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Model identification and control of prosthetic leg

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Model identification and control of prosthetic leg

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Abstract. Prosthesis is an artificial device that replaces missing body parts due to trauma, disease /accident. Prosthesis are mainly intended for restoring the normal functions of moving body part. Double pendulum structure is taken into consideration for modelling the system. In this system thigh and shank represents the pendulum links. Hip and knee joints create linkage between upper part of the body through thigh and shank. The muscles of both thigh and shank are initiated by external torques provided by stepper motor at hip and knee joints. The process variable to be controlled in the system is the position of both hip and knee. The work deals with the identification of system model along with the implementation of controllers like PID controller, Sliding mode controller (SMC) and Linear quadratic regulatory (LQR) controller and Linear quadratic tracking (LQT) controller. The results are compared.

1. Introduction

Prosthetic technology is used for persons who lost their limbs and can regain the same by means of artificial limb. Artificial limb usually depends upon type of amputation and can be either an arm/leg. Basic thing to be considered in designing a system is identification of system model. Mathematical modeling of system along with various phases of gait cycle is depicted in [1]. GPC plus integral compensator based control approach is considered in [2]. Also focuses on model identification using GPC plus integral compensator. Modeling and parameter estimation of 2 DOF motion is discussed in [3]. They also consider force feedback mechanism which are required for producing ground reaction forces. Dampers are generally required as rehabilitation devices and are applied to artificial joints for operating in two phases i.e. stance phase where full support occurs on ground and swing phase indicating motion to step out. The damping force should be increased in case of stance phase and the same must be reduced during swing phase. MR Damper based knee prosthesis along with damper design, system model and tracking control of knee is indicated in [4]. [5],[6],[7] focuses on advancements in lower limb prosthetic technology. Conceptual design of a bench for simulating various gait phases is also discussed in [6]. ILC based algorithm is used in [7]. A standardized procedure based modeling and simulation of prosthetic foot is discussed in [8]. Here the mechanical testing of system is done and is simulated in ANSYS WB. PID algorithm based control of actuated prosthetic knee is discussed in [9]. Closed loop position servo mechanism, position, velocity and current feedback along with PD control algorithm is indicated in [10]. Forward kinematics and inverse kinematics along with prosthetic leg design is indicated in paper [11]. Multibody simulation analysis using simmechanics toolbox is indicated in paper [12]. Hydraulic damper design along with its application on swing phase control is indicated in [13]. The damper parameters are then identified through an optimization procedure. Mathematical modeling of human leg is discussed in [14]. It



highlights the method s adopted for designing and model of human leg system.SMC based control of VTOL system is focused in [15]. It also focuses on design procedures of SMC and comparison of the performances of SMC and PIV Controller on VTOL system. Transformation of PI Controller into state feedback controller and solving the design by employing convex constraint optimization is discussed in [16].It also highlights about performances on Distillation Column using PI Controller. The main objective of this paper is to design a suitable controller that would stabilize the system and achieve a desirable tracking. Various control algorithms like PID, SMC and LQR were implemented and their performances are compared.

2. Prosthetic C Leg

Prosthesis indicate an artificial replacement for any/ all parts of lower / upper extremities. The present day best known artificial leg is C leg. Main feature which differentiates C leg from other type is its 2 mode operation i.e. one for walking and other for bicycling/any other preprogramed activity. The main problem in the C leg is the switching between the modes.10 programmable modes that are switchable using small remote control are present.



Figure 1. Prosthetic C leg

The C leg structure has inbuilt microprocessor that usually interrupts the movement of users along with anticipation of actions is shown in Figure 1.The system here is actuated through the legs hydraulic movement which gives users greater flexibility for changing speed/ direction without sacrificing stability.

3. System Modelling

Modelling of human leg system can be done by assuming double pendulum structure. Figure 2 shows the double pendulum with thigh and shank as two links with their masses m_1 and m_2 .The length of two links are represented by l_1 and l_2 . θ_1 and θ_2 represents hip angle and knee angle respectively. Torques responsible for movement of links are provided by stepper motor and are denoted as τ_1 and τ_2 [14]

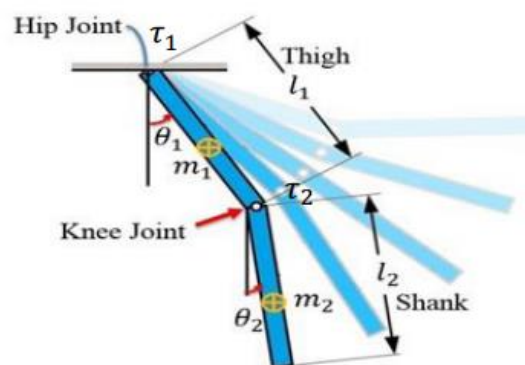


Figure 2. Representation of human leg

The system exhibits certain dynamics and can be represented using Lagrangian method. Dynamics representing system torques are denoted by following equation

$$\tau_1 = \frac{m_1+3m_2}{3} l_1^2 \ddot{\theta}_1 + \frac{m_2 l_1 l_2 \ddot{\theta}_2}{2} \cos(\theta_1 - \theta_2) + \frac{m_2 l_1 l_2 \dot{\theta}_2^2}{2} \sin(\theta_1 - \theta_2) + \frac{m_1+2m_2}{2} g l_1 \sin \theta_1 \quad (1)$$

$$\tau_2 = \frac{m_2}{3} l_2^2 \ddot{\theta}_2 + \frac{m_2 l_1 l_2 \ddot{\theta}_1}{2} \cos(\theta_1 - \theta_2) + \frac{m_2 l_1 l_2 \dot{\theta}_1^2}{2} \sin(\theta_1 - \theta_2) + \frac{m_2}{2} g l_2 \sin \theta_2 \quad (2)$$

By rearranging these dynamical equations the equations for double pendulum can be written as

$$M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) = \tau \quad (3)$$

Where $\theta, \dot{\theta}, \ddot{\theta}$ represents vectors of joint angle, joint angular velocity and joint angular acceleration which is of the order 2×1 . Here $M(\theta)$ indicates positive definite matrix, $C(\theta, \dot{\theta}) \dot{\theta}$ represents vector of Coriolis, $G(\theta)$ indicates gravitational torques and τ represents the joint torques produced by actuator.

$$M(\theta) = \begin{bmatrix} \frac{m_1+3m_2}{3} l_1^2 & \frac{m_2 l_1 l_2}{2} \cos(\theta_1 - \theta_2) \\ \frac{m_2 l_1 l_2}{2} \cos(\theta_1 - \theta_2) & \frac{m_2}{3} l_2^2 \end{bmatrix}$$

$$C(\theta, \dot{\theta}) \dot{\theta} = \begin{bmatrix} 0 & \frac{m_2 l_1 l_2 \dot{\theta}_2}{2} \sin(\theta_1 - \theta_2) \\ \frac{m_2 l_1 l_2 \dot{\theta}_1}{2} \sin(\theta_1 - \theta_2) & 0 \end{bmatrix}, G(\theta) = \begin{bmatrix} \frac{m_1+2m_2}{2} g l_1 \sin \theta_1 \\ \frac{m_2}{2} g l_2 \sin \theta_2 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (4)$$

The equations for $\ddot{\theta}_1$ and $\ddot{\theta}_2$ can be obtained on simplifying the torque equations τ_1 and τ_2

$$\ddot{\theta}_1 = \frac{K_4(\tau_1 - K_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - K_3 \sin \theta_1) - K_2 \cos(\theta_1 - \theta_2)(\tau_2 + K_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - K_5 \sin \theta_2)}{(K_1 K_4 - K_2^2 \cos(\theta_1 - \theta_2)^2)} \quad (5)$$

$$\ddot{\theta}_2 = \frac{K_1(\tau_2 + K_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - K_5 \sin \theta_2) - K_2 \cos(\theta_1 - \theta_2)(\tau_1 - K_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - K_3 \sin \theta_1)}{(K_1 K_4 - K_2^2 \cos(\theta_1 - \theta_2)^2)} \quad (6)$$

Where

$$K_1 = \frac{m_1+3m_2}{3} l_1^2, K_2 = \frac{m_2 l_1 l_2}{2}, K_3 = \frac{m_1+2m_2}{2} g l_1, K_4 = \frac{m_2}{3} l_2^2, K_5 = \frac{m_2}{2} g l_2 \quad (7)$$

By assuming state variables as

$$\begin{aligned} x_1 &= \theta_1 - \text{Upper link angular position} \\ x_2 &= \theta_2 - \text{Lower link angular position} \\ x_3 &= \dot{\theta}_1 - \text{Upper link angular velocity} \\ x_4 &= \dot{\theta}_2 - \text{Lower link angular velocity} \end{aligned}$$

Which can be represented as

$$\dot{x}_1 = x_3 \quad (8)$$

$$\dot{x}_2 = x_4 \quad (9)$$

$$\dot{x}_3 = \frac{K_4(\tau_1 - K_2 x_4^2 \sin(x_1 - x_2) - K_3 \sin x_1) - K_2 \cos(x_1 - x_2)(\tau_2 + K_2 x_3^2 \sin(x_1 - x_2) - K_5 \sin x_2)}{(K_1 K_4 - K_2^2 \cos(x_1 - x_2)^2)} \quad (10)$$

$$\dot{x}_4 = \frac{K_1(\tau_2 + K_2 x_3^2 \sin(x_1 - x_2) - K_5 \sin x_2) - K_2 \cos(x_1 - x_2)(\tau_1 - K_2 x_4^2 \sin(x_1 - x_2) - K_3 \sin x_1)}{(K_1 K_4 - K_2^2 \cos(x_1 - x_2)^2)} \quad (11)$$

Inputs for the system can be represented as

$$u_1 = \tau_1 = \text{Torque exerted on upper link}$$

$$u_2 = \tau_2 = \text{Torque exerted on lower link}$$

Outputs can be represented as

$$y_1 = \theta_1 = \text{Angular position of upper link}$$

$$y_2 = \theta_2 = \text{Angular position of lower link}$$

A,B,C,D matrices indicating system model can be found out by linearizing the above equations for state variables by Jacobians method with certain initial conditions. The resultant state space representation denoting system model can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (12)$$

$$y(t) = Cx(t) + Du(t) \quad (13)$$

where x indicates the state vector, \dot{x} indicates the state differential equation, y represents output equation and A,B,C,D are the matrices representing system model.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -31.45145 & 5.80626 & 0 & 0 \\ 25.854121 & -30.11465 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.9367 & -1.1241 \\ -1.1241 & 5.3082 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Parameters considered for designing human leg system is clearly depicted in Table 1.

Table1. Parameters Of Human Leg System

Parameters	Value	Unit
Mass, m_1	8	Kg
Mass, m_2	3	Kg
Length, l_1	0.5	Kg
Length, l_2	0.4	m
g	9.81	m/s ²

4. Controller Design

4.1. PID Controller

PID controller indicates a controller with proportional, integral and derivative action. Proportional action implies an action that is proportional to variable controlled or its error. Limitations of P controller tend to generate offset between process variable and set point which indicates the necessity of I controller. The I controller generally initiates necessary actions for eliminating steady state error. The derivative action indicates the derivative of controlled variable. In earlier days implementation of PID controller was done using analog devices/components. But in the present world because of the advent of computational technologies PID controller logic can be realized using microcontrollers. PD controller in a continuous domain can be written as

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d}{dt} e(t) \quad (14)$$

PID controller with the system must be tuned properly before working so as to suit with the dynamics of process to be controlled. Usually designers tend to give default values for P, I and D terms and these values usually leads to system instability and finally reduces system performance too. Various methods of tuning PID controller are

- i) Trial and error method
- ii) Process reaction curve method
- iii) Ziegler Nichols method

This work presents the tuning done using trial and error method to obtain controller gains as in Table 2.

Table 2 .PID Controller gains

Controller	K_p	K_i	K_d
PID	93.13	57.12	3.47

4.2. SMC (Sliding Mode Controller)

Sliding mode control technique is a non linear technique with remarkable properties of accuracy, robustness, easy tuning and implementation. Mainly designed for driving system state onto particular surface called sliding surface. After attaining sliding surface sliding mode control tends to keep the states on close neighbourhood of sliding surface. Usually it is concerned with two part controller design. The first part in this includes the design of sliding surface while the second is concerned with control law selection which can make switching surface attractive to system state. Parameters taken into consideration while designing sliding mode controller are sliding gain, sliding slope and sliding thickness. These parameters make the system follow the sliding surface. For monitoring the sliding surface $s(t)$ at a constant value we make

$$\dot{s} = \frac{d}{dt}(\text{constant}) = 0 \quad (15)$$

Sliding thickness is basically used for removal of chattering effects in the system. The sliding function s can be written as

$$s = \lambda e + \dot{e} \quad (16)$$

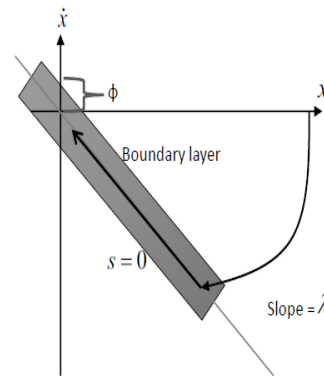


Figure 3. Sliding mode control with slope λ and sliding thickness \emptyset

Sliding phase trajectory along with a switching line of infinite frequency is indicated in Figure 3. Sliding thickness is increased for reducing chattering in the system. The switching function of sliding mode controller can be written as

$$\text{switching function} = \begin{cases} \text{sign}(s) \text{ for } |s| > \emptyset \\ \frac{s}{\emptyset} \text{ for } |s| < \emptyset \end{cases} \quad (17)$$

Sliding mode controller output is represented as

$$\text{output} = -(e - K * \text{switching function}) \quad (18)$$

Sliding mode parameters of the system is depicted in Table 3

Table 3. SMC parameters

Controller	Sliding Gain(K)	Slope(λ)	Sliding thickness(\emptyset)
SMC	40	2	0.0001

4.3. LQR Controller

LQR generally operates as a state feedback controller that is designed to minimize quadratic cost function for generating optimal control design. Main idea in LQR design is to minimize the quadratic cost function by driving state x to the origin

$$J = \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u \quad (19)$$

The main difference between LQR and LQT is in LQR the regulator aims to regulate and control linear system state variables about zero by considering an optimal control strategy in a finite/infinite horizon while in LQT the output tracks a favourite path as optimal. It makes the state variables converging to zero and makes system output track the reference input. LQR system can be designed using Algebraic Ricatti equation

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (20)$$

The state feedback gain for a matrix K is given by

$$K = R^{-1}B^T P \quad (21)$$

Quadratic cost function J can be written as

$$J = \int_0^T X^T Q X + u^T R u \quad (22)$$

Where Q and R are state control matrices. By using this boundary condition and integrating Ricatti equation backwards we can compute the optimal control online using the equation

$$u = -KX \quad (23)$$

Values of Q and R matrices along with the feedback gain K for the LQR controller are

$$Q = \begin{bmatrix} 100000 & 0 & 0 & 0 \\ 0 & 100000 & 0 & 0 \\ 0 & 0 & 0.00001 & 0 \\ 0 & 0 & 0 & 0.00001 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, K = \begin{bmatrix} 281.2 & -3.61 & 27.98 & 4.27 \\ -0.03 & 310.42 & 4.66 & 11.67 \end{bmatrix} \quad (24)$$

5. Simulation Results

After obtaining the model open loop response of the system is found out. Location of poles in the system are also noted for stability.



Figure 4. Step Response of system without controller

Step response without controller is indicated in Figure 4 for both hip and knee positions. System is stable with high oscillations and is also controllable since the system doesn't have singularity.

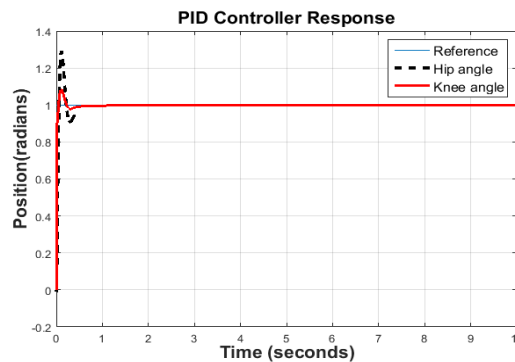


Figure.5. PID controller response

The system response using PID controller is indicated in Figure 5. PID controller exhibits a settling time of 2 sec, zero steady state error and a peak overshoot of 1.33 radians for hip angle and 1.15 radians for knee angle.

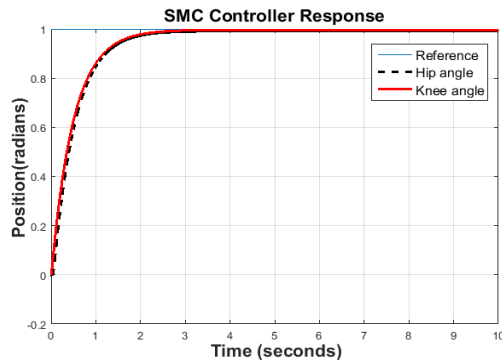


Figure.6.Position control response using SMC Controller

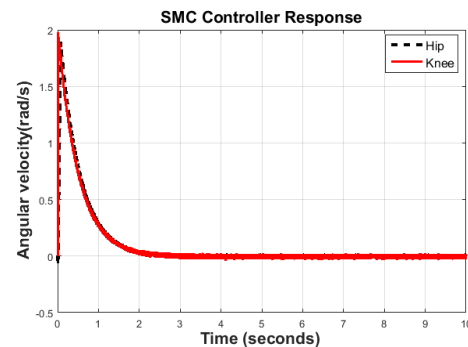


Figure.7.Velocity control response of system using SMC Controller

The system response using SMC controller is indicated in Figure 6. Design of SMC is based on certain parameters. Response shown by the system exhibits a settling time of 2.5 sec, zero steady state error and a peak overshoot of 1 radians for both hip and knee angle. The system response for velocity control using SMC controller is shown in Figure 7. Peak overshoot is higher in knee and both hip and knee settles to zero within 2.5 seconds.

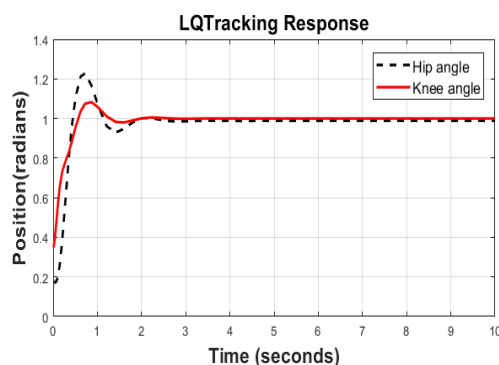


Figure.8.Response using LQ Tracker

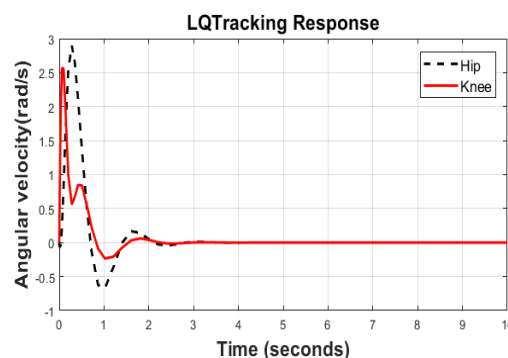
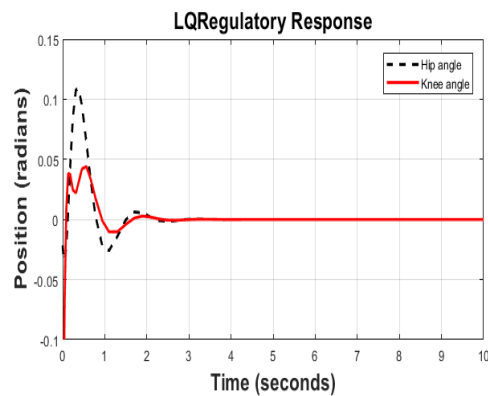
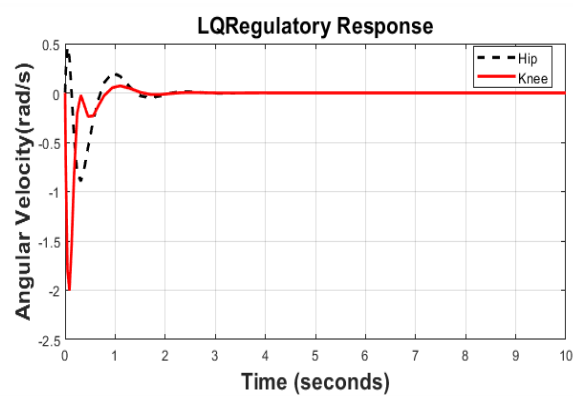


Figure.9.Velocity control response of system using LQ Tracker

The ability of the system to track specific controlled trajectories using LQ Tracker is shown in Figure 8. The response of the system using LQ Tracker exhibits a steady state error of 0.001, settling time of 0.5 sec and a peak overshoot of 1.21 radians for hip angle and 1.15 radians for knee angle. The velocity control response of the system using LQ Tracker is shown in Figure 9. Peak overshoot is higher in hip and velocities of both hip and knee settles to zero within 2 seconds.

**Figure.10.**Response using LQRegulator**Figure.11.**Velocity control response of system using LQ Regulator

Response of the system using LQRegulator is shown in Figure 10. The system response using LQRegulator exhibits a settling time of 2 sec and a peak overshoot of 0.22 radians for hip angle and 0.35 radians for knee angle with zero steady state error. The velocity control response of the system using LQ Regulator is shown in Figure 11. Peak overshoot is higher in hip and velocities of both hip and knee settles to zero within 2.5seconds.

The results are tabulated as in Table 4 based on the responses obtained from various controllers

Table 4. Comparison between different controllers

Controller	Peak Value		Settling Time	Steady state Error
	Hip angle	Knee angle		
PID	1.33	1.15	2	0
SMC	1	1	2.5	0
LQR	0.1	0.04	2	0
LQT	1.21	1.5	0.5	0.001

6. Conclusion

Linearized mathematical modelling of system is derived from the nonlinear model of system to improve the system performance. Different control algorithms were implemented on the system like PID control algorithm, SMC control and LQR with both regulatory and tracking control. All these control algorithms were simulated for the system in software using Matlab and SIMULINK. While comparing the performance of controllers it is seen that SMC controller takes more time to settle than PID and both LQR and LQT. Also while measuring peak overshoot response of system it was clear that SMC has better response than other two controllers PID and LQR. From all the responses it can be finalised that response of SMC is superior and also provides better control on process variable than other controllers

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