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MAT-107W

Part: A

Ans: to the que: NO-1(a)

given the matrice -

$$X = \begin{bmatrix} x & -x & 4 \\ x & 3 & -x \\ 6 & -3x & -5 \end{bmatrix}$$, where x is any real number.

If . $X = Y + Z$, where Y is symmetric and Z is skew-symmetric then,

$$y_{ij} = \frac{x_{ij} + x_{ji}}{2}, \quad z_{ij} = \frac{x_{ij} - x_{ji}}{2}, \text{ for all } 1 \leq i \,;\, j \leq n$$

$$\begin{bmatrix} x & -x & 4 \\ x & 3 & -x \\ 6 & -3x & -5 \end{bmatrix} = \begin{bmatrix} x & 0 & 5 \\ 0 & 3 & -2x \\ 5 & -2x & -5 \end{bmatrix} + \begin{bmatrix} 0 & -x & -1 \\ x & 0 & x \\ 1 & -x & 0 \end{bmatrix}$$

Ans: to the que: NO-1(b)

Given that,

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & -1 & 0 & 6 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

(i)

(i) Here,

$$R_2 \leftarrow R_2 - 2R_1 \quad \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

$$R_1 \leftarrow 5R_1 - 3R_4 \quad \begin{pmatrix} 14 & -9 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

$$R_1 \leftarrow R_1 - 9R_2 \quad \begin{pmatrix} 14 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} 14 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

(ii)

Here,

$$|B| = 5 \times |A|$$

$$\Rightarrow 14 \times (-1) \times 3 \times 5 = 5 \times |A|$$

$$\Rightarrow |A| = -42$$

$$\therefore |A| = -42$$

So, the determinant of $|A|$ is -42

(iii)

Here,

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & -1 & 0 & 6 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

Therefore, $A^T = \begin{pmatrix} 1 & 2 & 0 & -3 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ 3 & 6 & 0 & 5 \end{pmatrix}$

$$\xrightarrow{R_4 \leftarrow R_4 - 3R_1} \begin{pmatrix} 1 & 2 & 0 & -3 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 14 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 0 & -3 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 14 \end{pmatrix}$$

Here,

$|B| = |A^T|$

$\Rightarrow 1 \times (-1) \times 3 \times 14 = |A^T|$

$\Rightarrow |A^T| = -42$,

$\therefore |A^T| = -42$, which is equal to the $|A|$

Therefore, $|A^T| = -42$

(iv)

Here,

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & -1 & 0 & 6 \\ 0 & 2 & 3 & 0 \\ -3 & 3 & 0 & 5 \end{pmatrix}$$

Adjoining I_4 to A from the argumented matrix $[A|I_4]$

we have,

$$[A|I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 6 & 0 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 0 & 0 & 1 & 0 \\ -3 & 3 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$

$R_4 \leftarrow -R_4 - 3R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & -3 & 0 & -14 & -3 & 0 & 0 & -1 \end{array}\right]$$

$R_3 \leftarrow R_3 + 2R_2$

$R_4 \leftarrow -R_4 + 3R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 0 & 14 & -3 & 3 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow 14R_1 - 3R_4$ $\left[\begin{array}{cccc|cccc} 14 & 0 & 0 & 0 & 23 & -9 & 0 & -3 \\ 0 & -1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 0 & 14 & -3 & 3 & 0 & 1 \end{array}\right]$

$R_1 \leftarrow \frac{1}{14}R_1$

$R_2 \leftarrow -R_2$

$R_3 \leftarrow \frac{1}{3}R_3$

$R_4 \leftarrow \frac{1}{14}R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{23}{14} & -\frac{9}{14} & 0 & -\frac{3}{14} \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{4}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{3}{14} & \frac{3}{14} & 0 & \frac{1}{14} \end{array}\right]$$

Apply $[A \mid I_4] \sim [I_4 \mid A^{-1}]$ to find A^{-1}

Then,

$$A^{-1} = \begin{bmatrix} \frac{23}{14} & -\frac{9}{14} & 0 & -\frac{3}{14} \\ 2 & -1 & 0 & 0 \\ -\frac{4}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ -\frac{3}{14} & \frac{3}{14} & 0 & \frac{1}{14} \end{bmatrix}$$

Transforming A to row echelon form.

$R_1 \leftarrow 14R_1$

$R_3 \leftarrow 3R_3$

$R_4 \leftarrow 14R_4$

$$\begin{bmatrix} 23 & -9 & 0 & -3 \\ 2 & -1 & 0 & 0 \\ -4 & 2 & 1 & 0 \\ -3 & 3 & 0 & 1 \end{bmatrix}$$

$R_2 \leftarrow 23R_2 - 2R_1$

$R_3 \leftarrow 23R_3 + 4R_1$

$R_4 \leftarrow 23R_4 + 3R_1$

$$\begin{bmatrix} 23 & -9 & 0 & -3 \\ 0 & -5 & 0 & 6 \\ 0 & 10 & 23 & -12 \\ 0 & 42 & 0 & 14 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 2R_2$

$R_4 \leftarrow 5R_4 + 42R_2$

$$\begin{bmatrix} 23 & -9 & 0 & -3 \\ 0 & -5 & 0 & 6 \\ 0 & 0 & 23 & 0 \\ 0 & 0 & 0 & 322 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 23 & -9 & 0 & -3 \\ 0 & -5 & 0 & 6 \\ 0 & 0 & 23 & 0 \\ 0 & 0 & 0 & 322 \end{bmatrix}$$

Here,

$$14 \times 3 \times 14 \times 23 \times 23 \times 23 \times 5 \times |A^{-1}| = |B|$$

$$\therefore |A^{-1}| = -\frac{1}{42}$$

Therefore the determinant of inverse A,

$$|A^{-1}| = -\frac{1}{42}$$

Ans: to the que; NO-2 (a)

Rewriting the linear system of equations in matrice form.

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Making on Augmented matrix.

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 0 \\ 2 & -3 & 1 & -1 \\ 1 & -2 & -2 & 1 \end{array}\right]$$

Using row operations on the augmented matrix -

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 0 \\ 2 & -3 & 1 & -1 \\ 1 & -2 & -2 & 1 \end{array}\right] \begin{array}{l} \\ R_2' \leftarrow R_2 - 2R_1 \\ R_3' \leftarrow R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc:c} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & -1 \\ 0 & -3 & -1 & 1 \end{array}\right] \begin{array}{l} \\ R_2' \leftarrow R_2 - 2R_1 \\ R_3' \leftarrow -R_1 + R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & -1 \\ 0 & 1 & -\frac{14}{5} & 8/5 \end{array}\right] R_3' \leftarrow R_3 - \frac{3}{5} R_2$$

The corresponding system.

$$\left.\begin{aligned} x+y-z&=0 \\ -5y+3z&=-1 \\ -\frac{14}{5} z &= \frac{8}{5} \end{aligned}\right\}$$

Here,

$$-\frac{14}{5} z = \frac{8}{5} \Rightarrow z = -\frac{4}{7}$$

again,

$$-5y+3\left(-\frac{4}{7}\right) = -1 \Rightarrow -5y = -1 + \frac{12}{17}$$

$$\Rightarrow -5y = \frac{5}{7} \quad \Rightarrow y = -\frac{1}{7}$$

Also,

$$x - \frac{1}{7} + \frac{4}{7} = 0$$

$$\therefore x = -\frac{3}{7}$$

As the system has a unique solution, the system is consistent.

Ans: to the que: NO-2 (b)

The rank of a matrix is the dimension of the vector space generated by its columns. This corresponds to the minimum number of linearly independent columns of the matrix.

Given,

$$M = \begin{bmatrix} 2 & -1 & -1 & 3 \\ 4 & -3 & -1 & 1 \\ 6 & -2 & -1 & -1 \\ 2 & -1 & 2 & -12 \end{bmatrix}$$

Transforming the matrix M to row echelon form.

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

$R_4 \leftarrow R_4 - R_1$

$$\begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & -1 & 1 & -5 \\ 0 & 1 & 2 & -10 \\ 0 & 0 & 3 & -15 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & -1 & 1 & -5 \\ 0 & 0 & 3 & -15 \\ 0 & 0 & 3 & -15 \end{bmatrix}$$

$$R_4 \leftarrow R_4 - R_3 \quad \begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & -1 & 1 & -5 \\ 0 & 0 & 3 & -15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We know, rank of a matrix equals to the number of nonzero rows in the echelon form of the matrix.

Therefore, rank of M is 3.