Multi-Objective Optimization Applied to the Matching of a Specified Torque-Speed Curve for an Internal Permanent Magnet Motor

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Traditionally, the objective function used for optimizing the design of an internal permanent magnet motor (IPM) has maximized efficiency or torque for a particular current or volume. Creating a particular torque speed curve can be considered to be a multi-objective problem and would usually be expressed in terms of a single objective by minimizing the average error over the curve. In this paper, it is proposed to treat this directly as a multi-objective problem thus allowing the designer to decide which tradeoffs in the torque–speed performance are most acceptable after the analysis has been performed rather than before. Additionally, a new variant of a multi-objective evolutionary algorithm using mixed elitism, intended for this problem, is described.

***Index Terms*—Genetic algorithms, optimization methods, permanent magnet motors.**

I. Introduction

PERMANENT magnet electrical machines have been growing in popularity over the past decade due to the decreasing cost of rare earth magnets. Such machines are finding applications in areas ranging from computer disk drives to electric vehicles. In particular, the internal permanent magnet motor (IPM) has found application in several commercial hybrid electric vehicles (HEVs) and one of the major design issues here is to create a motor with a torque–speed curve which matches that required by an electric vehicle in everyday use, i.e., high torque at low speed to provide significant acceleration but operation over a substantial speed range to cover both city and highway driving. In effect, this is a multi-objective problem; each point on the desired torque–speed curve can be considered to be a separate objective. The alternate approach is to treat this conventionally as a single objective problem and minimize the average error in the torque–speed curve. However, this approach provides little information for the designer in terms of tradeoffs that can be made to match particular parts of the curve, e.g., the point at which the constant torque operation switches over to constant power. In treating the problem as a single objective (SOP), both stochastic [1] and direct search [2] techniques have been used [3], [4].

The intention of this paper is to examine the design of a specific torque–speed curve as a multi-objective problem and to formulate a stochastic optimizer which is tuned to the features of this problem.

II. Torque–Speed Requirements of an HEV

One of the primary design objectives of a hybrid electric vehicle is a specific torque–speed curve. While, in practice, this can be modified through the use of a gearbox, this introduces inefficiencies. The critical requirements for any design are the range of the constant torque region, i.e., the maximum speed up to which the torque stays constant, and the ratio of the maximum to minimum speed over the constant power region, i.e., the area of field weakening in the case of a permanent magnet machine. This leads to three critical points in the torque speed curve, which form the basis for a multi objective optimization. The parameters which control these points in an IPM are related to the position and size of the permanent magnets, as well as the overall dimensions of the machine.

III. Multi-Objective Evolutionary Algorithms

Multi-objective evolutionary algorithms (MOEAs) have evolved as the preferred choice for solving MOPs since MOEAs deal with multiple possible solutions simultaneously which makes it possible to find the tradeoff surface consisting of nondominated solutions [5], i.e., solutions which are better in one objective than all the other solutions and at least as good in all the others. This set of solutions is known, in objective function space, as the Pareto front. The construction of these algorithms has to address certain critical issues. The first is how the search can be guided towards the Global Pareto front and how to get a good distribution of solutions along the front. Second, but not as critical, are the determination of an appropriate initial population size and a termination criterion. While several MOEAs currently exist, the "No Free Lunch Theorem" [6] states that no one optimization algorithm will be optimal for all problems. In effect, this says that it is always possible to produce an algorithm which is optimal for a particular class of problems but will be suboptimal on all the others. Thus, the intention here is to propose an algorithm which will perform better than existing algorithms for the problem described above.

Any stochastic optimization embeds two distinct phases in its operation: exploration and exploitation. In exploration, the goal is to search the possible objective function space to find possible areas for the optimal solution; in exploitation, local knowledge is used to determine the exact location of the optimal solution. All algorithms provide some form of balance between these two phases and how the balance is applied depends on the shape of the objective space. In a MOP, this has to be applied for each objective. The critical issues are as considered below.

Guiding the search towards the Global Pareto front: Any search algorithm might get stuck in a local Pareto optimal point due to a deceptive front [5] also referred to as multimodality (if there are more than two local optima), i.e.,

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a lack of an effective exploration component. In order to solve this, ranking and selection strategies are generally used.

Achieving a well distributed tradeoff front: To determine a useful Pareto front, it is necessary to find solutions which are as diverse as possible. If all the solutions are clustered together on the front, the purpose of multi-objective optimization (MOO) is not served. This issue is related to convexity or nonconvexity of the front and a clustering on the front is dependent on the fitness assignment scheme. If the fitness is proportional to the number of solutions dominated then the sampling of convex shaped functions is better or more biased than in the case of nonconvex functions [5]. Such difficulties are often resolved by fitness sharing and elitism.

Determining the correct initial population size: Most MOEAs guess an initial population size, but if the population is too small there can be a possibility of premature convergence; if it is too large, it will incur an increased computation time and the fitness improvement will be slower [7]. Instead of guessing the optimal population, algorithms such as the incrementing multi-objective evolutionary algorithm (IMOEA) [7] employ the concept of a dynamic population.

Terminating the search process: In multi-objective problems, the greater the number of points on the Pareto front, the better it is for the final design decision. The termination of a multi-objective algorithm (MOA) means making a choice between having enough solutions and the desire to have more solutions. Most algorithms either halt processes after a fixed number of generations or monitor the population at certain intervals and interpret it visually to determine if the search should be halted.

IV. Mixed Elitist Multi-Objective Evolutionary Algorithm (mE-MOEA)

The proposed algorithm employs elitism to both explore and exploit. It achieves elitism in two different ways: first by maintaining an archive of nondominated solutions, which take part during nondominated sorting, and, second, by using the selection of the fittest. Nondominated individuals are treated as one identical group to be used in crossover, but all individuals, including the dominated ones, are ranked for selection before mutation. A second archive of dominated individuals is maintained so that no information is lost and duplication can be avoided, thus reducing computational costs, Fig. 1. The fitness (strength) formula for a chromosome is different from the conventional one and is calculated based on

$$s = \frac{n_d + n_{nd}}{n} \quad 0 \leq s < 1$$

where n_d is the number chromosomes dominated by that chromosome, n_{nd} is the number of chromosomes it neither dominates nor gets dominated by, and n is the current population size. A selection method is implemented during mutation, but unlike a rank selection method, it is partly random and partly guided due to the fact that the chance of a nondominated individual getting selected increases as the percentage of nondominated ones in the main population increases with generations. Moreover, the starting point for choosing individuals for mutation is randomly chosen for an elitism-based ranking of the entire population.

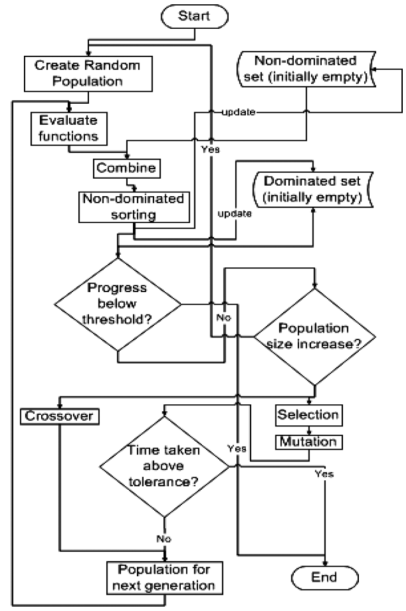

Fig. 1. Basic flow chart of the algorithm.

While chromosomes are based on real-valued numbers, reproduction is executed at a binary level and this reduces the search space to a finite size. The algorithm can handle any number of objectives and parameters. The strategy of maintaining a dynamic population adds weight to the exploration capabilities. Due to the growth in the population as the algorithm runs, a small initial population is actually desirable. The proposed scheme of tracking the relative change in the size of the nondominated set, with respect to the current size of that set, over a couple of generations, is a way of measuring progress towards the Pareto front.

V. Advantages of mE-MOEA

1) The ability of mE-MOEA to come out of deceptive Pareto front very quickly unlike NSGA [8].
2) The ability to save time during finite-element analysis by having a mechanism for keeping track of all chromosomes evaluated unlike SPEA [9].
3) A mechanism to preserve information unlike IMOEA [7].
4) The ability to implement effective diversity without fitness sharing method unlike NSGA.
5) An effective mechanism to tackle both convex and nonconvex functions without bias unlike MOGA [10].

VI. Testing the Algorithm

Before applying the proposed algorithm to the IPM problem, it was tested on several standard problems to determine its capabilities in terms of finding the true Global Pareto front. The first test was analytic and the performance appears to match or

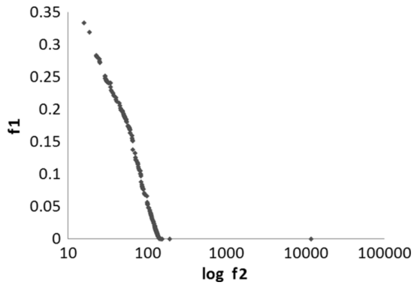

Fig. 2. Pareto front of the optimization of the SMES.

TABLE I
FINAL RESULT FROM SMES OPTIMIZATION

R *m*	H *m*	D *m*	Energy *MJ*	$\lvert B_{\mathrm{stray}}\rvert$ *T*	Volume m^3
3.1361	0.5200	0.3336	180.30	0.00237	3.4182

TABLE II
RESULT OF SMES OPTIMIZATION IN [13]

R *m*	H *m*	D *m*	Energy *MJ*	$\lvert B_{\mathrm{stray}}\rvert$ *T*	Volume m^3
3.4000	0.4397	0.2945	184.53	0.00256	2.7665

TABLE III
BEST RESULT OF SMES OPTIMIZATION IN [13]

R *m*	H *m*	D *m*	Energy *MJ*	$\lvert B_{\mathrm{stray}}\rvert$ *T*	Volume m^3
3.0800	0.4780	0.3940	179.80	0.00089	3.6446

exceed existing algorithms. The second test involved an electromagnetic problem: the TEAM SMES Problem 22 This involves determining the parameters for one of the two superconducting coils such that a particular amount of energy is stored in the system while the stray fields are minimized. The objectives are given in [11] and [12], and the problem is defined as

$$\text{Minimize} \quad f_1 = \frac{|E - E_{\mathrm{ref}}|}{E_{\mathrm{ref}}}, \quad f_2 = \frac{B_{\mathrm{stray}}^2}{B_{\mathrm{norm}}^2}$$

$$\text{where} \quad B_{\mathrm{stray}}^2 = \frac{\sum \left|B_{\mathrm{stray}_i}\right|^2}{22}, \quad i = 1, 2, \ldots, 22$$

$$\text{given} \quad E_{\mathrm{ref}} = 180\,\mathrm{MJ},\ B_{\mathrm{norm}} = 2 \times 10^{-4}\ T$$

$$R_1 = 2\,m,\ h_1/2 = 0.8\,m,\ d_1 = 0.27\,m$$

$$\text{Parameters are :} \quad R_2, \frac{h_2}{2}, d_2$$

$$\text{given } 2.6 \le R_2 \le 3.4,\ 0.204 \le \frac{h_2}{2} \le 1.1,\ 0.1 \le d_2 \le 0.4,$$

$$J_1 = 22.5\ \mathrm{MA/m^2},\ J_2 = -22.5\ \mathrm{MA/m^2}.$$

The representation of each parameter in a chromosome was handled by a 16-bit number. Thus, there were a finite number of possible representations for this problem, which led to a search space containing 108×10^9 possible solutions. The number of points in the parameter space evaluated during the execution of the algorithm was 14 564 and it took 95 iterations to converge to the Pareto Optimal set. This set has 195 optimal solutions, Fig. 2. To make the final decision on a particular solution a further set of requirements is needed. Using the procedure given in [13], the final solution is given in Table I, and compares well with the published results.

TABLE IV
OPTIMAL DESIGN OF THE IPM

w_m	$l_m/2$	a	T_1	T_2	T_3	T_{cogging}
5.6 mm	20.9 mm	55^0	274.43 Nm	313.78 Nm	157.36 Nm	6.5%

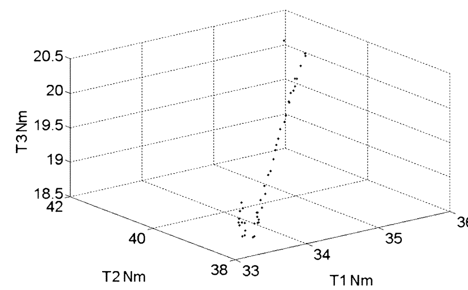

Fig. 3. Torque plot solutions derived from the Pareto front.

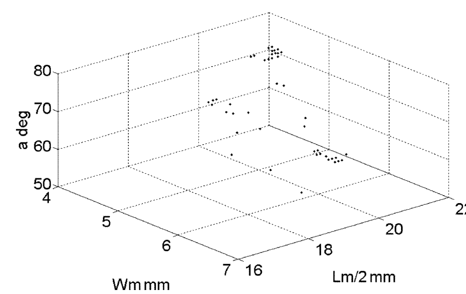

Fig. 4. 3-D plot of the nondominated solutions in parameter space.

VII. INTERNAL PERMANENT MAGNET MOTOR

The principle of operation of an IPM is similar to that of a conventional synchronous machine. However, since the IPM is a voltage-driven motor, the back emf opposes the voltage drop due to the direct axis current component of the stator current, leading to a decrease in the output power. By flux weakening, a constant output power can be achieved for speeds above the base speed. Embedded magnets in the IPM give rise to a reluctance difference between the d-axis and the q-, or inter-polar, axis which generates a second torque component. The sum of the magnetic and the reluctance torque is the total torque.

Due to the cost of permanent magnet material, the main parameters considered in the final optimization work for the IPM were the magnet size and pole-arc angle. The reference machine specification chosen for experiment, which is close to that for the motor used by Toyota, is given in [14]. However, since speed and current are closely related in a permanent magnet machine during flux weakening, the optimization was performed for a specified torque–current curve rather than torque–speed. The model that was used required some experimentation to determine suitable values for some critical aspects such as the winding, spread angle of the embedded magnets, air gap length,

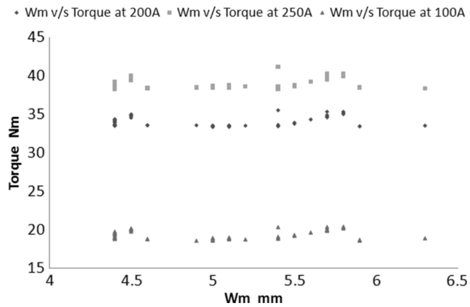

Fig. 5. Half of the length of the magnet from nondominated solutions against the respective torques.

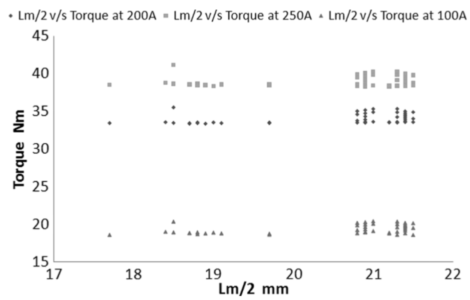

Fig. 6. Width of the magnets from nondominated solutions against the respective torques.

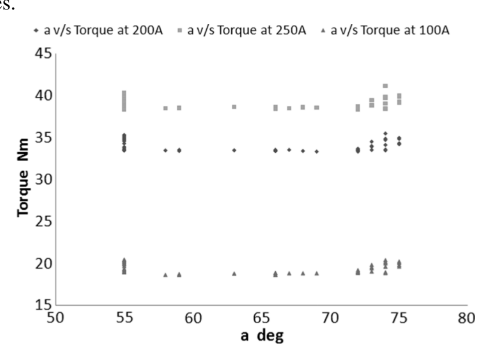

Fig. 7. Half of the pole-arc angle from nondominated solutions against the respective torques.

magnet material, stator and rotor core material, etc. The optimization problem was then defined as

$$\text{Minimize } f_1 = \frac{|T_1 - T_{200}|}{T_{200}}, \quad f_2 = \frac{|T_2 - T_{250}|}{T_{250}}$$
$$f_3 = \frac{|T_3 - T_{100}|}{T_{100}}$$
$$17.5 \le l_m/2 \le 21.5,\ 4.0 \le w_m \le 6.5,\ 55 \le a \le 75$$

where T_{200}, T_{250}, T_{100} are the reference torques at 200 A, 250 A, and 100 A, respectively; l_m is the length of the magnet, w_m is the width of the magnet, and a is the half pole-arc angle or half spread angle of the magnets.

From the Pareto front, the "best" design can be chosen and two steps were taken to make this choice.

1) Filter out those errors that are greater than 5%.
2) After filtering out, the best design is the one having the least cogging torque.

VIII. Conclusion

This paper has described the creation of a modified MOEA intended for use in the design of IPMs and validated its performance. By choosing currents at three critical regions of the torque–speed profile for the optimization process, the results shown in Figs. 3–7 were obtained from which we can conclude that the optimization parameters affect torque in the same way at all currents, and therefore at all speeds, and that torque is directly proportional to the pole-arc angle. Also, there appears to be no visible relationship between the length or width of the magnet and the torque. Therefore, instead of treating l_m and w_m as two separate entities we may consider them as the area of the magnet. On comparing the slope of the plot in Fig. 7 with that in Fig. 8, it can be observed that the area of the magnet has more affect on torque than the spread angle which also explains the nature of the Pareto plot in Fig. 3, i.e., the torque is dependent on two independent parameters.

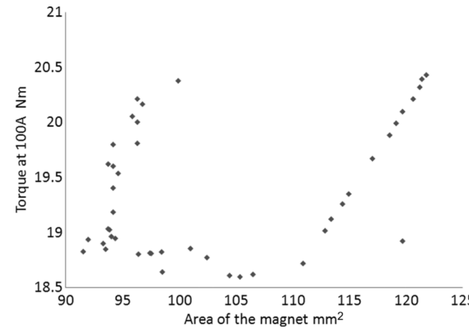

Fig. 8. Effect of the area of the magnet on the torque.

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