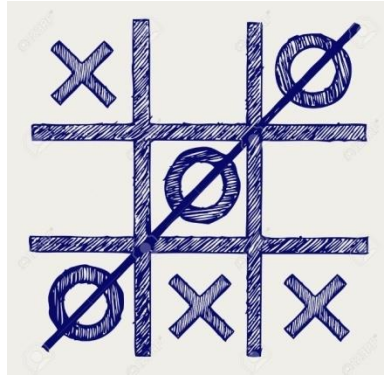


CSC 422

Class #3

Problem Solving as Search

Why searching in AI?



Maybe you all are familiar with the board game Tic-Tac-Toe.

As an AI student, you want to design an intelligent agent, which can play this game with you as an opponent.

Why searching in AI?

Intelligent agent should know the current state of the Tic-Tac-Toe board.

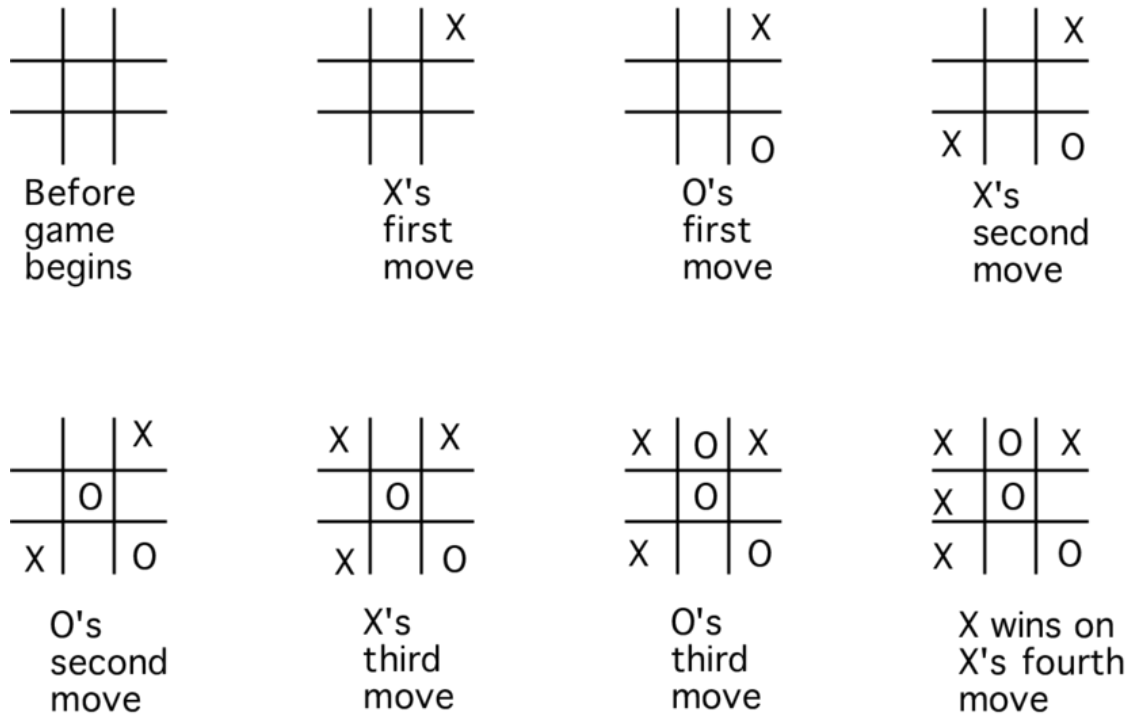


Fig: Some states of Tic-Tac-Toe board.

Why searching in AI?

Intelligent agent should explore the next possible movement and search for the best move to win the game.

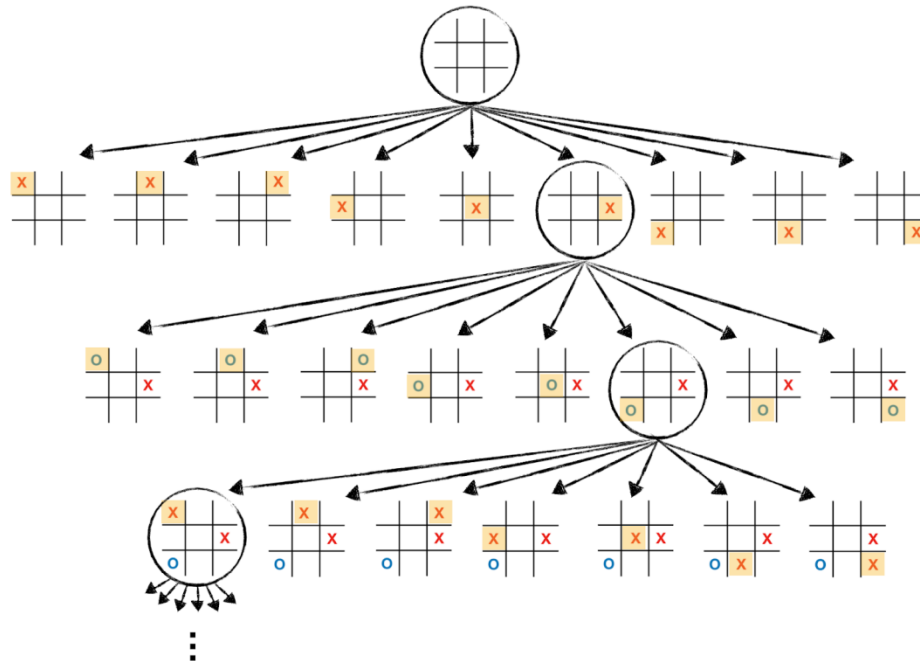
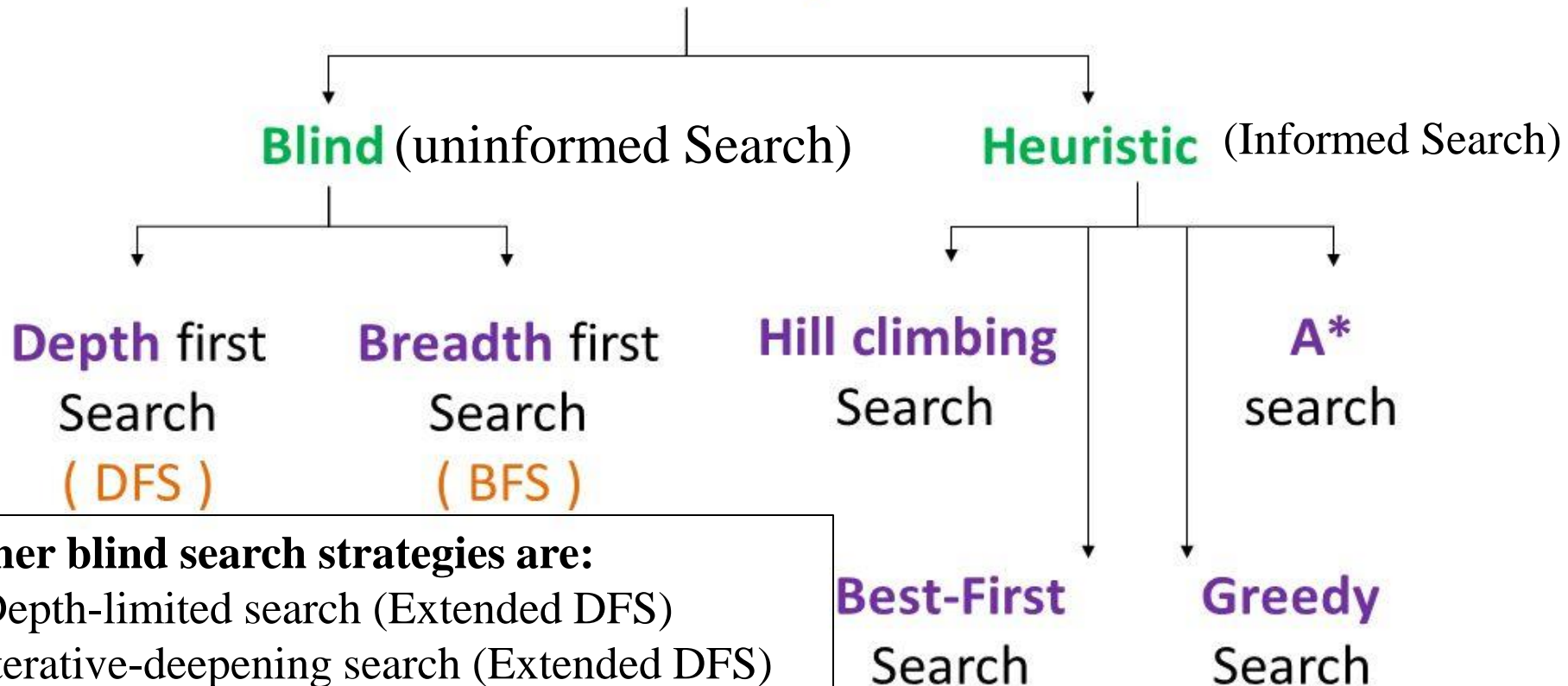


Fig: Game search tree for Tic-Tac-Toe board (partial view)

SEARCH TECHNIQUES

Search techniques



Other blind search strategies are:

- Depth-limited search (Extended DFS)
- Iterative-deepening search (Extended DFS)
- Uniform cost search
- Bi-directional search

Uninformed Vs Informed Search

Uninformed search: Use only the information available in the problem definition. Example: breadth-first, depth-first, depth limited, iterative deepening, uniform cost and bidirectional search

Informed search: Use domain knowledge or heuristic to choose the best move. Example. Greedy best-first, A*, IDA*, and beam search

Additional Note:

optimization in which the search is to find an optimal value of an objective function: hill climbing, simulated annealing, genetic algorithms, Ant Colony Optimization

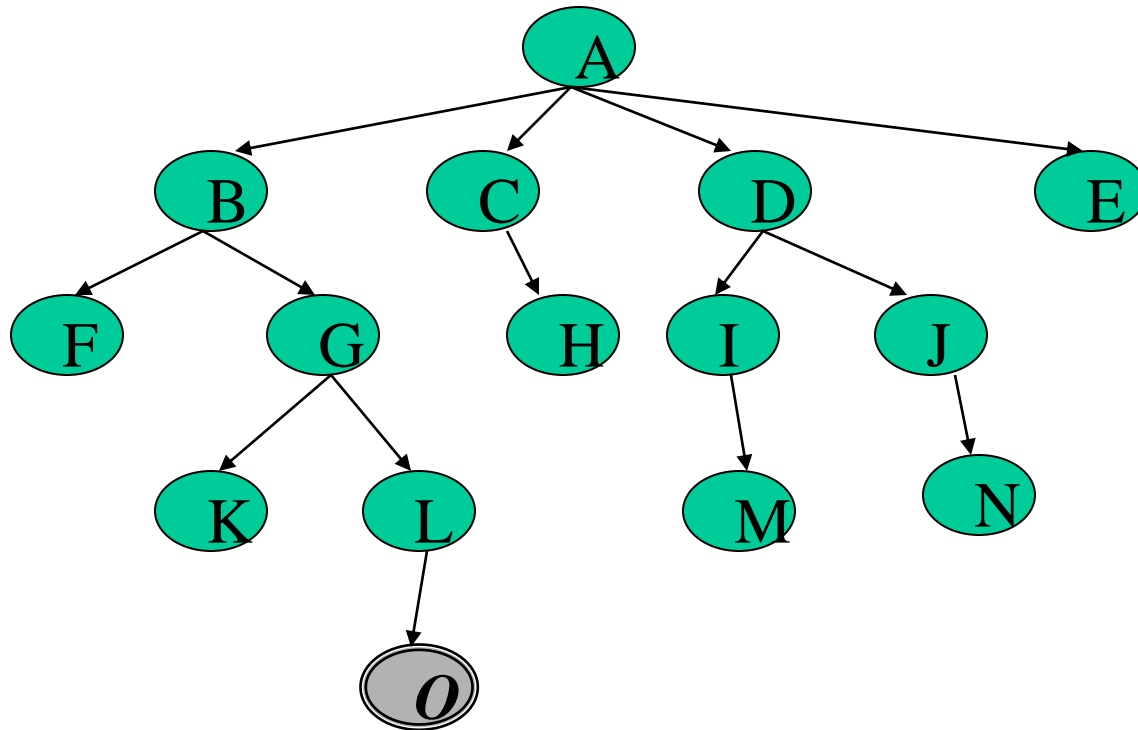
Game playing, an adversarial search: minimax algorithm, alpha-beta pruning

Uninformed Search

Breadth First Search

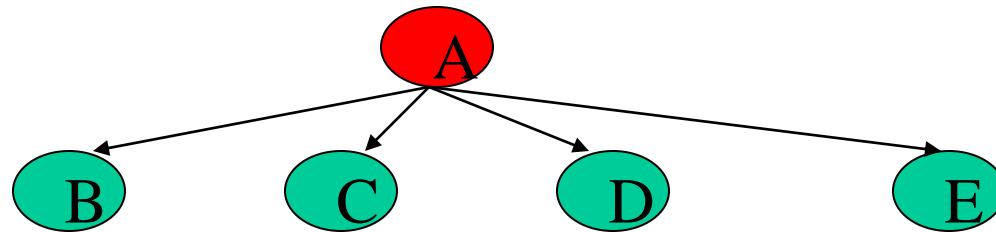
- Application1:

Given the following state space (tree search), give the sequence of visited nodes when using BFS (assume that the node *O* is the goal state):



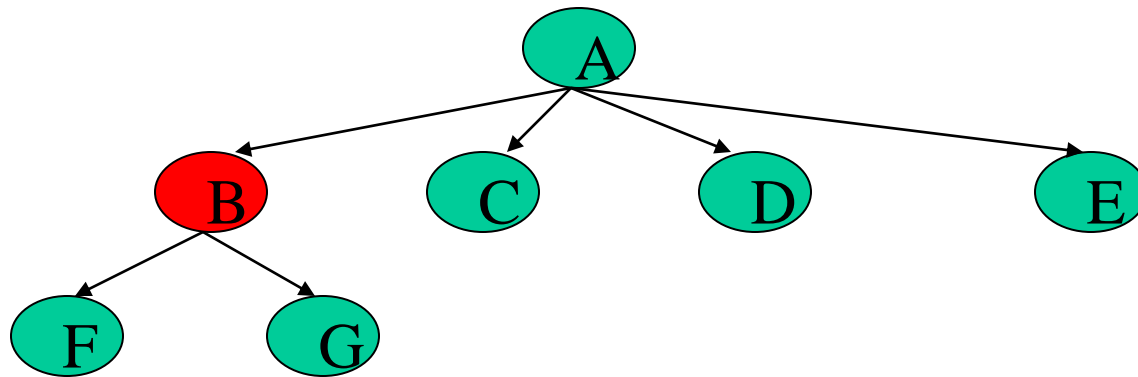
Breadth First Search

- A,



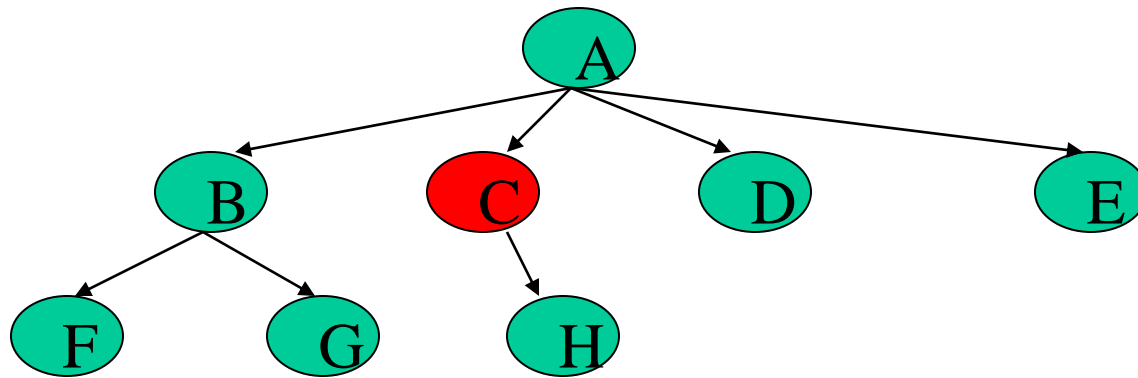
Breadth First Search

- A,
- B,



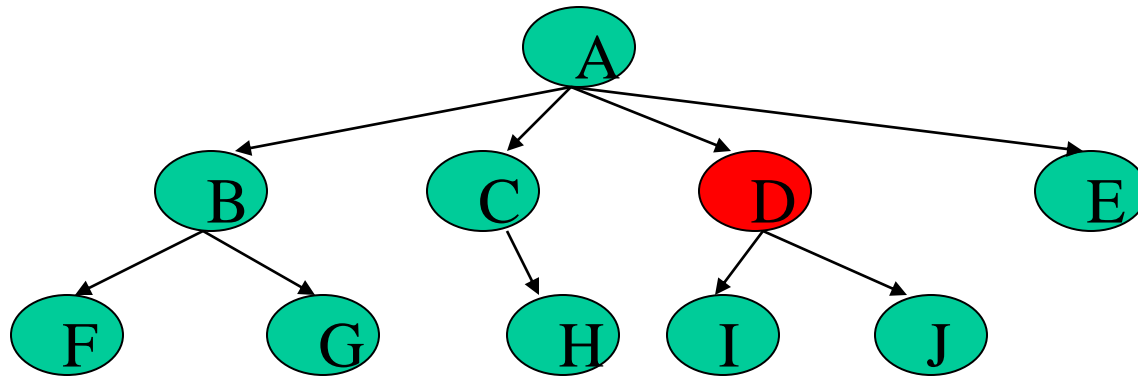
Breadth First Search

- A,
- B,C



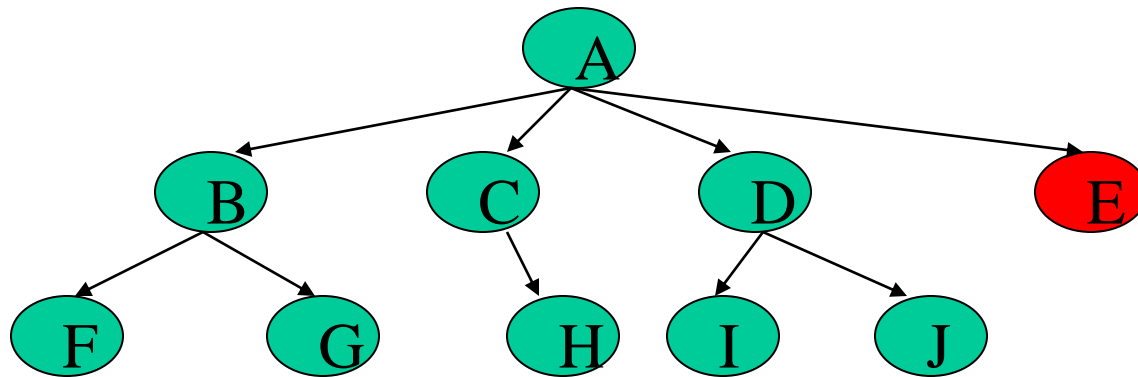
Breadth First Search

- A,
- B,C,D



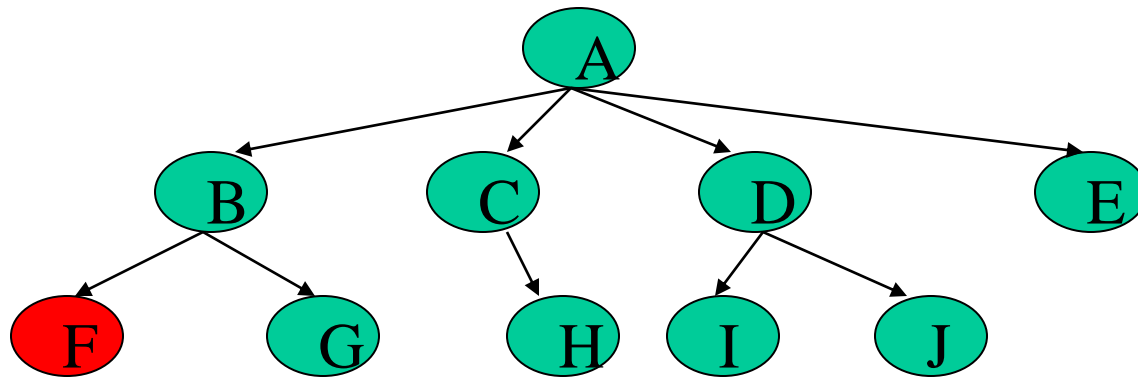
Breadth First Search

- A,
- B,C,D,E



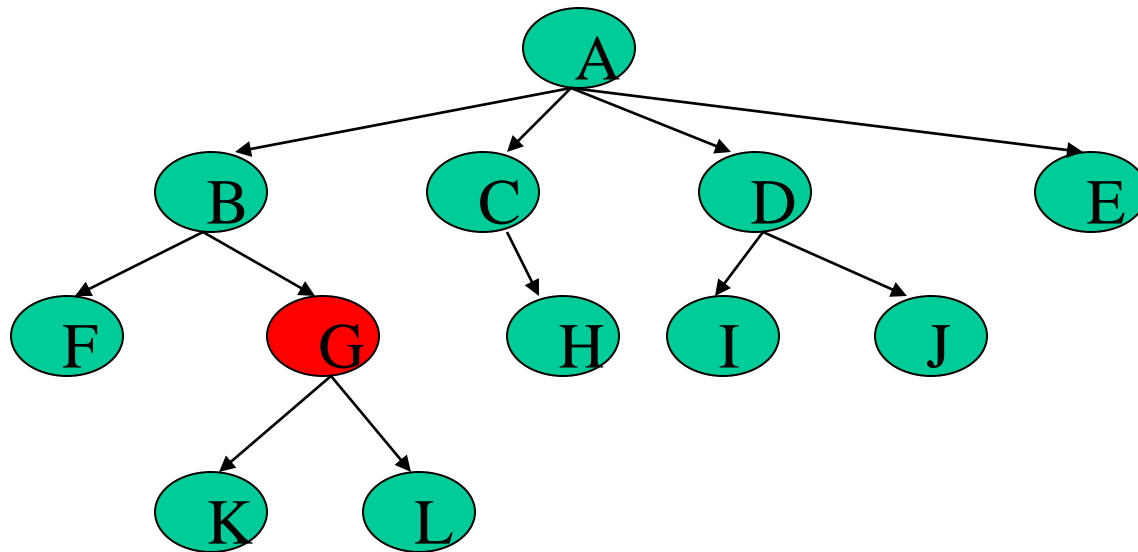
Breadth First Search

- A,
- B,C,D,E,
- F,



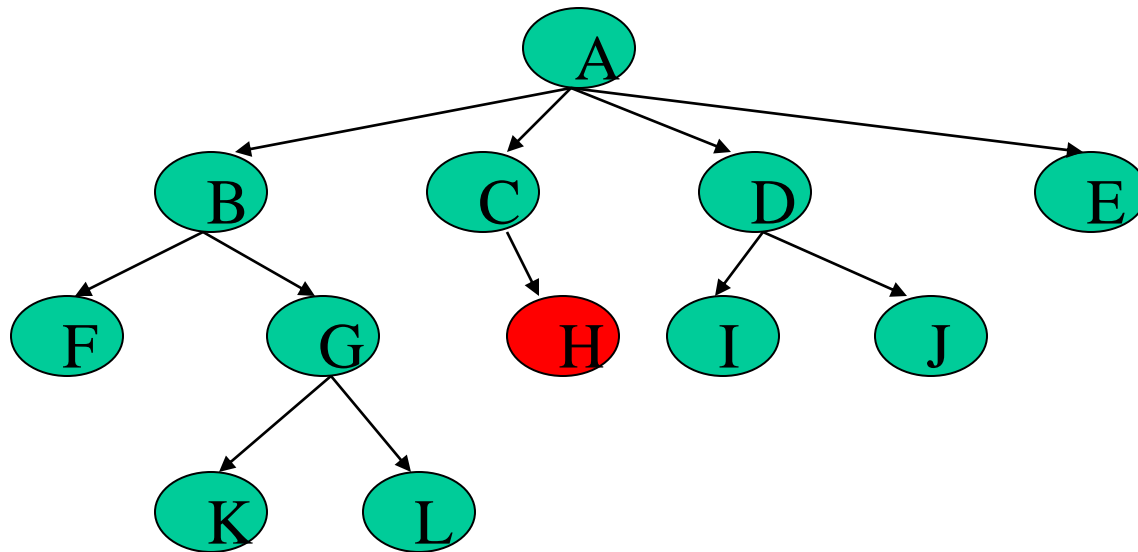
Breadth First Search

- A,
- B,C,D,E,
- F,G



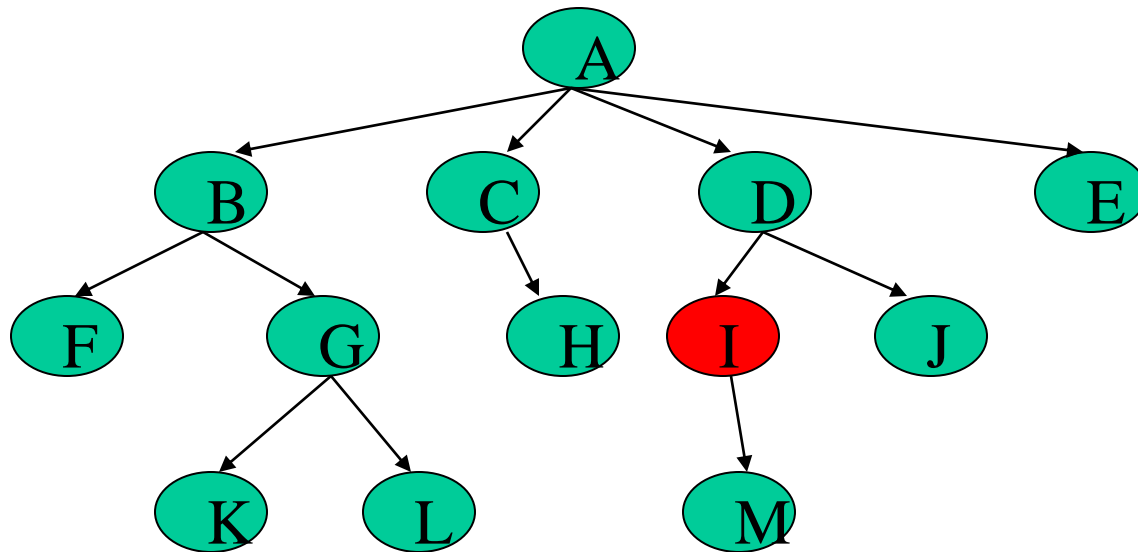
Breadth First Search

- A,
- B,C,D,E,
- F,G,H



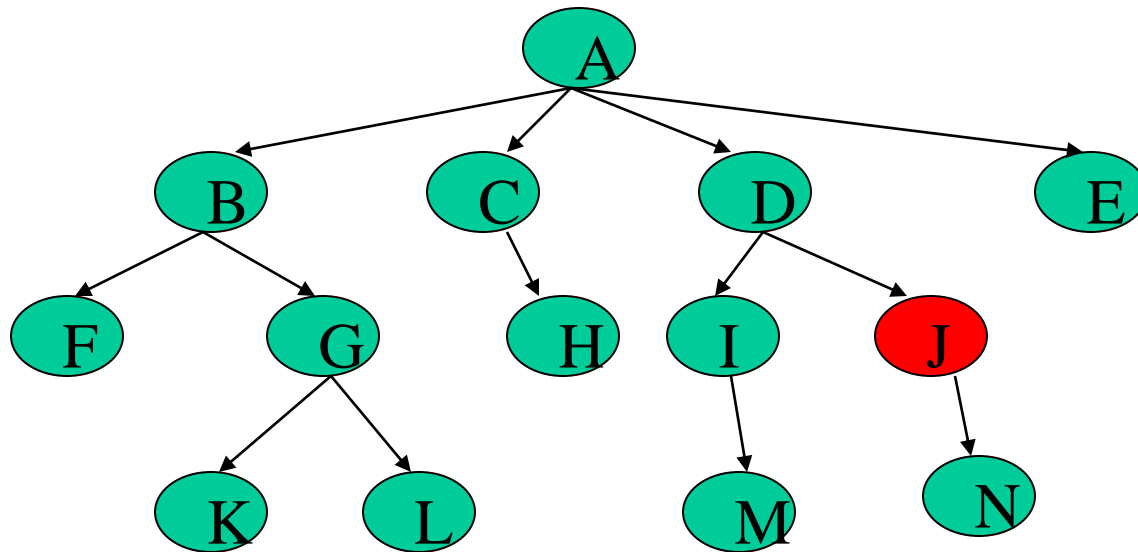
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I



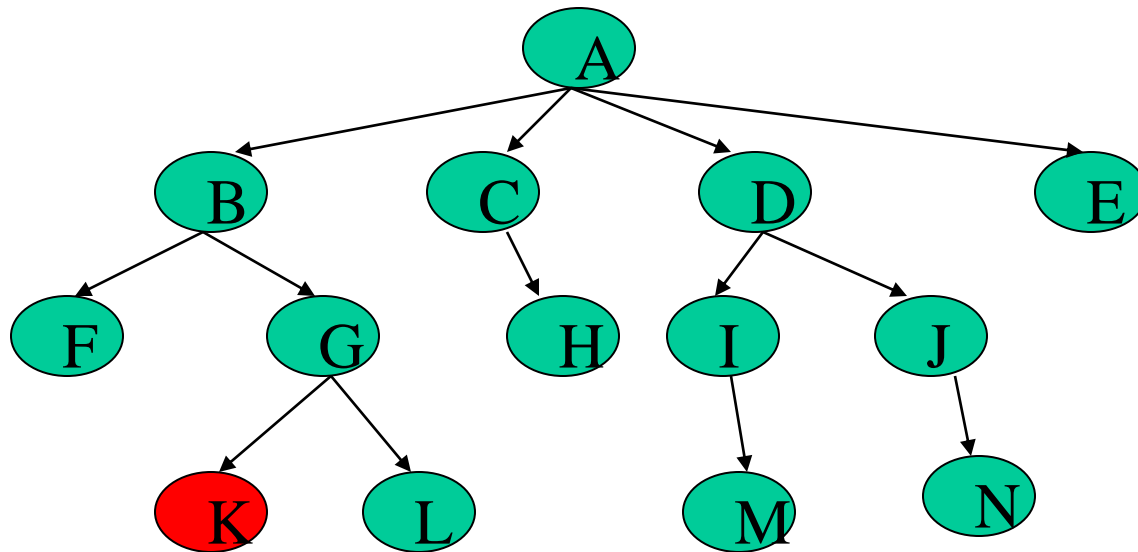
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I,J,



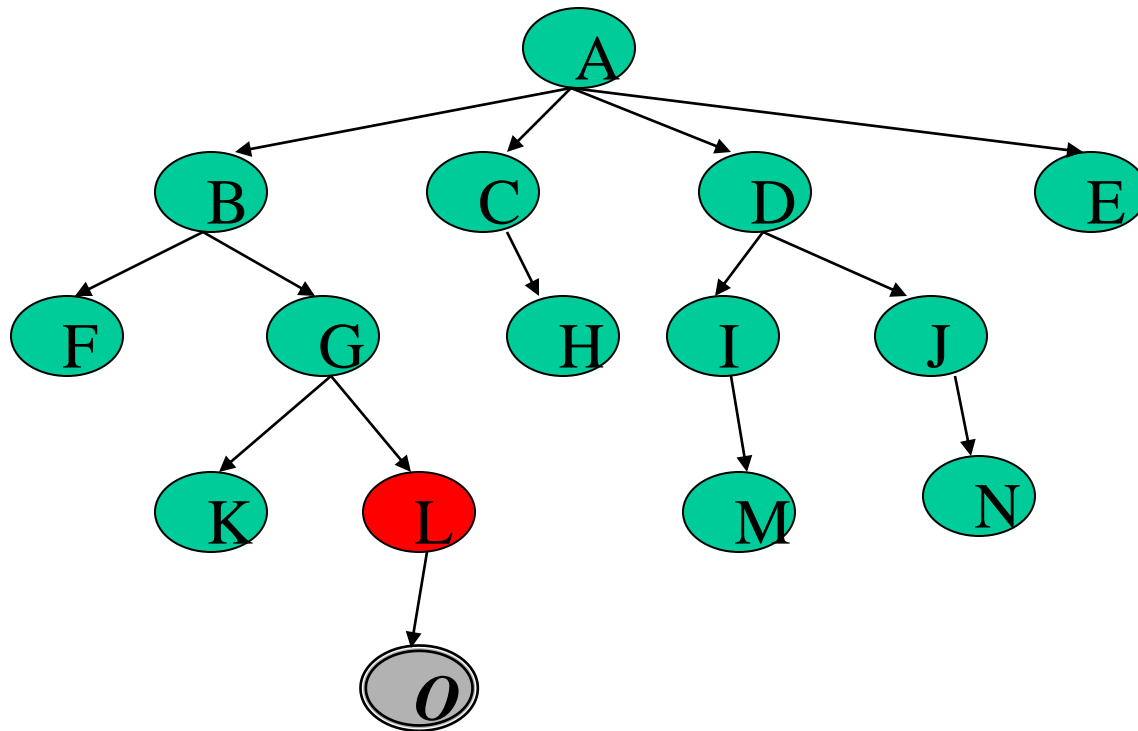
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I,J,
- K,



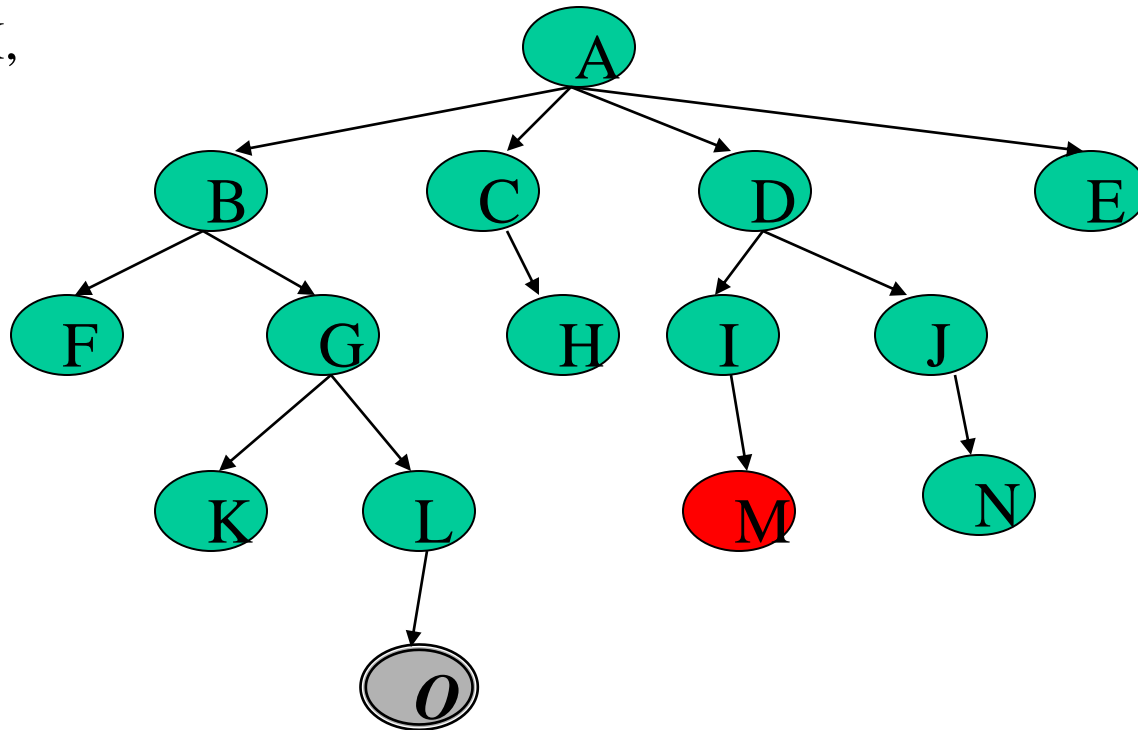
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I,J,
- K,L



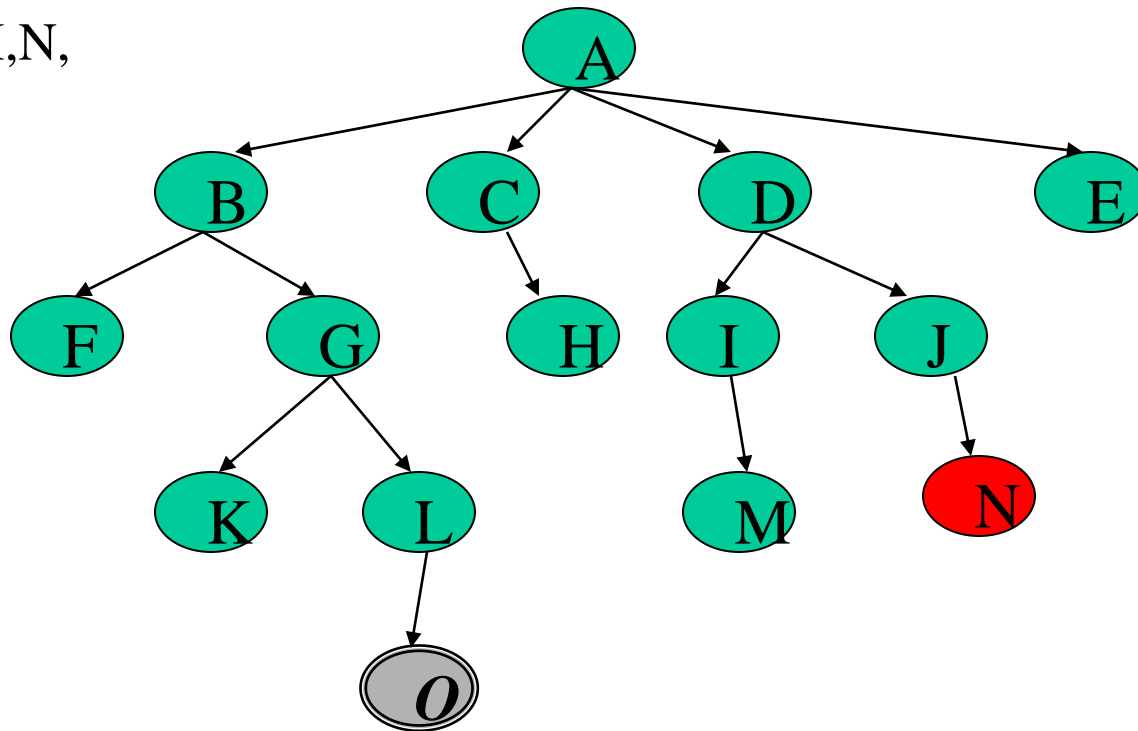
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I,J,
- K,L, M,



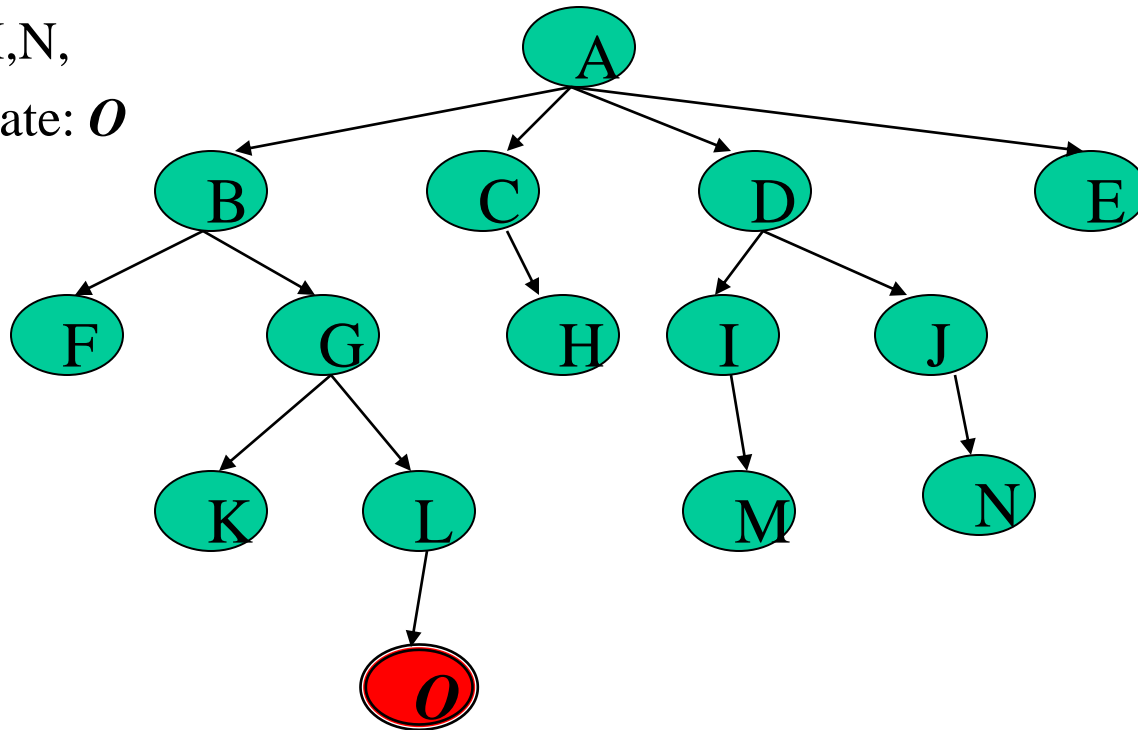
Breadth First Search

- A,
- B,C,D,E,
- F,G,H,I,J,
- K,L, M,N,



Breadth First Search

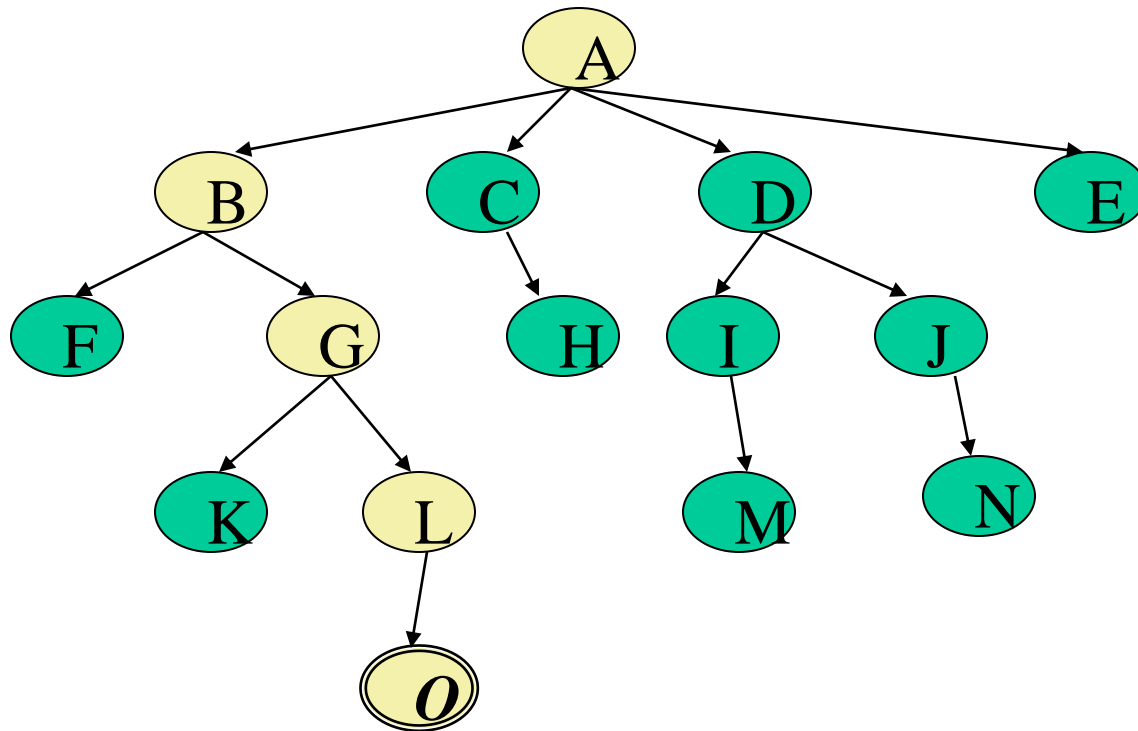
- A,
- B,C,D,E,
- F,G,H,I,J,
- K,L, M,N,
- Goal state: *O*



Breadth First Search

- The returned solution is the sequence of operators in the path:

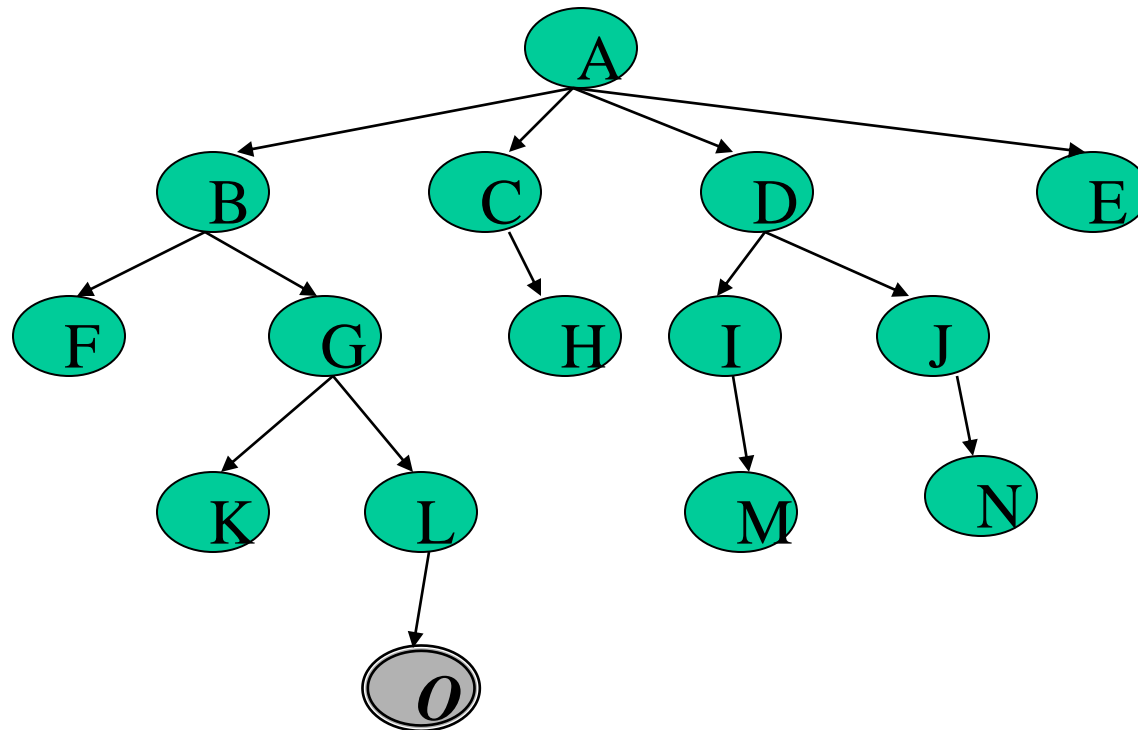
A, B, G, L, O



Depth First Search (DFS)

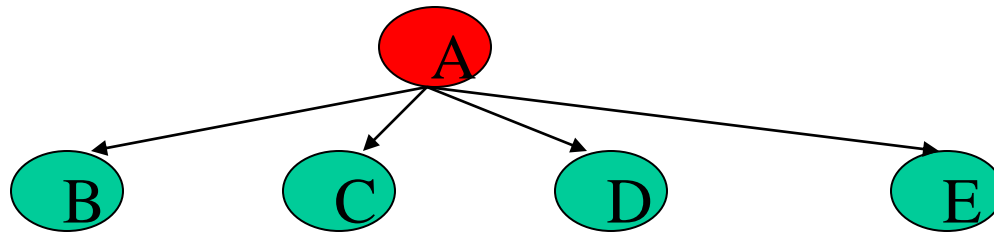
- Application2:

Given the following state space (tree search), give the sequence of visited nodes when using DFS (assume that the node *O* is the goal state):



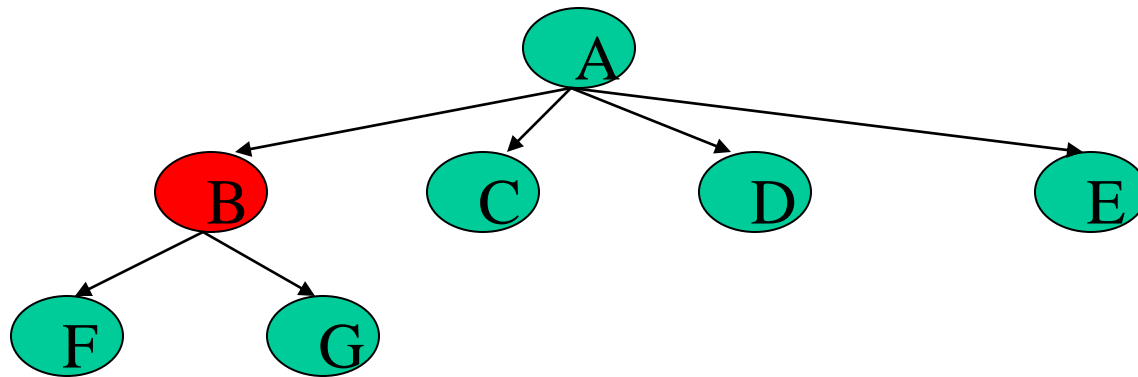
Depth First Search

- A,



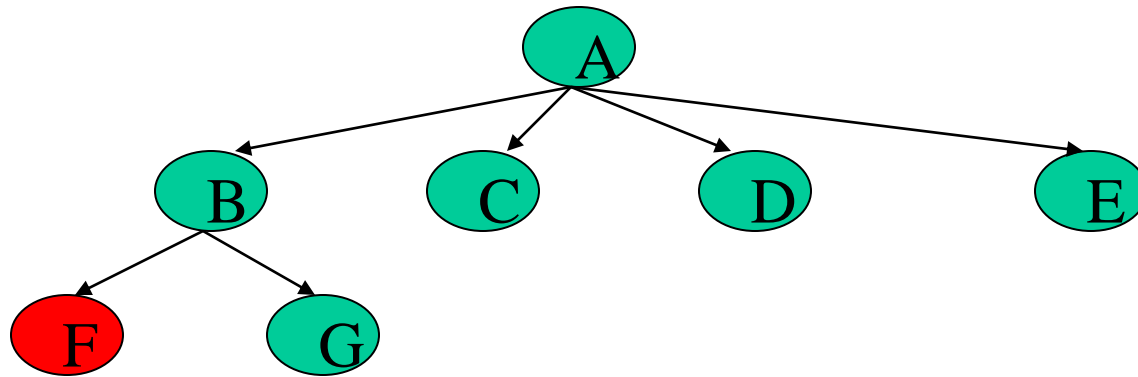
Depth First Search

- A,B,



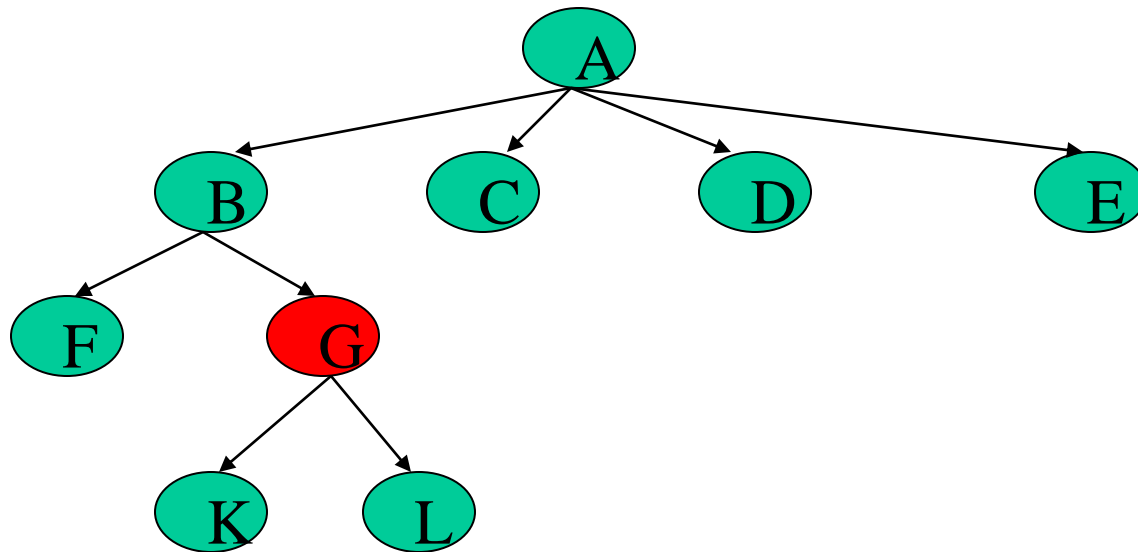
Depth First Search

- A,B,F,



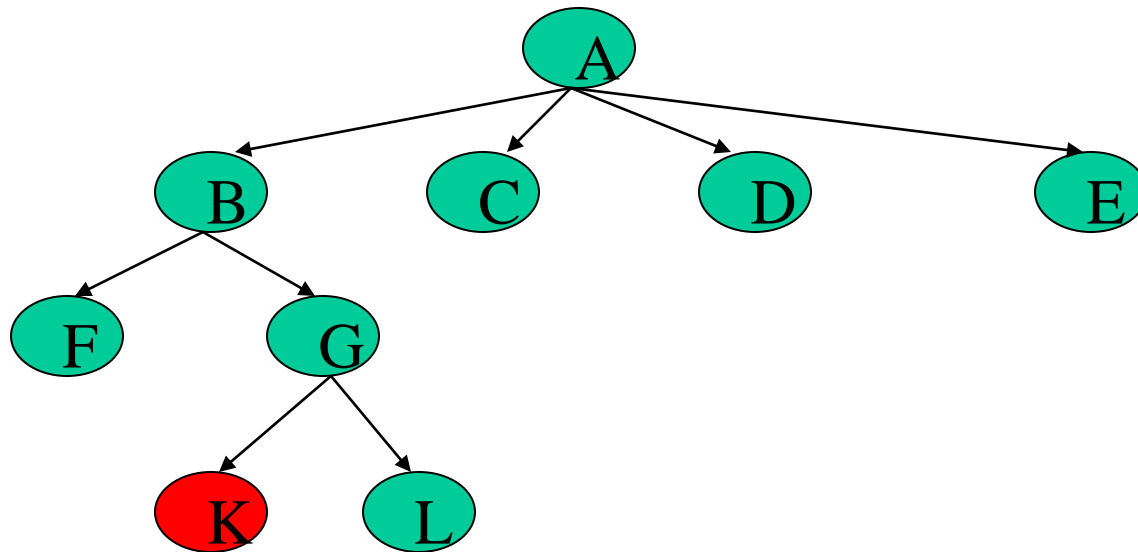
Depth First Search

- A,B,F,
- G,



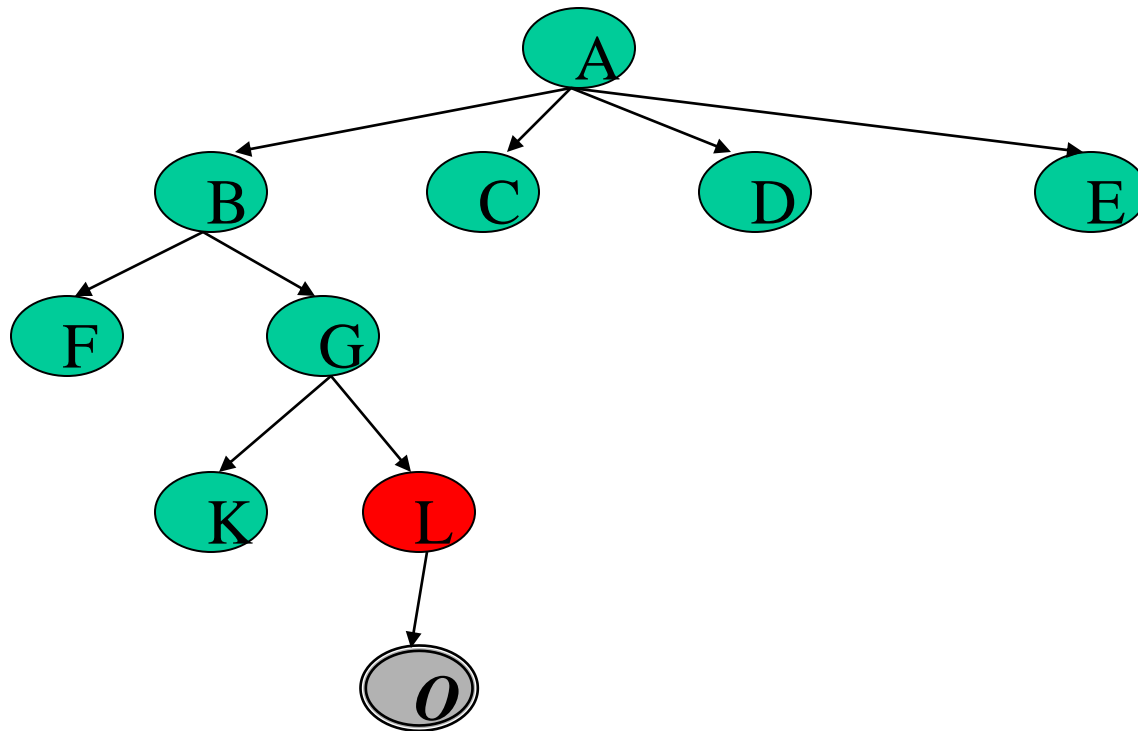
Depth First Search

- A,B,F,
- G,K,



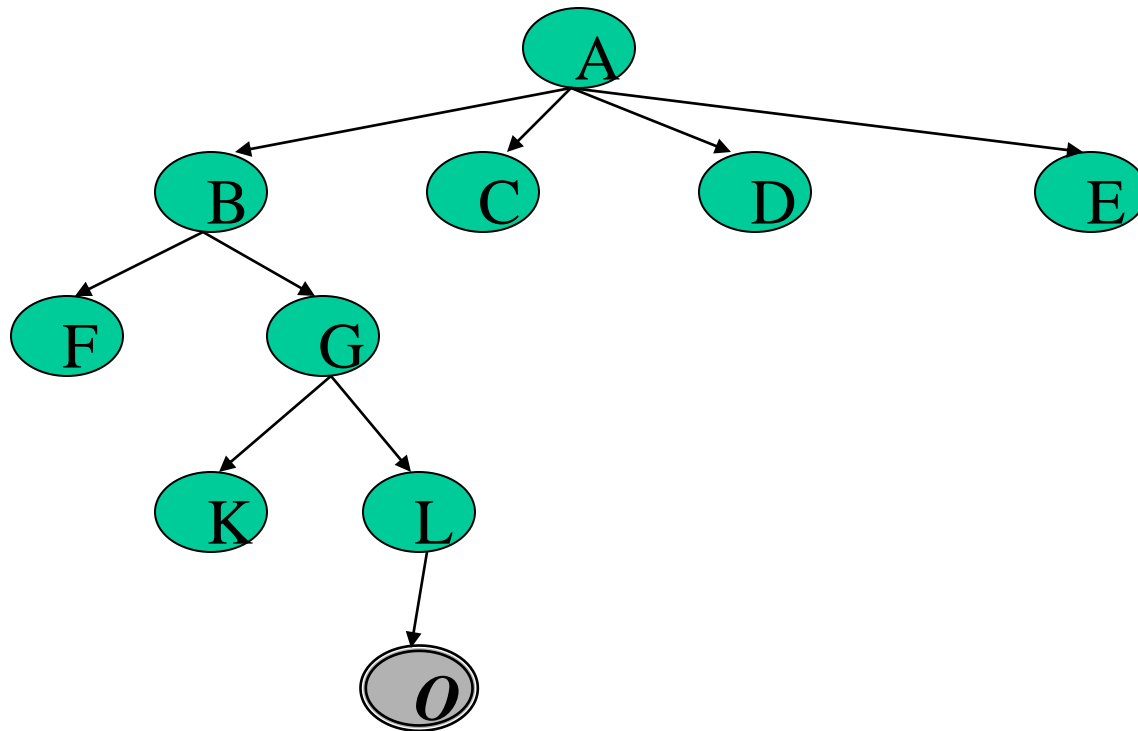
Depth First Search

- A,B,F,
- G,K,
- L,



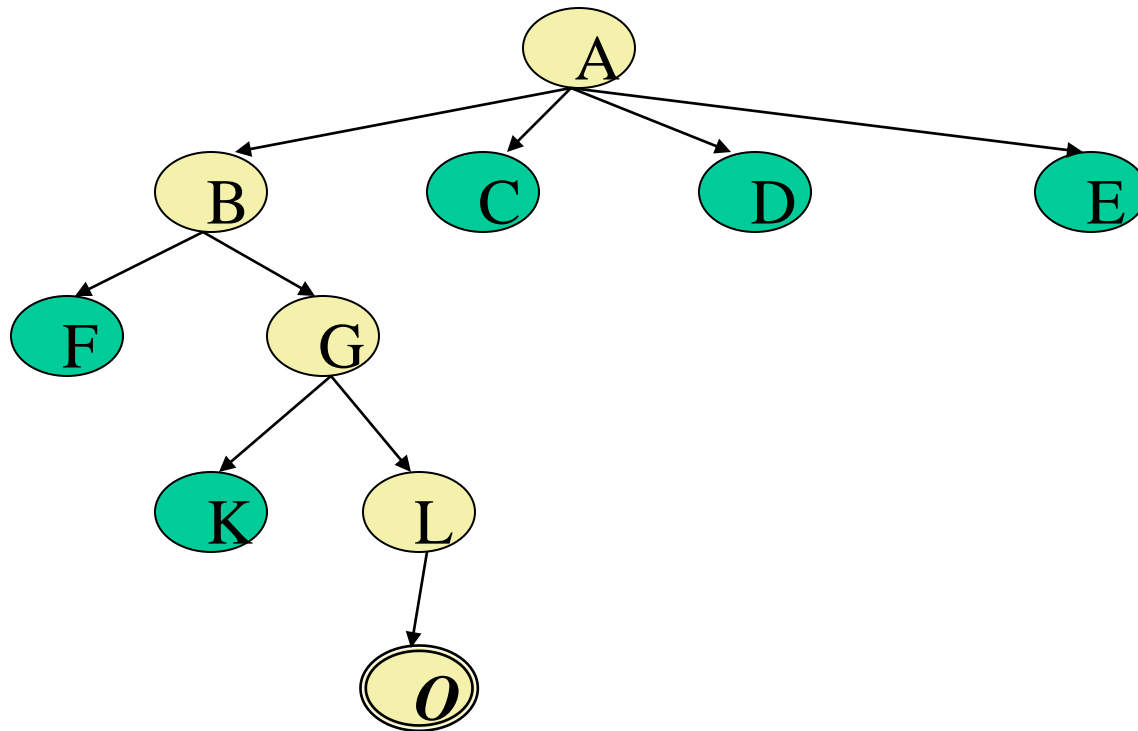
Depth First Search

- A,B,F,
- G,K,
- L, *O*: Goal State



Depth First Search

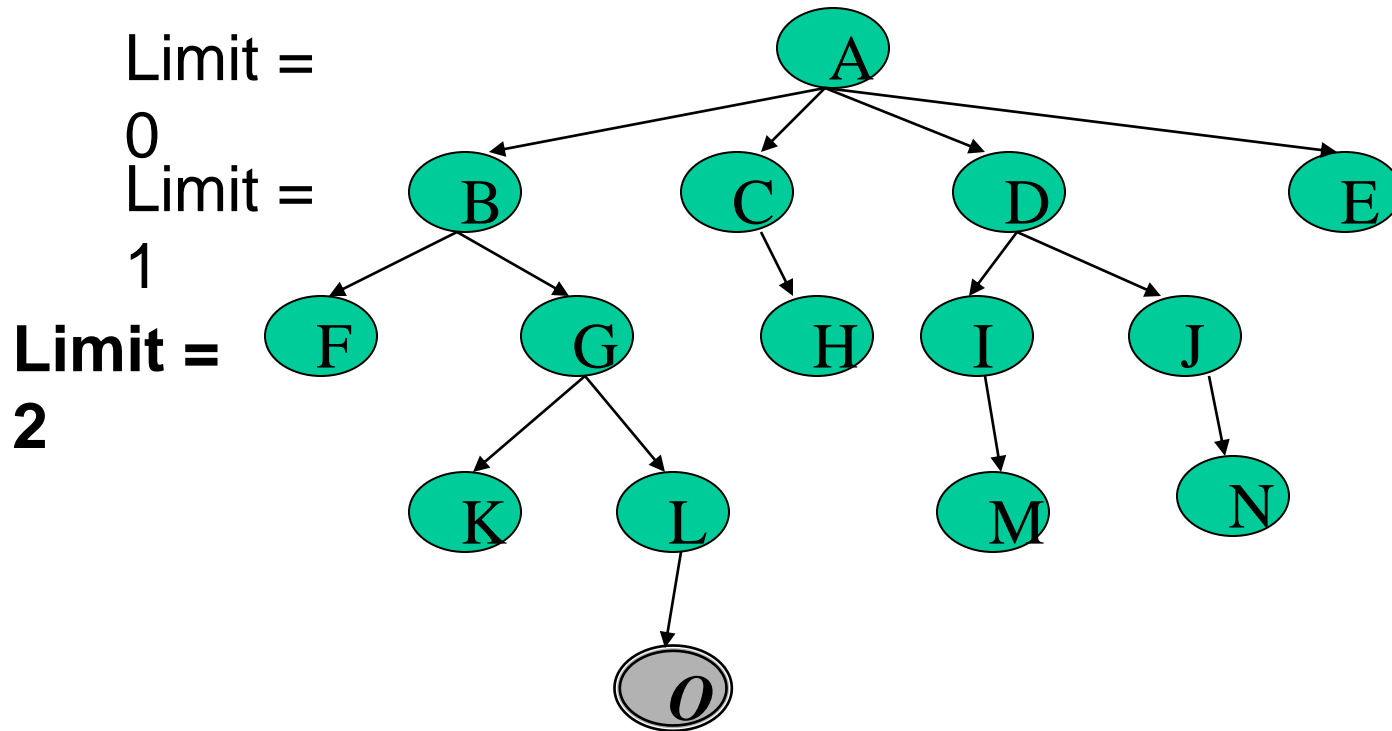
The returned solution is the sequence of operators in the path:
A, B, G, L, O



Depth-Limited Search (DLS)

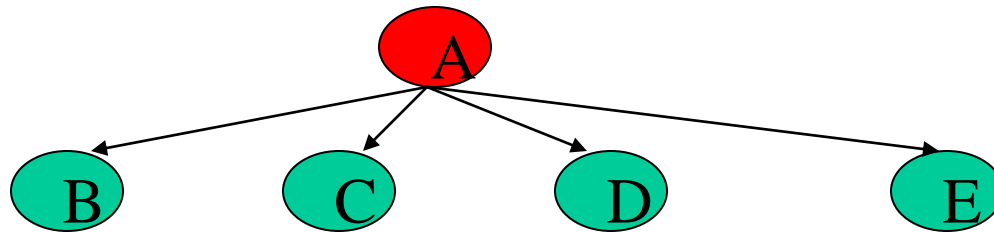
- Application3:

Given the following state space (tree search), give the sequence of visited nodes when using DLS (Limit = 2):



Depth-Limited Search (DLS)

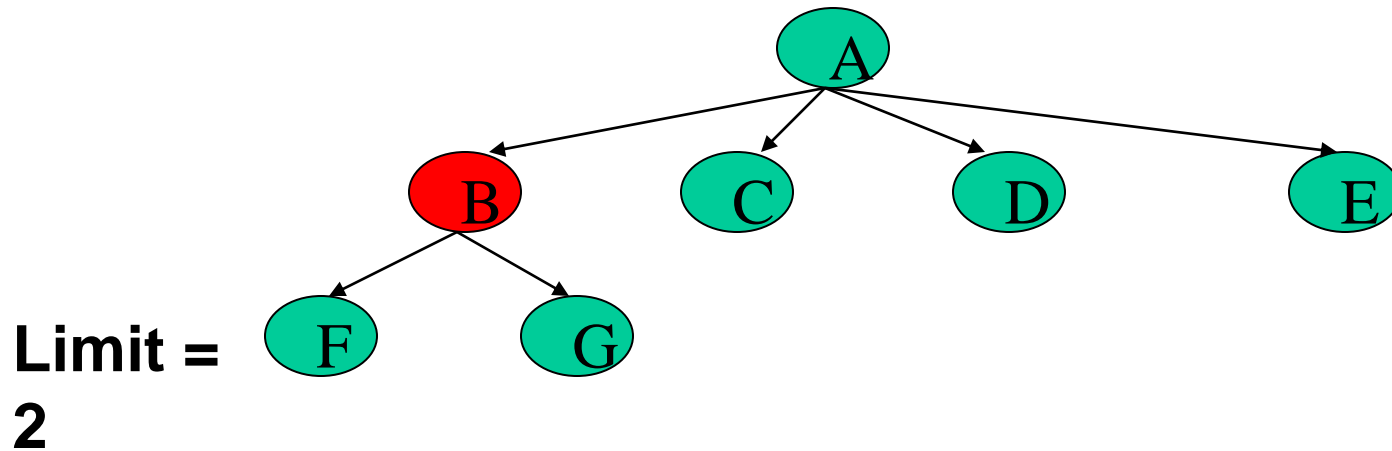
- A,



Limit =
2

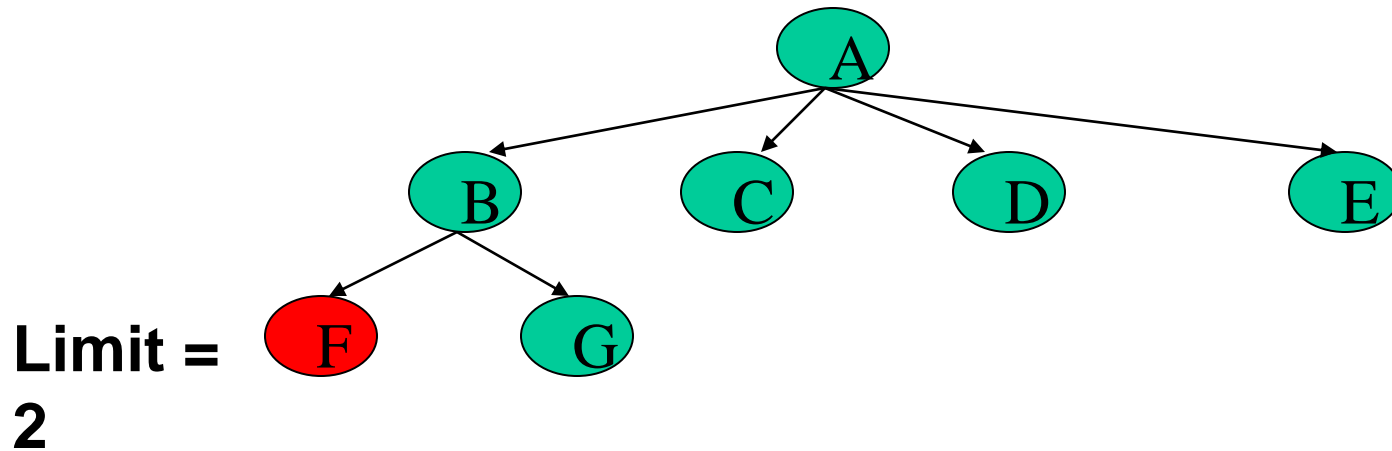
Depth-Limited Search (DLS)

- A,B,



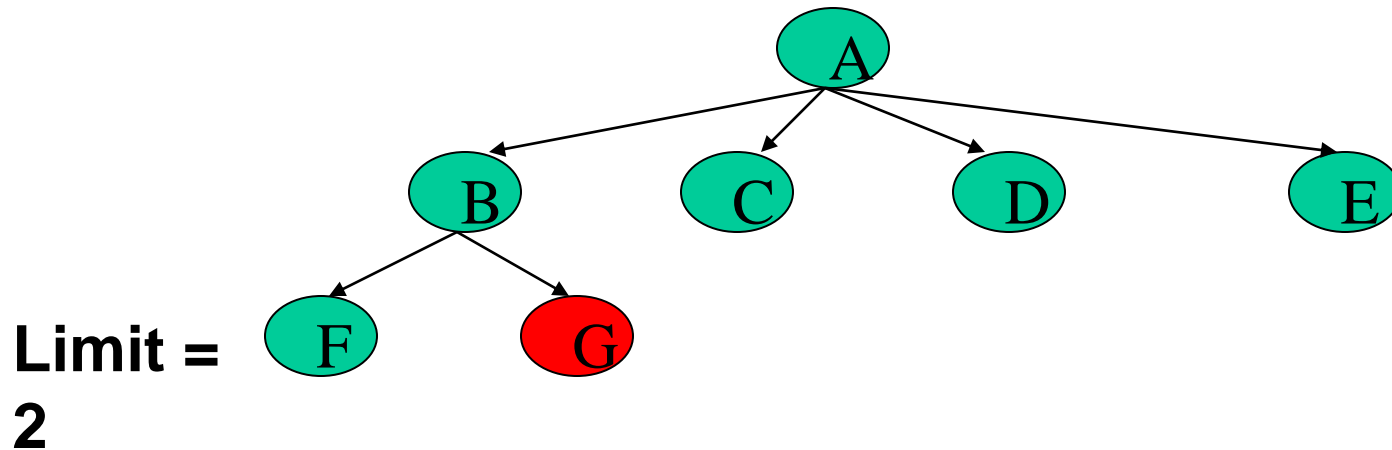
Depth-Limited Search (DLS)

- A,B,F,



Depth-Limited Search (DLS)

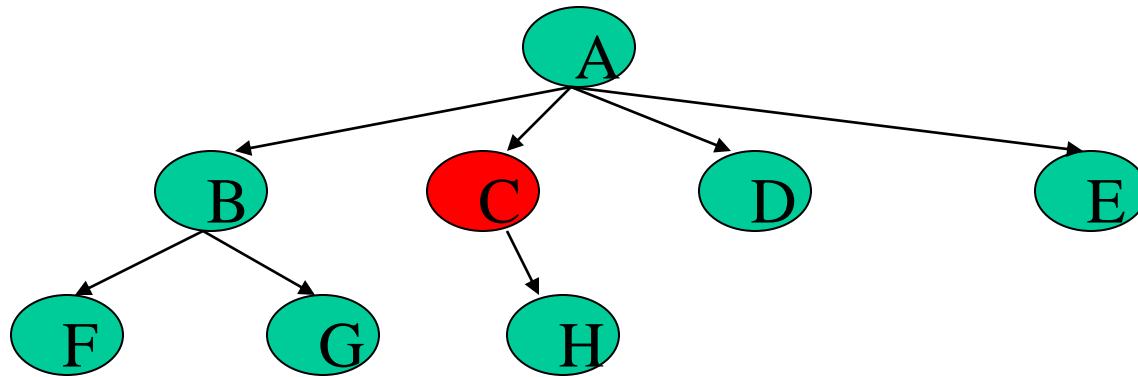
- A,B,F,
- G,



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,

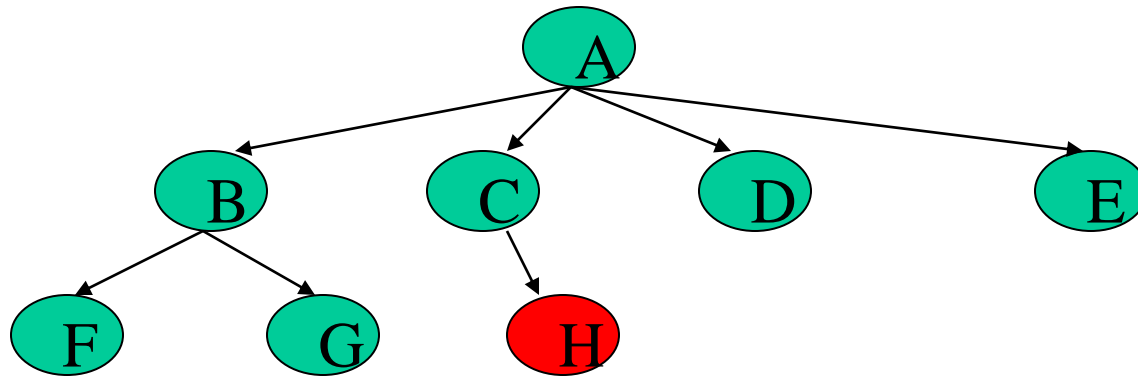
Limit =
2



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,H,

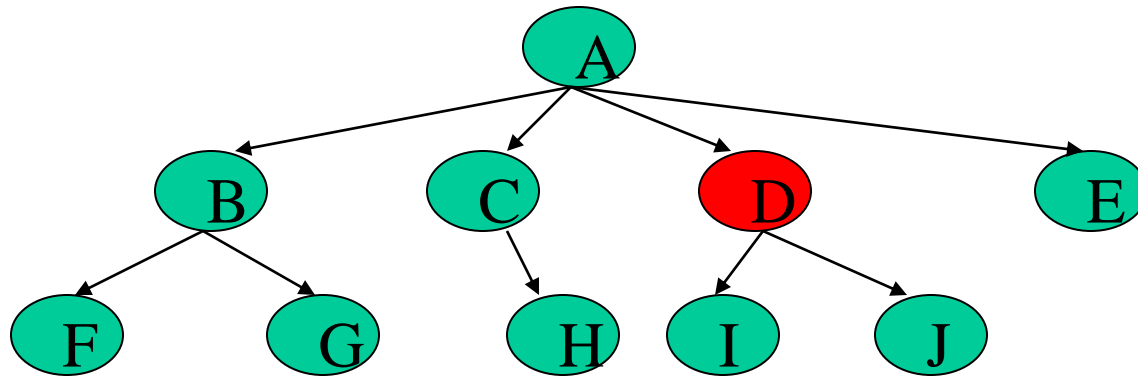
**Limit =
2**



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,H,
- D,

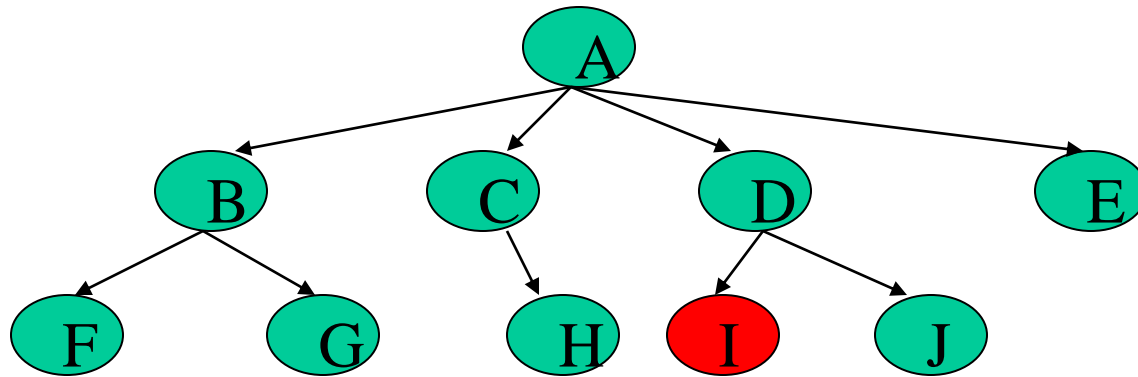
Limit =
2



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,H,
- D,I

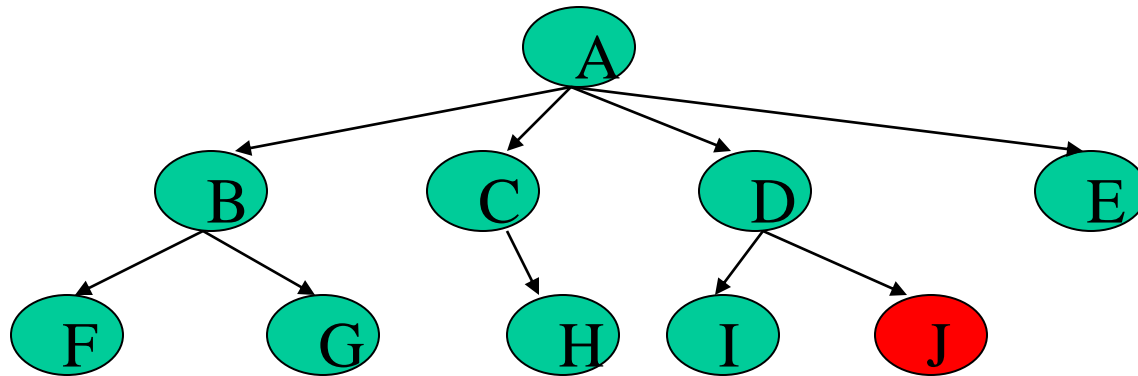
Limit =
2



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,H,
- D,I
- J,

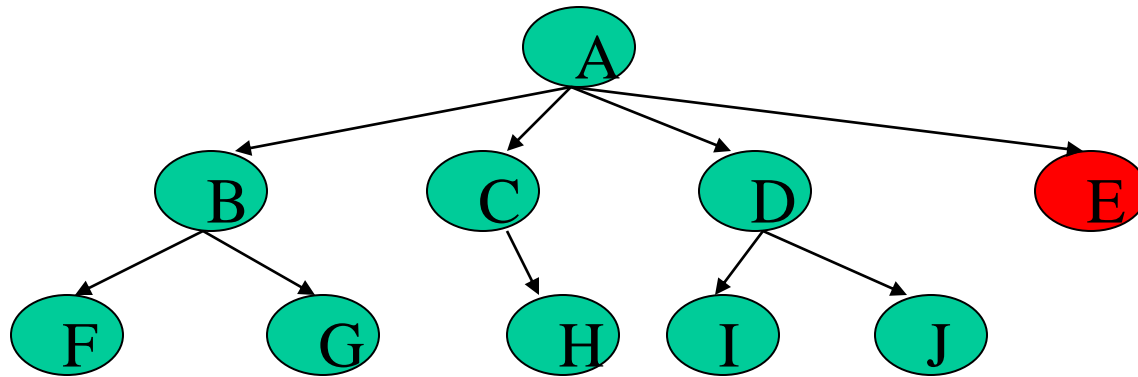
Limit =
2



Depth-Limited Search (DLS)

- A,B,F,
- G,
- C,H,
- D,I
- J,
- E

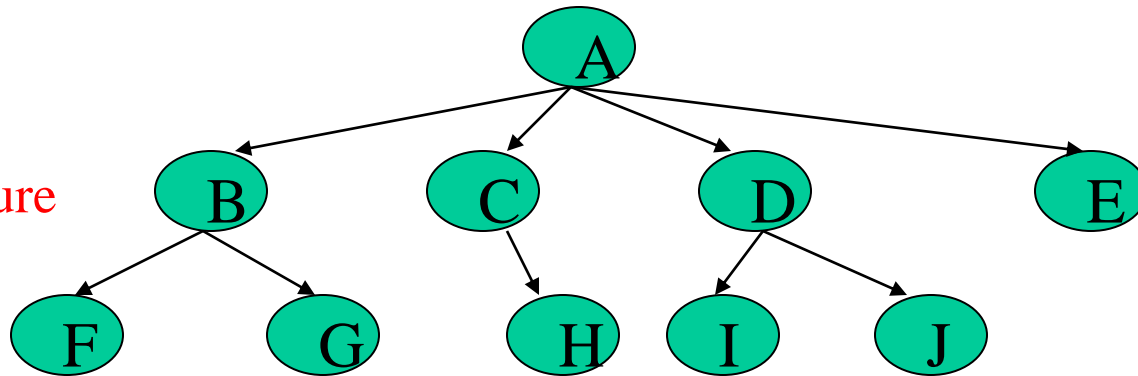
Limit =
2



Depth-Limited Search (DLS)

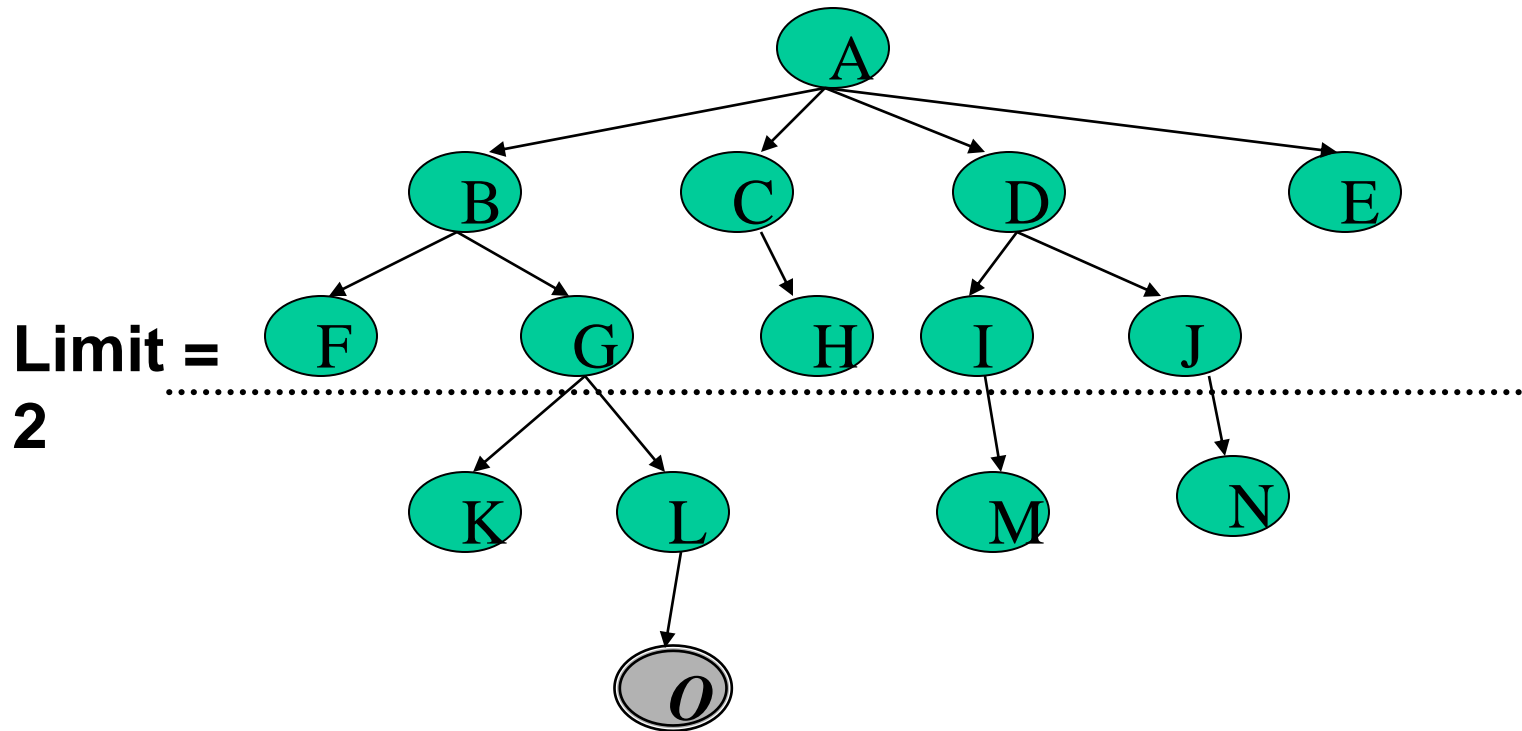
- A,B,F,
- G,
- C,H,
- D,I
- J,
- E, Failure

Limit =
2



Depth-Limited Search (DLS)

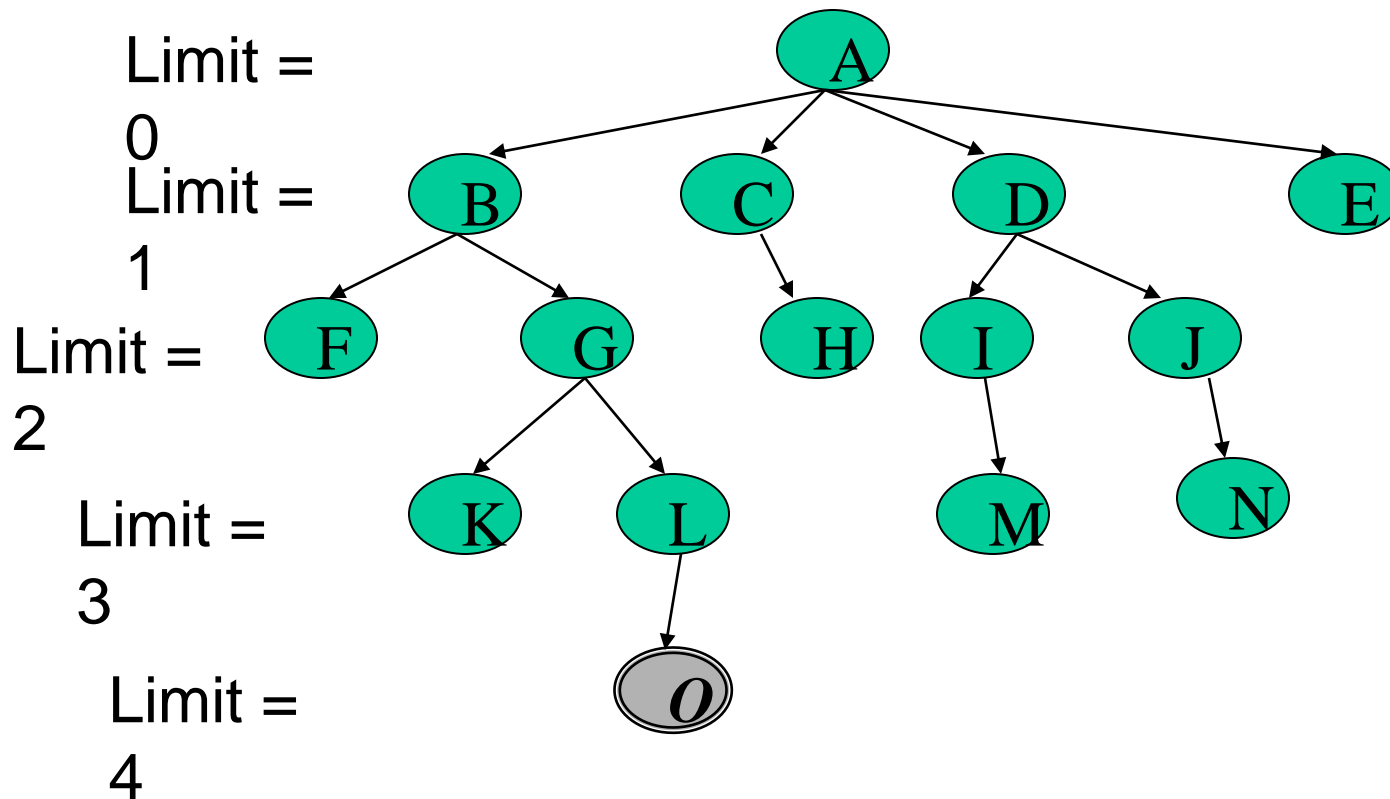
- DLS algorithm returns **Failure (no solution)**
- The reason is that the goal is beyond the limit (Limit = 2): the goal depth is (d=4)



Iterative Deepening Search (IDS)

- Application4:

Given the following state space (tree search), give the sequence of visited nodes when using IDS:



Iterative Deepening Search (IDS)

DLS with bound = 0

Iterative Deepening Search (IDS)

- A,

Limit =
0



Iterative Deepening Search (IDS)

- A, Failure

Limit =
0



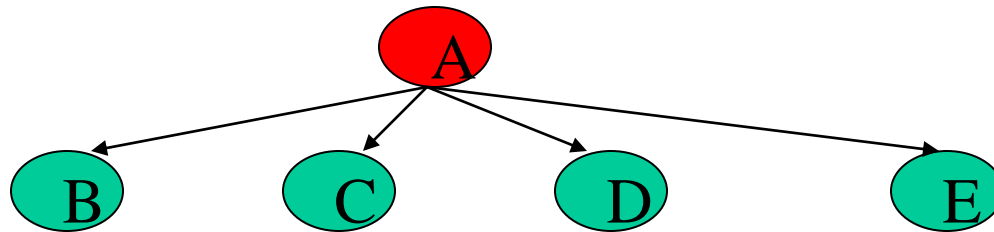
Iterative Deepening Search (IDS)

DLS with bound = 1

Iterative Deepening Search (IDS)

- A,

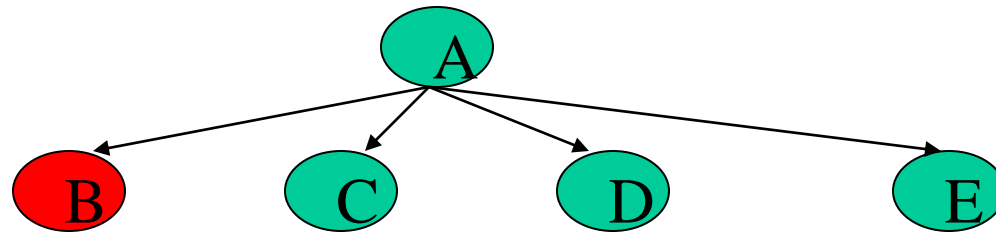
Limit =
1



Iterative Deepening Search (IDS)

- A,B,

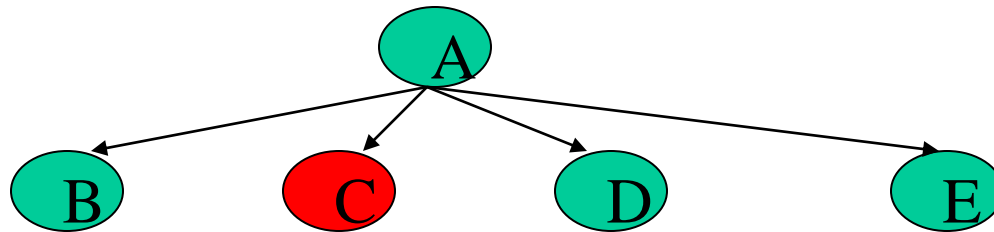
Limit =
1



Iterative Deepening Search (IDS)

- A,B,
- C,

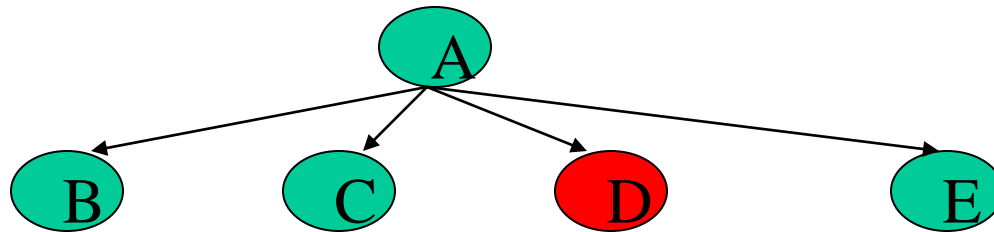
Limit =
1



Iterative Deepening Search (IDS)

- A,B,
- C,
- D,

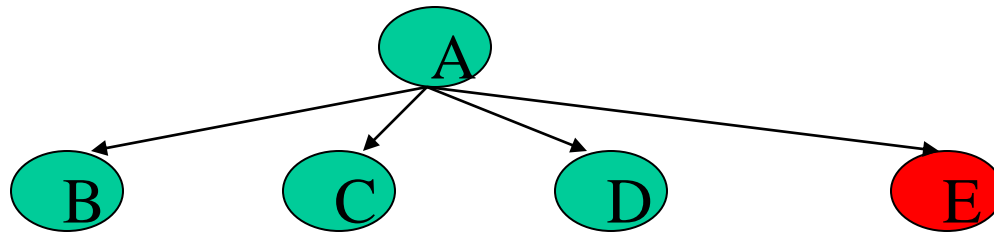
Limit =
1



Iterative Deepening Search (IDS)

- A,B
- C,
- D,
- E,

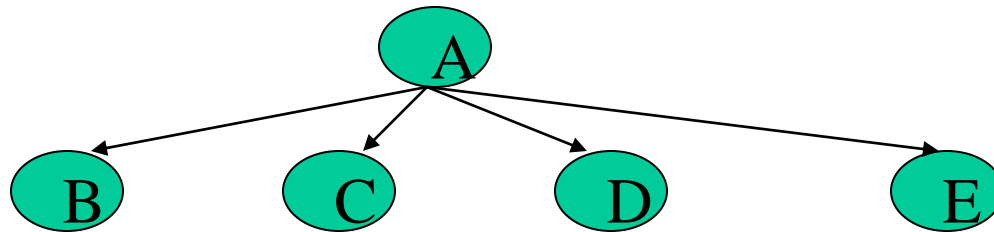
Limit =
1



Iterative Deepening Search (IDS)

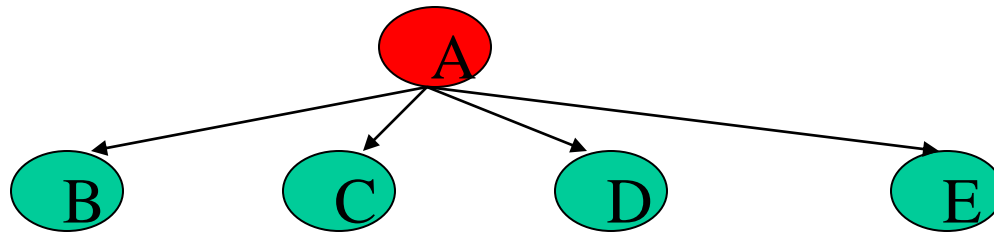
- A,B,
- C,
- D,
- E, **Failure**

Limit =
1



Iterative Deepening Search (IDS)

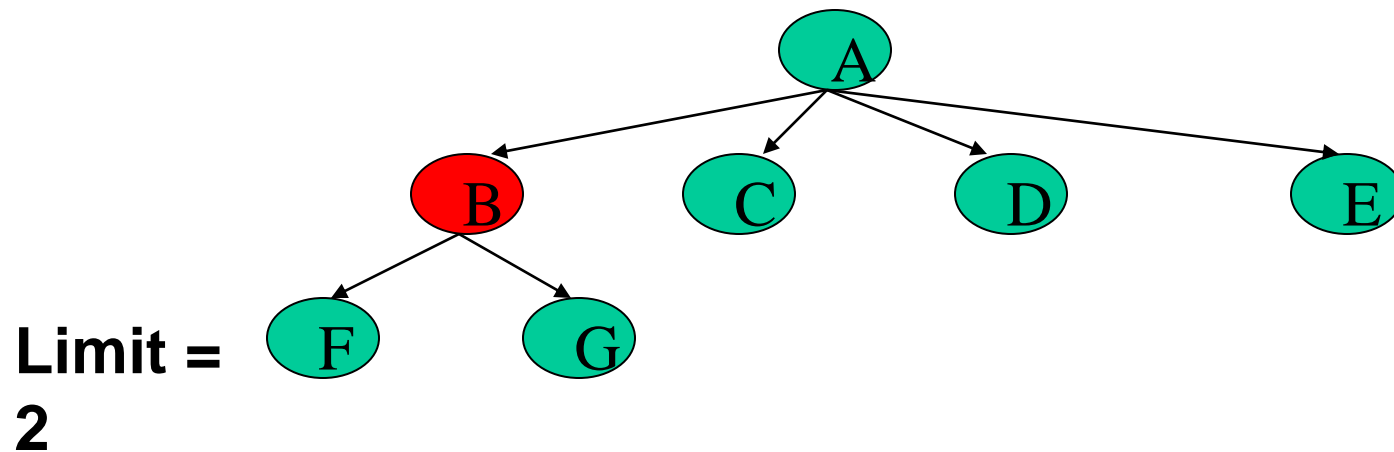
- A,



Limit =
2

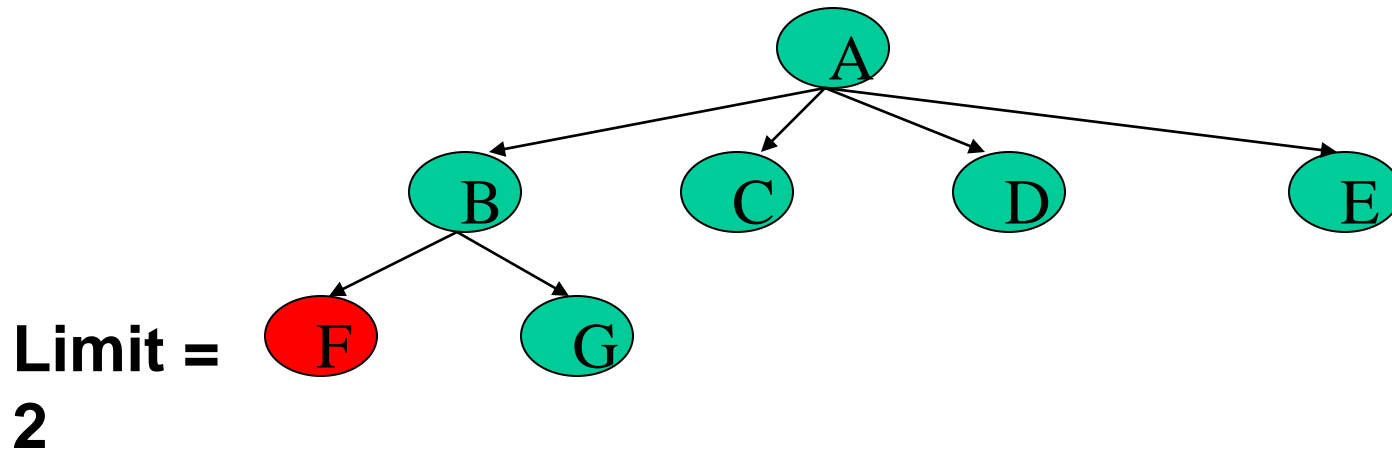
Iterative Deepening Search (IDS)

- A,B,



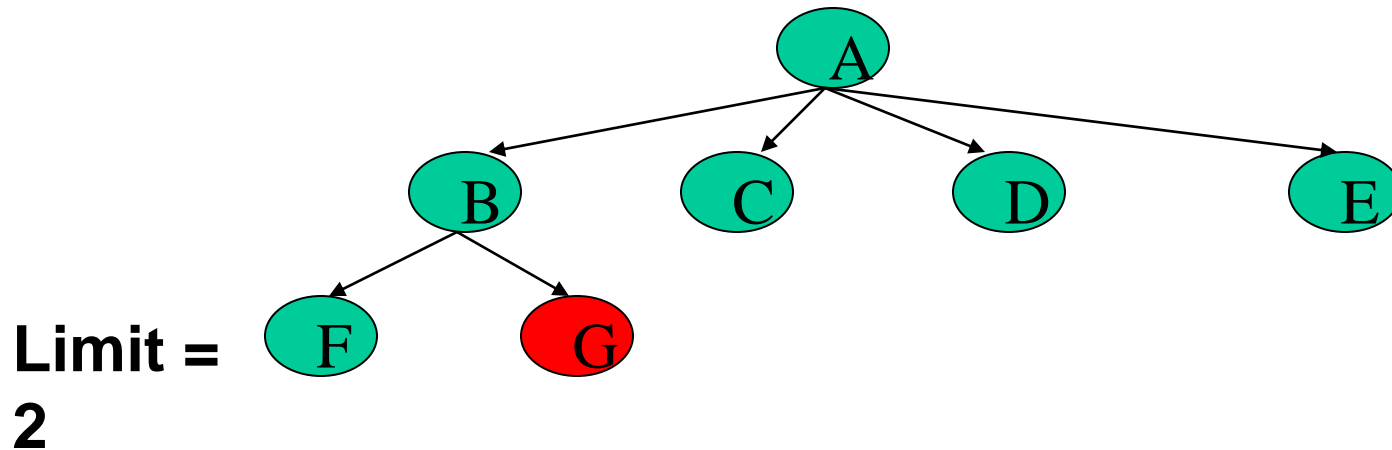
Iterative Deepening Search (IDS)

- A,B,F,



Iterative Deepening Search (IDS)

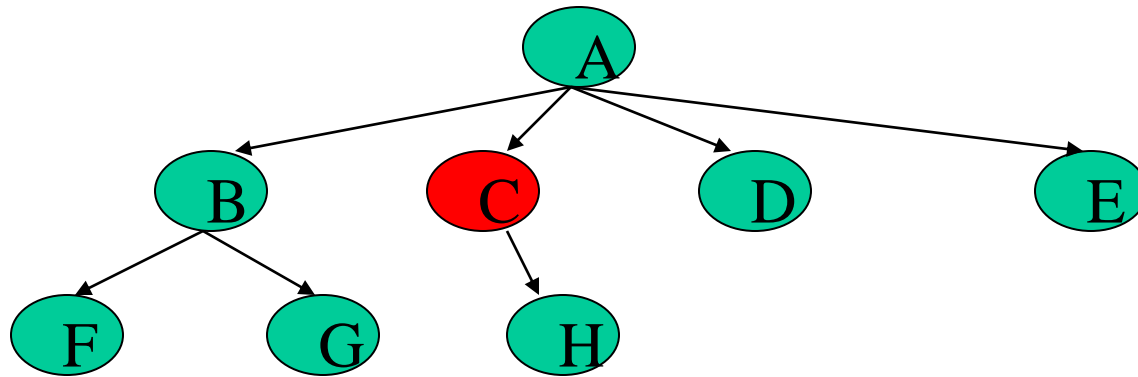
- A,B,F,
- G,



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,

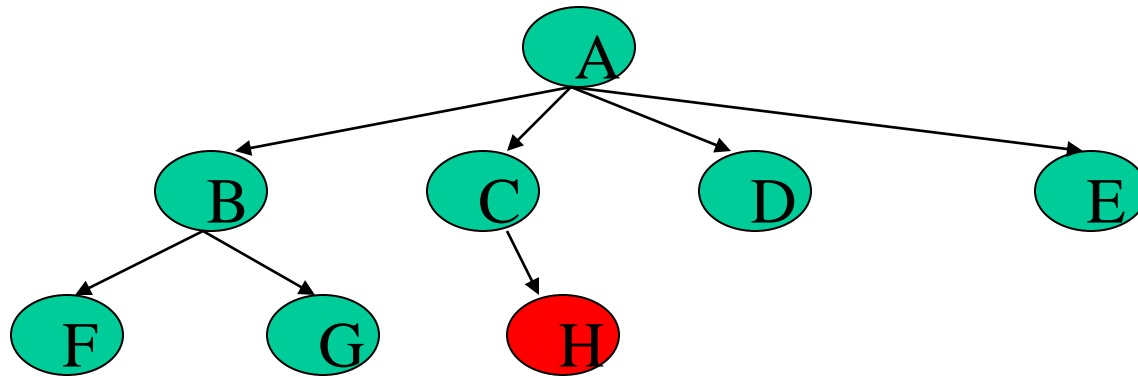
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,

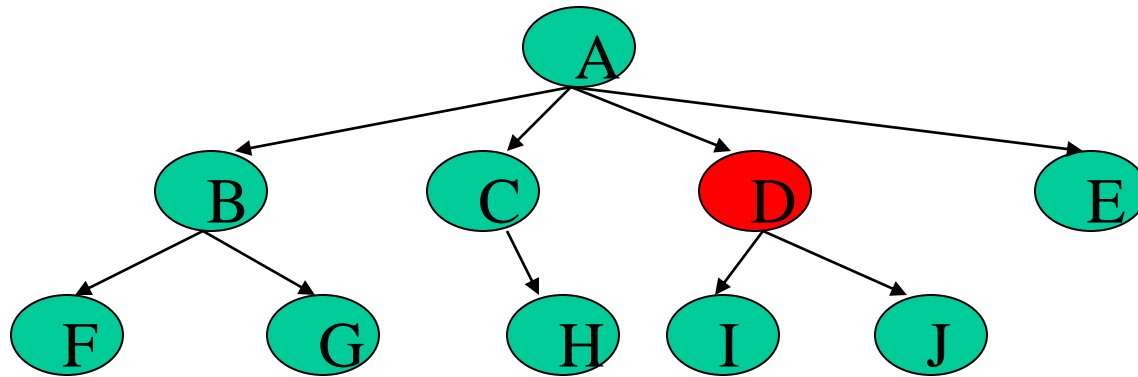
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,
- D,

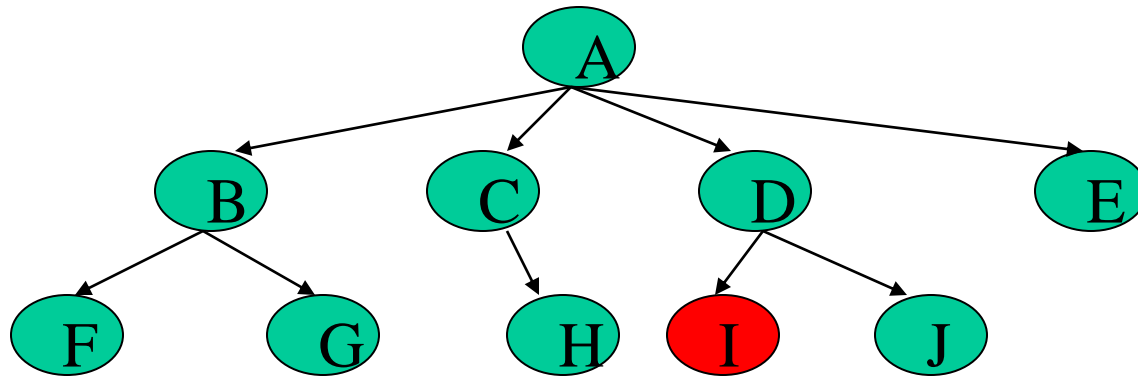
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,
- D,I

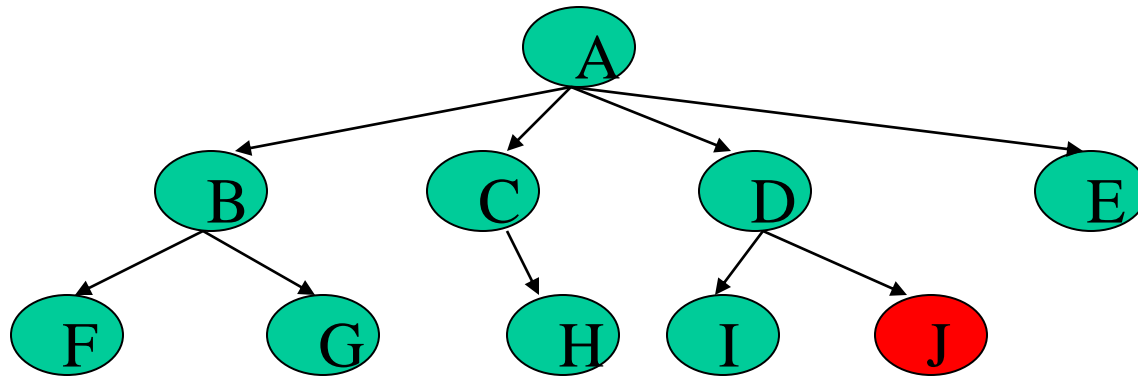
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,
- D,I
- J,

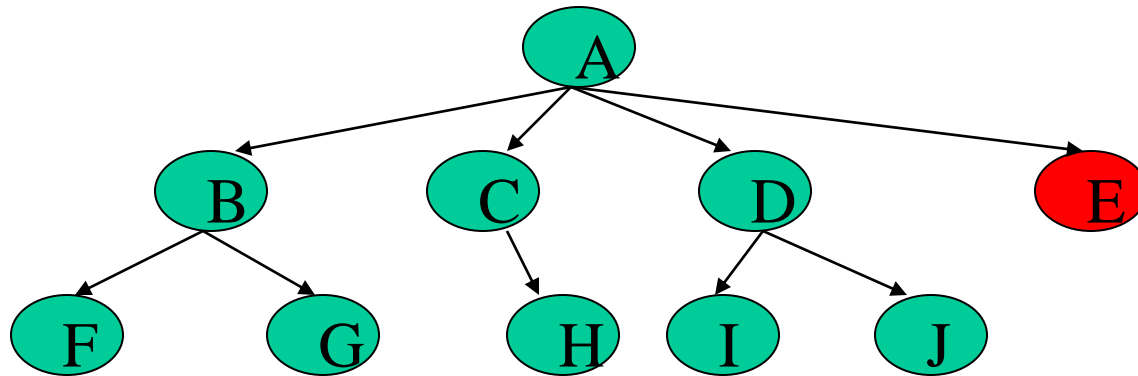
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,
- D,I
- J,
- E

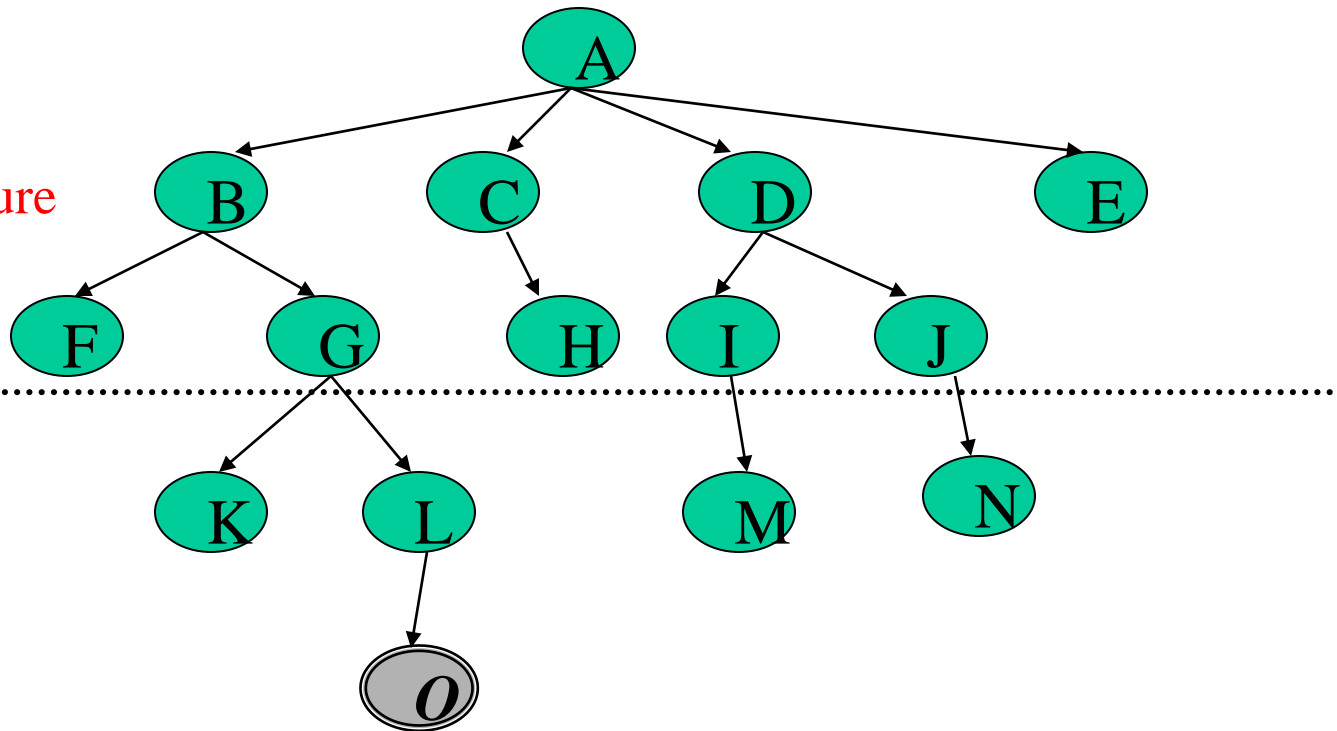
Limit =
2



Iterative Deepening Search (IDS)

- A,B,F,
- G,
- C,H,
- D,I
- J,
- E, Failure

Limit =
2

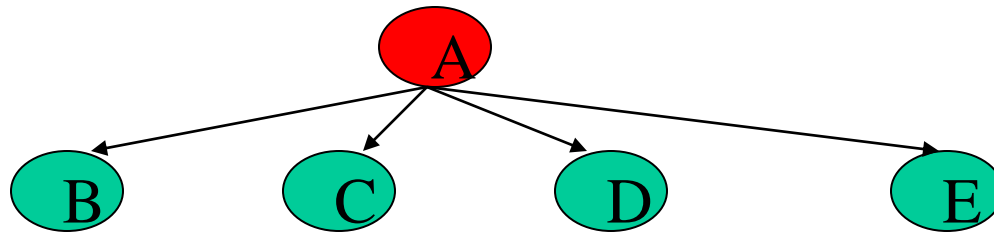


Iterative Deepening Search (IDS)

DLS with bound = 3

Iterative Deepening Search (IDS)

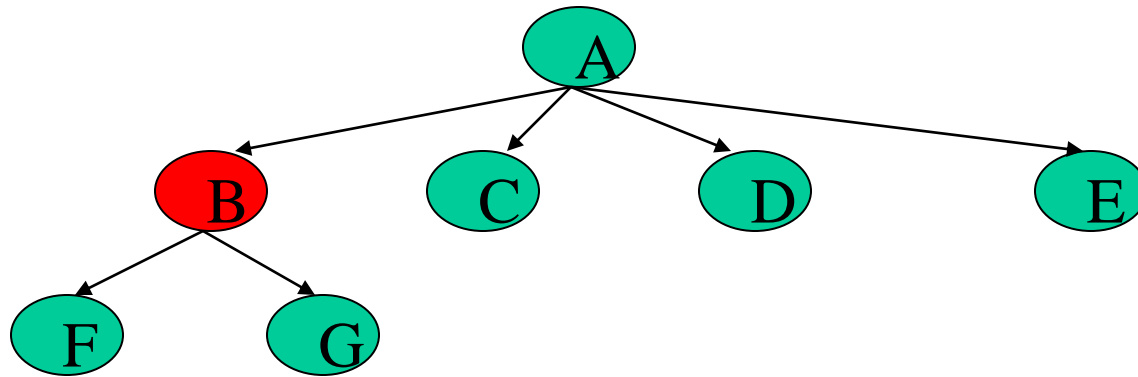
- A,



Limit =
3

Iterative Deepening Search (IDS)

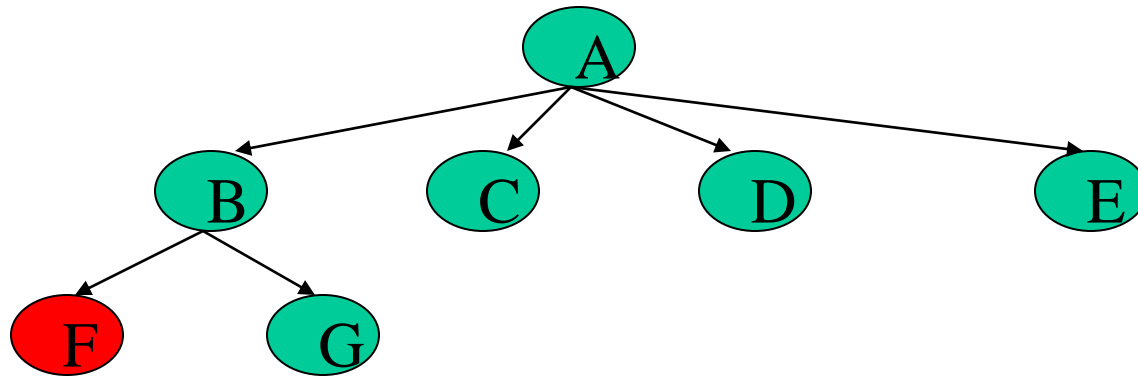
- A,B,



Limit =
3

Iterative Deepening Search (IDS)

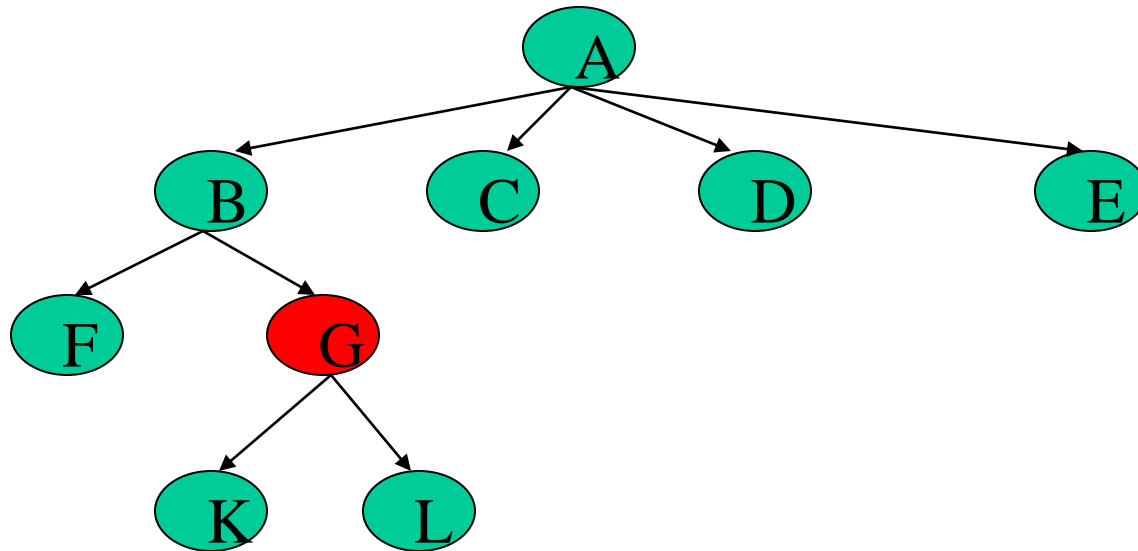
- A,B,F,



Limit =
3

Iterative Deepening Search (IDS)

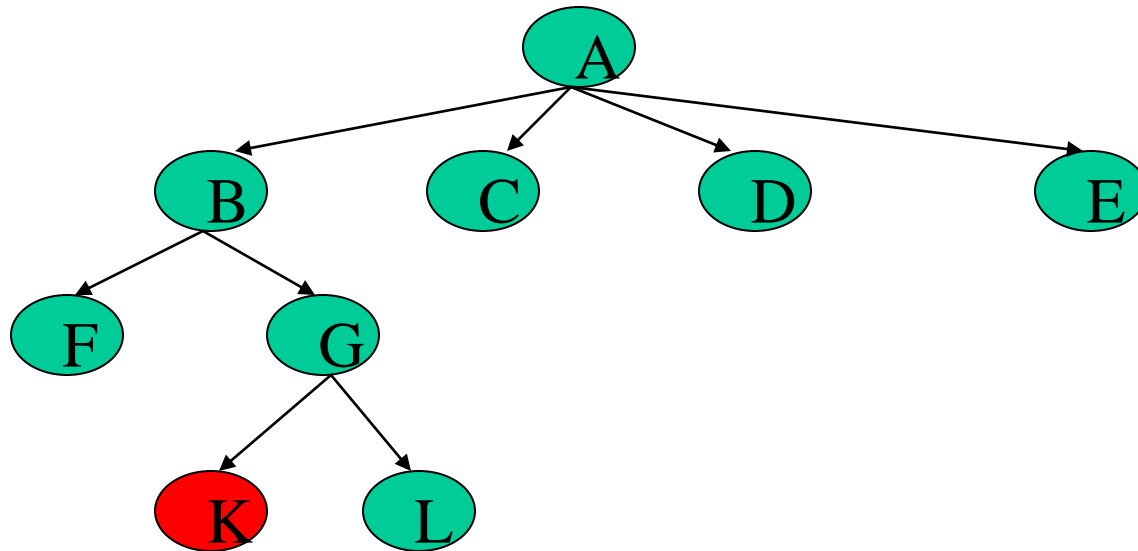
- A,B,F,
- G,



Limit =
3

Iterative Deepening Search (IDS)

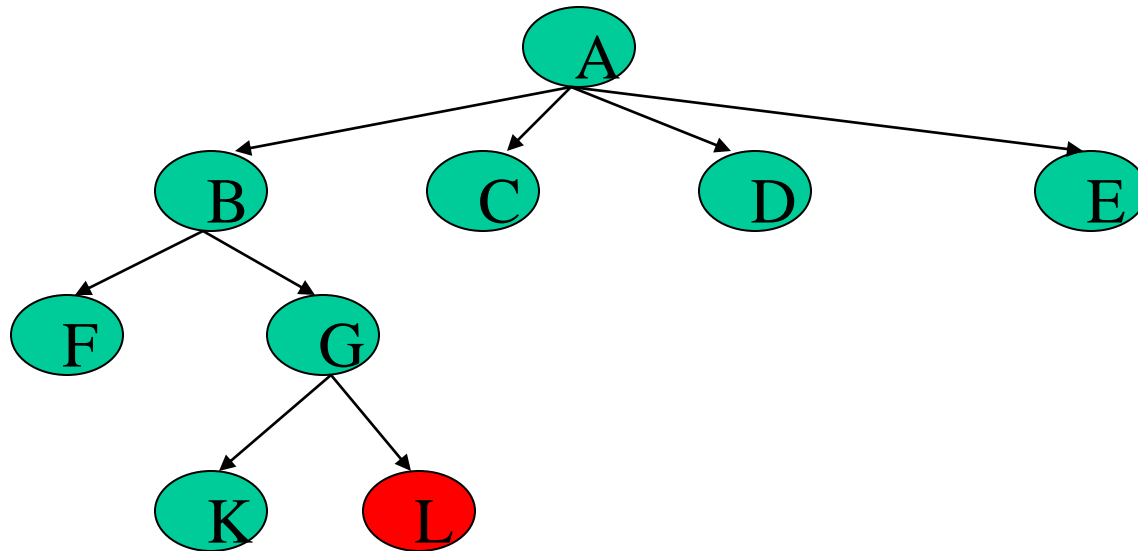
- A,B,F,
- G,K,



Limit =
3

Iterative Deepening Search (IDS)

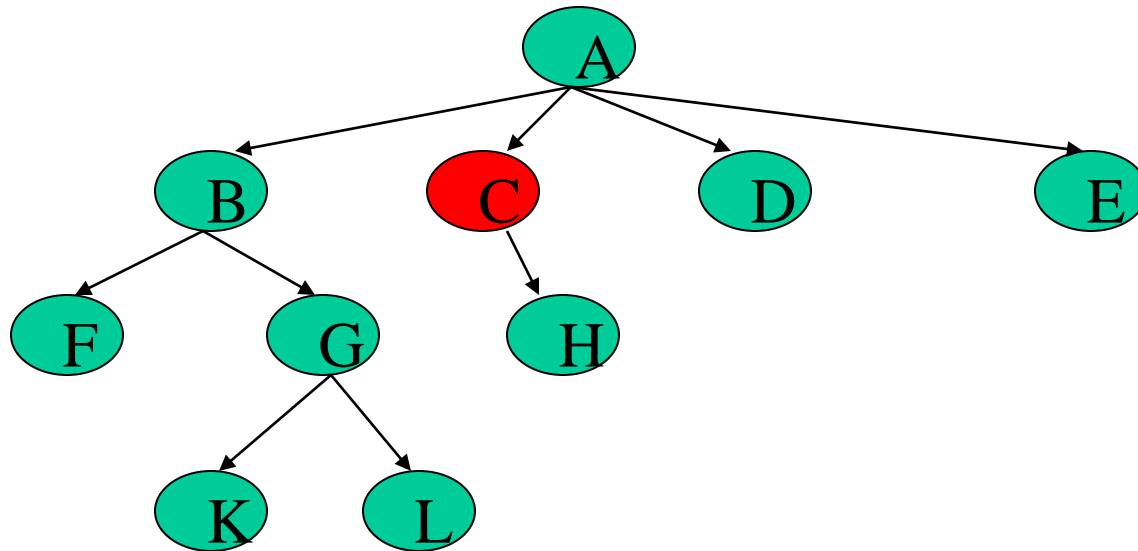
- A,B,F,
- G,K,
- L,



Limit =
3

Iterative Deepening Search (IDS)

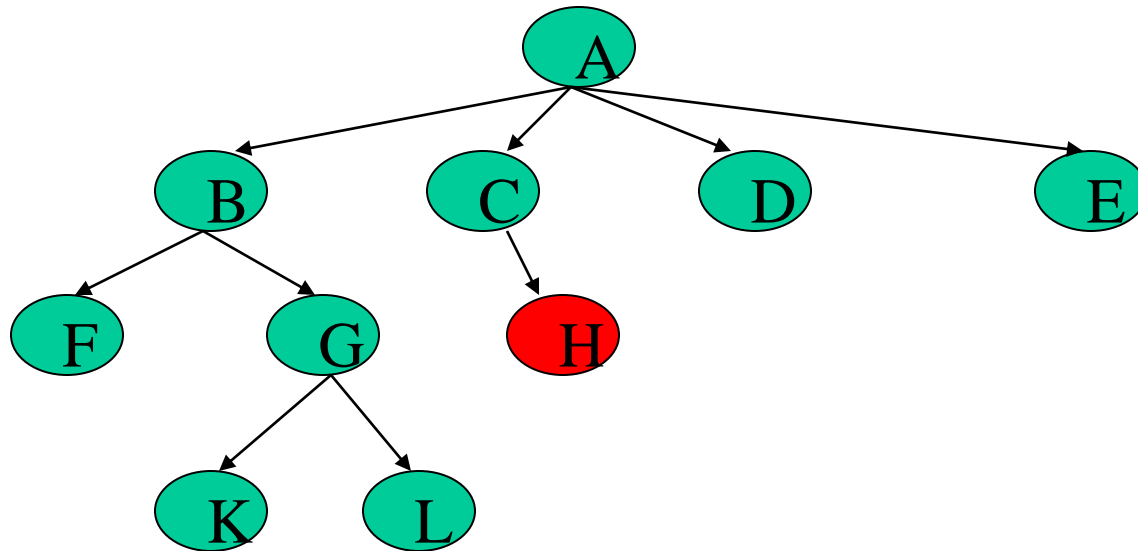
- A,B,F,
- G,K,
- L,
- C,



Limit =
3

Iterative Deepening Search (IDS)

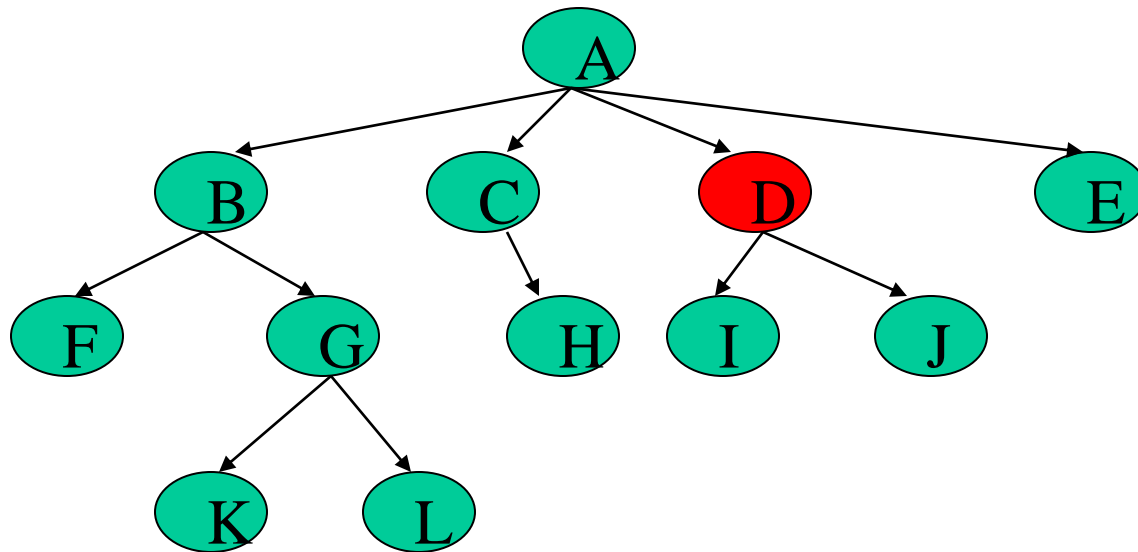
- A,B,F,
- G,K,
- L,
- C,H,



Limit =
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Iterative Deepening Search (IDS)

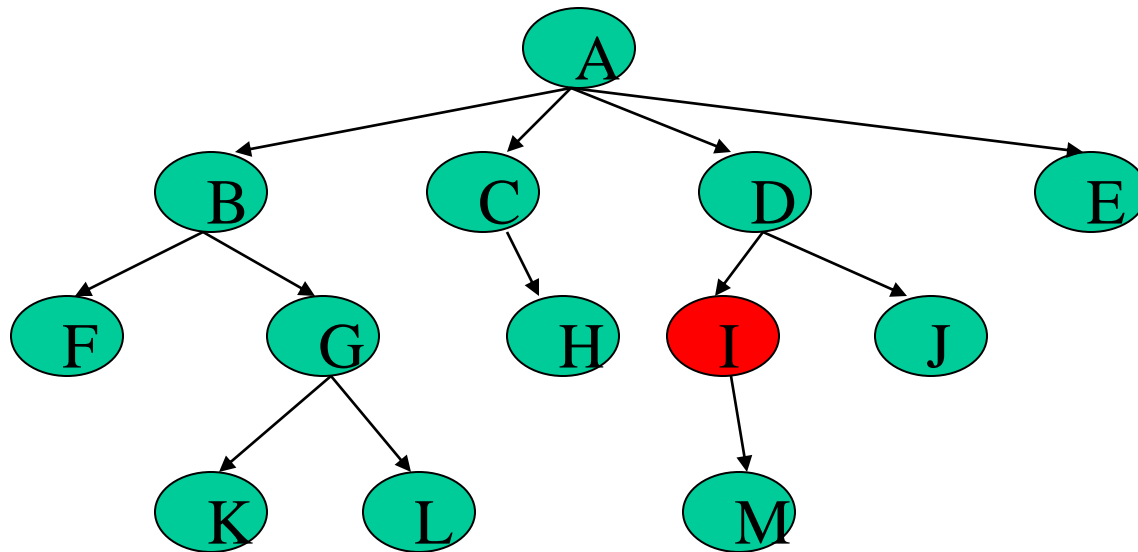
- A,B,F,
- G,K,
- L,
- C,H,
- D,



Limit =
3

Iterative Deepening Search (IDS)

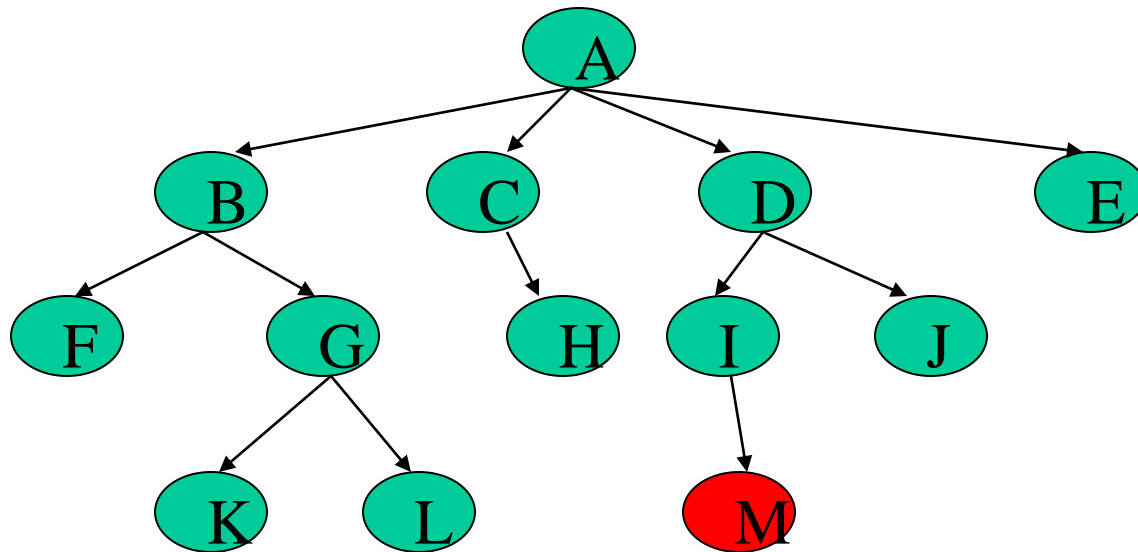
- A,B,F,
- G,K,
- L,
- C,H,
- D,I,



Limit =
3

Iterative Deepening Search (IDS)

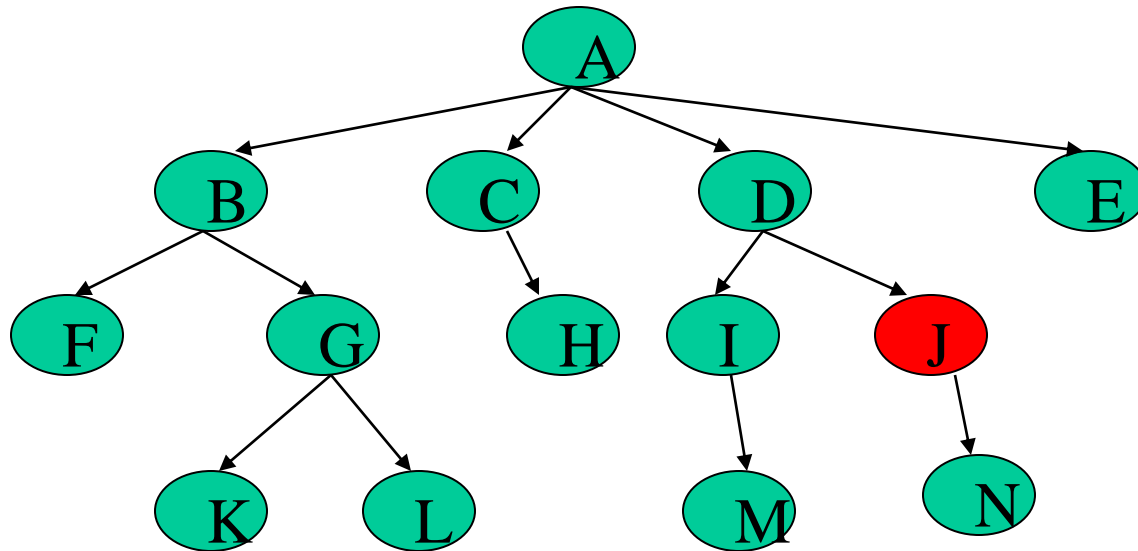
- A,B,F,
- G,K,
- L,
- C,H,
- D,I,M,



Limit =
3

Iterative Deepening Search (IDS)

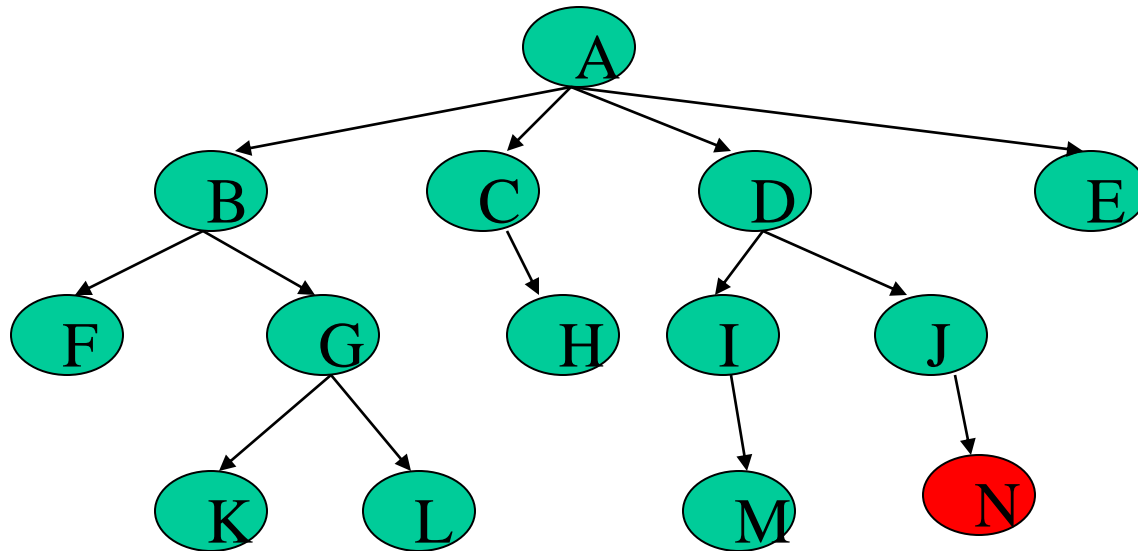
- A,B,F,
- G,K,
- L,
- C,H,
- D,I,M,
- J,



Limit =
3

Iterative Deepening Search (IDS)

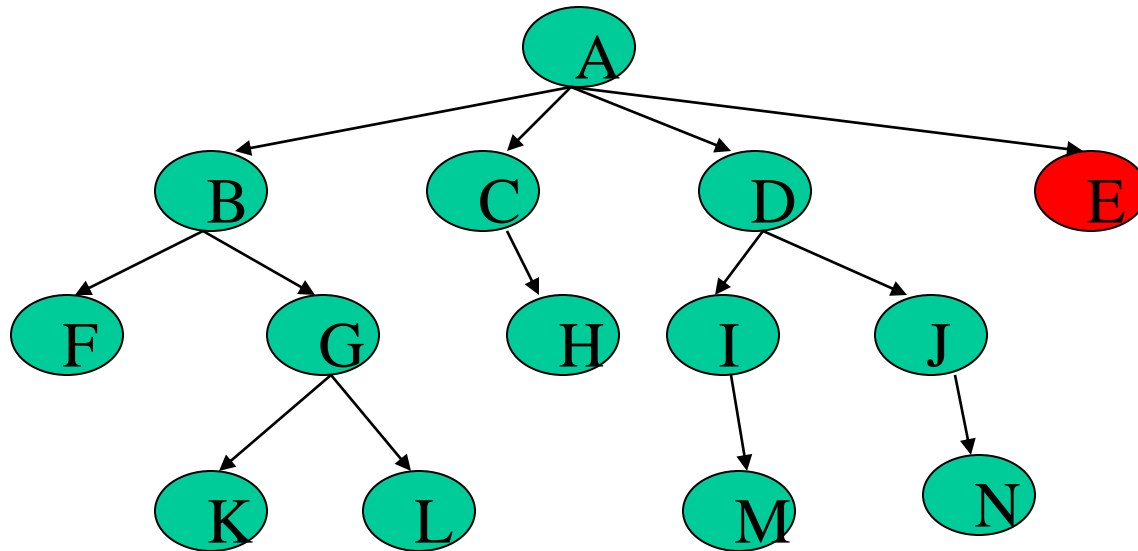
- A,B,F,
- G,K,
- L,
- C,H,
- D,I,M,
- J,N,



Limit =
3

Iterative Deepening Search (IDS)

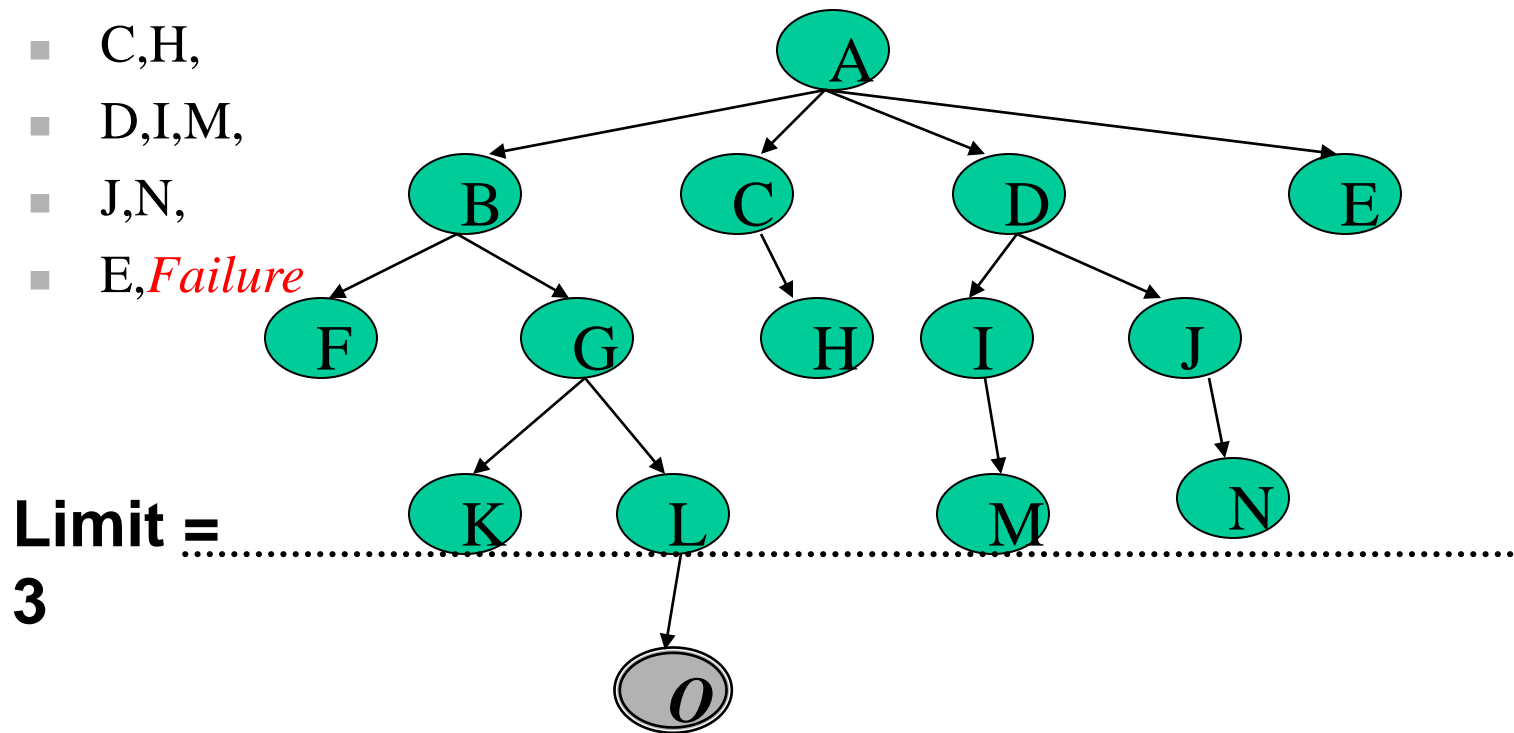
- A,B,F,
- G,K,
- L,
- C,H,
- D,I,M,
- J,N,
- E,



Limit =
3

Iterative Deepening Search (IDS)

- A,B,F,
- G,K,
- L,
- C,H,
- D,I,M,
- J,N,
- E,*Failure*

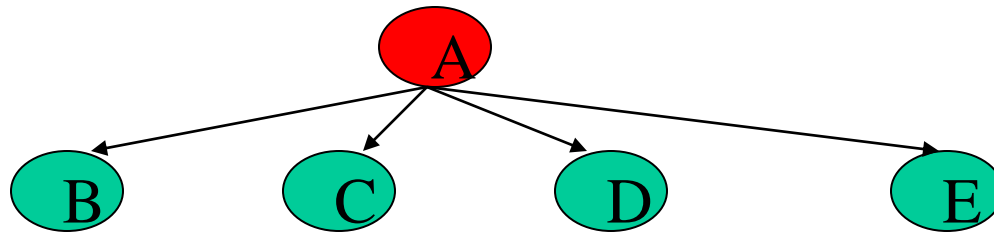


Iterative Deepening Search (IDS)

DLS with bound = 4

Iterative Deepening Search (IDS)

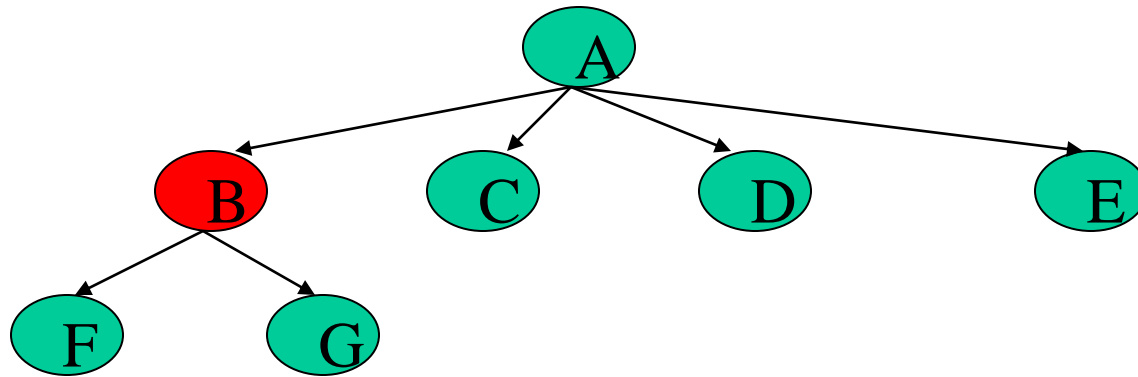
- A,



Limit =
4

Iterative Deepening Search (IDS)

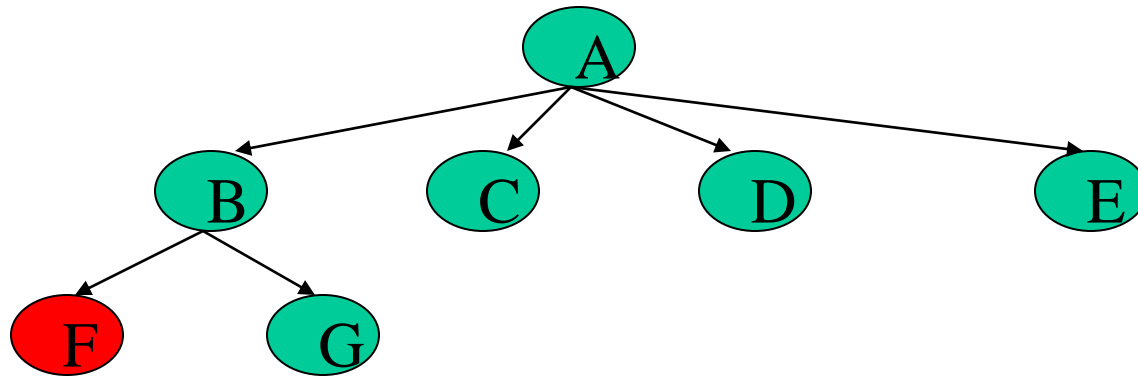
- A,B,



Limit =
4

Iterative Deepening Search (IDS)

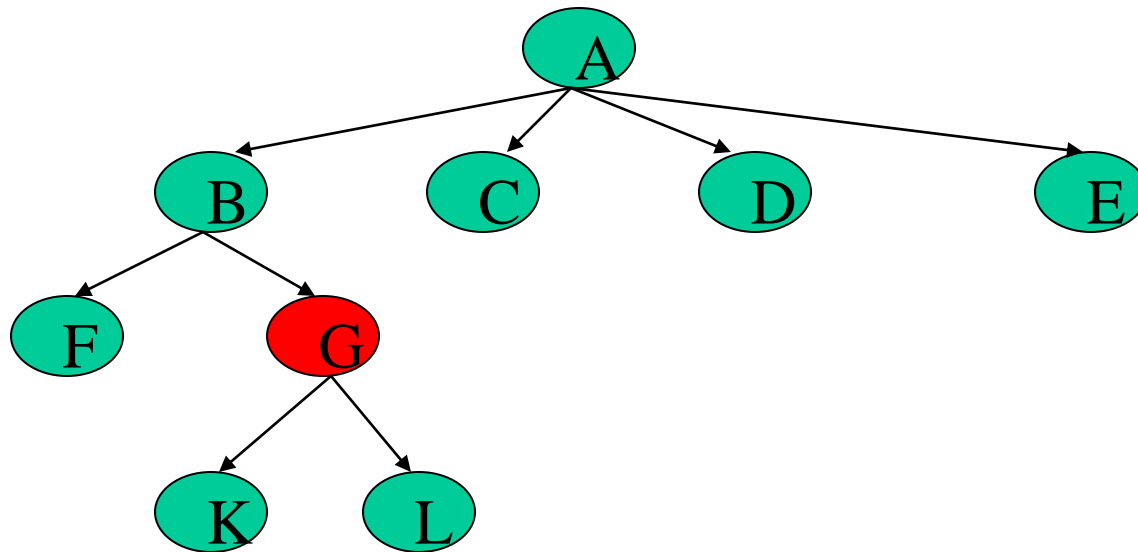
- A,B,F,



Limit =
4

Iterative Deepening Search (IDS)

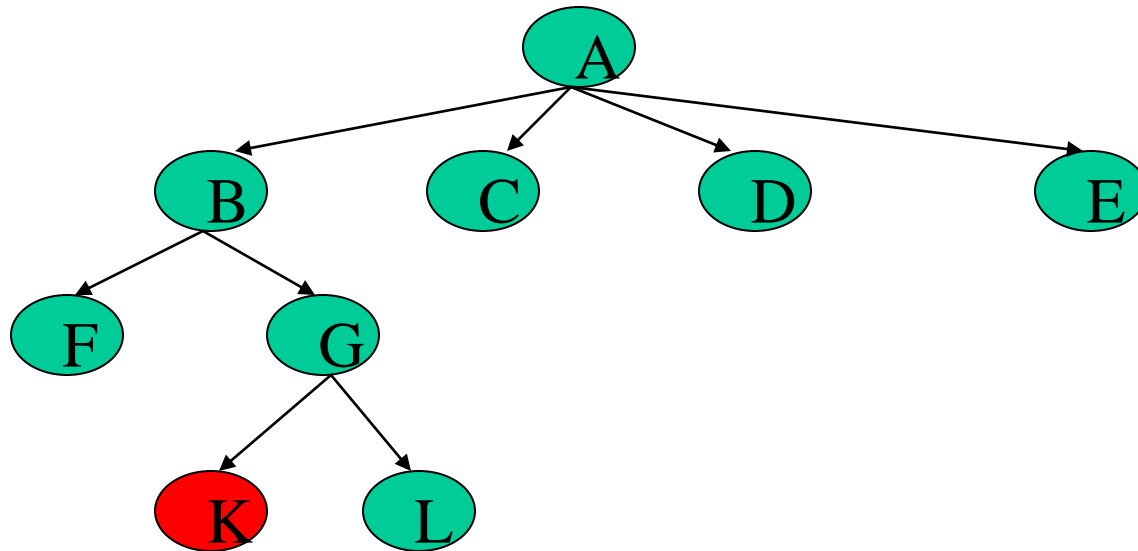
- A,B,F,
- G,



Limit =
4

Iterative Deepening Search (IDS)

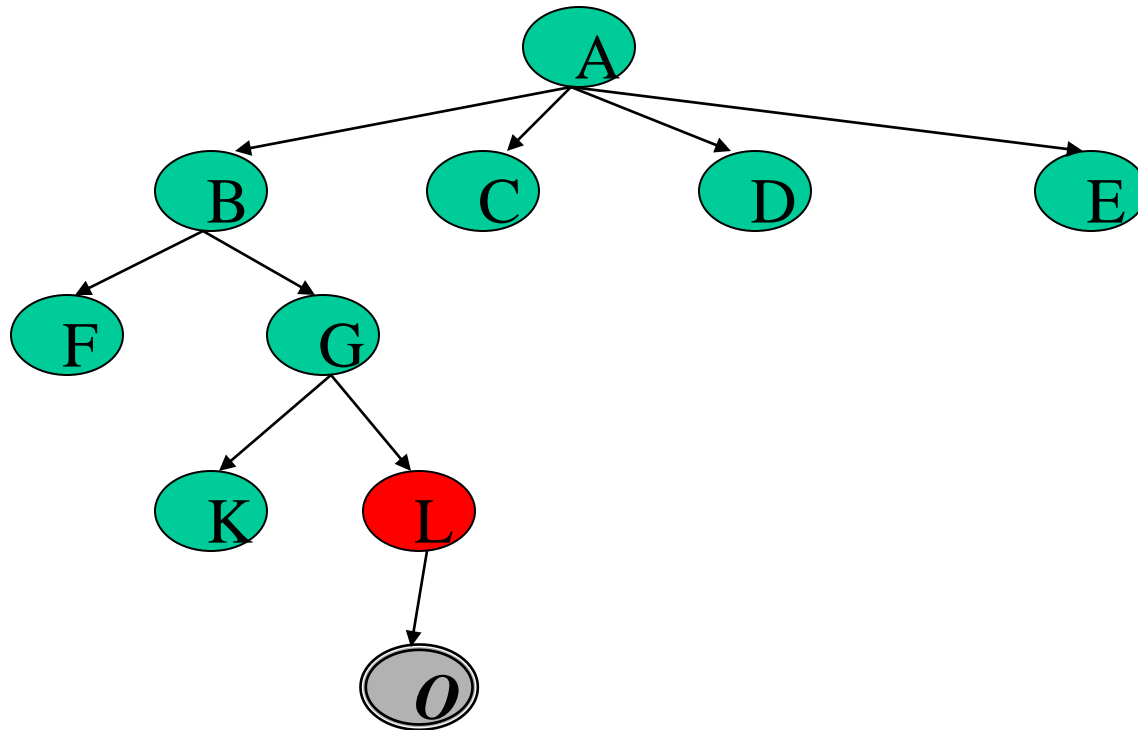
- A,B,F,
- G,K,



Limit =
4

Iterative Deepening Search (IDS)

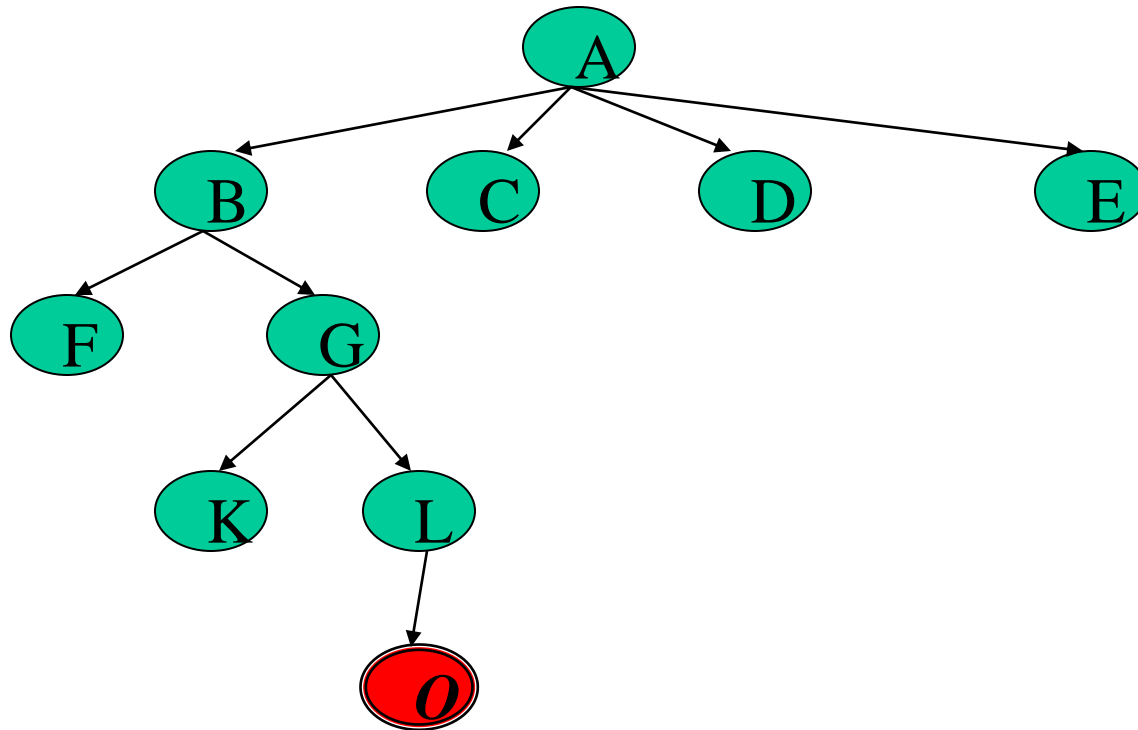
- A,B,F,
- G,K,
- L,



Limit =
4

Iterative Deepening Search (IDS)

- A,B,F,
- G,K,
- L, O: *Goal State*

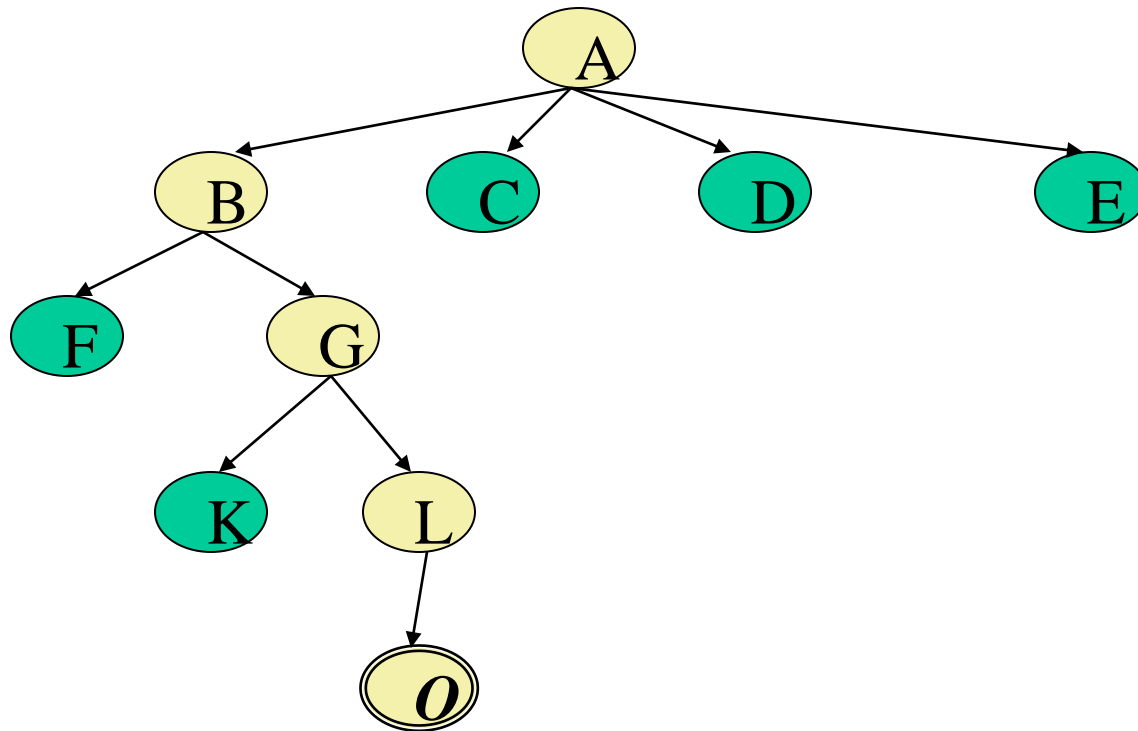


Limit =
4

Iterative Deepening Search (IDS)

The returned solution is the sequence of operators in the path:

A, B, G, L, O



Uniform Cost Search (UCS)

Main idea: **Uniform-cost Search:** Expand node with smallest path cost $g(n)$.

- **Implementation:**

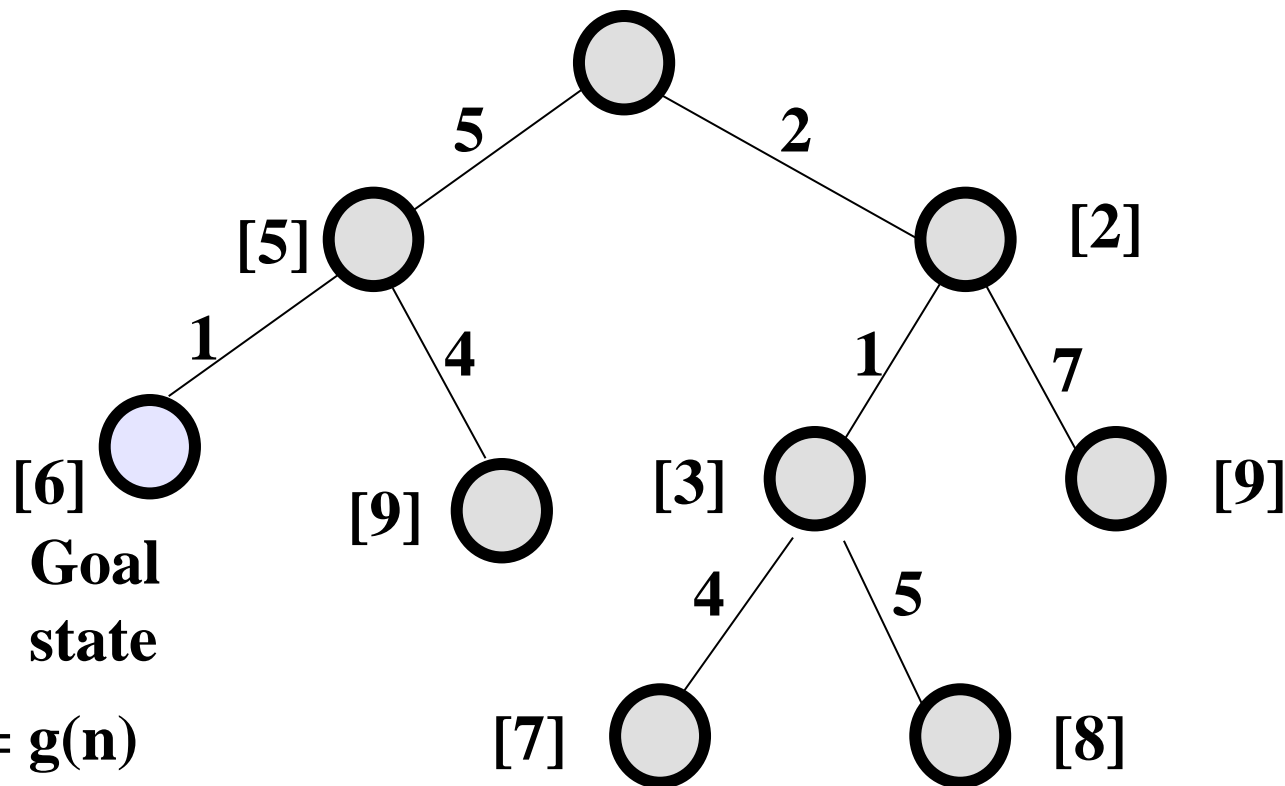
Enqueue nodes in order of cost $g(n)$.

QUEUING-FN:- insert in order of increasing path cost.

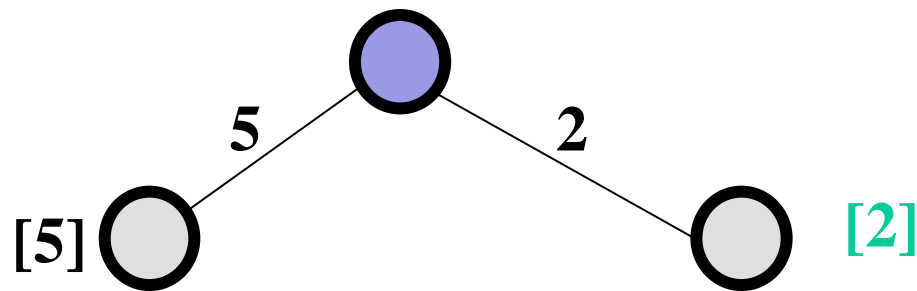
Enqueue new node at the appropriate position in the queue so that we dequeue the cheapest node.

- Complete? Yes.
- Optimal? Yes, if path cost is nondecreasing function of depth
- Time Complexity: $O(b^d)$
- Space Complexity: $O(b^d)$, note that every node in the fringe keep in the queue.

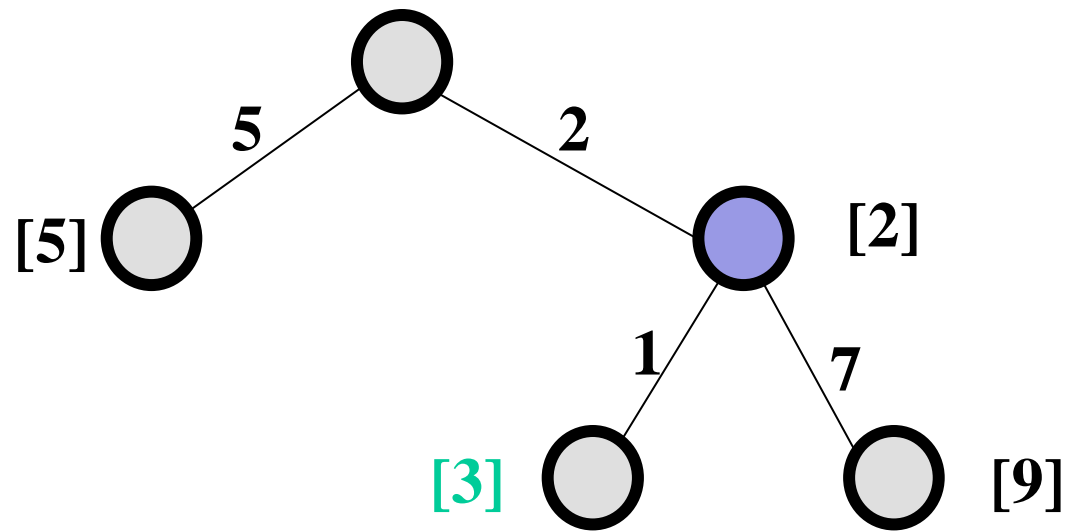
Uniform Cost Search (UCS)



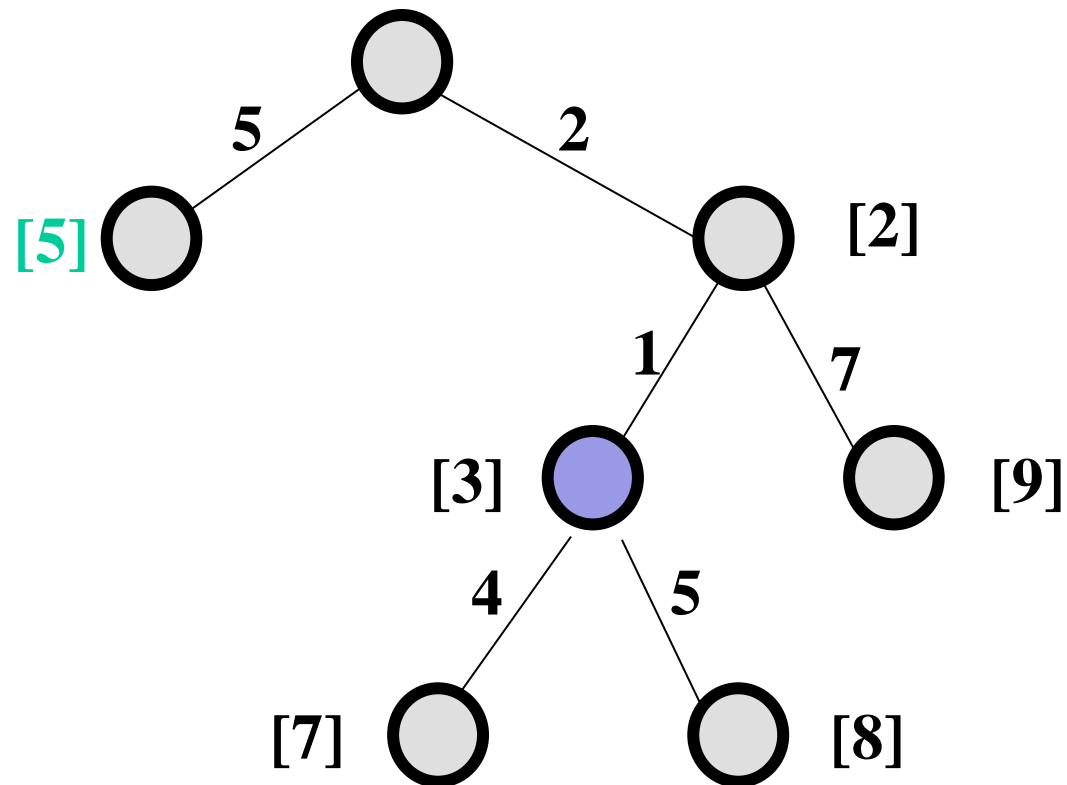
Uniform Cost Search (UCS)



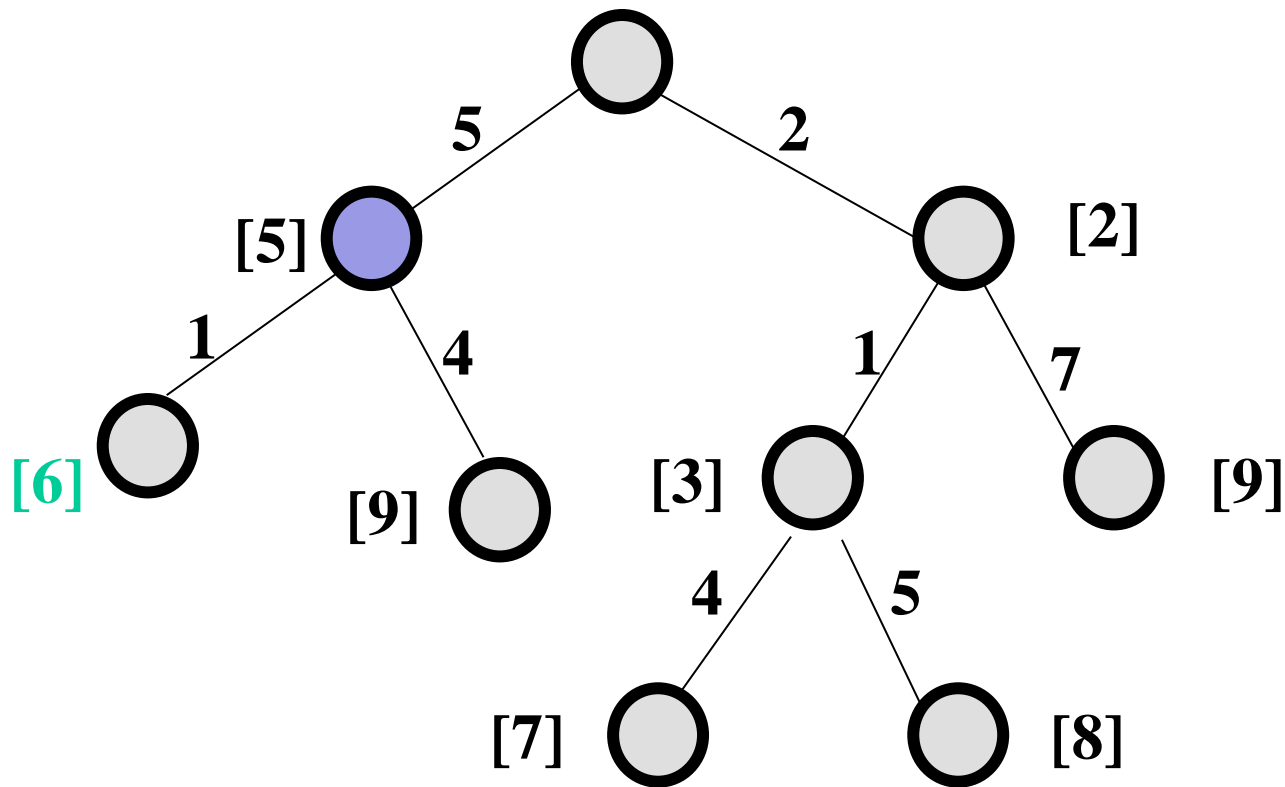
Uniform Cost Search (UCS)



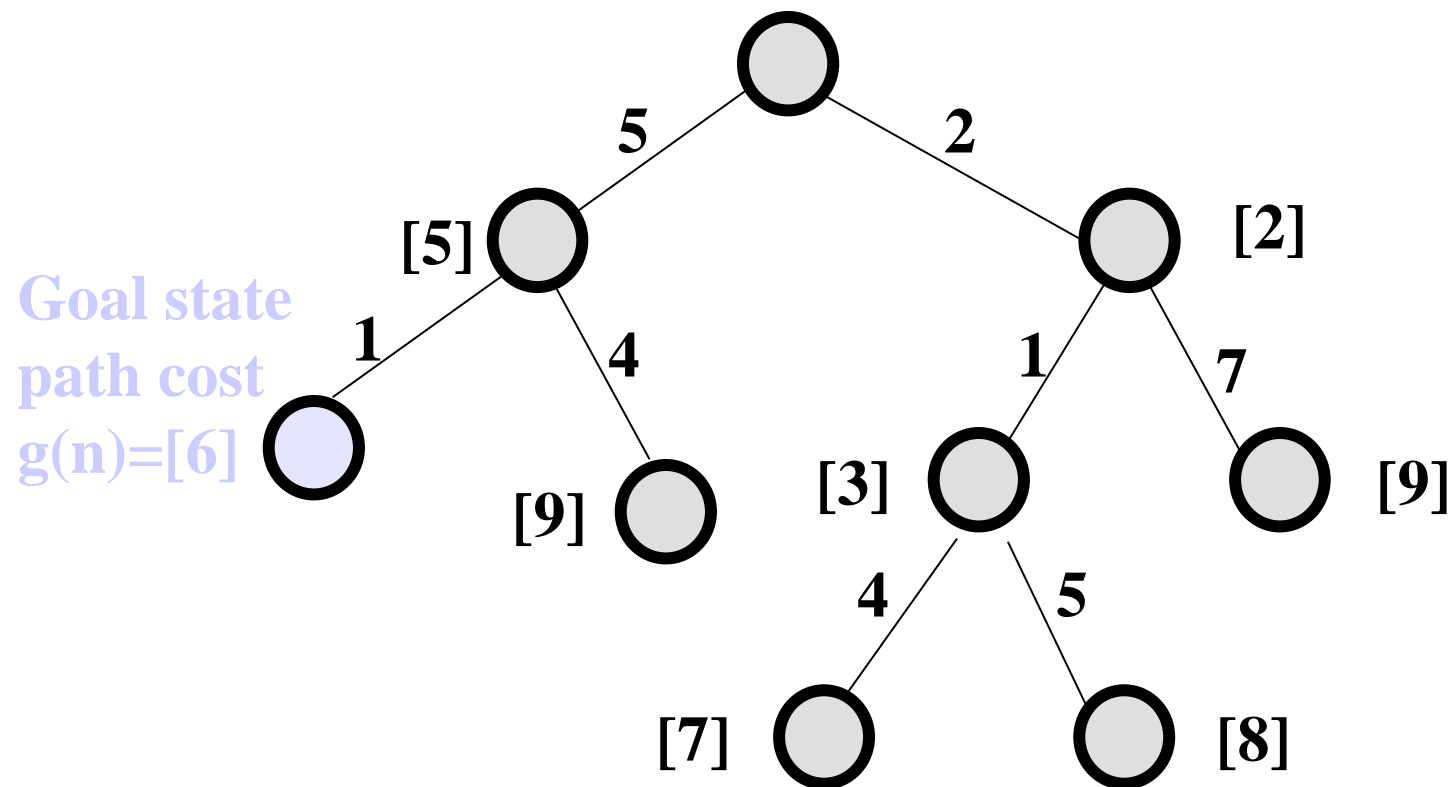
Uniform Cost Search (UCS)



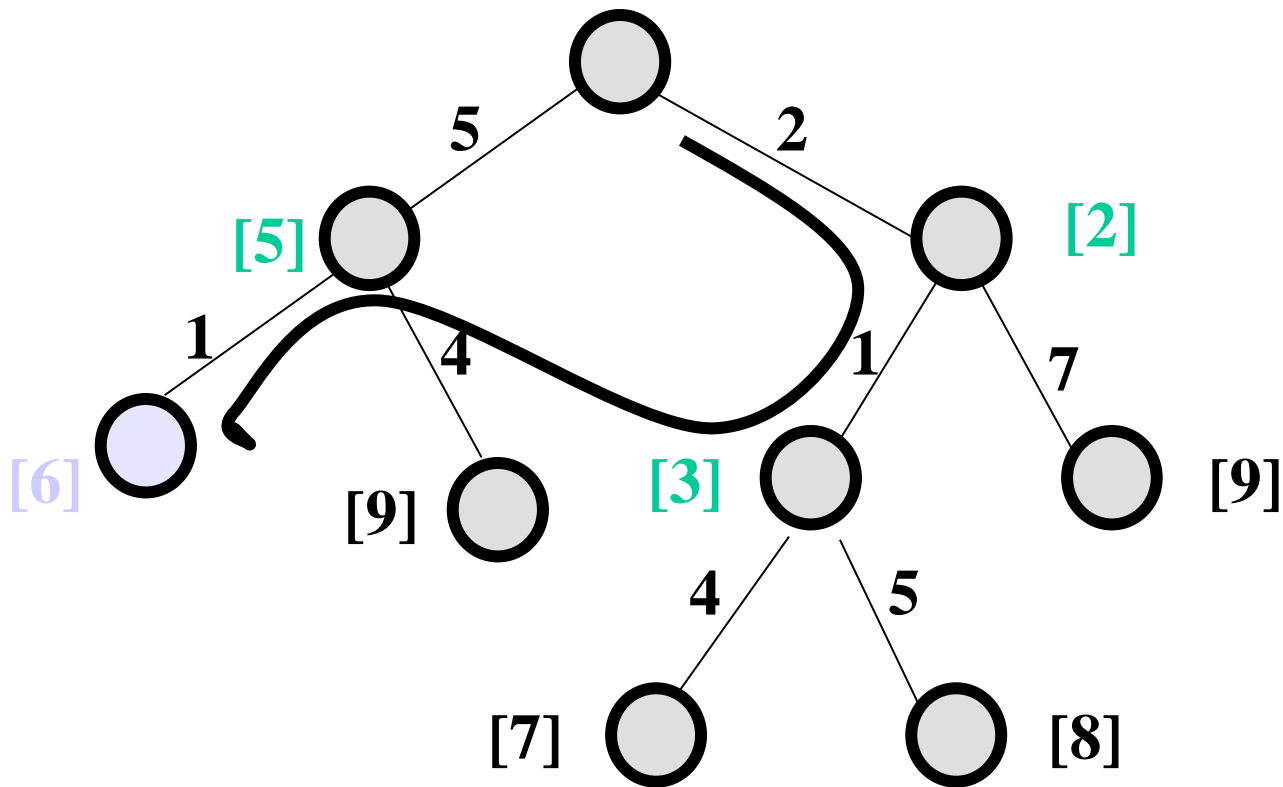
Uniform Cost Search (UCS)



Uniform Cost Search (UCS)



Uniform Cost Search (UCS)



Uniform-cost search

Breadth-first is only optimal if step costs is increasing with depth (e.g. constant). Can we guarantee optimality for any step cost?

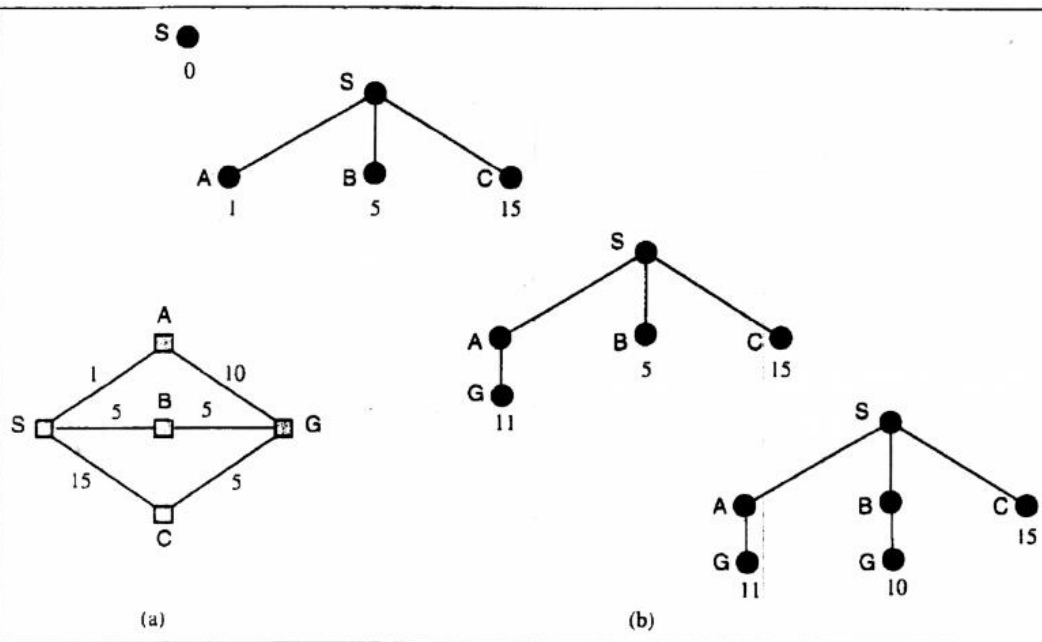
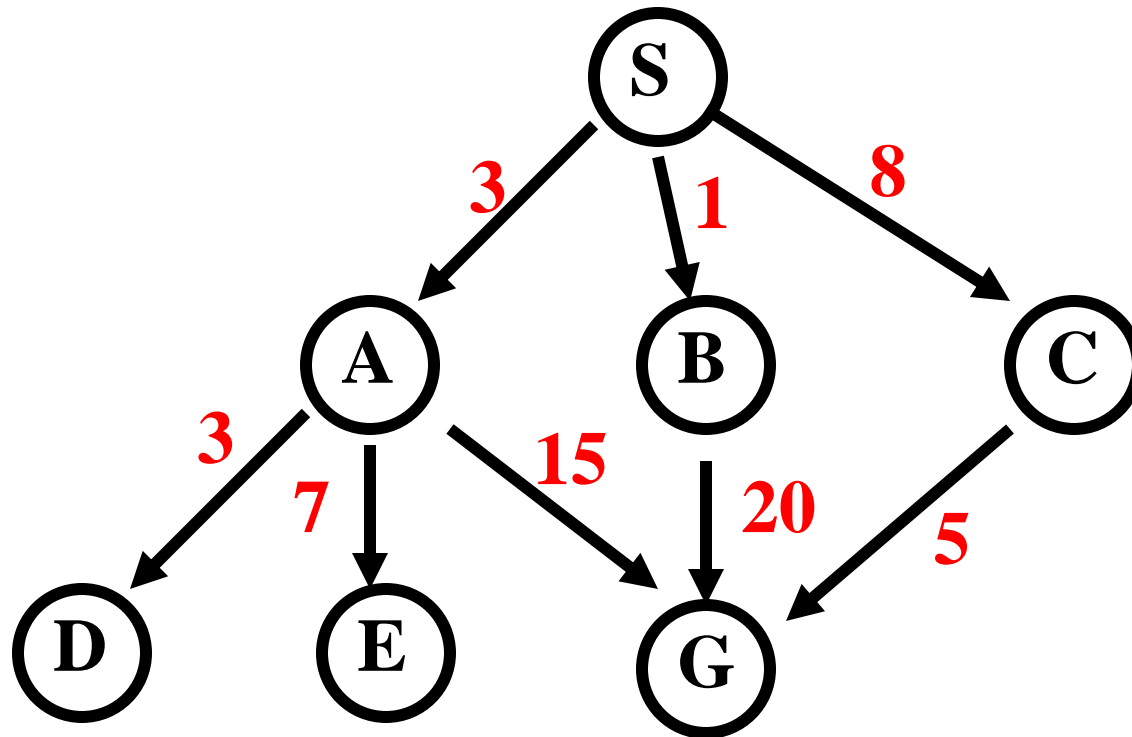


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Example for Illustrating Search Strategies



Depth-First Search

Expanded node

Nodes list

S^0

$\{ S^0 \}$

A^3

$\{ A^3 B^1 C^8 \}$

D^6

$\{ D^6 E^{10} G^{18} B^1 C^8 \}$

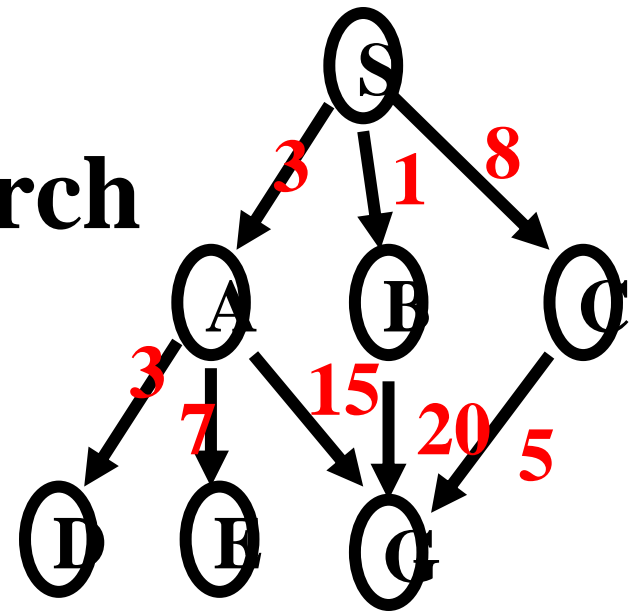
E^{10}

$\{ E^{10} G^{18} B^1 C^8 \}$

G^{18}

$\{ G^{18} B^1 C^8 \}$

$\{ B^1 C^8 \}$



Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

Breadth-First Search

Expanded node

Nodes list

S^0

$\{ S^0 \}$

A^3

$\{ A^3 B^1 C^8 \}$

B^1

$\{ B^1 C^8 D^6 E^{10} G^{18} \}$

C^8

$\{ C^8 D^6 E^{10} G^{18} G^{21} \}$

D^6

$\{ D^6 E^{10} G^{18} G^{21} G^{13} \}$

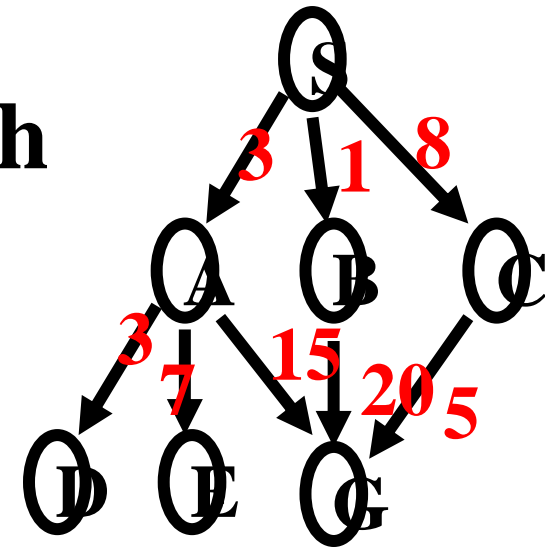
E^{10}

$\{ E^{10} G^{18} G^{21} G^{13} \}$

G^{18}

$\{ G^{18} G^{21} G^{13} \}$

$\{ G^{21} G^{13} \}$



Solution path found is S A G , cost 18

Number of nodes expanded (including goal node) = 7

Uniform-Cost Search

Expanded node

Nodes list

S^0

$\{ S^0 \}$
 $\{ B^1 A^3 C^8 \}$

B^1

$\{ A^3 C^8 G^{21} \}$

A^3

$\{ D^6 C^8 E^{10} G^{18} \}$

D^6

$\{ C^8 E^{10} G^{18} \}$

C^8

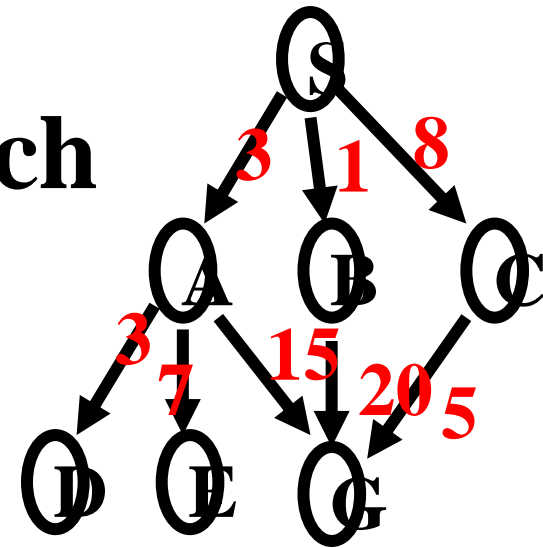
$\{ E^{10} G^{13} \}$

E^{10}

$\{ G^{13} \}$

G^{13}

$\{ \}$



Solution path found is S C G, cost 13

Number of nodes expanded (including goal node) = 7

Bidirectional Search

- Idea
 - simultaneously search forward from S and backwards from G
 - stop when both “meet in the middle”
 - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
 - need a way to specify the predecessors of G
 - this can be difficult,
 - e.g., predecessors of checkmate in chess?
 - what if there are multiple goal states?
 - what if there is only a goal test, no explicit list?

What Criteria are used to Compare different search techniques ?

As we are going to consider different techniques to search the problem space, we need to consider what criteria we will use to compare them.

- **Completeness:** Is the technique guaranteed to find an answer (if there is one).
- **Optimality/Admissibility :** does it always find a least-cost solution?
 - an admissible algorithm will find a solution with minimum cost
- **Time Complexity:** How long does it take to find a solution.
- **Space Complexity:** How much memory does it take to find a solution.

Time and Space Complexity ?

Time and space complexity are measured in terms of:

- The average number of new nodes we create when expanding a new node is the (effective) branching factor **b**.
- The (maximum) branching factor **b** is defined as the maximum nodes created when a new node is expanded.
- The length of a path to a goal is the depth **d**.
- The maximum length of any path in the state space **m**.

Properties of breadth-first search

- Complete? Yes it always reaches goal (if b is finite)
- Time? $1 + b + b^2 + b^3 + \dots + b^d + (b^{d+1} - b) = O(b^{d+1})$
(this is the number of nodes we generate)
- Space? $O(b^{d+1})$ (keeps every node in memory, either in fringe or on a path to fringe).
- Optimal? Yes (if we guarantee that deeper solutions are less optimal, e.g. step-cost=1).
- **Space** is the bigger problem (more than time)

Properties of depth-first search

- Complete? No: fails in infinite-depth spaces
Can modify to avoid repeated states along path
- Time? $O(b^m)$ with m =maximum depth
- terrible if m is much larger than d
 - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space! (we only need to remember a single path + expanded unexplored nodes)
- Optimal? No (It may find a non-optimal goal first)

Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d b^1 + (d-1)b^2 + \dots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1 or increasing function of depth.

Uniform-cost search

Implementation: *fringe* = queue ordered by path cost
Equivalent to breadth-first if all step costs all equal.

Complete? Yes, if step cost $\geq \epsilon$
(otherwise it can get stuck in infinite loops)

Time? # of nodes with *path cost* \leq cost of optimal solution.

Space? # of nodes on paths with path cost \leq cost of optimal solution.

Optimal? Yes, for any step cost.

Bi-Directional Search

Complexity: time and space complexity are: $O(b^{d/2})$

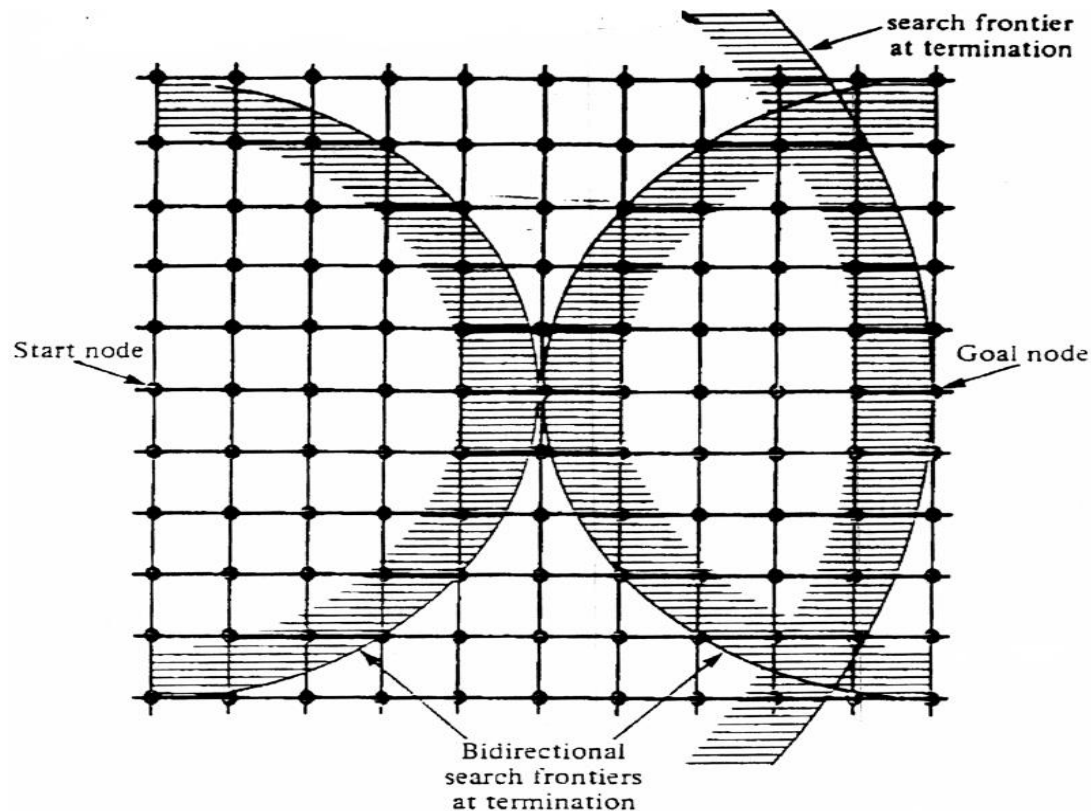


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes