SINUSOIDAL ALTERNATING WAVEFORMS

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Outline

- ***Introduction to Alternating Waveforms***
- ***Sinusoidal AC Voltage Characteristics & Definitions***
- ***General Format for the Sinusoidal Voltage or Current***
- ***Determination of Phase Relation between Waveforms of the same Frequency***
- ***Definition and Determination of Average Value of an Alternating Wave***
- ***Definition and Determination of Effective (rms) Value of an Alternating Wave***

Objectives

- ***Become familiar with the characteristics of a sinusoidal waveform including its general format, average value, and effective value.***
- ***Be able to determine the phase relationship between two sinusoidal waveforms of the same frequency.***
- ***Understand how to calculate the average and effective values of any waveform.***

Introduction to Alternating Waveforms

An *Alternating waveform* constantly changes its polarity every half cycle, *alternating* between a positive maximum value and a negative maximum value respectively with regards to time.

There are various types of Alternating waveforms. Such as:

- Sinusoidal
- Square wave
- Triangular wave

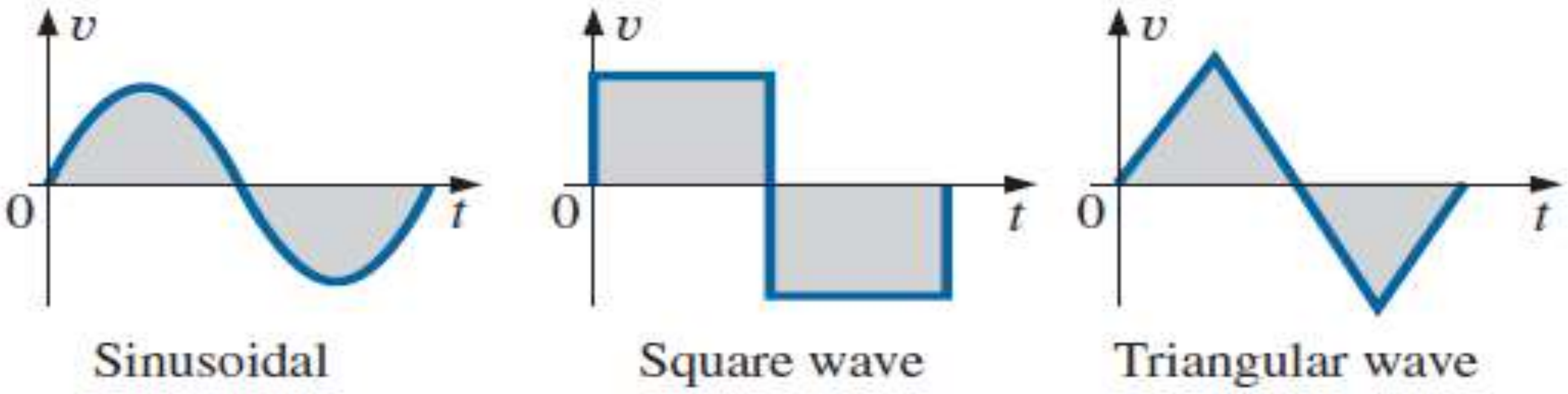


Fig-1: Alternating waveforms.

Our discussion will mostly be about the **Sinusoidal Alternating waveform**.

Sinusoidal AC Voltage Characteristics & Definitions

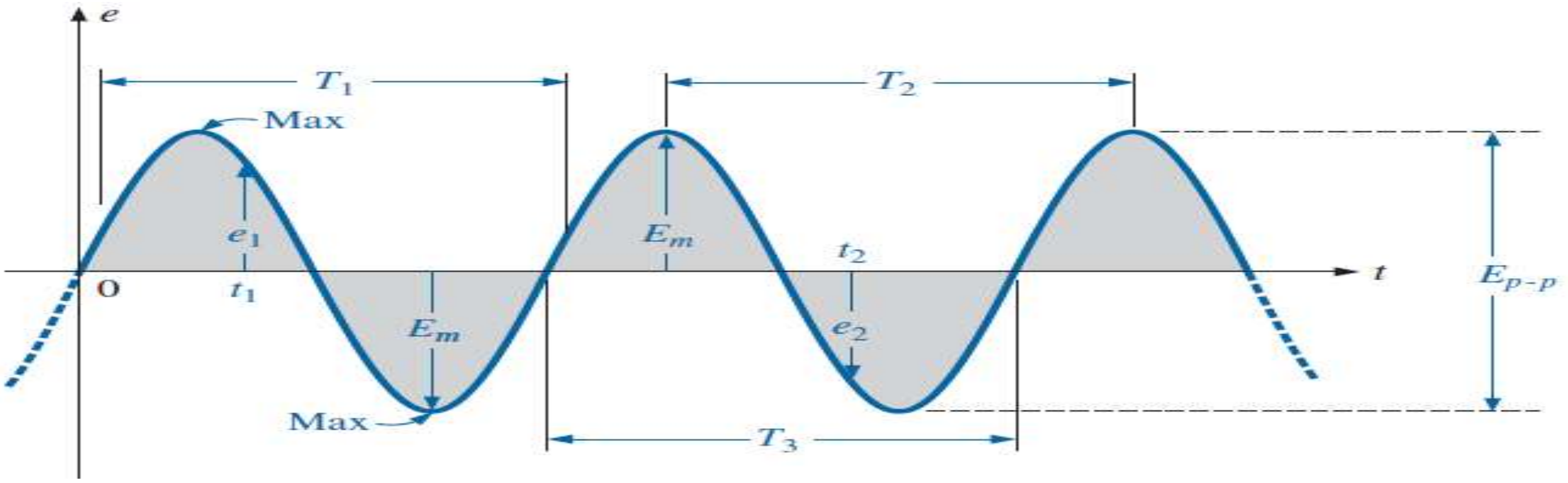


Fig-2: Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage (as in Fig-2), plotted as a function of some variable such as time (as above).

Instantaneous value: The magnitude of a waveform at any instant of Time. (denoted by e_1, e_2 in Fig-2)

Peak amplitude: The maximum value of a waveform as measured from its *average,* or *mean,* value.

(denoted by E_m in Fig-2)

Peak value: The maximum instantaneous value of a function as measured from the zero volt level. For the waveform in Fig-2, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Sinusoidal AC Voltage Characteristics & Definitions (cont.)

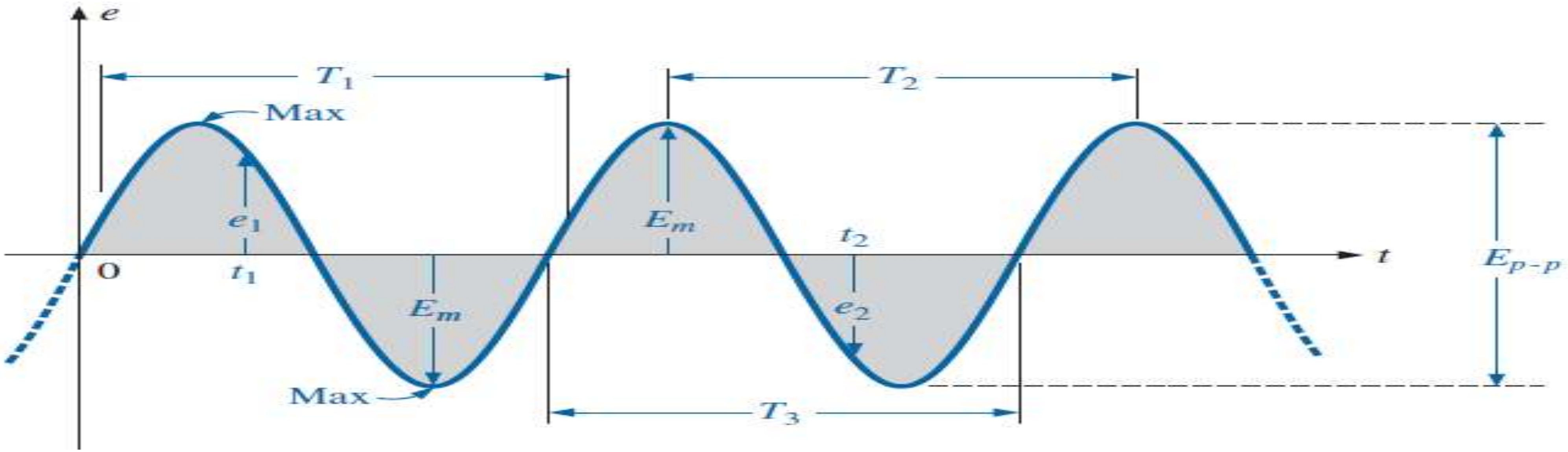


Fig-3: Important parameters for a sinusoidal voltage.

Peak-to-peak value: Denoted by $E_{p\text{-}p}$ or $V_{p\text{-}p}$ (as shown in Fig-3), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform in Fig. 13.3 is a periodic waveform.

Period (*T*): The time of a periodic waveform.

Sinusoidal AC Voltage Characteristics & Definitions (cont.)

Cycle: The portion of a waveform contained in one period of time. The cycles within T_1, T_2, and T_3 in Fig-4 they are all bounded by one period of time and therefore satisfy the definition of a cycle.

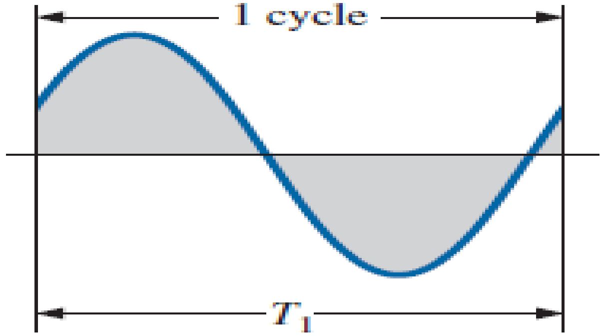


Fig-3: Cycle of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform in Fig-4(a) is 1 cycle per second, and for Fig-4(b) is $2\frac{1}{2}$ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig-4(c)], the frequency would be 2 cycles per second.

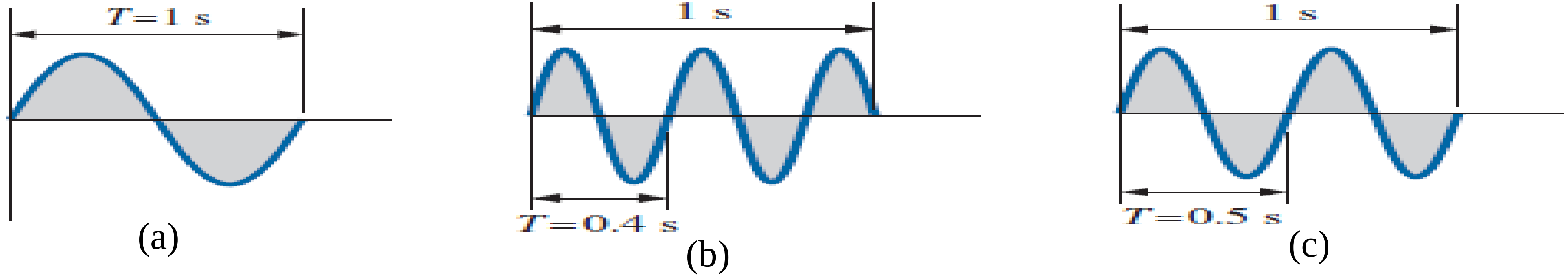


Fig-4: *Changing frequency on the period of a sinusoidal waveform.*

Sinusoidal AC Voltage Characteristics & Definitions (cont.)

For the sinusoidal waveform in Figure: 1,

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

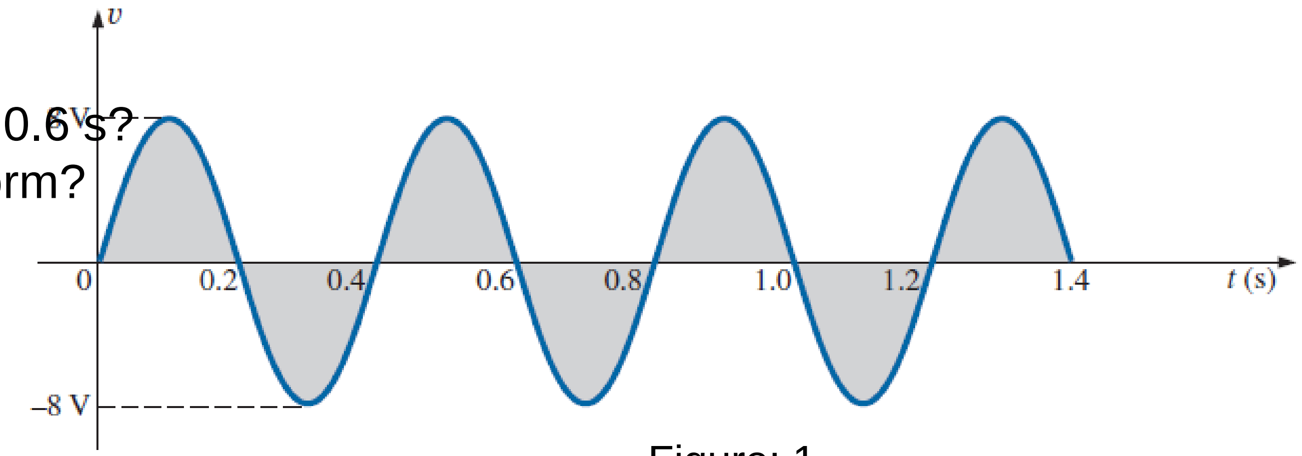


Figure: 1

a. **8 V.**
b. At 0.3 s, **8 V;** at 0.6 s, **0 V.**
c. **16 V.**
d. **0.4 s.**
e. **3.5 cycles.**
f. **2.5 cps,** or **2.5 Hz.**

General Format for the Sinusoidal Voltage or Current

- The basic mathematical format for the
  Sinusoi [illegible] is

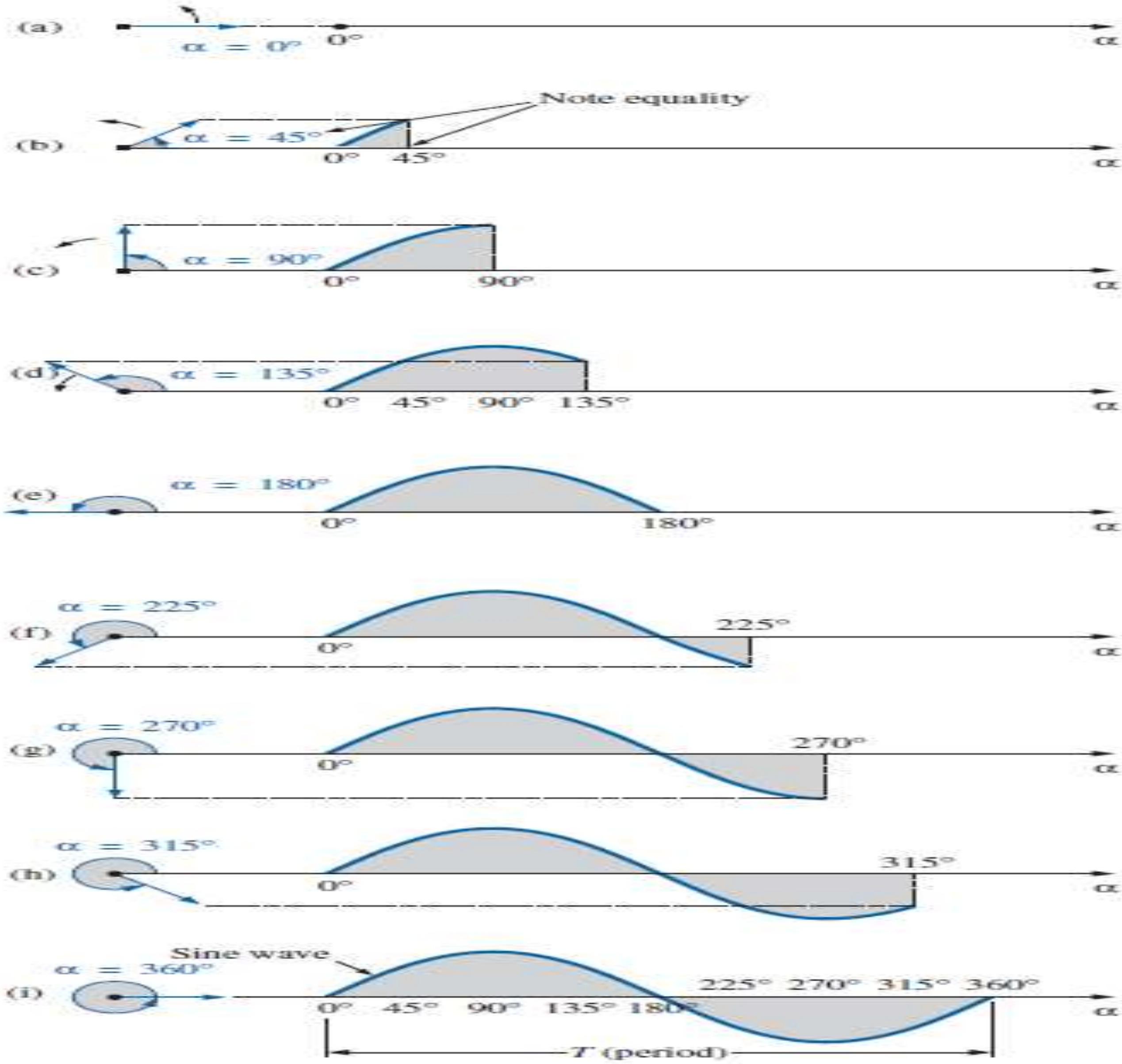
$$\boxed{A_m \sin \alpha}$$

- Since, a sinusoidal wave can be generated through the vertical projection of a rotating vector, α (angular displacement) can be written as

$$\boxed{\alpha = \omega t}$$

ω= Angular Velocity of the rotating vector

$$\boxed{\omega = \frac{2\pi}{T}} \qquad \text{(rad/s)}$$



General Format for the Sinusoidal Voltage or Current (cont.)

- For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript *m* represent the amplitude, and the lowercase letters *i* and *e* represent the instantaneous value of current and voltage, respectively, at any time *t.*

- The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha$$

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\boxed{\alpha = \sin^{-1} \frac{e}{E_m}}$$

Similarly, for a particular current level,

$$\boxed{\alpha = \sin^{-1} \frac{i}{I_m}}$$

Determination of Phase Relation between Waveforms of the same Frequency

- Thus far, we have considered only sine waves that have maxima at π/2 and 3π /2, with a zero value at 0, π, and 2π, as shown in figure:1.

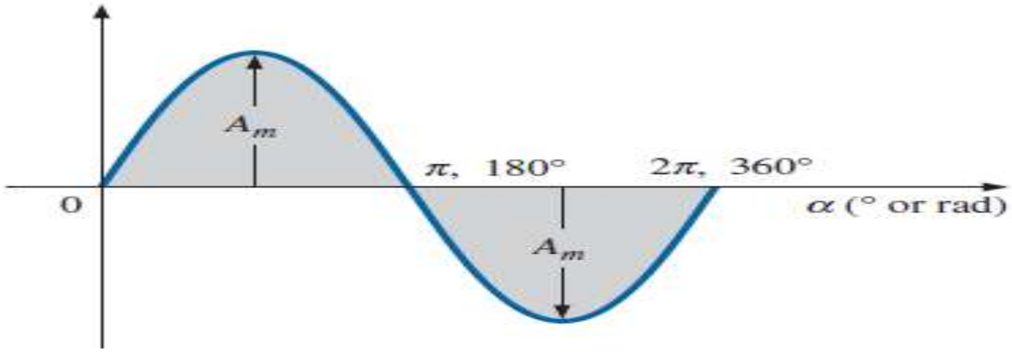


Figure: 1

- If the waveform is shifted to the right or left of 0°, the expression becomes,

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

In case of, $\boxed{A_m \sin(\omega t + \theta)}$

At $\omega t = 0°$, the magnitude is determined by $\boldsymbol{A_m \sin\theta}$ (Figure:1).

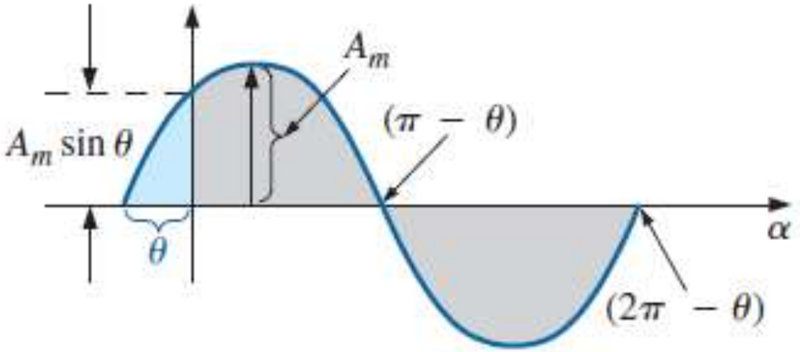


Figure: 1

In case of, $\boxed{A_m \sin(\omega t - \theta)}$

At $\omega t = 0°$, the magnitude is determined by $\boldsymbol{A_m \sin(\theta -) = -A_m \sin\theta}$ (Figure:2).

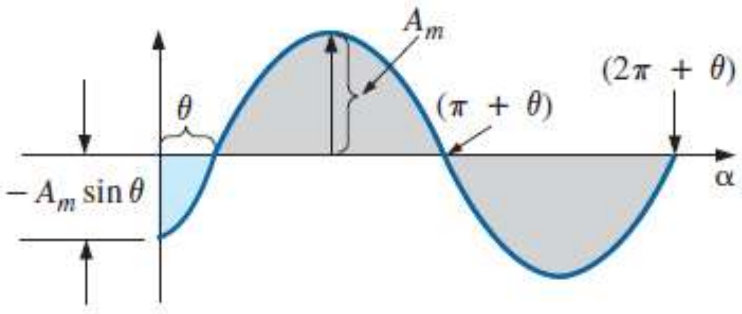


Figure: 2

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

- If we compare the previous two cases with a sinusoidal wave that originates from 0,

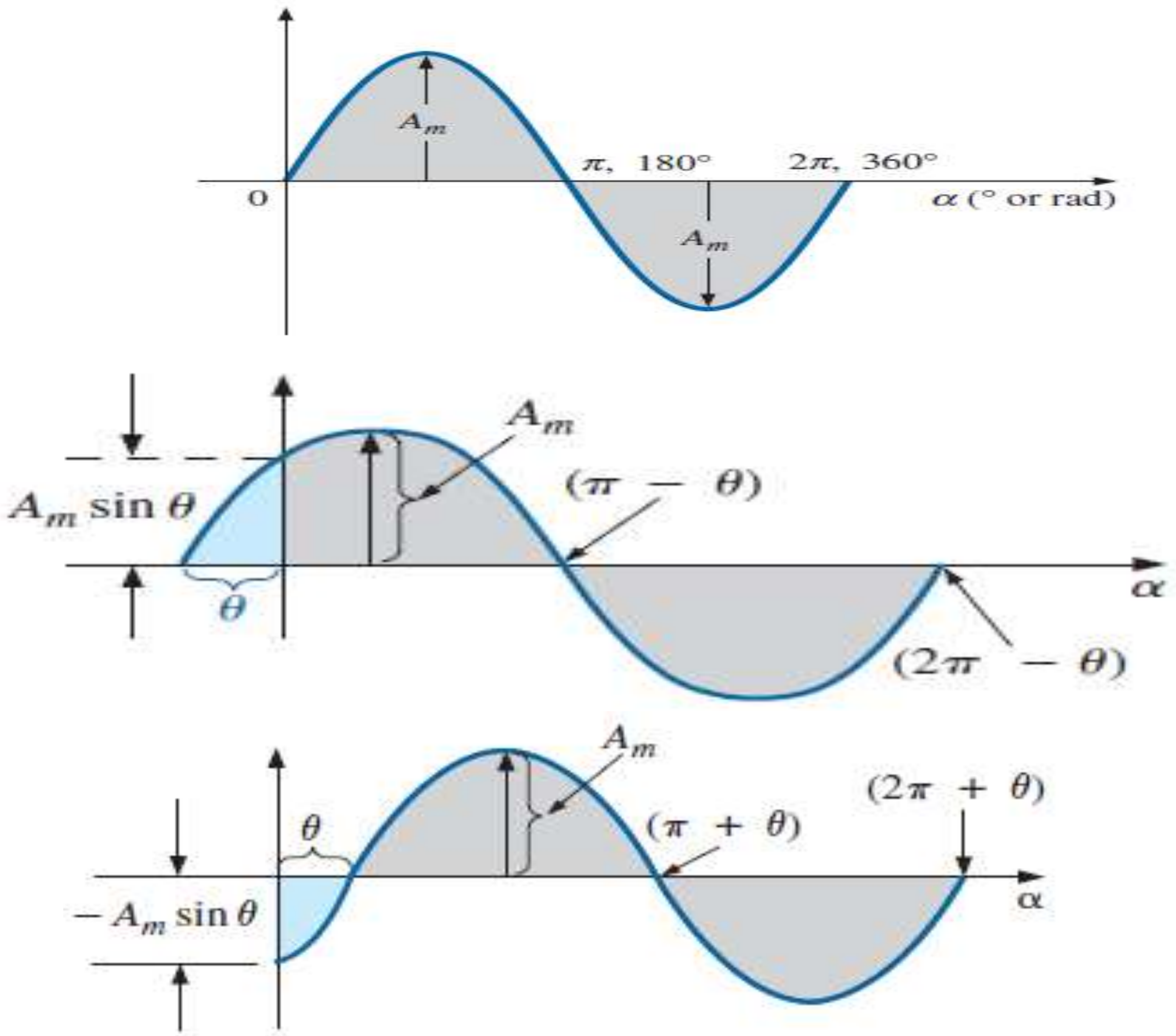


Figure: 1

Figure: 2

Figure: 3

- The waveform in Figure: 2 begins an angle θ ***before*** the waveform in Figure: 1. Therefore, it ***leads*** the waveform in Figure: 1 by an angle θ.

- The waveform in Figure: 3 begins an angle θ ***after*** the waveform in Figure: 1. Therefore, it ***lags*** the waveform in Figure: 1 by an angle θ.

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

- The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the ***same frequency*** plotted on the same set of axes.
- If both waveforms cross the axis at the same point with the same slope, they are ***in phase***.
- Figure: 1 shows the phase relationship between a ***sine wave*** and ***cosine wave.***

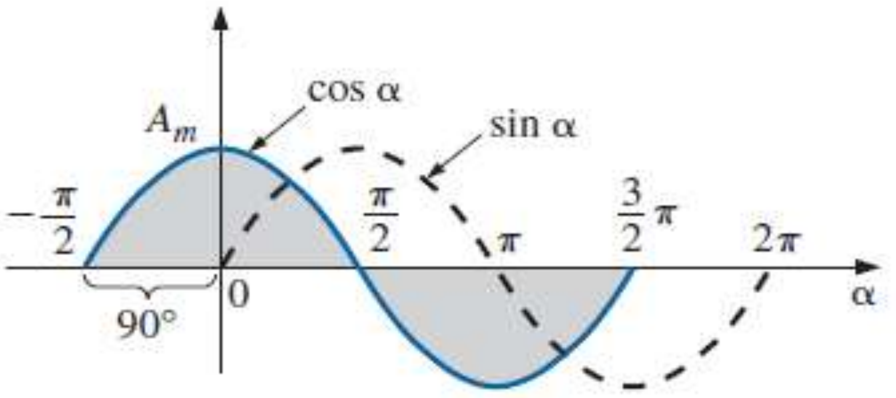


Figure: 1

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

- The cosine wave leads the sine wave by 90 degrees.

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

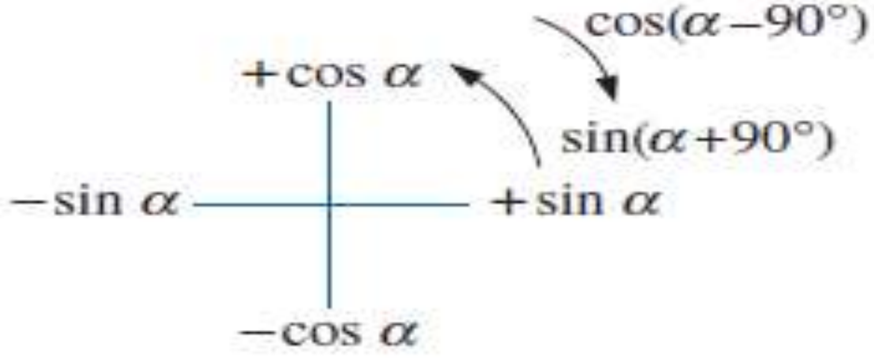


Figure: Graphic tool for finding the relationship between specific sine and cosine functions.

$$
\begin{aligned}
\cos\alpha &= \sin(\alpha + 90^\circ)\\
\sin\alpha &= \cos(\alpha - 90^\circ)\\
-\sin\alpha &= \sin(\alpha \pm 180^\circ)\\
-\cos\alpha &= \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)\\
&\text{etc.}
\end{aligned}
$$

In addition, note that

$$
\begin{aligned}
\sin(-\alpha) &= -\sin\alpha\\
\cos(-\alpha) &= \cos\alpha
\end{aligned}
$$

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

- ***How to determine the phase relationship between the sinusoidal waveforms?***

- Make sure the waveforms are of the same frequency.

- When determining the phase relationship between two waveforms make sure they are both in the same format. That is, both should be either in sine or cosine format.

- Both waveforms should either have positive or negative amplitude. If one has positive amplitude and the other has negative, any one of them should be shifted 180 degrees to make both positive (or negative).

Determination of Phase Relation between Waveforms of the same Frequency (cont.)

- Let's workout an example,

What is the phase relationship between the sinusoidal waveforms of the following set?

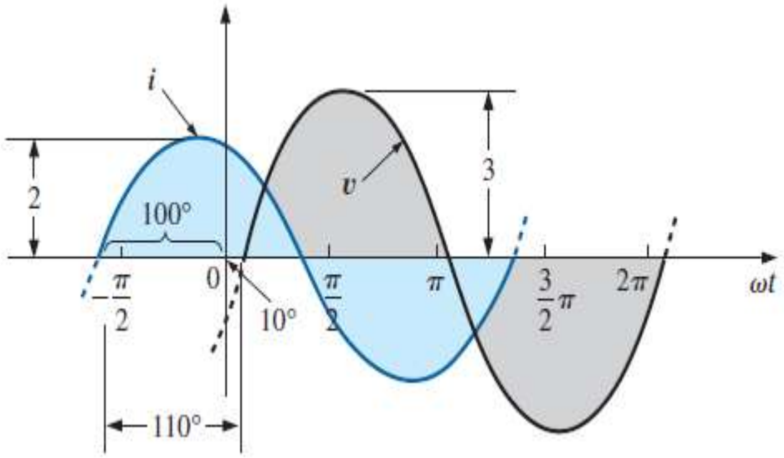
$$i = 2\cos(\omega t + 10°)$$
$$v = 3\sin(\omega t - 10°)$$

$$\cos\alpha = \sin(\alpha + 90°)$$
$$\sin\alpha = \cos(\alpha - 90°)$$
$$-\sin\alpha = \sin(\alpha \pm 180°)$$
$$-\cos\alpha = \sin(\alpha + 270°) = \sin(\alpha - 90°)$$
etc.

We can write,

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

i **leads** ***v*** **by 110º, or** ***v*** **lags** ***i*** **by 110º.**



Determination of Phase Relation between Waveforms of the same Frequency (cont.)

- Let's workout another example,

What is the phase relationship between the sinusoidal waveforms of the following set?

$$i = -\sin(\omega t + 30°)$$
$$v = 2\sin(\omega t + 10°)$$

We can write,

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - 180°)$$
$$= \sin(\omega t - 150°)$$

***v* leads *i* by 160º, or *i* lags *v* by 160º.**

Or using

Note

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° + 180°)$$
$$= \sin(\omega t + 210°)$$

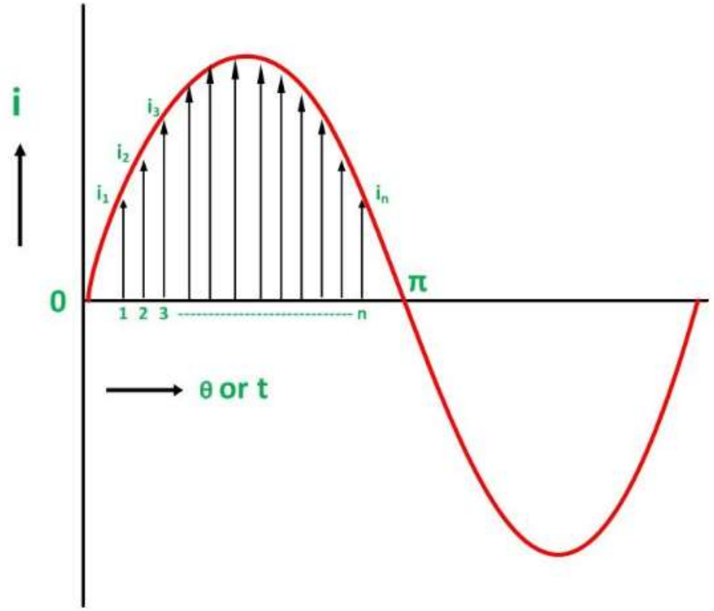
***i* leads *v* by 200º, or *v* lags *i* by 200º.**

$$\cos\alpha = \sin(\alpha + 90°)$$
$$\sin\alpha = \cos(\alpha - 90°)$$
$$-\sin\alpha = \sin(\alpha \pm 180°)$$
$$-\cos\alpha = \sin(\alpha + 270°) = \sin(\alpha - 90°)$$

etc.

Average Value of Alternating Voltage/Current

The average of all the instantaneous values of an alternating voltage and currents over one complete cycle is called **Average Value.** That is, the ratio of the sum of all considered instantaneous values to the number of instantaneous values in one alternation period.

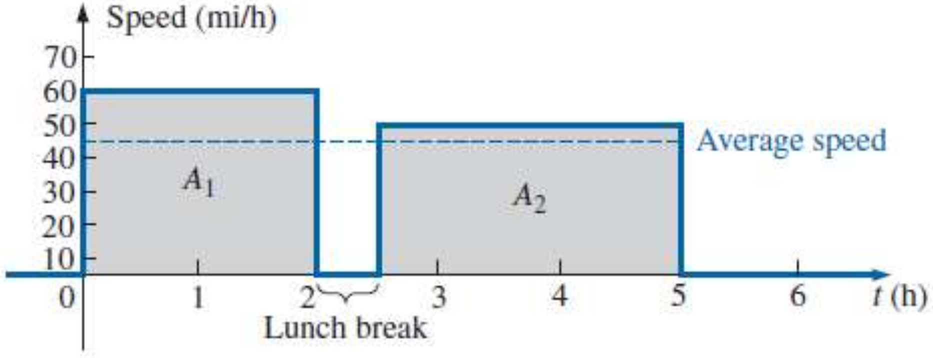


$$I_{av} = \frac{i_1 + i_2 + i_3 + \dots\dots\dots + i_n}{n} = \frac{\text{Area of alternation}}{\text{Base}}$$

Average Value of Alternating Voltage/Current (Cont.)

To understand the concept better, let's workout an example.

- The figure below shows ***speed versus time*** for an automobile excursion.



- We have to determine the average speed of the car through the entire journey.
- In order to measure the Average speed, we need to sum all the instantaneous speeds through the entire trip (5 hours) and divide the sum by the total number of the instances (total time of the trip).
- As we observe the figure carefully, we see that the sum of all the instantaneous speeds upto 2 hrs is actually the integration of all the values with limit 0 to 2 , the result of which is the area A_1.
- Therefore, by finding the total area under the curve for the 5 h and then dividing the area by 5 h (the total time for the trip), we can obtain the average speed of the car.

Average Value of Alternating Voltage/Current (Cont.)

We can use the same principle to find the average value of a voltage/current waveform. The formula for that can be written as:

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

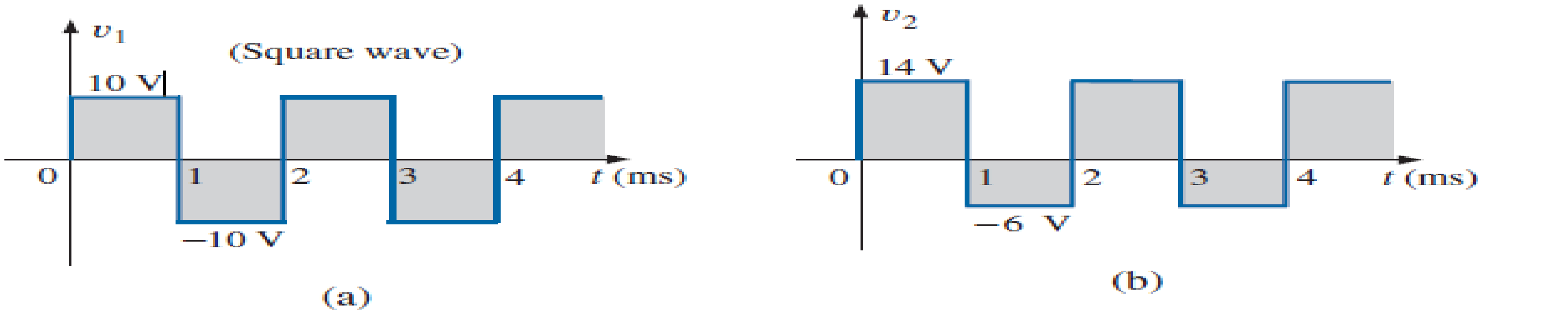


Fig: Voltage vs Time Curve of two Alternating Voltage waveforms

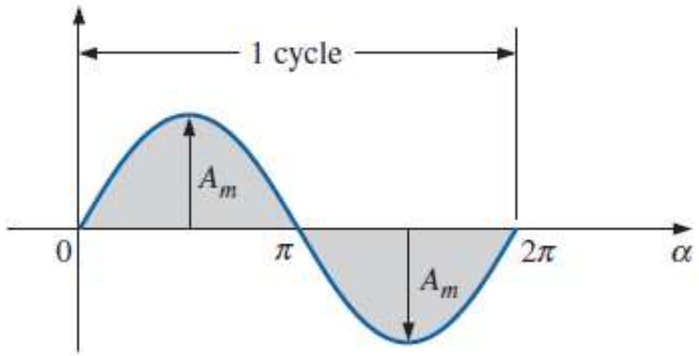
By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using the above equation,

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

Using the above equation for Fig (b),

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

Average Value of Alternating Voltage/Current (Cont.)



- If we consider ***symmetrical waves*** like sinusoidal current or voltage waveform, the positive half cycle will be exactly equal to the negative half cycle. Therefore, the average value over a complete cycle will be **zero**.

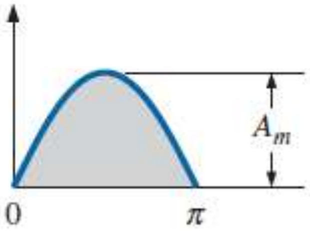
- The area and average value of the positive or negative half cycle can be determined by integration.

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

Integrating, we obtain

$$\begin{aligned}\text{Area} &= A_m[-\cos\alpha]_0^{\pi}\\ &= -A_m(\cos\pi - \cos 0^\circ)\\ &= -A_m[-1-(+1)] = -A_m(-2)\end{aligned}$$

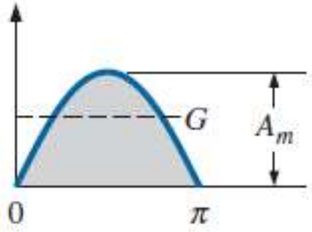
$$\boxed{\text{Area} = 2A_m}$$



$$G = \frac{2A_m}{\pi}$$

and

$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m}$$

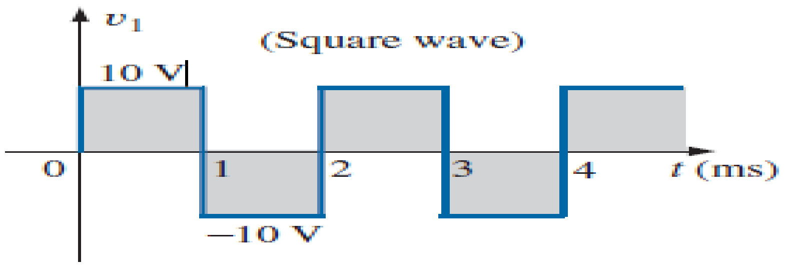


Average Value of Alternating Voltage/Current (Cont.)

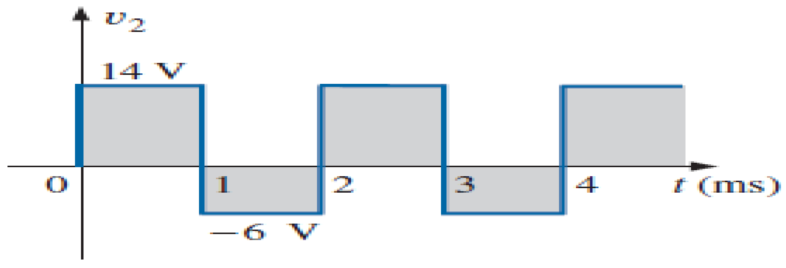
- What will happen when the waveform is ***periodic but NOT symmetrical***?

Average Value of Alternating Voltage/Current (Cont.)

For example, let’s observe a previous example,



(a)

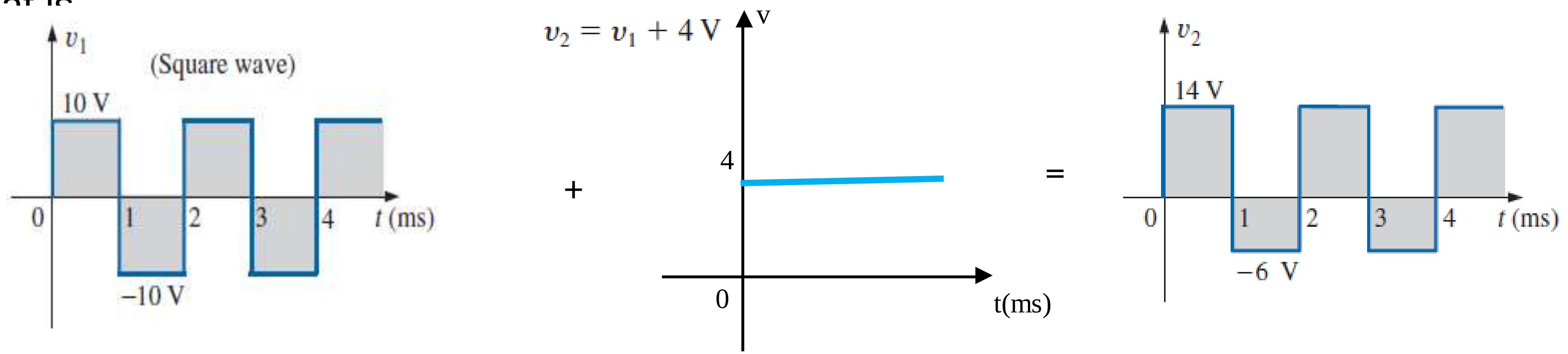


(b)

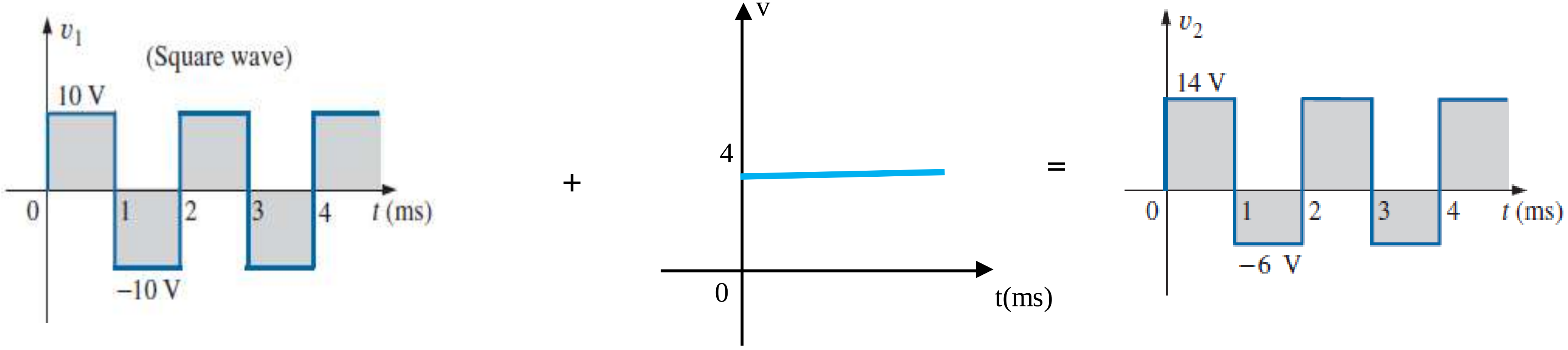
- Average value for these waveforms are,

$$G_a = 0\text{ V} \;\&\; G_b = 4\text{ V}$$

- In reality, the waveform in Fig (b) is simply the square wave in Fig (a) with a dc shift of 4 V; that is



Average Value of Alternating Voltage/Current (Cont.)



$$v_2 = v_1 + 4\text{ V}$$

- If we remove the dc value (pull the waveform of v_2 down 4V) from v_2 , the wave form will return to its original form(v_1).
- We can conclude that, the average value of ***any*** current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value.

Average Value of Alternating Voltage/Current (Cont.)

Let's workout another example,

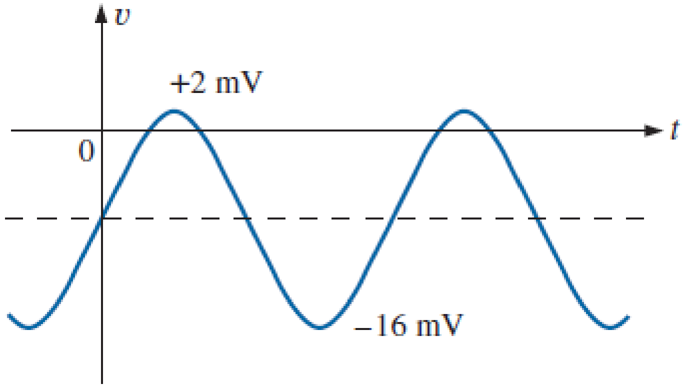


Figure: 1

- This is a sinusoidal waveform but it is not symmetrical. So, the average of the waveform will not be 0.
- The peak-to-peak value of the sinusoidal function is 16 mV +2 mV= 18 mV.
- The peak amplitude (A_m) of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV
- That means, if the waveform were to be symmetrical it would've had 9 mV above and below the horizontal axis (Figure:2).

- If the waveform of figure: 2 is pulled down 7V along the negative y-axis, it will resemble the waveform of figure: 1. Therefore, the average value of the waveform in figure: 1 is -7V.

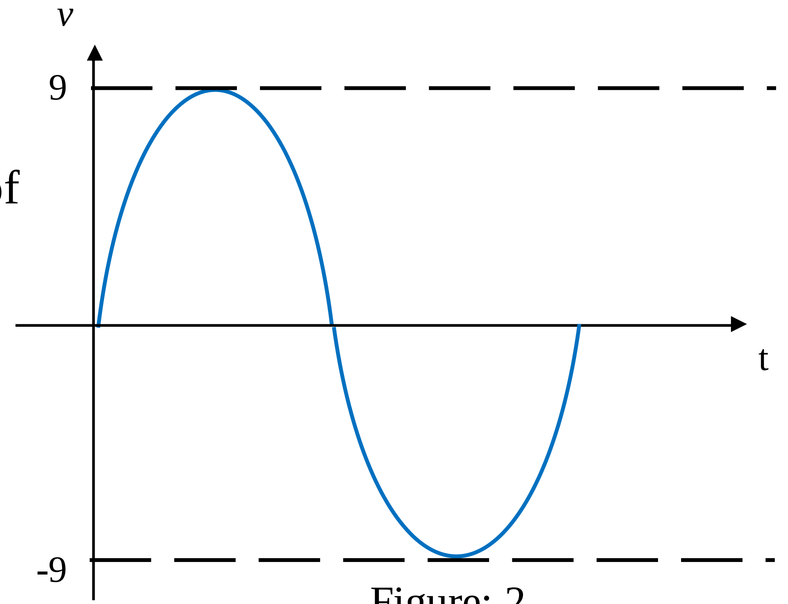


Figure: 2

Effective Value of Alternating Voltage/Current

- ***The equivalent dc value of any alternating voltage or current defined as its Effective value.***

Which means,

- Effective value of the alternating current/voltage is that value of the ***dc current/voltage*** for which the amount of net power delivered to a load of known value for a given period of time equals to the amount of net power delivered by the alternating current/voltage to the same load for the same period of time

Effective Value of Alternating Voltage/Current (cont.)

- ***How is it possible for a sinusoidal ac quantity to deliver a net power if, over a full cycle, the net current in any one direction is zero (average value 0)?***

- It would almost appear that the power delivered during the positive portion of the sinusoidal waveform is withdrawn during the negative portion, and since the two are equal in magnitude, the net power delivered is zero.
- However, understand that regardless of *direction*, current of any magnitude through a resistor delivers power *to that resistor.* We know,

$$P = I^2 R$$

- Therefore, a net power will be delivered to the load regardless the direction of the current flow.
- The power delivered at each instant, of course, varies with the magnitude of the sinusoidal ac current, but there will be a net flow during either the positive or the negative pulses with a net flow over the full cycle.
- The net power flow equals twice that delivered by either the positive or the negative regions of sinusoidal quantity.

Effective Value of Alternating Voltage/Current (cont.)

A fixed relationship between ac and dc voltages and currents can be derived from the experimental setup shown in Figure: 1.

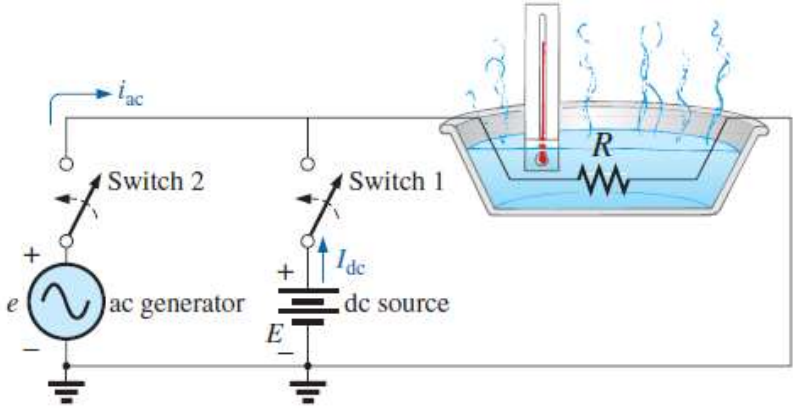


Figure: 1

- When switch 1 is closed the load R receives power from the DC source.
- When switch 2 is closed the load R receives power from the AC source.
- The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2R = (I_m \sin \omega t)^2R = (I_m^2 \sin^2 \omega t)R$$

However,

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonometric identity)}$$

Therefore,

$$P_{ac} = I_m^2\left[\frac{1}{2}(1 - \cos 2\omega t)\right]R$$

and

$$\boxed{P_{ac} = \frac{I_m^2R}{2} - \frac{I_m^2R}{2}\cos 2\omega t}$$

Effective Value of Alternating Voltage/Current (cont.)

$$P_{\text{ac}} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

- The *average power* delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform.
- In order to find the effective value we equate the average power delivered by the ac generator to that delivered by the dc source,

$$P_{\text{av(ac)}} = P_{\text{dc}}$$

$$\frac{I_m^2 R}{2} = I_{\text{dc}}^2 R$$

and

$$I_{\text{dc}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

The equivalent dc value is called the **rms** or **effective value** of the sinusoidal quantity.

Effective Value of Alternating Voltage/Current (cont.)

➢ Steps to determine RMS value of a non-sinusoidal waveform.

Consider the waveform of Figure: 1.

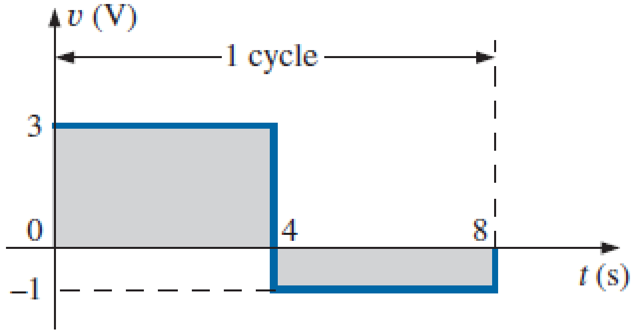


Figure: 1

Step 1: Obtain the square of the waveform (Figure: 2).
Step 2: Obtain the average (mean) of the squared waveform.
Step 3: Obtain the square root of the average (mean) of the squared waveform.

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$

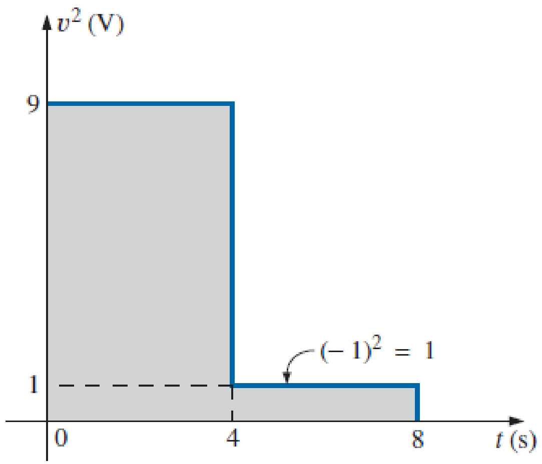


Figure: 2

Numerical Problems/ Home Work

- Practice all the related exercises and examples.

Reference: Chapter 13 (Sinusoidal Alternating Waveforms)
Introductory Circuit Analysis
Robert L Boylestad

Any Questions?