Multi-Objective Optimization of PM AC Machines Using Computationally Efficient - FEA and Differential Evolution

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***Abstract* – A total of eleven independent stator and rotor variables are simultaneously employed for the optimization of a generic example IPM motor design. The multi-objective criterion maximizes efficiency, while minimizing torque ripple at the rated output condition. A Pareto-based differential evolution (DE) algorithm with 100 generations, each with a population of 100 individuals, is presented. Computationally efficient FEA (CE-FEA), which is based on a reduced number of magnetostatic solutions for a motor model in the *abc* reference frame, is employed. As a result, a total of 10,000 candidate motor designs, which are included in the comprehensive study, are evaluated in a record short time on a typical PC-based workstation. The paper includes an engineering trade-off discussion based on a typical-reference motor, two optimum designs in terms of average torque and torque ripple, and a best-compromise solution. For the case-study, an order of magnitude reduction of the rated-load torque ripple and open-circuit cogging torque has been achieved. This is while, at the same time, the specific torque output has been increased by as much thirty seven percent.**

***Index Terms* – permanent magnet machines, PMAC, PMSM, IPM, multi-objective optimization, simplified FEA, differential evolution.**

I. Introduction

DESIGN of electric machinery is complicated by a large number of design variables related to complex geometries and material properties. In addition, the designer is confronted by a large number of oftentimes conflicting design objectives that further complicate the design process. In a typical design flow, initial device sizing is performed using analytical and magnetic equivalent circuit models, and FEA is used in the final steps of the design process for verification and parameter fine tuning. In this approach, several iterations may be performed to satisfy a very limited number of objectives. Furthermore, a majority of the design variables are predetermined based on the analytically derived models and then a small number of variables is fine tuned using FEA. In this approach only a small number of variables are varied using FEA and optimality of the final design is not guaranteed. On the other hand, a design process that relies solely on conventional FEA models to investigate a large number of candidate designs may be inadequate due to the prohibitively long computer execution times. This limitation of FEA-based models has prevented their application in large scale design optimization studies, where a significant number of design parameters are varied in the search for optimum designs.

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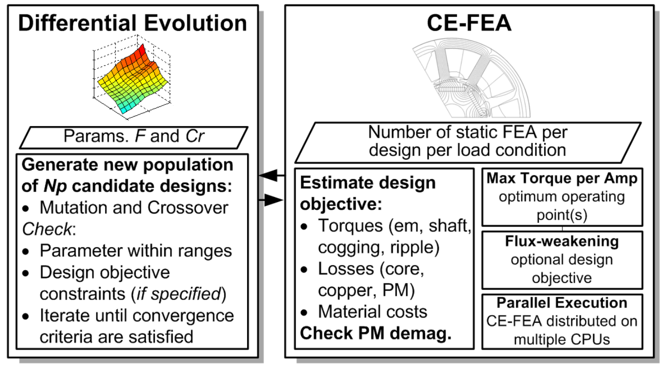

Fig. 1. Block diagram showing model-based DE design optimization employing CE-FEA.

Several reduced-effort FEA modeling approaches have been proposed in the literature, e.g. [1]-[5]. These techniques aim to minimize the computational effort associated with FEA while maximizing the amount of available machine performance information. Previous methods were able to estimate the average torque and the torque ripple [1], [2] *or* the stator core losses [3]. Computationally-efficient FEA (CE-FEA) was introduced by these authors in [4] and [5]. In this new approach, both magnetic and electric circuit symmetries of PM AC machines were utilized to reduce the number of required static FE solutions. In CE-FEA, a comprehensive set of performance indices, including the back emf waveforms, average torque, profiles of cogging torque and torque ripple, flux density waveforms, as well as core losses are extracted from a minimum number of magnetostatic FE solutions considering a motor model in the *abc* reference frame. This makes CE-FEA especially useful in comprehensive, large scale design optimization studies that involve evaluation of thousands of candidate designs.

In this paper, CE-FEA is used in a model-based multi-objective design optimization study that employs a differential evolution (DE) optimization algorithm [6]. A large number of stator and rotor variables are used to automatically optimize the average torque, efficiency, the cogging and the ripple torques. The method is demonstrated on a generic IPM machine.

The block diagram of the proposed model-based design optimization is shown in Fig. 1. In Fig. 1, a DE algorithm is used to dynamically update and improve the candidate design population, and the CE-FEA is used to extract the objectives (average and ripple torques, core losses, etc.). It should be noted that for the case-study IPM machine parameters, such as average electromagnetic and shaft torques, torque ripples, and efficiency, at a single load point, are extracted from several magnetostatic solutions that take several seconds to execute using a typical PC-based workstation.

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II. Design Optimization

A. Optimum Operating Point (*Maximum Torque per Ampere - MTPA*)

Design optimization of IPM machines is complicated by the fact that the maximum torque production is a function of the current advance angle, which in turn is a function of the design parameters. This means that for a systematic comparison of candidate designs, an additional search/optimization of the optimum operating point (MTPA) has to be performed for every candidate design. Determination of MTPA operating condition requires additional model iterations to obtain the optimum operating condition, hence increasing execution time. This, however, is not of significant concern with CE-FEA, where several (2-3) static FE solutions are sufficient for the estimation of the average torque [5].

B. Differential Evolution (DE) Optimization Algorithm

The Differential Evolution algorithm belongs to the wide-class of meta-heuristic optimizers that attempts to find a global minimum/maximum by iteratively improving a population of candidate designs until the convergence criteria are satisfied [6]. The "differential" part of the DE algorithm implies that unlike other derivative-free population based evolutionary algorithms (Genetic Algorithm, Particle Swarm, Simulated Annealing, etc.) DE utilizes a weighted difference between candidate designs to facilitate the improvement of future generations. DE has been shown to outperform most population based evolutionary algorithms on a number of bench mark test functions [6].

In DE's most basic form, see Fig. 2, the parameters, u_i, of the new trial member, $\mathbf{u}$, are updated based on mutation and crossover ideas given in the following:

$$u_i = \begin{cases} x_{r0} + F(x_{r1} - x_{r2}), & \text{if}(\text{rand}(0,1) \le C_r) \\ x_i & \end{cases} \quad (1)$$

where, x_i, is the parameter of the current population member, $x_{r0} + F(x_{r1} - x_{r2})$, is the mutation operation applied to the parameters of the three randomly selected current population members x_{ro}, x_{r1} and x_{r2}, with F being a positive real difference scale factor. The mutation operation is carried out only if the following crossover condition is satisfied, $\text{rand}(0,1) \le C_r$, where Cr, is a predefined crossover probability. Once the trial member, $\mathbf{u}$, is created its objective function, $f(\mathbf{u})$, is evaluated and compared to the objective function of the current member, $f(\mathbf{x})$. The trial vector, $\mathbf{u}$, is allowed to enter the population only if it outperforms the current member, $\mathbf{x}$. The population size is heuristically chosen to be ten times the number of design variables [6].

There are two approaches to multi-objective optimization. The most straightforward approach is based on a weighted sum of objectives that transforms a multi-objective problem into a simple single-objective case, as shown in (2):

$$f_1 = \sum_{n=1}^{N} w_n f_n(\mathbf{x}) \quad (2)$$

However, in this approach a choice of weights, w_n, may have a significant impact on the optimality of the final design. Moreover, the dual nature of some objectives (conflicting/non-conflicting) may lead to problems with proper weight assignment. For a given set of weights, this approach leads to a single optimal design that either maximizes or minimizes the objective function given in (2). The second approach to multi-objective optimization problems, used in this work, is based on the Pareto-dominance selection criteria. This approach typically results in the family of the best-compromise designs that provide the designer with a clear view of various trade-offs between a number of Pareto-optimal designs. Pareto-dominance selection criteria can be incorporated into the DE using several approaches outlined in [6] and [7].

While (convergence criteria are not satisfied)
    For (every population member)
        choose random members, $\mathbf{x_{r0}}$, $\mathbf{x_{r1}}$, and $\mathbf{x_{r2}}$

        Generate a trial member (**crossover** and **mutation**)

$$u_i = \begin{cases} x_{r0} + F(x_{r1} - x_{r2}), & \text{if}(\text{rand}(0,1) \le C_r) \\ x_i & \end{cases}$$

    End
    For (every population member)

        Select the new generation member (if it outperforms the existing)

$$\mathbf{x} = \begin{cases} \mathbf{u}, & if\ (f(\mathbf{u}) \le f(\mathbf{x})) \\ \mathbf{x} & \end{cases}$$

    End
End

Fig. 2. Simplified pseudo-code of DE implementation for a single objective *minimization* problem.

C. Problem Statement

The multi-objective definition of the PM AC machine design used in this paper can be summarized as follows:

- *minimize* torque ripple and total losses (core and copper), while
- *maximizing* the torque production per unit volume at rated load.

One way to achieve this is by considering two objectives given by the following:

$$\begin{aligned} &minimize: \ f_1 = T_{em(pk-pk)} \\ &maximize: \ f_2 = \frac{T_{em}}{\sqrt{P_{Cu} + P_{Fe}}} \end{aligned} \quad (3)$$

where, f_1, is corresponds to peak-to-peak torque ripple and, f_2, corresponds to "goodness," a measure of average torque production with respect to total losses. The outside diameter and axial length are fixed during optimization. The axial length is then scaled to achieve the desired output ratings of 15Nm at 3600r/min, which corresponds to the power output rating of 7.5hp. Current density and slot fill are fixed at $7\text{A}_{\text{rms}}/\text{mm}^2$ and 0.3, respectively. Every candidate design is evaluated at MTPA condition. Permanent magnet material and steel used in the core construction are fixed. A purely sinusoidal current regulated sine-wave drive is assumed. A total of eleven geometric variables (7-stator and 4-rotor) are varied during optimization. The resulting parameter vector is given in (4), below:

$$\mathbf{x} = [D_{Si}, g, w_T, l_S, d_S, d_T, \theta_T, w_{PM}, h_{PM}, \tau_P, w_q] \quad (4)$$

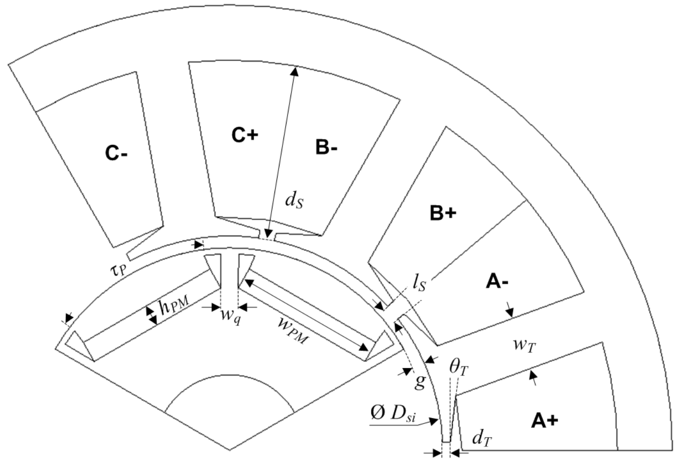


Fig. 3. Cross-section of the case-study machine showing *11* geometric variables used in multi-objective optimization.

TABLE I. OPTIMIZATION PARAMETERS: LIMITS AND VALUES CORRESPONDING TO MACHINES OF FIGS. 5. VALUES ARE PER UNIT OF POLE-PITCH ARC LENGTH.

Optimized Parameters (7-stator, 4-rotor)		Fractional-Slot (9-slot, 6-pole)					
		Limits		Machine			
		Lower	*Upper*	M-1	M-2	M-3	M_{TYP}
Stator	Stator inner diam., D_{si}	1.940	2.156	1.941	1.941	1.965	1.979
	Air-gap height, g	0.027	0.069	0.028	0.040	0.062	0.034
	Tooth width, w_T	0.213	0.355	0.266	0.266	0.253	0.284
	Slot opening, l_S	0.062	0.275	0.072	0.070	0.084	0.156
	Slot depth, d_S	0.550	0.824	0.801	0.820	0.815	0.742
	Tip depth, d_T	0.029	0.048	0.028	0.036	0.036	0.038
	Tip angle, θ_T [deg. mech.]	7.500	30.000	9.019	8.825	12.419	18.750
Rotor	PM width, w_{PM}	0.550	0.687	0.661	0.683	0.686	0.687
	PM height, h_{PM}	0.055	0.103	0.099	0.102	0.103	0.079
	Pole-arc length, τ_P	0.548	0.940	0.641	0.627	0.600	0.778
	q-axis bridge width, w_q	0.027	0.096	0.092	0.091	0.072	0.062

where the variable definitions are specified in Fig. 3 and Table I. The search for the MTPA and objective function evaluations are automatically carried out using CE-FEA that is scripted in a commercially available magnetostatic FE solver MagNet [8].

III. OPTIMIZATION STUDY

In this section, an optimization study of the IPM machine with fractional-slot ($q = 0.5$) shown in Fig. 3 is discussed. The machine has been optimized following the procedure outlined in Section II. C, with design parameter limits set to the values provided in Table I. The population size of a single generation

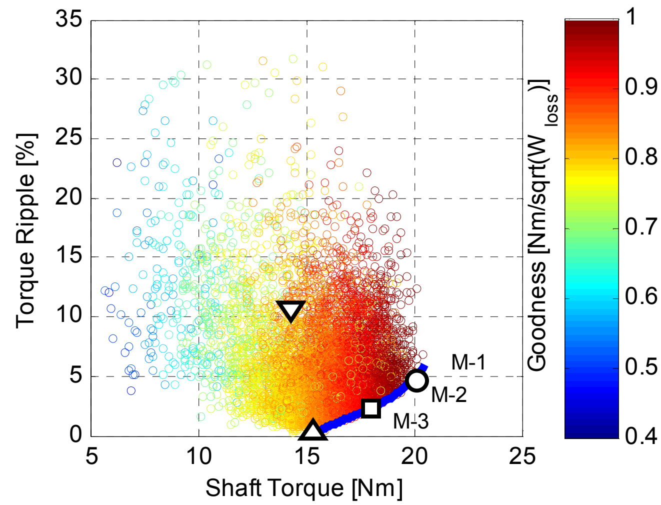


Fig. 4. Design objectives showing: Pareto-optimal set, three optimal machines M-1F, M-2F, M-3F, and the conventional machine. Shaft torque corresponds to the output torque after the core losses.

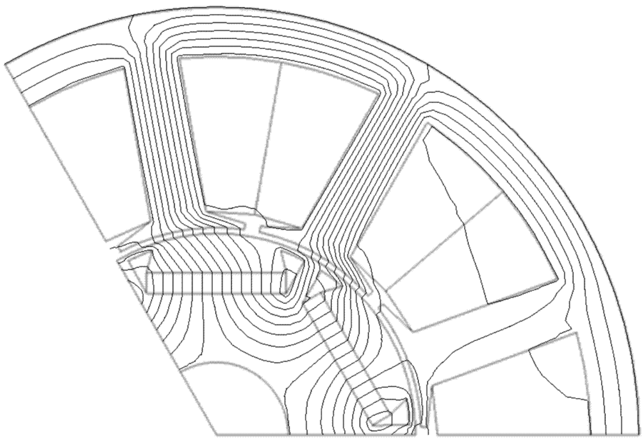
was set to *100*, and the DE algorithm was executed for *100*. These settings resulted in the total of *10,000* candidate design evaluations. In CE-FEA, *6* magnetostatic FE solutions are used per design per load condition.

A. *Case-Study Machine* (*9-slot, 6-pole*)

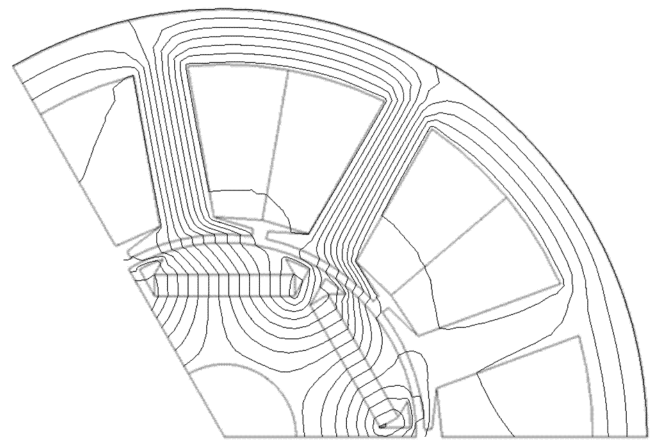
Shown in Fig. 4 are the values of the design objectives for *10,000* candidate designs as well as the objective function values corresponding to the final Pareto-optimal set of designs. From Fig. 4 it should be observed that a large variation between the torque ripple and the average torque production occurs from values as low as 0.4% to 32%, with the shaft torque varying from 6Nm to 21Nm. Three Pareto-optimal machines are selected from the Pareto-optimal set:

- M-1, highest shaft torque (specific torque),
- M-2, compromise between high torque and low ripple,
- M-3, low ripple machine.

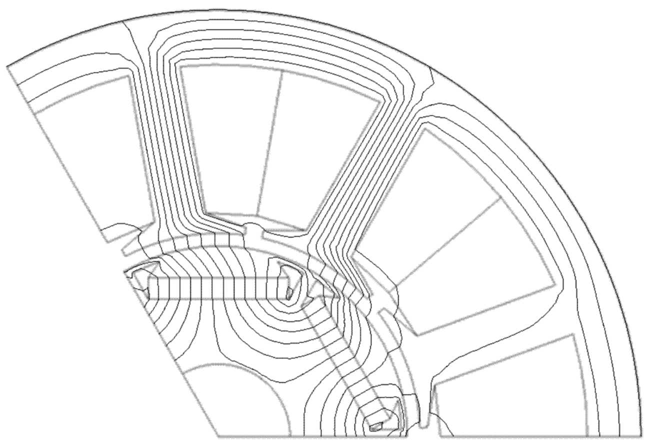
Cross-sections corresponding to machines M-1, M-2, and M-3 are provided in Fig. 5. Also shown in Fig. 5 is a typical design used for reference. Values of design variables corresponding to all machines are provided in Table I. Depicted in Figs. 6a and 6b, are the torque profiles corresponding to the four machines operating at rated-load (MTPA), and open-circuit (cogging torque) conditions, respectively. A significant reduction of both on-load and open-circuit ripple (cogging) is observed for optimized designs. For machines M-1 and M-2 a significant increase in the average torque production is observed. More specifically, a 37% increase of the average torque is observed for machine M-1. From Fig. 6b, a significant reduction of cogging torque is observed in the case of the optimized machines. Cogging torque is reduced by as much 90% in the case of the optimized machine M-3. It should be noted that results provided in Fig. 6 are verified with a detailed FEA that employs Maxwell stress tensor and 2nd order finite elements for the calculation of the electromagnetic torque. Open-circuit and on-load induced voltages for the four machines are provided in Figs. 7 and 8.



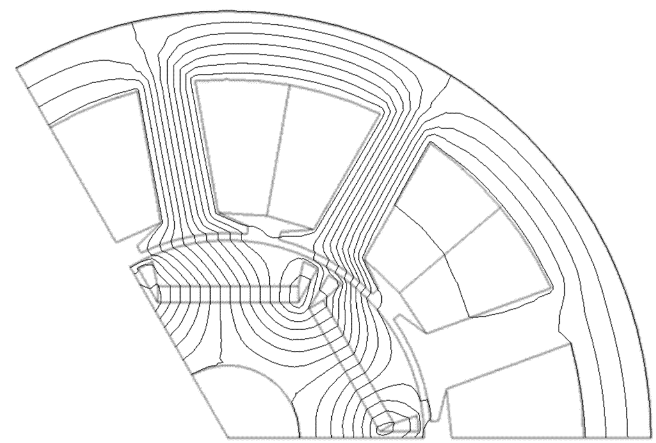
(a) M-1 High average torque
(max. $B_{mid\text{-}tooth}$ = 1.75T, min. B_{PM} = 0.76T)



(b) M-2 Compromise between average torque, and ripple
(max. $B_{mid\text{-}tooth}$ = 1.71T, min. B_{PM} = 0.75T)

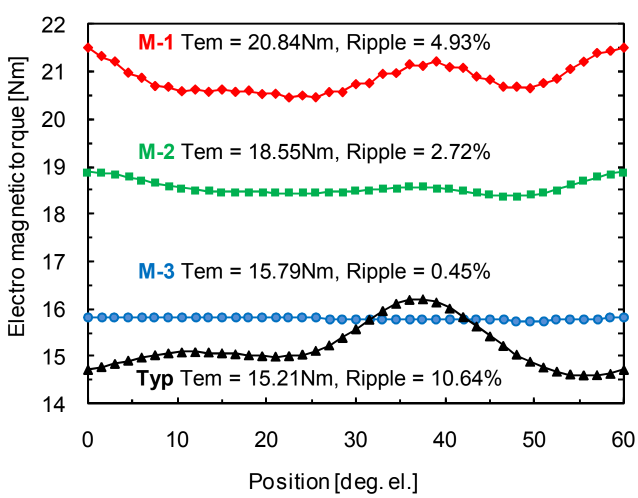


(c) M-3 Low ripple
(max. $B_{mid\text{-}tooth}$ = 1.65T, min. B_{PM} = 0.73T)

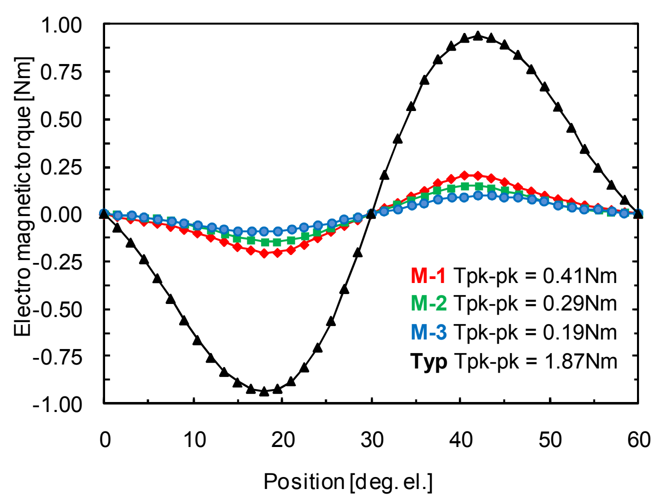


(d) Typical design
(max. $B_{mid\text{-}tooth}$ = 1.67T, min. B_{PM} = 0.78T)

Fig. 5. Cross-sections of optimized machines corresponding to Fig. 4 and Table I. Also shown are the flux plots for the rated-load conditions and the maximum flux density in mid-tooth and minimum flux density in the permanent magnet indicating demagnetization proximity.

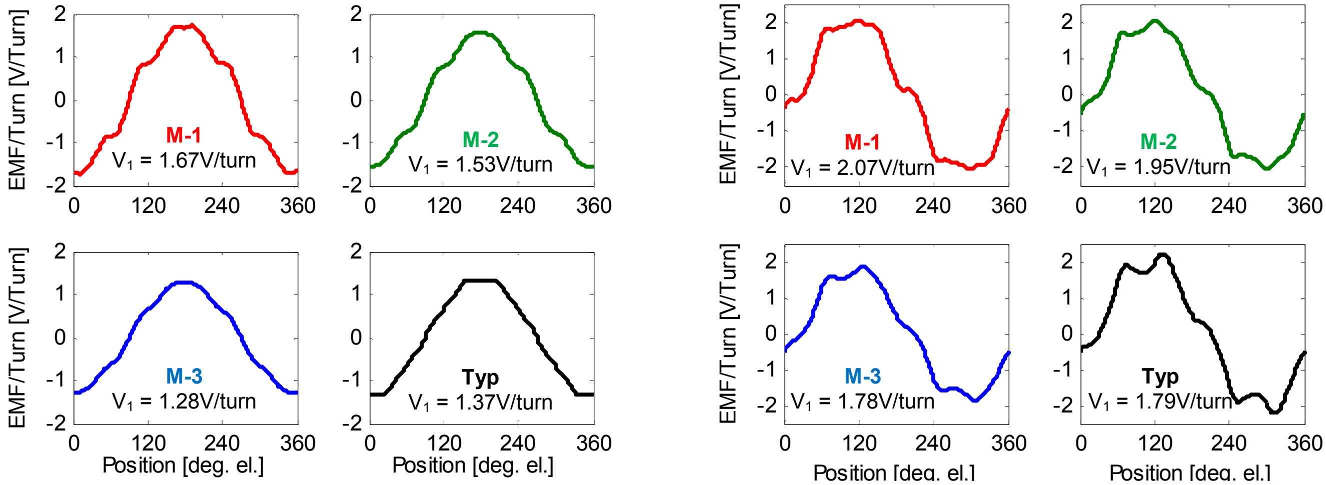


(a) Torque at rated-load conditions



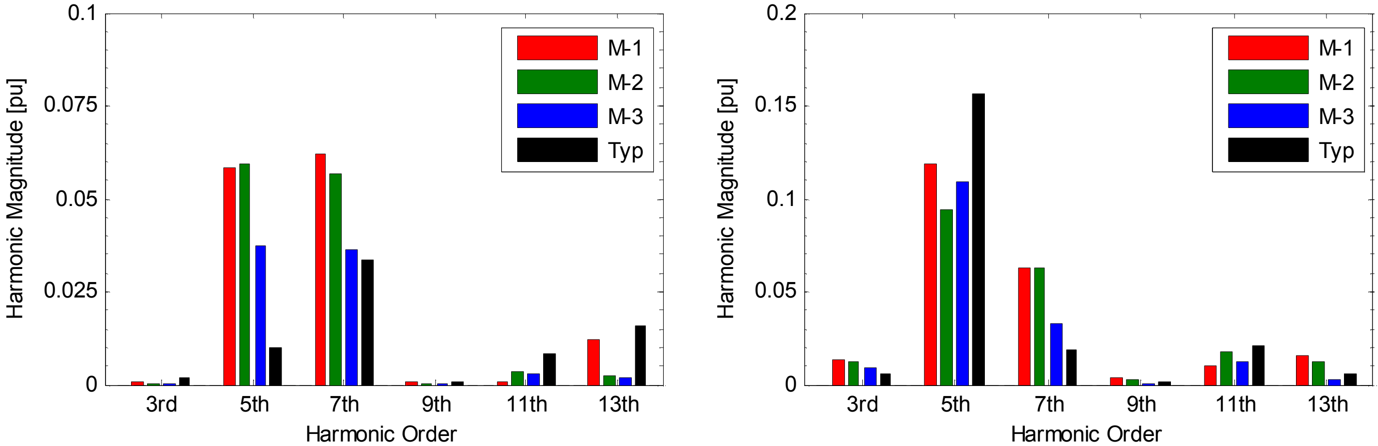
(b) Cogging torque

Fig. 6. Electromagnetic torque for machines of Fig. 5 supplied by a current regulated sine-wave drive (purely sinusoidal currents are assumed). Torque profiles are verified/obtained using detailed FEA with Maxwell stress tensor and 2nd order elements.



(a) Open-circuit (b) Rated-load (MTPA)

Fig. 7. Induced voltages at open-circuit and rated-load for machines of Fig. 5. Also shown are the peak values of the fundamental components.



(a) Open-circuit (b) Rated-load (MTPA)

Fig. 8. Harmonic analysis of induced voltages shown in Fig. 7. Fundamental component is not shown for clarity.

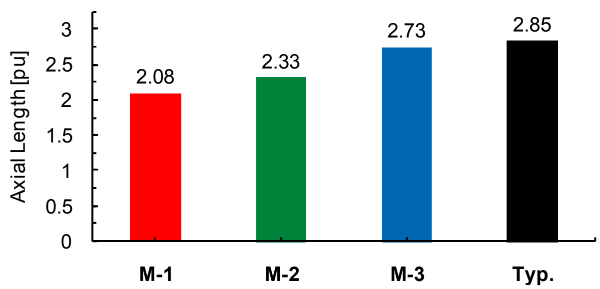


Fig. 9. Axial length for 7.5hp rating. Per unit length is defined with respect to the pole-pitch arc length.

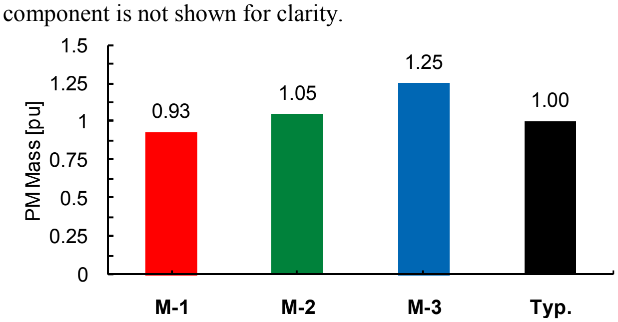


Fig. 10. PM mass per machine, values are per unitized to typical design (Fig. 5d).

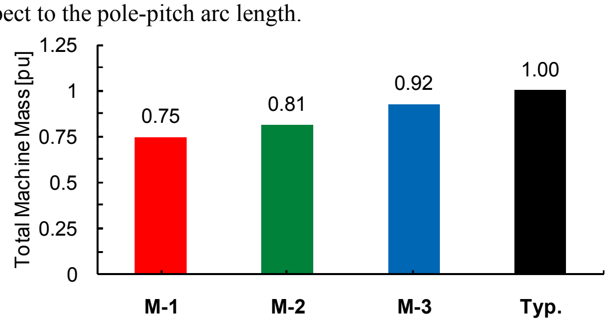


Fig. 11. Total machine mass, values are per unitized to typical design (Fig. 5d).

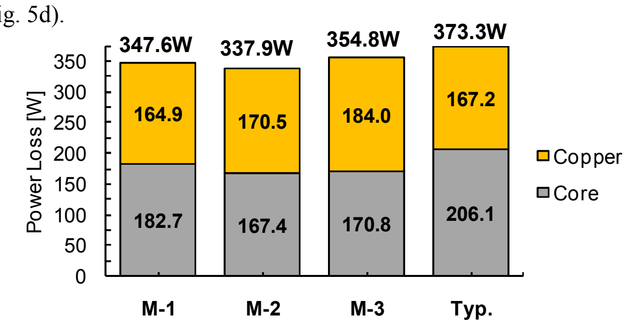


Fig. 12. Separation of loss components: core loss (hysteresis and eddy), copper winding loss.

B. *Design to Achieve Desired Ratings of 7.5hp (15Nm, 3600r/min)*

Here, three optimized machines (M-1, M-2, M-3) and the conventional machine are scaled to achieve the desired 7.5hp output rating. The axial length, l_{Fe}, is used to achieve this. Depicted in Fig. 9 are the corresponding axial lengths for the four machines per unit pole-pitch arc length. Also, reported in Figs. 10 and 11 are the permanent magnet and total machine masses per unitized to the machine of typical proportions. As expected, machine M-1, having the highest specific torque (Fig. 6a), results in the shortest and lightest machine that utilizes the least permanent magnet material for the given rating. More specifically, for machine M-1 the mass of permanent magnet material is reduced by 7% while the total machine mass and length are reduced by 25% and 23%, respectively. This is while the on-load torque ripple and open-circuit cogging torque are reduced by 54% and 79%, respectively. This practically eliminates the need for stator and/or rotor skewing, hence reducing manufacturing costs. In the case of applications demanding higher torque production quality, further reductions of torque ripple can be achieved with machines M-2 and M-3. Shown in Fig. 12 are the separations of loss components for the four machines. Here, it should be observed that machine M-2 results in the lowest overall loss and hence the highest efficiency of 94%. An almost equal split between copper and core losses for machine M-2 should also be noted. Overall, machine M-2 offers the best balance between the output torque quality, efficiency, material and manufacturing costs.

C. *Discussion*

The optimization results presented in this work show several design trends that may seem unconventional. First, the optimized machines have a significant reduction, of up to an order of magnitude, of both on-load and open-circuit (cogging) torque ripples, see Fig. 6. This is while the induced voltage waveforms at both open-circuit and rated-load conditions are rich in their harmonic content, as can be observed from Figs. 7 and 8. In the optimized machines, the reduction in the torque ripple is achieved through a combination of design variables, namely, air-gap height, slot opening and tooth-tip shape (tip depth and tip angle), and pole arc length. In the optimized machines, these parameters result in a balance between the torque ripple produced by the back emf harmonics and the torque ripple resulting from the position-dependent variation of the stored magnetic energy as discussed in [5]. It should be noted that significant reduction of torque ripple levels in the optimized machines eliminates the need for stator and/or rotor skewing. Ripple-free operation of non-skewed optimized machines can be achieved with a well tuned sine-wave current regulated drive, capable of regulating/eliminating higher order current harmonics (5^{th} and 7^{th}) that may result from the back emf harmonics.

Second, three optimized machines tend to have lower pole-arc coverage (0.62 average) resulting in higher permanent magnet flux concentration/focusing. As can be seen from Fig. 7, this results in an increase of the fundamental component of the induced voltage of the optimized machines. For example, for machine M-1, operating under open-circuit conditions, the induced voltage is increased by 22% in comparison to the machine that has a conventional pole-arc coverage of 0.78.

IV. Conclusions

In this paper a comprehensive large-scale FEA-based multi-objective design optimization study has been presented. The recently developed CE-FEA modeling method has been used for fast evaluation of *10,000* candidate designs on a typical PC-based workstation. The total simulation time for *10,000* designs that included the search of the maximum torque per ampere condition for every candidate design was *51* hours on a typical workstation. This shows that application of simplified FEA models, such as the CE-FEA, to large-scale optimization studies eliminates the need for use of significantly less accurate analytical and lumped parameter equivalent circuit models in comprehensive electric machine optimization studies.

A total of eleven stator and rotor independent design variables have been simultaneously optimized to satisfy multiple design criteria. For the case-study fractional-slot machine, an order of magnitude reduction of the rated-load torque ripple and open-circuit cogging torque has been achieved. At the same time the specific torque output has been increased by as much as thirty seven percent. These results show that significant improvements in the performance (torque production quality, efficiency, size, and cost) are achievable through comprehensive multi-objective optimization of electric machinery.

Acknowledgment

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References

[1] D. Staton, R. Deodhar, W. Soong, and T. Miller, “Torque prediction using the flux-MMF diagram in AC, DC, and reluctance motors,” IEEE Trans. Ind. Appl., vol. 32, no. 1, pp. 180–188, Jan./Feb. 1996.

[2] T.J.E. Miller, M. Popescu, C. Cossar, M. I. McGlipp, M. Olaru, A. Davies, J. Sturgess, A. Sitzia, “Embedded finite-element solver for computation of brushless permanent-magnet motors,” IEEE Trans. on Ind. Apps., vol. 44, no. 4, pp. 1124-1133, July/August 2008.

[3] F. A. Fouad, T. W. Nehl, and N. A. O. Demerdash, “Magnetic field modeling of permanent magnet type electronically operated synchronous machines using finite elements,” IEEE Trans. on PAS, vol. PAS-100, no. 9, pp. 4125-4133, 1981.

[4] D. M. Ionel and M. Popescu, “Finite element surrogate model for electric machines with revolving field - application to IPM motors”, IEEE Trans on Ind. Apps., vol. 46, no.6, pp. 2424-2433, Nov/Dec 2010.

[5] G. Y. Sizov, D. M. Ionel, and N.A.O. Demerdash, “Modeling and design optimization of PM AC machines using computationally efficient – finite element analysis,” IEEE Energy Conversion Congress and Exposition ECCE, pp. 578-585, Atlanta, Georgia, September 2010.

[6] K. V. Price, R. M. Storn, and J. A. Lampinen, *Differential Evolution - A Practical Approach to Global Optimization*, Springer-Verlag Berlin Heidelberg, 2005.

[7] E. Mezura-Montes, M. Reyes-Sierra, and C. A. Coello Coello. Multi-Objective Optimization using Differential Evolution: A Survey of the State-of-the-Art. In U. K. Chakraborty, editor, *Advances in Differential Evolution*, pages 173–196. Springer, Berlin, 2008. ISBN 978-3-540-68827-3.

[8] Infolytica Corporation, MagNet, ElecNet, ThermNet, FullWave, Scripting Manual, 2004.