In [1]:
```
%autosave 0
```

```
Autosave disabled
```

Chapter 29: Some Classical Differential Equations. Introduction to four famous and often used ODEs: Chebyshev, Hermite, Laguerre, and Legendre.

Note: When transcribing text for entry into Wolfram Alpha the "fenceposts" !| ... |! are excluded from the text.

29.1 Let $n = 2$ in the Hermite DE. Use the Rodrigues formula to find the polynomical solution. (The Hermite becomes $y'' - 2xy' + 4y = 0$.)

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| y'' - 2 x y' + 4 y = 0 |!

and the answer returned is:

$$y(x) = c_2 \left(\sqrt{\pi}\,(2x^2 - 1)\,\mathrm{erfi} - 2e^{x^2} x\right) + c_1 \left(x^2 - \frac{1}{2}\right)$$

Note: Wolfram Alpha identifies the equation as a Hermite equation.

Instead of solving the Hermite equation itself, the problem question was actually requesting the Hermite polynomial of degree 2. That can be obtained from sympy as shown in the cell below.

In [20]:
```
import sympy as sp
import numpy as np
from scipy import special as spe
from sympy import polys as po

po.hermite_poly(2)
```

Out[20]: $4x^2 - 2$

29.20 **a** : Determine whether the five following differential equations have two polynomial solutions; if they do, give the degrees of the solutions: *a)* $(1 - x^2)y'' + 5xy' - 5y = 0$

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| (1 - x^2) y'' + 5 x y' - 5 y = 0 |!

and the answer returned is:

$$y(x) = \frac{c_2 \sqrt{x^2 - 1}\; {}_2F_1\left(-\frac{5}{2}, -\frac{1}{2}; \frac{1}{2}; x^2\right)}{\sqrt{1 - x^2}} + c_1 x$$

Note: Wolfram Alpha identifies the equation as a Sturm-Liouville equation.

29.20 **b** : Determine whether the five following differential equations have two polynomial solutions; if they do, give the degrees of the solutions: *b)* $(1 - x^2)y'' + 8xy' - 18y = 0$

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| (1 - x^2) y'' + 8 x y' - 18 y = 0 |!

and the answer returned is:

$$y(x) = c_2\, x\,(5x^2 + 3) + c_1\,(x + 5)\,(x - 1)^5$$

Note: Wolfram Alpha identifies the equation as a Sturm-Liouville equation.

29.20 **c** : Determine whether the five following differential equations have two polynomial solutions; if they do, give the degrees of the solutions: *c)* $(1 - x^2)y'' + 2xy' + 10y = 0$

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| (1 - x^2) y'' + 2 x y' + 10 y = 0 |!

and the answer returned is:

$$y(x) = c_1\, x\,(x^2 - 1)^2 + c_2\,(x^2 - 1)\left(\frac{15}{2}\, x\,(x^2 - 1)\right.$$

$$\left.(\log(1 - x) - \log(x + 1)) + \frac{(1 - x^2)(-15x^4 + 25\,x^2 - 8)}{(x^2 - 1)^2}\right)$$

Note: Wolfram Alpha identifies the equation as a Sturm-Liouville equation.

29.20 **d** : Determine whether the five following differential equations have two polynomial solutions; if they do, give the degrees of the solutions: *d)* $(1 - x^2)y'' + 14xy' - 56y = 0$

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| (1 - x^2) y'' + 14 x y' - 56 y = 0 |!

and the answer returned is:

$$y(x) = \frac{c_1 (x^2 - 1)^{7/2} (1 - x)^{9/2}}{(x + 1)^{7/2}} +$$
$$\frac{c_2 (x^2 - 1)^{7/2} ((x^4 + 7x^2 + 7) x^3 + x) (1 - x)^{9/2}}{(x - 1)^8 (x + 1)^{7/2}}$$

Note: Wolfram Alpha identifies the equation as a Sturm-Liouville equation.

29.20 **e** : Determine whether the five following differential equations have two polynomial solutions; if they do, give the degrees of the solutions: $e)\ (1 - x^2)y'' + 12xy' - 22y = 0$

Note: This equation can be input into Wolfram Alpha. The entry line is:

!| (1 - x^2) y'' + 12 x y' - 22 y = 0 |!

and the answer returned is:

$$y(x) = c_1 \left(x^2 + \frac{1}{11}\right) +$$
$$c_2 ((7x^8 - 55 x^6 + 198 x^4 - 462 x^2 + 1155) x^3 + 693 x)$$

Note: Wolfram Alpha identifies the equation as a Sturm-Liouville equation.