Uniform Circular Motion

A particle is in **uniform circular motion** if it travels around a circle or a circular arc at constant (*uniform*) speed. Although the speed does not vary, *the particle is accelerating* because the velocity changes in direction. Figure :1 shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion. Both vectors have constant magnitude, but their directions change continuously. The velocity is always directed tangent to the circle in the direction of motion. The acceleration is always directed *radially inward.* Because of this, the acceleration associated with uniform circular motion is called a **centripetal** (meaning "center seeking") **acceleration.**

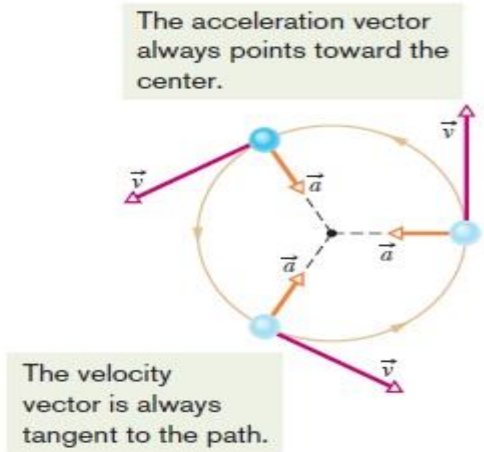


Figure:1

- **In Projectile motion- The acceleration is constant in both magnitude and direction, But velocity changes in both magnitudes and direction.**
- **In Uniform Circular motion- both velocity and acceleration are constant in magnitude but both changes their directions continuously.**

To find the magnitude and direction of the acceleration for uniform circular motion, we consider Fig. 2. In Fig. 2*a*, particle *p* moves at constant speed *v* around a circle of radius *r*. At the instant shown, *p* has coordinates x_p and y_p. The velocity of a moving particle is always tangent to the particle's path at the particle's position. In Fig. 2*a*, that means is perpendicular to a radius *r* drawn to the particle's position. Then the angle u that makes with a vertical at *p* equals the angle u that radius *r* makes with the *x* axis. The scalar components of are shown in Fig. 2*b*. With them, we can write the velocity as

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-v\sin\theta)\hat{\imath} + (v\cos\theta)\hat{\jmath} \ldots\ldots\ldots (1)$$

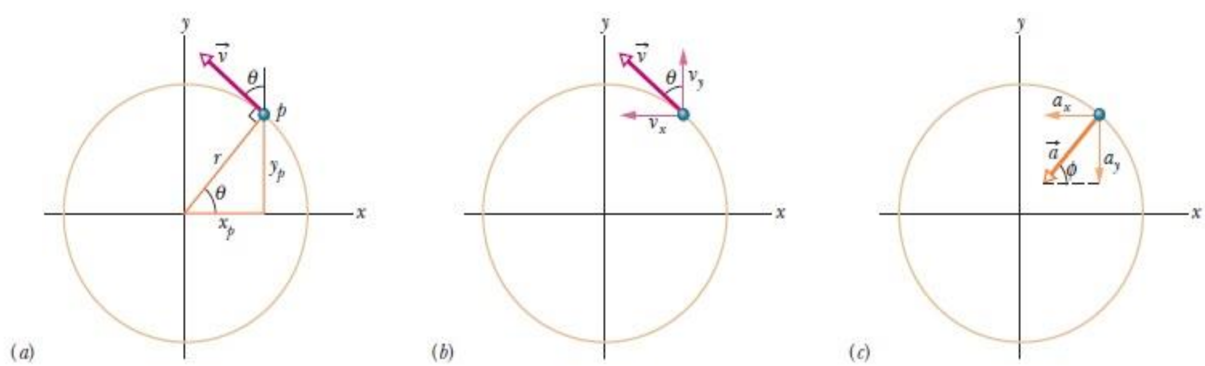


Fig.2: Particle *p* moves in counterclockwise uniform circular motion. (*a*) Its position and velocity: *v* at a certain instant. (*b*) Velocity: v. (*c*) Acceleration: *a*.

Now, using the right triangle in Fig. 2*a*, we can replace sin**θ** with y_p/r and
cos**θ** with x_p/r to write

$$\vec{v} = \left(-v\frac{y_p}{r}\right)\hat{\imath} + \left(v\frac{x_p}{r}\right)\hat{\jmath} \ldots\ldots\ldots (2)$$

To find the acceleration of particle *p*, we must take the time derivative of this equation. Noting that speed *v* and radius *r* do not change with time, we obtain

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{y_p}{dt}\right)\hat{\imath} + \left(\frac{v}{r}\frac{x_p}{dt}\right)\hat{\jmath} \ldots\ldots\ldots (3)$$

Now note that the rate dy_p/dt at which y_p changes is equal to the velocity component v_y. Similarly, $dx_p/dt = v_x$, and, again from Fig. 2*b*, we see that $v_x = -v \sin \Theta$ and $v_y = v \cos \Theta$. Making these substitutions in Eq. 3, we find

$$\vec{a} = \left(-\frac{v^2}{r}sin\theta\right)\hat{\imath} + \left(-\frac{v^2}{r}cos\theta\right)\hat{\jmath} \ldots\ldots\ldots (4)$$

Now,

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{\cos^2\theta + \sin^2\theta} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r} \ldots . (5)$$

Let's find the angle Φ shown in Fig:2c.

$$tan\emptyset = \frac{a_x}{a_y} = \frac{\left(-\frac{v^2}{r}sin\theta\right)}{\left(-\frac{v^2}{r}cos\theta\right)} = tan\theta \ldots\ldots\ldots\ldots . (6)$$

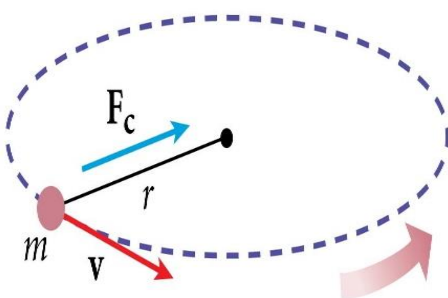
Thus, Φ=Θ which means that a is directed along the radius *r* of Fig. 2*a*, toward the circle's center.

Centripetal Force:

A centripetal force is a net force that acts on an object to keep it moving along a circular path.

Any object traveling along a circular path of radius r with velocity v experiences an acceleration directed toward the center of its path, which is called centripetal acceleration.

$$a = \frac{v^2}{r}$$



This centripetal acceleration, for a particle in uniform circular motion must be caused by a force directed toward center also.

Newton's 1st law tells us that an object will continue moving along a straight path unless acted on by an external force. The external force here is the centripetal force.

The tension force in the string of a swinging tethered ball and the gravitational force keeping a satellite in orbit are both examples of centripetal forces. Multiple individual forces can even be involved as long as they add up (by vector addition) to give a net force towards the center of the circular path. According to Newton's 2nd Law:

$F = ma$

Putting the formula of centripetal acceleration, we get centripetal force as-

$$F_c = ma = m\frac{v^2}{r}$$

This force is always directed towards the center of the circular path. Equivalently, if ω is the angular velocity then because $v=r\omega$

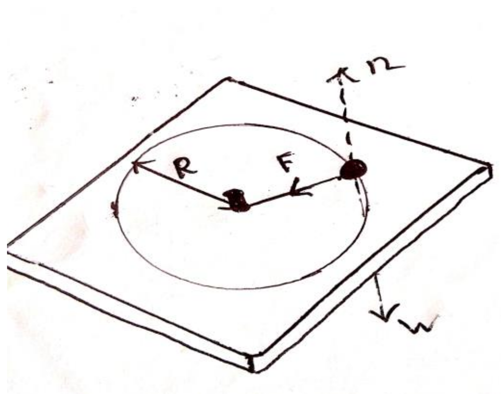
$$F_c = ma = mr\omega^2$$

Compare ⤇ Centrifugal force and Centripetal force

#Mathematical problem:

A small body of mass 0.2 kg revolves uniformly in a circle on a horizontal frictionless surface, attached by a cord 0.2m long to a pin set in that surface. If the body makes two complete rotations per second, find the force F exerted on it by the cord.

Solution: Circumference of the circle is 2π (0.2m) = 0.4m



So, for 2 revolutions per sec its speed will be 0.8m/s. hence the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{R} = \frac{(0.8ms^{-1})^2}{0.2m} = 31.6ms^{-2}$$

Since the body has no vertical acceleration, the forces n and w are equal and opposite and the force F is the resultant force. Therefore

F= ma= (0.2 kg) ($31.6m.s^{-2}$)= 6.32N