Rotation

Angular Position To describe the rotation of a rigid body about a fixed axis, called the **rotation axis,** we assume a **reference line** is fixed in the body, perpendicular to that axis and rotating with the body. We measure the **angular position ϴ** of this line relative to a fixed direction. When u is measured in **radians,**

$$\theta = \frac{s}{r} \text{(radian measure)}$$

where *s* is the arc length of a circular path of radius *r* and angle **ϴ** Radian measure is related to angle measure in revolutions and degrees by

$$1 \text{ rev} = 360^0 = 2\pi \text{ rad}$$

In Fig.1 The **angular position** of this line is the angle of the line relative to a fixed direction, which we take as the **zero angular position.** In Fig. 2, the angular position **ϴ**is measured relative to the positive direction of the *x* axis.

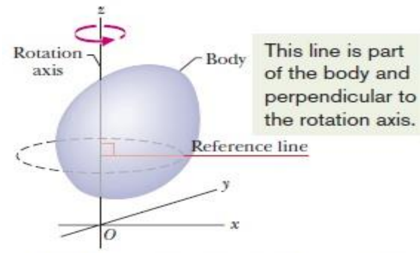

Fig. 1 A rigid body of arbitrary shape in pure rotation about the *z* axis of a coordinate system. The position of the *reference line* with respect to the rigid body is arbitrary, but it is perpendicular to the rotation axis. It is fixed in the body and rotates with the body.

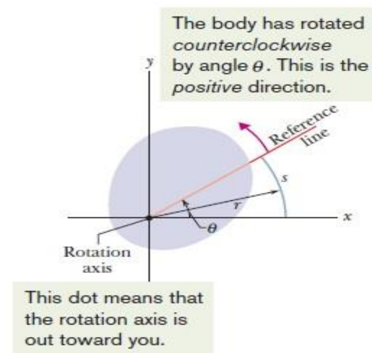

Fig. 2 The rotating rigid body of Fig. 1 in cross section, viewed from above. The plane of the cross section is perpendicular to the rotation axis, which now extends out of the page, toward you. In this position of the body, the reference line makes an angle θ with the *x* axis.

Angular Displacement A body that rotates about a rotation axis, Fig.3changing its angular position from $\Theta_1$to Θ_2, undergoes an **angular displacement**

$$\Delta\Theta = \Theta_2 - \Theta_1$$

where **Δϴ** is positive for counterclockwise rotation and negative for clockwise rotation.

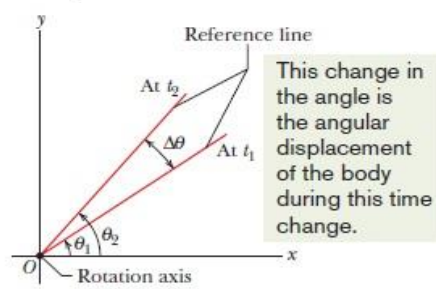

Fig. 3 The reference line of the rigid body of Figs. 1 and 2 is at angular position θ_1 at time t_1 and at angular position θ_2 at a later time t_2. The quantity $\Delta\theta (= \theta_2 - \theta_1)$ is the angular displacement that occurs during the interval $\Delta t (= t_2 - t_1)$. The body itself is not shown.

Angular Velocity and Speed If a body rotates through an angular displacement **Δϴ** in a time interval **Δ***t*, its **average angular velocity**

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

The **(instantaneous) angular velocity** ω of the body is

$$\omega = \frac{d\theta}{dt}$$

these two equations hold not only for the rotating rigid body as a whole but also for *every particle of that body* because the particles are all locked together. The unit of angular velocity is commonly the radian per second (rad/s) or the revolution per second (rev/s). Both $\boldsymbol{\omega_{avg}}$ and **ω** are vectors. The magnitude of the body's angular velocity is the **angular speed.**

Angular Acceleration If the angular velocity of a body changes from $\boldsymbol{\omega_1}$ to $\boldsymbol{\omega_2}$ in a time interval $\Delta t = t_2 - t_1$, the **average angular acceleration** $\boldsymbol{\alpha_{avg}}$ of the body is

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The **(instantaneous) angular acceleration** a of the body is

$\alpha = \frac{d\omega}{dt}$, Both $\boldsymbol{\alpha_{avg}}$ and $\boldsymbol{\alpha}$ are vectors.

Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

The Kinematic Equations for Constant Acceleration

Linear Equation	Angular Equation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Linear and Angular Variables Related A point in a rigid rotating body, at a *perpendicular distance r* from the rotation axis, moves in a circle with radius *r*. If the body rotates through an angle u, the point moves along an arc with length *s* given by $\boxed{s=\Theta r \text{ (radian measure)}}$ where Θ is in radians.

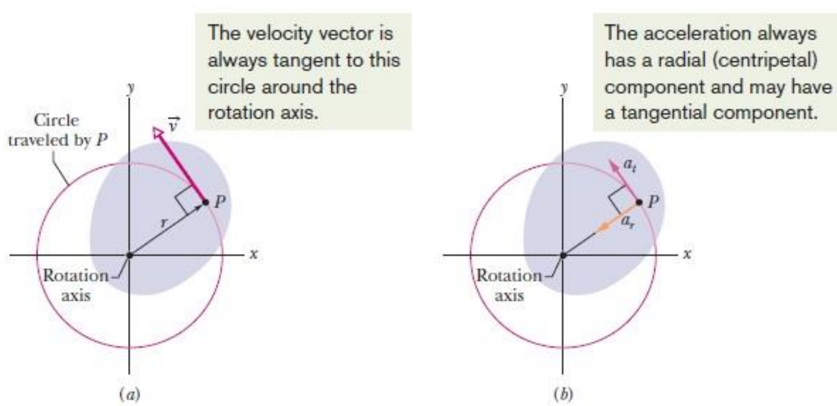

Fig. 4 The rotating rigid body of Fig. 1 shown in cross section viewed from above. Every point of the body (such as *P*) moves in a circle around the rotation axis. (*a*) The linear velocity $\vec{v}$ of every point is tangent to the circle in which the point moves. (*b*) The linear acceleration $\vec{a}$ of the point has (in general) two components: tangential a_t and radial a_r.

The linear velocity of the point is tangent to the circle; the point's linear speed *v* is given by

$$v = \boldsymbol{\omega} r \text{ (radian measure)}$$

where $\boldsymbol{\omega}$ is the angular speed (in radians per second) of the body. Figure : 4 (a) reminds us that the linear velocity is always tangent to the circular path.

The linear acceleration of the point has both *tangential* and *radial* components (Figure: 4b). The tangential component is

$$a_t = \alpha r \text{ (radian measure)}$$

where a is the magnitude of the angular acceleration (in radians per second-squared) of the body. The radial component of is

$$a_r = \frac{v^2}{r} = \omega^2 r \text{(radian measure)}$$

If the point moves in uniform circular motion, the period *T* of the motion for the point and the body is $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$(radian measure).

Rotational Kinetic Energy and Rotational Inertia The

Consider a rigid body rotating about a fixed axis. We shall treat the table saw (and any other rotating rigid body) as a collection of particles with different speeds. We can then add up the kinetic energies of all the particles to find the kinetic energy of the body as a whole.

In this way we obtain, for the kinetic energy of a rotating body,

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \cdots \cdots = \sum \frac{1}{2} m_i v_i^2$$

in which *mi* is the mass of the *i*th particle and v_i is its speed. The sum is taken over all the particles in the body. Using $v = \boldsymbol{\omega} r$ we get

$K = \sum \frac{1}{2} m_i \omega^2 r_i^2 = \frac{1}{2}(\sum m_i r_i^2)\omega^2$, where $\boldsymbol{\omega}$ is same for all particles.

The quantity in parentheses on the right side of the above equation tells us how the mass of the rotating body is distributed about its axis of rotation. We call that quantity the **rotational inertia** (or **moment of inertia**) I of the body with respect to the axis of rotation. It is a constant for a particular rigid body and a particular rotation axis.

$I = \sum \frac{1}{2} m_i r_i^2$

Hence the kinetic energy can be written as

$$K = \frac{1}{2} I \omega^2$$

The Parallel-Axis Theorem The *parallel-axis theorem* relates the rotational inertia *I* of a body about any axis to that of the same body about a parallel axis through the center of mass:

$$I = I_{cm} + Mh^2$$

Here, M is the total mass of the body and *h* is the perpendicular distance between the two axes, and I_{cm} is the rotational inertia of the body about the axis through the center of mass (cm).We can describe *h* as being the distance the actual rotation axis has been shifted from the rotation axis through the cm.

Torque *Torque* is a turning or twisting action on a body about a rotation axis due to a force **F.** If **F** is exerted at a point given by the position vector **r** relative to the axis, then the magnitude of the torque is

$$\tau = rF_t = rF sin\theta$$

where F_t is the component of **F** perpendicular to **r** and $\varphi$is the angle between **r** and **F**. The quantity is the perpendicular distance

To determine how results in a rotation of the body around the rotation axis, we resolve into two components(Fig:5). One component, called the *radial component* F_r, points along **r**. This component does not cause rotation. The other component of , called the *tangential component* F_t, is perpendicular to and has magnitude $F_t = F \sin \varphi$. This component *does* cause rotation.

Two equivalent ways of computing the torque are

$$\tau = rF_t = (r)(F sin\varphi)$$
$$\tau = r_{\perp}F = (r sin\varphi)F$$

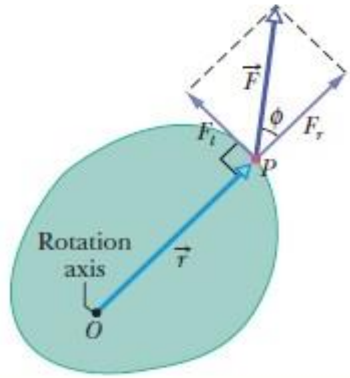

Figure:5 only the *tangential* component of the force causes the rotation.

Newton's Second Law in Angular Form

A torque can cause rotation of a rigid body. we want to relate the net torque **τ**net on a rigid body to the angular acceleration **α** that torque causes about a rotation axis. We do so by analogy with Newton's second law ($F_{net} = ma$) for the acceleration *a* of a body of mass *m* due to a net force F_{net} along a coordinate axis.

A force acts on the particle. However, because the particle can move only along the circular path, only the tangential component *Ft* of the force (the component that is tangent to the circular path) can accelerate the particle along the path. We can relate *Ft* to the particle's tangential acceleration *at* along the path with Newton's second law, writing

$$F_t = ma_t$$

The torque acting on the particle is

$$\tau = F_t r = ma_t r$$

Again $a_t = \boldsymbol{\alpha} r$

So, $\tau = m(\alpha r)r = mr^2\alpha$

The quantity in parentheses on the right is the rotational inertia I of the particle about the rotation axis. Finally, we get

$$\tau = I\alpha$$

For the situation in which more than one force is applied to the particle, we can generalize

$$\tau_{net} = I\alpha$$

Work and Rotational Kinetic Energy The equations used for calculating work and power in rotational motion correspond to equations used for translational motion and are

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

$$\text{and } P = \frac{dw}{dt} = \tau\omega$$

When τ is constant,

$$W = (\theta_f - \theta_i)\tau$$

The form of the work–kinetic energy theorem used for rotating bodies is

$$\Delta K = k_f - k_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$$