Problem Set 7

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```r
library(tidyverse)     # For ggplot, mutate(), filter(), and friends
library(broom)         # For converting models to data frames
library(estimatr)      # For lm_robust() and iv_robust()
library(modelsummary)  # For showing side-by-side regression tables
```

Task 1: Education, wages, and kids

Let's look once again at the effect of education on earnings. You'll use data from the 1976 Current Population Survey run by the US Census. The data is available as `wage` in the **wooldridge** R package—here I've just taken a subset of variables and renamed them. There are three columns:

Variable name	Description
`wage`	Average hourly earnings (in 1976 dollars)
`education`	Years of education
`n_kids`	Number of dependents living at home

You're interested in estimating β_1 in:

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + \epsilon_i$$

However, there is an issue with omitted variable bias and endogeneity. Instrumental variables can potentially help address the endogeneity.

Step 1

Load and look at the dataset

```r
wages <- read_csv("../data/wages.csv")
```

Step 2

We need an instrument for education, since part of it is endogenous. Do you think the variable `n_kids` (the number of children) would be a valid instrument? Does it meet the three requirements of a valid instrument?

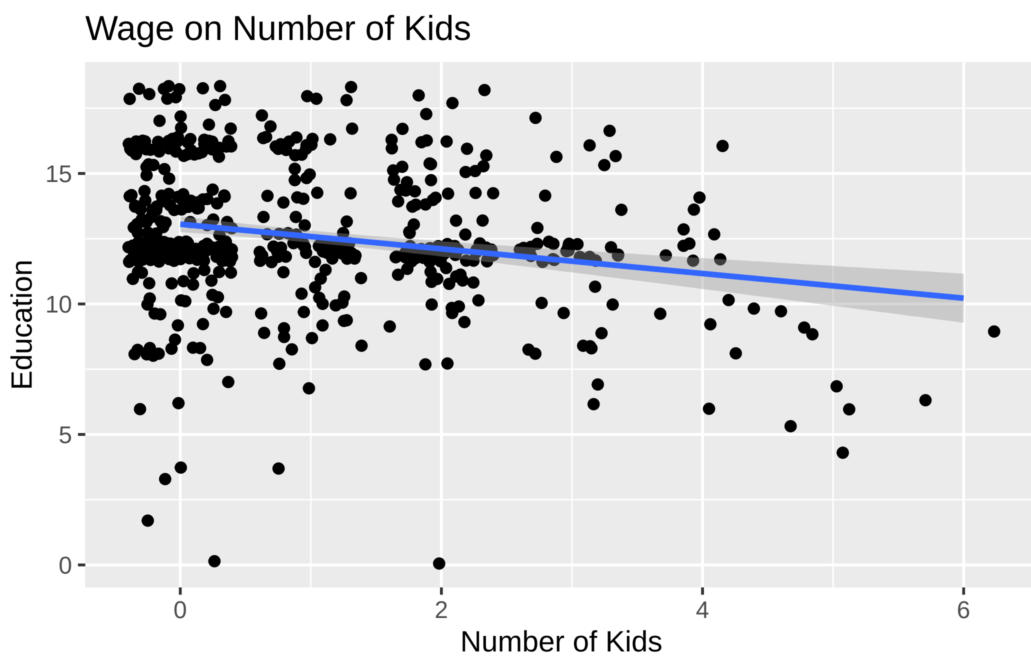
`Education` is endogenous because of association with ability. Unfortunately, ability is not measurable, and when using the Naive model, left in the error term. To account for this, we can use the instrumental variable `n_kids`.

Below are tests for the efficacy of the Instrument Variable.

Relevance

`n_kids` is correlated with `education` (r = -0.22) and has a regression coefficient of -0.47.

```r
ggplot(wages, aes(n_kids, education)) +
  geom_jitter() +
  geom_smooth(method = "lm") +
  labs(title = "Wage on Number of Kids", x = "Number of Kids", y = "Education")
```




```r
check_relevance <- lm(education ~ n_kids, data = wages)
tidy(check_relevance)
```

```
# A tibble: 2 x 5
  term        estimate std.error statistic   p.value
  <chr>          <dbl>     <dbl>     <dbl>     <dbl>
1 (Intercept)   13.1      0.153      85.2  3.22e-309
2 n_kids        -0.472    0.0936     -5.05 6.21e-  7
```

```r
glance(check_relevance)
```

```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic     p.value    df logLik   AIC   BIC
      <dbl>         <dbl> <dbl>     <dbl>       <dbl> <dbl>  <dbl> <dbl> <dbl>
1    0.0464        0.0445  2.71      25.5 0.000000621     1 -1269. 2544. 2557.
# ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

The tables above show that there is a significant negative assosication between the number of kids and education (F>10). We can assume relevance.

Exclusion

`n_kids` must only effect `wage` through `education`. We can test this in the following way:

$$Wage = \beta_0 + \beta_1 education Education = \gamma_0 + \gamma_1 nkids Wage = \alpha_0 + \alpha_1 nkids H_a : \text{Exclusion if } \alpha_1 = \beta_1 \gamma_1$$

Mathematical testing:

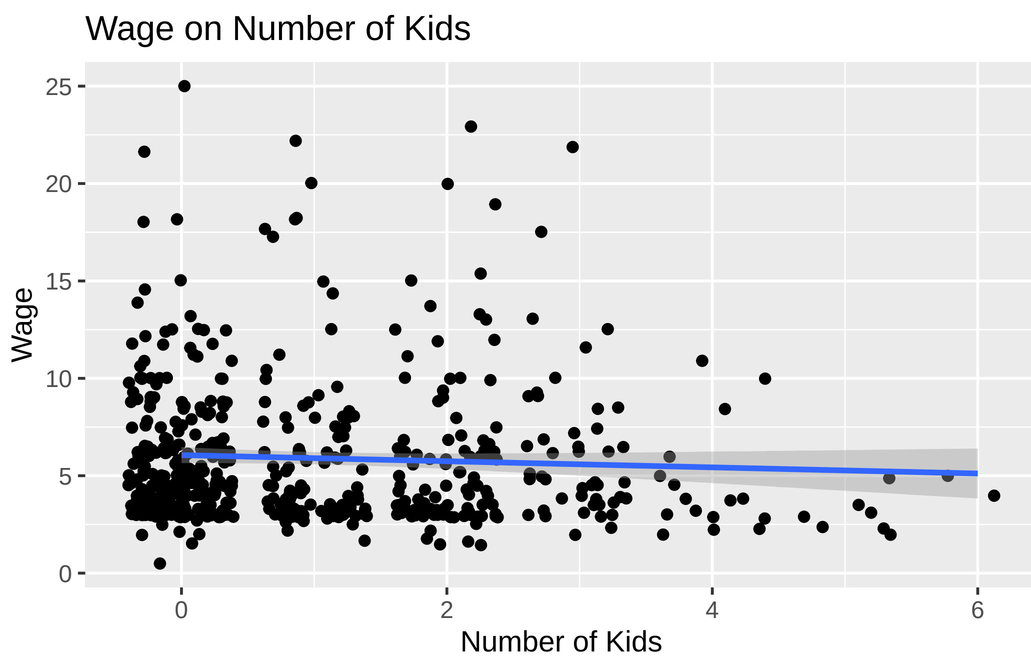
The Instrument Variable does not pass the Exclusion, although the difference is small.

```r
equation.1 <- lm(wage ~ education, data = wages)
equation.2 <- lm(education ~ n_kids, data = wages)
equation.3 <- lm(wage ~ n_kids, data = wages)

beta_1 <- coef(equation.1)["education"]
gamma_1 <- coef(equation.2)["n_kids"]
alpha_1 <- coef(equation.3)["n_kids"]

tibble("beta" = beta_1,
       "gamma" = gamma_1,
       "alpha"=alpha_1,
       "beta * gamma"=beta_1 * gamma_1,
       "diff"=as.numeric(beta_1 * gamma_1 - alpha_1))
```

```
# A tibble: 1 x 5
   beta  gamma  alpha `beta * gamma`    diff
  <dbl>  <dbl>  <dbl>          <dbl>   <dbl>
1 0.541 -0.472 -0.157         -0.256 -0.0982
```

Graphical Testing

```r
ggplot(wages, aes(n_kids, wage)) +
  geom_jitter() +
  geom_smooth(method = "lm") +
  labs(title = "Wage on Number of Kids", x = "Number of Kids", y = "Wage")
```



Exogeneity

From economic intuition, we can argue that `n_kids` is not correlated with `ability`. Ability or IQ is likely randomly distributed at birth and not a result of the number of offspring.

Step 3

Assume that the number of children is a valid instrument (regardless of whatever you concluded earlier). Using the number of children (`n_kids`) as an instrument for education (`education`), estimate the effect of education on wages via two-stage least squares (2SLS) instrumental variables (IV).

Do this by hand: create a first stage model, extract the predicted education, and use predicted education in the second stage.

```
first_stage <- lm(education ~ n_kids, data = wages)
wages_fitted <- augment_columns(first_stage, wages) %>%
  rename(education.fitted = .fitted)

second_stage <- lm(wage ~ education.fitted, data = wages_fitted)
tidy(second_stage)
```

```
# A tibble: 2 x 5
  term              estimate std.error statistic p.value
  <chr>                <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)          1.71      3.40      0.503   0.615
2 education.fitted     0.333     0.270     1.23    0.218
```

From the 2SLS model above, as education level increases by 1, average hourly earnings will increase by 33 cents. However the relationship is insignificant (p>.05), so we are speaking in trends.

Another way to do it:

```r
model_2sls <- iv_robust(wage ~ education | n_kids, data = wages)
tidy(model_2sls)
```

```
         term  estimate std.error statistic   p.value   conf.low conf.high  df
1 (Intercept) 1.7093781 2.8030095 0.6098367 0.5422343 -3.7971383 7.2158945 524
2   education 0.3331669 0.2221214 1.4999318 0.1342343 -0.1031909 0.7695247 524
  outcome
1    wage
2    wage
```

Step 4

Run a naive model predicting the effect of education on wages (i.e. without any instruments). How does this naive model compare with the IV model?

```r
naive_model <- lm(wage ~ education, data = wages)
```

Comparable Table

Table 2: From the model it looks like omitting the ability variable biases up the coefficient, so that the Naive coefficient appears larger and more significant than the 2SLS.

Model	Education Coeifficent	p.value
2SLS	0.33	0.21
Naive	0.54	0.00

Step 6

I would trust the Naive model more because the 2SLS model goes not pass the Exclusion Restriction. This means `n_kids` is mediating or confounding through another variable besides `education` to effect wage.

Task 2: Public housing and health

Economic research shows that there is a potential (albeit weak) connection between health outcomes and residency in public housing. You are interested in finding the effect of public housing assistance on health outcomes. In the absence of experimental data, you must use observational data collected by the Georgia Department of Public Health. You have access to a dataset of 1,000 rows with the following columns:

Variable name	Description
`HealthStatus`	Health status on a scale from 1 = poor to 20 = excellent
`HealthBehavior`	Omitted variable (you can't actually measure this!)
`PublicHousing`	Number of years spent in public housing
`Supply`	Number of available public housing units in the city per 100 eligible households
`ParentsHealthStatus`	Health status of parents on a scale from 1 = poor to 20 = excellent
`WaitingTime`	Average waiting time before obtaining public housing in the city (in months)
`Stamp`	Dollar amount of food stamps (SNAP) spent each month
`Age`	Age
`Race`	Race; 1 = White, 2 = Black, 3 = Hispanic, 4 = Other
`Education`	Education; 1 = Some high school, 2 = High school, 3 = Bachelor's, 4 = Master's
`MaritalStatus`	Marital status; 1 = Single, 2 = Married, 3 = Widow, 4 = Divorced

(This is simulated data, but it's based on analysis by Angela R. Fertig and David A. Reingold)

Your goal is to measure the effect of living in public housing (`PublicHousing`) on health (`HealthStatus`). There is omitted variable bias, though, since people who care more about their health might be more likely to self-select into public housing and report a better health status score. The magic variable `HealthBehavior` measures this omitted variable, and you can use it as reference to make sure you get the models right (this is the same as "ability" in the examples in class), but don't include it in any of your actual models, since it's not real.

This data includes four potential instruments:

- `Supply`: Number of available public housing units in the city per 100 eligible households
- `ParentsHealthStatus`: Health status of parents on a scale from 1 = poor to 5 = excellent
- `WaitingTime`: Average waiting time before obtaining public housing in the city (in months)
- `Stamp`: Dollar amount of food stamps (SNAP) spent each month

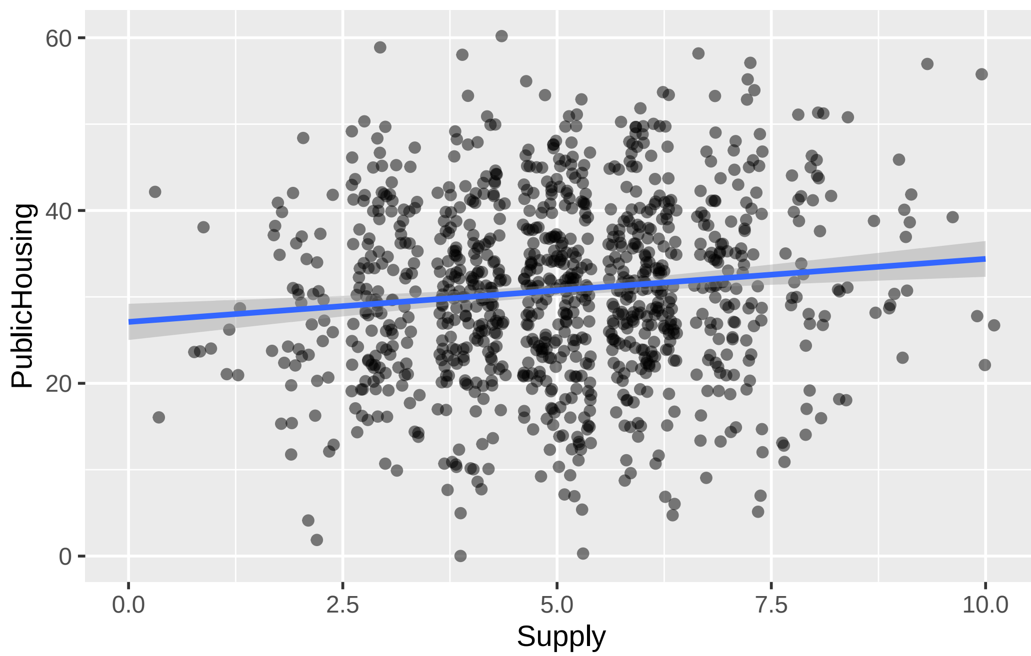
```
housing <- read_csv("../data/public_housing.csv")
```

$$HealthStatus = \beta_0 + \beta_1 PublicHousing$$

Relevance

Supply

```
ggplot(housing, mapping = aes(Supply, PublicHousing)) +
  geom_jitter(alpha = .5) +
  geom_smooth(method = "lm")
```




```r
check_relevance_supply <- lm(PublicHousing ~ Supply, data = housing)
tidy(check_relevance_supply)
```

```
# A tibble: 2 x 5
  term        estimate std.error statistic   p.value
  <chr>          <dbl>     <dbl>     <dbl>     <dbl>
1 (Intercept)   27.1       1.07      25.4  3.86e-110
2 Supply         0.729     0.202      3.61 3.23e-  4
```

```r
glance(check_relevance_supply)
```

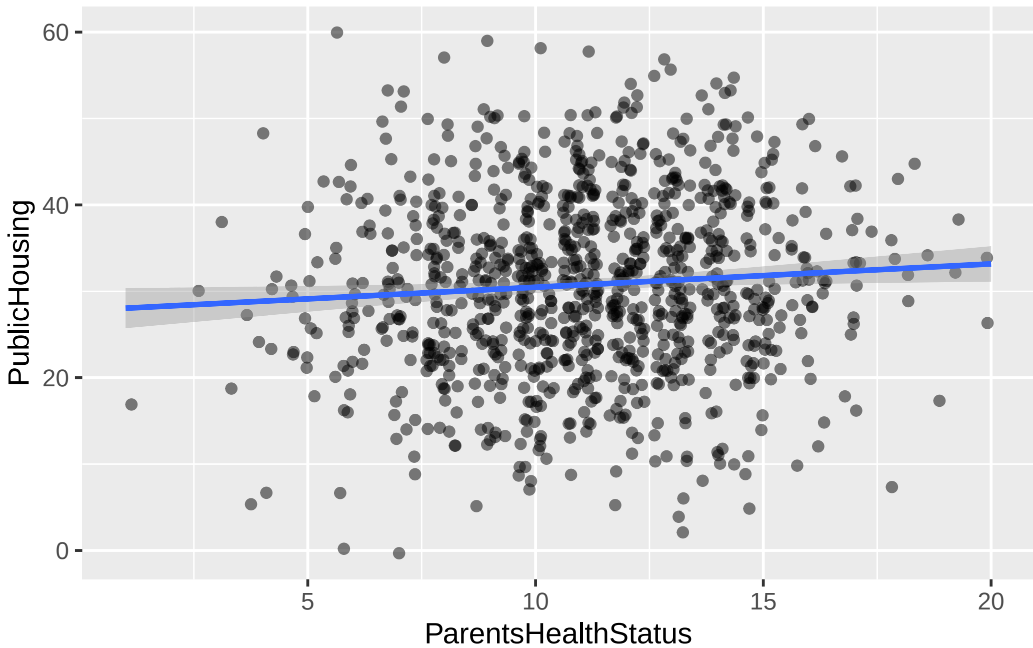
```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
      <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
1    0.0129        0.0119  10.3      13.0 0.000323     1 -3753. 7512. 7527.
# ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

`Supply` and `PublicHousing` have a significant and positive relationship, and a F-statistic greater than 10, so we can assume that `Supply` is Relevant.

Parents Health Status

```r
ggplot(housing, mapping = aes(ParentsHealthStatus, PublicHousing)) +
  geom_jitter(alpha = .5) +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
check_relevance_ParentsHealthStatus <- lm(PublicHousing ~ ParentsHealthStatus, data = hous
tidy(check_relevance_ParentsHealthStatus)
```

```
# A tibble: 2 x 5
  term                estimate std.error statistic  p.value
  <chr>                  <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)           27.8       1.29      21.6  6.66e-85
2 ParentsHealthStatus    0.270     0.112      2.41 1.61e- 2
```

```r
glance(check_relevance_ParentsHealthStatus)
```

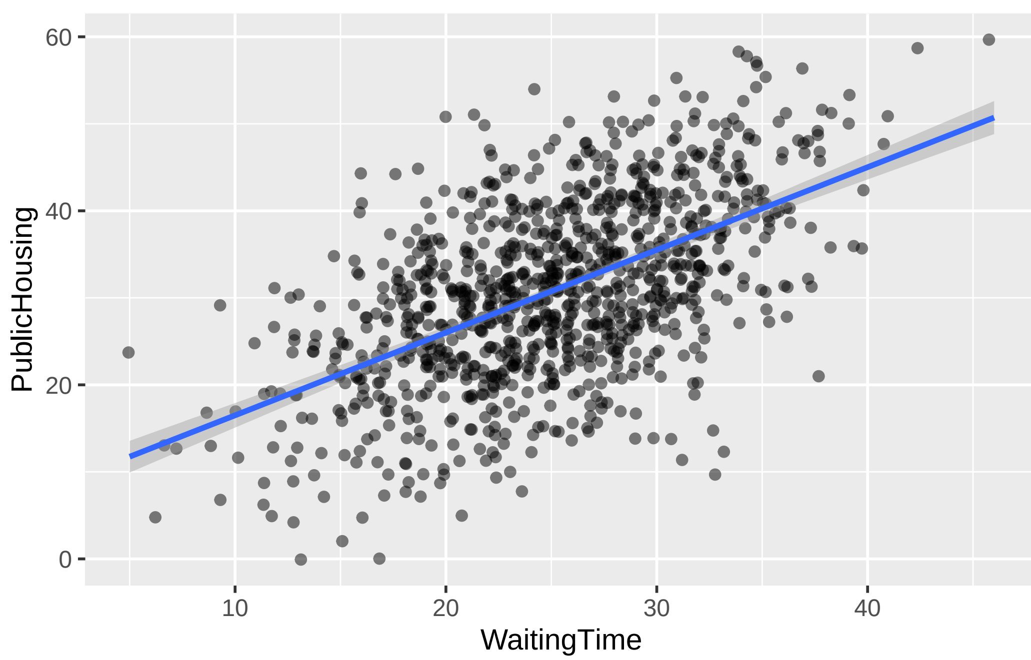
```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
      <dbl>         <dbl> <dbl>     <dbl>   <dbl> <dbl>  <dbl> <dbl> <dbl>
1   0.00579       0.00479  10.4      5.81  0.0161     1 -3757. 7519. 7534.
# ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

`ParentsHealthStatus` and `PublicHousing` are significant and positively correlated, but have a F-statistic less than 10, so we cannot assume relevancy.

Waiting Time

```r
ggplot(housing, mapping = aes(WaitingTime, PublicHousing)) +
  geom_jitter(alpha = .5) +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
check_relevance_WaitingTime <- lm(PublicHousing ~ WaitingTime, data = housing)
tidy(check_relevance_WaitingTime)
```

```
# A tibble: 2 x 5
  term        estimate std.error statistic  p.value
  <chr>          <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)    6.99     1.14        6.13 1.28e- 9
2 WaitingTime    0.951    0.0443     21.5  2.50e-84
```

```r
glance(check_relevance_WaitingTime)
```

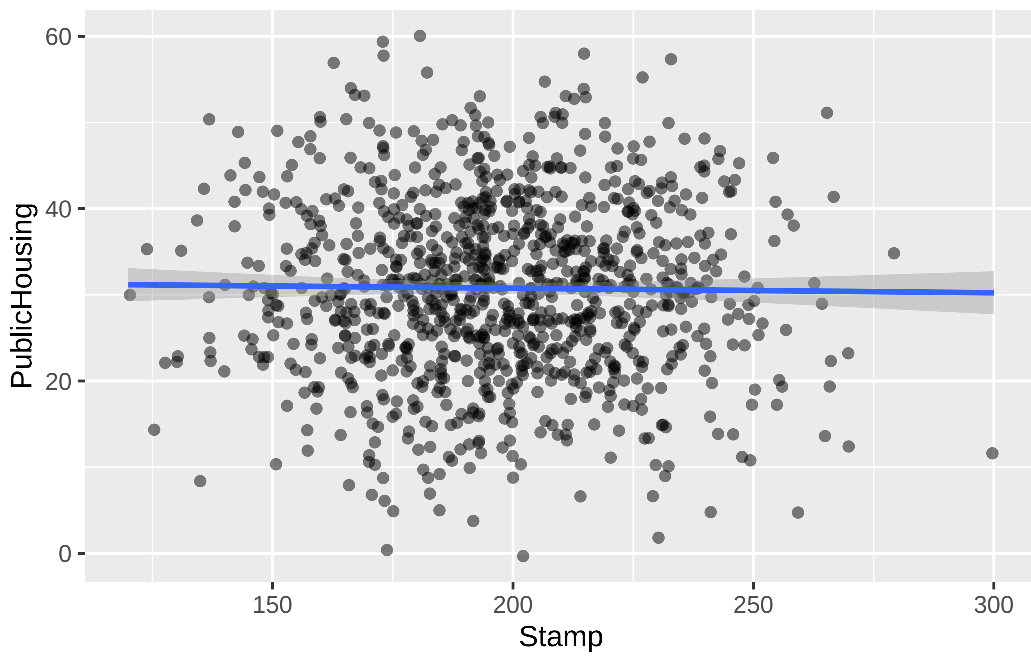
```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
      <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
1     0.316         0.315  8.60      461. 2.50e-84     1 -3570. 7146. 7160.
# ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

`WaitingTime` and `PublicHousing` are significantly and positively correlated, and the F-statistic in above 10, so we can assume relevancy.

Stamp

```r
ggplot(housing, mapping = aes(Stamp, PublicHousing)) +
  geom_jitter(alpha = .5) +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
check_relevance_Stamp <- lm(PublicHousing ~ Stamp, data = housing)
tidy(check_relevance_Stamp)
```

```
# A tibble: 2 x 5
  term        estimate std.error statistic  p.value
  <chr>          <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept) 31.8        2.38      13.3   1.74e-37
2 Stamp       -0.00522    0.0120    -0.436 6.63e- 1
```

```r
glance(check_relevance_Stamp)
```

```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
      <dbl>         <dbl> <dbl>     <dbl>   <dbl> <dbl>  <dbl> <dbl> <dbl>
1  0.000191     -0.000811  10.4     0.190   0.663     1 -3760. 7525. 7540.
# ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

`Stamp` and `PublicHousing` are insignificantly correlated, and the F-statistic is less than 10, so we cannot assume relevancy.

Exclusion

Creating a function to evaluate exclusion:

```
exclusion <- function(Y, X, Z) {
  equation.1 <- lm(Y ~ X)
  equation.2 <- lm(X ~ Z)
  equation.3 <- lm(Y ~ Z)

  beta_1 <- coef(equation.1)[2]
  gamma_1 <- coef(equation.2)[2]
  alpha_1 <- coef(equation.3)[2]

  mylist <- list()
  mylist["beta x gamma"] <- gamma_1 * beta_1
  mylist["alpha"] <- alpha_1
  mylist["diff"] <- (beta_1*gamma_1) - alpha_1

  return(mylist)
}
```

Supply

```
exclusion(housing$HealthStatus, housing$PublicHousing, housing$Supply)
```

```
$`beta x gamma`
[1] 0.234959

$alpha
[1] 0.1900004

$diff
[1] 0.04495858
```

The difference is close to zero, so we can assume that is passes the exclusion restriction.

Parents Health Status

```
exclusion(housing$HealthStatus, housing$PublicHousing, housing$ParentsHealthStatus)
```

```
$`beta x gamma`
[1] 0.08696487

$alpha
[1] 0.09953293

$diff
[1] -0.01256806
```

The difference is close to 0, so it passes the exclusion restrictions.

Waiting Time

```r
exclusion(housing$HealthStatus, housing$PublicHousing, housing$WaitingTime)
```

```
$`beta x gamma`
[1] 0.3063208

$alpha
[1] 0.2128377

$diff
[1] 0.09348304
```

The difference is small enough where an exclusion argument can be made, but it is possible that `WaitingTime` effects `HealthStatus` through a mediator or confounding variable.

Stamp

```r
exclusion(housing$HealthStatus, housing$PublicHousing, housing$Stamp)
```

```
$`beta x gamma`
[1] -0.001683272

$alpha
[1] -0.001567504

$diff
[1] -0.000115768
```

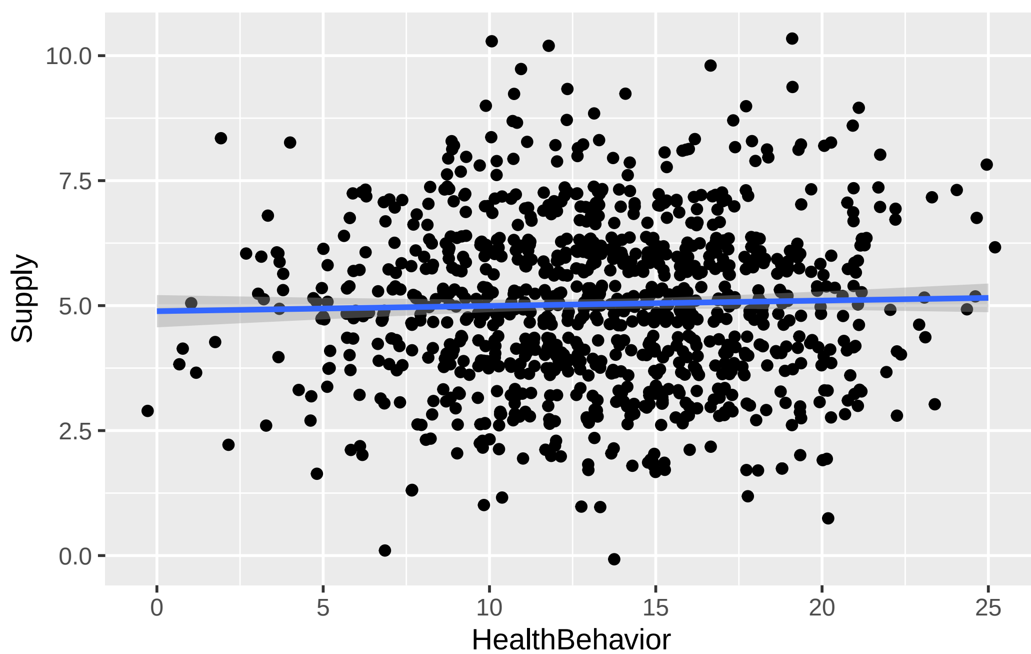
The difference is very small so we can assume exclusion.

Exogeneity

Supply

```r
ggplot(housing, aes(x = HealthBehavior, y = Supply)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
cor(housing$HealthBehavior, housing$Supply)
```

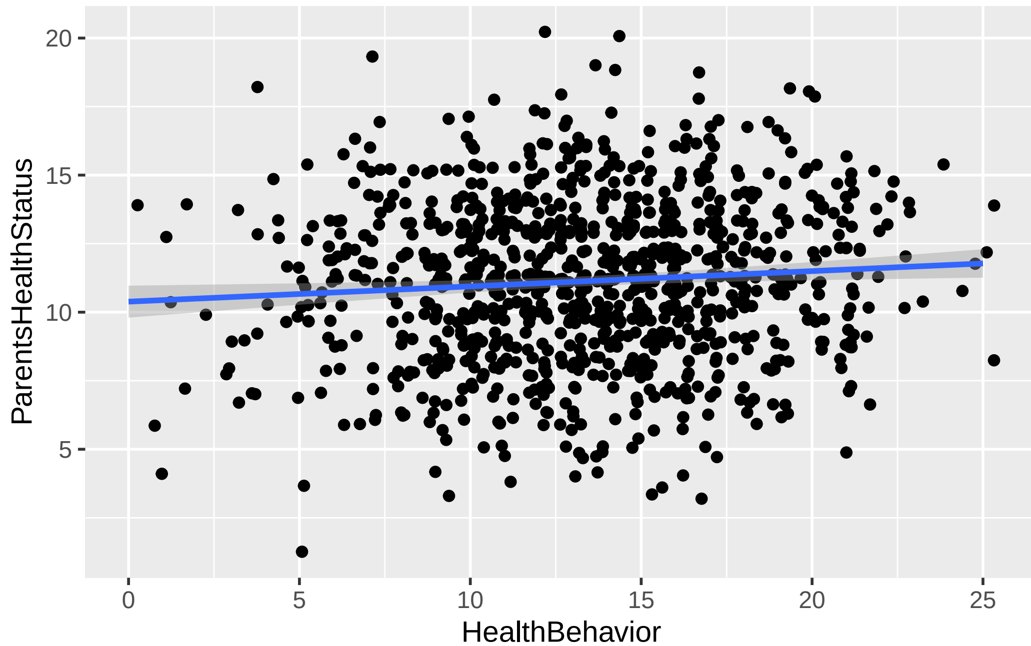
```
[1] 0.02868536
```

The correlation is close to zero, so we can assume it passes exogeneity criterion.

Parents Health Status

```r
ggplot(housing, aes(x = HealthBehavior, y = ParentsHealthStatus)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
cor(housing$HealthBehavior, housing$ParentsHealthStatus)
```

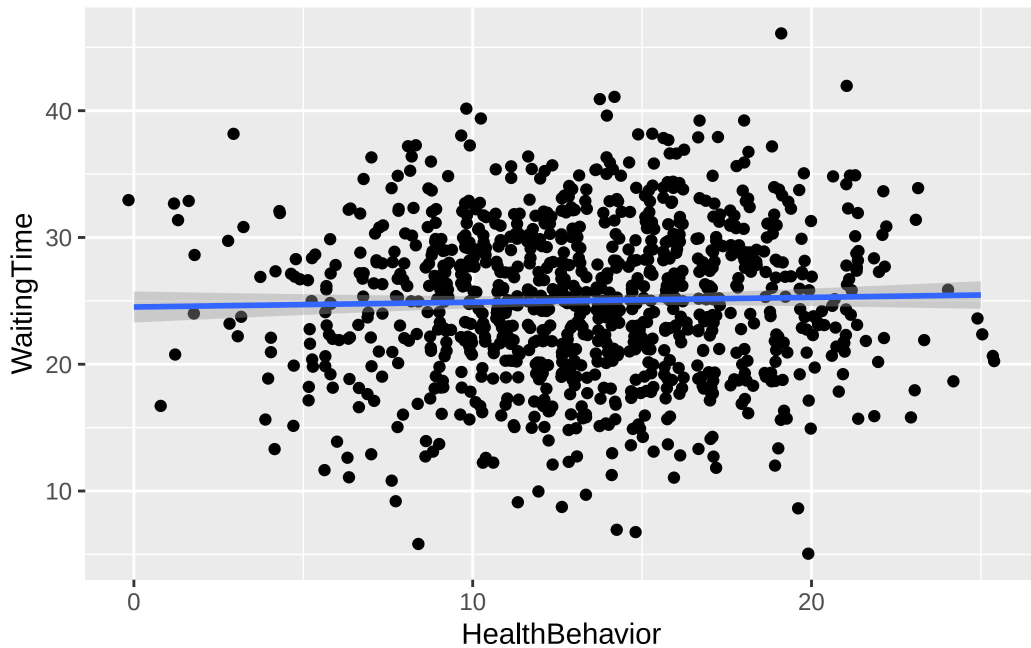
```
[1] 0.0830035
```

The correlation is close to zero, so we can assume it passes exogeneity criterion.

Waiting Time

```r
ggplot(housing, aes(x = HealthBehavior, y = WaitingTime)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
cor(housing$HealthBehavior, housing$ParentsHealthStatus)
```

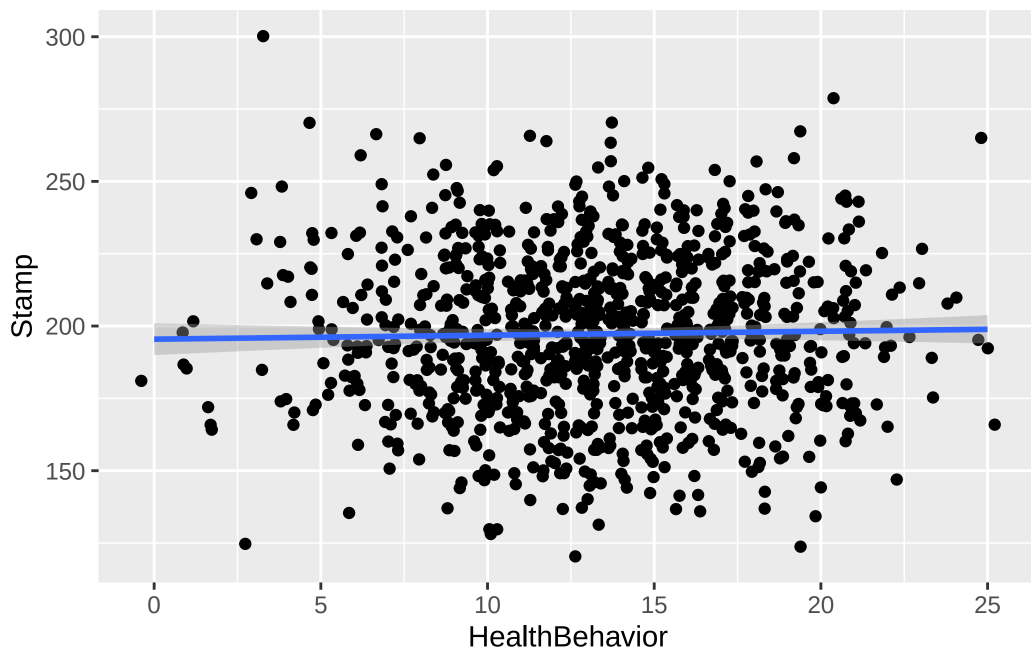
```
[1] 0.0830035
```

The correlation is close to zero, so we can assume it passes exogeneity criterion.

Stamp

```r
ggplot(housing, aes(x = HealthBehavior, y = Stamp)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```

```
`geom_smooth()` using formula 'y ~ x'
```




```r
cor(housing$HealthBehavior, housing$ParentsHealthStatus)
```

```
[1] 0.0830035
```

The correlation is close to zero, so we can assume it passes exogeneity criterion.

Naive Model

```r
naive_model <- lm(HealthStatus ~ PublicHousing , data = housing)
tidy(naive_model)
```

```
# A tibble: 2 x 5
  term          estimate std.error statistic  p.value
  <chr>            <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)      1.25    0.0885       14.2 1.09e-41
2 PublicHousing    0.322   0.00272     118.  0
```

The Naive Model above shows the number of years spent in public housing (`PublicHousing`) increases by 1, health status increase by .32 (p<.01).

2SLS

```r
first_stage <- lm(PublicHousing ~ WaitingTime, data = housing)
housing_fitted <- augment_columns(first_stage, housing) %>%
  rename(PublicHousing.fitted = .fitted)

second_stage <- lm(HealthStatus ~ PublicHousing.fitted, data = housing_fitted)
tidy(second_stage)
```

```
# A tibble: 2 x 5
  term                 estimate std.error statistic  p.value
  <chr>                   <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)             4.28     0.545       7.85 1.04e-14
2 PublicHousing.fitted    0.224    0.0174     12.9  3.55e-35
```

Model Summary

```r
modelsummary::modelsummary(list(
  "Naive" = naive_model,
  "First Stage" = first_stage,
  "Second Stage" = second_stage
))
```

Both results show that the duration spent in public housing significantly increases health status, therefore we can reject the null hypothesis that public housing does not increase health status. I would argue that the 2SLS has more authenticity than the naive model. An individuals Health Behavior is correlated with their living environment and their health status. Since Health Behavior is unobserved, this leaves endogeneity in the model. The variable `WaitingTime` is a good instrument because is passed the three assumptions mentions above. Intuitively, we can expect that `WaitingTime` is randomly assigned to people, so it cannot be correlated with behavior. Therefore, I would suggest that the 2SLS is more unbiased than the naive model.

	Naive	First Stage	Second Stage
(Intercept)	1.254	6.991	4.280
	(0.088)	(1.141)	(0.545)
PublicHousing	0.322		
	(0.003)		
WaitingTime		0.951	
		(0.044)	
PublicHousing.fitted			0.224
			(0.017)
Num.Obs.	1000	1000	1000
R2	0.933	0.316	0.142
R2 Adj.	0.933	0.315	0.141
AIC	2619.3	7145.7	5175.7
BIC	2634.1	7160.5	5190.5
Log.Lik.	−1306.668	−3569.868	−2584.864
F	14 000.142	460.710	165.618
RMSE	0.89	8.60	3.21