Math for DSA

Bitwise Operator

> Note : Bitwise operator follows associative property i.e. (a*b)c = a(bc)

1. AND (&) - Both true

   > Note : Finding even odd number using AND

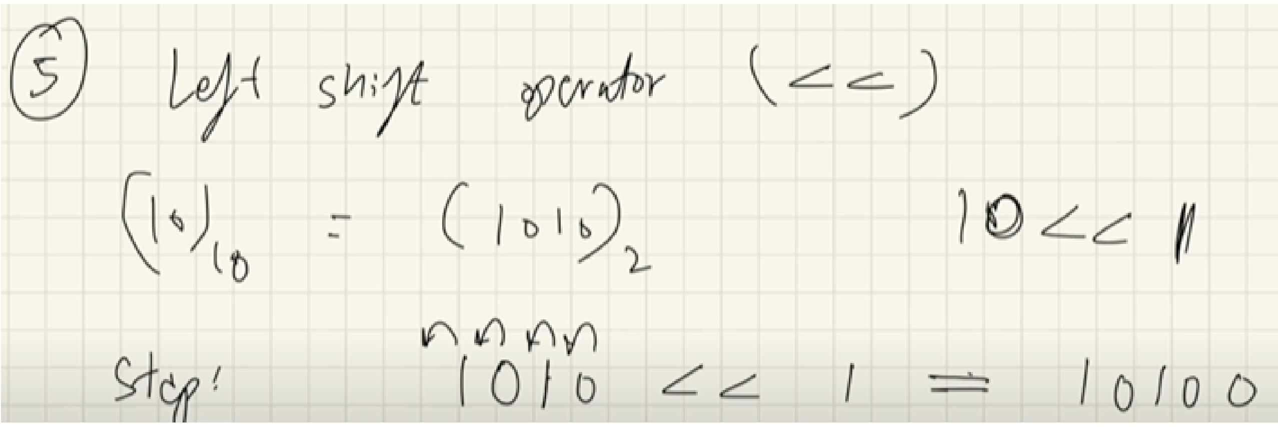
2. OR (|) - Either one true

3. XOR (^) - If and only if one ture

   > Note : XOR any with 1 => compliment's the number

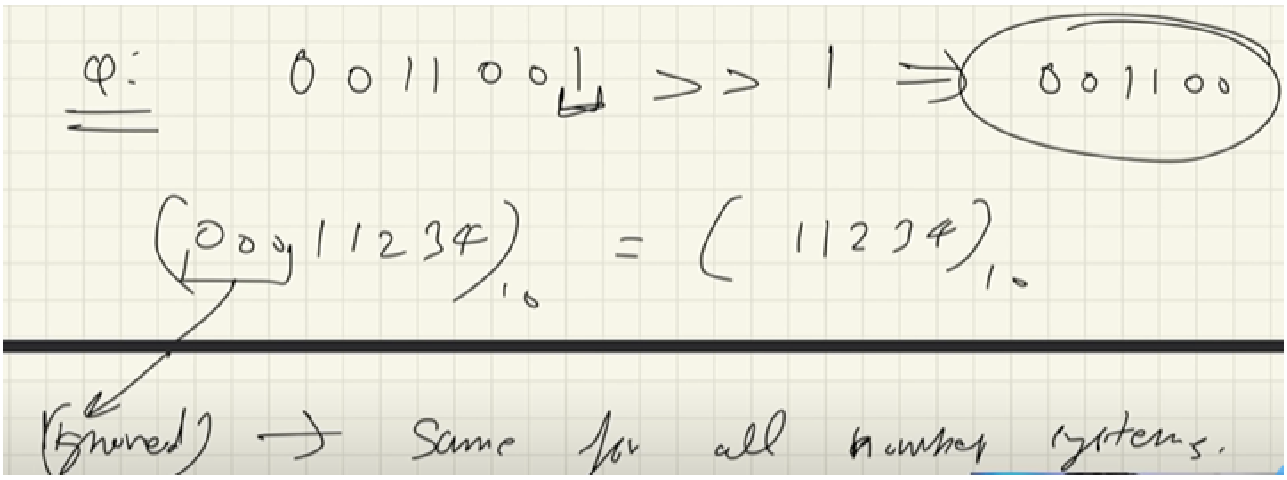
4. Compliment (~) - Negates the number

5. Left Shift Operator (<<) -

   > Note : a<<b = a*(2^b)

6. Right Shift Operator (>>) -

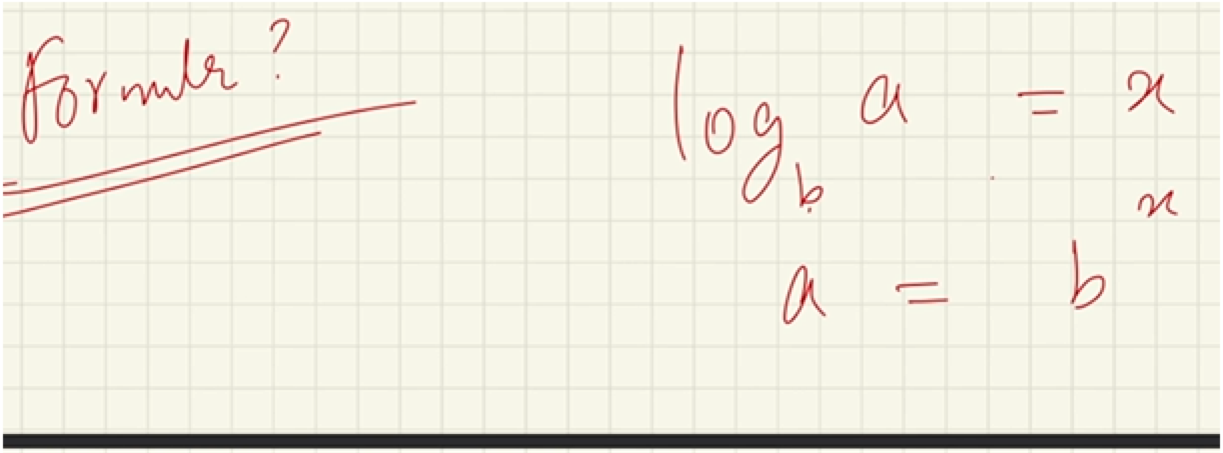
   > Note : a>>b = a/(2^b)

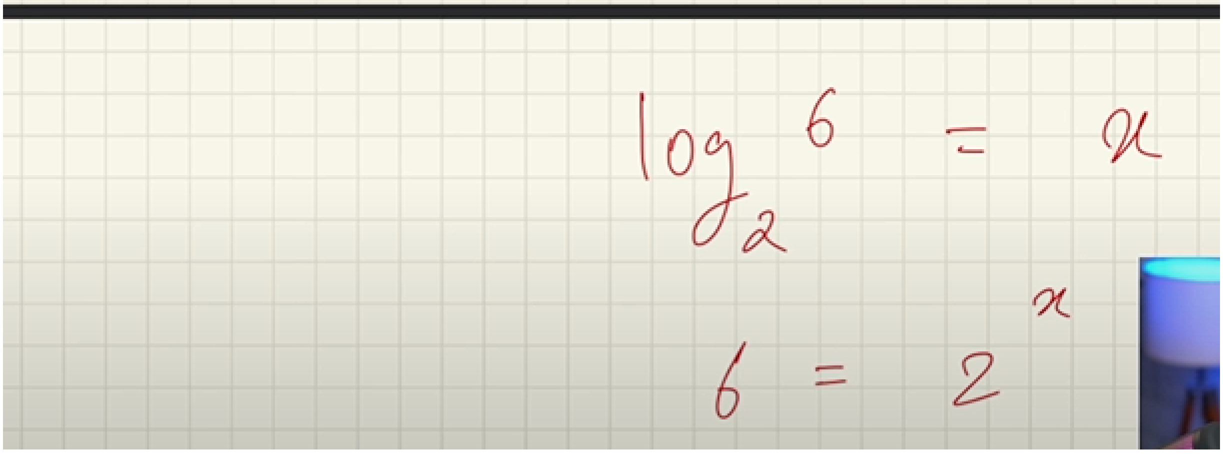
Number system

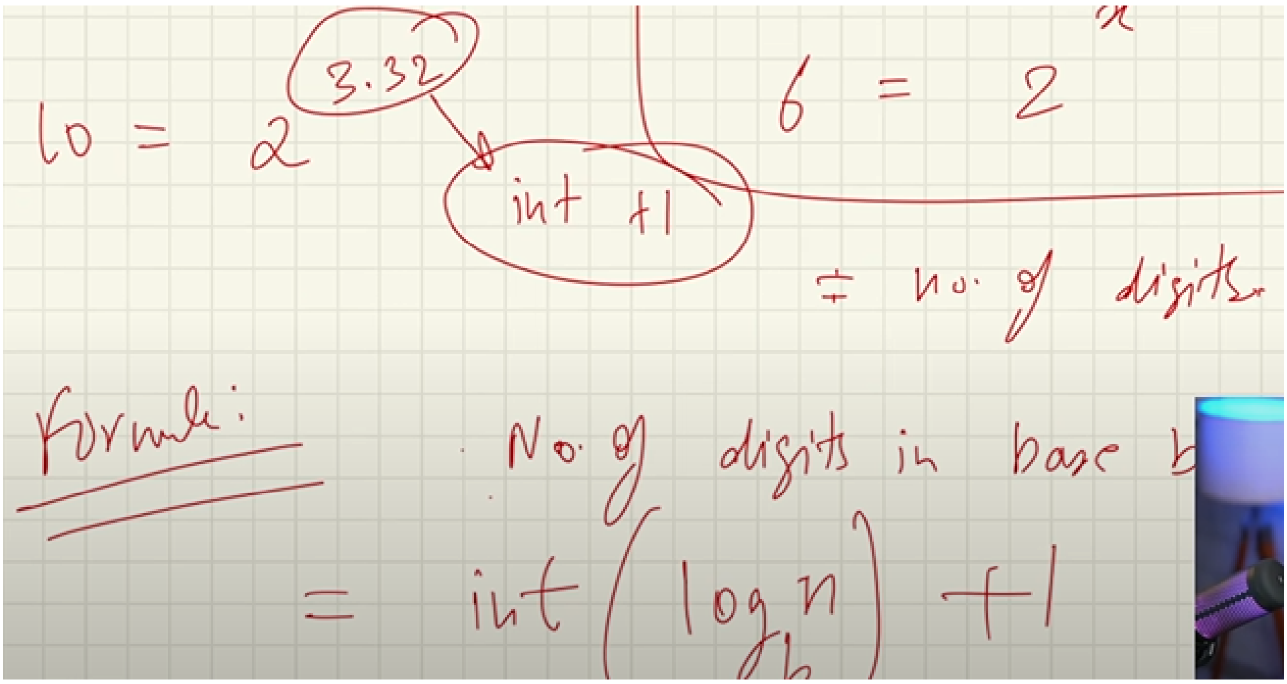
1. Decimal (Base 10) - 1 to 9
2. Binary (Base 2) - 0 & 1

> Note : Range - [-(base)^(n-1) to (base)^(n-1) - 1] where n is number of bits coz considering base = 2 & n - 8 including 0 - (base)^(n-1) - 1 we get 128 numbers and -1 to -(base)^(n-1)128 numbers. Total unique numbers - (base)^(n) according to example 128 + 128 = 256.

Number of binary bits in an integer







Negative -

Most Significant bit i.e. leftmost bit is set to be 1.

Steps - 2's compliment

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a. Complement of number.
b. Add 1 to it.
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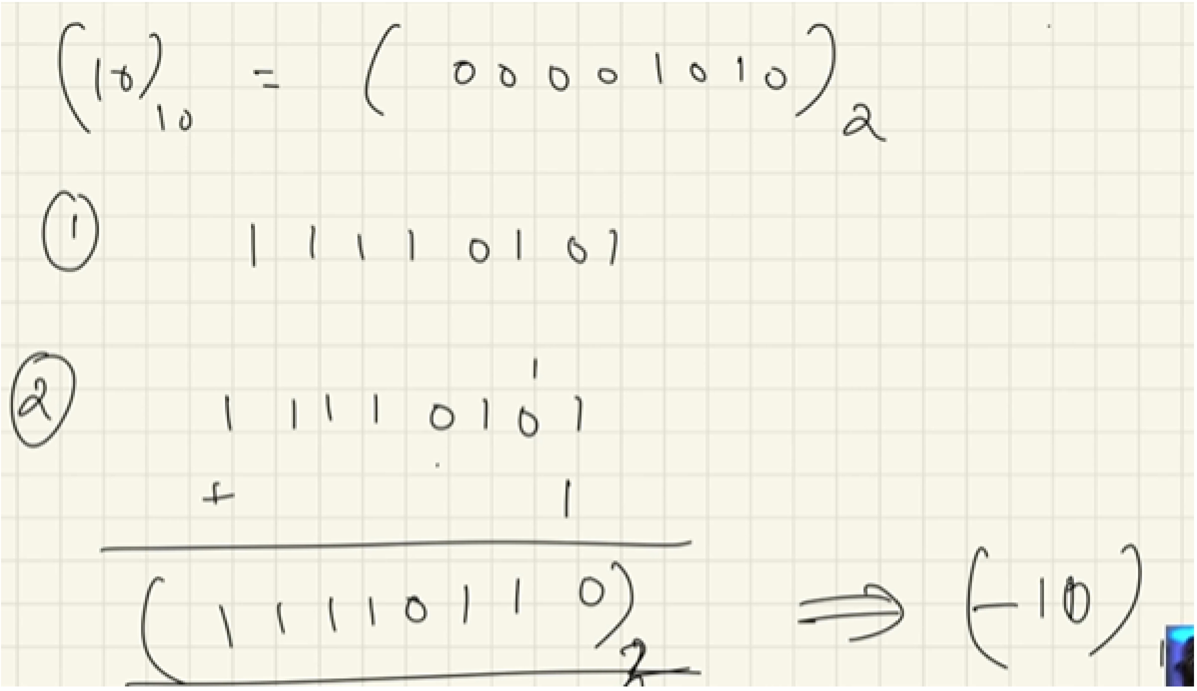


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Why?
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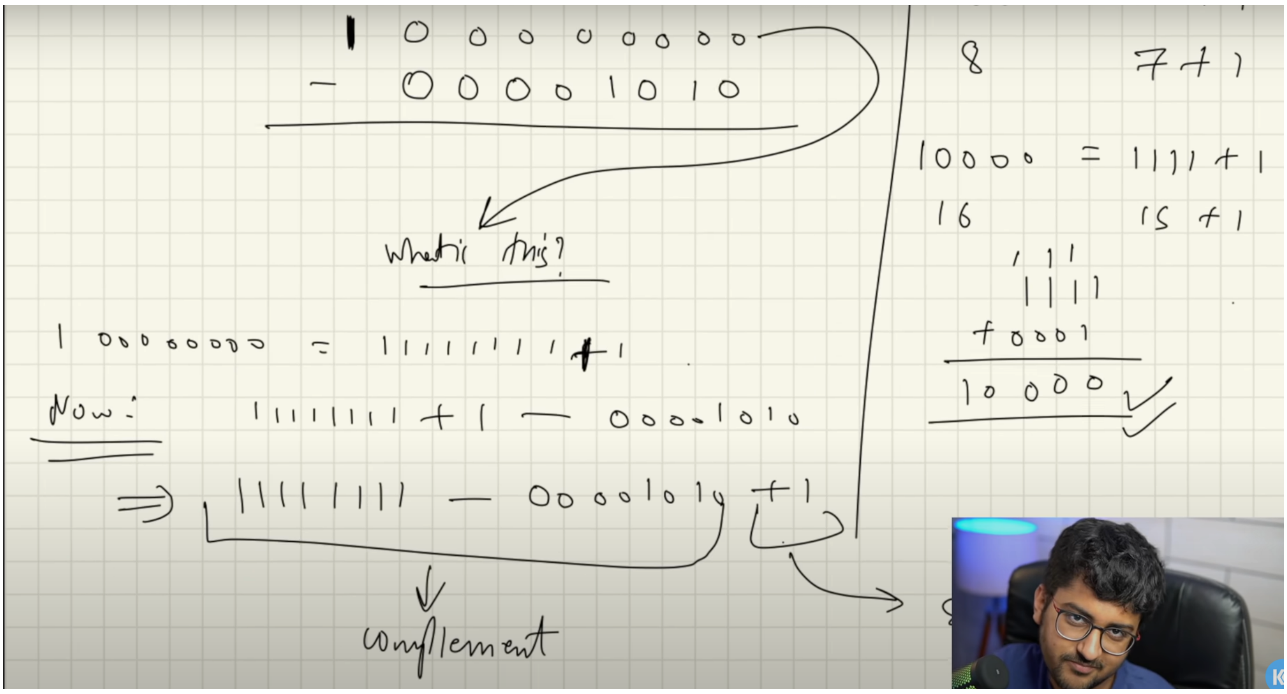


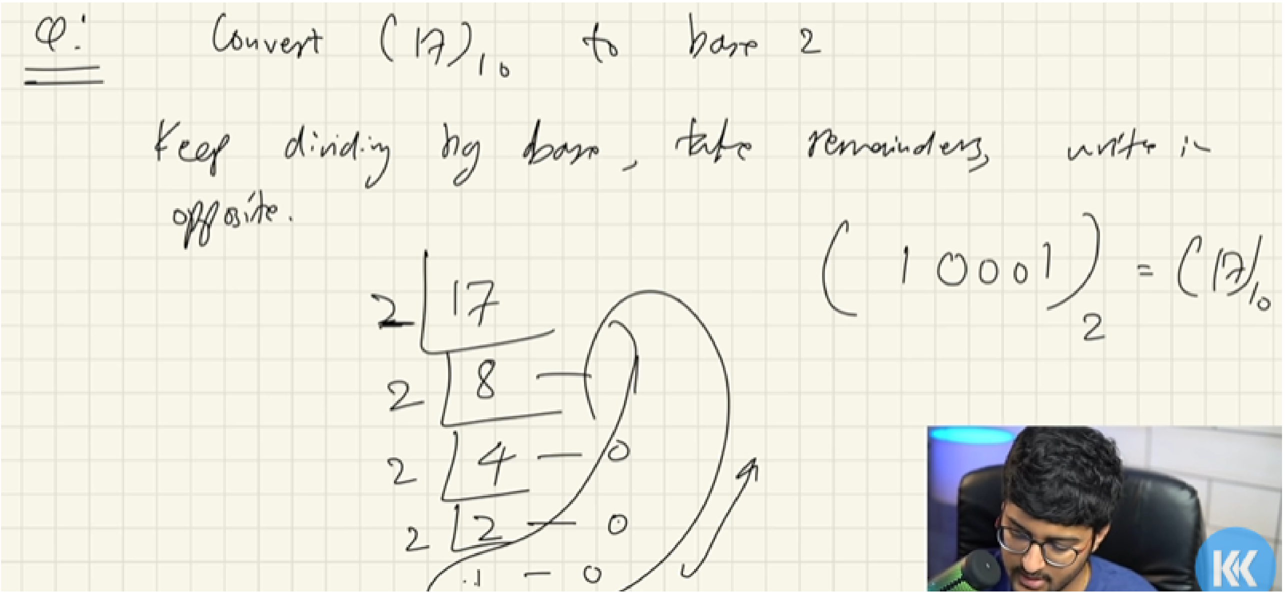
3. Octal (Base 8) - 0 to 7

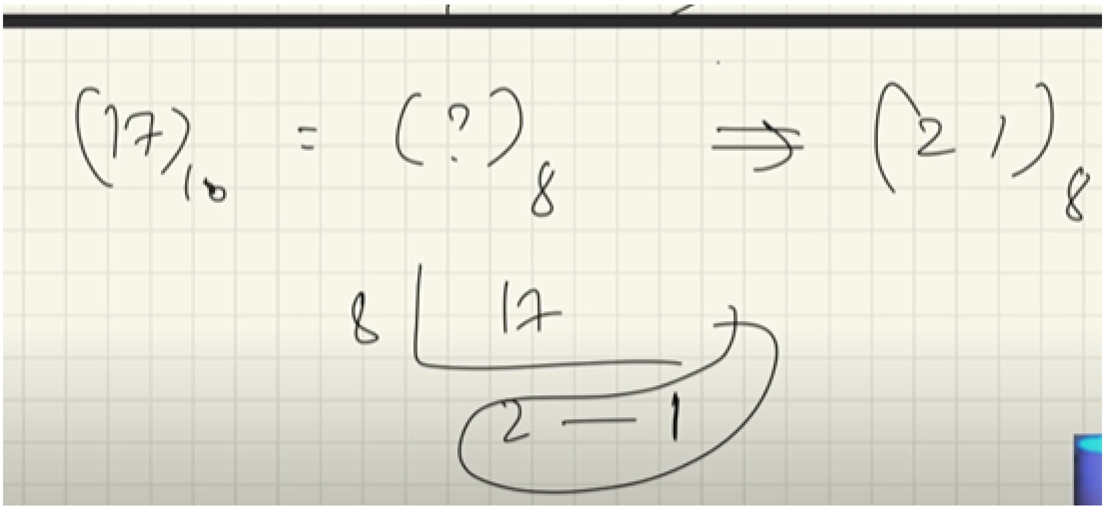
4. Hexadecimal (Base 16) - 0 to 9 & A to F

Conversion

1. Decimal to base b

   Keep dividing by base, take reminder and write in opposite.





2. Base b to decimal

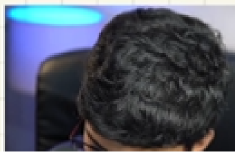
   Multiply and add power of base with digit

$(10001)_2 = (\quad)_{10}$?

Steps: multiply & add the power of base with digit.

$= 1*2^4 + 0*2^3 + 0*2^2 + 0*2^1 + 1*2^0$

$= 16 + 0 + 0 + 0 + 1 = (17)_{10}$



Q: $(21)_8 = (\quad)_{10}$

$= 2*8^1 + 1*8^0$

$= 2*8+1 = (17)_{10}$

Code

[Sum of numbers of nth row of Pascal's Triangle]

Ans: Sum of each row =

$$^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

For n^{th} row, sum = 2^{n-1}

Ans: $1 << (n-1) = 1 * 2^{n-1}$

Ans

Power of Two or not

```
  1 0 0 0 0 0 0 0        1 0 0 1 0
& 0 1 1 1 1 1 1 1      & 0 1 1 1 1
-----------------      -----------
        0                0 0 0 1 0
```

If $n \,\&\, (n-1) == 0$ // It is powers of 2.

A Power B

$3^6 \Rightarrow 3 * 3 * 3 * 3 * 3 * 3$ // O(b)

$3^6 = 3^{110} = 3^{2+4} = 3^2 * 3^4$

ans = ~~1~~ 9

base = 3

base = 9

81

base = base * base

O(log(b))

n = 110

n & 1 ⇒ 0

n >> 1 ⇒ 11&1 ⇒ 1

n = 11>>1 ⇒ 1&1 = 1

n = 1 >> 1 = 0

$3^{110} \Rightarrow 3^4 * 3^2 * 3^0$

ans = ans * base

Imp to understand this question

Number of set bits

Q: Given a number n, find the no. of set bits in it.

$n = 9$

$n = 1001$ Ans = 2

$n \,\&\, (-n) = 0001$

$n - [n \,\&\, (-n)] = 1000 \Rightarrow 1$

$n = 1001$

$\&\ 1000$

$1000 \rightarrow 1$

$8 \,\&\, 7 \Rightarrow$

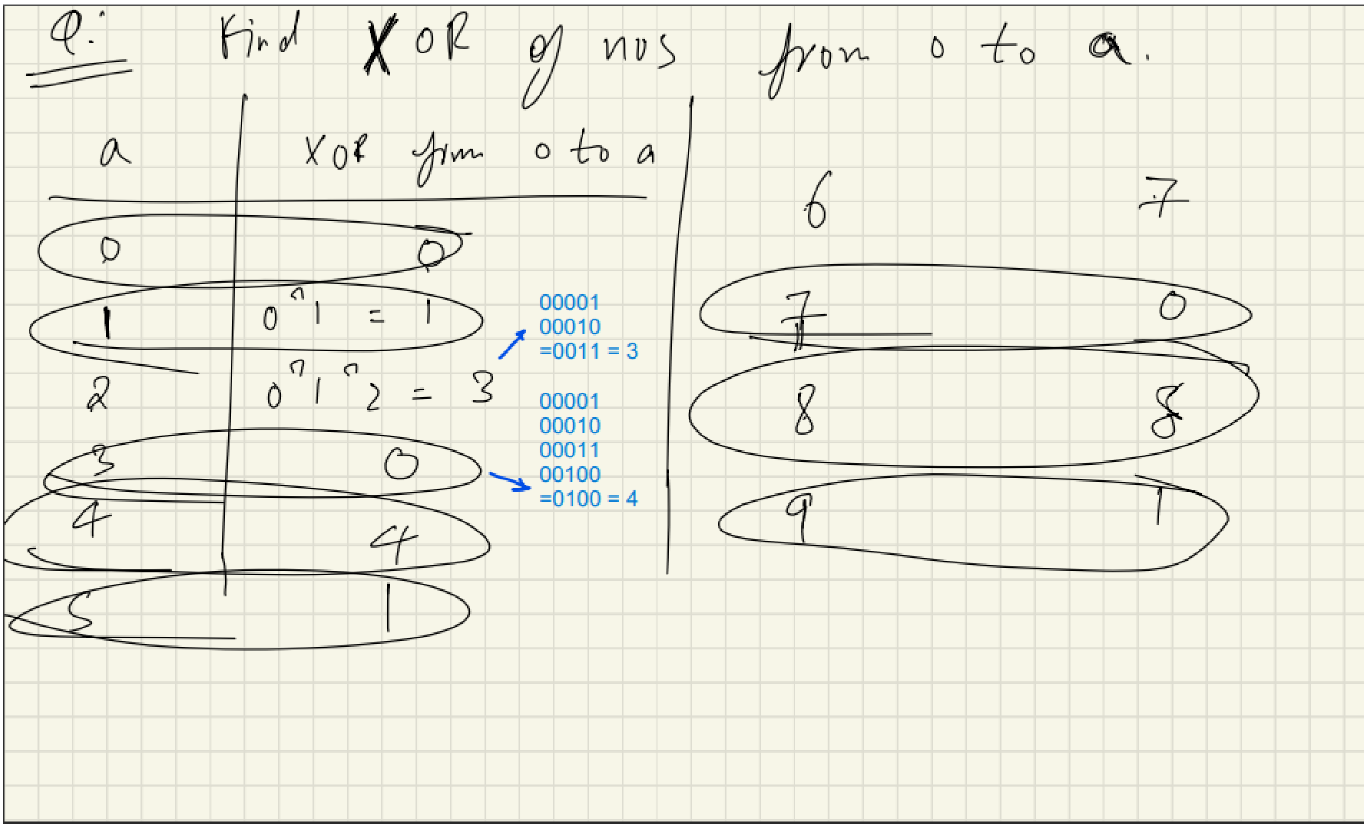
1000

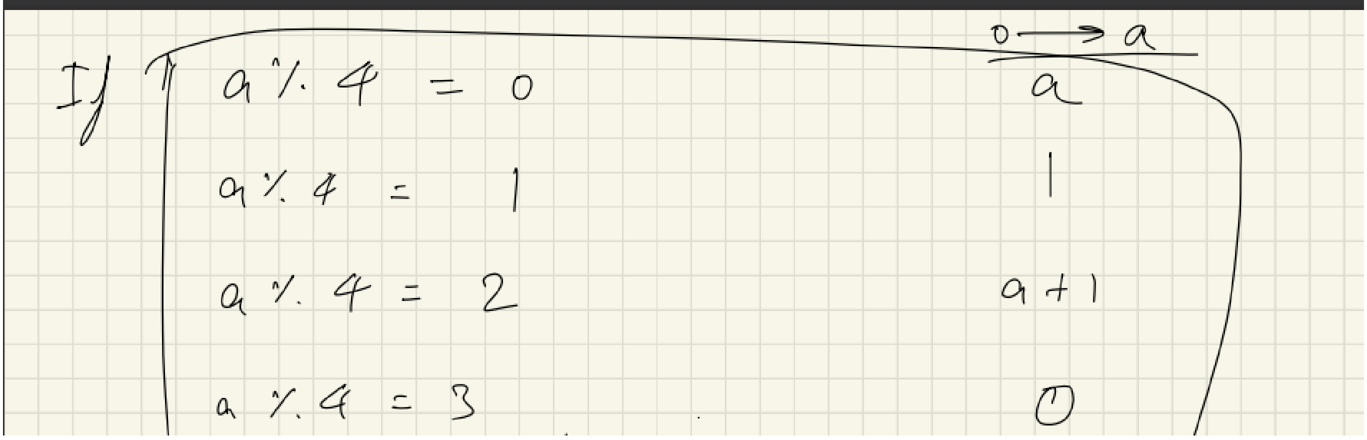
$\&\ 111$

$0 \quad 2$

No. of set bits = no. of iterations.

XoR form 0 to n





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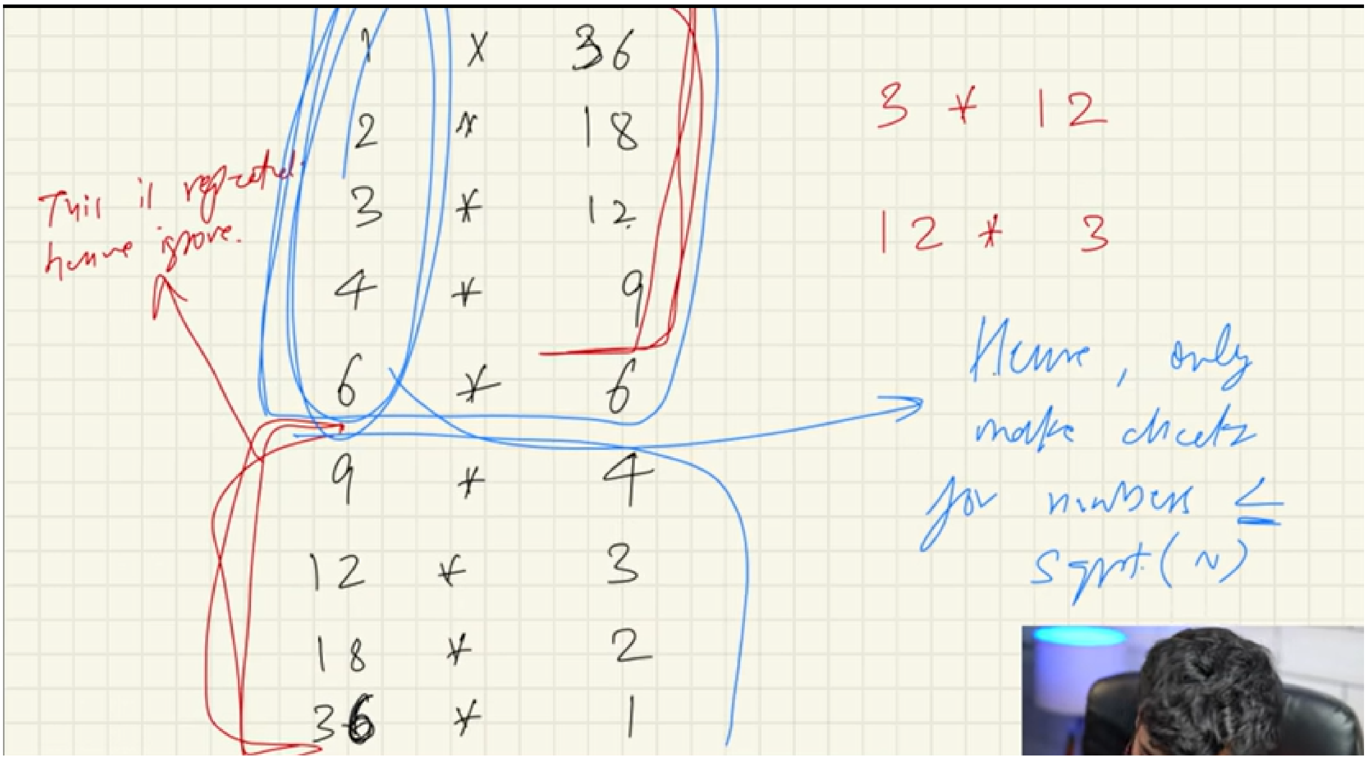
Prime Number

Check till sqrt of the number.



Prime till N - **Sieve of Eratosthenes**

Check this also only till square root

Time Complexity -

Time Complexity:

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \cdots$$

$$\underbrace{n\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots\right)}_{\text{Harmonic progression for primes}}$$

$$= \log(\log N)$$

Total time complexity: $O(N * \log(\log N))$

Finding Square Root of a Number

Using Binary Search, Then for precision itterate over 1 to 9 and add decimal.

Time Complexity - O(log(n))

Newton Raphson MethodS

$$root = X + \frac{N}{X}$$

actual sq root

Sqrt you have assumed

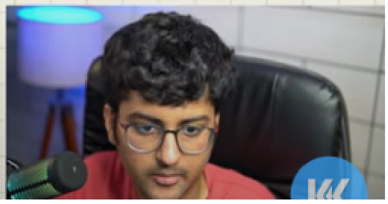
error = |root - X|

You will find your ans when error < 1

(1) Assign X to N

(2)

(2) Update the value of X = root



> *Note:* Complexity - FFT - O(log(n)*f(n)) where f(n) = cost of complexity f(n)/f'(n) with n digit precision.

Factor of a number

Check the divisiblity only till Square root of n as post that numbers are only interchanged.

But the answer wont be in sorted order.

Modulo Properties

Properties of modulo (%)

- $(a+b)\%m = ((a\%m) + (b\%m))\%m$
- $(a-b)\%m = ((a\%m) - (b\%m) + m)\%m$
- $(a*b)\%m = ((a\%m) * (b\%m))\%m$
- $\left(\frac{a}{b}\right)\%m = ((a\%m) * (b^{-1}\%m))\%m$

$b^{-1}\%m \Rightarrow$ Multiplicative modulo inverse (mmi)

Ex:

$(6*y)\%7 = 1$

$y =$ mmi for 6 & $y = 6$

$(6*6)\%7 = 36\%7 = 1$

mmi $= b^{-1}\%m$ means that b & m are co-primes.

Co-primes has only factor 1 as common factor

* $(a \% m) \% m = a \% m$
* $m^x \% m = 0 \quad \forall x \in$ +ve Integers.

Extra: If p is prime no. which is not a divisor of b, then $ab^{p-1} \% p = a \% p$ due to Fermat's Little theorem.

How? will be covered in advance DS course :)

Die-hard example

Measure 4 Gallons of 3 and 5 gallons jug.

Die-hard Example:

$3_a \quad 5_b \quad = \quad 4$

$1^{st} \rightarrow (0, 0) \rightarrow (3, 0) \rightarrow (0, 3)$ (a, b)

$2^{nd} \rightarrow (0, 3) \rightarrow (3, 3) \rightarrow (1, 5) \rightarrow (1, 0) \rightarrow (0, 1)$

$3^{rd} \rightarrow (0, 1) \rightarrow (3, 1) \rightarrow (0, 4)$

Ans!

Ans

jug a $\rightarrow$ s^1 times

jub b $\rightarrow$ s^2 times

$$r = as^1 - bs^2$$
$$r = as^1 + (-bs^2)$$

$$L = s'a + t'b$$
$$s'a = L - t'b$$

$$r = s'a + t'b - t'b - bs^2$$

$$r = L - (t' + u)b$$

If $t' + u \neq 0 \Rightarrow [r < 0 \text{ or } r > b]$

which is not true

$$t' + u = 0 \Rightarrow u = -t'$$

$$r = s'a + t'b = L$$

aka → $r = ax + by$

$$3x + 5y = 4 \quad ?$$

Put x & y as integers, what is the minimum +ve value you can have of eqⁿ.

$x = -3$, $y = 2$

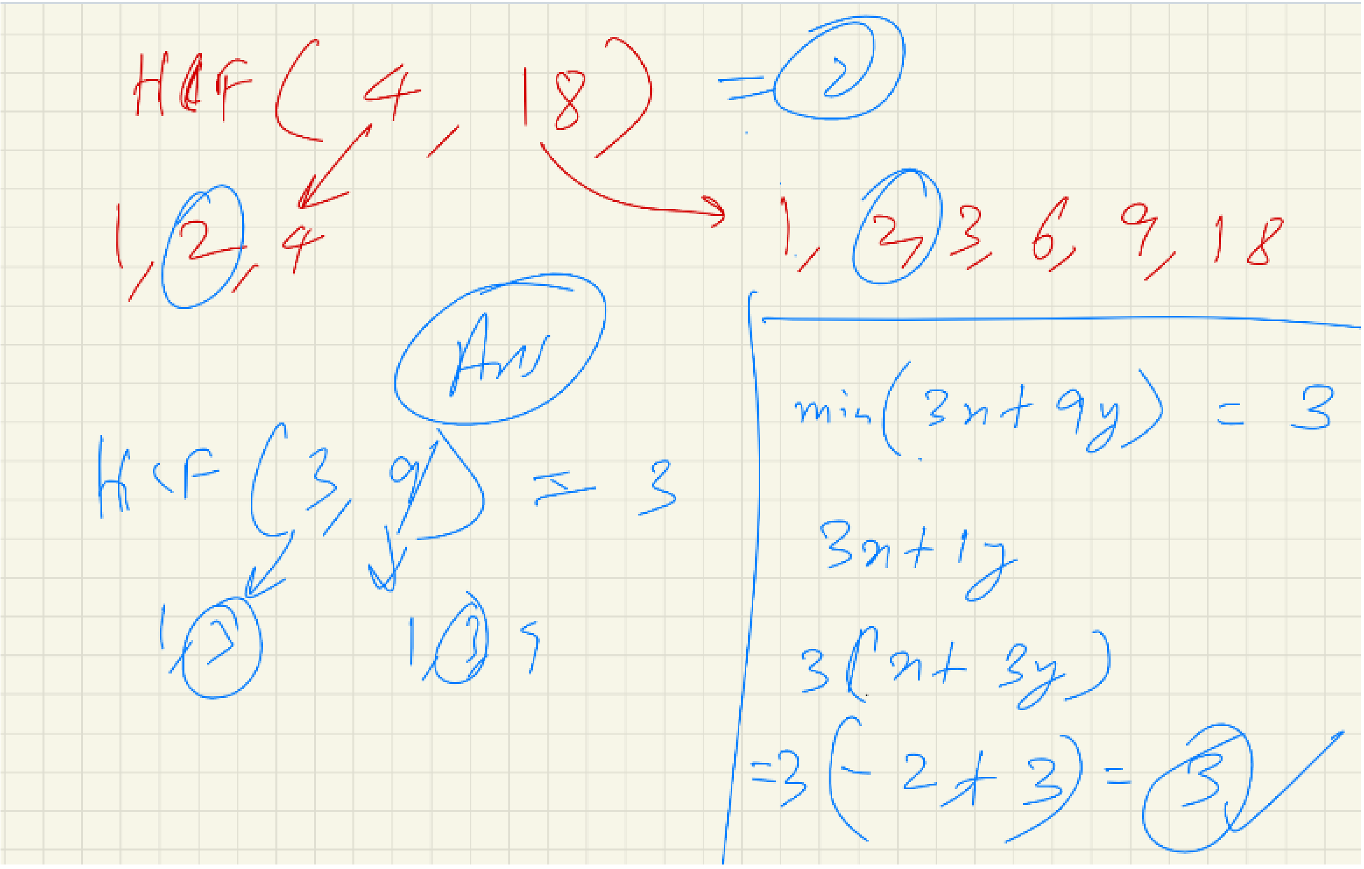
$$3x + 5y = 1$$

minimum +ve value that I can form

★ this is called hcf:

HCF/GCD of a & b = min +ve value of eqⁿ $ax + yb$ when x & y are ints.

HCF



Joining concept of die-hard and HCF

a, b

$$ax + by = L$$

$$2x + 4y = 5$$

$$2(x + 2y) = 5$$

$$\cancel{x + 2y = 2.5}$$

note:

whatever x, y you will get, that will come out as common.

$$3x + 6y = 9$$

$$3(x + 6y) = 1$$

$$x + 2y = 3$$

$$\underbrace{3x + 5y} = 17$$

$$1(3x + 5y) = 17$$

2 and 4 can't make 5 gallon water but 3 & 6 can make 9 also 3 & 5 can make 17

Euclidean's Algorithm

Euclid's Algorithm:

$$\gcd(a, b) = \gcd(\mathrm{rem}(b, a), a)$$

$$\gcd(105, 224) = \gcd(\mathrm{rem}(224, 105), 105)$$

$$= \gcd(14, 105)$$

Why?

$$105x + 224y$$

$\downarrow$ why subtract?

$$14x + 105y$$

i.e. because the gcd of (105, 224) also divides a linear combination of 105 & 224.

Ex: $224 - 2 * 105 = 14$ (rem)

LCM

LCM:

$lcm(a, b) =$ min. no. divisible by both a &

$lcm(2, 4) = 4$

$(3, 7) = 21$

Note: Say we have a, b

$d = gcd(a, b)$

$f = \frac{a}{d}, \quad g = \frac{b}{d}$

$\Rightarrow a = fd, \quad b = gd$

$lcm = c$ $\star$ $lcm(a, b) = lcm(fd, gd)$

* We know that f & g will have no other common factor.

$a = 9$, $b = 18$ $d = 9$

$f = 1$, $g = 2$

Say, $h \cdot y = \cancel{3}*3 = 9$ wrong!

↓ bigger $f = \frac{\cancel{9}}{\cancel{3}}\ 3$ $g = \frac{\cancel{18}}{\cancel{3}}\ 6$

* $a = fd$ $b = gd$

$lcm = f * g * d$ ⟹ This is how above conditions are satisfied.

more info:

are satisfied.

more info: $= a * b$

$= fd * g * d \rightarrow$ d is repeating, hence remove

17, 19

$lcm = f * g * d$

$a * b = fd * gd$

$= d * dfg$

$= hcf * lcm$

$hcf * lcm = a \times b$

Formula!

$$\boxed{LCM(a,b) = \frac{a \times b}{HCF(a,b)}}$$