

# Math for DSA

## Bitwise Operator

Note : Bitwise operator follows associative property i.e.  $(ab)c = a(bc)$

1. AND (&) - Both true

Note : Finding even odd number using AND

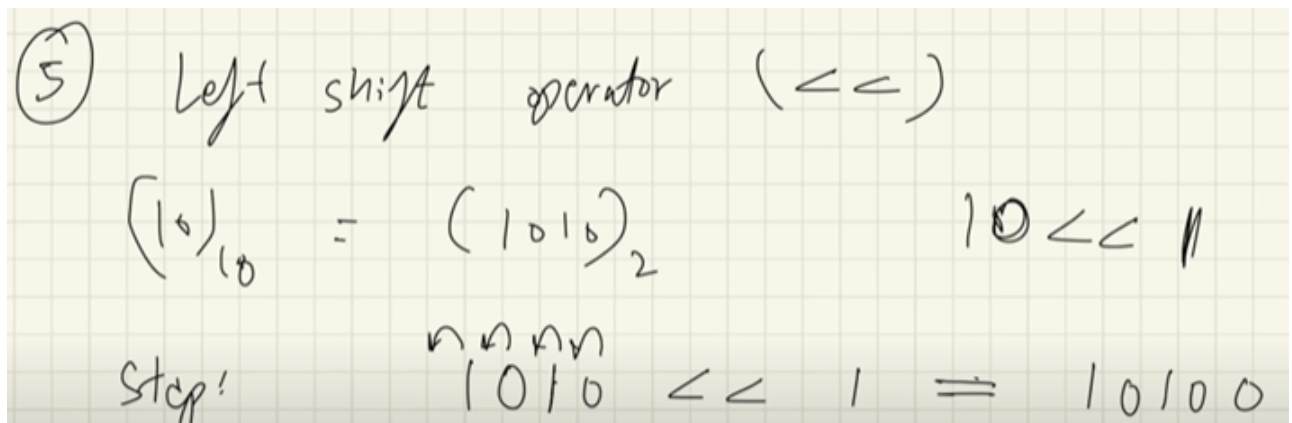
2. OR (|) - Either one true

3. XOR (^) - If and only if one true

Note : XOR any with 1 => compliment's the number

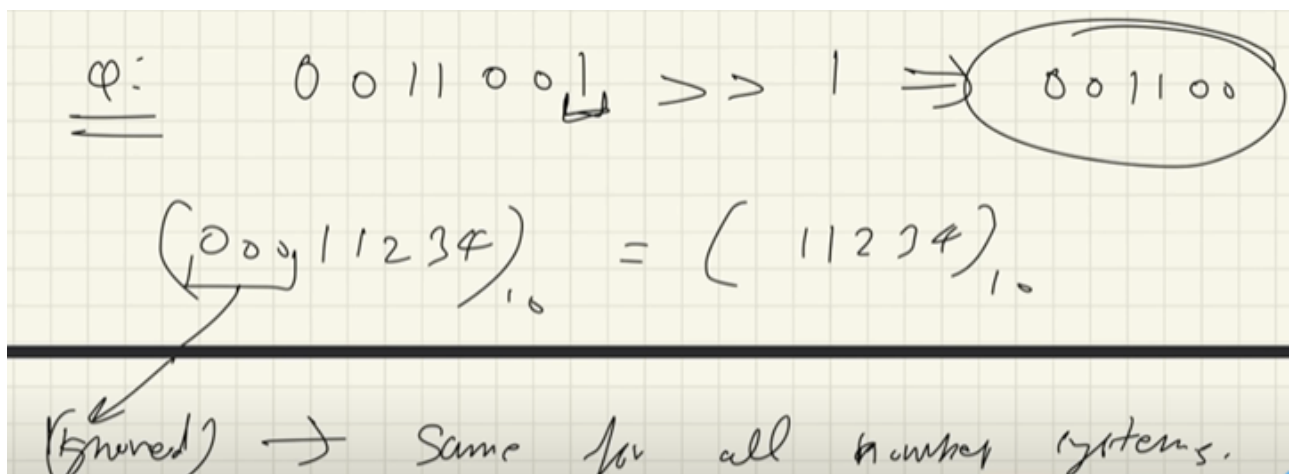
4. Complement (~) - Negates the number

5. Left Shift Operator (<<) -



Note :  $a << b = a * (2^b)$

6. Right Shift Operator (>>) -



Note :  $a >> b = a / (2^b)$

## Number system

1. Decimal (Base 10) - 1 to 9

2. Binary (Base 2) - 0 & 1

Note : Range -  $[-(\text{base})^{(n-1)} \text{ to } (\text{base})^{(n-1)} - 1]$  where  $n$  is number of bits coz considering base = 2 &  $n = 8$  including 0 -  $(\text{base})^{(n-1)} - 1$  we get 128 numbers and -1 to  $-(\text{base})^{(n-1)}$  128 numbers. Total unique numbers -  $(\text{base})^n$  according to example  $128 + 128 = 256$ .

Number of binary bits in an integer

Formula?

$$\log_b a = x$$

$$a = b^x$$


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$$\log_2 6 = x$$

$$6 = 2^x$$

$\log = 2^{3.32}$

$6 = 2^x$

$\text{int} + 1$

$\hat{=}$  no. of digits

Formula:

No. of digits in base  $b$

$$= \text{int} \left( \log_b n \right) + 1$$

## Negative -

Most Significant bit i.e. leftmost bit is set to be 1.

Steps - 2's complement

- Complement of number.
- Add 1 to it.

$$(10)_{10} = (00001010)_2$$

(1)  $11110101$

(2)

$$\begin{array}{r} 11110101 \\ + \quad \quad \quad 1 \\ \hline (11110110)_2 \Rightarrow (-10) \end{array}$$

Why?

$$\begin{array}{r} 10000000 \\ - 00001010 \\ \hline \end{array}$$

What's this?


$10000000 = 11111111 + 1$

Now:  $11111111 + 1 - 00001010$

$$\Rightarrow (11111111 - 00001010) + 1$$

↓ complement

$$\begin{array}{r} 8 \quad 7+1 \\ 10000 = 1111 + 1 \\ 16 \quad 15+1 \\ \begin{array}{r} 111 \\ 1111 \\ +0001 \\ \hline 10000 \end{array} \end{array}$$



3. Octal (Base 8) - 0 to 7

4. Hexadecimal (Base 16) - 0 to 9 & A to F

## Conversion

### 1. Decimal to base b


Keep dividing by base, take remainder and write in opposite.

Q: Convert  $(17)_{10}$  to base 2

Keep dividing by base, take remainders, write in opposite.

$$\begin{array}{r} 2 \overline{) 17} \\ \underline{2 \phantom{0}} 8 \\ 2 \overline{) 8} \\ \underline{2 \phantom{0}} 4 \\ 2 \overline{) 4} \\ \underline{2 \phantom{0}} 2 \\ 2 \overline{) 2} \\ \underline{2 \phantom{0}} 0 \end{array}$$

$(10001)_2 = (17)_{10}$



$$(17)_{10} = (?)_8 \Rightarrow (21)_8$$

$$\begin{array}{r} 8 \overline{) 17} \\ \underline{8 \phantom{0}} 9 \\ 8 \overline{) 9} \\ \underline{8 \phantom{0}} 1 \end{array}$$

### 2. Base b to decimal

Multiply and add power of base with digit

$$(10001)_2 = (\quad)_{10}?$$

Steps: multiply & add the power of base with digits.

$$= 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 16 + 0 + 0 + 0 + 1 = (17)_{10}$$

Q:  $(21)_8 = (\quad)_{10}$

$$= 2 \times 8^1 + 1 \times 8^0$$

$$= 2 \times 8 + 1 = (17)_{10} \checkmark$$

Code

Sum of numbers of nth row of Pascal's Triangle

Ans: Sum of each row =

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

For  $n^{\text{th}}$  row, sum =  $2^{n-1}$

Ans:  $1 \ll (n-1) = 1 \times 2^{n-1}$

Ans

Power of Two or not

$  \begin{array}{cccccccc}  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\  \& 0 & 1 & 1 & 1 & 1 & 1 & 1 \\  \hline  & & & 0 & & & &   \end{array}  $	$  \begin{array}{cccccc}  1 & 0 & 0 & 1 & 0 & \\  \& 0 & 1 & 1 & 1 & 1 \\  \hline  0 & 0 & 0 & 1 & 0 &   \end{array}  $
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If  $n \& (n-1) = 0$  // It is power of 2.

A Power B

$$3^6 \Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$3^6 = 3^{110} = 3^{2+4} = 3^2 \times 3^4$$

$$\text{ans} = \cancel{9}$$

$$\text{base} = 3$$

$$\text{base} = 9$$

$$81$$

$$\text{base} = \text{base} \times \text{base}$$

$$b(\log(b))$$

$$n = \cancel{11}0$$

$$n \& 1 \Rightarrow 0$$

$$n \gg 1 \Rightarrow 11 \& 1 \Rightarrow 1$$

$$n = 11 \gg 1 \Rightarrow 1 \& 1 \Rightarrow 1$$

$$n = 1 \gg 1 \Rightarrow 0$$

$$110$$

$$3 \Rightarrow 3^4 \times 3^2 \times 3^0$$

$$\text{ans} = \text{ans} \times \text{base}$$

Imp to understand this question

Number of set bits

Q: Given a number  $n$ , find the no. of set bits in it.

$$n = 9$$

$$n = 1001$$

$$\text{Ans} = 2$$

$$n \& (-n) = 0001$$

$$n - [n \& (-n)] = 1000 \Rightarrow \textcircled{1}$$

$$\begin{array}{r} n = \quad 1001 \\ \& \quad 1000 \\ \hline \quad 1000 \end{array} \quad \textcircled{1}$$

$$\begin{array}{r} \swarrow \\ 8 \& 7 \Rightarrow \begin{array}{r} 1000 \\ \& 111 \\ \hline 0 \end{array} \quad \textcircled{2} \end{array}$$

No. of set bits = no. of iterations.

XoR from 0 to n



Q. Find XOR of nos from 0 to a.

a	XOR from 0 to a
0	0
1	$0 \oplus 1 = 1$
2	$0 \oplus 1 \oplus 2 = 3$
3	0
4	4
5	1

00001  
 00010  
 =0011 = 3  
 00001  
 00010  
 00011  
 00100  
 =0100 = 4

6	7
7	0
8	8
9	1

If  $\uparrow$

$a \% 4 = 0$	$0 \rightarrow a$
$a \% 4 = 1$	a
$a \% 4 = 2$	1
$a \% 4 = 3$	a+1
	0

## Lec 26


### Prime Number

Check till sqrt of the number.

This is repeated hence ignore.

1	X	36
2	X	18
3	X	12
4	X	9
6	X	6
9	X	4
12	X	3
18	X	2
36	X	1

Hence, only make checks for numbers  $\leq \sqrt{n}$



### Prime till N - Sieve of Eratosthenes

	②	③	X	⑤	X	7	X	X	X
11	X	13	X	X	X	17	X	19	X
21	X	23	X	X	X	25	X	27	X
31	X	X	X	X	X	37	X	X	X

○ → True  
 X → False

Check this also only till square root

Time Complexity -

Time Complexity:

$$\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \dots$$

$$N \left( \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots \right)$$

Harmonic progression for primes.

$\log(\log N)$

Total time complexity:  $O(N * \log(\log N))$

## Finding Square Root of a Number

Using Binary Search, Then for precision iterate over 1 to 9 and add decimal.

Time Complexity -  $O(\log(n))$


## Newton Raphson Methods

$$\text{root} = \left[ \text{X} + \frac{N}{X} \right]$$

actual sq. root  $\rightarrow$  X  
 sqrt you have assumed  $\rightarrow$  N

$\text{error} = |\text{root} - X|$   
 You will find your ans when  $\text{error} < 1$

- ① Assign X to N
- ②  $\leftarrow$
- ③ Update the value of  $X = \text{root}$



Note: Complexity - FFT -  $O(\log(n) * f(n))$  where  $f(n)$  = cost of complexity  $f(n)/f'(n)$  with  $n$  digit precision.

## Factor of a number

Check the divisibility only till Square root of  $n$  as post that numbers are only interchanged.

But the answer wont be in sorted order.

## Modulo Properties

Properties of modulo (%)

$$\star (a+b) \% m = ((a \% m) + (b \% m)) \% m$$

$$\star (a-b) \% m = ((a \% m) - (b \% m) + m) \% m$$

$$\star (a * b) \% m = ((a \% m) * (b \% m)) \% m$$

$$\star \left(\frac{a}{b}\right) \% m = ((a \% m) * (b^{-1} \% m)) \% m$$

$b^{-1} \% m \Rightarrow$  multiplicative modulo inverse (mm1)

Ex:  $(6 * y) \% 7 = 1$

$$y = \text{mm1 for } 6 \quad \& \quad y = 6$$

$$(6 * 6) \% 7 = 36 \% 7 = \textcircled{1}$$

$\text{mm1} = b^{-1} \% m$  means that

$b$  &  $m$  & co-prime.

Co-primes has only factor 1 as common factor

$$\star (a \% m) \% n = a \% m$$

$$\star m^a \% m = 0 \quad \forall a \in \text{+ve Integers.}$$

Extra:

If  $p$  is prime no. which is not a divisor of  $a$ , then  $a b^{p-1} \% p = a \% p$  due to Fermat's Little theorem.

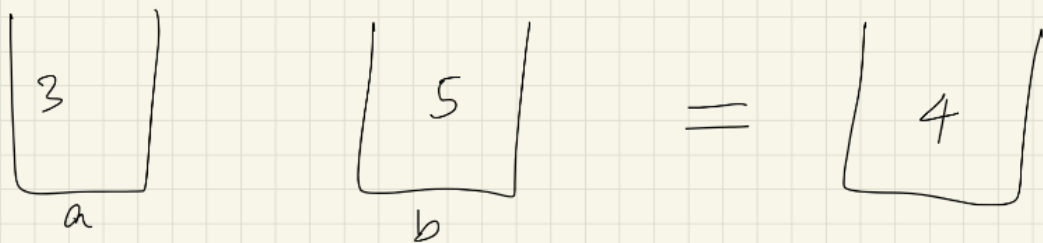
How?

will be covered in advance D1 course :)

## Die-hard example

Measure 4 Gallons of 3 and 5 gallons jug.

Die-hard Example:



1<sup>st</sup>  $\rightarrow$   $\begin{matrix} a & b \\ (0, 0) \end{matrix} \rightarrow (3, 0) \rightarrow (0, 3)$

2<sup>nd</sup>  $\rightarrow (0, 3) \rightarrow (3, 3) \rightarrow (1, 5)$

$(0, 1) \leftarrow (1, 0)$

3<sup>rd</sup>  $\rightarrow (0, 1) \rightarrow (3, 1) \rightarrow (0, 4)$

Ans!

$$\begin{aligned} \text{jug } a &\rightarrow s^1 \text{ times} \\ \text{jug } b &\rightarrow s^2 \text{ times} \end{aligned}$$

(Ans')

$$\begin{aligned} r &= as' - bs^2 \\ r &= as' + (-bs^2) \end{aligned} \quad \left| \begin{aligned} L &= s'a + t'b \\ s'a &= L - t'b \end{aligned} \right.$$

$$r = s'a + t'b - t'b - bs^2$$

$$r = L - (t' + u)b$$

If  $t' + u \neq 0 \Rightarrow [r < 0 \text{ or } r > b]$   
 which is not true

$$t' + u = 0 \Rightarrow u = -t'$$

$$r = s'a + t'b = L$$

aka

$$\rightarrow r = an + by$$

$$3x + 5y = 4$$

?

Put  $x$  &  $y$  as integers, what is the minimum value you can have of left.

$$x = -3, y = 2$$

$$3x + 5y = 1$$

minimum value that I can form

This is called hcf!

HCF/ of  $a$  &  $b$  = min +ve value  
 of  $eq^n$   $(an + by)$   
 where  $x$  &  $y$  are ints.

HCF

$HCF(4, 18) = 2$   
 $1, 2, 4 \rightarrow 1, 2, 3, 6, 9, 18$

$HCF(3, 9) = 3$   
 $1, 3 \rightarrow 1, 3$

**Ans**  
 $\min(3x + 9y) = 3$   
 $3x + 12$   
 $3(x + 3y)$   
 $= 3(-2 + 3) = 3$

Joining concept of die-hard and HCF



$a, b$

$$ax + by = L$$

$$2x + 4y = 5$$

$$2(x + 2y) = 5$$

$$x + 2y = 2.5$$

note:

What one  $x$ ,  
you will get,  
that will  
come out  
as common.

$$3x + 6y = 9$$

$$3(x + 2y) = 9$$

$$x + 2y = 3$$

$$3x + 5y = 17$$

$$1(3x + 5y) = 17$$

2 and 4 can't make 5 gallon water but 3 & 6 can make 9 also 3 & 5 can make 17

Euclidean's Algorithm

## Euclid's Algorithm:

$$\gcd(a, b) = \gcd(\text{rem}(b, a), a)$$

$$\gcd(105, 224) = \gcd(\text{rem}(224, 105), 105) \\ = \gcd(14, 105)$$

Why?

$$\Rightarrow 105x + 224y$$

↓ why subtract?

$$14x + 105y$$

(i.e.) because the gcd of  $(105, 224)$  also divides a linear combination of  $105$  &  $224$ .

Ex:  $224 - 2 \times 105 = 14 \text{ (rem)}$

LCM

LCM:

$\text{lcm}(a, b) =$  min. no. divisible by both  $a$  &  $b$

$$\text{lcm}(2, 4) = 4$$

$$(3, 7) = 21$$

Note:-

Say we have  $a, b$

$$d = \text{gcd}(a, b)$$

$$f = \frac{a}{d}, \quad g = \frac{b}{d}$$

$$\Rightarrow a = fd, \quad b = gd$$

$$\text{lcm} = c \quad \star \text{lcm}(a, b) = \text{lcm}(fd, gd)$$

★ We know that  $f$  &  $g$  will have no other common factor.

$$a = 9, \quad b = 18$$

$$f = \textcircled{1}, \quad g = \textcircled{2}$$

$$d = 9$$

Say,  $h = 9 = 3 \times 3 = \textcircled{9}$

↓ bigger

$$f = \frac{9}{3} = 3$$

$$g = \frac{18}{3} = 6$$

(wrong!)

★  $a = fd$

$b = gd$

$$\text{Lcm} = f \times g \times d$$

⇒ This is how above conditions are satisfied.

more info:

more info:  $a \times b$  one condition.

$$= f \times d \times g \times d \rightarrow d \text{ is repeating, hence remove}$$

17, 19

$$\text{Lcm} = f \times g \times d$$

$$\begin{aligned} \star a \times b &= f \times d \times g \times d \\ &= d \times d \times f \times g \\ &= h \times f \times \text{Lcm} \end{aligned}$$

$$h \times f \times \text{Lcm} = a \times b$$

Formula!

$$\text{Lcm}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$