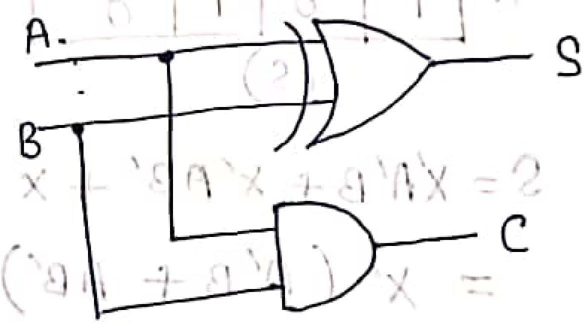
Combinational Logic Circuits

- n number of input → 2^n possible combinations → 2^n possible output combinations

HALF ADDER:



A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

K-Map:

	B	B′
A′	1	0
A	0	1

(S)

	B	B′
A′	0	0
A	1	0

(C)

$$S = A'B + AB'$$
$$= A \oplus B$$

$$C = AB$$

FULL ADDER:

X	A	B	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

K-Map: (For S)

	$A'B'$	$A'B$	AB	AB'
X'	0	1	0	1
X	1	0	1	0

(S)

$$S = X'A'B + X'AB' + XA'B' + X \cdot AB$$
$$= X'(A'B + AB') + X(AB + A'B')$$
$$= X'(A \oplus B) + X(A \oplus B)'$$
$$= \boxed{X \oplus A \oplus B}$$

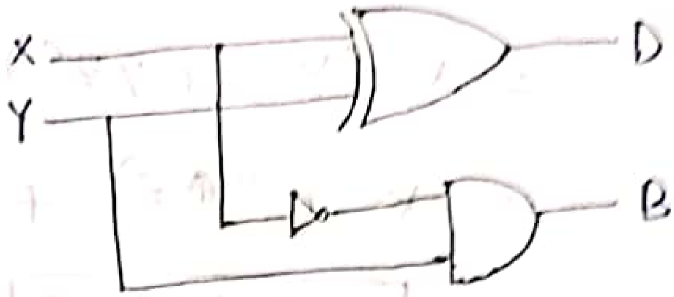
K-Map: (For C)

	$A'B'$	$A'B$	AB	AB'
X'	0	0	1	0
X	0	1	1	1

$$C = XB + XA + AB$$
$$= AB + X(A + B)$$
$$= AB + X\{A(B + B') + B(A + A')\}$$
$$= AB + X(AB + AB' + AB + A'B)$$
$$= AB + XAB + X(AB' + A'B)$$
$$= AB(X + 1) + X \cdot (A \oplus B)$$

$$\boxed{= AB + X(A \oplus B)} \qquad [\because X + 1 = 1]$$

Half - Subtractor :



X	Y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

D :

	Y	Y'
X'	1	0
X	0	1

$D = XY' + X'Y$

$= X \oplus Y$

B :

	Y	Y'
X'	1	0
X	0	0

$B = X'Y$

Full - Subtractor :

X	Y	Z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

D :

	Y'Z'	Y'Z	YZ	YZ'
X'	0	1	0	1
X	1	0	1	0

$D = X'(Y'Z + YZ') + X(Y'Z' + YZ)$

B :

	Y'Z'	Y'Z	YZ	YZ'
X'	0	1	1	1
X	0	0	1	0

$B = X'(Y + Z) + YZ$

Modifying the Equation of D and B:

$$D = x'y'z + x'yz' + xy'z' + xyz$$

$$= x'(y'z + yz') + x(yz + y'z')$$

$$= x'(y \oplus z) + x(y \oplus z)'$$

$$= \boxed{x \oplus y \oplus z}$$

$$B = x'z + x'y + yz$$

$$= x'y + z(x' + y)$$

$$= x'y + z\{x'(y + y') + y(x + x')\}$$

$$= x'y + z(x'y + x'y' + xy + x'y)$$

$$= x'y + z(x'y + x'y' + xy)$$

$$= x'y + z(xy + x'y')$$

$$= x'y + x'yz + z(xy + x'y')$$

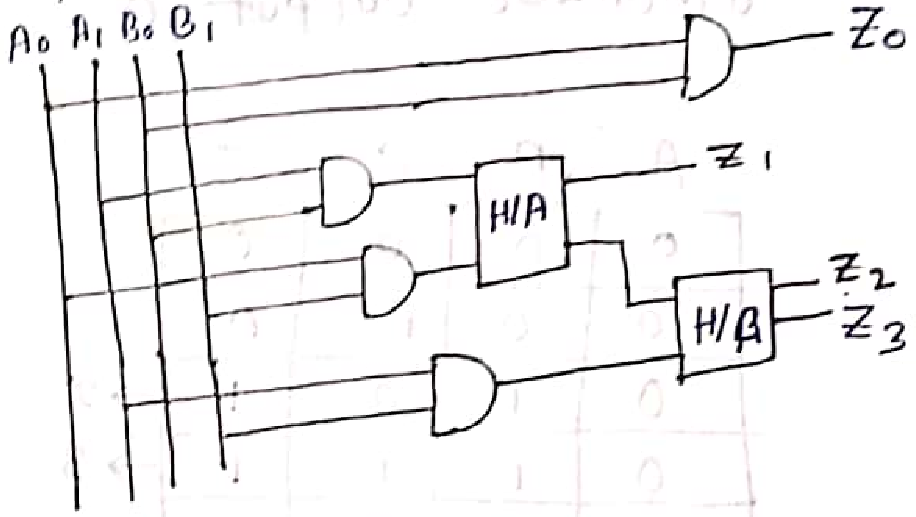
$$= x'y(1 + z) + z(x \oplus y)'$$

$$= \boxed{x'y + z \cdot (x \oplus y)'}$$

Design a combinational circuit that multiplies two 2 bit numbers.

Input: $A = A_1 A_0$ and $B = B_1 B_0$

$$
\begin{array}{cccc}
 & & A_1 & A_0 \\
 & \times & B_1 & B_0 \\
\hline
 & & A_1 B_0 & A_0 B_0 \\
 & + & & \\
 & A_1 B_1 & A_0 B_1 & \\
\hline
 & Z_2 & Z_1 & Z_0
\end{array}
$$



* Carry of the first HA flows to the next HA as an input bit.

* The sum bit of the second HA is Z_2 and carry bit is Z_3.

2) Design a circuit that has a 3-bit binary input and a single output that output 1 if it is a prime number. (2, 3, 5, 7); otherwise output 0.

A	B	C	O	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\rightarrow 2$
0	1	1	1	$\rightarrow 3$
1	0	0	0	
1	0	1	1	$\rightarrow 5$
1	1	0	0	
1	1	1	1	$\rightarrow 7$

K-Map

	$B'C'$	$B'C$	BC	BC'
A'	0	0	1	1
A	0	1	1	0

$$O = BC + AC + A'B$$

$$O = AC + B\cdot(A' + C)$$

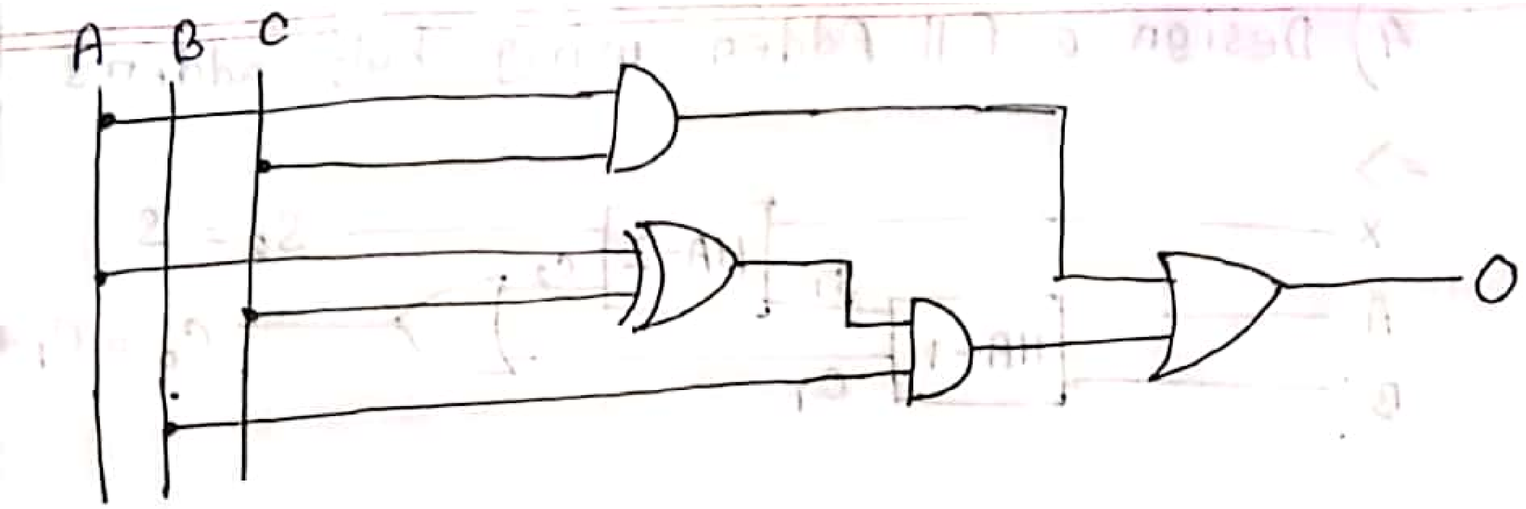
$$= AC + B\{A'(C+C') + C(A+A')\}$$

$$= AC + B(A'C + AC' + AC)$$

$$= AC + ABC + B(A'C + AC')$$

$$= AC(1+B) + B(A \oplus C)$$

$$= AC + B(A \oplus C)$$



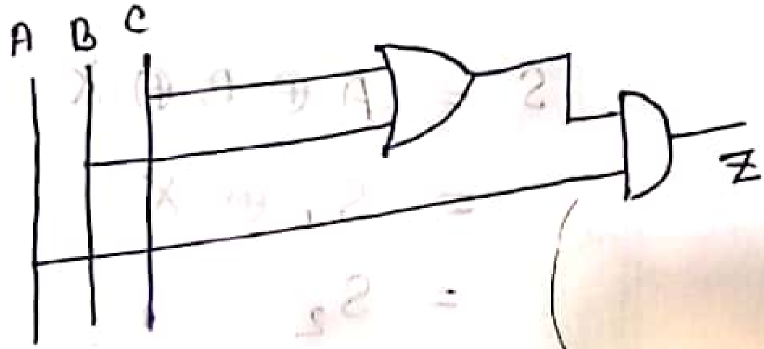
3) Design a circuit that has a 3-bit binary input and a single output (z) specified as follows

$Z = 0$, when the input is less than 5_{10}

$Z = 1$, otherwise

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

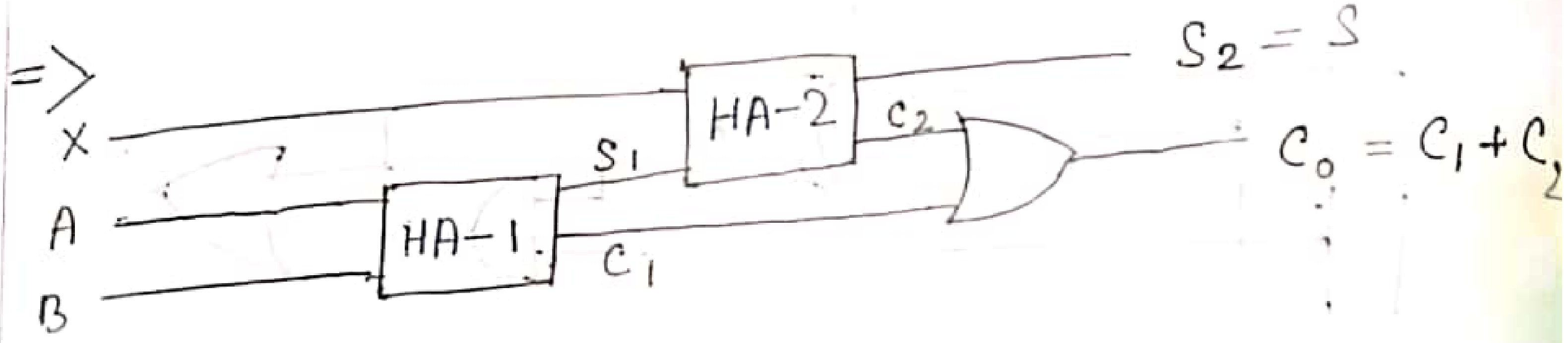
$$Z = A \cdot C + AB$$
$$= A(B+C)$$



	B'C'	B'C	BC	BC'
A'	0	0	0	0
A	0	1	1	1

4) Design a Full Adder using half adders

=>



HA 1 :

$S_1 = A \oplus B$ —— (1)

$C_1 = AB$ —— (2)

HA 2 :

$S_2 = S_1 \oplus X$ —— (3)

$C_2 = S_1 \cdot X$ —— (4)

from the equation of full-adder, we get,

$\boxed{S} = A \oplus B \oplus X$

$= S_1 \oplus X$

$= S_2$

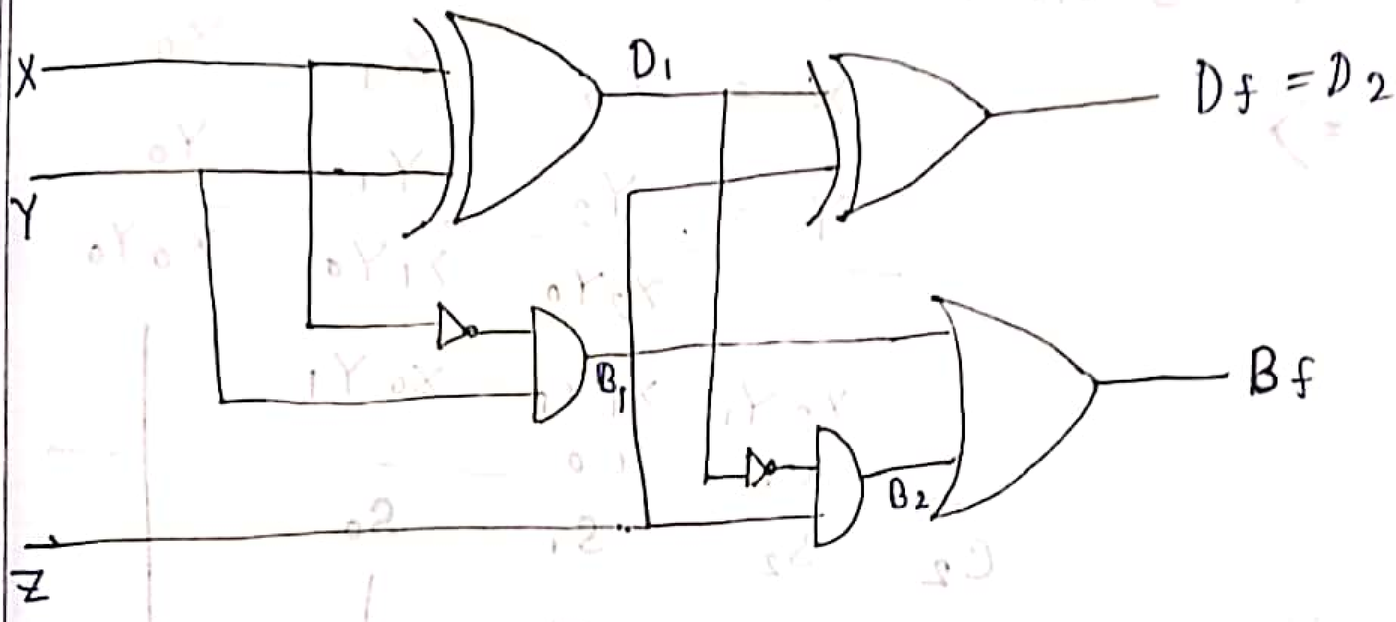
$C_0 = X(A \oplus B) + AB$

$= X \cdot S_1 + C_1$

$= C_1 + C_2$

5) Implement a full subtractor using half subtractors

=>



HS 1 : $D_1 = X \oplus Y$

$B_1 = X' Y$

HS 2 : $D_2 = D_1 \oplus Z$

$B_2 = D_1' Z$

from the equation of full-subtractor, we get,

$D_f = X \oplus Y \oplus Z = D_1 \oplus Z = D_2$

$B_f = X'Y + Z(X \oplus Y)'$

$= B_1 + D_1' Z$

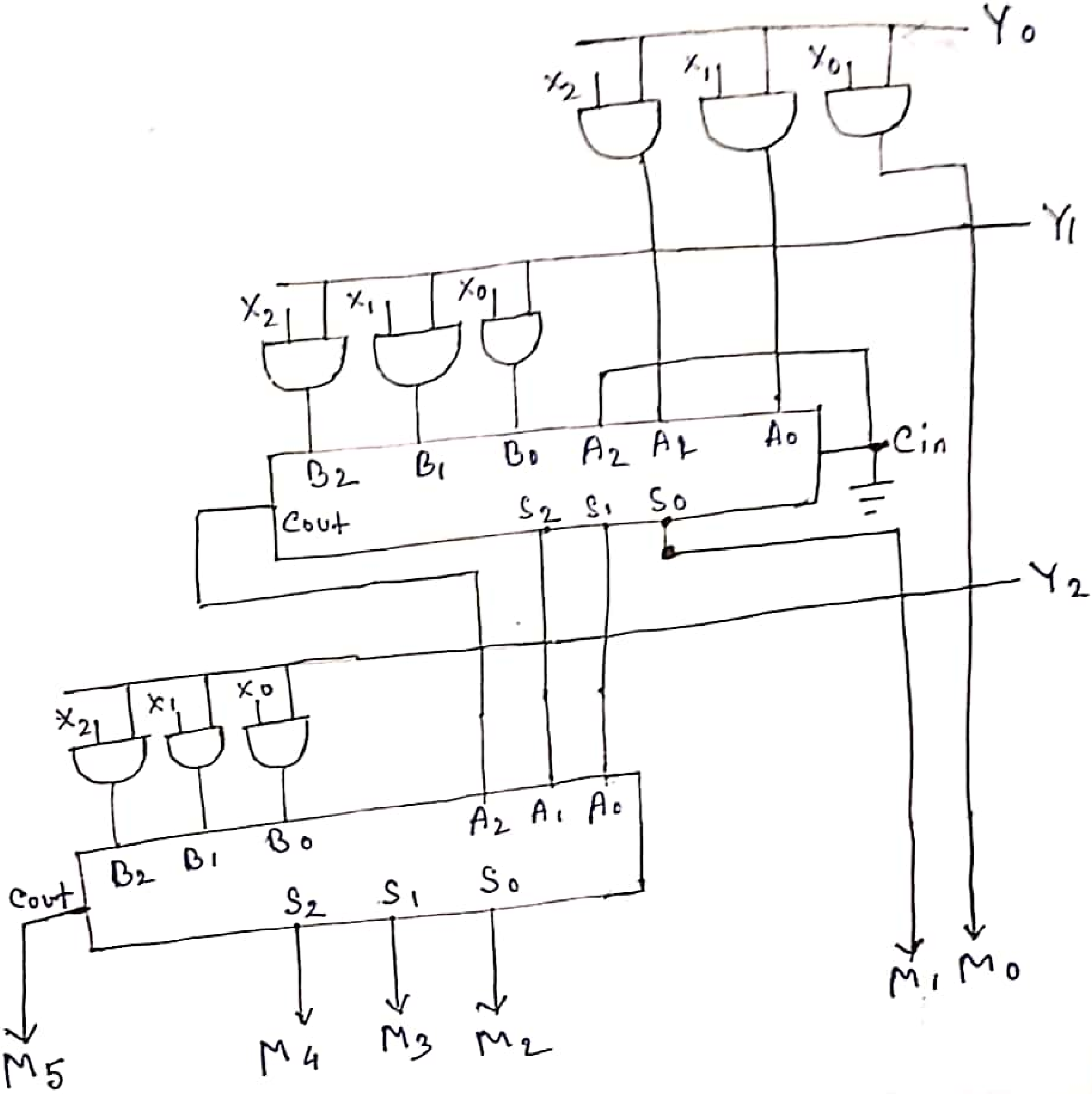
$= B_1 + B_2$

6) Design a combinational circuit that multiplies two 3 bit numbers.

$\Rightarrow$

$$
\begin{array}{cccccc}
 & & X = X_2 & X_1 & X_0 & \\
 & & Y = Y_2 & Y_1 & Y_0 & \\
\hline
 & & X_2Y_0 & X_1Y_0 & X_0Y_0 & \\
 & X_2Y_1 & X_1Y_1 & X_0Y_1 & & \\
 & C_1 & C_0 & & & \\
\hline
C_2 & S_2 & S_1 & S_0 & & \\
X_2Y_2 & X_1Y_2 & X_0Y_2 & & & \\
C_4 & C_3 & & & & \\
\hline
\end{array}
$$

$$
\begin{array}{cccccc}
C_5 & S_5 & S_4 & S_3 & \downarrow & \downarrow \\
\downarrow & \downarrow & \downarrow & \downarrow & & \\
M_5 & M_4 & M_3 & M_2 & M_1 & M_0
\end{array}
$$



:(figure)