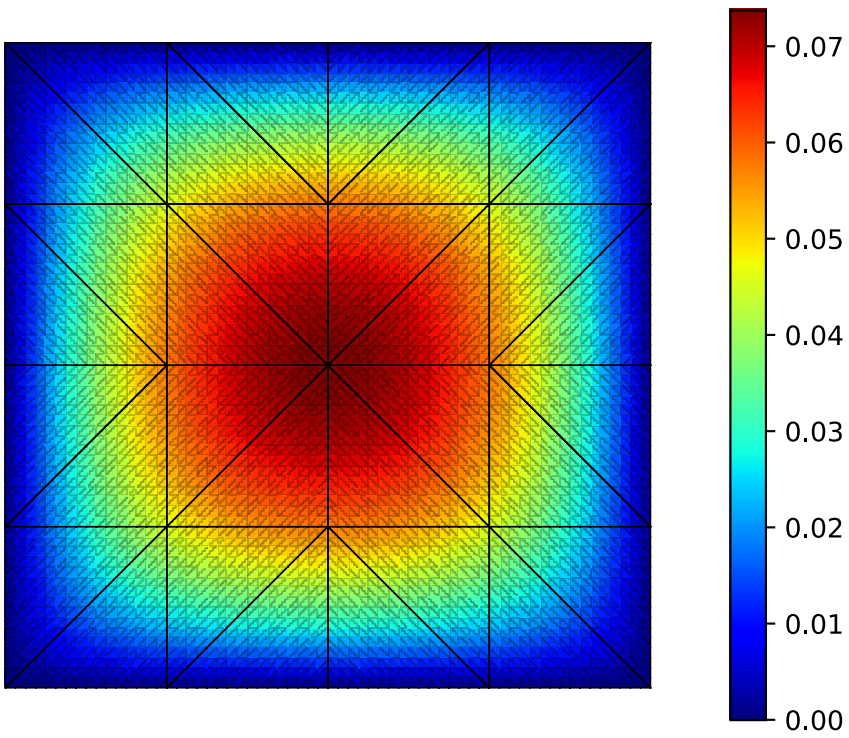
In [17]:

```
"""Discontinuous Galerkin method."""

from skfem import *
from skfem.helpers import grad, dot, jump
from skfem.models.poisson import laplace, unit_load

m = MeshTri.init_sqsymmetric().refined()
e = ElementTriDG(ElementTriP4())
alpha = 1e-3

ib = Basis(m, e)
bb = FacetBasis(m, e)
fb = [InteriorFacetBasis(m, e, side=i) for i in [0, 1]]

@BilinearForm
def dgform(u, v, p):
    ju, jv = jump(p, u, v)
    h = p.h
    n = p.n
    return ju * jv / (alpha * h) - dot(grad(u), n) * jv - dot(grad(v), n) * ju

@BilinearForm
def nitscheform(u, v, p):
    h = p.h
    n = p.n
    return u * v / (alpha * h) - dot(grad(u), n) * v - dot(grad(v), n) * u

A = asm(laplace, ib)
B = asm(dgform, fb, fb)
C = asm(nitscheform, bb)
b = asm(unit_load, ib)

x = solve(A + B + C, b)

M, X = ib.refinterp(x, 4)

def visualize():
    from skfem.visuals.matplotlib import plot, draw
    %config InlineBackend.figure_formats = ['svg']

    ax = draw(M, boundaries_only=True)
    return plot(M, X, shading="gouraud", ax=ax, colorbar=True)

if __name__ == "__main__":
    visualize().show()

```



The Github repository of *gdmcbain/fenics-tuto-in-skfem* contains eight exercises from the FEnics tutorial book. FEnics's great popularity no doubts stems largely from the book by Langtangen and Logg, entitled *Solving PDEs in Python: The FEnics Tutorial I*, a full-length book about using FEnics for finite element solutions of PDEs. This book can be accessed on line or downloaded for free. Since FEnics is such a popular package, it is natural for someone to wonder if the new, little-known and somewhat brash program scikit-fem could reproduce the results of the FEnics tutorials. Below are six of the eight problems of the collection. (Note: problems 2 and 6 could not be attempted because they require a commercial program for execution.)

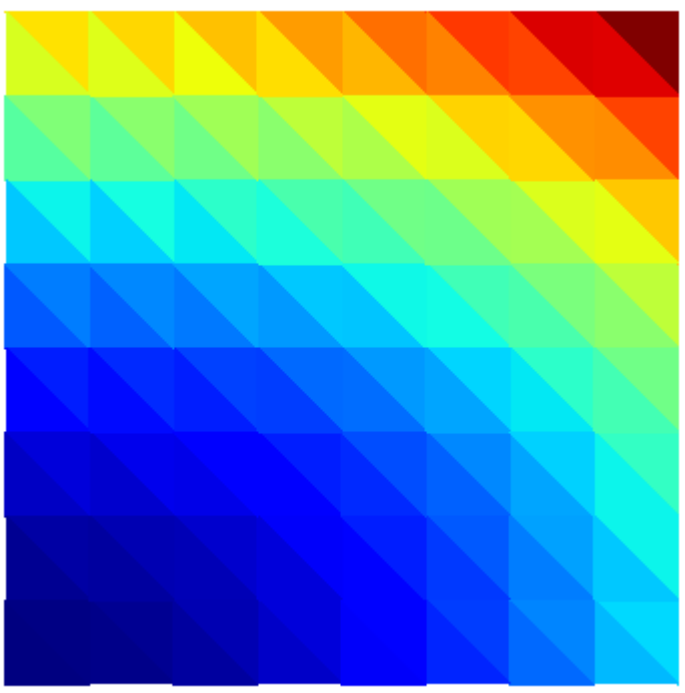
FEnics exercise #1 below, executed by scikit-fem.

In [1]:

```
import numpy as np

from skfem import *
from skfem.models.poisson import laplace, unit_load, mass

mesh = MeshTri().refined(3)
mesh

V = InteriorBasis(mesh, ElementTriP1())

u_D = 1 + [1, 2] @ mesh.p ** 2

boundary = mesh.boundary_nodes()

u = np.zeros_like(u_D)
u[boundary] = u_D[boundary]

a = asm(laplace, V)
L = -6.0 * asm(unit_load, V)

u = solve(*condense(a, L, u, D=boundary))

ax = mesh.plot(u)
ax.get_figure().savefig("poisson.svg")

mesh.save("fenics01.ply")

error = u - u_D
print("error_L2  =", np.sqrt(error.T @ asm(mass, V) @ error))
print("error_max =", np.linalg.norm(error, np.inf))

```

```
Warning: PLY doesn't support 64-bit integers. Casting down to 32-bit.
```

```
error_L2  = 3.090730095650652e-16
error_max = 1.1102230246251565e-15
```



Type *Markdown* and LaTeX: α^2

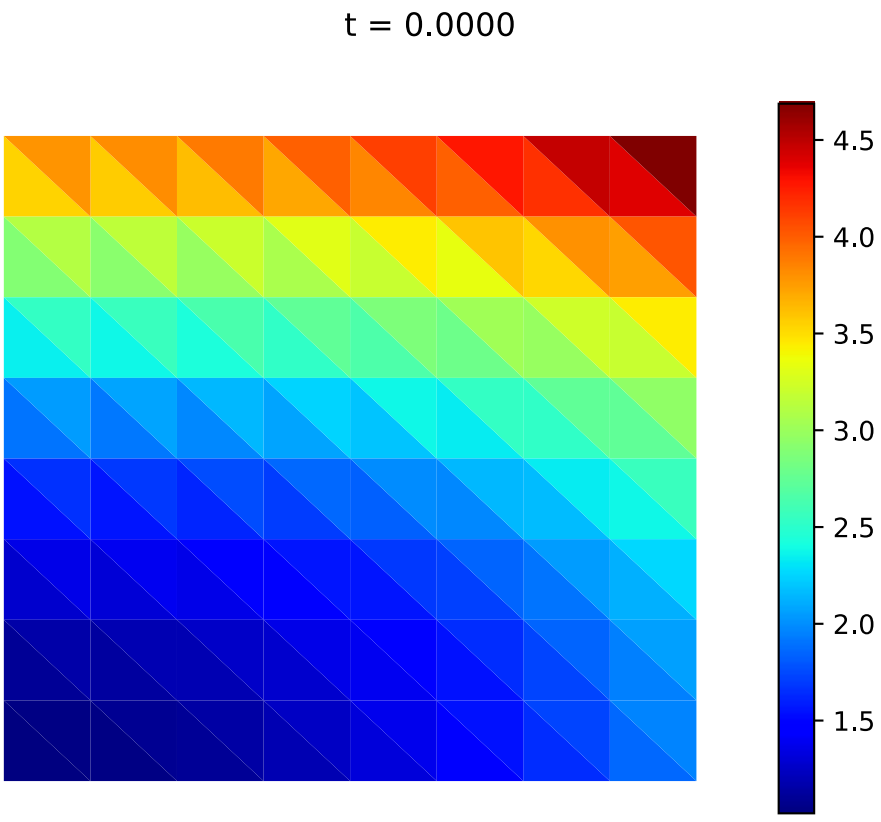
FEnics exercise #3 below, executed by scikit-fem.

In [2]:

```
from matplotlib.pyplot import subplots, pause
# %matplotlib qt
import numpy as np
%config InlineBackend.figure_formats = ['svg']

from skfem import *
from skfem.models.poisson import laplace, mass, unit_load

alpha = 3.0
beta = 1.2

nx = ny = 2 ** 3

time_end = 2.0
```

```
num_steps = 10
dt = time_end / num_steps

mesh = (
    MeshLine(np.linspace(0, 1, nx + 1)) * MeshLine(np.linspace(0, 1, ny + 1))
).to_meshtri()
basis = InteriorBasis(mesh, ElementTriP1())

boundary = basis.get_dofs().all()
interior = basis.complement_dofs(boundary)

M = asm(mass, basis)
A = M + dt * asm(laplace, basis)
f = (beta - 2 - 2 * alpha) * asm(unit_load, basis)

fig, ax = subplots()


def dirichlet(t: float) -> np.ndarray:
    return 1.0 + [1.0, alpha] @ mesh.p ** 2 + beta * t


t = 0.0
u = dirichlet(t)

zlim = (0, np.ceil(1 + alpha + beta * time_end))

for i in range(num_steps + 1):

    ax.cla()
    ax.axis("off")
    fig.suptitle("t = {:.4f}".format(t))
    mesh.plot(u, ax=ax, zlim=zlim)
    if t == 0.0:
        fig.colorbar(ax.get_children()[0])
    fig.show()
    pause(1.0)

    t += dt
    b = dt * f + M @ u

    u_D = dirichlet(t)
    u = solve(*condense(A, b, u_D, D=boundary))
    error = np.linalg.norm(u - u_D)
    print("t = %.2f: error = %.3g" % (t, error))

```

```
C:\Users\gary\AppData\Local\Temp/ipykernel_7460/3832573378.py:50: UserWarning: Matplotlib is curren
tly using module://matplotlib_inline.backend_inline, which is a non-GUI backend, so cannot show the
figure.
  fig.show()
```




```
t = 0.20: error = 9.44e-15
t = 0.40: error = 1.25e-14
t = 0.60: error = 1.45e-14
t = 0.80: error = 1.7e-14
t = 1.00: error = 1.68e-14
t = 1.20: error = 1.89e-14
t = 1.40: error = 2.01e-14
t = 1.60: error = 2.01e-14
t = 1.80: error = 2.16e-14
t = 2.00: error = 2.34e-14
t = 2.20: error = 2.46e-14
```

FEnics exercise #4 below, executed by scikit-fem.

In [3]:

```python
from pathlib import Path

from matplotlib.pyplot import subplots, pause
import numpy as np

from skfem import *
from skfem.models.poisson import laplace, mass

a = 5.0

nx = ny = 30

time_end = 2.0
num_steps = 50
dt = time_end / num_steps

mesh = (
    MeshLine(np.linspace(-2, 2, nx + 1)) * MeshLine(np.linspace(-2, 2, ny + 1))
).to_meshtri()
basis = InteriorBasis(mesh, ElementTriP1())

boundary = basis.get_dofs().all()
interior = basis.complement_dofs(boundary)

M = asm(mass, basis)
A = M + dt * asm(laplace, basis)

fig, ax = subplots()

t = 0.0
u = np.exp(-a * (np.sum(mesh.p ** 2, axis=0)))  # initial condition, P1 only

output_dir = Path("heat_gaussian")
try:
    output_dir.mkdir()
except FileExistsError:
    pass

for i in range(num_steps + 1):

    ax.cla()
    ax.axis("off")
    fig.suptitle("t = {:.4f}".format(t))
    mesh.plot(u, ax=ax, zlim=(0, 1))
    if t == 0.0:
        fig.colorbar(ax.get_children()[0])
        fig.savefig("initial.png")
    fig.show()
    pause(0.01)

    t += dt
    b = M @ u

    u = solve(*condense(A, b, D=boundary))

    mesh.save(str(output_dir.joinpath(f"solution{i:06d}.msh")), {"temperature": u})

```

```
C:\Users\gary\AppData\Local\Temp/ipykernel_7460/3973729471.py:48: UserWarning: Matplotlib is curren
tly using module://matplotlib_inline.backend_inline, which is a non-GUI backend, so cannot show the
figure.
  fig.show()
```