Homework0_e12329556_Yash_Waware

Plate with Piezoelectric patches

Problem description

A plate with piezoelectric patches is considered. The plate is fixed on the sides (Γ_s) and has 3 piezoelectric patches attached on the front, as shown in the figures 1 and 2, to measure the response of the plate under different loading conditions. The piezoelectric patches are made up of the piezoceramic material PIC 255. Electrodes (of negligible thickness) are located on the top and bottom of the piezoceramic material, and it is insulated by an insulating material as shown in the figure 3. The plate is made of polymethyl methacrylate (PMMA), and the dimensions are l = 750 mm, w = 450 mm, t = 8 mm. The dimensions of the simplified model (used for modeling) of the piezoelectric patch are l_p = 61 mm, w_p = 35 mm, t_p = 0.4 mm as shown in the figure 4.

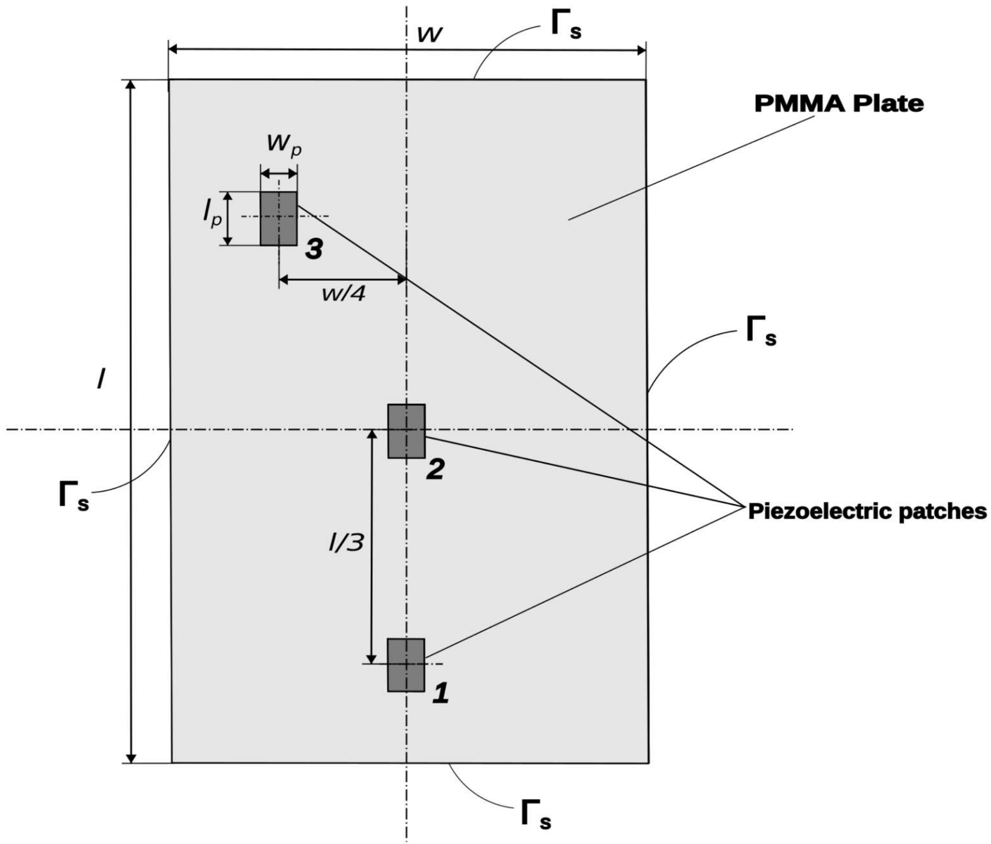


Figure 1: Front view

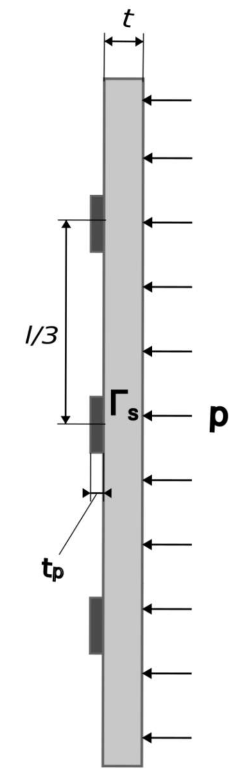


Figure 2: Side view

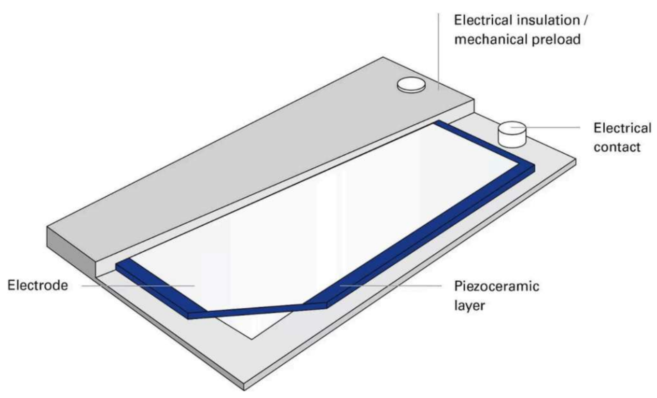


Figure 3: Actual piezoelectric patch

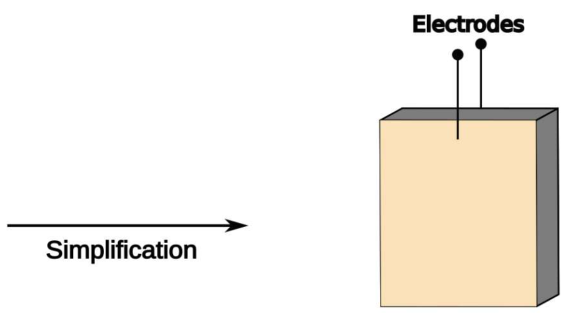


Figure 4: Simplified model

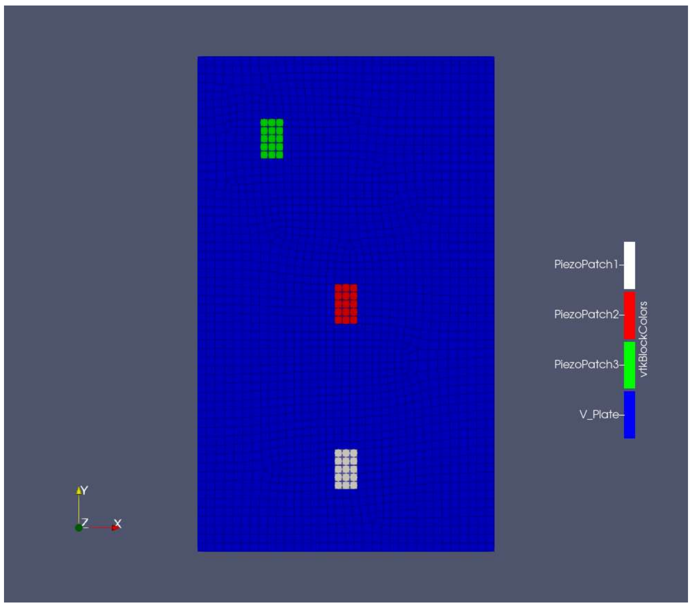
Material data for PMMA	Value
Density (kg/m³)	1180
Young's modulus E, (GPa)	4.5
Poisson's ratio	0.4

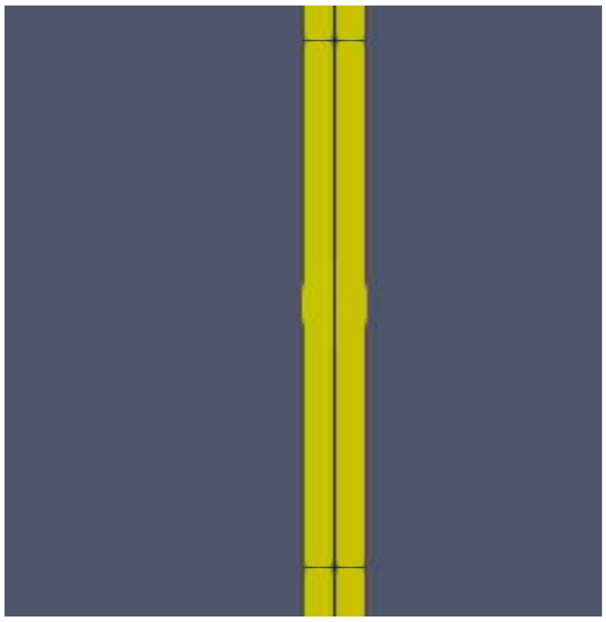
To complete the homework assignment follow the tasks and questions below.

Assignment tasks

1. Create a suitable mesh for the problem

- Create hexahedral mesh (2 elements per layer in thickness direction should be sufficient). **(1 Point)**.
- Use 2nd order elements (element type HEX20). **(0.5 Points)**.
- Define the necessary regions, assign meaningful names and show the mesh. **(0.5 Points)**





2. Modelling assumptions

- What are the modelling assumptions in the conducted simulation? Consider PDEs, boundary conditions, material models, and analysis type, ... (**2 Points**)

Answer:

Material Defination

- PMMA
  - Linear elatsic material
  - Isotropic in nature
- PIC255
  - Homogenous polarization
  - Assumed zero resistance in electrodes
  - Electrodes are considered infinitely thin

Analysis type

- Static: Provides a time-independent solution
- Harmonic: Applies perfect harmonic forcing and analyzes steady-state response
- Eigenvalue: Solves the coupled free oscillation eigenvalue problem
- Transient: Numerically integrates the coupled ODE system for each time step

PDE's

- Mechanics
  - Small strains are assumed
- Electrostatics
  - Satisfying Faraday's and Gauss' law

Volume Coupling

- Establishes a linear relationship between deformation/tension and electric potential, representing linear piezoelectricity

Boundary Conditions

- Mechanics
  - All four sides of the plate are fixed (in x-, y- and z-direction)
  - When mechanical forcing is needed, pressure is applied on the plate surface opposite to where the piezoelectric patches are mounted -Piezoelectricity
  - The surface of each piezoelectric patch, glued to the plate, is set to the ground potential (zero-point)

3. Material data

The material data of the piezoceramic PIC 255 are given below,

Compliance tensor:

$$\boldsymbol{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12}) \end{bmatrix}$$

with $S_{11} = 16.1 \cdot 10^{-12} \frac{m^2}{N}$, $S_{12} = -4.7 \cdot 10^{-12} \frac{m^2}{N}$, $S_{13} = -8.5 \cdot 10^{-12} \frac{m^2}{N}$, $S_{33} = 20.7 \cdot 10^{-12} \frac{m^2}{N}$, $S_{44} = 42 \cdot 10^{-12} \frac{m^2}{N}$,

$d_{31} = -180 \cdot 10^{-12} \frac{C}{N}$, $d_{33} = 400 \cdot 10^{-12} \frac{C}{N}$, and $d_{15} = 550 \cdot 10^{-12} \frac{C}{N}$,

$\epsilon_{11} = 1.4069 \cdot 10^{-8} \frac{As}{Vm}$, $\epsilon_{33} = 1.5494 \cdot 10^{-8} \frac{As}{Vm}$,

Density $\rho = 7800 \frac{kg}{m^3}$

The piezoceramic PIC255 is polarized in the thickness (3) direction. The blocking force of the piezoelectric patch for an applied voltage of $500V$ is $F_{max} = 256N$ in the length direction and the strain in the width direction is $-650 \mu m/m$. The stress-free nominal displacement of the patch under the applied voltage is $\Delta L_0 = -27 \mu m$ in the length direction.

Calculate the significant effective material properties d_{31}^{simp}, S_{11}^{simp}, S_{12}^{simp} and ϵ_{33}^{simp} for the simplified model of the patch. Calculate the percentage change in effective material properties from the PIC255 material data and apply the change to other unknown material properties using the hints below:

- Find d_{31}^{simp} using the nominal displacement and the applied electric field E_3. Apply the change to d_{33}^{simp} and d_{15}^{simp} (Hint: Use the d-form of the constitutive relation). (**1 point**)

Answer:

To find d_{31}^{simp} we need to use d-form: $\mathbf{s} = \mathbf{S}\sigma + \mathbf{dE}$, here $\mathbf{S}\sigma$ = 0 because no stress, so we are left with $\mathbf{s}_{11} = \mathbf{d}_{11}^{simp} \mathbf{E}_3$

With the applied voltage, the electric field intensity in thickness direction is; $\mathbf{E}_3 = \frac{V}{t\ p} = 1.25 \cdot 10^6 \frac{V}{m}$, $\mathbf{s}_{11}$ = $\frac{\nabla L_0}{L_0} = \frac{\nabla L_0}{l_0} = -442.6 \cdot 10^{-6}$, $\mathbf{d}_{11}^{simp} = \frac{s_{11}}{E_3} = -354.1 \cdot 10^{-12} \frac{C}{N}$,

For values lead to $\mathbf{d}_{33}^{simp} = 786.8 \cdot 10^{-12} \frac{C}{N}$ and $\mathbf{d}_{15}^{simp} = 1081.85 \cdot 10^{-12} \frac{C}{N}$

- Calculate the material coefficients S_{11}^{simp} and S_{12}^{simp} and the effective youngs modulus E_L^{simp} in the plane normal to thickness direction. Apply the change in young's modulus to other material coefficients. (Hint: Use the d-form and the blocking force). (**1 point**)

Answer:

For S_{11}^{simp} because of S_{max} there is no strain in s_{11} which means that;

$s_{11} = S_{11}^{simp}\sigma_{11} + d_{31}^{simp}E_3$ = 0 = $S_{11}^{simp} \cdot \frac{F_{max}}{t_p w_p} + d_{31}^{simp}E_3$

$S_{11}^{simp} = 2.421 \cdot 10^{11} \frac{m^2}{N}$

for S_{12}^{simp}, we have strain in width direction, so using d-form $S_{22} = S_{12}^{simp}\sigma_{22} + d_{13}^{simp}E_3$, here $\sigma_{22} = \sigma_{11} = \frac{F_{max}}{t_p w_p}$

$S_{12}^{simp} = 11.34 \cdot 10^{11} \frac{m^2}{N}$

The effective young's modulus $E_L^{simp} = \frac{1}{S_{12}^{simp}} = 4.131 \cdot 10^{10} \frac{N}{m^2}$

Applying the changes in young's modulus to the other material coefficients leads to:

- $S_{13}^{simp} = -12.78 \cdot 10^{12} \frac{m^2}{N}$
- $S_{33}^{simp} = 31.12 \cdot 10^{12} \frac{m^2}{N}$
- $S_{44}^{simp} = 63.16 \cdot 10^{12} \frac{m^2}{N}$

- The capacitance of the piezoelectric patch is $90nF$. Calculate the relative permittivity ϵ_{33}^{simp} and apply the change to ϵ_{11}. (**1 point**)

Answer:

The relative permittivity is calculated by $C = \epsilon_{33}^{simp} \cdot \frac{l_p w_p}{t_p}$

$\epsilon_{33}^{simp} = \frac{C \cdot t_p}{l_p w_p} = 1.6862 \cdot 10^{-8} \frac{A \cdot s}{V \cdot m}$

Applying change $\% = \frac{\epsilon_{33}^{simp} - \epsilon_{33}}{\epsilon_{33}} = 8.83\%$

$\epsilon_{11}^{simp} = \epsilon_{11} + 8.83\% \cdot \epsilon_{11} = 1.531 \cdot 10^{-8} \frac{A \cdot s}{V \cdot m}$

- Compute $\boldsymbol{C}$ (stiffness tensor), $\boldsymbol{\epsilon}$ (permittivity tensor) ,$\boldsymbol{e}$ (couple tensor in e-Form) and use your results for the material definition (`mat_eff.xml`) of the patches. (**1.5 points**)

Answer:

$\boldsymbol{C}$ (stiffness tensor) is;

$$C^E = S^{E^{-1}} = \begin{bmatrix} 22.59 & 19.78 & 17.41 & 0 & 0 & 0 \\ 19.78 & 22.59 & 17.41 & 0 & 0 & 0 \\ 17.41 & 17.41 & 17.52 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.577 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.577 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.401 \end{bmatrix} \cdot 10^{10}\text{Pa}$$

$\boldsymbol{\epsilon}$ (permittivity tensor) is;

$$\epsilon = \begin{bmatrix} 1.531 \times 10^{-8} & 0 & 0 \\ 0 & 1.531 \times 10^{-8} & 0 \\ 0 & 0 & 1.531 \times 10^{-8} \end{bmatrix} \frac{A \cdot s}{V \cdot m}$$

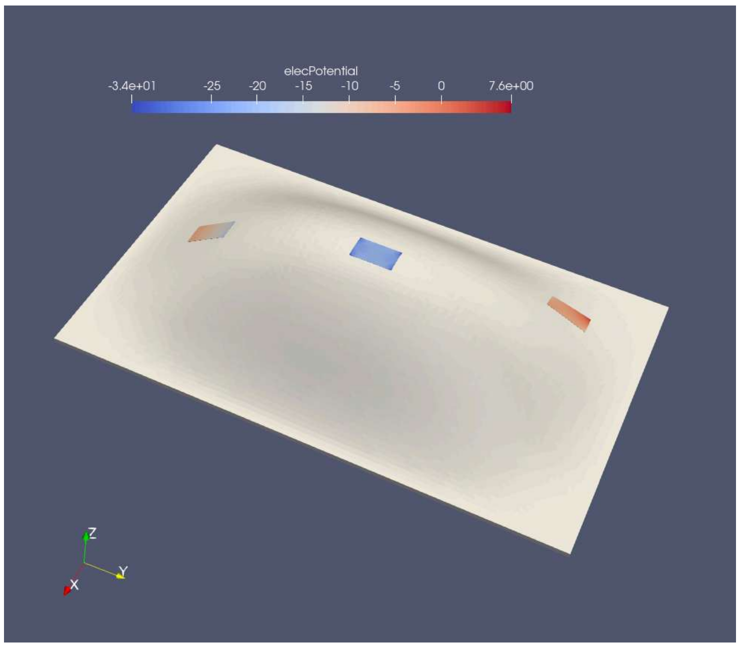
$\boldsymbol{e}$ (couple tensor in e-Form) is;

$$\boldsymbol{e} = \left(C^E \cdot d^T\right)^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 17.042 & 0 \\ 0 & 0 & 0 & 17.042 & 0 & 0 \\ -13.086 & -13.086 & -14.539 & 0 & 0 & 0 \end{bmatrix} \frac{C}{m^2}$$

4. Static analysis

- Test the model in static analysis by applying a pressure of magnitude **p** = 1000 N/m^2 on the back of the plate as shown in figure 2. **(1 point)**

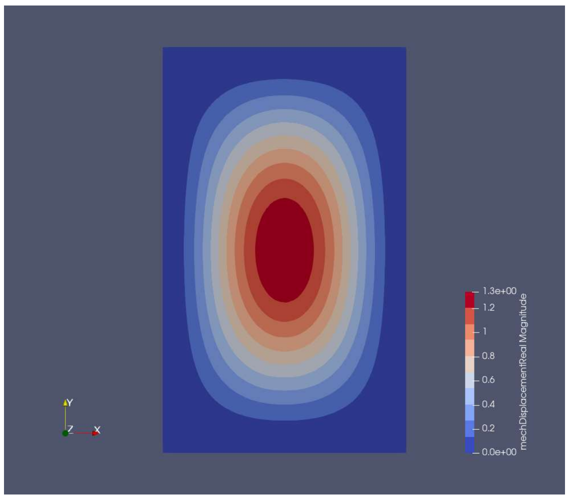
- Show the deformed shape (appropriately scaled) and visualize the computed electric potential in the piezoelectric patches. **(1 point)**



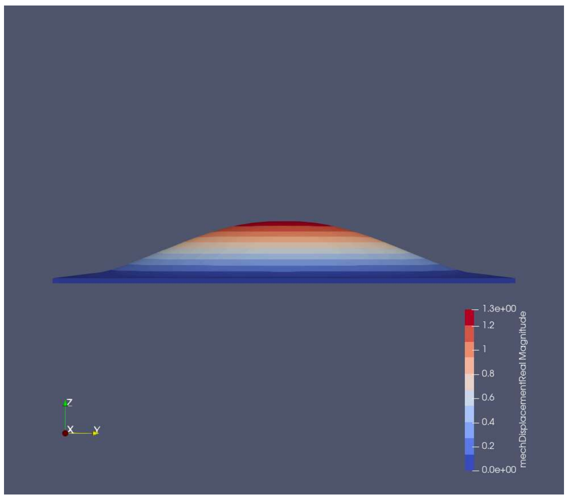
Deformed shape

5. Eigenvalue analysis

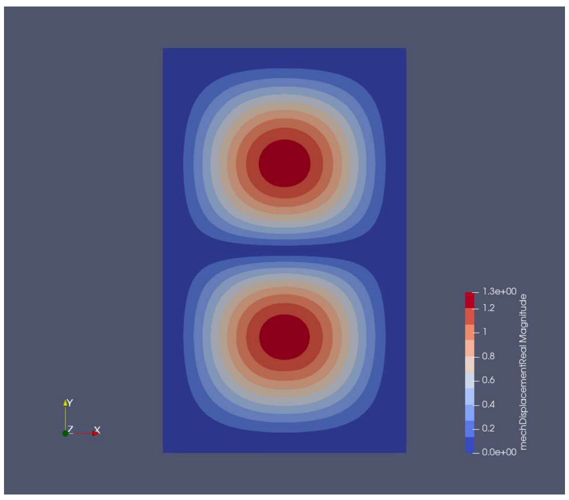
- Compute the first 10 eigenmodes of the PMMA plate with piezoelectric patches for open electrode configuration. Visualize the eigenmodes and provide the corresponding natural frequency **(1 point)**



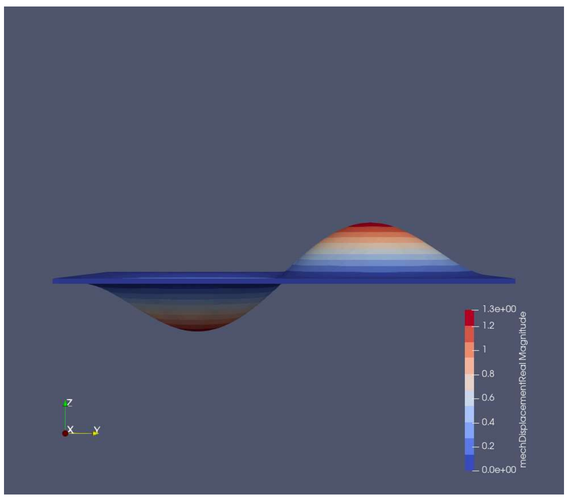
1st mode 101.038 Hz



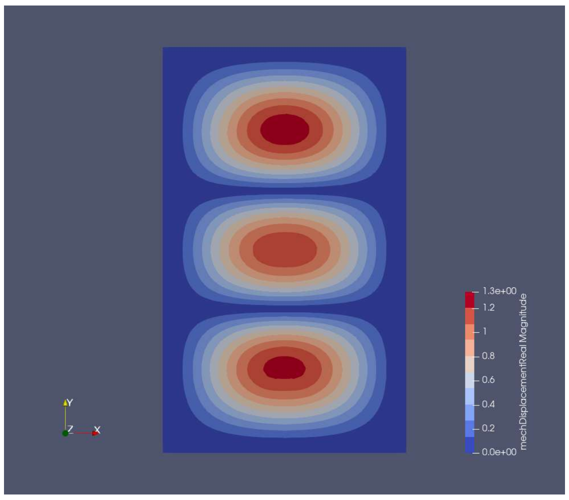
Deformation in the direction of mode 1



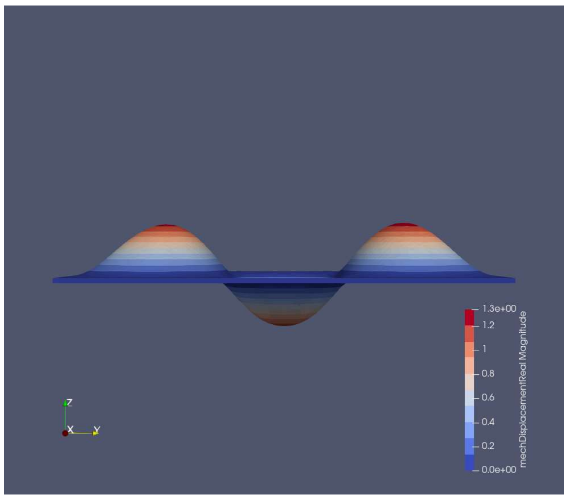
2nd mode 144.88 Hz



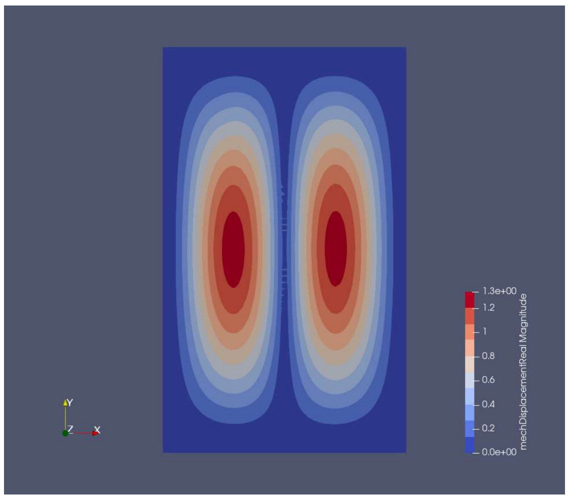
Deformation in the direction of mode 2



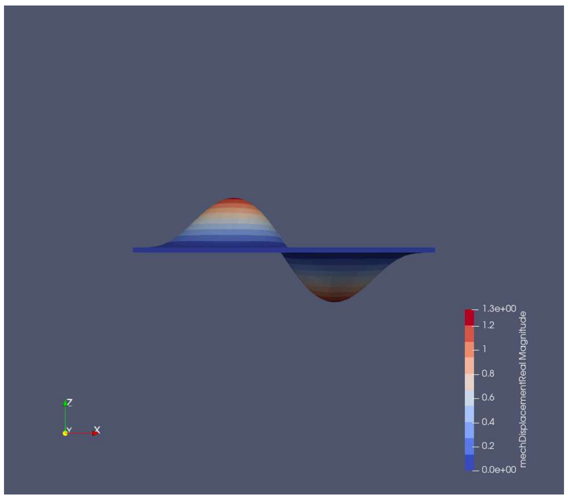
3rd mode 222.07 Hz



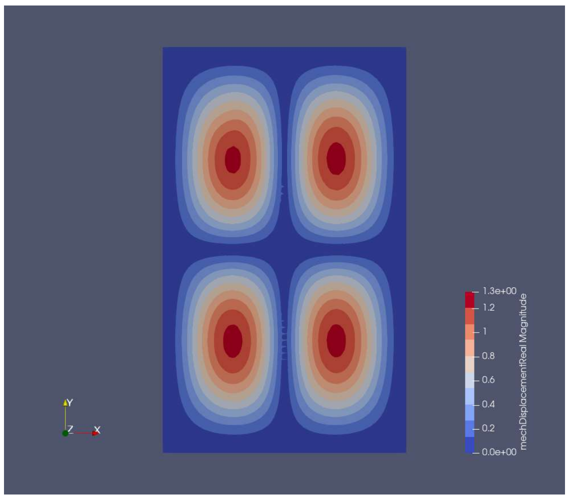
Deformation in the direction of mode 3



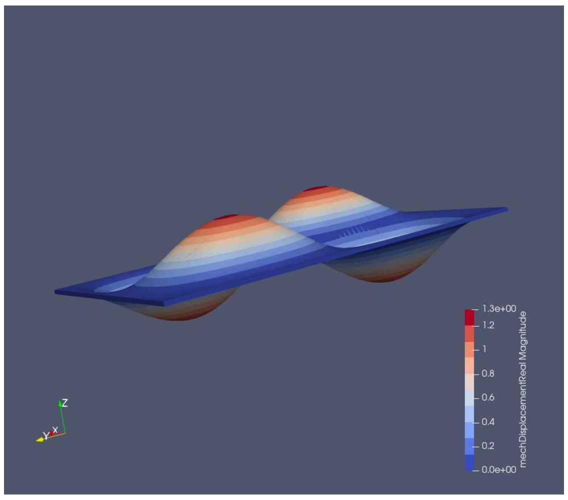
4th mode 253.54 Hz



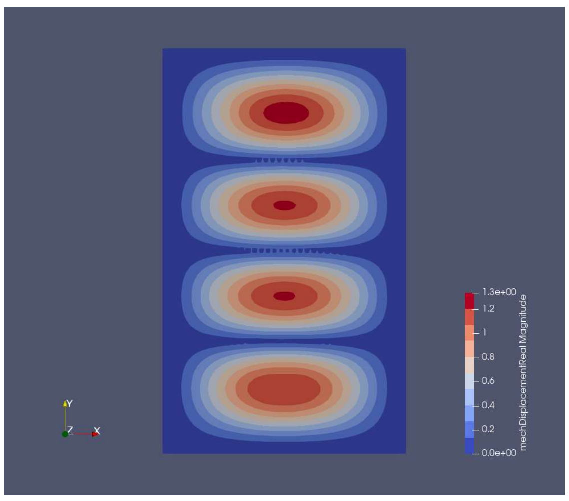
Deformation in the direction of mode 4



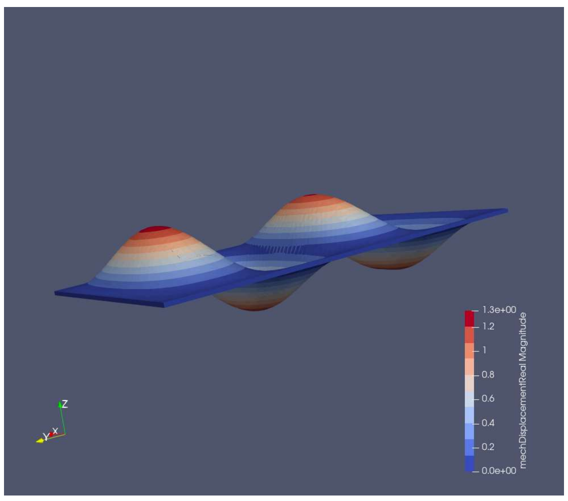
5th mode 295.84 Hz



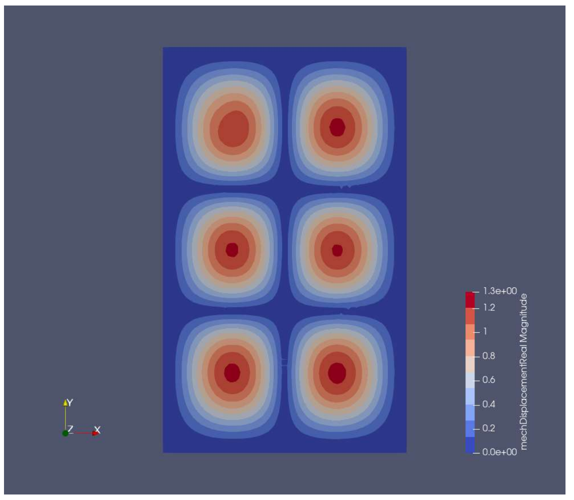
Deformation in the direction of mode 5



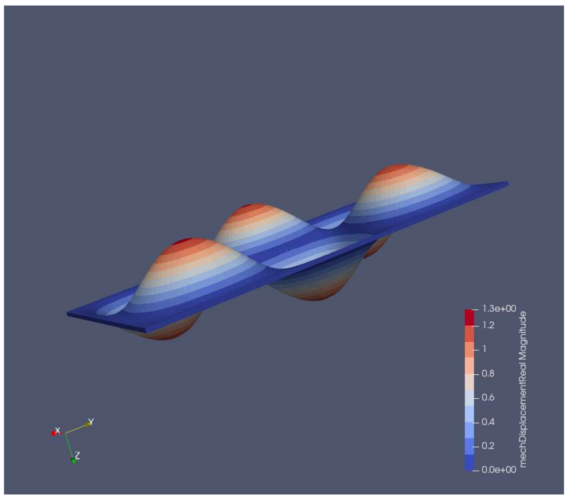
6th mode 328.25 Hz



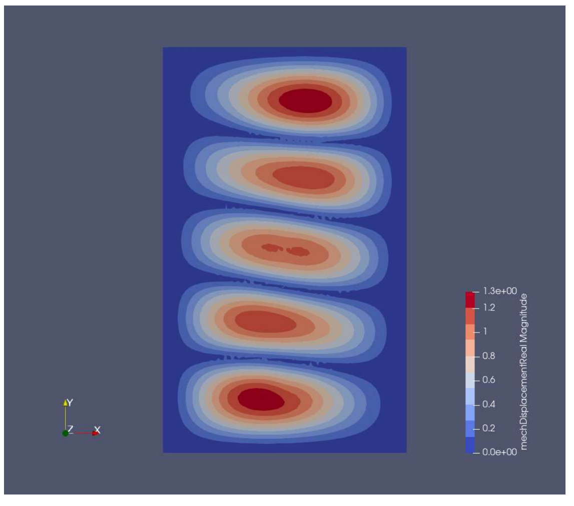
Deformation in the direction of mode 6



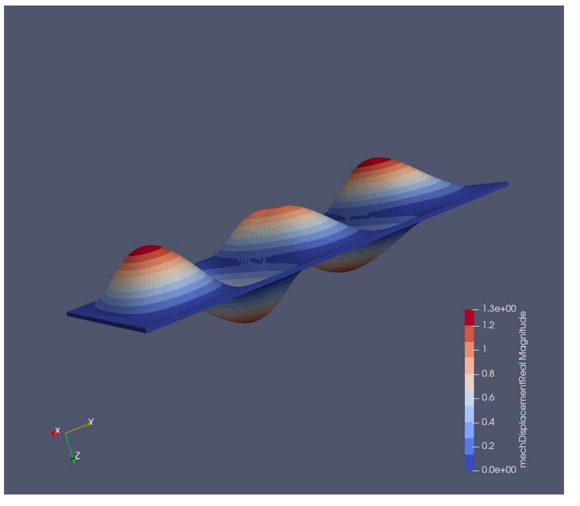
7th mode 365.703 Hz



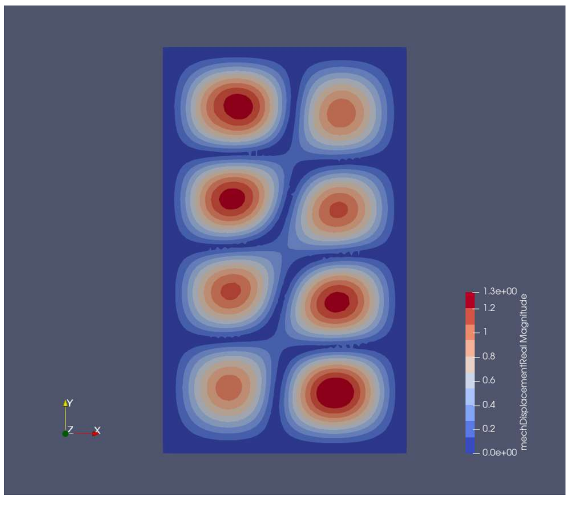
Deformation in the direction of mode 7



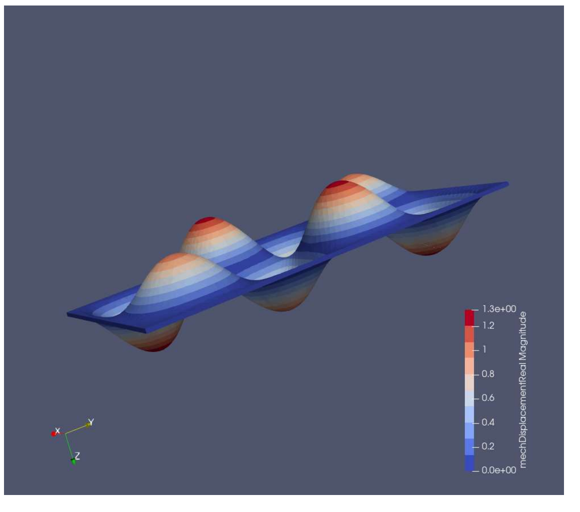
8th mode 465.38 Hz



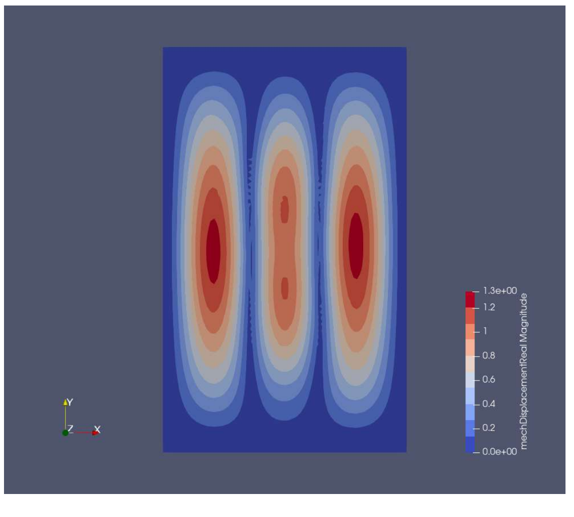
Deformation in the direction of mode 8



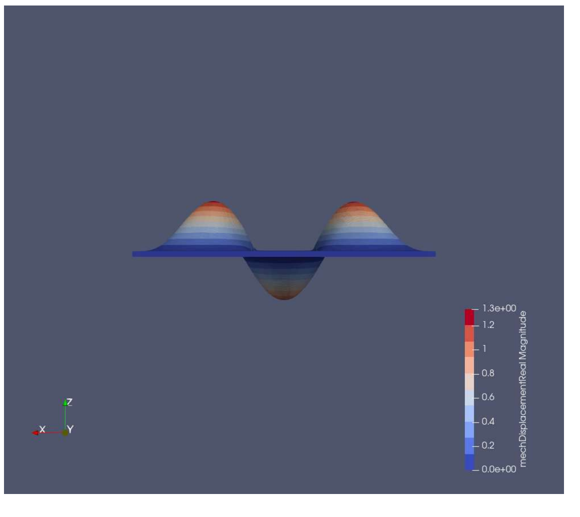
9th mode 468.099 Hz



Deformation in the direction of mode 9



10th mode 484.69 Hz



Deformation in the direction of mode 10

- Consider the piezoelectric patches as sensors. Briefly discuss how we can use them to detect the eigenmodes of the structure. **(1 point)**

Answer: A sensor patch can detect eigenmodes effectively when the mode shape induces deformation in the sensor. For example, the third patch can identify the fifth eigenmode (295.84Hz), while the first and second patches are unable to capture this mode due to their positioning along a nodal line of its shape. Likewise, the first and third patches can strongly identify the sixth eigenmode (328.25z), whereas the second patch cannot, as it is situated on a nodal line of this particular mode. A comprehensive visualization of all eigenmodes for each patch can be plotted for better understanding.

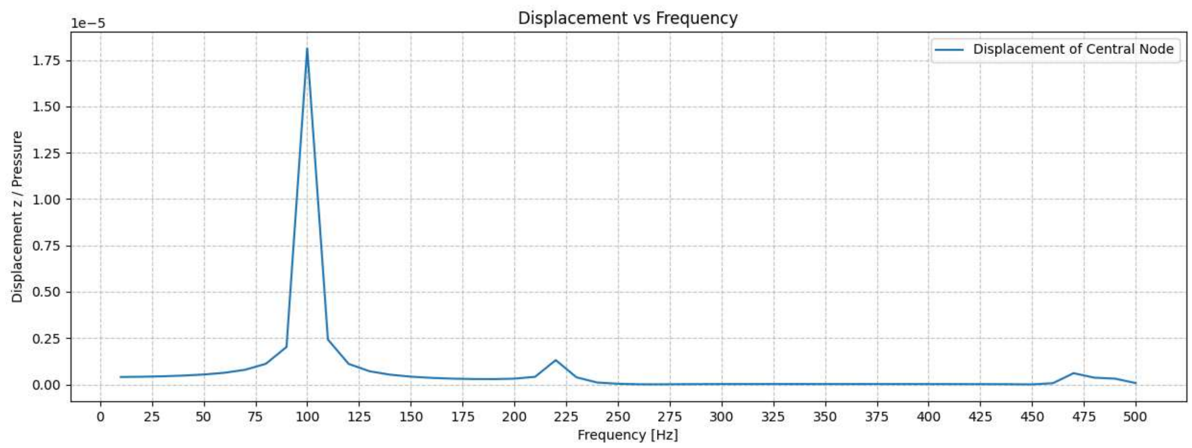
6. Harmonic analysis

- For a harmonic excitation with the pressure **p**, compute the transfer function (up to 500Hz) for the voltage at all 3 patches and the displacement of the central node of the PMMA plate. **(1 point)**

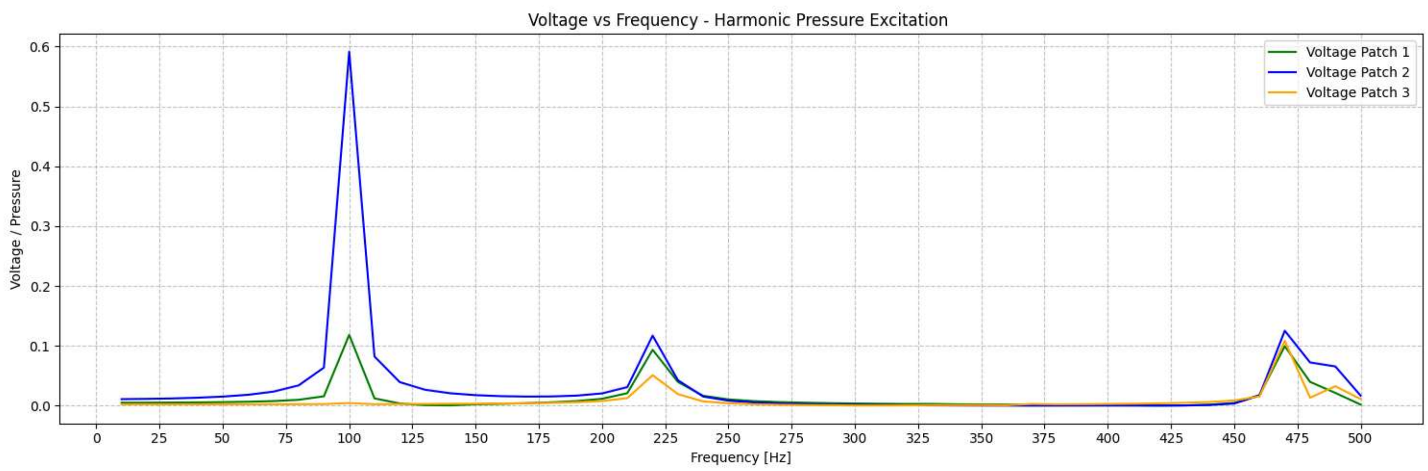
Answer: Harmonic excitation using pressure **p** has been applied, and the corresponding transfer function for voltage across all three patches, as well as the displacement of the central node, has been calculated up to 500Hz. The resulting values have been stored for future plotting. Please refer job3 files for reference

- Plot the transfer functions and explain the peaks observed in the plot **(1 point)**

Answer:



Harmonic displacement

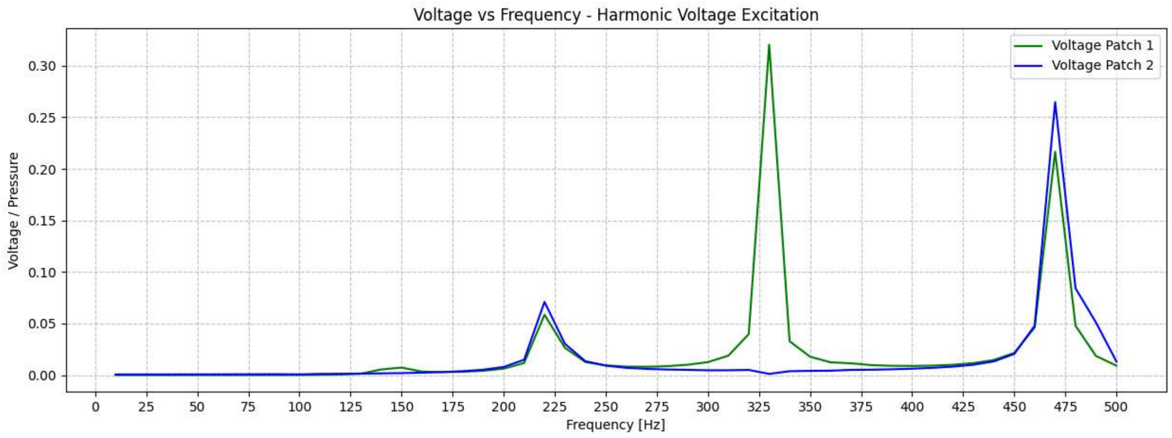


Harmonic pressure excitation

Due to the double symmetry of the harmonic excitation, only the modes that exhibit this symmetry are effectively excited. Specifically, modes 1, 3, 8, and 10 are stimulated, representing the only frequency components where both voltage and displacement transfer functions display peaks. These peaks occur at frequencies of 101.038Hz, 222.07Hz, 465.38Hz, and 484.69Hz. Notably, the third piezoelectric patch lies outside the deformed zone for mode 1, resulting in no observable voltage change at these frequency. At these frequency, patch 2 is positioned at the center of the mode shape, leading to the highest voltage output.

- Instead of harmonic pressure, apply a harmonic voltage of 500V on patch 3 and compute the transfer function (up to 500Hz) for the voltage at patches 1 and 2. Plot the transfer functions and briefly discuss what you observe. **(1.5 points)**

Answer:



Harmonic voltage excitation

These two transfer funtion shows patch 1 and patch 3 can detect same mode shapes. For instance, the first mode has a frequency of 101.038 Hz, but there is no peak for patch 3 because it is positioned outside the deformation zone of first mode shape, preventing it from exciting the plate at this frequency. The second mode can be slightly excited by patch 3, but only the first patch can detect this mode shape, as patch 2 is situated on a nodal line. The third mode shape at 222.07Hz can be significantly excited by patch 3, and both patch 1 and patch 2 can identify this mode shape.

7. Transient analysis

- Calculate the rayleigh damping coefficients α and β for a damping ratio ζ of 0.01 at the eigenmodes 1 and 3 (up to 500 Hz) and use them as damping inputs in the material definition (Refer Mechanic PDE in the CFS user documentation) **(1 point)**

Answer:

The formula for Rayleigh damping is $\zeta = \frac{\alpha}{2} \cdot \omega_i + \beta \cdot \omega_i^2$, where ω_i being eigenfrequency.

The frequency used are $\omega_1 = 101.038 \frac{1}{s}$ and $\omega_3 = 222.07 \frac{1}{s}$.

$\alpha = 1.378$

$\beta = \frac{2 \cdot \zeta}{\omega_1} - \frac{\alpha}{\omega_1^2} = 6.199 \cdot 10^{-5}$

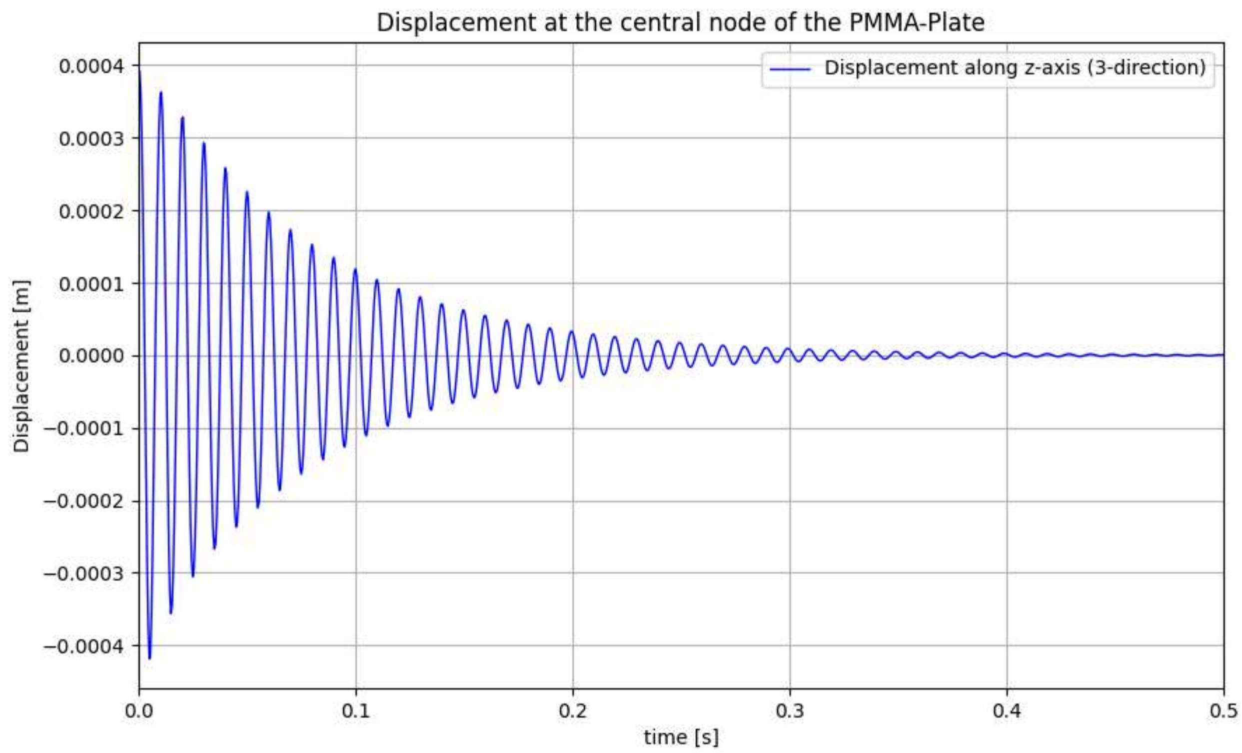
- Set up a transient analysis of the plate from a preloaded configuration and simulate by releasing the load under rayleigh damping for 0.5 seconds with at least 1000 time steps. (Hint: Use the results from the static analysis as the preloaded configuration) **(1.5 points)**

Answer: To initiate a transient analysis, it is essential to have the plate initially configured under a preload, and the deformation results from static Analysis in sequenceStep 1.

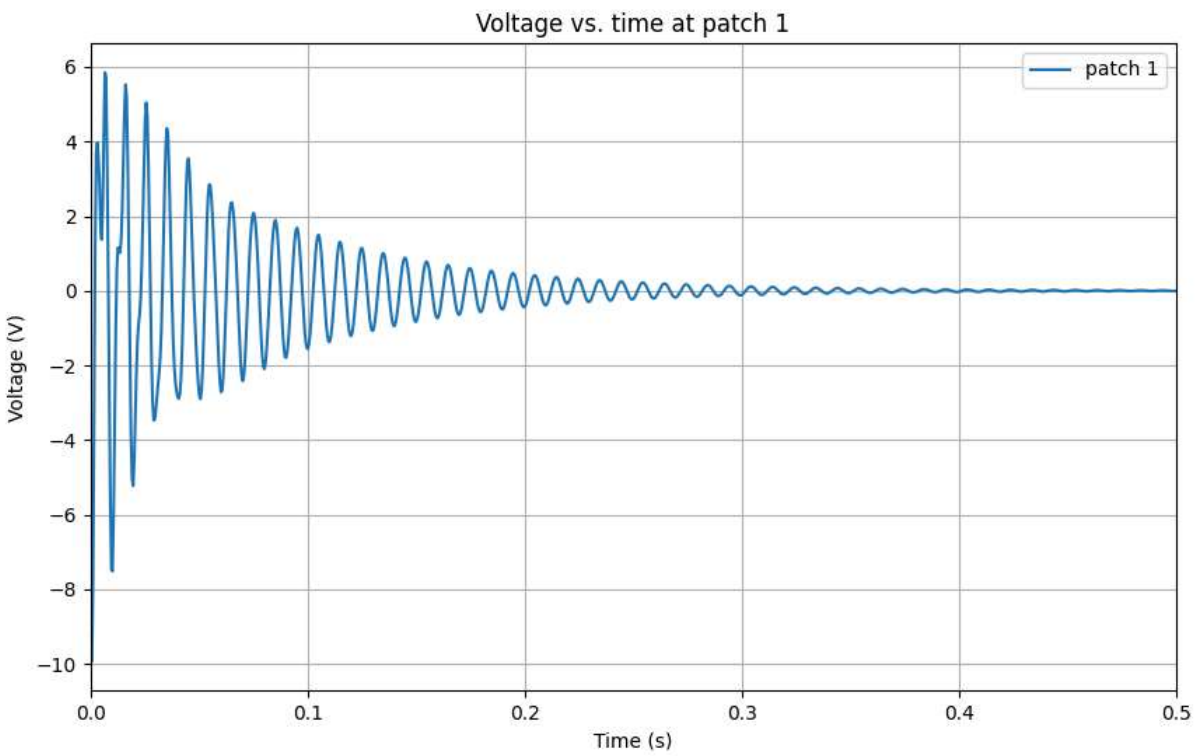
- Observe the damped response by plotting the displacement of the central node and the voltage at patch 1. Briefly discuss the frequency components present in the response and why? (Hint: FFT) **(1.5 points)**

Answer:

The displacement of the central node

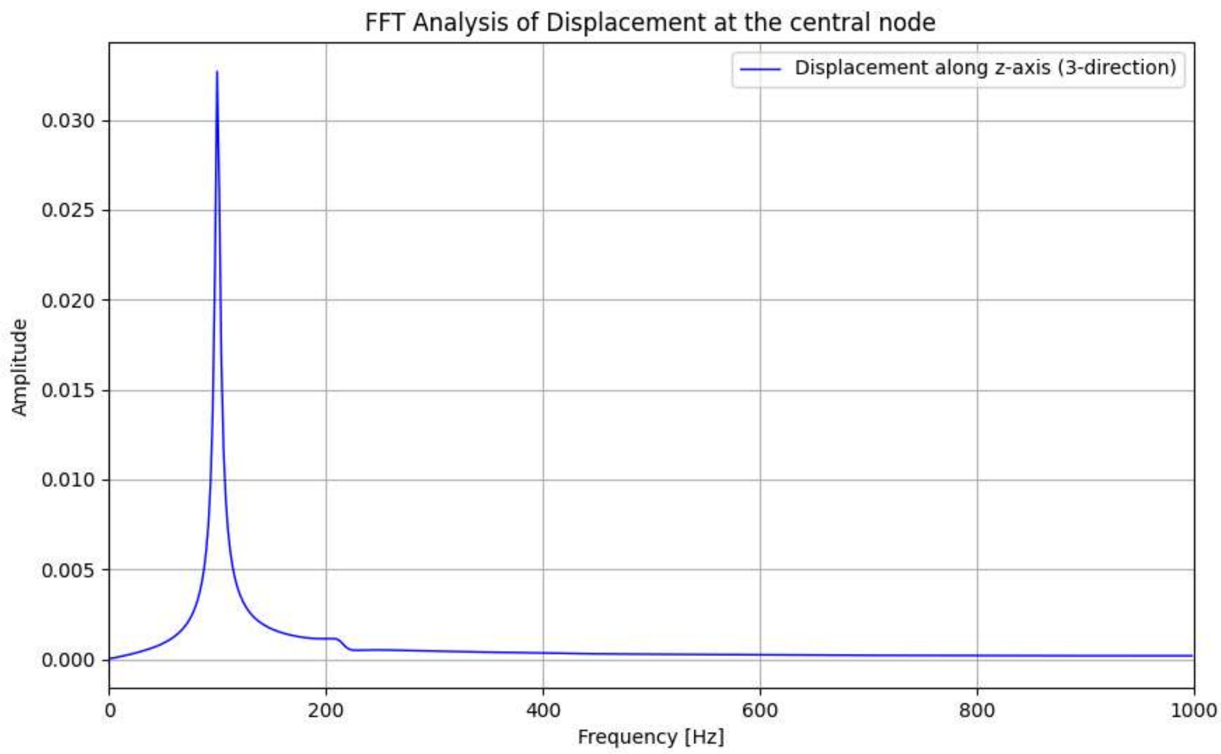


Voltage at patch 1

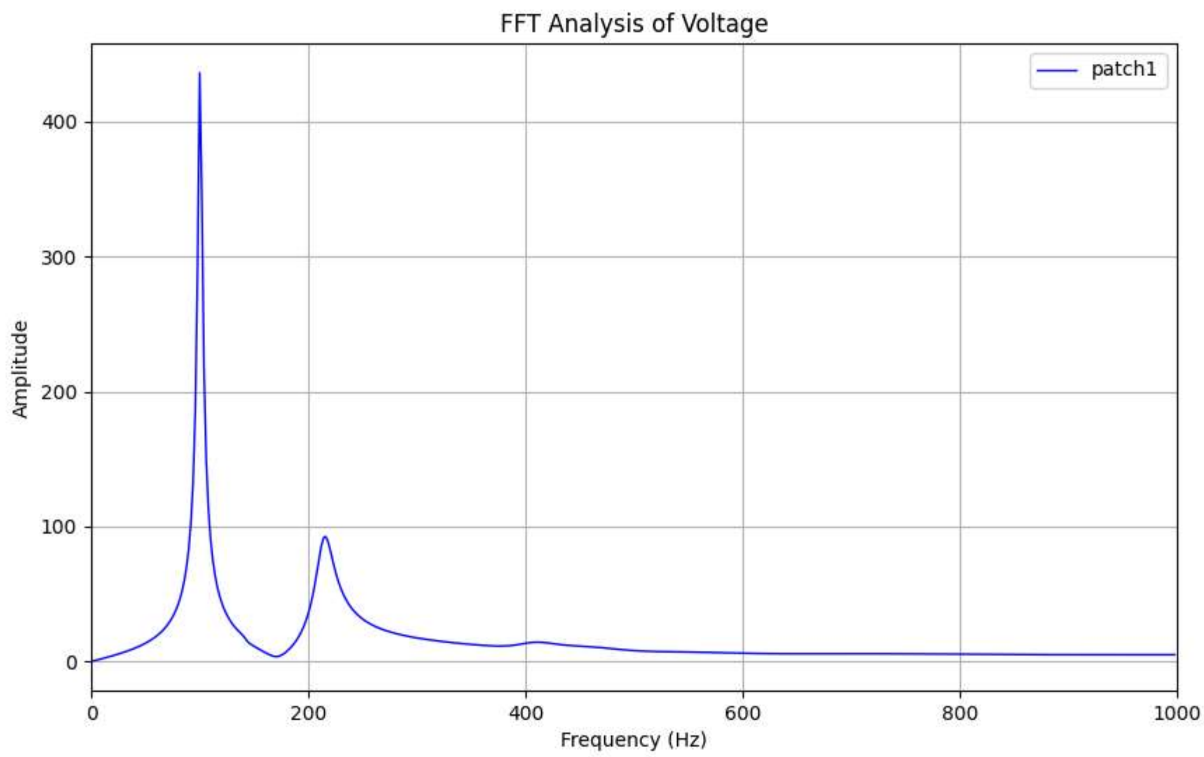


The system experiences oscillations in both displacement and voltage; however, it ultimately stabilizes at zero in an equilibrium state. The oscillations occur due to the dynamic response of the piezoelectric material to external forces. This equilibrium state suggests that the piezoelectric patches have responded to the applied loads but the net effect results in a balanced, leading to zero displacement and voltage.

FFT central node displacement



FFT Voltage



Conducting a Fourier analysis on the time signal reveals that the frequency components in the signals correspond to the first and third modes. As a pressure is initially applied to the plate during the transient analysis, the force field exhibits double symmetry, resulting in the excitation of only the first and third modes, similar to the harmonic. The proportion of each frequency is more significant for lower eigenfrequencies, but due to the detection of the third eigenmode by the first patch, the peaks appear.