Name: Omar Faruk, Roll: 2019831055

Problem 1: $M = \begin{bmatrix} 2 & 0 & -1 \\ 5 & x+y & 0 \\ 0 & -y & 3 \end{bmatrix}$

$$\det(M) = 2\{3(x+y) - 0\} + (-1)\{-5y\}$$
$$= 2\{3x + 3y + 5y\}$$
$$= 6x + 11y$$

$\therefore 6x + 11y \neq 0$

So

(i) x and y both can not be 0.

(ii) At a time x and y can be both positive (+ve) or negative (-ve). Then,

$$6x + 11y \neq 0$$

(iii) let 'a' be the arbitary value of x.

$$\therefore y = \frac{-6a}{11}$$

So, in case of singular matrix,

$$(x, y) = \left\{a, \frac{-6a}{11}\right\}$$

$\therefore$ For non-singular matrix,

$$(x, y) = \mathbb{R} - \left\{a, \frac{-6a}{11}\right\}$$ [where a belongs to $\mathbb{R}$]

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Let, $M = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ [given that $x = 2$, $y = -1$]

Let, assume, $M = A$

Applying Cayley-Hamilton theorem,

$$|A - \lambda I| = 0$$

$\therefore A - \lambda I = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 & -1 \\ 5 & 1-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{bmatrix}$

So, $|A - \lambda I| = 0$

$\Rightarrow (2-\lambda)(1-\lambda)(3-\lambda) - 5 = 0$

$\Rightarrow -\lambda^3 + 6\lambda^2 - 11\lambda + 1 = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 1 = 0$

Let, $\lambda = A$

$$A^3 - 6A^2 + 11A - I = 0$$

$\Rightarrow A^2 A A^{-1} - 6AAA^{-1} + 11AA^{-1} - IA^{-1} = 0$

$\Rightarrow A^2 - 6A + 11I - A^{-1} = 0$ $\quad [AA^{-1} = I, \quad AI = A]$

$\Rightarrow A^{-1} = A^2 - 6A + 11I$

$$= \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -6 \\ 30 & 6 & 0 \\ 0 & 6 & 18 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

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$\therefore A^{-1} = M^{-1}$ $\left[\because \text{assuming } A = M\right]$

$$M^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Problem 2:

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-y \\ z \\ x \end{bmatrix}$$

$$L\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdots\cdots (i)$$

$$L\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \cdots\cdots (ii)$$

$$L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdots\cdots (iii)$$

construct argument matrix:

$$\left[T_1\ T_2\ T_3\ T_4 \,\middle|\, L(S_1)\ L(S_2)\ L(S_3)\right]$$

$$\left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{array}\right] R_4' = R_1(-1) + R_4$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0 \end{array}\right] R_4' = R_1 - R_4$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \cancel{R_3' = R_3 - R_1}$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_3 - R_1$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_3(-1)$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 2 & 0 \\ 0 & -1 & 0 & -1 & -2 & -2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_2' = R_2 - R_1$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_2' = R_2 + R_4$$

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$$= \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] (R_2 \times -1)$$

$$= \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_1' = R_1 - R_4$$

$$= \left[\begin{array}{cccc:ccc} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_1' = R_1 - R_3$$

$$= \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] R_1' = R_1 - R_2$$

$\therefore$ Transformation matrix,

$$L = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\therefore$ $[L]$ is a 4×3 matrix