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MAT 105W

Part : A

Ans: to the que: NO - 1

a(i)

$f(x) = \ln(x-1)$

Here, the value f(x) will be real if

$x-1>0$

$\Rightarrow x>1$

$\therefore D_f = \{x : x>1\}$
$= (1,\infty)$

Again,

$y = f(x) = \ln(x-1)$

$\Rightarrow \ln(x-1) = y$

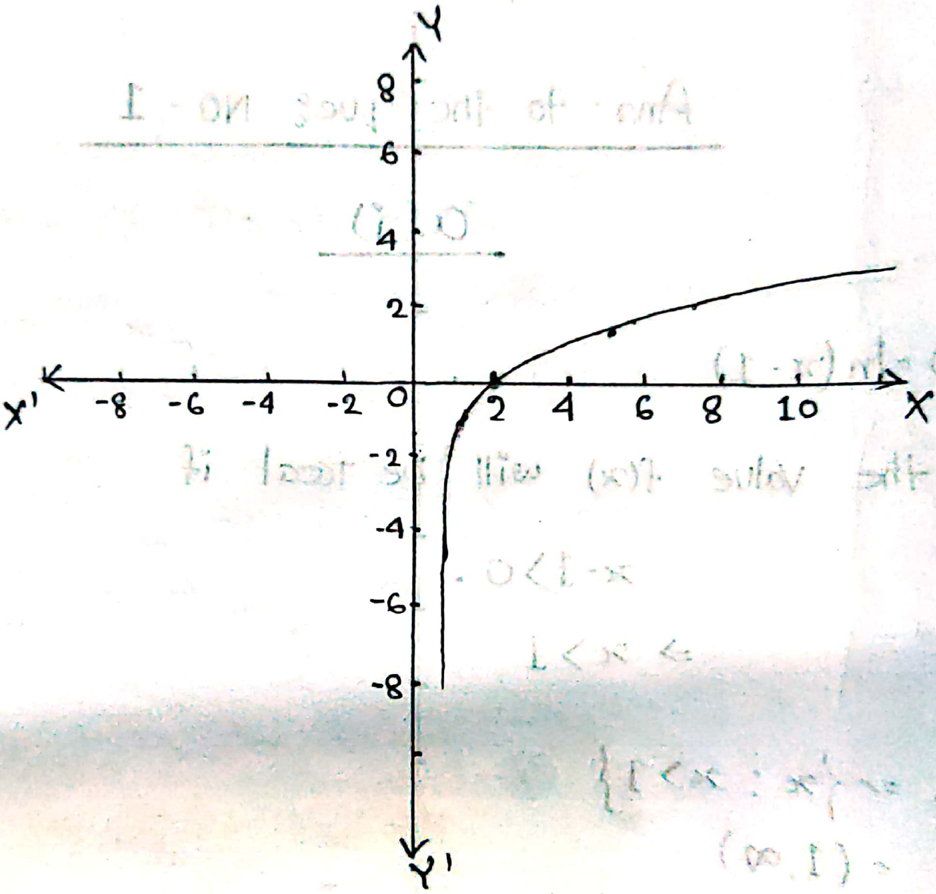
$\Rightarrow x-1 = e^y$

$\Rightarrow x = e^y + 1$

Here, the value of y is real for all real values of x.

$\therefore R_f = \mathbb{R}$
$= (-\infty, \infty)$

x	2	5	7	1.5
$f(x)$	0	1.38	1.79	-.69



a (ii)

$f(x) = e^x + 1$

since e^x is continuous for all real number hence the domain is,

$D_f = \mathbb{R}$
$= (-\infty, \infty)$

Again,

$y = f(x) = e^x + 1$

$\Rightarrow e^x + 1 = y$

$\Rightarrow e^x = y - 1$

$\Rightarrow x = \ln(y-1)$

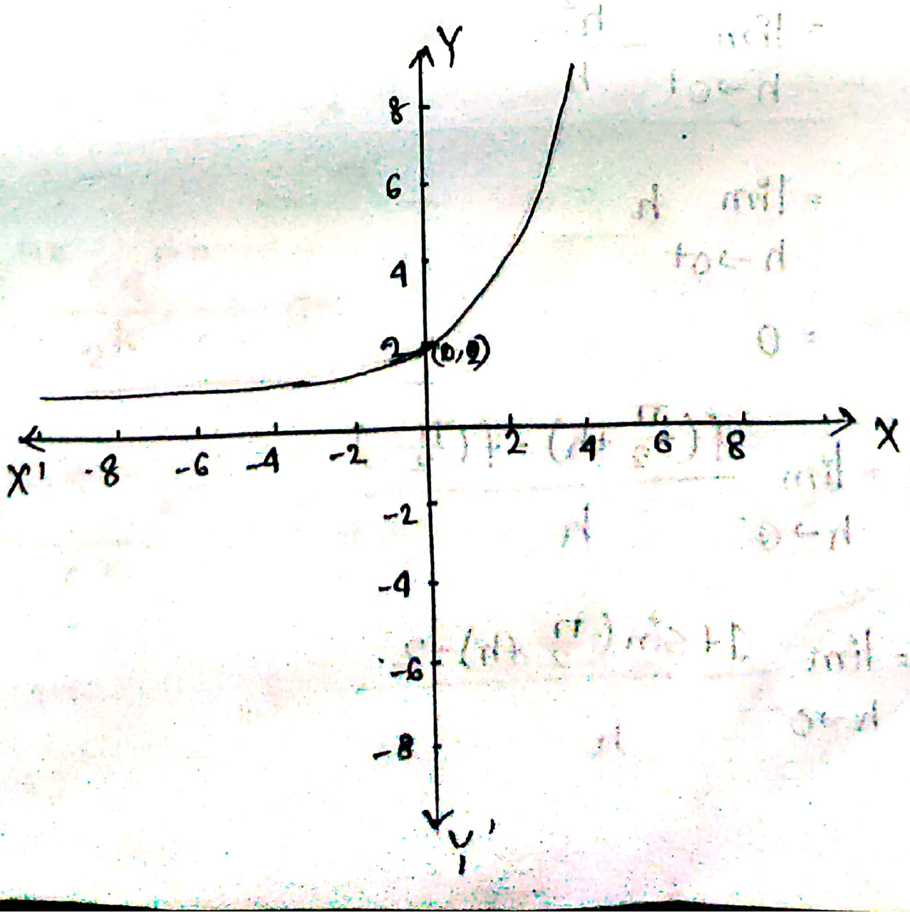
Here the value of x will be real if,

$y - 1 > 0$

$y > 1$

So, the range in $R_f = \{y : y \in \mathbb{R}, y > 1\}$

$= (1, \infty)$



Ans: to the que: NO - 1

b

$$f(x) = \begin{cases} 1 ; & x<0 \\ 1+\sin x ; & 0 \leq x \leq \pi/2 \\ 2+(x-\pi/2)^2 ; & x \geq \pi/2 \end{cases}$$

for differentiability at $x=\pi/2$

$$Rf'\left(\frac{\pi}{2}\right) = \lim_{h\to 0^+} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0^+} \frac{2+\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2 - 2}{h}$$

$$= \lim_{h\to 0^+} \frac{h^2}{h}$$

$$= \lim_{h\to 0^+} h$$

$$= 0$$

$$Lf'\left(\frac{\pi}{2}\right) = \lim_{h\to 0^-} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0^-} \frac{1+\sin\left(\frac{\pi}{2}+h\right) - 2}{h}$$

$$= \lim_{h \to 0^-} \frac{\cos h - 1}{h}$$

$$= \lim_{h \to 0^-} \frac{-2\sin^2 h/2}{h}$$

$$= -2 \lim_{h \to 0^-} \left(\frac{\sin \frac{h}{2}}{h/2}\right)^2 \cdot \frac{h}{4}$$

$$= -2 \cdot 1 \cdot \frac{0}{4}$$

$$= 0$$

$\therefore Rf'(\frac{\pi}{2}) = Lf'(\frac{\pi}{2})$

So, $f(x)$ is differentiable at $x = \frac{\pi}{2}$

Ans: to the que: NO-3

a

$$\int \frac{8^{1+x} + 4^{1-x}}{2^x} dx$$

$$= \int \frac{2^{3+3x} + 2^{2-2x}}{2^x} dx$$

$$= \frac{2^{3+2x}}{2 \ln 2} + \frac{2^{2-3x}}{-3 \ln 2} + c$$

$$= \frac{\cancel{4}\cdot 2^{2x}}{1}$$

$$= \int \frac{2^{3+3x}}{2^x}\,dx + \int \frac{2^{2-2x}}{2^x}\,dx$$

$$= \int 2^{3+2x}\,dx + \int 2^{2-3x}\,dx$$

$$= \int 8\cdot 2^{2x}\,dx + \int 4\cdot 2^{-3x}\,dx$$

$$= 8\,\frac{2^{2x}}{2\ln 2} + 4\,\frac{2^{-3x}}{-3\ln 2} + c$$

$$= \frac{4}{\ln 2}\left[2^{2x} - \frac{1}{3}\,2^{-3x}\right] + c$$

Ans: to the que: No - 3

(b)

Given that

$$\int_0^{\infty} \frac{x\,dx}{(x^2+a^2)(x^2+b^2)}$$

$$= \lim_{\epsilon \to \infty} \int_0^{\epsilon} \frac{x\,dx}{(x^2+a^2)(x^2+b^2)}$$

Now,

$$x^2 = z$$

$$\Rightarrow 2x\,dx = dz$$

$$\Rightarrow x\,dx = \tfrac{1}{2}dz$$

when,

$$x = \epsilon, \quad z = \epsilon^2$$

$$x = 0, \quad z = 0$$

So, the equation stand,

$$\lim_{\epsilon \to \infty} \frac{1}{2} \int_0^{\epsilon^2} \frac{dz}{(z+a^2)(z+b^2)}$$

$$= \frac{1}{2(a^2-b^2)} \lim_{\epsilon \to \infty} \left\{ \int_0^{\epsilon^2} \left(\frac{1}{z+b^2} - \frac{1}{z+a^2} \right) dz \right\}$$

$$= \frac{1}{2(a^2-b^2)} \lim_{\epsilon \to \infty} \Big[\ln(z+b^2) - \ln(z+a^2) \Big]_0^{\epsilon^2}$$

$$= \frac{1}{2(a^2-b^2)} \lim_{\epsilon \to \infty} \left[\ln \frac{z+b^2}{z+a^2} \right]_0^{\epsilon^2}$$

$$= \frac{1}{2(a^2-b^2)} \lim_{\epsilon \to \infty} \left(\ln \frac{\epsilon^2+b^2}{\epsilon^2+a^2} - \ln \frac{b^2}{a^2} \right)$$

$$= \frac{1}{2(a^2-b^2)} \left[\lim_{\epsilon \to \infty} \ln \frac{1+\left(\frac{b^2}{\epsilon^2}\right)}{1+\left(\frac{a^2}{\epsilon^2}\right)} + \ln\left(\frac{a^2}{b^2}\right) \right]$$

$$= \frac{1}{2(a^2-b^2)}\left[\ln 1 + 2\ln\frac{a}{b}\right]$$

$$= \frac{1}{2(a^2-b^2)}\left(0 + 2\ln\frac{a}{b}\right)$$

$$= \frac{1}{a^2-b^2}\ln\frac{a}{b}$$