EEE 101W

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Charge:

Charge is the fundamental property of forms of matter that exhibit electrostatic attraction or repulsion in the presence of other matter.

Charge is measured in coulombs (C).

- The coulomb is a large unit for charges. In 1 C of charge, there are $\frac{1}{1.602\times10^{-19}} = \mathbf{6.24\times10^{18}}$ electrons. Thus, realistic or laboratory values of charges are on the order of $\boldsymbol{pC, nC, or\ \mu C}$.
- According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $\boldsymbol{e = -1.602\times10^{-19}C}$.
- The law of conservation of charge states that charge can neither be created nor destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.

Current:

Electric current is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i, charge q, and time t is

$$\boldsymbol{i = \frac{dq}{dt}} \qquad \dots(\mathbf{1})$$

where current is measured in amperes (A), and

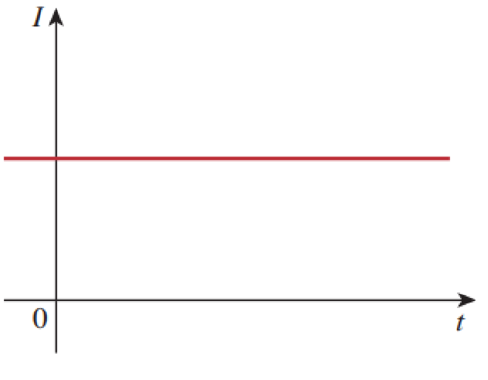
1 ampere = 1 coulomb/second

The charge transferred between time and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$\boldsymbol{Q = \int_{t_0}^{t} i\,dt} \qquad \dots(\mathbf{2})$$

DC Current:

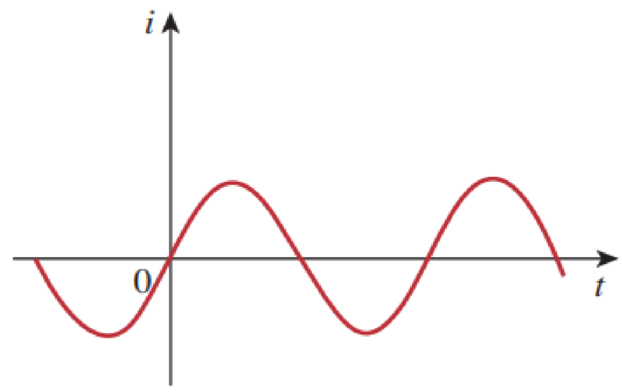
A direct current (dc) is a current that remains constant with time.



(a)

AC Current:

An alternating current (ac) is a current that varies sinusoidally with time.



(b)

Voltage:

Voltage, **electric potential difference**, **electric pressure** or **electric tension** is the difference in electric potential between two points, which is defined as the work needed per unit of charge to move a test charge between the two points.

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. below. This emf is also known as voltage or potential difference. The voltage between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$v_{ab} = \frac{dw}{dq} \qquad \dots (3)$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery.

1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

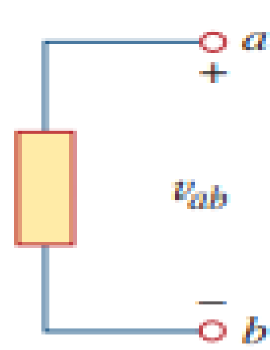
if a total of 1 joule (J) of energy is used to move the negative charge of 1 coulomb (C), there is a difference of 1 volt (V) between the two points.

Thus,

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

The defining equation is;

$$W = \frac{V}{Q} \qquad \dots (4)$$



This figure shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity.

The V_{ab} can be interpreted in two ways: (1) Point a is at a potential of V_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is V_{ab}. It follows logically that in general:

$$V_{ab} = -V_{ba}$$

Power:

Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as,

$$P = \frac{dw}{dt} \qquad \ldots(5)$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From equation (1), (3) and (5), it follows that

$$P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt} = vi \qquad \ldots(6)$$

The electrical unit of measurement for power is the watt (W) defined by

$$1\ watt\ (W) = 1\ joule\ /\ second\ (js^{-1})$$

In equation form, power is determined by

$$P = \frac{W}{t} \qquad \ldots(7)$$

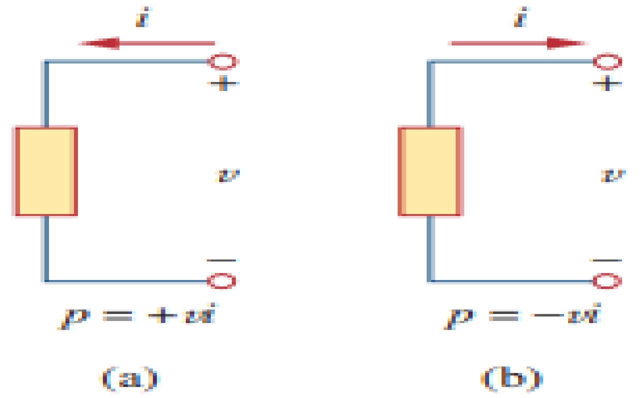
$$P = \frac{QV}{t} = V \times \frac{Q}{t} = VI \qquad \ldots(8)$$

$$P = VI = V \times \frac{V}{R} = \frac{V^2}{R} \qquad \ldots(9)$$

$$P = VI = (IR)I = I^2R \qquad \ldots(10)$$

1 horsepower = 746 watts

Passive sign convention is satisfied when the current enters through the positive terminal of an element and p = +vi. If the current enters through the negative terminal, p = -vi.



+ Power absorbed = - Power supplied

Energy:

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

1 Wh = 3,600 J

The energy (W) lost or gained by any system is therefore determined by

$$W = Pt \qquad \ldots(11)$$

Since power is measured in watts (or joules per second) and time in seconds, the unit of energy is the wattsecond or joule. The wattsecond, however, is too small a quantity for most practical purposes, so the watthour(Wh) and the kilowatthour(kWh) are defined, as follows:

$$Energy(Wh) = power(W) \times time(h) \qquad \ldots(12)$$

$$Energy(kWh) = \frac{power(W) \times time(h)}{1000} \qquad \ldots(13)$$

Dependent and independent sources

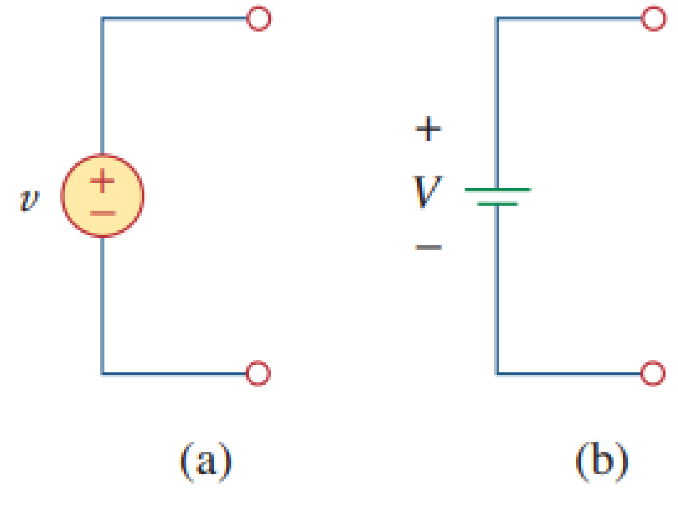
There are two kinds of sources: independent and dependent sources.

Independent Source:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

ideal independent voltage source:

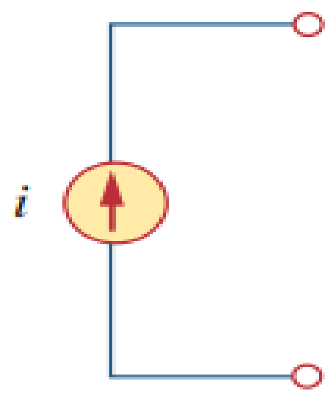
an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources.



Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

ideal independent current source:

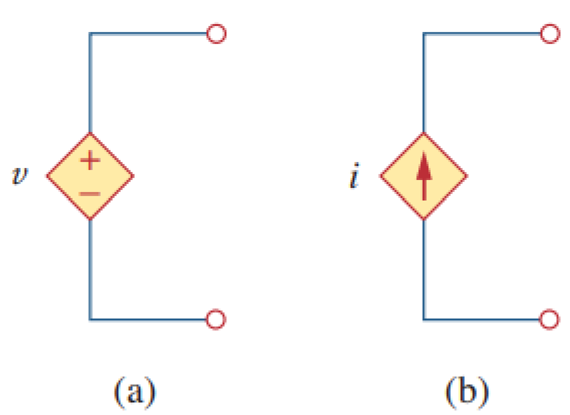
an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current.



Symbol for independent current source.

Dependent Source:

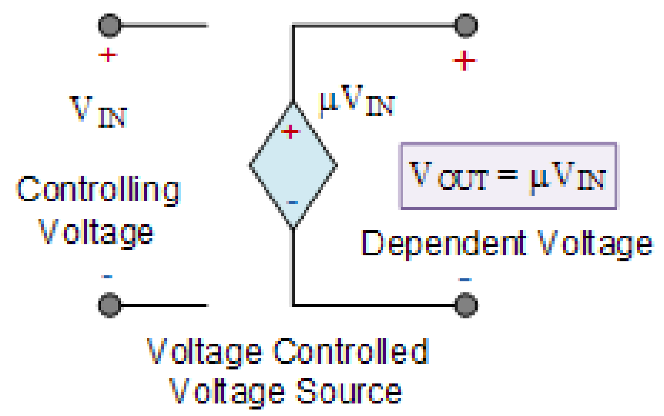
An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.



Symbols for: (a) dependent voltage source, (b) dependent current source.

Dependent sources are usually designated by diamond-shaped symbols. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

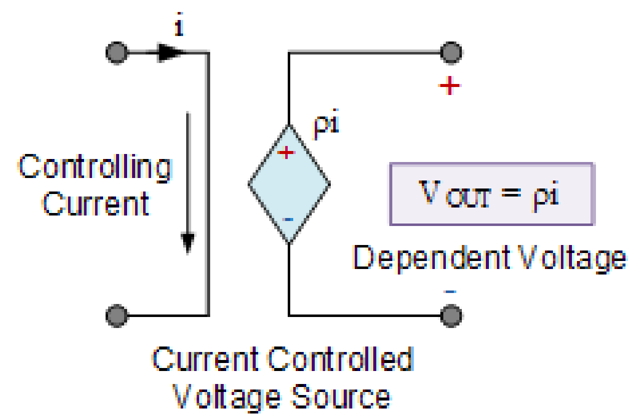
1. A voltage-controlled voltage source (VCVS).



An ideal dependent voltage-controlled voltage source, VCVS, maintains an output voltage equal to some multiplying constant (basically an amplification factor) times the controlling voltage present elsewhere in the circuit. As the multiplying constant is, well, a constant, the controlling voltage, V_{IN} will determine the magnitude of the output voltage, V_{OUT}. In other words, the output voltage "depends" on the value of input voltage making it a dependent voltage source and, in many ways, an ideal transformer can be thought of as a VCVS device with the amplification factor being its turns ratio.

Then the VCVS output voltage is determined by the following equation: $V_{OUT} = \mu V_{IN}$. Note that the multiplying constant μ is dimensionless as it is purely a scaling factor because $\mu = V_{OUT}/V_{IN}$, so its units will be volts/volts.

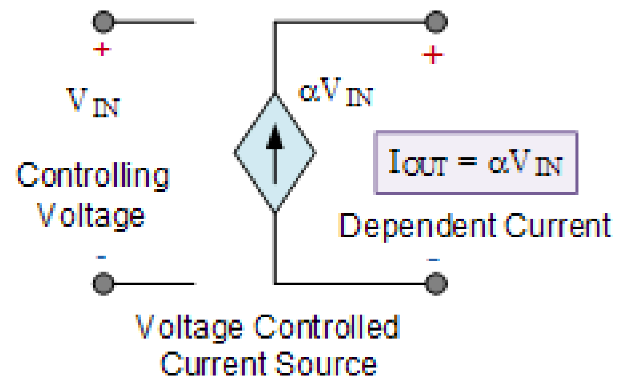
2. A current-controlled voltage source (CCVS).



An ideal dependent current-controlled voltage source, CCVS, maintains an output voltage equal to some multiplying constant (rho) times a controlling current input generated elsewhere within the connected circuit. Then the output voltage "depends" on the value of the input current, again making it a dependent voltage source.

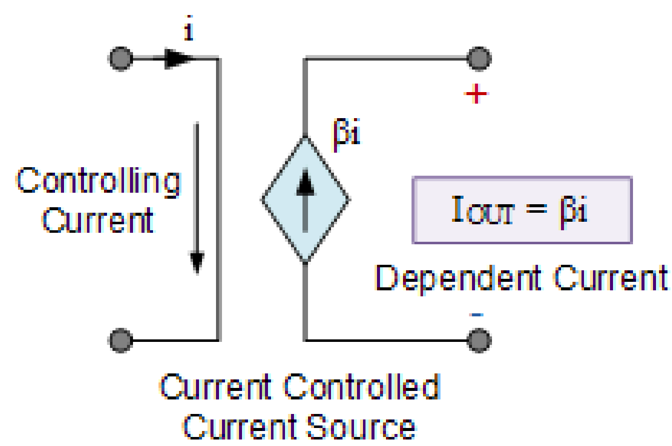
As a controlling current, I_{IN} determines the magnitude of the output voltage, V_{OUT} times the magnification constant ρ (rho), this allows us to model a current-controlled voltage source as a trans-resistance amplifier as the multiplying constant, ρ gives us the following equation: $V_{OUT} = \rho I_{IN}$. This multiplying constant ρ (rho) has the units of Ohm's because $\rho = V_{OUT}/I_{IN}$, and its units will therefore be volts/amperes.

3. A voltage-controlled current source (VCCS).



An ideal dependent voltage-controlled current source, VCCS, maintains an output current, I_{OUT} that is proportional to the controlling input voltage, V_{IN}. In other words, the output current "depends" on the value of input voltage making it a dependent current source.
Then the VCCS output current is defined by the following equation: $I_{OUT} = \alpha V_{IN}$.
This multiplying constant α (alpha) has the SI units of mhos, ℧ (an inverted Ohms sign) because $\alpha = I_{OUT}/V_{IN}$, and its units will therefore be amperes/volt.

4. A current-controlled current source (CCCS).



An ideal dependent current-controlled current source, CCCS, maintains an output current that is proportional to a controlling input current. Then the output current "depends" on the value of the input current, again making it a dependent current source.

As a controlling current, I_{IN} determines the magnitude of the output current, I_{OUT} times the magnification constant β (beta), the output current for a CCCS element is determined by the following equation: $I_{OUT} = \beta I_{IN}$. Note that the multiplying constant β is a dimensionless scaling factor as $\beta = I_{OUT}/I_{IN}$, so therefore its units would be amperes/amperes.

Ohm's Law:

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \qquad \dots (14)$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (14) becomes

$$v = iR \qquad \dots (15)$$

which is the mathematical form of Ohm's law. R in Eq. (15) is measured in the unit of ohms, designated (Ω). Thus,

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

We may deduce from Eq(15) that

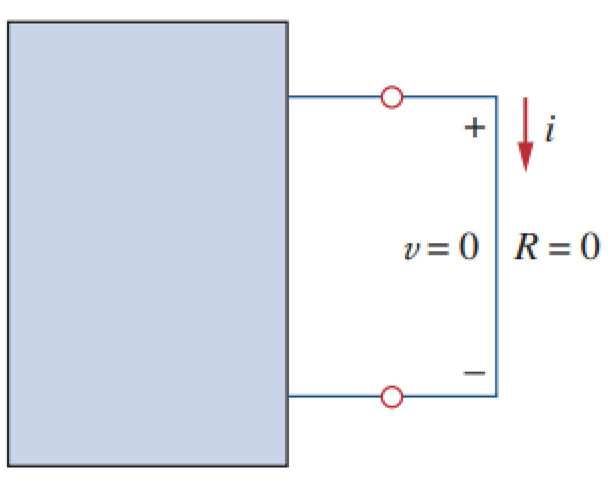
$$R = \frac{v}{i} \qquad \dots (16)$$

So that,

$$1\,\Omega = 1\,V/A$$

Short Circuit:

A short circuit is a circuit element with resistance approaching zero.



(a)

(a) Short circuit $(R = 0)$

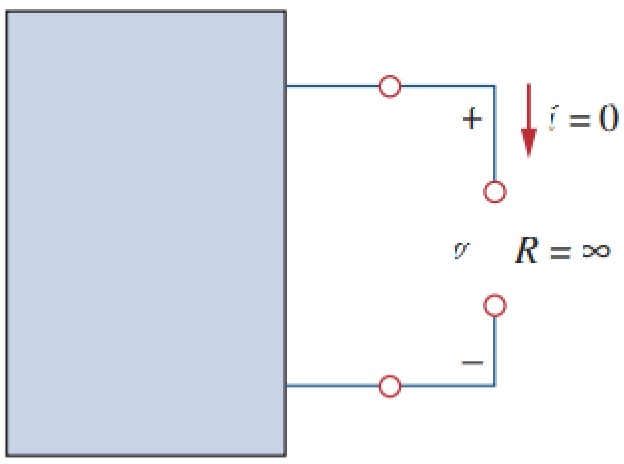
An element with $R = 0$ is called a short circuit. For a short circuit,

$$v = iR = 0 \qquad \dots (17)$$

showing that the voltage is zero but the current could be anything. In practice, a short circuit is usually a connecting wire assumed to be a perfect conductor.

Open Circuit:

An open circuit is a circuit element with resistance approaching infinity.



(b)

(b) Open circuit $(R = \infty)$

an element with $R = \infty$ is known as an open circuit. For an open circuit,

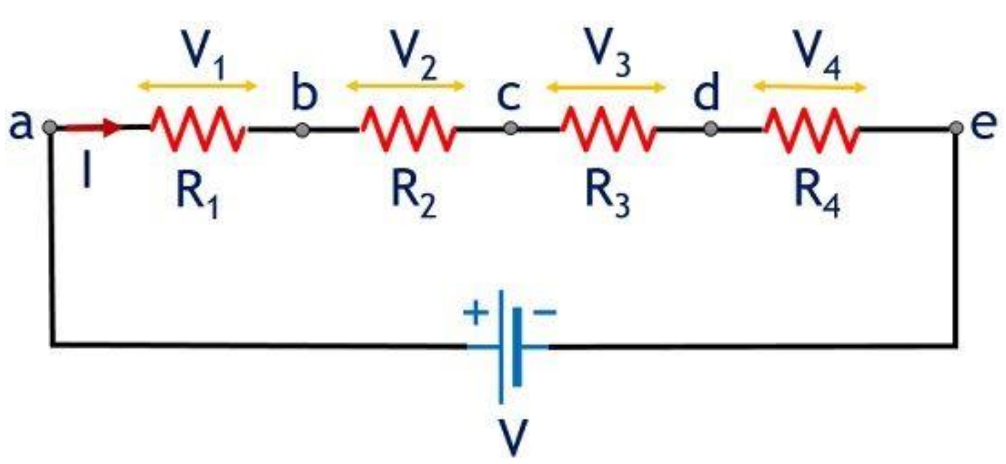
$$i = \lim_{R \to \infty} \frac{v}{R} = 0 \qquad \dots (18)$$

indicating that the current is zero though the voltage could be anything.

Series/ Parallel Circuit:

Series Circuit:

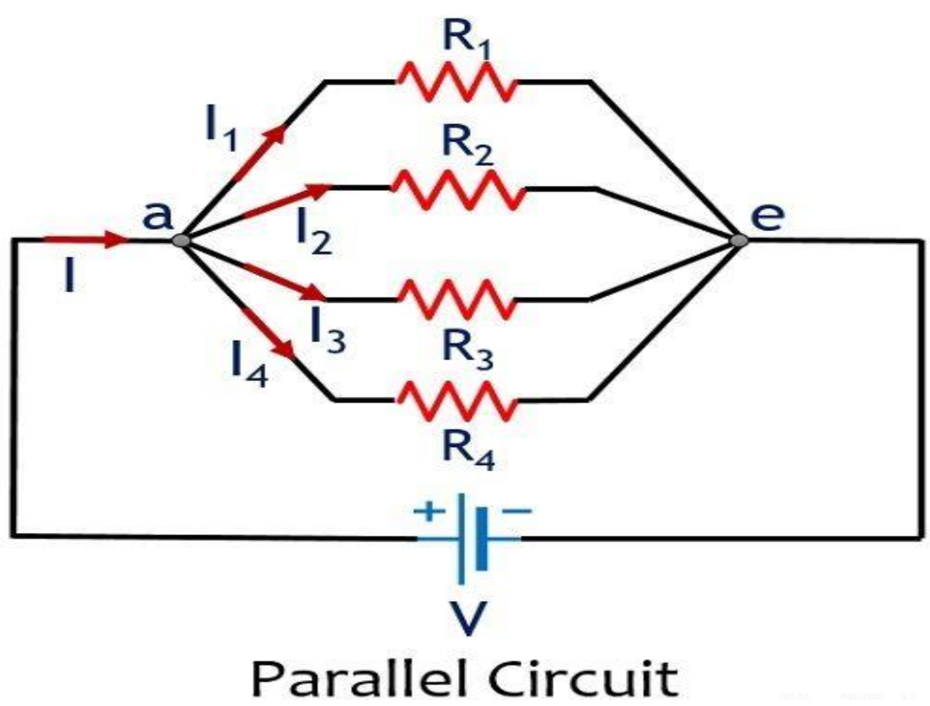
Two or more elements are in series if they exclusively share a single node and consequently carry the same current.



Series Circuit

Parallel Circuit:

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.



Voltage and Current Division:

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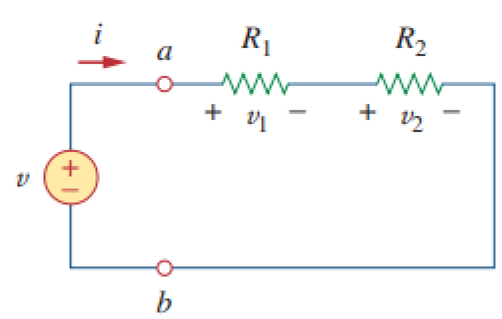
Series Resistors and Voltage Division:



The two resistors are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \qquad v_2 = iR_2 \qquad \dots(19)$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \qquad \dots(20)$$

Combining Eq. (19) and (20), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \qquad \dots(21)$$

Or,

$$i = \frac{v}{R_1 + R_2} \qquad \dots(22)$$

Notice that Eq. (2.26) can be written as

$$v = iR_{eq} \qquad \dots(23)$$

implying that the two resistors can be replaced by an equivalent resistor; that is,

$$R_{eq} = R_1 + R_2 \qquad \dots(24)$$

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n \qquad \ldots(25)$$

To determine the voltage across each resistor in Fig. 2.29, we substitute Eq. (21) into Eq. (19) and obtain

$$v_1 = \frac{R_1}{R_1 + R_2} v\,, \qquad v_2 = \frac{R_2}{R_1 + R_2} v \qquad \ldots(26)$$

In general, if a voltage divider has N resistors $(R_1, R_2, \ldots, R_N)$ in series with the source voltage v, the nth resistor (R_N) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v \qquad \ldots(27)$$

Parallel Resistors and Current Division:

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

Or

$$i_1 = \frac{v}{R_1}\,, \qquad i_2 = \frac{v}{R_2} \qquad \ldots(28)$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \qquad \ldots(29)$$

Substituting Eq. (28) into Eq. (29), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \qquad \ldots(30)$$

where is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \ldots(31)$$

Or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

Or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad \ldots(32)$$

Thus,

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (32), if $R_1 = R_2$, then $R_{eq} = R_1/2$.

We can extend the result in Eq. (31) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \qquad \ldots (33)$$

Note that Req is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \qquad \ldots (34)$$

To determine the current across each resistor i1 and i2 we get

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \qquad \ldots (35)$$

$$i_1 = \frac{R_2 i}{R_1 + R_2}, \qquad i_2 = \frac{R_1 i}{R_1 + R_2} \qquad \ldots (36)$$

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig. 2.31 is known as a current divider. Notice that the larger current flows through the smaller resistance.

Kirchhoff's Laws:

BF-64 and BI-160

KCL:

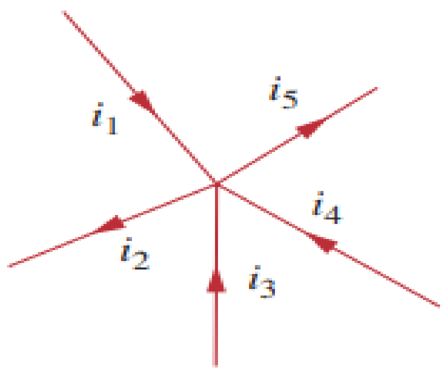
Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0 \qquad \ldots (37)$$

where N is the number of branches connected to the node and is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.



Consider the node in Fig. 2.16. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

The law can also be stated in the following way:

The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

In equation form, the above statement can be written as follows:

$$\sum I_i = \sum I_o \qquad \ldots (38)$$

By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5$$

KVL:

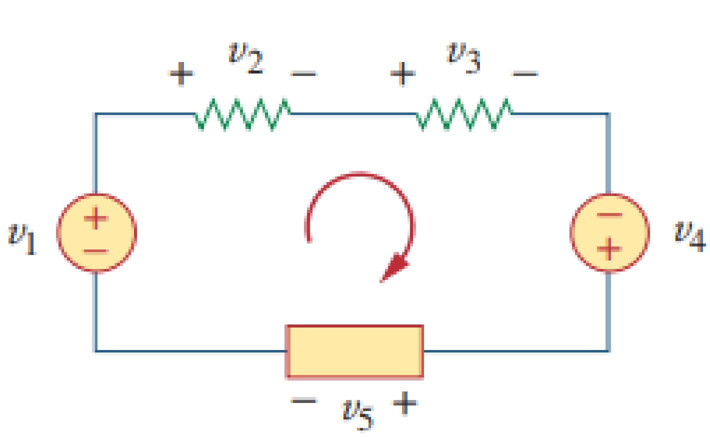
Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0 \qquad \ldots (39)$$

where M is the number of voltages in the loop (or the number of branches in the loop) and is the mth voltage.



To illustrate KVL, consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1, +v_2, +v_3, -v_4,\ and + v_5$ in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$.Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1, -v_5, +v_4, -v_3, and - v_2$, which is the same as before except that the signs are reversed.

Mesh analysis:

A mesh is a loop which does not contain any other loops within it.

Mesh Current Analysis is a technique used to find the currents circulating around a loop or mesh with in any closed path of a circuit.

Mesh analysis applies the Kirchhoff's Voltage Law (KVL) to determine the unknown currents in a given circuit.

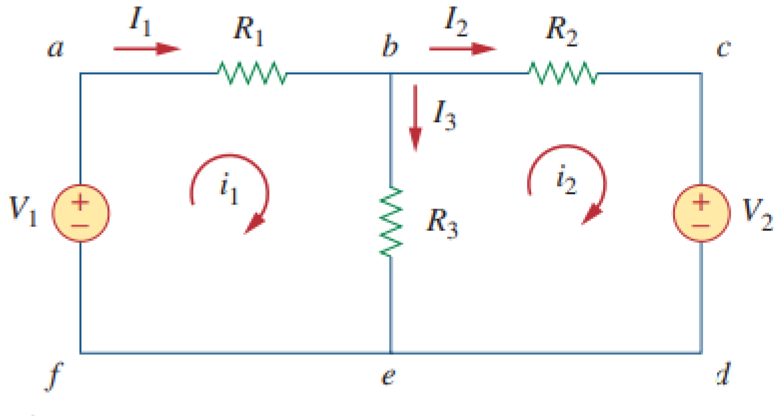


Figure-11: A circuit with two meshes

In Fig. 11, for example, paths abefa and bcdeb are meshes, but path abcdefa is not a mesh. The current through a mesh is known as mesh current. In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit. In this section, we will apply mesh analysis to planar circuits that do not contain current sources.

Steps to determine Mesh Currents:

1. Assign a mesh current to each mesh
2. Apply Kirchhoff's Voltage Law (KVL) around each mesh, in the same direction as the mesh currents.
3. Solve the resulting loop equations for the mesh currents.
4. Determine any requested current or voltage in the circuit using the mesh currents.

To illustrate the steps, consider the circuit in Fig. 3.17. The first step requires that mesh currents and are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

Or,

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1 \qquad \dots(40)$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

Or,

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \qquad \dots(41)$$

Then solving these equations using Cramer's rule, we get

$$I_1 = i_1 \qquad I_2 = i_2 \qquad I_3 = i_1 - i_2$$

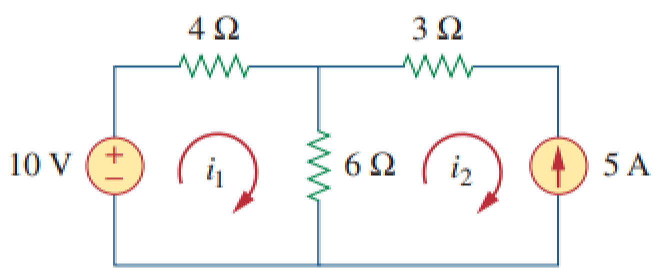
Mesh Analysis with Current Sources (Supermesh):

A supermesh occurs when a current source is contained between the essential meshes. The circuit is first treated as if the current source is not there. This leads to one equation that incorporates two mesh currents.

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

■ CASE 1

When a current source exists only in one mesh: Consider the circuit in Fig. 12, for example. We set i2= -5 A and write a mesh equation for the other mesh in the usual way; that is,

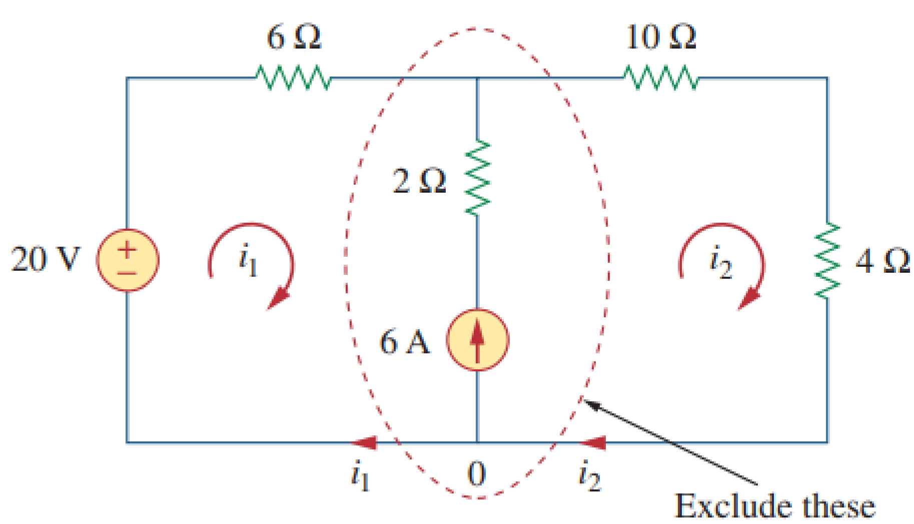


$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \qquad => \quad i_1 = -2\,A \qquad \ldots(42)$$

■ CASE 2

When a current source exists between two meshes: Consider the circuit in Fig. 13. for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. 13. Thus,

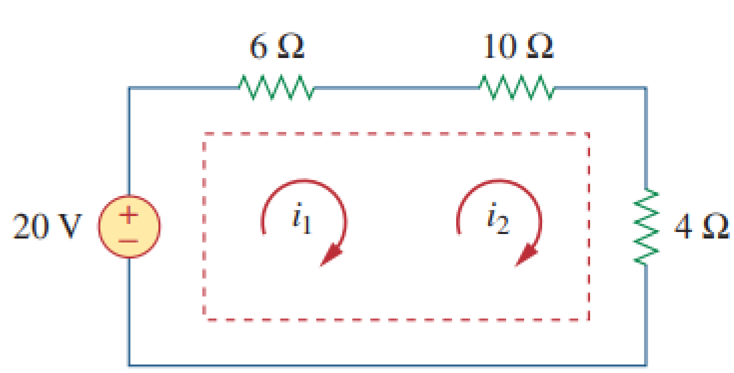
A supermesh results when two meshes have a (dependent or independent) current source in common.



(a)

Procedure:

1. Identify the total number of meshes
2. Assign the mesh current and check for supermesh in the circuit
3. If supermesh found, develop KVL for it.
4. Solve the equations to find the mesh currents.



(b)

As shown in Fig. 3.23(b), we create a supermesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.) Why treat the supermesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh in Fig. 3.23(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \qquad \text{... (43)}$$

We apply KCL to a node in the branch where the two meshes intersect. Applying KCL to node 0 in Fig. 3.23(a) gives

$$i_2 = i_1 + 6 \qquad \text{... (44)}$$

Solving Eqs. (43) and (44), We get

$$i_1 = -3.2\,A, \qquad i_2 = 2.8\,A$$

Note the following properties of a supermesh:

1. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

Nodal analysis:

A **node** is the point of connection between two or more branches.

Nodal analysis is a method that provides a general procedure for analyzing circuits using node vas the voltage circuit variables.

Nodal Analysis is based on the application of the Kirchhoff's Current Law (KCL).

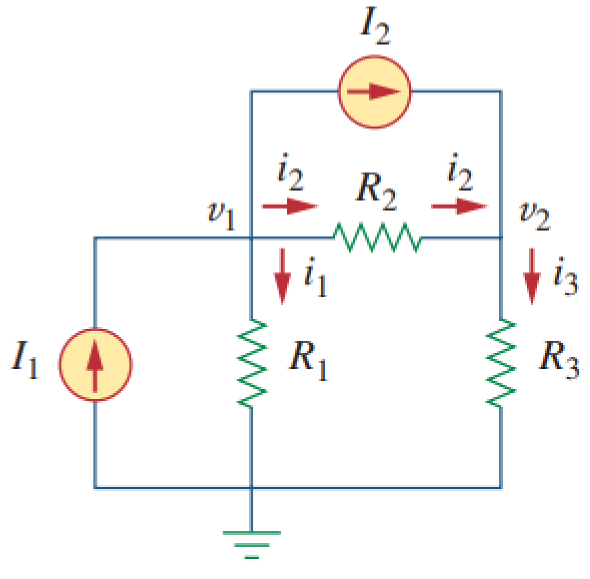


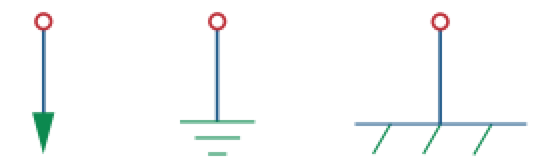
Figure 12: Nodal circuit

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \dots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.

2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

The first step in nodal analysis is selecting a node as the reference or datum node. The reference node is commonly called the ground since it is assumed to have zero potential.



Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in Fig. 12. Node 0 is the reference node while nodes 1 and 2 are assigned voltages and respectively. Keep in mind that the node voltages are defined with respect to the reference node.

As the second step, we apply KCL to each nonreference node in the circuit. Applying KCL;

At node V1:

$$I_1 = I_2 + i_1 + i_2 \qquad \dots(42)$$

At node V2:

$$I_2 + i_2 = i_3 \qquad \dots(43)$$

We now apply Ohm's law to express the unknown currents $i_1, i_2 \; and \; i_3$ in terms of node voltages. The key idea is:

Current flows from a higher potential to a lower potential in a resistor.

We can express this principle as

$$i = \frac{v_{higher} - v_{lower}}{R} \qquad \dots(44)$$

Ekta chobi dibo voltage ar current er ekhane

We obtain from Figure 12;

$$i_1 = \frac{v_1 - 0}{R_1}$$

$$i_2 = \frac{v_1 - v_2}{R_2} \qquad \dots(45)$$

$$i_3 = \frac{v_2 - 0}{R_3}$$

Substituting Eq. (45) in Eqs. (42) and (43) results, respectively, in

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \qquad \dots(46)$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \qquad \dots(47)$$

Nodal Analysis with Voltage Sources (Super Node)

A supernode is a large node that connects two non-reference nodes with a known voltage source.

We now consider how voltage sources affect nodal analysis. Consider the following two possibilities.

■ CASE 1

If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 13, for example,

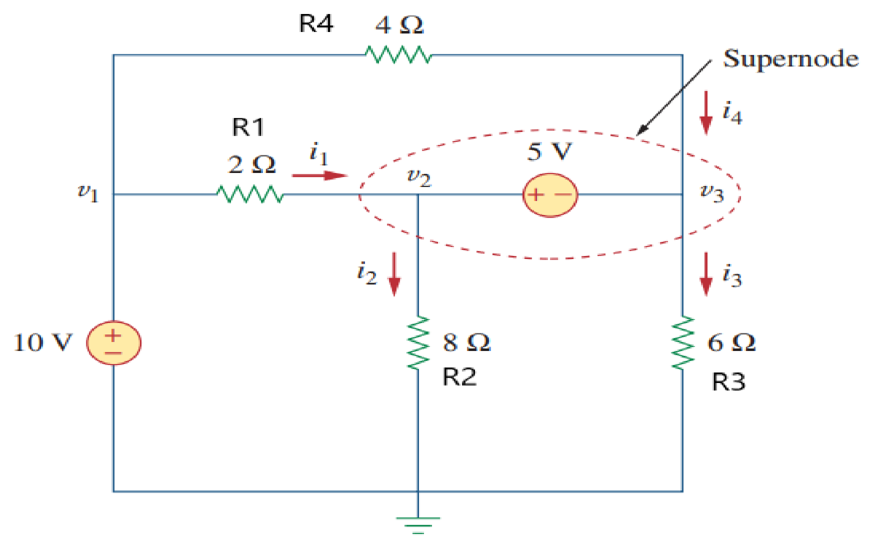
$$v_1 = 10\ V \qquad \dots (48)$$

Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

■ CASE 2

If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages.

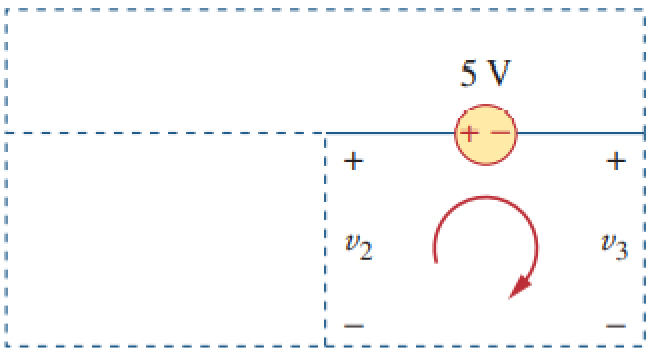
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.



Procedure:

1. Identify the total number of nodes and pick a reference node.
2. Assign node voltage to all the nodes and v0=0 V to the reference node.
3. Identify super node and find the voltage difference between the nodes of the independent / known voltage source.

4. Replace the known voltage source with a short circuit and redraw the circuit indicating supernode.
5. Apply KCL to supernode and remaining nodes.
6. Solve system of linear equation to find node voltages.



In Fig. 13, nodes 2 and 3 form a supernode. We analyze a circuit with supernodes using the same three steps mentioned in the previous section except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, KCL must be satisfied at a supernode like any other node. Hence, at the supernode in Fig. 13,

$$i_1 + i_4 = i_2 + i_3 \qquad \dots (49)$$

Or,

$$\frac{v_1 - v_2}{R_1} + \frac{v_1 - v_3}{R_4} = \frac{v_2 - 0}{R_2} + \frac{v_3 - 0}{R_3} \qquad \dots (50)$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

$$12v_1 - 12v_2 + 6v_1 - 6v_3 = 3v_2 + 4v_3$$

$$18v_1 - 15v_2 - 10v_3 = 0$$

$$3v_2 + 2v_3 = 36$$

To apply Kirchhoff's voltage law to the supernode in Fig. 13, we redraw the circuit as shown in Fig. 14. Going around the loop in the clockwise direction gives

$$-v_2 + 5 + v_3 = 0 \qquad => \; v_2 - v_3 = 5 \qquad \dots (51)$$

From Eqs. (48), (50), and (51), we obtain the node voltages.

$$v_1 = 10\,V, \qquad v_2 = 9.2\,V, \qquad v_3 = 4.2\,V$$

Note the following properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.