Name: Omar Faruk

Registration Number: 201983 1055

MAT 105W

Part - B

Ans: to the que: No-4

a

Let, $I_n = \int \cos^n x\,dx$

$\Rightarrow I_n = \int \cos^{n-1}x \cdot \cos x\,dx$

$= \cos^{n-1}x \int \cos x\,dx - \int \left\{ \frac{d}{dx}(\cos^{n-1}x) \int \cos x\,dx \right\} dx$

$= \cos^{n-1}x \cdot \sin x - \int (n-1)\cos^{n-2}x(-\sin x) \cdot \sin x\,dx$

$= \cos^{n-1}x \cdot \sin x + (n-1)\int \cos^{n-2}x\,(1-\cos^2 x)\,dx$

$= \cos^{n-1}x \cdot \sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx$

$= \cos^{n-1}x \cdot \sin x + (n-1) I_{n-2} - (n-1) I_n$

$\Rightarrow I_n + (n-1) I_n = \cos^{n-1}x \cdot \sin x + (n-1) I_{n-2}$

$$\Rightarrow I_n(1+n-1) = \sin x \cdot \cos^{n-1}x + (n-1)\, I_{n-2}$$

$$\Rightarrow I_n = \frac{\sin x \cos^{n-1}x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

Now, let, $I = \int \cos^8 x\,dx \quad \ldots (i)$

from the reduction formula,

$$I = \frac{\sin x \cdot \cos^7 x}{8} + \frac{7}{8}\int \cos^6 x\,dx$$

$$= \frac{\sin x \cdot \cos^7 x}{8} + \frac{7}{8}\left[\frac{\sin x \cdot \cos^5 x}{6} + \frac{5}{6}\int \cos^4 x\,dx\right]$$

$$= \frac{\sin x \cdot \cos^7 x}{8} + \frac{7\sin x \cdot \cos^5 x}{48.} + \frac{35}{48}\int \cos^4 x\,dx$$

$$= \frac{\sin x \cdot \cos^7 x}{8} + \frac{7\sin x \cdot \cos^5 x}{48} + \frac{35\sin x \cdot \cos^3 x}{192}$$

$$+ \frac{105}{192}\int \cos^2 x\,dx$$

$$= \frac{\sin x \cdot \cos^7 x}{8} + \frac{7\sin x \cdot \cos^5 x}{48} + \frac{35\sin x \cdot \cos^3 x}{192} +$$

$$\frac{105}{192}\left[\frac{\sin x \cdot \cos x}{2} + \frac{1}{2}\int dx\right]$$

$$= \frac{\sin x \cdot \cos^7 x}{8} + \frac{7\sin x \cdot \cos^5 x}{48} + \frac{35 \sin x \cdot \cos^3 x}{192} +$$

$$\frac{35 \sin x \cdot \cos x}{128} + \frac{35x}{128} + c$$

Ans:

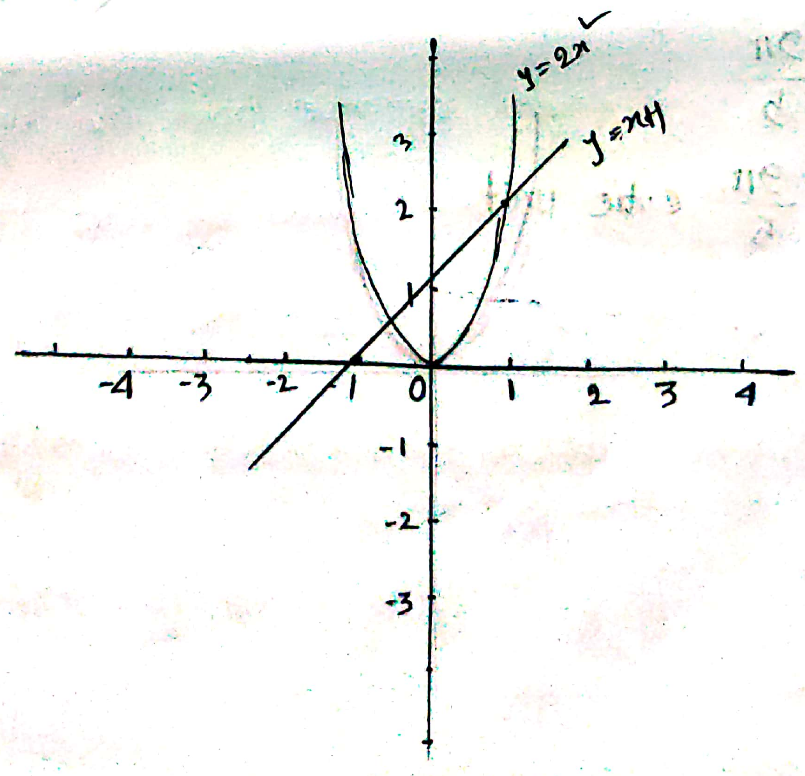
Ans to the que: NO-4

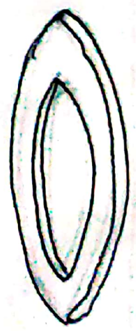
(b)

Given that,

$y = 2x^2$

and, $y = x+1$





$x+1 = 2x^2 \Rightarrow 2x^2 - x - 1 = 0$

$\therefore x = -\frac{1}{2}$ or $x = 1$

Now,

$dv = A\,dx$

$\Rightarrow dv = \pi \left\{ (x+1)^2 - (2x^2)^2 \right\} dx$

$\Rightarrow \int dv = \int_{-1/2}^{1} \pi \left\{ (x+1)^2 - 4x^4 \right\} dx$

$\Rightarrow V = \pi \left\{ \int_{-1/2}^{1} (x^2 + 2x + 1)\,dx - \int_{-1/2}^{1} 4x^4\,dx \right\}$

$\Rightarrow V = \pi \left\{ \left[\frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + x \right]_{-1/2}^{1} - 4 \left[\frac{x^5}{5} \right]_{-1/2}^{1} \right\}$

$\Rightarrow V = \pi \left\{ \left(\frac{1}{3} + 1 + 1 \right) - \left(-\frac{1}{24} + \frac{1}{4} - \frac{1}{2} \right) - 4 \cdot \left(\frac{1}{5} + \frac{1}{160} \right) \right\}$

$\Rightarrow V = \frac{9\pi}{5}$

$\therefore V = \frac{9\pi}{5}$ cubic unit.

Ans: to the que: NO - 5

(a)

The given equation of the conic is -

$$8x^2+4xy+5y^2-24x-24y=0 \quad \text{...(i)}$$

comparing the equation with the second degree equation -

$ax^2+2hxy+by^2+2gx+2fy+c=0$, we get,

$a=8,\ h=2,\ b=5,\ g=-12,\ f=-12,\ c=0.$

$\therefore \Delta = abc+2fgh-af^2-bg^2-ch^2$

$= 8\cdot5\cdot0+2(-12)\cdot(-12)\cdot2-8\cdot(-12)^2-5(-12)^2-0\cdot2^2$

$= 0+576-1152-720-0$

$= -1296 \neq 0$

and, $ab-h^2=40-4=36>0$

Hence, the given equation represent an ellipse.

Again, $\frac{\delta s}{\delta x}=16x+4y-24=0 \Rightarrow 4x+y-6=0 \quad \text{...(ii)}$

$$\frac{\delta s}{\delta x} = 4x + 10y - 24 = 0 \Rightarrow 2x + 5y - 12 = 0 \dots (iii)$$

Solving (ii) and (iii) we get the coordinates of the centre of the conic which are $(1,2)$.

Shifting the origin to $(1,2)$ and remaining the direction of the axes unaltered, the equation (i) takes the form.

$$8x^2 + 4xy + 5y^2 + c' = 0 \dots (iv)$$

where,

$$c' = gx_1 + fy_1 + c = (-12)\cdot 1 + (-12)\cdot 2 + 0 = -36$$

Putting $c' = -36$, in (iv), we get,

$$8x^2 + 4xy + 5y^2 = 36 \dots (v)$$

when the xy term remove from the equation (v) by the rotation of axes, let the reduce equation be $a_1x^2 + b_1y^2 = 36 \dots (vi)$

The by the inveriant propertise, we have,

$$a_1 + b_1 = a + b = 8 + 5 = 13$$

and, $a_1b_1 = ab - h^2 = 40 - 4 = 36$

$b_1 = 13 - a_1$

$a_1 b_1 = a_1(13 - a_1) = 36$

$\Rightarrow 13a_1 - a_1^2 = 36$

$\Rightarrow a_1^2 - 13a_1 + 36 = 0$

$\Rightarrow (a_1 - 4)(a_1 - 9) = 0$

$\therefore a_1 = 4, \; b_1 = 9 \;\; \Big\} \;\; a_1 = 9, \; b_1 = 4$

Since c' is negative. So we choose $a_1 < b_1$

$\therefore a_1 = 4, \; b_1 = 9$

putting the values of a_1 and b_1 in (vi), we get

$$4x^2 + 9y^2 = 36$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

this is the standraad form of an ellipse.

Ans: to the que: No - 5

(b)

The given equation is,

$$x^2+2\sqrt{3}xy+3y^2+2\sqrt{3}x-2y=0 \quad \ldots (i)$$

Comparing it to the second degree general equation -

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

here,

$a=1,\ b=3,\ h=\sqrt{3},\ g=\sqrt{3},\ f=-1.$

If we want to eliminate xy term, the required rotation is -

$$\tan 2\theta = \frac{2h}{a-b}$$

$$\Rightarrow 2\theta = \tan^{-1}\left(\frac{2h}{a-b}\right)$$

$$\Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{2\times\sqrt{3}}{1-3}\right)$$

$$\Rightarrow \theta = \frac{1}{2}\times(-60^\circ)$$

$$\Rightarrow \theta = -30^\circ$$

A