(MAT-105W)

All Questions

Contents

Limit and Continuity

Every differentiable function is continuous, but every continuous function need not to be differentiable.

(1) Show that the function $f(x) = x^2 + 2$ is continuous and differentiable at $x = 1$.

(2) If $f(x) = \begin{cases} x & ;\ 0 \le x \le \frac{1}{2} \\ 1 - x & ;\ \frac{1}{2} \le x \le 1 \end{cases}$

Show that $f(x)$ is continuous at $x = \frac{1}{2}$ but $f(x)$ is not differentiable at that point.

(3) If $f(x) = \begin{cases} 1 & when\ x \le 1 \\ x & when\ x > 1 \end{cases}$

test the continuity and differentiability at x=1.

(4) Discuss the continuity and differentiability at $x = 0$ of the function

$$f(x) = \begin{cases} 1 & when\ \ x < 0 \\ 1 + sinx & when\ 0 \le x \le \dfrac{\pi}{2} \end{cases}$$

(5) Discuss the continuity and differentiability at $x = 0$ and $x = \frac{\pi}{2}$ of the function

$$f(x) = \begin{cases} 1 & ; \ x < 0 \\ 1 + \sin(x) & ; \ 0 \le x < \dfrac{\pi}{2} \\ 2 + \left(x - \dfrac{\pi}{2}\right)^2 & ; \ x \ge \dfrac{\pi}{2} \end{cases}$$

(6) Discuss the continuity and differentiability at $x = 0$ and $x = 1$ of the function

$$f(x) = \begin{cases} x^2 + 1 & ; \ x \le 0 \\ x & ; \ 0 < x < 1 \\ \dfrac{1}{x} & ; \ x \ge 1 \end{cases}$$

(7) A function f is defined as follows:

$$f(x) = \begin{cases} x & when\ x < 1 \\ 2 - x & when\ 1 \le x \le 2 \\ -2 + 3x - x^2 & when\ x > 2 \end{cases}$$

Show that f is continuous at $x = 1$ and $x = 2$ both; it is derivable at $x = 2$ but not at $x = 1$.

(8) If $f(x) = \begin{cases} 3 + 2x & when\ -\frac{3}{2} < x \le 0 \\ 3 - 2x & when\ 0 < x < \frac{3}{2} \\ -3 - 2x & when\ x \ge \frac{3}{2} \end{cases}$

Discuss the continuity and differentiability of the functions at $x = 0$ and $x = \frac{3}{2}$.

(9) Show that the function $f(x) = |x| + |x-1|$ is continuous at $x = 1$ but not differentiable.

(10) A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 1+x & \text{when } x < 0 \\ x & \text{when } 0 < x < 1 \\ 2-x & \text{when } 1 \le x \le 2 \\ 2x - x^2 & \text{when } x > 2 \end{cases}$$

Show that the function $f(x)$is continuous at the points $x = 1$ and $x = 2$ but $f'(x)$ does not exist at that point.

<u>Graph of a function</u>

(1) Sketch the graphs of the following functions:

(i) $$f(x) = \begin{cases} x \ ; \ x < 1 \\ e^x \ ; \ 1 \le x \le 2 \\ 2 \ ; \ x > 2 \end{cases}$$

(ii) $$f(x) = \begin{cases} \sin(x) \ ; \ x < \frac{\pi}{2} \\ 0 \ ; \ x = \frac{\pi}{2} \\ x \ ; \ x > \frac{\pi}{2} \end{cases}$$

<u>Derivative</u>

Formula
$\frac{d}{dx}(u^v) = u^v \cdot \frac{d}{dx}\{v \cdot \ln(u)\}$

(1) $\sin y = x \cdot \sin(a+y)$; $\frac{dy}{dx} = ?$

(2) $y = \sqrt{x}\, e^x secx$; $\frac{dy}{dx} = ?$

(3) $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$; $\frac{dy}{dx} = ?$

(4) $y = e^{sinx} \sin(a^x)$; $\frac{dy}{dx} = ?$

(5) $y = \tan(lnx^2)$; $\frac{dy}{dx} = ?$

(6) $y = x^{(\sin^{-1} x)}$; $\frac{dy}{dx} = ?$

(7) $y = x^{lnx} + x^{\cos^{-1} x}$; $\frac{dy}{dx} = ?$

(8) $y = (sinx)^{sinx}$; $\frac{dy}{dx} = ?$

(9) $\ln(x+y) = xy$; $\frac{dy}{dx} = ?$

(10) $y = x^{\cos^{-1} x}$; $\frac{dy}{dx} = ?$

(11) $y = x^{sinx}$; $\frac{dy}{dx} = ?$

(12) $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$; $\frac{dy}{dx} = ?$

(13) $e^x + e^y = 2xy$; $\frac{dy}{dx} = ?$

(14) If $y = e^{-x} \cdot \sin x$, show that $y_4 + 4y = 0$.

(15) If $y = ae^{mx} + be^{-mx}$, then show that $y_2 = m^2 y$

Successive Differentiation

(1) If $y = x^n$, then show that $y_n = n!$

(2) If $y = (ax+b)^n$ and $n \in N$, then show that $y_n = n!\, a^n$

(3) If $y = a^{bx+c}$, then show that $y_n = a^{bx+c} \cdot \{\ln(a)\}^n \cdot b^n$

(4) If $y = \ln(ax+b)$, then show that $y_n = (-1)^{n-1} \cdot (n-1)! \cdot (ax+b)^{-n} \cdot a^n$

Leibnitz's Theorem
$(uv)_n = u_n v + n_{c_1} \cdot u_{n-1} v_1 + \cdots + n_{c_r} \cdot u_{n-r} v_r + \cdots + u v_n$

(1) If $y = e^{a\sin^{-1}x}$ then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

(2) If $y = \cot^{-1} x$ then show that

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

(3) If $y = a\cos(\ln x) + b\sin(\ln x)$ then show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(4) If $y = \cos\{ln(1+x)\}$ then show that

$$(1+x^2)y_{n+2} + (2n+1)(1+x)y_{n+1} + (n^2+1)y_n = 0$$

(5) If $y = \tan^{-1}(x)$, show that

$$(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$$

(6) If $\ln y = \cot^{-1} x$, show that

$$(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$$

(7) If $y = \sin(m\sin^{-1}x)$, prove that

$$(1-x^2)y_{n+2} - (2n+1)x \cdot y_{n+1} + (m^2-n^2)y_n = 0$$

Maxima – Minima

(1) Investigate for what values of x, $f(x) = 5x^6 - 18x^5 + 15x^4 - 10$ is minimum or maximum.

(2) Find for what values of x, the following expression is maximum or minimum respectively

$$2x^3 - 21x^2 + 36x - 20$$

Find out the maximum and minimum values of the expression.

Rolle's Theorem

(1) Verify Rolle's theorem for $f(x) = x^3 - 12x$ in the interval $0 \leq x \leq 2\sqrt{3}$.

(2) Does Rolle's theorem apply to the function $f(x) = 1 - (x-3)^{\frac{2}{3}}$

(3) State Rolle's theorem. Verify it for $f(x) = 2x^3 + x^2 - 4x - 2$

Mean Value Theorem

(No Math Found)

Euler's Theorem

$$\boldsymbol{x \cdot \frac{\delta F}{\delta x} + y \cdot \frac{\delta F}{\delta y} + \cdots = nF}$$

(1) If $u = \tan^{-1}\frac{x^3+y^3}{x+y}$, show that

$$x \cdot \frac{\delta u}{\delta x} + y \cdot \frac{\delta u}{\delta y} = \sin(2u)$$

(2) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x \cdot \frac{\delta u}{\delta x} + y \cdot \frac{\delta u}{\delta y} + \frac{1}{2}(\cot u) = 0$$

(3) If $u = \tan^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x \cdot u_x + y \cdot u_y = \frac{1}{4}\sin 2u$$

Improper Integral

(1) Is the area under the curve $y = \frac{1}{\sqrt{x}}$ from $x = 0$ to $x = 1$ finite? If so, what is it?

(2) Evaluate $\int_2^{\alpha} \frac{x+3}{(x-1)(x^2+1)} dx$

Taylor & Maclaurin's Polynomial

(1) Find Maclaurin's series for

(i) e^x

(ii) $\sin x$
(iii) e^{mx}
(iv) $\cos x$
(v) $\ln(1+x)$
(vi) $\sin 2x$
(vii) $\cos 2x$
(viii) a^x

(2) Find Taylor series for $f(x) = \ln x$ at $x = 2$.

Integration: As an Inverse Process Of Differentiation

- Find the anti-derivative of –

(1) $\sin 2x + e^{5x}$

(2) $\frac{8^{1+x} + 4^{1-x}}{2^x}$

- Find the Indefinite Integrals –

(1) $\int \frac{sin^8 x - cos^8 x}{1 - 2sin^2 x cos^2 x}\, dx$

(2) $\int \frac{dx}{x\{10 + 7\ln(x) + (lnx)^2\}}$

(3) $\int \frac{e^x\, dx}{e^{2x}+2e^x+5}$

(4) $\int \frac{dx}{(x^2-16)\sqrt{x+1}}$

(5) $\int \frac{dx}{3+2cosx}$

(6) $\int \frac{\cos x}{2\cos x+3}\, dx$

(7) $\int \frac{dx}{3sinx+2cosx+5}$

(8) $\int \frac{dx}{5+4cosx}$

(9) $\int e^{2x} \sin^3 x \ dx$

(10) $\int \frac{x(\tan^{-1} x)^2}{(1+x^2)^{\frac{1}{2}}(1+x^2)} dx$

(11) $\int \frac{xdx}{(x+1)\sqrt{x^2+1}}$

- Fundamental theorem of integral calculus and its application to definite integrals –

(1) $\int \frac{dx}{(a^2+x^2)^{\frac{3}{2}}}$

(2) $\int \frac{x\sqrt{2-x^2}}{\sqrt{1+x^2}} dx$

Definite Integral As The Limit Of A Sum

(1) $\lim_{n\to\infty} [\frac{n}{n^2} + \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+(n-1)^2}]$

(2) $\lim_{n\to\infty} [\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \cdots + \frac{n}{n^2+n^2}]$

(3) $\lim_{n\to\infty} [\frac{\sqrt{n}}{n^{\frac{3}{2}}} + \frac{\sqrt{n}}{(n+3)^{\frac{3}{2}}} + \frac{\sqrt{n}}{(n+6)^{\frac{3}{2}}} + \cdots + \frac{\sqrt{n}}{\{n+3(n-1)\}^{\frac{3}{2}}}]$

(4) $\lim_{n\to\infty} [\frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + \frac{3^2}{3^3+n^3} + \cdots + \frac{n^2}{n^3+n^3}]$

(5) $\lim_{n\to\infty} [\frac{1}{n} + \frac{\sqrt{n^2-1^2}}{n^2} + \cdots \frac{\sqrt{n^2-(n-1)^2}}{n^2}]$

(6) $\lim_{n\to\infty} [\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \cdots \frac{1}{n}]$

Reduction Formula

(1) Apply reduction formula for $\int cos^n x\, dx$ and then evaluate $\int cos^8 x\, dx$.

(2) Apply reduction formula for $\int sin^n x\, dx$ and then evaluate $\int sin^8 x\, dx$.

(3) Apply reduction formula for $\int tan^n x\, dx$ and then evaluate $\int tan^8 x\, dx$.

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