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On the Role of Robustness in Multi-Objective Robust Optimization: Application to an IPM Motor Design Problem

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This paper discusses the different roles that robustness can assume when solving a multi-objective design optimization problem. A new role for robustness in multi-objective design optimization problems is proposed based on which an approach to conducting multi-objective robust optimization and finding the robust optima is suggested. This new approach is then tested on an Internal Permanent Magnet (IPM) motor design problem and the results are presented.

Index Terms— **Design Optimization, Pareto Optimization, Robustness.**

I. Introduction

Multi-objective optimization is concerned with optimizing more than one objective function under a set of constraints. The general problem is formulated as:

$$\underset{x}{\text{Minimize}}\, F(x) = [F_1(x), \dots, F_m(x)]^T \tag{1}$$

$$subject\ to: \begin{cases} g_i(x) \leq 0, & i = 1, \dots, k \\ h_j(x) = 0, & j = 1, \dots, l \end{cases} \tag{2}$$

Where m is the number of objectives, k is the number of inequality constraints, and l is the number of equality constraints. The solution to the formulation above is usually presented in the form of a set of non-dominated points in the feasible region of the objective space. The optima are known as the Pareto front since this definition of optimality and the relations of dominance were first introduced by Vilfredo Pareto.

When dealing with a design optimization problem, the primary concern is to find the optimal solution(s) that returns the best value(s) for the performance objective function(s). However, some of these solutions may seem less interesting when some effects that come with the real world implementation of the designs are factored in. These effects are usually uncontrollable and/or unpredictable factors that affect the performance of the design hence causing perturbations to the performance values which were calculated during the design optimization process. Examples of these factors include design parameter uncertainties, uncertainties in the operating environment, uncertainties in performance evaluation, and time dependent effects and variables. This unpredictability and uncontrollable differences between the design and the implementation of the design are highly undesirable. Therefore, one of the highly desired characteristics of a good design is the amount of resistance (to change) its important performance properties show in face of uncertainties. This concept of high resistance or low sensitivity is the most popular approach to defining robustness. A design is defined as robust if its performance objectives show low sensitivity with respect to changes caused by the aforementioned factors.

Robust optimization refers to the act of performing optimization when some degree of robustness is part of the criteria for optimality. Issues such as defining metrics for robustness and performing robust optimization have been well studied in the context of single-objective optimization [1], [2]; however, there has been little research work that deals with robustness in multi-objective problems and most of the existing work [3]-[5] is extensions of measures and methods used in single-objective problems. Considering that using most of the existing single-objective robustness measures and approaches in multi-objective problems would lead to significant increases in computational cost, it seems necessary to pay more attention to the concept of robustness from a multi-objective perspective.

The biggest problem in performing robust optimization is the high computational cost of obtaining robustness information. This is an even bigger problem in electromagnetics since the cost of solution evaluation (using numerical methods) is high. A number of studies have tried to address this issue in the recent years [6]-[9]. This paper is concerned with the different roles that robustness can play in multi-objective optimization, rather than how to measure robustness in multi-objective spaces.

II. Possible Roles of Robustness

In multi-objective robust optimization having robustness as a part of the criteria for optimality is a must, but where and to what extent should it be integrated into these criteria? Generally speaking, this is a matter of preference and should be indicated by the decision maker. However, there is usually a lack of sufficient and clear information from the decision maker's end. Therefore, attempting to come up with scenarios that make few assumptions about his/her preferences which can be applied to a large class of real world problems is very important. The role that robustness assumes in the process of optimization is usually one or a mixture of the following:

A. *Robustness as extra objective(s)*

Suitable metrics can be chosen in order to measure robustness. It is then introduced to the objective space in the form of one or more objectives alongside the performance objectives. An example of this can be found in [10].

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B. Robustness as modifier to performance objectives

There are multiple techniques for modifying performance objectives in order to account for robustness. The new robust performance objectives can then be used to perform regular optimization. Examples of modifying the performance objectives with robustness information include substituting the nominal values of the performance objective functions with their expected values [3] and using a robustness measure to penalize the values of the performance objective functions.

C. Robustness as extra constraint(s)

Measured values for robustness can be used as additional constraints which reduce the size of the feasible objective space. An example of this can be found in [3].

D. Robustness as modifier to dominance relations

In some cases the definition of dominance is modified in order to account for robustness when comparing solutions in the objective space. Examples of this can be found in [11], [12].

Role *A* has the advantage of keeping the trade-off information between robustness and performance but it increases the dimensionality of the objective space which makes the problem much more computationally expensive especially when the number of performance objectives is already high. Most of the modification methods in role *B* can be classified as either worst case methods or probability distribution based methods. The former is too conservative for high numbers of objectives and the latter requires probability distribution information which is very expensive to construct if not already present. Role *C* has the major drawback of not preserving the trade-off information between robustness and performance. It also usually requires one or more prescribed values for robustness constraints. There are not many instances of robustness being used in role *D* in the literature. The reason is that integrating preference information into the dominance relations in a meaningful sense is difficult. It is worth mentioning that roles *B* and *D* for robustness are very close since many instances of using one can be interpreted as using the other (e.g. using the worst value in the worst case method).

III. Proposed Role for Robustness

All of the possible roles of robustness mentioned in the previous section require the evaluation of robustness information on a wide range of the objective space. Considering that the evaluation of robustness is usually the most computationally expensive part in multi-objective robust optimization, it is very important that all types of unnecessary robustness calculations be avoided.

The assumption made here about the decision maker's preferences is twofold. (i) Robustness is desirable only in optimal or close to optimal solutions and it is considered irrelevant in very sub-optimal solutions. In other words, it is assumed that in many real world multi-objective robust optimization problems optimality with regards to performance takes precedence over robustness and solutions located far from the optima in the objective space are considered undesirable even if they are highly robust. (ii) The trade-off information between robustness and performance is important and should be presented to the decision maker.

The proposed role for robustness is based on the previous assumptions which imply that the evaluation of robustness and the preservation of trade-off information is only necessary close to the performance optima in the objective space. This turns robustness into a semi-objective, meaning that it is treated as an objective only in the vicinity of the performance optima and ignored everywhere else. Using robustness in this role has the advantage of keeping the useful information while limiting the calculation of robustness to a sub-region of the objective space and avoiding increasing the dimensionality of this space.

IV. Implementation

There is more than one way to use robustness in the proposed role. There needs to be a formulation which defines the region in the vicinity of the performance optima where the robustness information should be kept. This region is considered the optimum in this context and the goal of the multi-objective robust optimization process is to find it. The implementation approach suggested here is based on creating a balance between the deterioration of performance and the gain in robustness as one moves away from the Pareto front. In order to implement this balance, two unary metrics are used. One is a sensitivity based metric known as $PRHV_{dominated}$ (Perturbation Region Hyper-Volume) [13] which can be used to measure the robustness of a solution and the other is the hyper-volume (S) metric [14] which is used to measure the degree of sub-optimality of a solution. Note that higher values of $PRHV_{dominated}$ correspond to lower robustness and vice versa. Two inequalities are used in order to define the optimal region:

$$PRHV_{Dominated} \leq PRHV_{Dominated}^{min} \tag{3}$$

$$\frac{S}{S_{ref}} \leq \frac{\alpha}{1-\alpha}\left(1-\frac{PRHV_{Dominated}}{PRHV_{Dominated}^{avr}}\right) \tag{4}$$

Fig. (1a) shows an arbitrary two dimensional objective space used to demonstrate the evaluation of inequalities (3) and (4) for the solution point specified with a cross. $PRHV_{Dominated}^{min}$ is the minimum value of $PRHV_{Dominated}$ among the solution points that dominate the specified solution point. S is the hyper-volume of the area enclosed by the Pareto front and the hyper-planes that contain the specified solution point. S_{ref} is the reference hyper-volume which is the hyper-volume of the area enclosed by the Pareto front and the hyper-planes that contain the nadir point. $PRHV_{Dominated}^{avr}$ is the average value of $PRHV_{Dominated}$ over the solution points that dominate the specified solution point. If and only if a solution point satisfies both inequalities (3) and (4) then it belong to the optimal region.

Inequality (3) assures that the specified solution point belongs to the robust front according to the definition proposed in [2]. In other words, if there exists a solution that dominates the specified solution in both performance and robustness then there is no point in keeping the specified solution since it is dominated in both aspects. Inequality (4) is the one that creates the balance. The left hand side term serves as a normalized indicator as to how far the specified solution point is from the Pareto front and the right hand side term shows how much more

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robust the specified solution is compared to the solution points that dominate it. Note that both S and $PRHV_{Dominated}$ belong to $[0, +\infty)$. This inequality ensures that the gain in robustness is greater than the loss in performance optimality. Also, α is a tunable parameter between 0 and 1 which indicates to what degree the trade-off between performance and robustness is allowed. Setting α to 0 reduces the optimal region to the Pareto front whereas setting it to 1 results in obtaining the whole robust front. In the following optimization problems α is always 0.5.

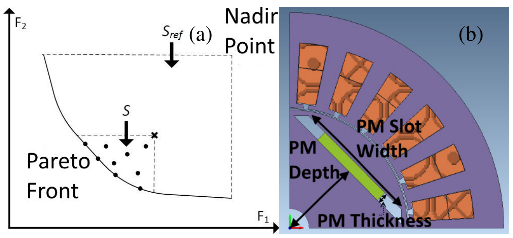

Fig. 1. Arbitrary objective space (a) and IPM shape design problem (b).

The optimization approach adopted here consists of two steps. The first step is a generic multi-objective optimization run in order to find the Pareto optima. The second step tries to evolve the end population of the first step into a spread of points over the optimal region defined by (3) and (4).

V. Test Problems and Results

The suggested approach is applied to the analytical test problem defined and explained in [13]:

$$Minimize\ F = [f_1, f_2]^T \quad (5)$$

$$\begin{cases} f_1(r,\theta) = \big(1+g(r)\big)cos\theta \\ f_2(r,\theta) = \big(1+g(r)\big)sin\theta \end{cases} \quad (6)$$

$$g(r) = r = 10x_2^3 - 15x_2^2 + 7.5x_2 \quad (7)$$

$$\theta = \frac{\pi}{2}x_1^5\,, \quad 0 \le x_1, x_2 \le 1 \quad (8)$$

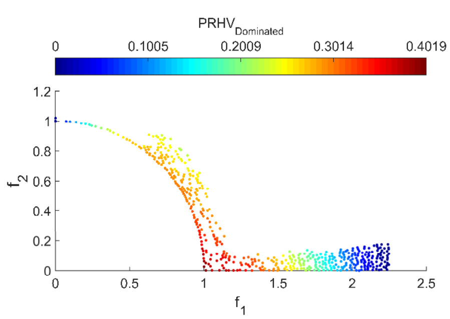

Fig. 2. Optimal region of the analytical test problem.

Where f_1 and f_2 are objectives, x_1 and x_2 are design variables, and r and θ are intermediate parameters. As equations (7) and (8) suggest, heavy bias has been introduced into the mapping from the design space to the objective space of this test problem in order to build a simple test problem with variations in sensitivity across the objective space. The results of the optimization approach are shown in Fig. 2 where a color map is used to show the values of robustness for the points in the optimal region. As can be seen in Fig. 2, the optimal region shows a larger recession away from the Pareto front where the gain in robustness is higher. The main reason why color is chosen to visualize robustness is that the values of $PRHV_{Dominated}$ (or any existing multi-objective robustness measure) have no physical meaning and can mostly be helpful in comparing robustness of different solutions. Therefore, having an extra axis for robustness would be redundant in addition to making the visualization more difficult.

The proposed approach is then applied to an IPM (Interior Permanent Magnet) motor rotor design problem. As shown in Fig. (1b), a simple topology with three design variables namely PM (Permanent Magnet) depth, PM thickness, and PM slot width $(PM\ Width = (^2/_3) \times PM\ Slot\ Width)$ is to be optimized in two bi-objective scenarios. In the first scenario, the IPM is optimized for average torque and torque ripple. Figs. (3) and (4) show the obtained optimal region in the objective space and design space, respectively. The objectives for the second scenario of optimization are efficiency and PM volume. Efficiency (η) is calculated using:

$$\eta = {}^{P_{out}}/_{(P_{out} + Iron\ Loss + Copper\ Loss)} \quad (9)$$

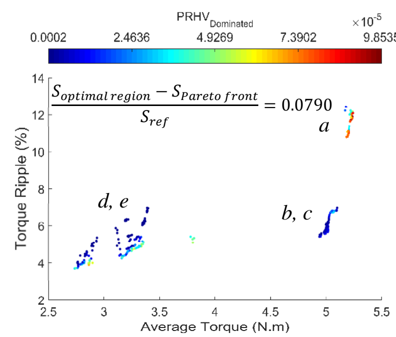

Fig. 3. Optimal region in the objective space for the first scenario.

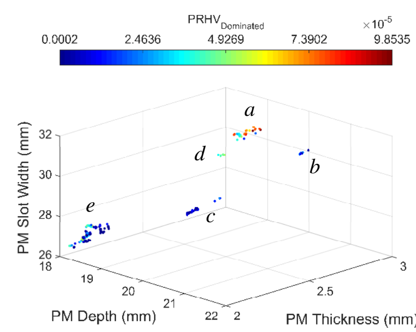

Fig. 4. Optimal region in the design space for the first scenario.

It should be noted that since iron losses are underestimated, efficiency is overestimated but the trend of change (which is important for the optimization results to maintain validity) is expected to remain intact. Figs. (6) and (7) show the optimal region in the objective space and in the design space for the second scenario, respectively.

As evident in Fig. (3), the optimal region for the first scenario consists of three main clusters in the objective space. The top right cluster and the bottom right cluster are the most and the least robust clusters, respectively. The bottom right and bottom left clusters appear to be fairly robust but the gain in robustness is higher in the bottom left cluster as one moves away from the

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Pareto front. Fig. (4) shows that the optimal region for this problem is multi-modal in the design space. A simple clustering technique based on Euclidean distance is used to categorize the points into five clusters (*a* through *e*). The mapping of these five clusters to the clusters in the objective space is shown in Fig. (3). Fig. (5) shows the shape of the IPM motor at the centroids of these five clusters.

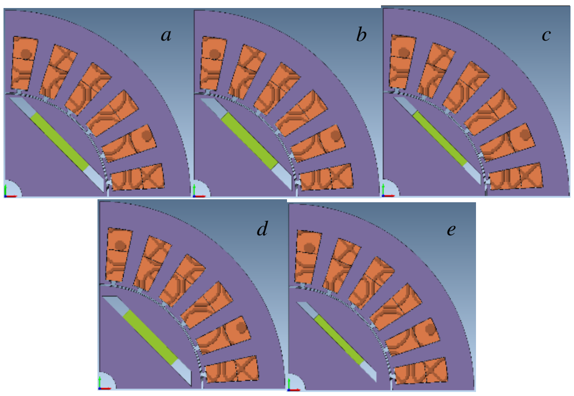

Fig. 5. IPM motor shapes at the centroids of the five clusters.

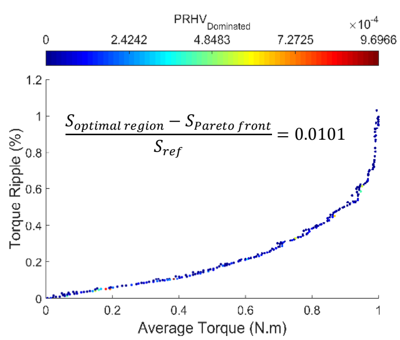

Fig. 6. Optimal region in the objective space for the second scenario.

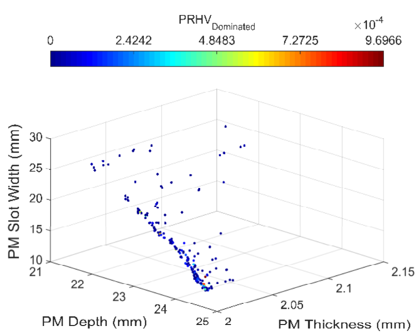

Fig. 7. Optimal region in the design space for the second scenario.

Fig. (6) shows the optimal region for the second scenario in the objective space. Here, the trade-off between performance and robustness is more uniform along the front compared to the first scenario (Fig. (3)). In fact, the optimal region barely recedes from the front and almost the entire front is very robust. Fig. (7) shows the optimal region in the design space. As can be seen, contrary to the first scenario the optimal region is unimodal. Also, in both scenarios the optimal region is very small in the objective space (the hyper-volume trapped between the optimal region and the front is indicated in Figs. (3) and (6).) which shows that the performance fronts are relatively robust.

VI. Conclusion

As demonstrated, keeping the information about optimality in performance and robustness and trade-off between the two, while limiting the region that needs to be explored for robustness is the key advantage of the proposed role. Although the number of function evaluations is the proper measure for comparing the computational cost of using robustness in different roles, the high dependency of this measure on other factors, the most important of which is the optimization algorithm, renders drawing such a contrast impossible. Nevertheless, one can expect a lower cost for the proposed role since the target region of the search is possibly much smaller.

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