Problem Set 6

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There is substantial research and evidence that class attendance has a positive and significant effect on student performance. Because of this, state and local government agencies and school districts have designed programs and policies that incentivize students to not miss school days. Examples include tangible prizes like colorful pendants and free tickets to events, automated calls from celebrities, or class policies that mandate attendance.

Existing research has used a range of methods to test the relationship between attendance programs and student performance, including simple regression analysis, randomized experiments, and regression discontinuity approaches.

In this assignment, you will use regression discontinuity approaches to measure the effect of a hypothetical program on hypothetical student grades (this data is 100% fake).

In this simulated program, high school students who have less than 80% attendance during their junior year (11th grade) are assigned to a mandatory school attendance program during their senior year (12th grade). This program requires them to attend school and also provides them with additional support and tutoring to help them attend and remain in school. At the end of their senior year, students take a final test to assess their overall learning in high school.

The dataset I've provided contains four columns:

- `id`: A randomly assigned student ID number
- `attendance`: The proportion of days of school attended during a student's junior year (ranges from 0 to 100)
- `treatment`: Binary variable indicating if a student was assigned to the attendance program during their senior year
- `grade`: A student's final test grade at the end of their senior year

```
library(tidyverse)
library(rdrobust)
library(rddensity)
```

```r
library(broom)
library(modelsummary)

# This turns off this message that appears whenever you use summarize():
# `summarise()` ungrouping output (override with `.groups` argument)
options(dplyr.summarise.inform = FALSE)

program <- read_csv("../data/attendance_program.csv")
```

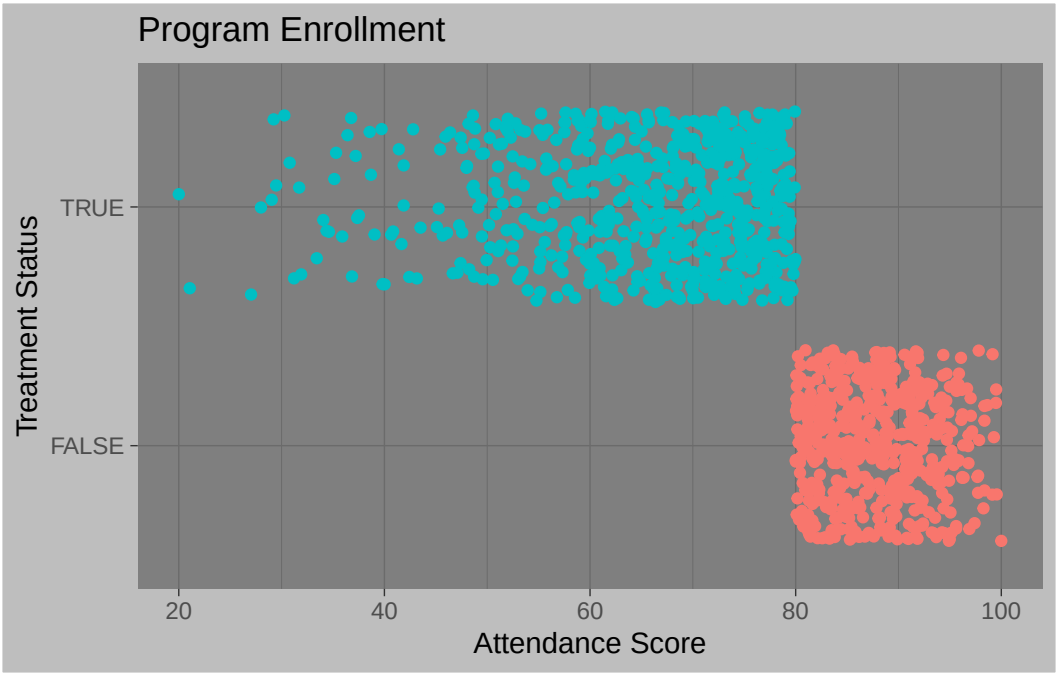
Step 1: Determine if process of assigning treatment is rule-based

Regression discontinuity can be used to measure the program effect because the program is rule based, with a hard cut off score. Students whose attendance is less than 80% receive the treatment and students above do not. There is no logical difference in students that have an attendance score right above or below the cut off point (i.e. Range [75,85]).

Step 2: Determine if the design is fuzzy or sharp

It appears that all observations with an attendance score below 80 are in the treatment group and that there is a sharp cut off. However, there may be some mixing right at the 80 vertical line.

```r
# Dot plot with attendance on the x-axis and treatment on the y-axis
ggplot(program, mapping =aes(attendance, treatment, color = treatment)) +
  geom_jitter(show.legend = F) +
  labs(title = "Program Enrollment", x="Attendance Score", y="Treatment Status") +
  theme_dark() +
  theme(plot.background = element_rect(fill = "gray"))
```



Below we see that there is no group leakage.

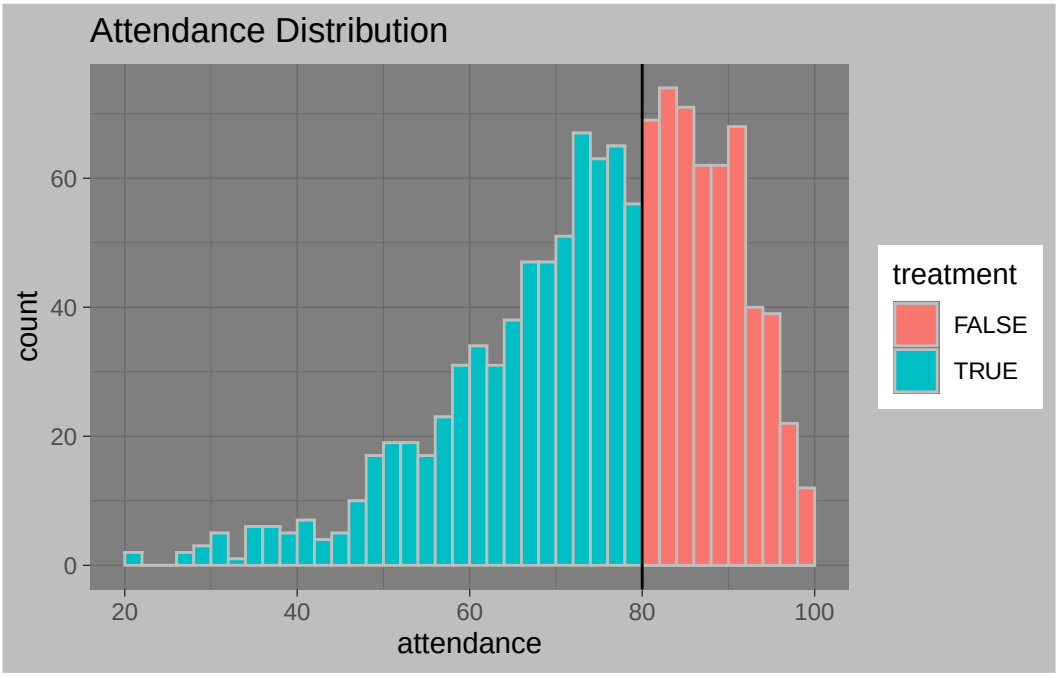
```
program %>%
  group_by(treatment, attendance <= 80) %>%
  summarise(count = n())
```

```
# A tibble: 2 x 3
# Groups:   treatment [2]
  treatment `attendance <= 80` count
  <lgl>     <lgl>              <int>
1 FALSE     FALSE                519
2 TRUE      TRUE                 681
```

Step 3: Check for discontinuity in running variable around cut-point

The histogram below provides evidence that teachers may be pushing students with actual attendance scores between 78 and 80 above 80. There is a dip in treatment density just before the 80 cut-off point that shows this.

```r
# Histogram of attendance
ggplot(program, mapping = aes(attendance, fill=treatment)) +
  geom_histogram(boundary = 80, binwidth = 2, color="gray") +
  geom_vline(xintercept = 80) +
  labs(title = "Attendance Distribution") +
  theme_dark() +
  theme(plot.background = element_rect(fill = "gray"))
```



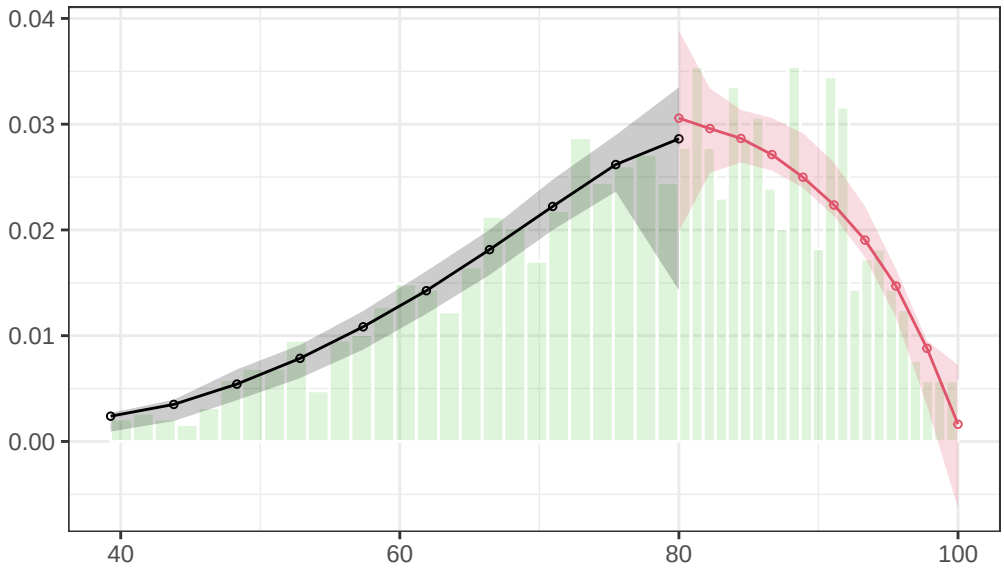
From the output below, in the "robust" line, we see that the p-value is not significant, so we can assume that there is not manipulation

```r
# McCrary test
test_density <- rddensity(program$attendance, c = 80)
summary(test_density)
```

```
Manipulation testing using local polynomial density estimation.

Number of obs =       1200
Model =               unrestricted
Kernel =              triangular
```

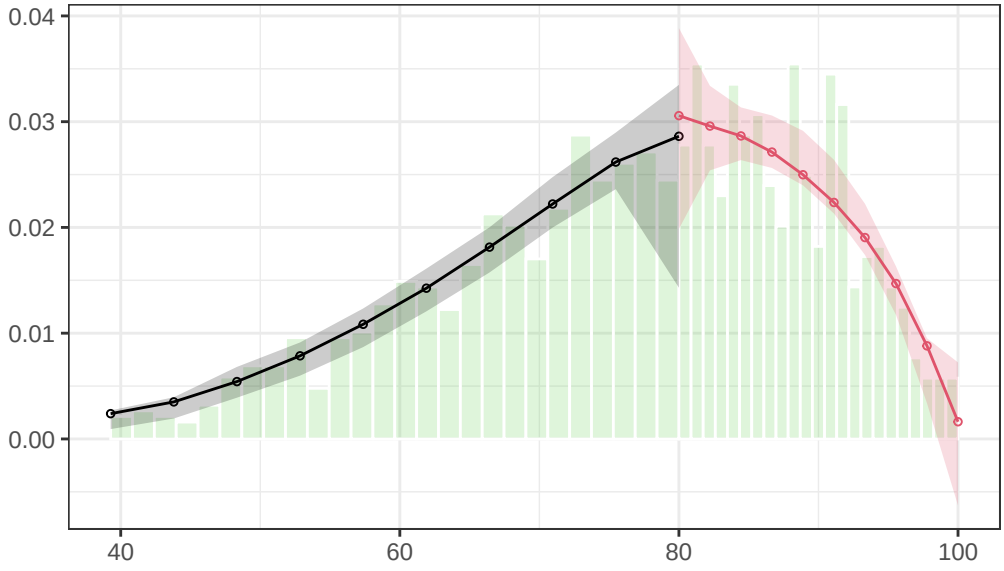
```
BW method =           estimated
VCE method =          jackknife

c = 80                Left of c           Right of c
Number of obs         681                 519
Eff. Number of obs    384                 421
Order est. (p)        2                   2
Order bias (q)        3                   3
BW est. (h)           13.574              12.521

Method                T                   P > |T|
Robust                0.7748              0.4384


P-values of binomial tests (H0: p=0.5).

Window Length / 2          <c     >=c    P>|T|
0.290                       6      14    0.1153
0.580                      12      23    0.0895
0.870                      22      29    0.4011
1.160                      31      37    0.5446
1.450                      42      55    0.2229
1.740                      52      66    0.2313
2.030                      58      70    0.3309
2.320                      72      84    0.3785
2.610                      74      95    0.1237
2.900                      88     103    0.3111
```

```r
rdplotdensity(rdd=test_density,
              X = program$attendance,
              type = "both")
```




```
$Estl
Call: lpdensity

Sample size                                        681
Polynomial order for point estimation    (p=)      2
Order of derivative estimated            (v=)      1
Polynomial order for confidence interval (q=)      3
Kernel function                                    triangular
Scaling factor                                     0.567139282735613
Bandwidth method                                   user provided

Use summary(...) to show estimates.

$Estr
Call: lpdensity

Sample size                                        519
Polynomial order for point estimation    (p=)      2
Order of derivative estimated            (v=)      1
Polynomial order for confidence interval (q=)      3
Kernel function                                    triangular
Scaling factor                                     0.432026688907423
Bandwidth method                                   user provided
```

```
Use summary(...) to show estimates.

$Estplot
```

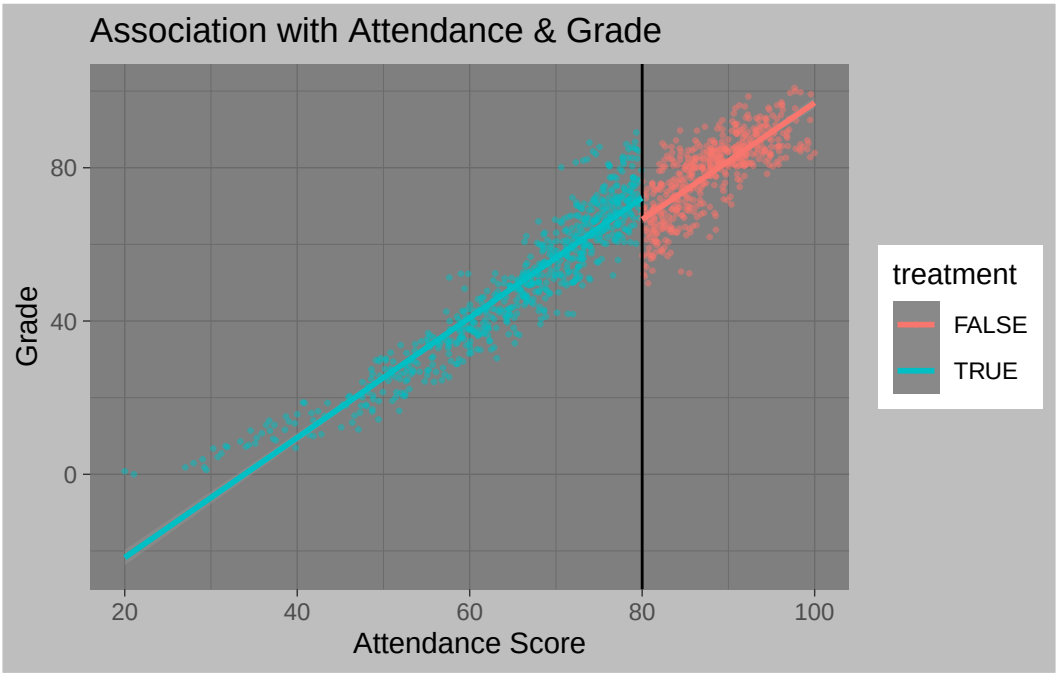


Step 4: Check for discontinuity in outcome across running variable

From the graph, we can see that there is discontinuity around the cut off point. It appears students who receive the treatment make higher scores.

```r
# Graph showing discontinuity in grades across levels of attendance
ggplot(program, mapping = aes(attendance, grade, color=treatment)) +
  geom_point(size = .5, alpha = .5) +
  geom_vline(xintercept = 80) +
  geom_smooth(method = "lm") +
  labs(title = "Association with Attendance & Grade",
       y = "Grade",
       x = "Attendance Score") +
    theme_dark() +
  theme(plot.background = element_rect(fill = "gray"))
```

```
`geom_smooth()` using formula 'y ~ x'
```



Step 5: Measure the size of the effect

Parametric estimation

Create a new dataset based on `program` that has a new variable in it named `attendance_centered`. This will be the value of `attendance` minus 80. This centers student attendance around the cutpoint (if a student had 85% attendance, they'd have a value of 5; if they had 70% attendance, they'd have a value of 10; etc.) and makes it easier to interpret the intercept coefficient in linear models since it shifts the y-intercept up to the cutpoint instead of zero.

```
# Add column to program that centers attendance
program.parametric <- program %>%
  mutate(attendance_centered = attendance - 80)
```

Regression model:

$$\text{Grade} = \beta_0 + \beta_1 \text{Attendance (centered)} + \beta_2 \text{Program} + \epsilon$$

For each additional point in the Attendance Score above 80, a students grade will be 1.56 points higher (p<.01). If a student is in the program, then on average, there grade is 5.88 points higher (p<.01).

```
# Linear model
program.parametric.model <- lm(grade ~ attendance_centered + treatment,
                               data = program.parametric)
tidy(program.parametric.model)
```

```
# A tibble: 3 x 5
  term                estimate std.error statistic  p.value
  <chr>                  <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)            66.2     0.330     201.   0
2 attendance_centered     1.56    0.0203     76.6  0
3 treatmentTRUE           5.88    0.595       9.89 3.07e-22
```

```
# Data and model with bandwidth = 5
program.parametric.bw5 <- program.parametric %>%
  filter(abs(attendance_centered) < 5)

program.parametric.model.bw5 <- lm(grade ~ attendance_centered + treatment,
                               data = program.parametric.bw5)
```

```
# Data and model with bandwidth = 10
program.parametric.bw10 <- program.parametric %>%
  filter(abs(attendance_centered) < 10)

program.parametric.model.bw10 <- lm(grade ~ attendance_centered + treatment,
                               data = program.parametric.bw10)
```

The coefficient of the treatment variable increases as the bandwidth decreases. A bandwidth of 10 produces a coefficient of 11.87, and a bandwidth of 5 produces a treatment coefficient of 12.43. This means that the program increases grades by 11.87 and 12.43 points, respectively. The number of observations seems to fall by a factor of 2 between each model. 1200 observations in model one, 640 in model two, and 330 in model three.

The advantage of the smaller bandwidth is we can assume the treatment and control group are identical with more certainty, so the most likely coeifficent is model 3.

```
# All three models
modelsummary(list(
```

	No Bandwidth	Bandwidth 10	Bandwidth 5
(Intercept)	66.191	64.195	64.050
	(0.330)	(0.601)	(0.859)
attendance_centered	1.560	2.026	2.148
	(0.020)	(0.097)	(0.272)
treatmentTRUE	5.884	11.869	12.340
	(0.595)	(1.094)	(1.575)
Num.Obs.	1200	640	330
R2	0.907	0.505	0.169
R2 Adj.	0.907	0.503	0.163
AIC	7924.3	4297.8	2228.8
BIC	7944.6	4315.6	2244.0
Log.Lik.	−3958.135	−2144.889	−1110.378
F	5823.048	324.837	33.144
RMSE	6.56	6.92	7.03

```
  "No Bandwidth" = program.parametric.model,
  "Bandwidth 10" = program.parametric.model.bw10,
    "Bandwidth 5" = program.parametric.model.bw5
))
```

Nonparametric estimation

The non-parametric approach yields similar results to the parametric approach. Form the table below, we see that the effect size is 12.013, meaning the local treatment effect yields 12.013 grade points higher.

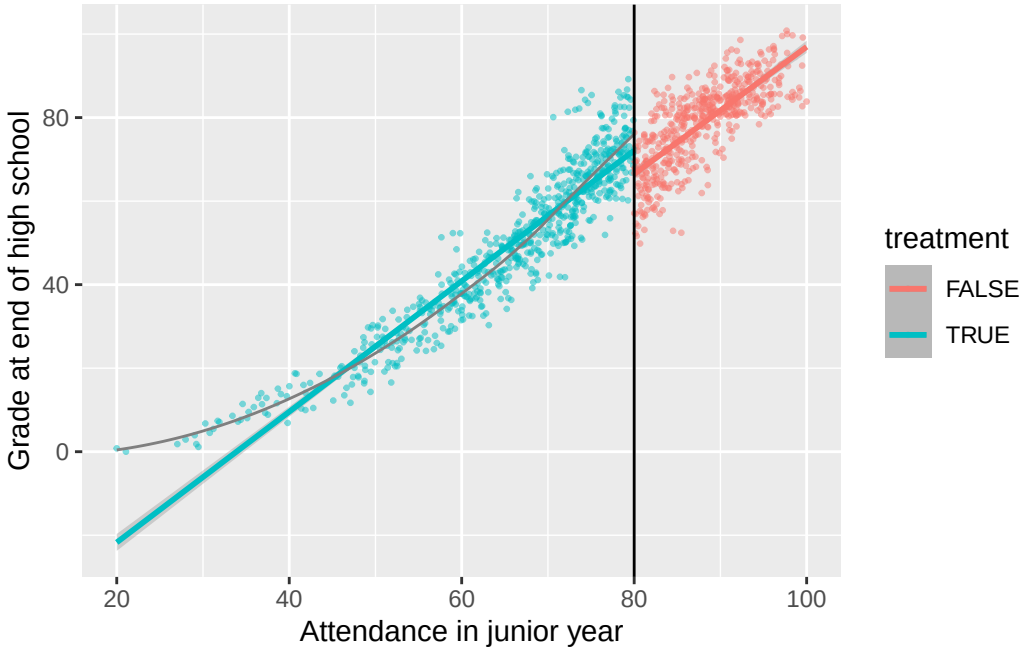
```
# rdrobust()
rdrobust(y = program$grade, x = program$attendance, c = 80) %>%
  summary()
```

```
Sharp RD estimates using local polynomial regression.

Number of Obs.                 1200
BW type                       mserd
Kernel                   Triangular
VCE method                       NN

Number of Obs.                  681          519
```

```
Eff. Number of Obs.              255          279
Order est. (p)                     1            1
Order bias  (q)                    2            2
BW est. (h)                    8.112        8.112
BW bias (b)                   12.449       12.449
rho (h/b)                      0.652        0.652
Unique Obs.                      627          451

===============================================================================
        Method     Coef. Std. Err.         z     P>|z|      [ 95% C.I. ]
===============================================================================
  Conventional   -12.013     1.394    -8.619     0.000   [-14.745 , -9.281]
        Robust         -         -    -7.244     0.000   [-15.473 , -8.883]
===============================================================================
```

```
# Plot
# rdplot(y = program$grade, x = program$attendance, c = 80)
ggplot(program, aes(x = attendance, y = grade, color = treatment)) +
  geom_point(size = 0.5, alpha = 0.5) +
  geom_smooth(data = filter(program, attendance < 80), method = "lm") +
  geom_smooth(data = filter(program, attendance < 80), method = "loess",
              size = 0.5, color = "grey50", se = FALSE) +
  geom_smooth(data = filter(program, attendance >= 80), method = "lm") +
  geom_vline(xintercept = 80) +
  labs(x = "Attendance in junior year", y = "Grade at end of high school")
```

```
`geom_smooth()` using formula 'y ~ x'
`geom_smooth()` using formula 'y ~ x'
`geom_smooth()` using formula 'y ~ x'
```



Nonparametric sensitivity checks

Now that we have an effect, we can adjust some of the default options to see how robust the effect size is.

First we'll play with the bandwidth. Find the ideal bandwidth with with `rdbwselect()`, then run `rdrobust` with twice that bandwidth and half that bandwidth (hint: use `h = SOMETHING`).

```
# Find the ideal bandwidth. Make sure rdbwselect() pipes into summary() so you
# can see the results: rdbwselect() %>% summary()
#
# You'll use the same y, x, and c as before
rdbwselect(y = program$grade, x = program$attendance, c = 80) %>%
  summary()
```

```
Call: rdbwselect

Number of Obs.                 1200
BW type                       mserd
Kernel                   Triangular
VCE method                       NN
```

```
Number of Obs.                  681         519
Order est. (p)                    1           1
Order bias  (q)                   2           2
Unique Obs.                     627         451

=============================================================
                  BW est. (h)    BW bias (b)
            Left of c Right of c  Left of c Right of c
=============================================================
     mserd     8.112      8.112     12.449     12.449
=============================================================
```

```r
# rdrobust() with half bandwidth
rdrobust(y = program$grade, x = program$attendance, c = 80, h = 8.112/2) %>%
  summary()
```

```
Sharp RD estimates using local polynomial regression.

Number of Obs.                1200
BW type                     Manual
Kernel                  Triangular
VCE method                      NN

Number of Obs.                 681         519
Eff. Number of Obs.            122         146
Order est. (p)                   1           1
Order bias  (q)                  2           2
BW est. (h)                  4.056       4.056
BW bias (b)                  4.056       4.056
rho (h/b)                    1.000       1.000
Unique Obs.                    681         519

================================================================================
        Method     Coef. Std. Err.         z     P>|z|      [ 95% C.I. ]
================================================================================
  Conventional   -12.761     2.000    -6.380     0.000   [-16.681 , -8.841]
        Robust         -         -    -3.913     0.000   [-16.492 , -5.485]
================================================================================
```

```
# rdrobust() with two times the bandwidth
rdrobust(y = program$grade, x = program$attendance, c = 80, h = 8.112*2) %>%
  summary()
```

```
Sharp RD estimates using local polynomial regression.

Number of Obs.                 1200
BW type                       Manual
Kernel                    Triangular
VCE method                        NN

Number of Obs.                  681          519
Eff. Number of Obs.             436          490
Order est. (p)                    1            1
Order bias  (q)                   2            2
BW est. (h)                  16.224       16.224
BW bias (b)                  16.224       16.224
rho (h/b)                     1.000        1.000
Unique Obs.                     681          519

===============================================================================
        Method     Coef. Std. Err.         z     P>|z|      [ 95% C.I. ]
===============================================================================
  Conventional   -11.327     0.980   -11.554     0.000   [-13.248 , -9.405]
        Robust         -         -    -8.613     0.000   [-15.499 , -9.753]
===============================================================================
```

Next we'll play with the kernel. Use the default ideal bandwidth and adjust the kernel to change how heavily weighted the observations right by the cutoff are. You already used a triangular kernel—that was the first `rdrobust()` model you ran, since triangular is the default. Try using Epanechnikov and uniform kernels (look at the help file for `rdrobust` or look at the in-class example to see how to specify different kernels):

```
# rdrobust() with an Epanechnikov kernel
rdrobust(y = program$grade, x = program$attendance, c = 80, kernel = "epanechnikov") %>%
  summary()
```

```
Sharp RD estimates using local polynomial regression.

Number of Obs.                 1200
```

```
BW type                       mserd
Kernel                   Epanechnikov
VCE method                         NN

Number of Obs.                   681          519
Eff. Number of Obs.              245          261
Order est. (p)                     1            1
Order bias  (q)                    2            2
BW est. (h)                    7.780        7.780
BW bias (b)                   12.498       12.498
rho (h/b)                      0.622        0.622
Unique Obs.                      627          451

===============================================================================
        Method     Coef. Std. Err.         z     P>|z|      [ 95% C.I. ]
===============================================================================
  Conventional   -11.910     1.377    -8.649     0.000   [-14.609 , -9.211]
        Robust         -         -    -7.313     0.000   [-15.348 , -8.860]
===============================================================================
```

```r
# rdrobust() with a uniform kernel
rdrobust(y = program$grade, x = program$attendance, c = 80, kernel = "uniform") %>%
  summary()
```

```
Sharp RD estimates using local polynomial regression.

Number of Obs.                  1200
BW type                        mserd
Kernel                       Uniform
VCE method                        NN

Number of Obs.                   681          519
Eff. Number of Obs.              195          231
Order est. (p)                     1            1
Order bias  (q)                    2            2
BW est. (h)                    6.441        6.441
BW bias (b)                   11.081       11.081
rho (h/b)                      0.581        0.581
Unique Obs.                      627          451

===============================================================================
```

```
        Method     Coef. Std. Err.         z     P>|z|      [ 95% C.I. ]
==============================================================================
  Conventional   -11.531     1.448    -7.965     0.000   [-14.368 , -8.694]
        Robust         -         -    -6.817     0.000   [-15.171 , -8.395]
==============================================================================
```

Step 6: Compare all the effects

I believe the most accurate model is the Parametric model with a bandwidth of 5. The data appears to be relatively linear, so the nonparametric line is unnecessary. Also the low bandwidth gives us confidences that the values are all similar.

Method	Bandwidth	Kernel	Estimate
Parametric	Full data	Unweighted	5.884
Parametric	10	Unweighted	11.869
Parametric	5	Unweighted	12.340
Nonparametric	8.112	Triangular	12.013
Nonparametric	4.056	Trianglar	12.761
Nonparametric	16.224	Triangular	11.327
Nonparametric	7.780	Epanechnikov	11.910
Nonparametric	6.441	Uniform	11.531

The program appears to have significant effect. Assuming the benefits out-way the cost, the new program could help students attain better grades. Further research should be done, if the program can benefit all students, and not just students around the cut-off point.