Name: Omar Faruk

Registration num: 2019831055

SWE-123

Part - A

Ans: to the que: No:-1

a

A minimum spanning tree is a special kind of tree that minimizes the lengths of the edges of the tree.

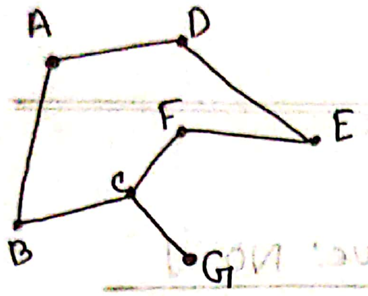
A spanning tree of a graph is a tree that:

(i) contains all the original graph's vertices.

(ii) Reach out to all vertices.

(iii) The graph doesn't have any ~~loop~~ nodes which loop back to itself.

Planar graph: A planar graph is a graph which can be drawn in the plane without any of the edges crossing over, that is, meeting at points Other than the vertices.



This is a Planar graph.

Ans: to the que: No-1

b

A relation R on a set A is ~~called~~ said to be reflexive if every element of A is related to itself.

R reflexive if $\Leftrightarrow (a,a) \in R \ \forall a \in A$

According to the definition, $A = \{1,2,3,4\}$ for both R_1 and R_2.

The relations R_1 and R_2 are reflexive because they both contain all pairs of the form (a,a), namely, (1,1), (2,2), (3,3), (4,4)

Symmetric: A relation R on a set A is said to be symmetric relation iff.

$(a,b) \in R \Rightarrow (b,a) \in R \ , \ \forall \ a,b \in A$

Let the matrix representation of R_1 and R_2 be M_{R_1} and M_{R_2} where,

$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$(M_{R_1})^t = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \text{ and } (M_{R_2})^t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Here, $M_{R_1} = (M_{R_1})^t$, therefore R_1 is symmetric.

and

$M_{R_2} \neq (M_{R_2})^t$

so, R_2 is not symmetric.

Anti-symmetric:

The matrix of an anti-symmetric relation has the Property that if $m_{ij} = 1$ with $i \neq j$, then $m_{ji} = 0$

M_{R_1} doesn't satisfy the condition for being anti-symmetric, because,

$$(M_{R_1})_{12} = (M_{R_1})_{21} = 1$$

$\therefore R_1$ is not anti-symmetric.

On the contrary, R_2 satisfies the condition for being anti-symmetric as there is no such i, j, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ for which $(M_{R_2})_{ij} = (M_{R_2})_{ji}$ with $i \neq j$

So, R_2 is anti-symmetric.

Transitive:

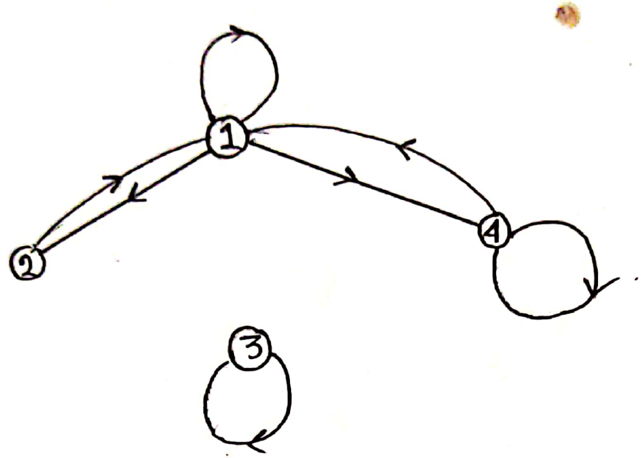
Let A be any set, A relation R on A is said to be a transitive relation iff

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$$
$$\forall\, a, b, c \in A$$

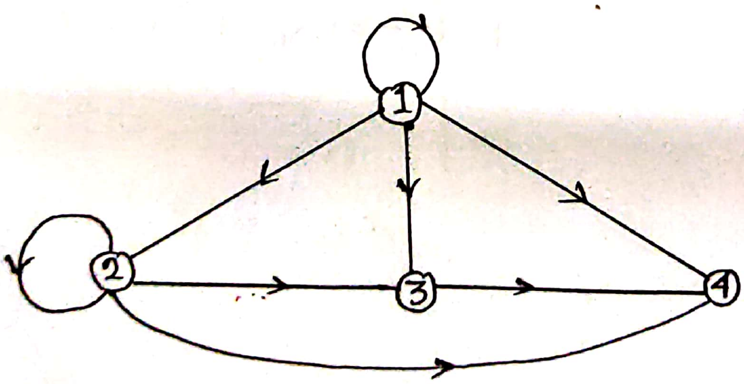
R_1 is not transitive because,
$(1,1) \in R_1$ and $(3,3) \in R_1$ but $(1,3) \notin R_1$

On the other hand, R_2 is transitive, because $(1,1)$ and $(1,2)$, $(1,1)$ and $(1,3)$, $(1,1)$ and $(1,4)$, $(1,2)$ and $(2,2)$, $(1,2)$ and $(2,3)$, $(1,2)$ and $(2,4)$, $(1,3)$ and $(3,3)$, $(1,3)$ and $(3,4)$, ~~and~~ $(1,4)$ and are the only such sets of pairs, and (1,2), (1,3), (1,4), (1,2), (1,3), (1,4), (1,3), (1,4) and (3,4) belong to R_2

The representation of the relation R_1 in directed is shown below:



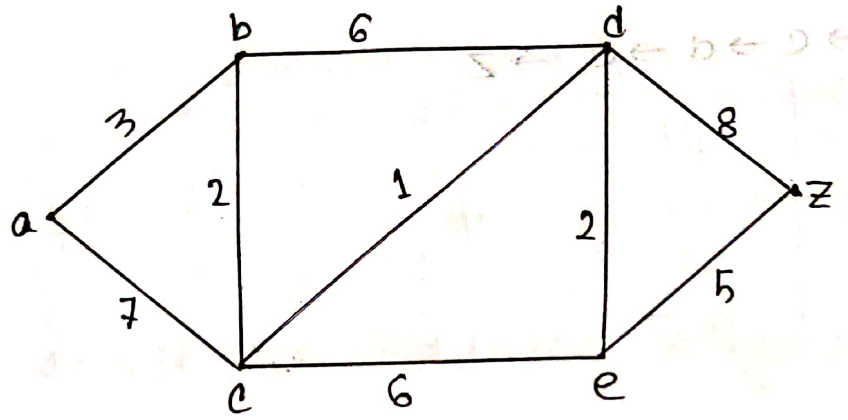
The representation of the relation R_2 in directed is ~~shown~~ shown below.



Ans: to the que: No - 1

c

Given graph in figure 2 is-



In Dijkstra's algorith, if G is graph and u and v are two adjacent vertices, then the distance of v will be distance of u and weight of (u,v), we will update the distance if -

$$(d(u)+c(u,v) < d(v))$$

	a	b	c	d	e	Z
a	$\boxed{0}$	∞	∞	∞	∞	∞
b		$\boxed{3}$	7	∞	∞	∞
c			$\boxed{5}$	9	∞	∞
d				$\boxed{6}$	11	∞
e					$\boxed{8}$	14
						$\boxed{13}$

So, using ~~dij~~ Dijkstra algorithm, the shortest path from a to z is 13, and the path is -

$z \rightarrow e \rightarrow d \rightarrow c \rightarrow b \rightarrow a$.

In reverse,

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow z$