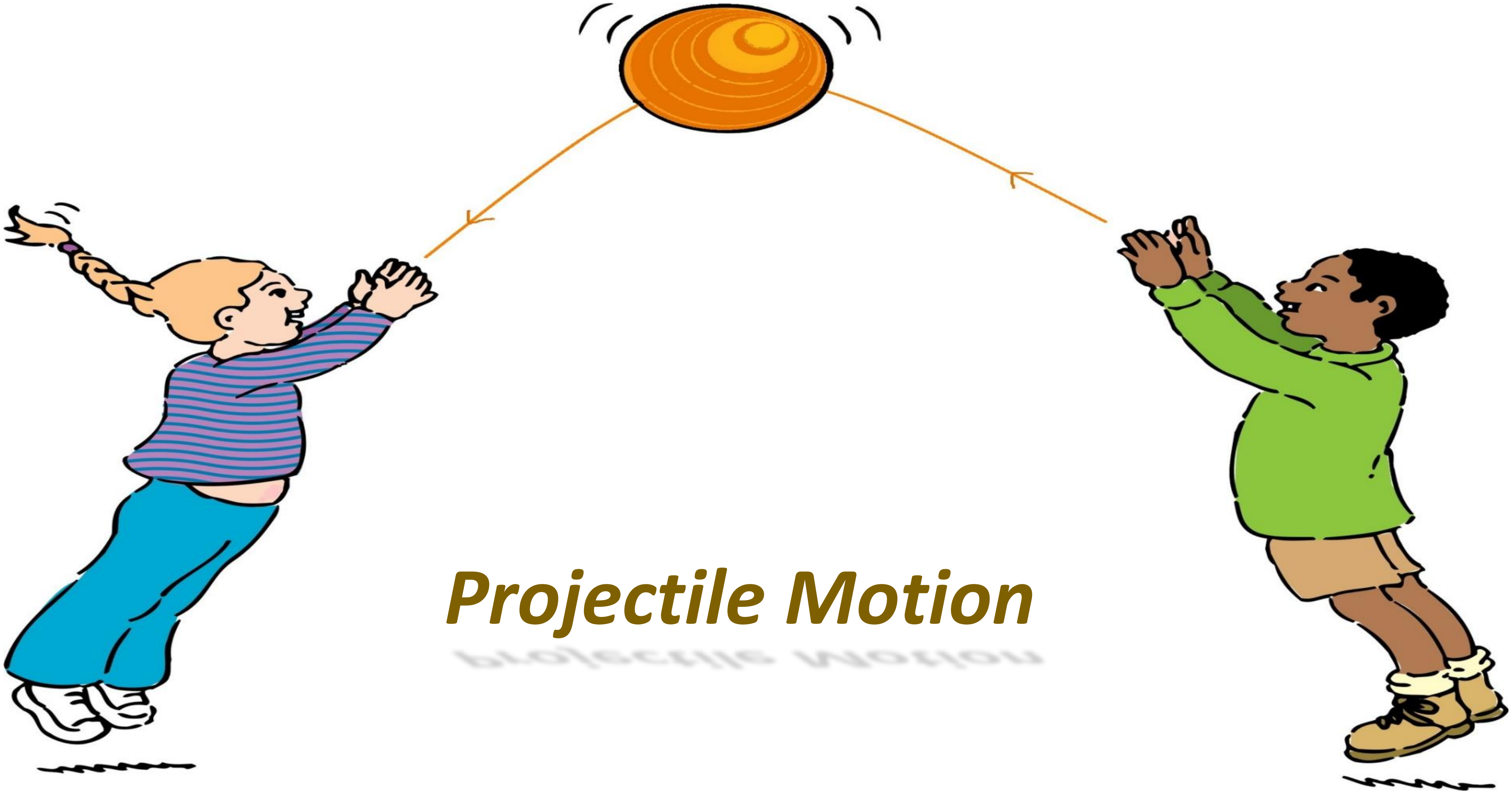


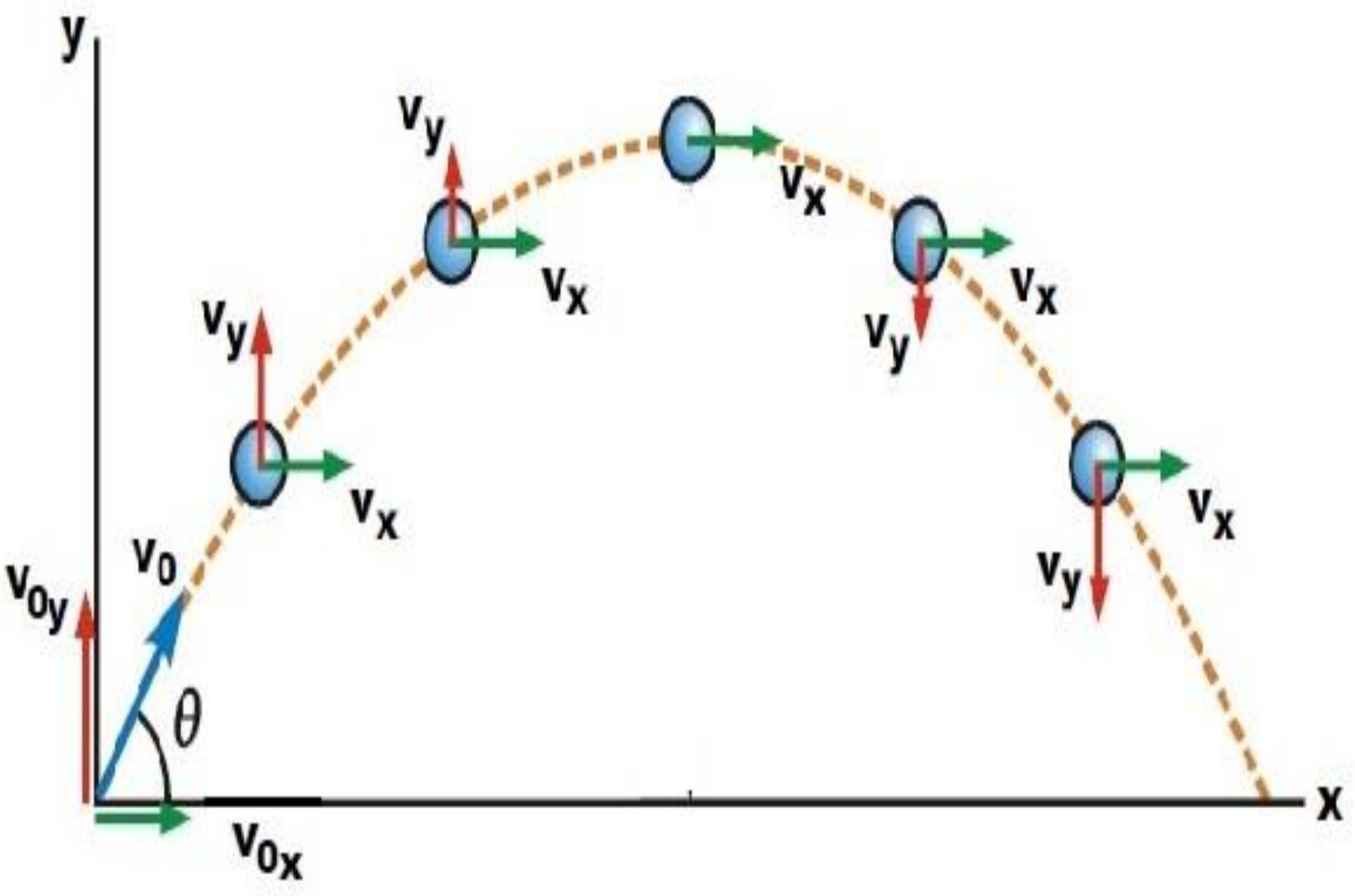
Projectile Motion

Projectile Motion ⤇ The two dimensional motion of a particle thrown obliquely into the air. The particle is called *projectile* and the path followed by a projectile is called its *trajectory*.

- **Example:** Motion of baseball or golf ball, missile shot from a gun, a rocket after its fuel is exhausted…. etc
- **Condition:** For simplicity, neglect the air resistance.

(when the air resistance or drag force is greater than the weight of the projectile, then it will affect its motion)

Trajectory is parabolic. It has two independent motion along x-direction and y-direction.
Since this is not going straight, there is an effect of unbalanced force in its motion.



In a x-y coordinate system choose the origin from where the projectile begins its flight.
The velocity at time t=0, the instant the projectile begins its flight, is $\overrightarrow{v_o}$, which makes an angle ϴ **with the positive x direction.**

Horizontal motion

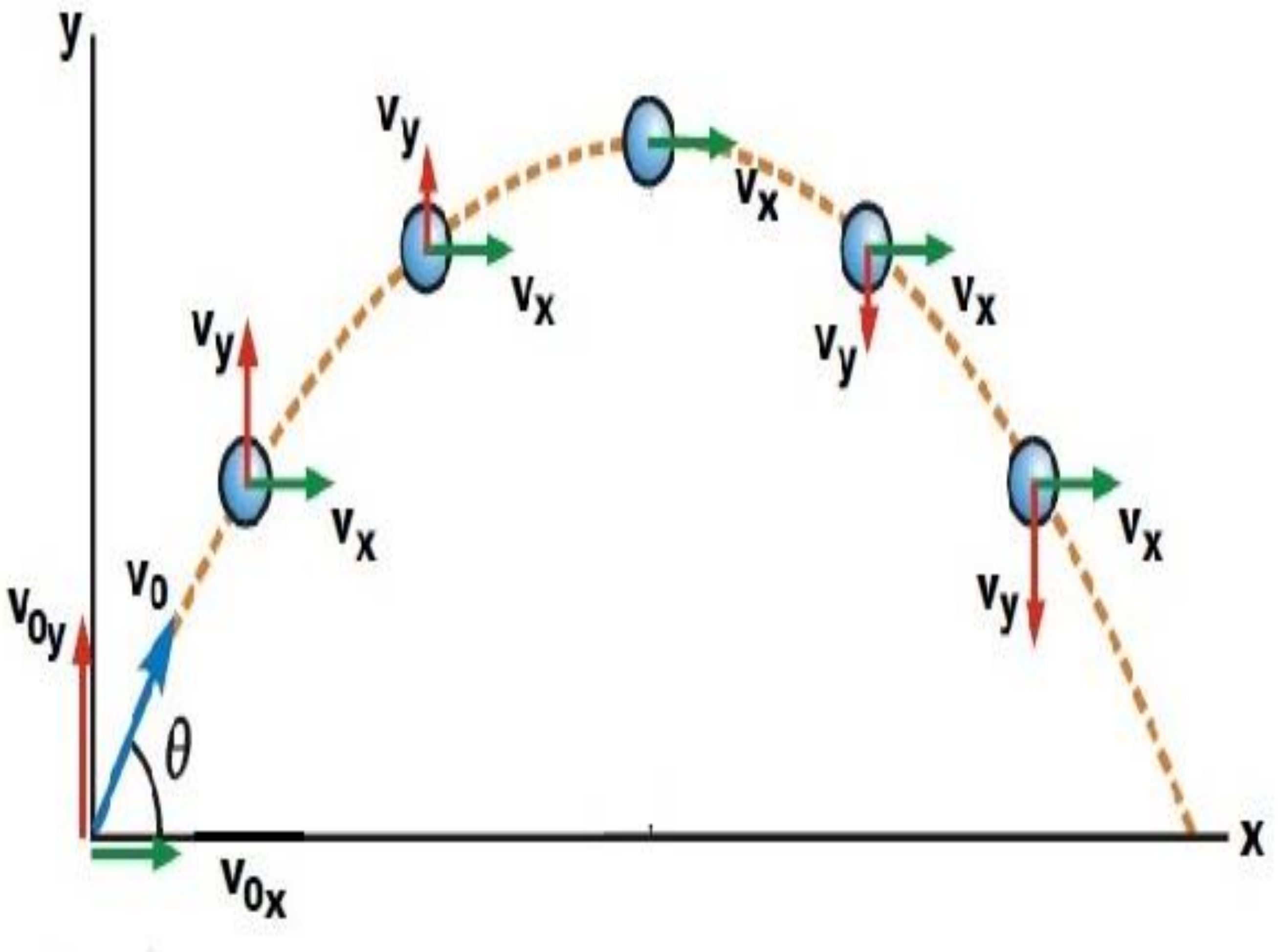
Balanced force $a_x = 0$

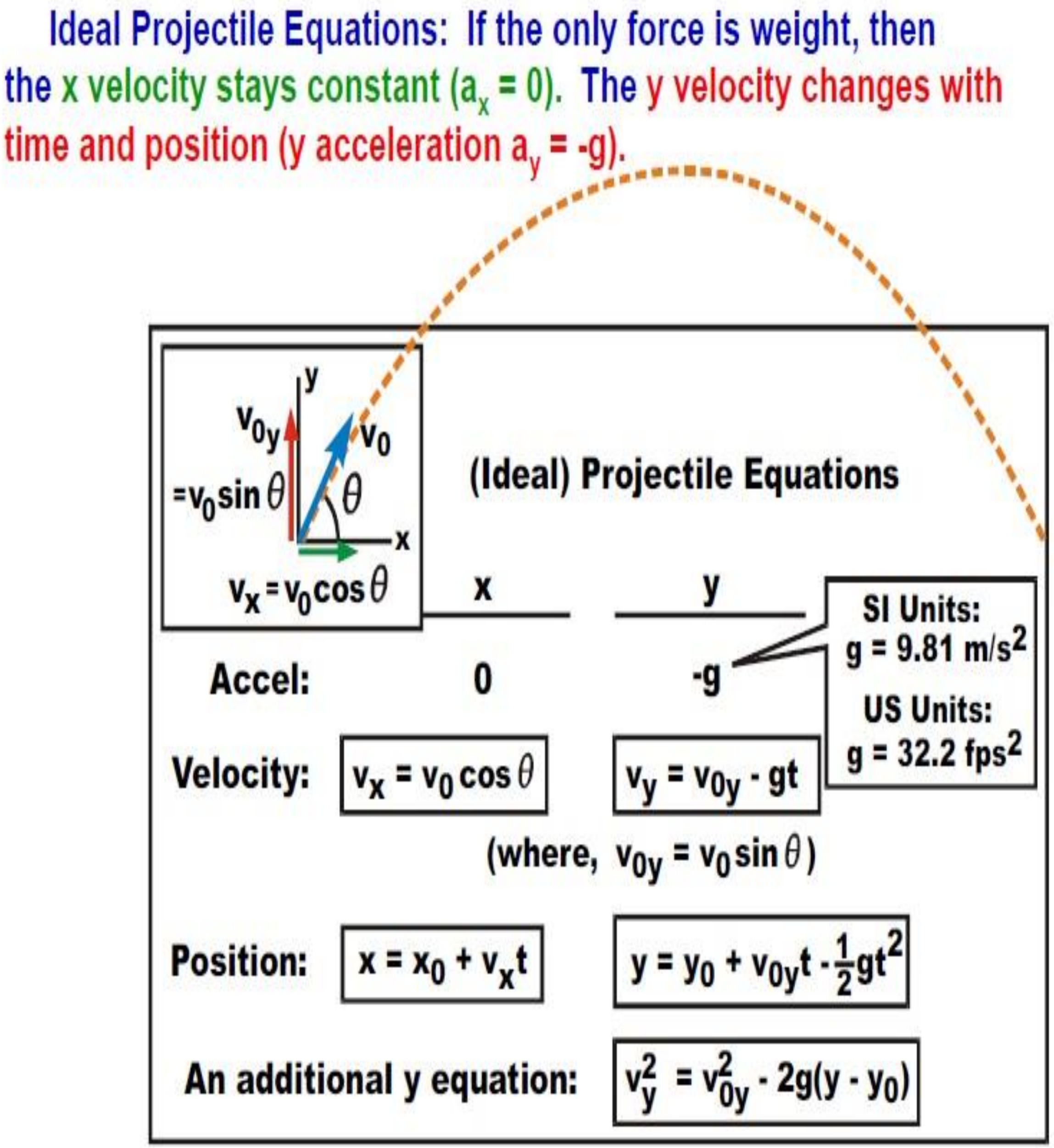
Velocity is constant $v_{ox} = v_x = v_o \cos\Theta$

Vertical Motion

Unbalanced force $a_y = -9.81 m/s$

Velocity v_y changes with time
At t=0, $v_{yo} = v_o \sin\Theta$
At=t, $v_y = v_{yo} + a_y t = v_o \sin\Theta - gt$





At t=0, particle's position X_o=0, Y_o=0

Particle's position at any time t-

X-direction:

$$x = x_o + v_{xo} t + \frac{1}{2} a_x t^2 = v_o cos\Theta t$$

Y-direction:

$$y = y_o + v_{yo} t + \frac{1}{2} a_y t^2 = v_o sin\Theta t - \frac{1}{2} g t^2$$

Putting value of t $= \frac{x}{v_o cos\Theta}$ from the equation of position in x-direction we get-

$$y = (\tan\Theta)x - \frac{g}{2(v_o cos\Theta)^2} x^2$$

This is the equation of trajectory of projectile which is parabolic.

Let's find:
Maximum height(h),Time at maximum height(t),
Time of flight(T),Horizontal range (R)

h: $v_y^2 = v_{oy}^2 - 2g(y - y_o) = v_{oy}^2 - 2gh$

$$0 = v_0^2\sin^2\Theta - 2gh$$

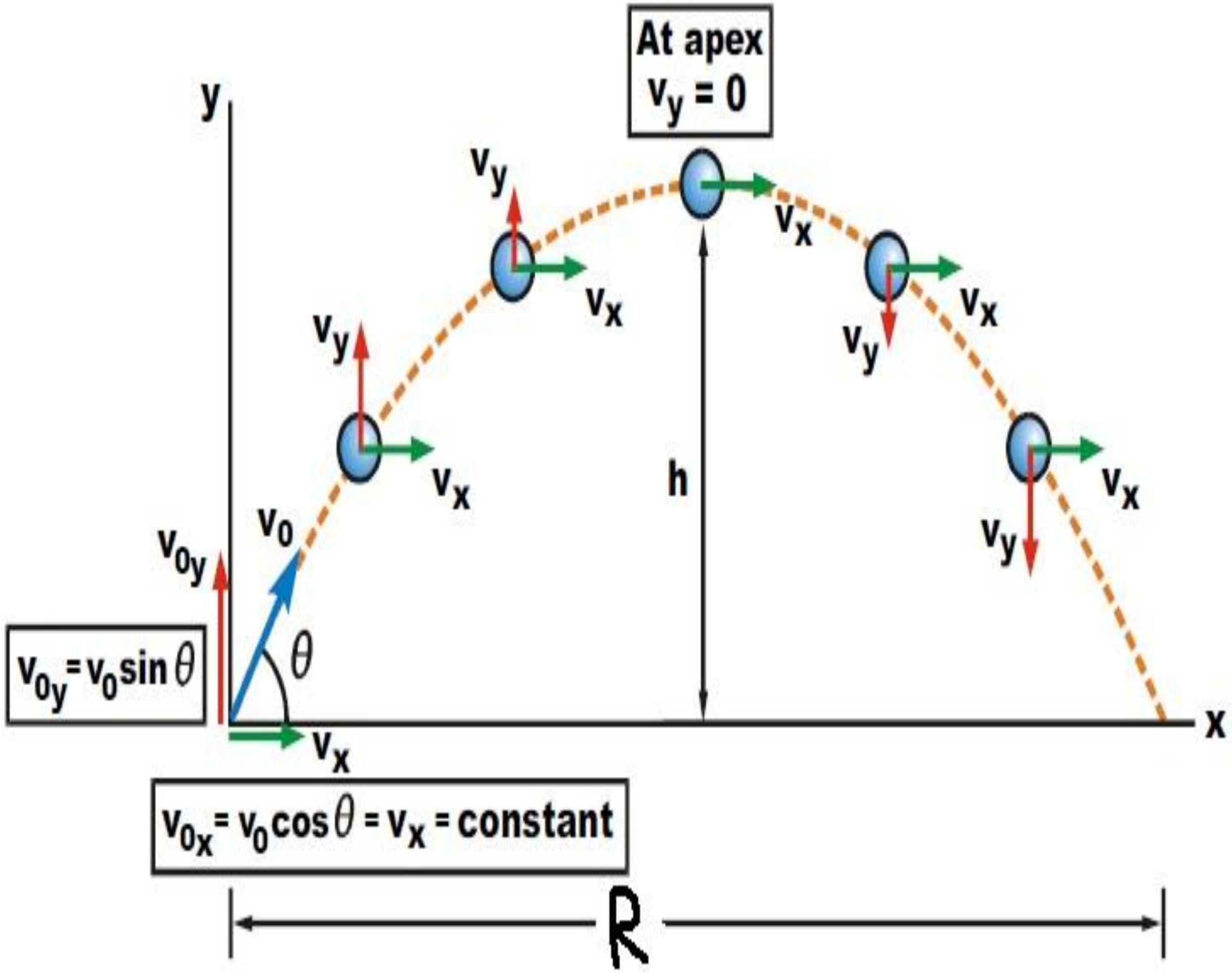
$$h = \frac{v_0^2\sin^2\Theta}{2g}$$

t: $v_y = v_{oy} - gt$

$$0 = v_o sin\Theta - gt$$

$$t = \frac{v_o\sin\Theta}{g}$$

T: $T = 2t = \dfrac{2v_o\sin\Theta}{g}$



R: $R = \text{Horizontal velocity} \times \text{Time of flight} = v_o\cos\Theta \times \dfrac{2v_o\sin\Theta}{g} = \dfrac{v_o^2\sin2\Theta}{g}$

When $\Theta = 45°$, R is maximum

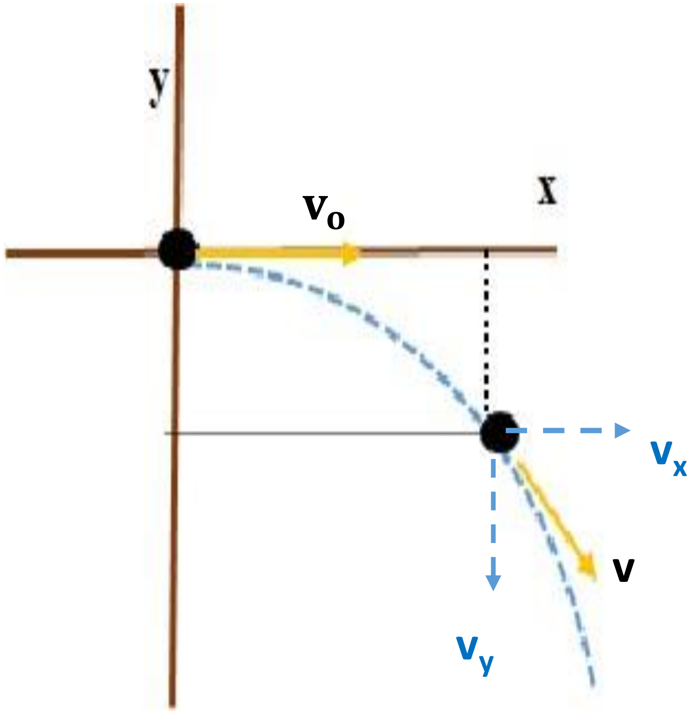
#Trajectory of a body projected Horizontally

Initial Velocity v_o
The departure angle $\theta=0^0$
At any time t, position
In x-direction

$$x = v_o t$$

In y direction

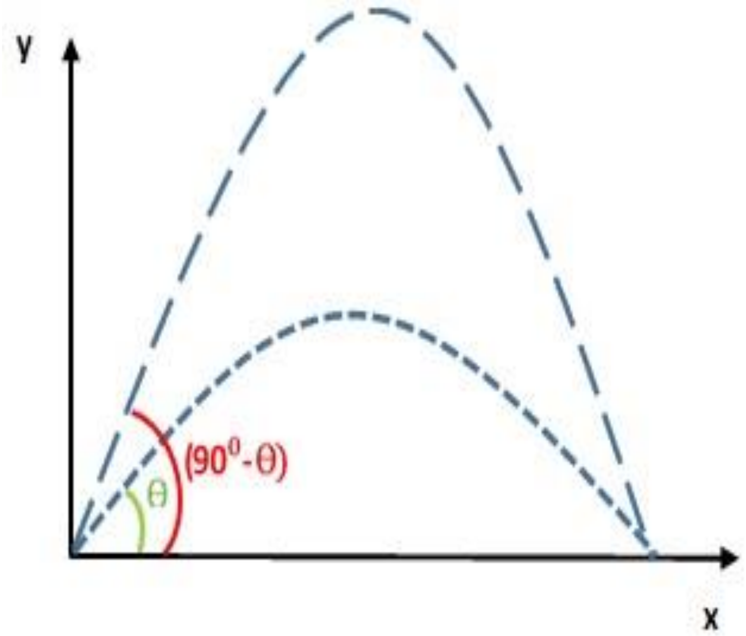
$$y = v_o sin0^0 t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2$$



Same Horizontal Range

For Departure Angle θ and $(90^0 - \theta)$ with same initial velocity, the Horizontal range will be equal.

$$R = \frac{v_o^2 \sin 2\theta}{g}$$



Projectile motion on an incline

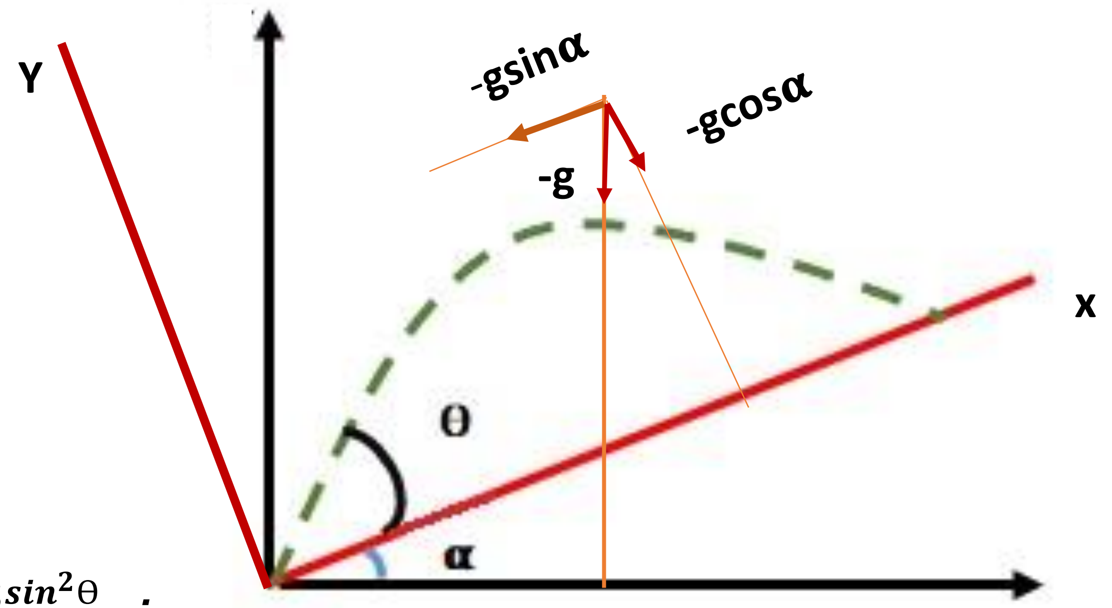
Components of acceleration due to gravity

$a_x = -g\sin\alpha; \quad a_y = -g\cos\alpha$

Maximum Height, $h = \dfrac{v_0^2 \sin^2\theta}{2g\cos\alpha}$

Time at maximum height, $t = \dfrac{v_0 \sin\theta}{g\cos\alpha}$

Time of flight, $T = \dfrac{2v_0 \sin\theta}{g\cos\alpha}$



Horizontal Range of Flight

$$R = x = v_{xo}T + \frac{1}{2}a_x T^2 = v_o\cos\theta\,\frac{2v_o\sin\theta}{g\cos\alpha} - \frac{1}{2}\frac{4v_o^2\sin^2\theta}{g\cos^2\alpha}\sin\alpha$$

$$R = \frac{v_o^2}{g\cos^2\alpha}(2\cos\theta\sin\theta\cos\alpha - \sin\alpha \times 2\sin^2\theta)$$

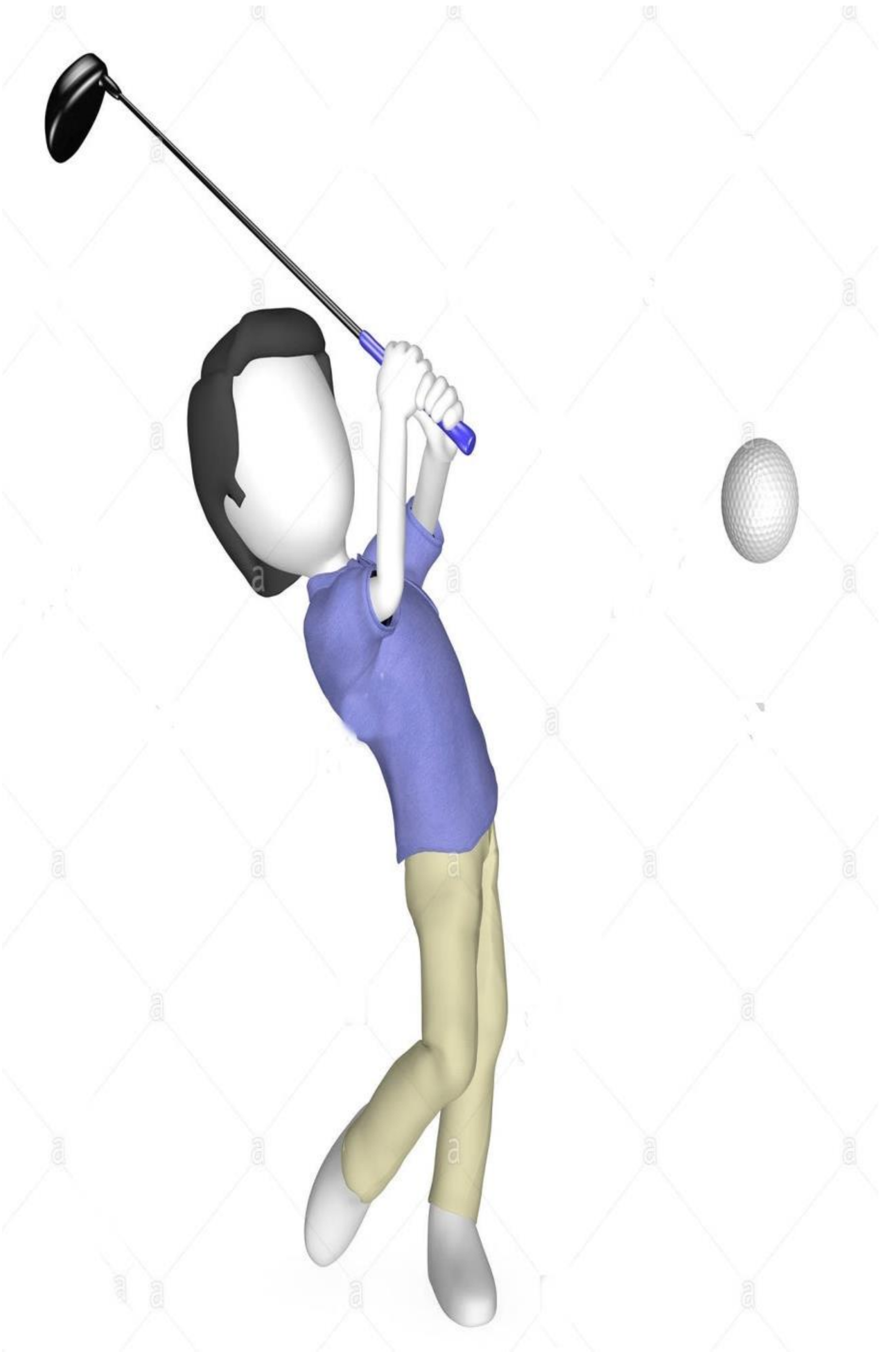
$$R = \frac{v_o^2}{g\cos^2\alpha}(\sin 2\theta\cos\alpha - \sin\alpha \times \{1 - \cos 2\theta\}\Big)$$

$$R = \frac{v_o^2}{g\cos^2\alpha}\{\sin(2\theta + \alpha) - \sin\alpha\}$$

$$R = \frac{2v_o^2}{g\cos^2\alpha}\{\sin\theta\cos(\theta + \alpha)\}$$

$$\mathrm{Sin}\,(A + B) = \mathrm{SinACosB} + \mathrm{CosASinB}$$

$$\mathrm{SinA} - \mathrm{SinB} = 2\mathrm{Sin}\frac{A - B}{2}\mathrm{Cos}\frac{A + B}{2}$$



Thank you

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