Concept of wave

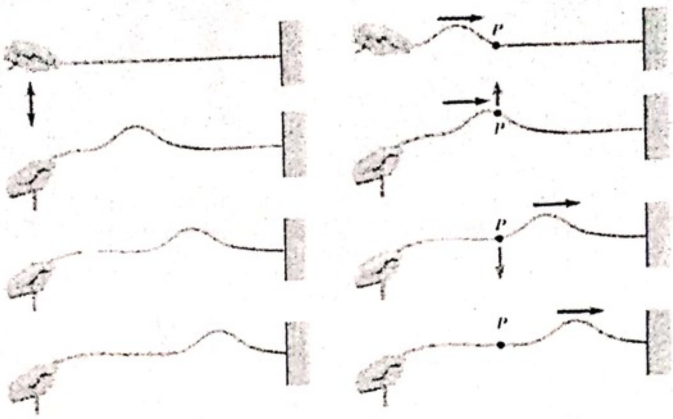

Figure 1: Pulse in a rope

Consider one end of a rope is attached with a rigid support and another end is hold by you, as shown in figure 1. If you create a pulse on the free end, you will see the pulse is moving toward the right as time being. Careful observation of the point P on rope will lead to you a conclusion, that the point is vibrating vertically but the pulse is moving horizontally. The pulse is one kind of energy which is continually moving far away from its point of origin. From this simple experiment we can define *wave as a process of transferring energy without permanently displacing the particles of a medium (rope) from its equilibrium position.*

Categories of Waves

One way to categorize waves is on the basis of the direction of movement of the individual particles of the medium relative to the direction that the waves travel. Categorizing waves on this basis leads to three notable categories: transverse waves, longitudinal waves, and surface waves.

A *transverse wave* is a wave in which particles of the medium move in a direction perpendicular to the direction that the wave moves. A light wave traveling through air is a classic example of a transverse wave.

A *longitudinal wave* is a wave in which particles of the medium move in a direction parallel to the direction that the wave moves. A sound wave traveling through air is a classic example of a longitudinal wave.

Waves traveling through a solid medium can be either transverse waves or longitudinal waves. Transverse waves require a relatively rigid medium in order to transmit their energy. As one particle begins to move it must be able to exert a pull on its nearest neighbor. If the medium is not rigid as is the case with fluids, the particles will slide past each other. This sliding action that is characteristic of liquids and gases prevents one particle from displacing its neighbor in a direction perpendicular to the energy transport. It is for this reason that only longitudinal waves are observed moving through the bulk of liquids such as our oceans. Earthquakes are capable of producing both transverse and longitudinal waves that travel through the solid structures of the Earth. When seismologists began to study earthquake waves they noticed that only longitudinal waves were capable of traveling through the core of the Earth. For this reason, geologists believe that the Earth's core consists of a liquid - most likely molten iron.

While waves that travel within the depths of the ocean are longitudinal waves, the waves that travel along the surface of the oceans are referred to as surface waves. *A surface wave is a wave in which particles of the medium undergo a circular motion.* Surface waves are neither longitudinal nor transverse. In a surface wave, it is only the particles at the surface of the medium that undergo the circular motion. The motion of particles tends to decrease as one proceeds further from the surface.

Another way to categorize waves is on the basis of their ability or inability to transmit energy through a vacuum (i.e., empty space). Categorizing waves on this basis leads to two notable categories: electromagnetic waves and mechanical waves.

An electromagnetic wave is a wave that is capable of transmitting its energy through a vacuum (i.e., empty space). Electromagnetic waves are produced by the vibration of charged particles. Electromagnetic waves that

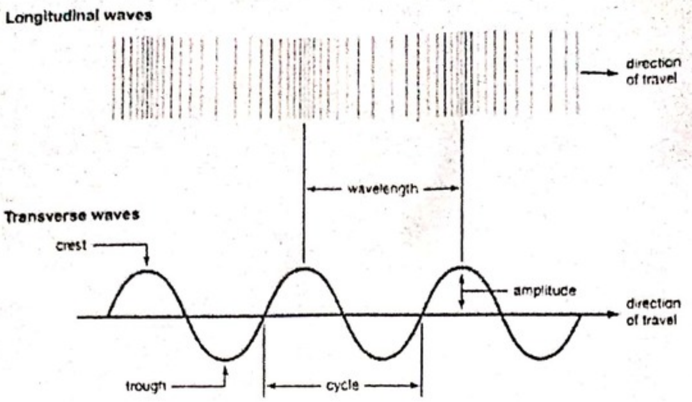

Figure 2: Longitudinal wave and Transeverse. wave

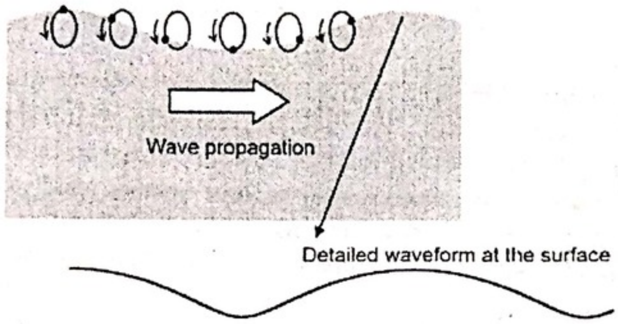

Figure 3: Surface wave

are produced on the sun subsequently travel to Earth through the vacuum of outer space. All light waves are examples of electromagnetic waves.

A mechanical wave is a wave that is not capable of transmitting its energy through a vacuum. Mechanical waves require a medium in order to transport their energy from one location to another. A sound wave is an example of a mechanical wave.

Describing waves

Consider a simple model for the propagation of a wave along the x axis which is represented pictorially as a sin function that depends both on time and position . The high points on the sine wave are called crests and the low points are called troughs as shown in figure 2.

The *amplitude* A of the wave is the maximum disturbance of the wave from the mid-point between the crest and trough to either the top of the crest or to the bottom of a trough. The amplitude is a positive number. The unit of amplitude is m.

The *wavelength* λ is the distance between two adjacent crests or two adjacent troughs or between any two successive identical parts of the wave.

The *frequency* f of the wave is the number of vibrations each part of the wave undergoes in one second. The unit of frequency is $Hertz$ or simply Hz

The *period* T is the time interval for one complete vibration. The unit of period is *seconds* or simply s. The relation between f and T is

$$T = \frac{1}{f} \tag{1}$$

The speed v of a wave is related to its wavelength and its period (frequency). The wave advances 1 wavelength in a time interval of 1 period, therefore,

$$v = \frac{\lambda}{T} = \lambda f \tag{2}$$

It is mathematically very convenient to define two other quantities in describing waves: the wave number or propagation constant k and the angular frequency ω.

$$k = \frac{2\pi}{\lambda} \qquad or \qquad \lambda = \frac{2\pi}{k} \tag{3}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad or \qquad T = \frac{2\pi}{\omega} \tag{4}$$

The Equation of a Traveling Wave or Progressive Wave

We will consider simple harmonic progressive wave.
A simple harmonic progressive is a wave which continually advances in a given direction without change of form and and the particle of the medium perform SHM about their mean position with the same amplitude and period, when the waves pass them.

Expression for One Dimensional Simple Harmonic Progressive Wave:

Consider a simple harmonic progressive wave traveling along the $+x$ direction. Let the vibration of the particles of the medium be parallel or perpendicular to the x-axis. At any instant t the displacement of the particles at the origin is given by,

$$y = a \sin \omega t \tag{5}$$

where a is the amplitude and $\frac{2\pi}{\omega}$ is the period of oscillation.

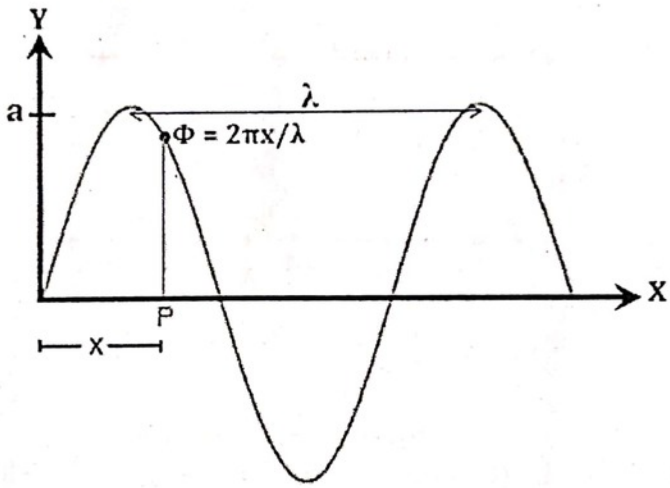

Figure 4:. Progressive wave

Now consider a particle of the medium at point P at a distance of x from O. This particle also performs SHM with same amplitude and period, however, the disturbance from O will reach P after some time. In other word, the particle at P *lags* behind the particle at O in phase. Hence, the displacement of the particles at P at the instant t is given by,

$$y = a \sin(\omega t - \phi) \tag{6}$$

where ϕ is the phase difference between O and P
The angle ϕ depends on the distance OP. In wave motion, two particles separated by a distance λ differ in phase 2π radians. In other words, the path difference λ corresponds to a phase difference 2π radians. Therefore, the path difference x will corresponds to a phase difference of $\phi = \frac{2\pi x}{\lambda} = kx$. Hence, the displacement of the particle at P with $\omega = \frac{2\pi}{T}$ is given by,

$$y = a\sin(\omega t - kx) \tag{7}$$

$$y = a\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \tag{8}$$

$$y = a\sin\left(2\pi f t - \frac{2\pi}{\lambda}x\right) = a\sin\frac{2\pi}{\lambda}(f\lambda t - x) = a\sin\frac{2\pi}{\lambda}(vt - x) \tag{9}$$

If the progressive wave is moving along $-x$ direction Equation 7 takes the form

$$y = a\sin(\omega t + kx) = a\sin\frac{2\pi}{\lambda}(vt + x) \tag{10}$$

Standing Waves and Normal Modes

Standing wave, also called stationary wave, combination of two waves moving in opposite directions, each having the same amplitude and frequency. Another way of saying: Standing wave is a wave which oscillates in time but whose peak amplitude profile does not move in space.

Suppose two waves reach a particle in a medium in the same time and y_1 is the displacement of the particle due to one wave and y_2 is the displacement due to another wave, you can find the resultant displacement y of the particle by the vector sum of individual displacements that is,

$$\vec{y} = \vec{y_1} + \vec{y_2} \tag{11}$$

The vector sum of the displacements of a particle at a point due to all waves that reach the particle at that point in the same time is called superposition.

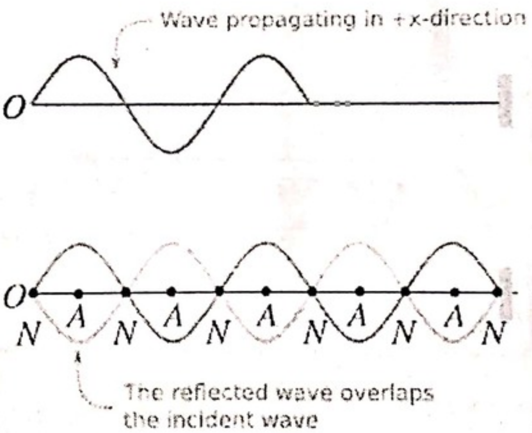

Figure 5: Standing wave

We consider a sinusoidal transverse wave along a string traveling in positive x-direction as in figure 5. When the wave is reflected at the fixed end or boundary, it overlaps the incident wave and their superposition turns into another wave called standing wave.
In Figure 5 the reflected wave is traveling in negative x-direction. When the incident and reflected waves overlap some points are developed in the medium. The points where a particle in the medium do not move at all and the displacement is zero are called nodes denoted by N. Other type of points where there is a maximum displacement of a particle are called antinodes denoted by A. The nodes and antinodes are shown in figure 5.

In standing wave the net transfer of energy in a particular direction is zero that is the same amount of energy which flows towards the right (towards $+x$-direction) also flows towards the left (towards $-x$-direction) and net transfer is zero. Therefore, energy is not transferred in a standing wave.
We know that the wave function of a simple harmonic wave traveling in positive $x-$diection is $y_1 = a\sin(\omega t - kx)$ and the wave function of a simple harmonic wave traveling in negative $x-$diection is $y_2 = a\sin(\omega t + kx)$. The resultant wave function of these two waves is $y = y_1 + y_2$. You may know the

trigonometric identities of cosine function such as $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\sin(A-B) = \sin A \cos B - \cos A \sin B$. Now can find the resultant wave function which is

$$y = y_1 + y_2 = (2a \sin \omega t)(\cos kx) \tag{12}$$

This equation describes a wave that oscillates in time, but has a spatial dependence that is stationary; at any point x the amplitude of the oscillations is constant with value $2a \cos\left(\frac{2\pi x}{\lambda}\right)$. The amplitude will be $2a$ or antinode will be found when

$$\cos \frac{2\pi x}{\lambda} = \pm 1 = \cos n\pi \tag{13}$$

$$\frac{2\pi x}{\lambda} = n\pi \Rightarrow x = \frac{n\lambda}{2} = 2n\frac{\lambda}{4} \tag{14}$$

Hence, antinodes will be found at locations which are even multiples of a quarter wavelength.
The amplitude will be 0 or node will be found when

$$\cos \frac{2\pi x}{\lambda} = 0 = \cos(2n+1)\frac{\pi}{2} \tag{15}$$

$$\frac{2\pi x}{\lambda} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\lambda}{4} \tag{16}$$

Hence, nodes will be found at locations which are odd multiples of a quarter wavelength.
https://www.physicskey.com/39/standing-waves-and-normal-modes