TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

ILSB Institute of Lightweight Design and Structural Biomechanics

Tissue Biomechanics UE 317.523

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Tutorial 2: Bone anisotropic elasticity

Total points: 10

Consider a cylindrical bone specimen, originated from the femoral neck with a nominal diameter of 40 mm. The thickness of the cortical bone is 2 mm and is also considered to be uniform across the length (L) of the bone specimen. The remaining space (light gray in figure 1) is trabecular bone.

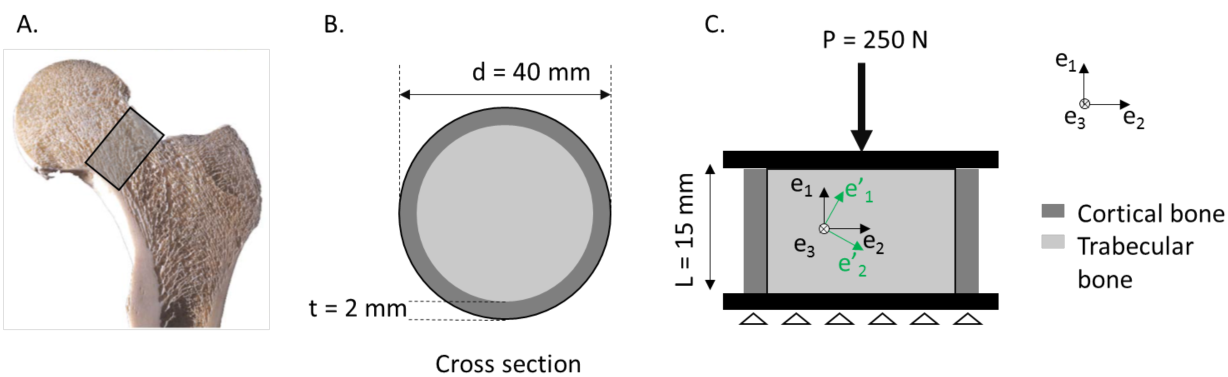

Figure 1: Anatomy of the proximal femur (A), abstract model of the femoral neck (B), abstracted loading case (C) during normal stance.

Assuming the cortical bone to be a transversely isotropic material the constitutive law relating stress and strain in the elastic regime is:

$$
\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{pmatrix}
=
\begin{pmatrix}
\frac{1}{E_1} & \frac{-\nu_1}{E_2} & \frac{-\nu_1}{E_2} & 0 & 0 & 0 \\
\frac{-\nu_1}{E_1} & \frac{1}{E_2} & \frac{-\nu_2}{E_2} & 0 & 0 & 0 \\
\frac{-\nu_1}{E_1} & \frac{-\nu_2}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2\mu_2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2\mu_1} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2\mu_1}
\end{pmatrix}
\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}
$$

With E_1 = 20 GPa, E_2= 15 GPa, μ_1 = 6 GPa μ_2 = 9 GPa, $\nu_1 = \nu_2$ = 0.3.

The constitutive law for trabecular bone, assuming an apparent orthotropic behavior is given by:

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{pmatrix} = \begin{pmatrix} \frac{1}{E\rho^k m_1^{2l}} & \frac{-v}{E\rho^k m_1^l m_2^l} & \frac{-v}{E\rho^k m_1^l m_3^l} & 0 & 0 & 0 \\ \frac{-v}{E\rho^k m_1^l m_2^l} & \frac{1}{E\rho^k m_2^{2l}} & \frac{-v}{E\rho^k m_2^l m_3^l} & 0 & 0 & 0 \\ \frac{-v}{E\rho^k m_1^l m_3^l} & \frac{-v}{E\rho^k m_2^l m_3^l} & \frac{1}{E\rho^k m_3^{2l}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\mu\rho^k m_2^l m_3^l} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2\mu\rho^k m_1^l m_3^l} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\mu\rho^k m_1^l m_2^l} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

Where the apparent density is ρ = 0.34. The principal values of the MIL tensor are m_1 = 1.25, m_2 = 1, m_3 = 0.8 and the constants *k* and *l* have the values *k* = 1.45 and *l* = 1.3. The isotropic elastic modulus of trabecular bone tissue is *E* = 7.5 GPa, the shear modulus μ = 3 GPa and Poisson's ratio *v* = 0.3. Moreover, the principal direction of the trabeculae is along the e'1 axis, rotated by 15° (degrees) clockwise as shown in Figure 1C; axes highlighted in green colour.

Assuming a uniaxial compressive load of 250 N (as shown in Figure 1C) results in a stress field of the form:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

i.e. normal stress along the e_1 direction:

Given this load case:

A. Compute the stress and strain encountered by the cortical and the trabecular bone compartments (Hint: start considering the strain encountered by each compartment). (2.0 pt)

B. Compute the total load carried by the cortical and trabecular bone compartment. (2.0 pt)

C. What would be the stress encountered by the cortical bone if the trabecular bone was completely removed? (1.5 pt)

D. To simulate aging let the diameter of the femoral neck increase with a concomitant thinning of neck wall thickness as well as a loss of trabecular bone. Assuming a cylindrical bone geometry at the femoral neck, with nominal outer diameter d_{aged} = 46 mm, cortical bone thickness of t_{aged} = 0.8 mm and trabecular bone apparent density ρ = 0.2, compute the stress and strain encountered by the cortical and the trabecular bone compartments. (2.5 pt)

E. Defining a fracture risk factor for cortical bone as:

$$RF = \frac{Max.\,stress\ (\mathrm{MPa})}{Yield\ \ stress\ (\mathrm{MPa})}$$

where the yield stress of the aged bone specimen is 50 MPa, what is RF in this case? (2.0 pt)