Math 244 Lecture Notes

Chapter 24: Chi-Squared Goodness of Fit

Overview: We return today to analyzing counts and proportions, much like the material from Exam I. Today, however, we will find a way to analyze multiple counts all at once.

Mr. Wherry is given a "fair" six sided die. He's not convinced in its fairness however, and after rolling it several times he gets the following outcomes:

Outcome of Die	1	2	3	4	5	6
# of trials	9	13	9	12	8	15

Example 1. (a) If the die is actually fair, what proportion of the time should Mr. Wherry get an outcome of a "1"? How many counts should he expect for the number of "1"s? Do this for all values.

(b) In terms of counts, how far off is the observed number of trials (table) from the expected number of trials (last part)?

(c) What is the relative/percentage error for each outcome? For example, maybe the observed outcome 2 is 50% more than what we were expecting [not actual numbers].

(d) Should we use a test or an interval to determine if the die is fair?

NOTE: Our total for the relative/percentage errors for each outcome is ___.

What happened?!

Let's create a new model:

$$\chi^2 =$$

Degrees of freedom are based off the number of groups. For example, our die problem has df= ____________. In general, df=.

The assumptions are...

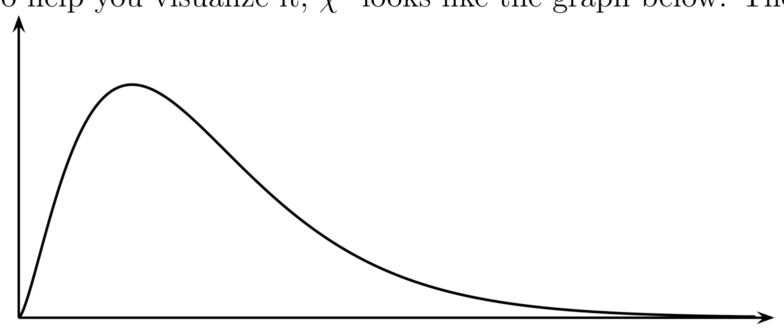
(a) Countable data.

(b)

(c)

(d)

To help you visualize it, χ^2 looks like the graph below. The **mean=df** and the mound is at df-2.



Example 2. Let's look at the six-sided die hypothesis test in detail. Recall that

Outcome of Die	1	2	3	4	5	6
OBSERVED # of trials	9	13	9	12	8	15
EXPECTED # of trials						

STEP I:

STEP II:

STEP III:

STEP IV:

Which way do we shade?

Assume that the die is perfectly fair...

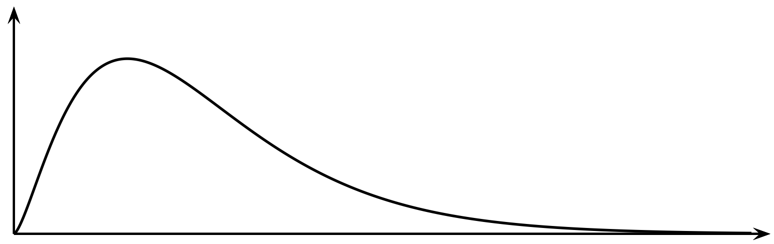
(a) How should the observed and expected compare?

(b) Should we have a large value for χ^2 or a small value?

(c) Do we want to reject or fail to reject the null-hypothesis?

(d) Do we want a large or small P-value?

(e) SHADE IN THE CURVE:



Assume that the die is extremely unfair...

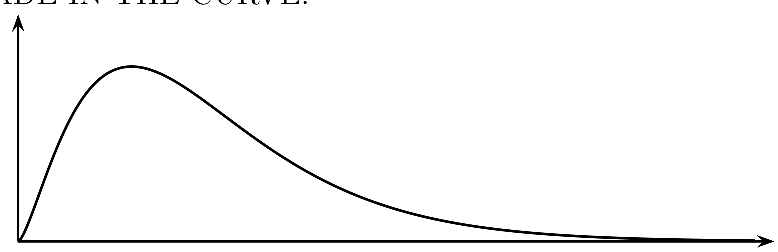
(a) How should the observed and expected compare?

(b) Should we have a large value for χ^2 or a small value?

(c) Do we want to reject or fail to reject the null-hypothesis?

(d) Do we want a large or small P-value?

(e) SHADE IN THE CURVE:



OBSERVATION:

TI-83 and TI-84: [2nd]→[Vars]→[chi-square cdf].
TI-89: [Apps]→[Stat/List]→[F5:Dist]→[chi-square cdf].

The general setup is the same for all calculators. We need to type in "chisquarecdf(low χ^2-score, high χ^2-score, df)".
Our P-value=____

Example 3. We're going to analyze a bag of M&M's. Let's test the claim that the color distribution is uniform. The colors are blue, brown, green, orange, red, and yellow.

	Blue	Brown	Green	Orange	Red	Yellow
OBSERVED # of trials						
EXPECTED # of trials						

STEP I:

STEP II:

STEP III:

STEP IV:

We call $Obs - Exp$ the residual.
We call $\frac{Obs-Exp}{\sqrt{Exp}}$ the standardized residual. It looks like the square-root of a single entry for our χ^2 and it works like a z-score!

Example 4. Calculate all the standardized residuals.

	Blue	Brown	Green	Orange	Red	Yellow
St. Res.						

Critical Value: Just like Z^* and t^*, we can find $(\chi^2)^*$ using a table. This can be thought of as the cutoff for what we consider a large or small χ^2-score.

Example 5. Let's reanalyze the bag of M&M's. This time, we will test the claim for the company itself. They claim that the percentages are as follows: blue= 24%, brown= 13%, green= 16%, orange= 20%, red= 13%, and yellow= 14%.

	Blue	Brown	Green	Orange	Red	Yellow
OBSERVED # of trials						
EXPECTED # of trials						

STEP I:

STEP II:

STEP III:

STEP IV:

Example 6. Recalculate all the standardized residuals.

	Blue	Brown	Green	Orange	Red	Yellow
St. Res.						