Trefethen p15 to p27.

This notebook showcases the second thirteen problems in Trefethen's classic book *Spectral Methods in MATLAB*. These problems have been ported to Python by Praveen Chandrashekar. Later problems in the set will have been ported to Python by Orlando Camargo Rodríguez.

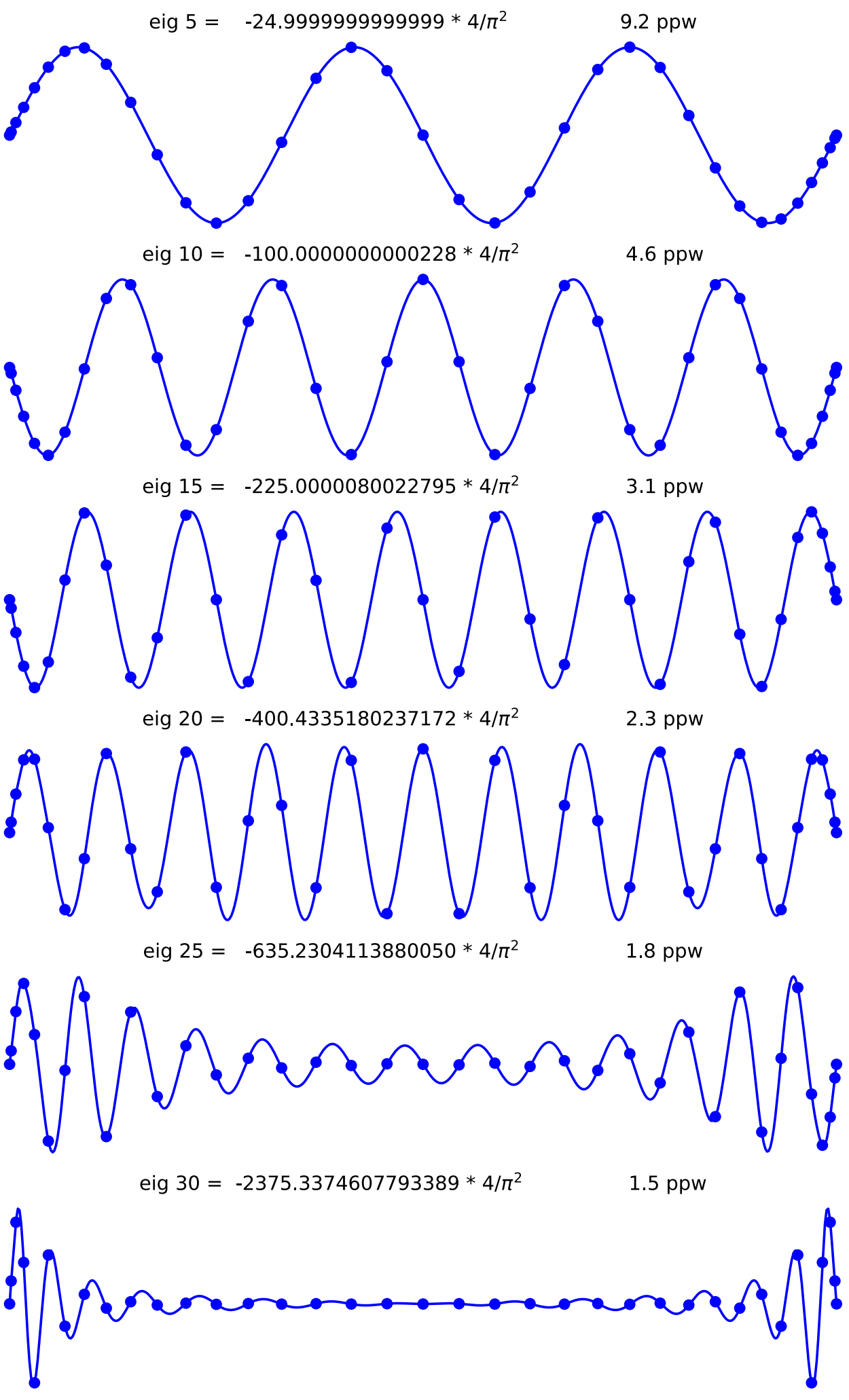
Program 15 : Solve eigenvalue BVP

In [38]:
```
%matplotlib inline
%config InlineBackend.figure_format='svg'
from numpy import dot,argsort,linspace,shape,zeros,polyval,polyfit,pi,real
#from chebPy import cheb
from scipy.linalg import solve,eig
from matplotlib.pyplot import figure,subplot,plot,title,axis
```

In [2]:
```
from numpy import pi,cos,arange,ones,tile,dot,eye,diag

def cheb(N):
    '''Chebushev polynomial differentiation matrix.
       Ref.: Trefethen's 'Spectral Methods in MATLAB' book.
    '''
    x      = cos(pi*arange(0,N+1)/N)
    if N%2 == 0:
        x[N//2] = 0.0 # only when N is even!
    c      = ones(N+1); c[0] = 2.0; c[N] = 2.0
    c      = c * (-1.0)**arange(0,N+1)
    c      = c.reshape(N+1,1)
    X      = tile(x.reshape(N+1,1), (1,N+1))
    dX     = X - X.T
    D      = dot(c, 1.0/c.T) / (dX+eye(N+1))
    D      = D - diag( D.sum(axis=1) )
    return D,x
```

In [40]:

```
N = 36
D,x = cheb(N)
D2 = dot(D,D)
D2 = D2[1:N,1:N]

lam,V = eig(D2)
ii = argsort(-lam)
lam = real(lam[ii])
V = V[:,ii]

fig = figure(figsize=(10,15))
for j in range(5,35,5):
    lv = shape(V)[0]+2
    u = zeros(lv)
    u[1:lv-1] = V[:,int(j)]
    subplot(6,1,j//5)
    plot(x,u,'bo')
    xx = linspace(-1.0,1.0,501)
    uu = polyval(polyfit(x,u,N),xx)     # interpolate grid data
    s = 'eig %d = %20.13f * 4/$\pi^2$' %(j,lam[j-1]*4/pi**2)
    s = s + '\t\t %4.1f ppw' % (4*N/(pi*j))
    title(s)
    plot(xx,uu,'b')
    axis('off')
```




Program 16 : Poisson equation on [-1,1]×[-1,1] with $u = 0$ on boundary

Solve the following Poisson problem

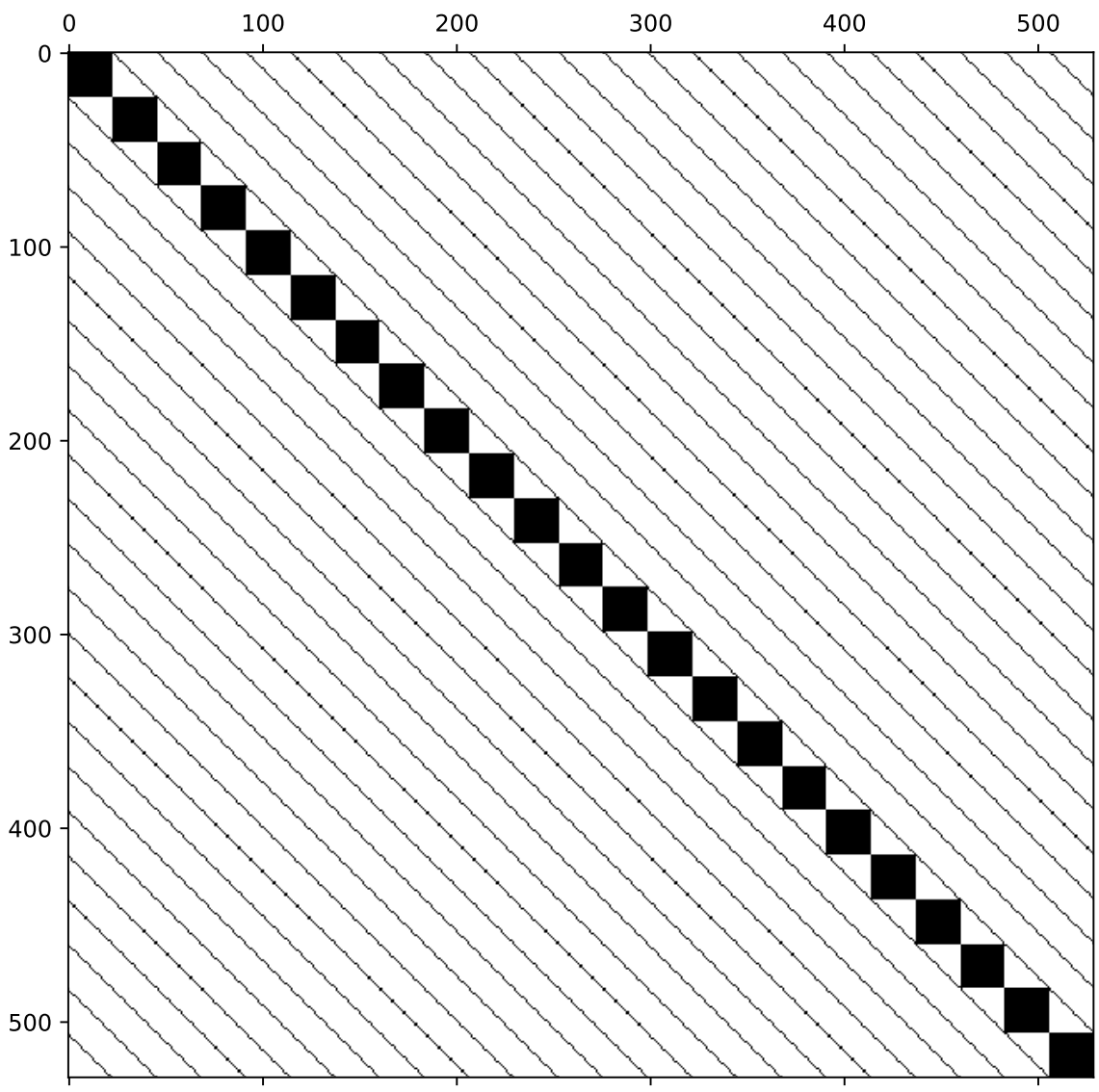
$$u_{xx} + u_{yy} = 10\sin(8x(y-1)), \qquad -1 < x, y < 1, \qquad u = 0 \quad \text{on boundary}$$

In [4]:

```
%matplotlib inline
%config InlineBackend.figure_format='svg'
#from chebPy import cheb
from numpy import meshgrid,sin,dot,eye,kron,zeros,reshape,linspace
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.pyplot import figure,subplot,plot,title,axis,xlabel,ylabel,spy
from matplotlib import cm
from scipy.linalg import solve
from scipy.interpolate import interp2d

```

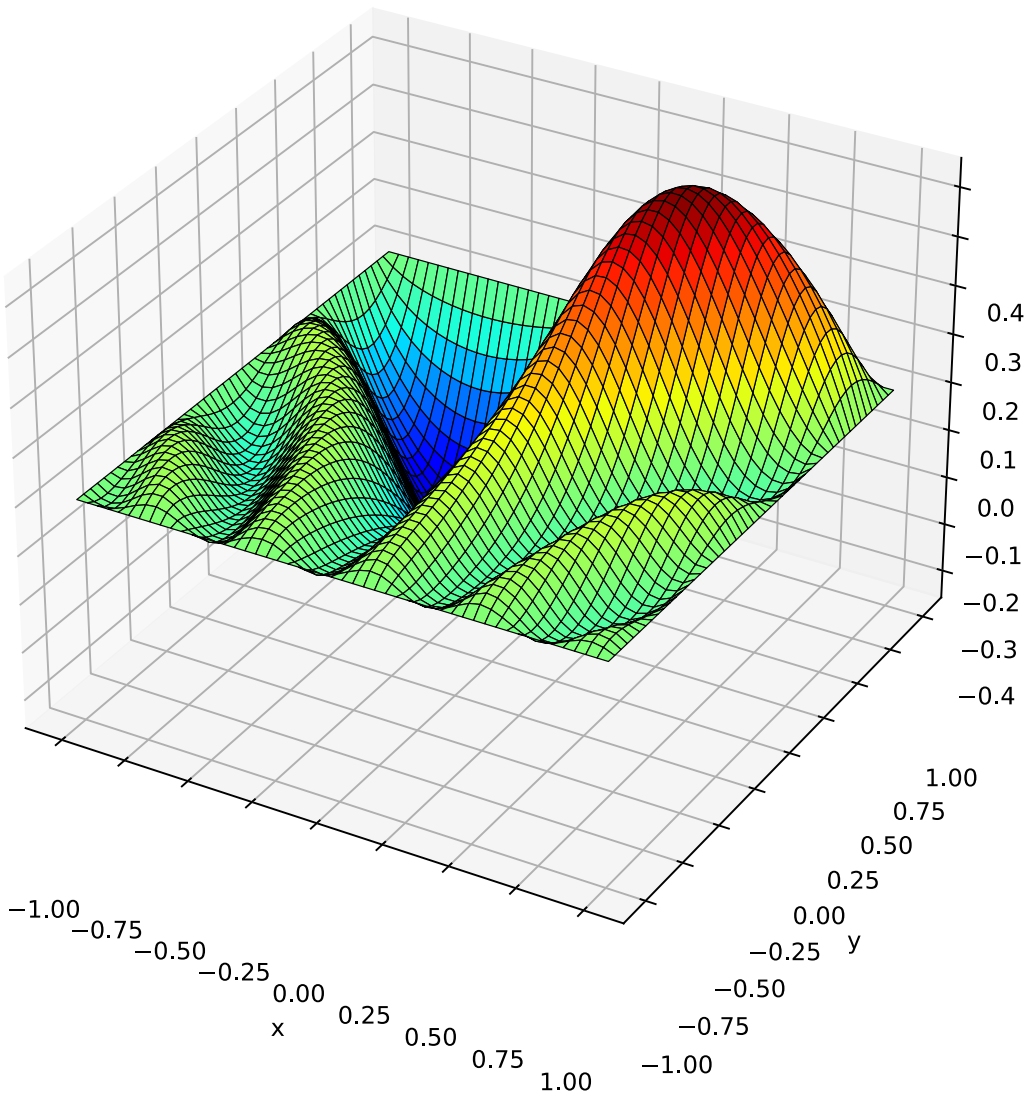
In [5]:
```
N = 24; D,x = cheb(N); y = x;
xx,yy = meshgrid(x[1:N],y[1:N])
xx = reshape(xx,(N-1)**2)
yy = reshape(yy,(N-1)**2)
f = 10*sin(8*xx*(yy-1))
D2 = dot(D,D); D2 = D2[1:N,1:N]; I = eye(N-1)
L = kron(I,D2) + kron(D2,I)
# Plot sparsity pattern
figure(figsize=(8,8)), spy(L)
# Solve Lu=f
u = solve(L,f)
# Convert 1-d vectors to 2-d
uu = zeros((N+1,N+1)); uu[1:N,1:N] = reshape(u,(N-1,N-1))
[xx,yy] = meshgrid(x,y)
value = uu[N//4,N//4]

# Interpolate to finer mesh just for visualization
f = interp2d(x,y,uu,kind='cubic')
xxx = linspace(-1.0,1.0,50)
uuu = f(xxx,xxx)
fig = figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')
X,Y = meshgrid(xxx,xxx)
ax.plot_surface(X,Y,uuu,rstride=1,cstride=1,cmap=cm.jet,edgecolor='black', linewidth=0.5)
title("$u(2^{-1/2},2^{-1/2})$="+str(value))
xlabel("x"); ylabel("y");

```

```
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\4152483477.py:18: DeprecationWarning: `interp2d` is deprecated!
`interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

For legacy code, nearly bug-for-bug compatible replacements are
`RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
scattered 2D data.

In new code, for regular grids use `RegularGridInterpolator` instead.
For scattered data, prefer `LinearNDInterpolator` or
`CloughTocher2DInterpolator`.

For more details see
`https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  f = interp2d(x,y,uu,kind='cubic')
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\4152483477.py:20: DeprecationWarning:         `interp2d` is depr
ecated!
        `interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

        For legacy code, nearly bug-for-bug compatible replacements are
        `RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
        scattered 2D data.

        In new code, for regular grids use `RegularGridInterpolator` instead.
        For scattered data, prefer `LinearNDInterpolator` or
        `CloughTocher2DInterpolator`.

        For more details see
        `https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  uuu = f(xxx,xxx)
```




$u(2^{-1/2}, 2^{-1/2})$=0.32071594510757084

In [ ]:




Program 17 : Hemholtz equation

$$u_{xx} + u_{yy} + k^2 u = f, \qquad \text{on} \qquad [-1, 1] \times [-1, 1]$$

A minor modification of p16 to solve such problem for the particular choices as follows:

$$k = 9, \qquad f(x, y) = \exp(-10[(y - 1)^2 + (x - 1/2)^2])$$

In [7]:

```
%matplotlib inline
%config InlineBackend.figure_format='svg'
#from chebPy import cheb
from numpy import meshgrid,sin,dot,eye,kron,zeros,reshape,exp,linspace
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.pyplot import figure,subplot,plot,title,axis,xlabel,ylabel,contour
from matplotlib import cm
from scipy.linalg import solve
from scipy.interpolate import interp2d

```

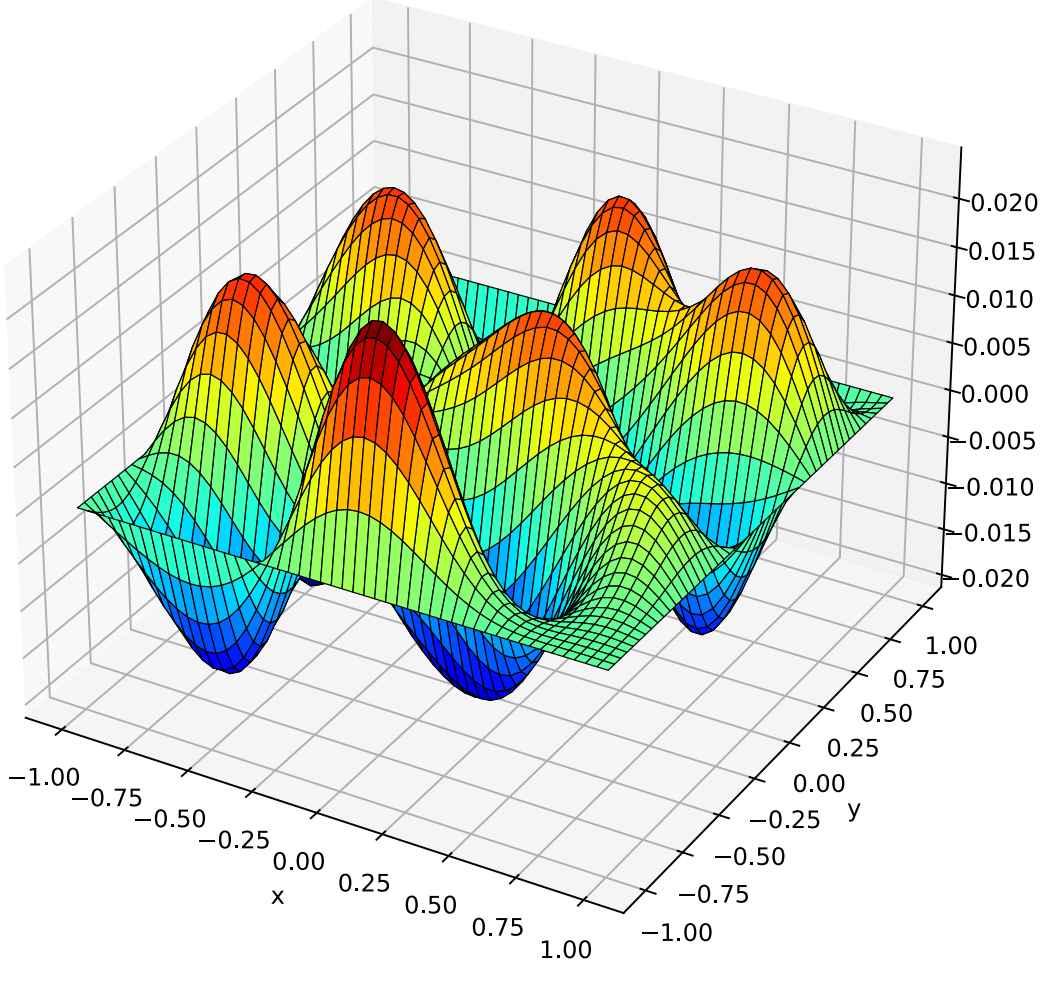
In [8]:

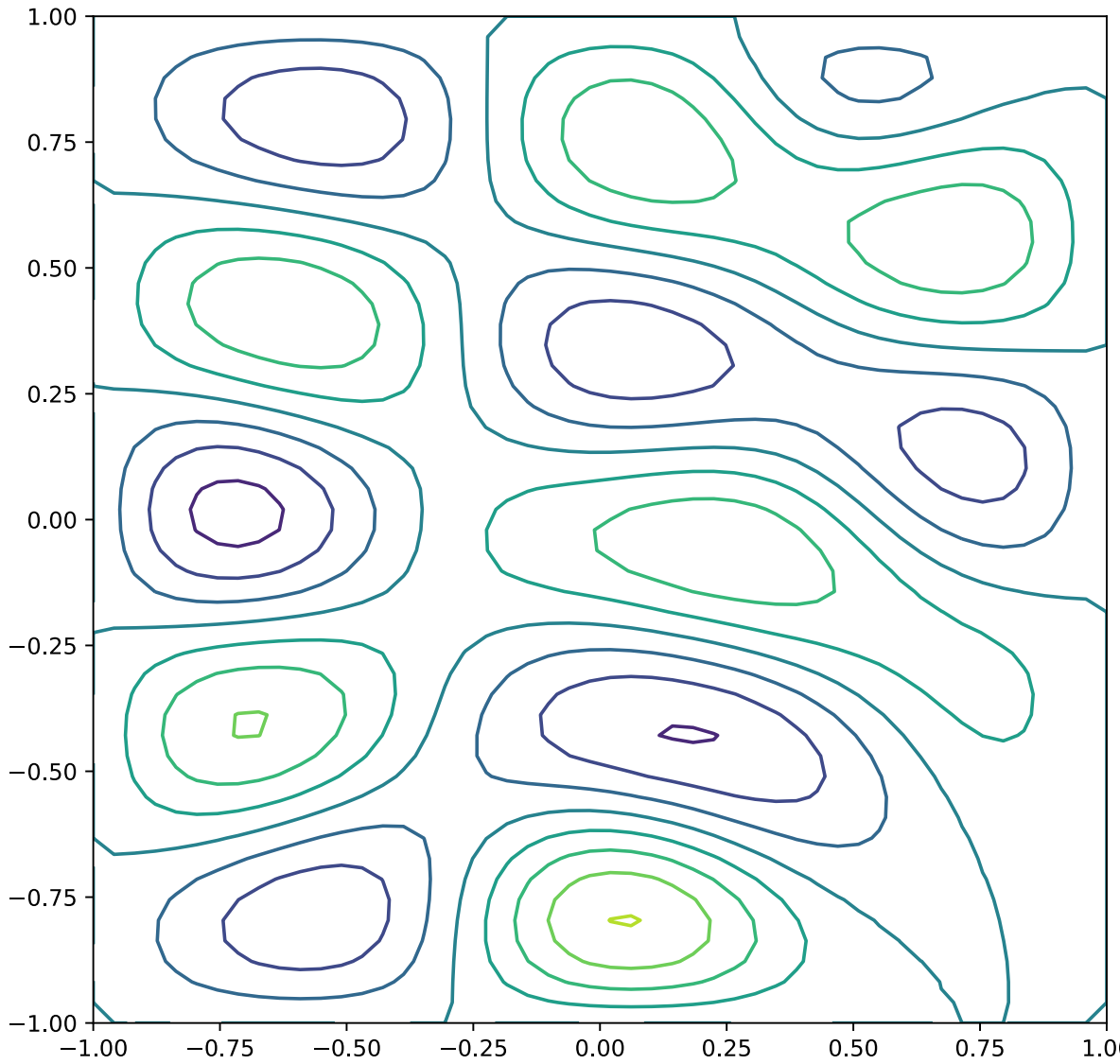
```
N = 24; D,x = cheb(N); y = x;
xx,yy = meshgrid(x[1:N],y[1:N])
xx = reshape(xx,(N-1)**2)
yy = reshape(yy,(N-1)**2)
f = exp(-10*((yy-1)**2 + (xx - 0.5)**2 ))
D2 = dot(D,D); D2 = D2[1:N,1:N]; I = eye(N-1)
k = 9
L = kron(I,D2) + kron(D2,I) + k**2*eye((N-1)**2)
# Solve Lu=f
u = solve(L,f)
# Convert 1-d vectors to 2-d
uu = zeros((N+1,N+1)); uu[1:N,1:N] = reshape(u,(N-1,N-1))
[xx,yy] = meshgrid(x,y)
value = uu[N//2,N//2]

f = interp2d(x,y,uu,kind='cubic')
xxx = linspace(-1.0,1.0,50)
uuu = f(xxx,xxx)

fig = figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')
```

```
[X ,Y] = meshgrid(xxx,xxx)
ax.plot_surface(X,Y,uuu,rstride=1,cstride=1,cmap=cm.jet,edgecolor='black', linewidth=0.5)
title("$u(0,0)$="+str(value))
xlabel("x"); ylabel("y");

figure(figsize = (8,8))
contour(X,Y,uuu);
```

```
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\1702318091.py:16: DeprecationWarning: `interp2d` is deprecated!
`interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

For legacy code, nearly bug-for-bug compatible replacements are
`RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
scattered 2D data.

In new code, for regular grids use `RegularGridInterpolator` instead.
For scattered data, prefer `LinearNDInterpolator` or
`CloughTocher2DInterpolator`.

For more details see
`https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  f = interp2d(x,y,uu,kind='cubic')
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\1702318091.py:18: DeprecationWarning:         `interp2d` is depr
ecated!
        `interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

        For legacy code, nearly bug-for-bug compatible replacements are
        `RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
        scattered 2D data.

        In new code, for regular grids use `RegularGridInterpolator` instead.
        For scattered data, prefer `LinearNDInterpolator` or
        `CloughTocher2DInterpolator`.

        For more details see
        `https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  uuu = f(xxx,xxx)
```

$u(0, 0)$=0.01172257000265278






## Program 18 : Chebyshev differentiation via FFT

In [49]:

```
%matplotlib inline
%config InlineBackend.figure_format = 'svg'
#from chebfftPy import chebfft
from numpy import pi,linspace,sin,cos,exp,round,zeros,arange,real, flipud
from numpy.fft import fft,ifft
from matplotlib.pyplot import figure,subplot,plot,grid,title,axis

```

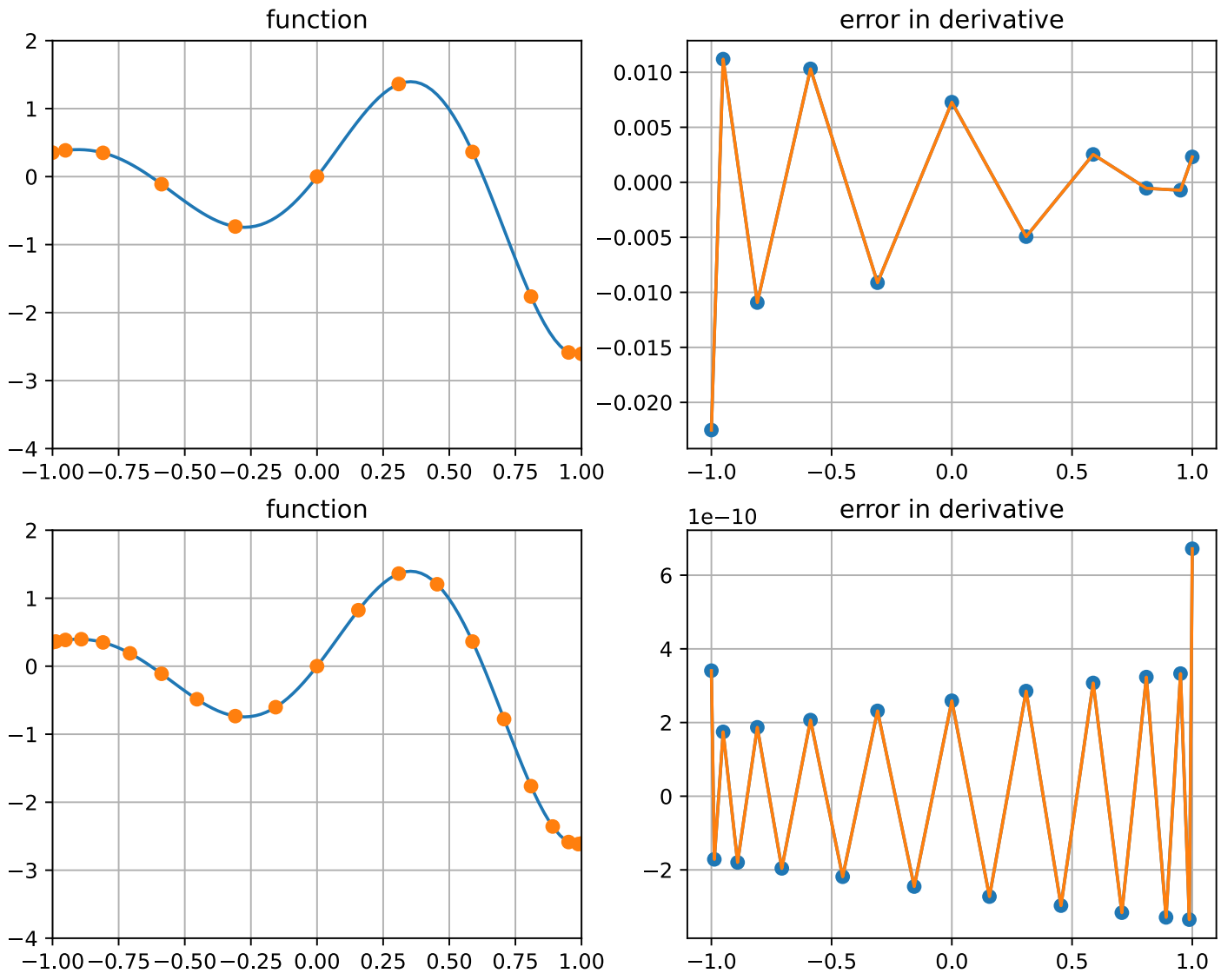
In [50]:

```
from numpy import pi,cos,arange,array, flipud,\
     real,zeros, sqrt
from numpy.fft import fft,ifft

def chebfft(v):
    '''Chebyshev differentiation via fft.
       Ref.: Trefethen's 'Spectral Methods in MATLAB' book.
    '''
    N = len(v)-1
    if N == 0:
        w = 0.0 # only when N is even!
        return w
    x  = cos(pi*arange(0,N+1)/N)
    ii = arange(0,N)
    V = flipud(v[1:N]); V = list(v) + list(V);
    U = real(fft(V))
    b = list(ii); b.append(0); b = b + list(arange(1-N,0));
    w_hat = 1j*array(b)
    w_hat = w_hat * U
    W = real(ifft(w_hat))
    w = zeros(N+1)
    w[1:N] = -W[1:N]/sqrt(1-x[1:N]**2)
    w[0] = sum(ii**2*U[ii])/N + 0.5*N*U[N]
    w[N] = sum((-1)**(ii+1)*ii**2*U[ii])/N + \
              0.5*(-1)**(N+1)*N*U[N]
    return w


```

In [51]:

```
figure(figsize=(10,12))
plot_count = 1

for N in [10,20]:
    xx = linspace(-1.0,1.0,100)
    ff = exp(xx)*sin(5*xx)
    x = cos(arange(0,N+1)*pi/N)
    f = exp(x)*sin(5*x)
    error = chebfft(f) - exp(x)*(sin(5*x)+5*cos(5*x))
    subplot(3,2,plot_count)
    plot_count +=1
    plot(xx,ff,'-',x,f,'o')
    grid(True)
    axis([-1, 1, -4,2])
    title('function')
    subplot(3,2,plot_count)
    plot_count +=1
    plot(x,error,'-o')
```

```
    title('error in derivative')
    plot(x,error)
    grid(True)

```




Program 19 : Second order Wave Equation on Chebyshev Grid

Solve

$$u_{tt} = u_{xx}, \qquad -1 < x < 1, \qquad t > 0$$

with boundary condition

$$u(\pm 1, t) = 0$$

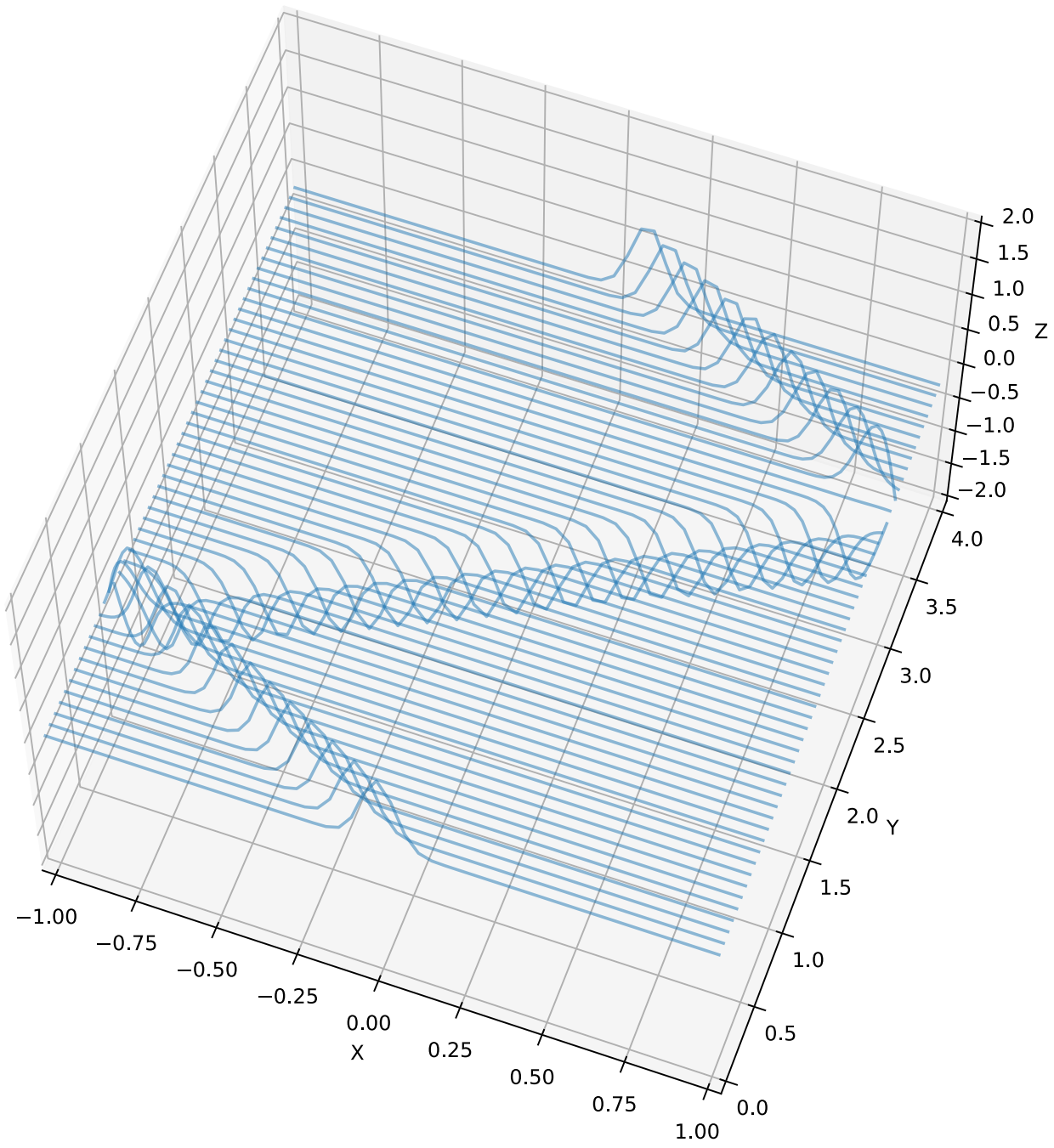
and initial condition

$$u(x, 0) = e^{-200x^2}$$

In [55]:
```
%matplotlib inline
%config InlineBackend.figure_format = 'svg'
#from chebfftPy import chebfft
from numpy import arange,cos,zeros,round,exp,pi
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.collections import LineCollection
from matplotlib.pyplot import figure

```

In [56]:
```
# Time-stepping by Leap Frog Formula:
N = 80; t = 0.0 ;x = cos(pi*arange(0,N+1)/N);dt = 8.0/(N**2);
v = exp(-200*x**2); vold = exp(-200*(x-dt)**2);
tmax = 4 ; tplot = 0.075;
plotgap = int(round(tplot/dt)); dt = tplot/plotgap;
nplots = int(round(tmax/tplot));
plotdata = []; plotdata.append(list(zip(x,v)));
tdata = []; tdata.append(0.0)
for i in range(1,nplots):
    for n in range(plotgap):
        t = t + dt
        w = chebfft(chebfft(v)); w[0] = 0.0; w[N] = 0.0;
        vnew = 2*v - vold + dt**2*w; vold = v; v = vnew;
    plotdata.append(list(zip(x,v)));
    tdata.append(t);

fig = figure(figsize=(10,12))
ax = fig.add_subplot(111,projection='3d')
poly = LineCollection(plotdata)
poly.set_alpha(0.5)
ax.add_collection3d(poly, zs=tdata, zdir='y')
ax.set_xlabel('X')
ax.set_xlim3d(-1, 1)
ax.set_ylabel('Y')
ax.set_ylim3d(0, tmax)
ax.set_zlabel('Z')
```

```
ax.set_zlim3d(-2, 2)
ax.view_init(60,-70)
```




## Program 20 : Second order Wave Equation using FFT

Solve the wave equation in 2-d

$$u_{tt} = u_{xx} + u_{yy}, \qquad -1 < x, y < 1, \qquad t > 0$$

with $u = 0$ on the boundary and initial condition

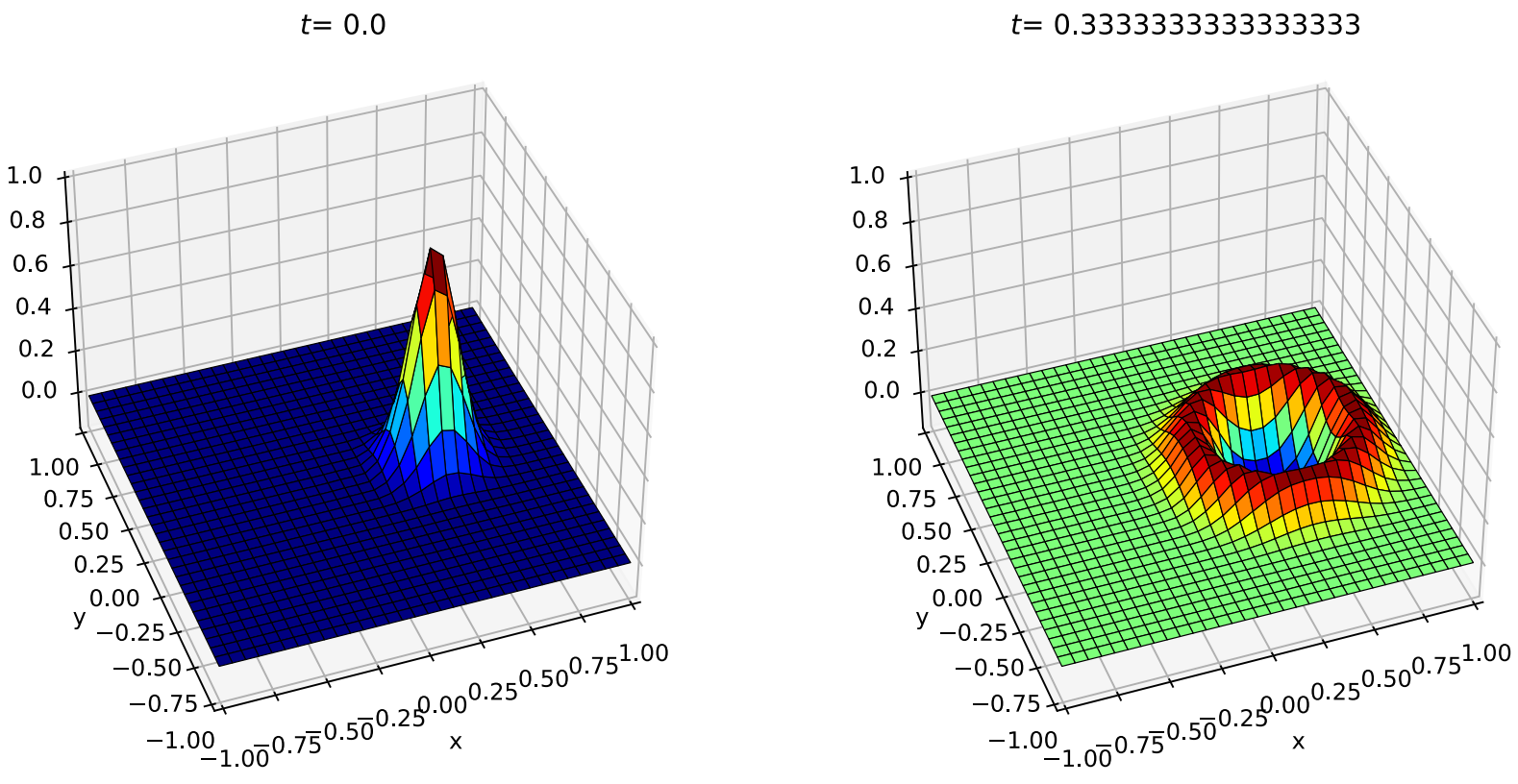
$$u(x, y, 0) = e^{-40((x-0.4)^2+y^2)}, \qquad u_t(x, y, 0) = 0$$

In [10]:
```
%matplotlib inline
%config InlineBackend.figure_format = 'svg'
from numpy import meshgrid,cos,pi,round,exp,real,remainder,zeros,fliplr,flipud,array,arange
from numpy.fft import fft, ifft
from matplotlib.pyplot import subplot, figure ,title,axis
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.pyplot import figure,subplot,plot,title,axis,xlabel,ylabel
from matplotlib import cm
from scipy.interpolate import interp2d

```

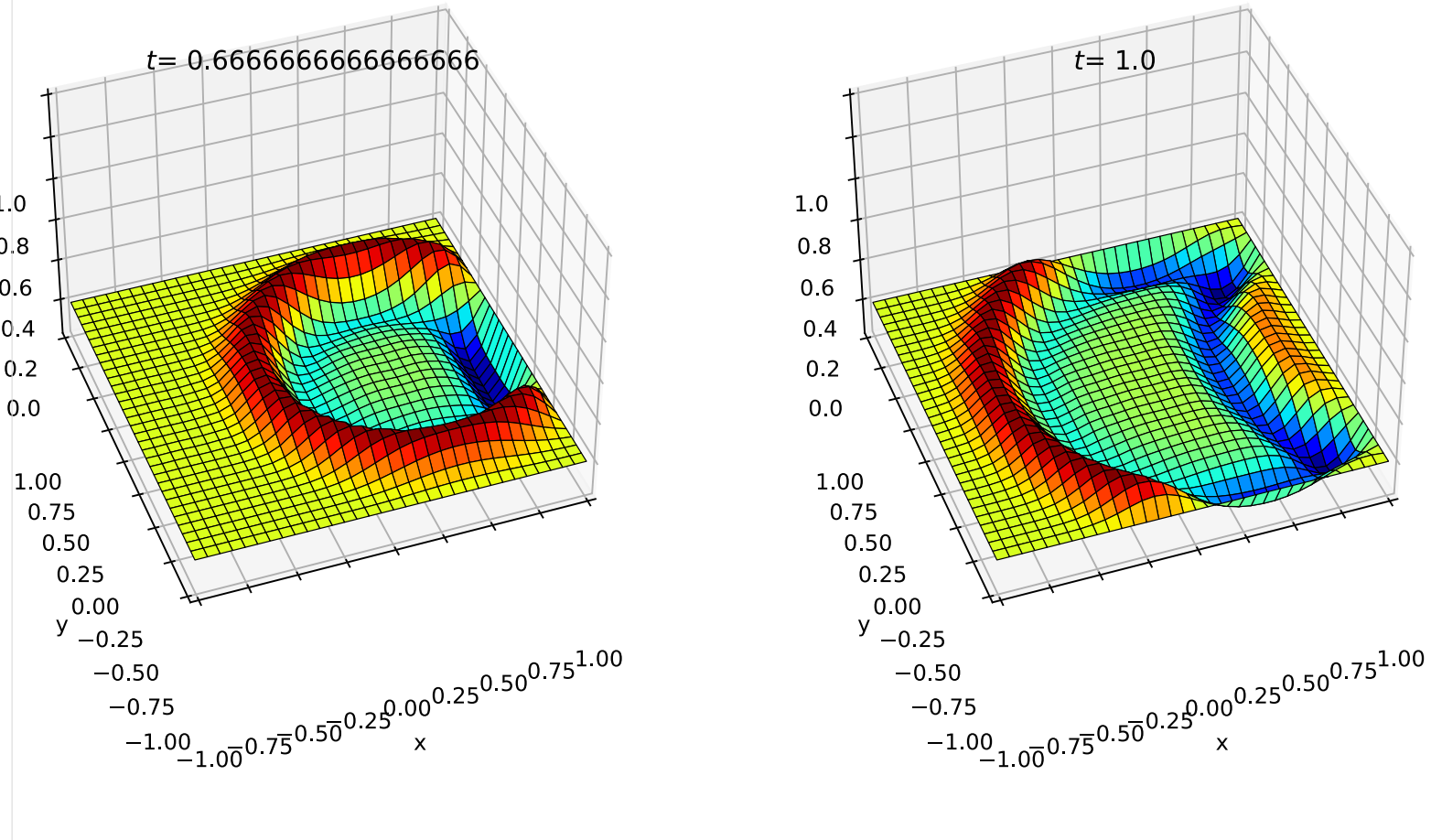
In [11]:
```
# Grid and inital Data:
N = 24; x = cos(pi*arange(0,N+1)/N); y = x;
t = 0.0; dt = (6.0)/(N**2)
xx, yy = meshgrid(x,y)
plotgap = int (round( (1.0/3.0) / (dt))); dt = (1.0/3.0)/(plotgap);
vv = exp(-40*((xx-0.4)**2 + yy**2));
vvold = vv;

#Time stepping Leapfrog Formula:
fig = figure(figsize=(12,12))
k = 1;
for n in range(0,(3*plotgap)+1):
    t = n*dt;
    if (remainder(n+0.5,plotgap) < 1):
        ax = fig.add_subplot(2,2,k,projection ='3d')
        f = interp2d(x,y,vv,kind='cubic');
        xxx = arange(-1.,1.+1./16,1./16);
        vvv = f(xxx,xxx)
        X,Y = meshgrid(xxx,xxx);
        ax.plot_surface(X,Y,vvv,rstride=1,cstride=1,cmap=cm.jet,edgecolor='black', linewidth=0.5)
```

```
        ax.set_zlim3d([-0.15,1])
        ax.set_xlim3d([-1,1])
        ax.set_ylim3d([-1,1])
        ax.view_init(elev=40., azim=250.)
        title("$ t $= " +str(t))
        xlabel("x"); ylabel("y");
        k = k+1;

    uxx = zeros((N+1,N+1)); uyy = zeros((N+1,N+1));
    ii = arange(1,N);

    for i in range(1,N):
        v = vv[i,:];
        V = list(v) + list(flipud(v[ii]));
        U = real(fft(V));
        w1_hat = 1j*zeros(2*N);
        w1_hat[0:N] = 1j*arange(0,N)
        w1_hat[N+1:] = 1j*arange(-N+1,0)
        W1 = real(ifft(w1_hat * U))
        w2_hat = 1j*zeros(2*N);
        w2_hat[0:N+1] = arange(0,N+1)
        w2_hat[N+1:] = arange(-N+1,0)
        W2 = real(ifft((-w2_hat**2) * U))
        uxx[i,ii] = W2[ii]/(1-x[ii]**2) - (x[ii]*W1[ii])/(1-x[ii]**2)**(3.0/2);
    for j in range(1,N):
        v = vv[:,j];
        V = list(v) + list(flipud(v[ii]));
        U = real(fft(V))
        w1_hat = 1j*zeros(2*N);
        w1_hat[0:N] = 1j*arange(0,N)
        w1_hat[N+1:] = 1j*arange(-N+1,0)
        W1 = real(ifft(w1_hat * U))
        w2_hat = 1j*zeros(2*N);
        w2_hat[0:N+1] = arange(0,N+1)
        w2_hat[N+1:] = arange(-N+1,0)
        W2 = real(ifft(-(w2_hat**2) * U))
        uyy[ii,j] = W2[ii]/(1-y[ii]**2) - y[ii]*W1[ii]/(1-y[ii]**2)**(3.0/2.0);
    vvnew = 2*vv - vvold + dt**2 *(uxx+uyy)
    vvold = vv ; vv = vvnew;

```

```
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\3484298906.py:16: DeprecationWarning: `interp2d` is deprecated!
`interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

For legacy code, nearly bug-for-bug compatible replacements are
`RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
scattered 2D data.

In new code, for regular grids use `RegularGridInterpolator` instead.
For scattered data, prefer `LinearNDInterpolator` or
`CloughTocher2DInterpolator`.

For more details see
`https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  f = interp2d(x,y,vv,kind='cubic');
C:\Users\gary\AppData\Local\Temp\ipykernel_8112\3484298906.py:18: DeprecationWarning:         `interp2d` is depr
ecated!
        `interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

        For legacy code, nearly bug-for-bug compatible replacements are
        `RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
        scattered 2D data.

        In new code, for regular grids use `RegularGridInterpolator` instead.
        For scattered data, prefer `LinearNDInterpolator` or
        `CloughTocher2DInterpolator`.

        For more details see
        `https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

vvv = f(xxx,xxx)
```



In [ ]:

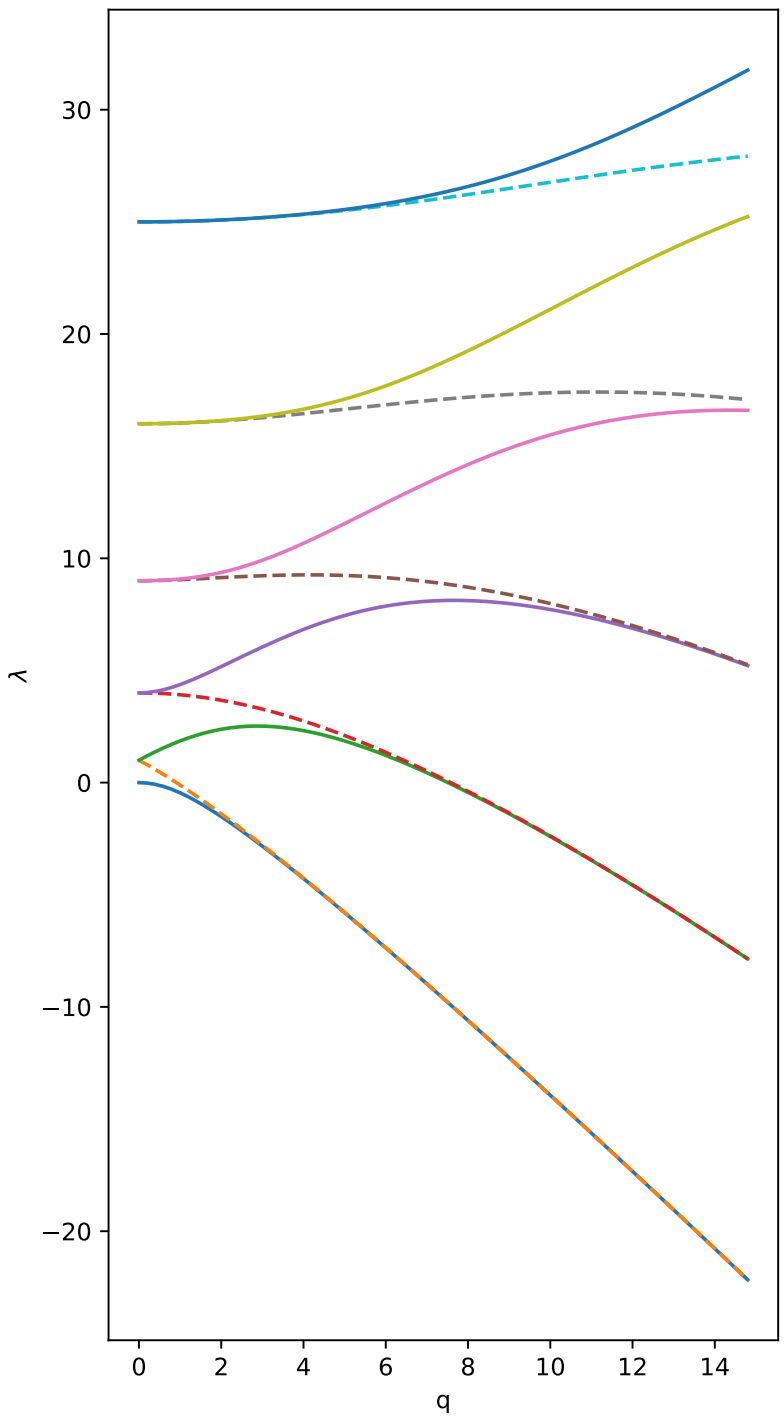



## Program 21 : Eigenvalues of Mathieu operator

In [13]:

```
%matplotlib inline
%config InlineBackend.figure_format='svg'
from numpy import pi,arange,sin,cos,zeros,diag,sort,real
from scipy.linalg import toeplitz
from numpy.linalg import eig
from itertools import cycle
from matplotlib.pyplot import figure,plot,xlabel,ylabel
%config InlineBackend.figure_formats = ['svg']

```

In [60]:
```
N = 42; h = 2.0*pi/N; x = h*arange(1,N+1)
col = zeros(N)
col[0] = -pi**2/(3.0*h**2) - 1.0/6.0
col[1:] = -0.5*(-1.0)**arange(1,N)/sin(0.5*h*arange(1,N))**2
D2 = toeplitz(col)

ne = 11 # number of eigenvalues to plot
qq = arange(0.0, 15.0, 0.2)
data= zeros((len(qq),ne))
i = 0
for q in qq:
    evals,evecs = eig(-D2 + 2.0*q*diag(cos(2.0*x)))
    e = real(sort(evals))
    data[i,:] = e[0:ne]
    i = i + 1

figure(figsize=(5,10))
lines=cycle(["-","--"])
for i in range(ne):
    plot(qq,data[:,i],next(lines))
xlabel("q")
ylabel("$\lambda$");

```

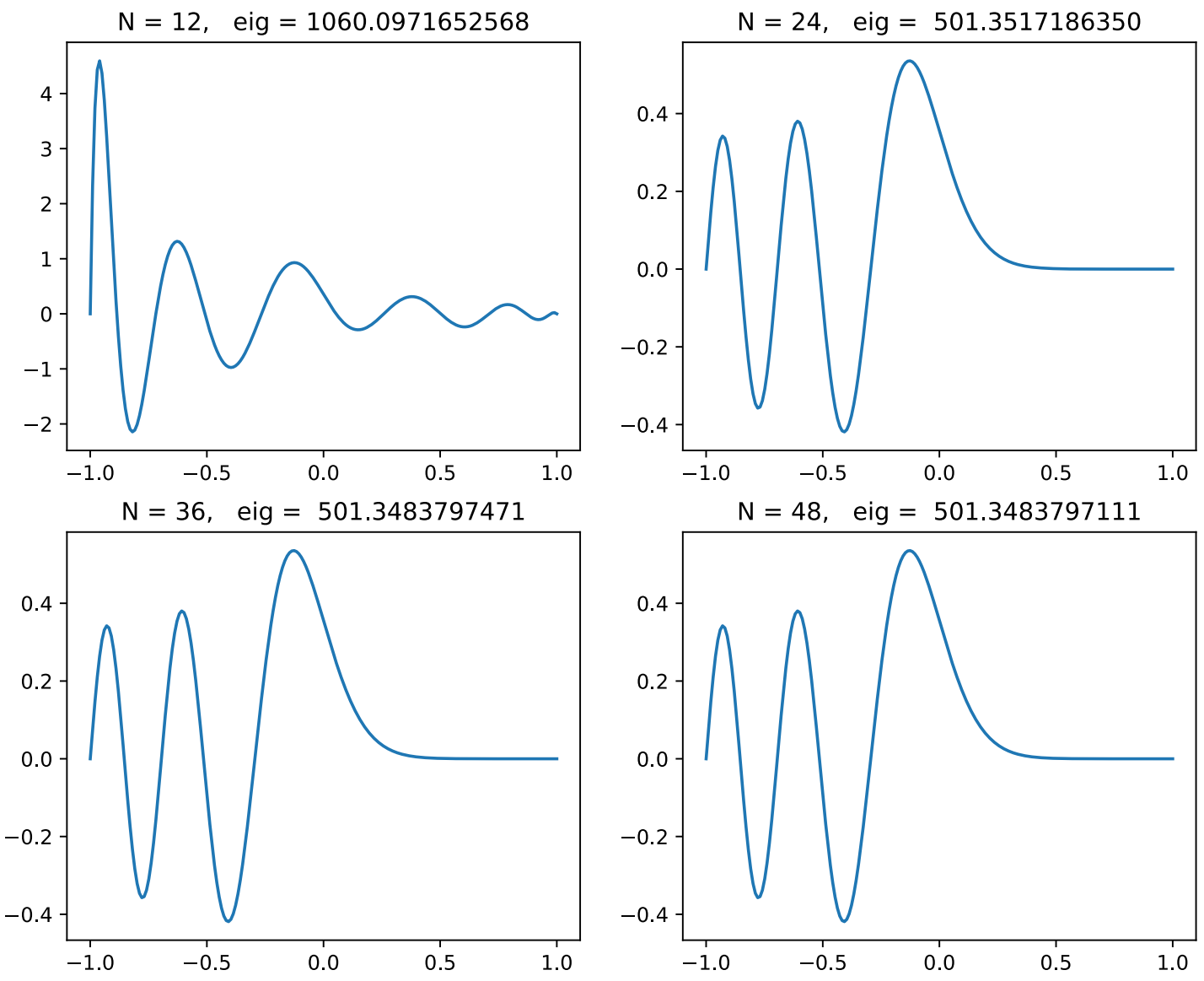



## Program 22 : 5th eigenvector of Airy equation

In [63]:
```
%matplotlib inline
%config InlineBackend.figure_format='svg'
#from chebPy import *
from numpy import dot,diag,real,argsort,zeros,linspace,polyval,polyfit,where
from scipy.linalg import eig
from scipy.special import airy
from matplotlib.pyplot import figure,subplot,plot,title

```

In [64]:
```
figure(figsize=(10,8))
for N in range(12,60,12):
    D,x = cheb(N); D2 = dot(D,D); D2 = D2[1:N,1:N]
    Lam,V = eig(D2,diag(x[1:N]))
    Lam = real(Lam); ii = where(Lam>0)[0]
    V = real(V[:,ii]); Lam = Lam[ii]
    ii = argsort(Lam); ii=ii[4]; Lam=Lam[ii]
    v = zeros(N+1); v[1:N] = V[:,ii]; v = v/v[N//2]*airy(0.0)[0]
    xx = linspace(-1.0,1.0,200); vv = polyval(polyfit(x,v,N),xx);
    subplot(2,2,N//12); plot(xx,vv)
    title("N = %d,   eig = %15.10f"%(N,Lam));

```

```
C:\Users\gary\AppData\Local\Programs\Python\Python39\lib\site-packages\IPython\core\interactiveshell.py:3460: Ra
nkWarning: Polyfit may be poorly conditioned
  exec(code obj, self.user global ns, self.user ns)
```




Program 23 : Eigenvalues of perturbed Laplacian

solve the following eigenvalue problem

$$-(u_{xx} + u_{yy}) + f(x, y)u = \lambda u, \qquad -1 < x, y < 1, \qquad u = 0 \quad \text{on boundary}$$

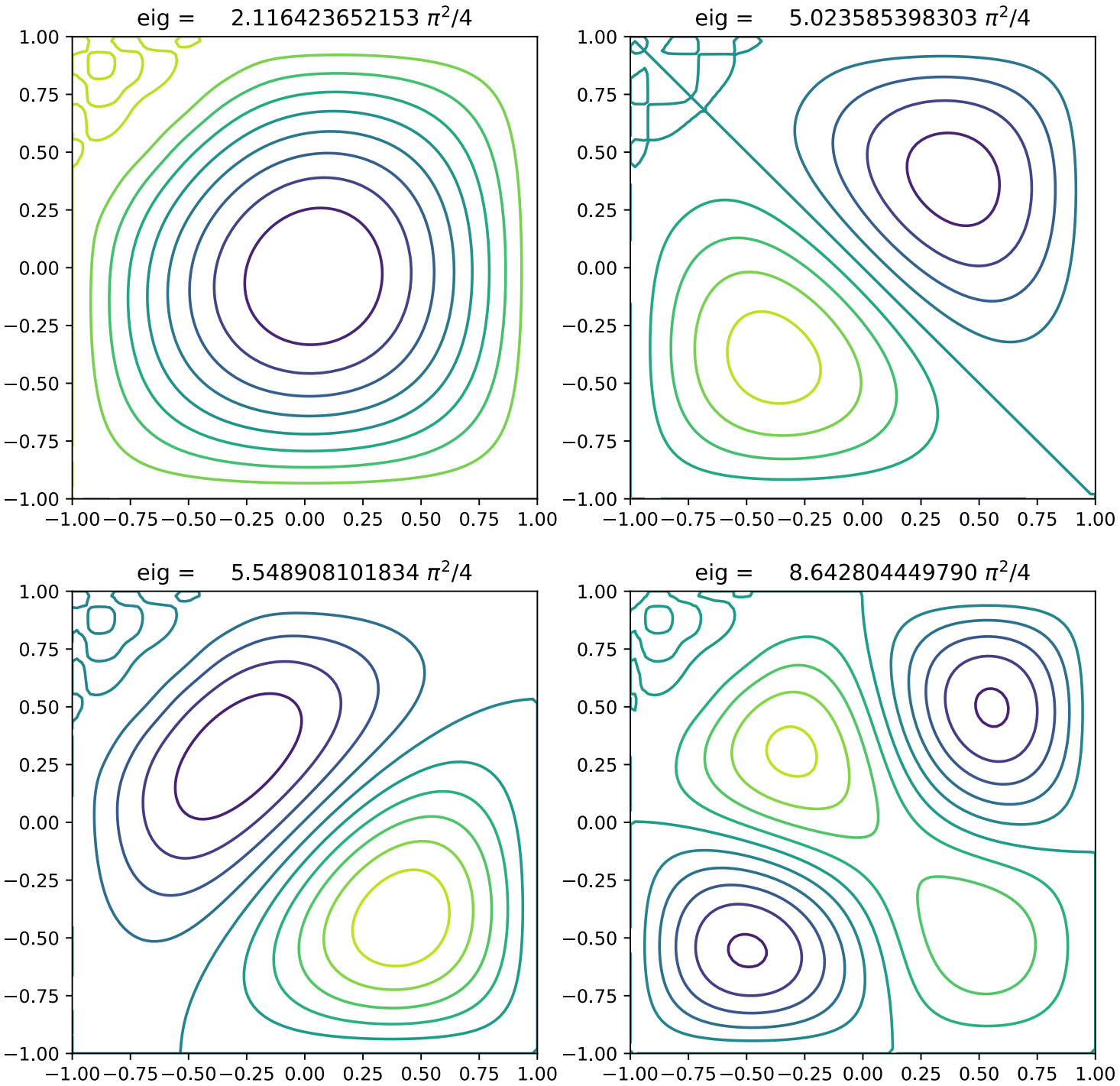
where

$$f(x, y) = \exp(20(y - x - 1))$$

In [66]:
```
%matplotlib inline
%config InlineBackend.figure_format='svg'
#from chebPy import cheb
from numpy import meshgrid,dot,eye,kron,zeros,reshape,pi,real,imag
from numpy import diag,exp,argsort,linspace,inf
from matplotlib.pyplot import figure,subplot,plot,title,contour
from scipy.linalg import eig,norm
from scipy.interpolate import interp2d

```

In [67]:
```
# Set up tensor product Laplacian and compute 4 eigenmodes
N = 16; D,x = cheb(N); y = x;
xx,yy = meshgrid(x[1:N],y[1:N])
xx = reshape(xx,(N-1)**2)
yy = reshape(yy,(N-1)**2)
D2 = dot(D,D); D2 = D2[1:N,1:N]; I = eye(N-1)
L = -kron(I,D2) - kron(D2,I)
L = L + diag(exp(20*(yy-xx-1)))
D,V = eig(L); D = real(D); V = real(V)
ii = argsort(D); ii = ii[0:4]; D = D[ii]; V = V[:,ii]

# Reshape them to 2D grid, interpolate to finer grid, and plot
fine = linspace(-1.0,1.0,100,True);
uu = zeros((N+1,N+1));

figure(figsize=(10,10))
for i in range(4):
    uu[1:N,1:N] = reshape(V[:,i],(N-1,N-1))
    uu = uu/norm(uu,inf)
    f = interp2d(x,y,uu,kind='cubic')
    uuu = f(fine,fine)
    subplot(2,2,i+1)
    contour(fine,fine,uuu,10)
    title("eig = %18.12f $\pi^2/4$"%(D[i]/(pi**2/4)))

```

```
C:\Users\gary\AppData\Local\Temp\ipykernel_7648\26879621.py:20: DeprecationWarning: `interp2d` is deprecated!
`interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

For legacy code, nearly bug-for-bug compatible replacements are
`RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
scattered 2D data.

In new code, for regular grids use `RegularGridInterpolator` instead.
For scattered data, prefer `LinearNDInterpolator` or
`CloughTocher2DInterpolator`.

For more details see
`https://gist.github.com/ev-br/8544371b40f414b7eaf3fe6217209bff`

  f = interp2d(x,y,uu,kind='cubic')
C:\Users\gary\AppData\Local\Temp\ipykernel_7648\26879621.py:21: DeprecationWarning:         `interp2d` is deprec
ated!
        `interp2d` is deprecated in SciPy 1.10 and will be removed in SciPy 1.12.0.

        For legacy code, nearly bug-for-bug compatible replacements are
        `RectBivariateSpline` on regular grids, and `bisplrep`/`bisplev` for
        scattered 2D data.

        In new code, for regular grids use `RegularGridInterpolator` instead.
        For scattered data, prefer `LinearNDInterpolator` or
        `CloughTocher2DInterpolator`.

        For more details see
```



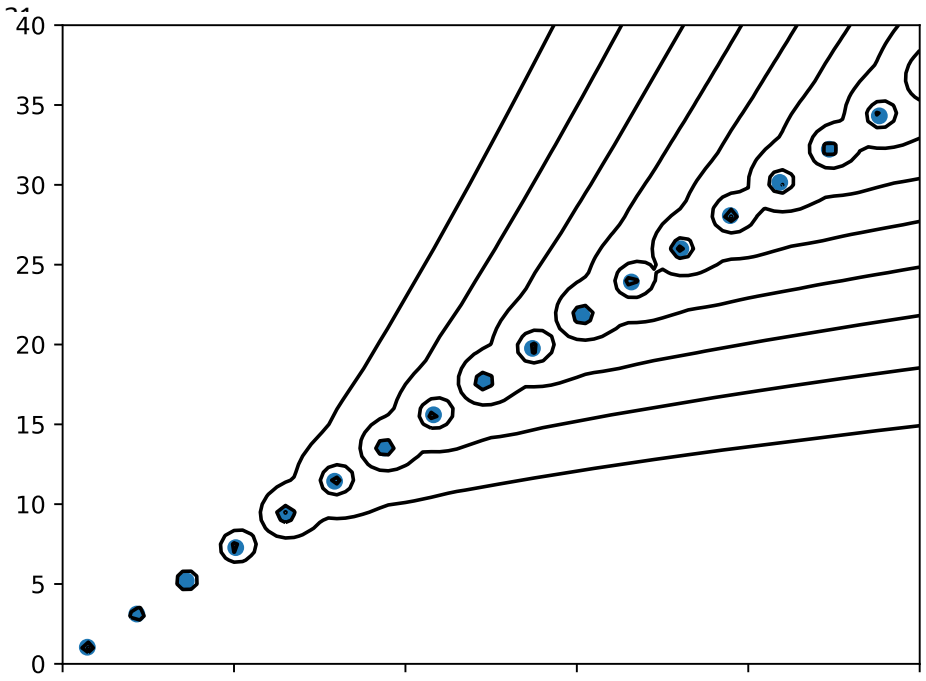

Program 24 : Pseudospectra of Davies complex harmonic oscillator

Note: This problem requires more than the average time allotment for processing.

In [74]:
```
%matplotlib inline
%config InlineBackend.figure_format = 'svg'
#from chebPy import cheb
from numpy import dot,argsort,zeros,real,imag,meshgrid,eye,diag,arange
from scipy.linalg import solve,eig,svd,svdvals
from matplotlib.pyplot import figure,plot,title,axis,contour

```

In [75]:
```
N = 70; D, x = cheb(N); x = x[1:N];
L = 6.0; x = L*x; D = D/L;
A = -dot(D,D);
A = A[1:N,1:N] + (1+3j)*diag(x**2);
lam, v = eig(A)
fig = figure()
plot(real(lam),imag(lam),"o")
```

```
axis([0, 50, 0, 40])

h = 0.5 #Smaller the value, finer the plot
x = arange(0,50+h,h); y = arange(0,40+h,h); xx,yy = meshgrid(x,y);
zz = xx + 1j*yy;
I = eye(N-1); sigmin = zeros((len(y),len(x)))
for j in range(0,len(x)):
    for i in range(0,len(y)):
        sigmin[i,j] = min(svdvals(zz[i,j]*I - A));

levels = 10.0**arange(-4.5,0.0,0.5);
contour(x,y,sigmin,levels,colors = 'k');

```



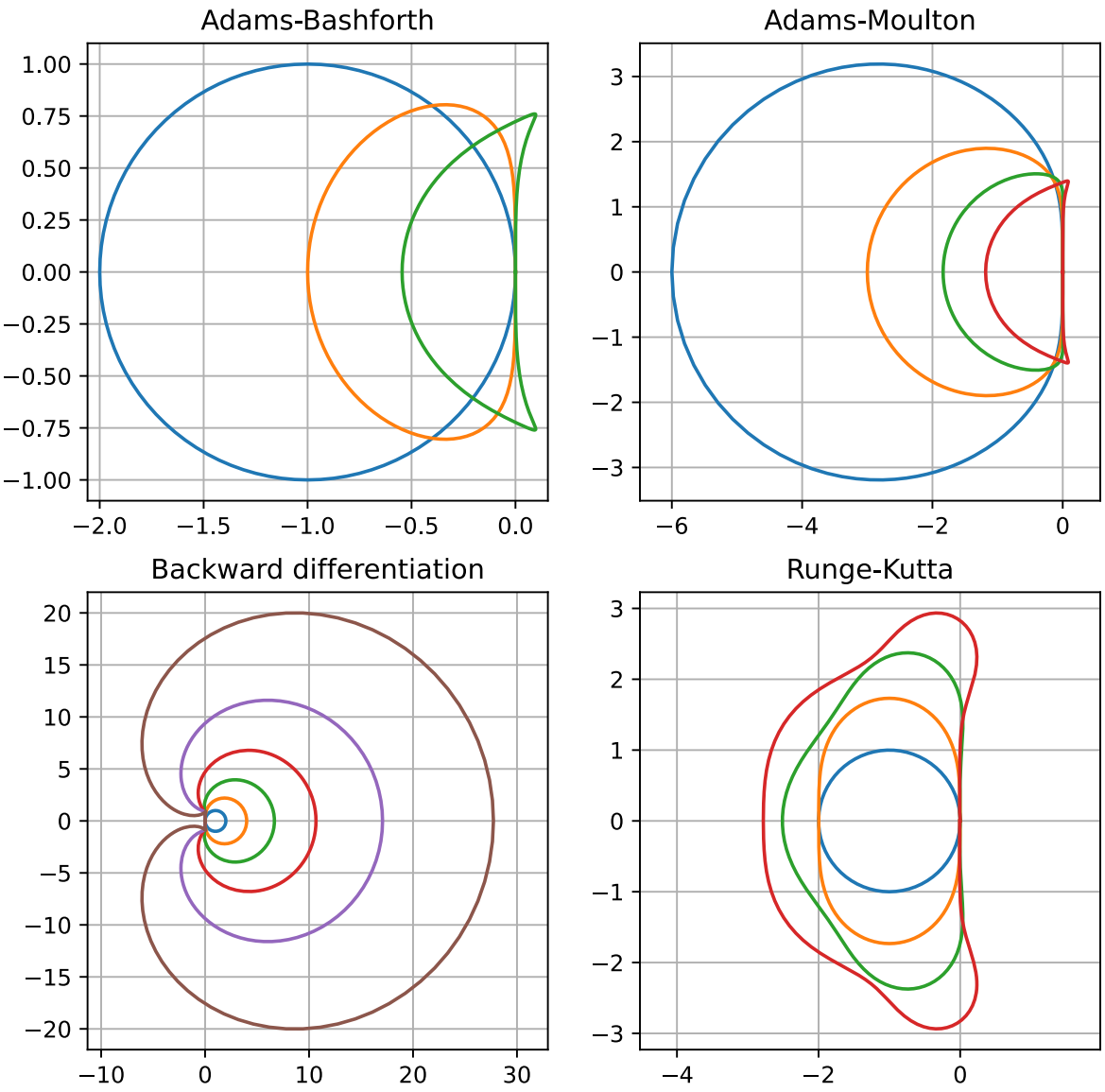

## Program 25 : Stability regions for ODE formulas

In [76]:

```
%matplotlib inline
%config InlineBackend.figure_format='svg'
from numpy import pi,real,imag,zeros,exp,arange
from matplotlib.pyplot import figure,subplot,plot,axis,grid,title

```

In [77]:

```
figure(figsize=(8,8))

# Adams-Bashforth
subplot(2,2,1)
z = exp(1j*pi*arange(0,201)/100); r = z - 1
s = 1; rr = r/s; plot(real(rr),imag(rr))
s = (3 - 1/z)/2; rr = r/s; plot(real(rr),imag(rr))
s = (23 - 16/z + 5/z**2)/12; rr = r/s; plot(real(rr),imag(rr))
axis('equal'); grid('on')
title('Adams-Bashforth')

# Adams-Moulton
subplot(2,2,2)
s = (5*z + 8 - 1/z)/12; rr = r/s; plot(real(rr),imag(rr))
s = (9*z + 19 - 5/z + 1/z**2)/24; rr = r/s; plot(real(rr),imag(rr))
s = (251*z + 646 - 264/z + 106/z**2 - 19/z**3)/720; rr = r/s; plot(real(rr),imag(rr))
d = 1 - 1/z
s = 1 - d/2 - d**2/12 - d**3/24 - 19*d**4/720 - 3*d**5/160; dd = d/s; plot(real(dd),imag(dd))
axis('equal'); grid('on')
title('Adams-Moulton')

# Backward differentiation
subplot(2,2,3)
r = 0
for i in range(1,7):
    r = r + d**i/i; plot(real(r),imag(r))
axis('equal'); grid('on')
title('Backward differentiation')

# Runge-kutta
subplot(2,2,4)
w = 0; W = 1j*zeros(len(z)); W[0] = w;
for i in range(1,len(z)):
    w = w - (1+w-z[i]); W[i] = w
plot(real(W),imag(W))
w = 0; W = 1j*zeros(len(z)); W[0] = w;
for i in range(1,len(z)):
    w = w - (1+w+0.5*w**2-z[i]**2)/(1+w); W[i] = w
plot(real(W),imag(W))
w = 0; W = 1j*zeros(len(z)); W[0] = w;
for i in range(1,len(z)):
    w = w - (1+w+0.5*w**2+w**3/6-z[i]**3)/(1+w+0.5*w**2); W[i] = w
plot(real(W),imag(W))
w = 0; W = 1j*zeros(len(z)); W[0] = w;
for i in range(1,len(z)):
    w = w - (1+w+0.5*w**2+w**3/6+w**4/24-z[i]**4)/(1+w+w**2/2+w**3/6); W[i] = w
plot(real(W),imag(W))
axis('equal'); grid('on')
title('Runge-Kutta');

```




## Program 26 : Eigenvalues of 2nd-order Chebyshev differential matrix

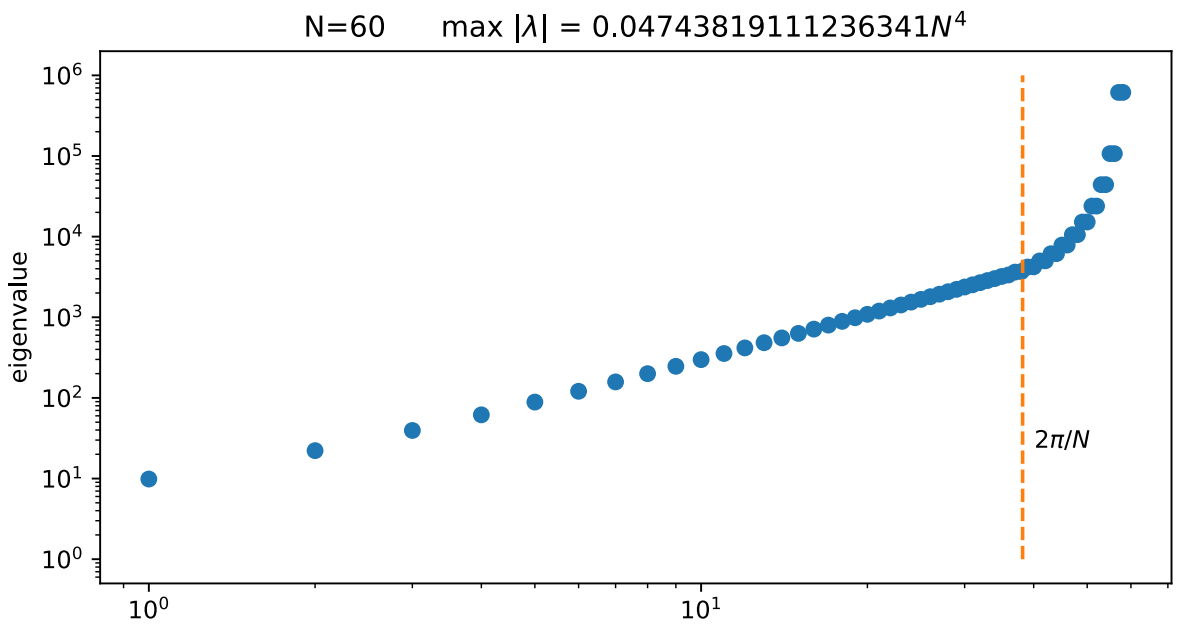
In [78]:

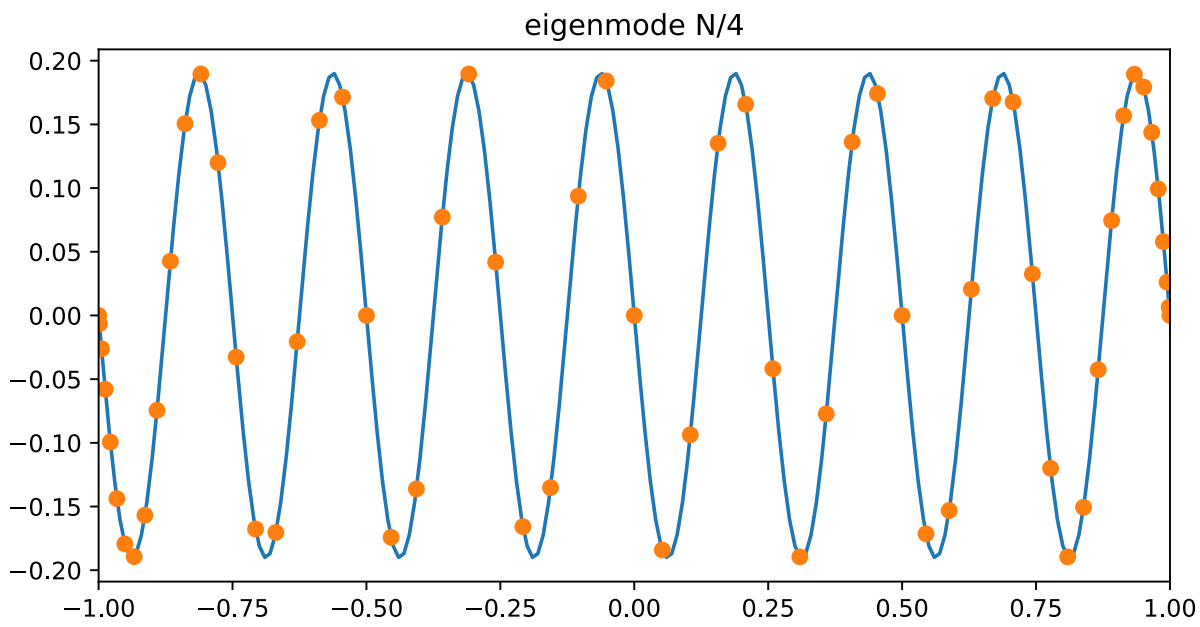
```
%matplotlib inline
%config InlineBackend.figure_format='svg'
#from chebPy import *
from numpy import dot,argsort,diag,real,imag,pi,array,polyfit,polyval,zeros
```

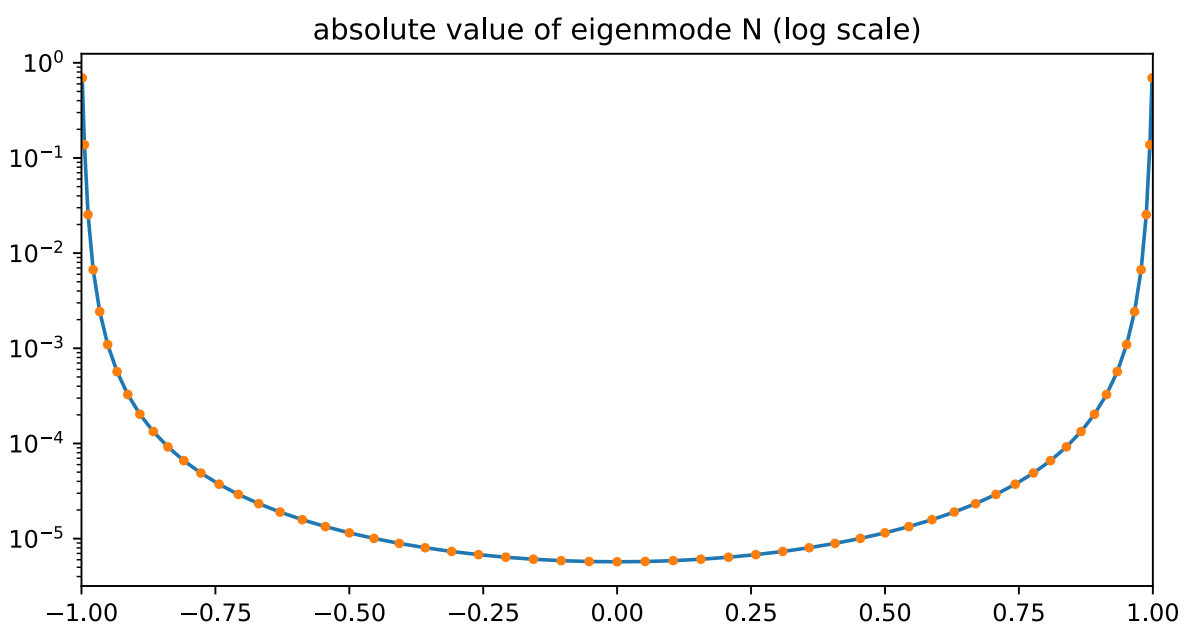
```
5 from numpy.linalg import eig
6 from matplotlib.pyplot import figure,loglog,semilogy,plot,title,ylabel,text,xlim
7
```

In [79]:
```
N = 60; D, x = cheb(N); D2 = dot(D,D); D2 = D2[1:N,1:N]
Lam, V = eig(D2)
ii = argsort(-Lam); e = Lam[ii]; V = V[:,ii]

# Plot eigenvalues
figure(figsize=(8,4))
loglog(-e,'o')
semilogy(2*N/pi*array([1,1]),array([1,1e6]),'--')
ylabel('eigenvalue')
title('N='+str(N)+'      max |$\lambda$| = '+str(max(-e)/N**4)+'$N^4$')
text(2.1*N/pi,24,'$2\pi/N$')

# Plot eigenmode N/4 (physical)
figure(figsize=(8,4))
vN4 = zeros(N+1)
vN4[1:N] = V[:,N//4];
xx = arange(-1.0,1.01,0.01)
vv = polyval(polyfit(x,vN4,N),xx)
plot(xx,vv,'-')
plot(x,vN4,'o')
xlim((-1.0,1.0))
title('eigenmode N/4')

# Plot eigenmode N (nonphysical)
figure(figsize=(8,4))
vN = V[:,N-2]
semilogy(x[1:N],abs(vN))
plot(x[1:N],abs(vN),'.')
xlim((-1.0,1.0))
title('absolute value of eigenmode N (log scale)');

```

```
C:\Users\gary\AppData\Local\Programs\Python\Python39\lib\site-packages\IPython\core\interactiveshell.py:3460: Ra
nkWarning: Polyfit may be poorly conditioned
  exec(code_obj, self.user_global_ns, self.user_ns)
```



## Program 27 : Solve KdV equation

Solve the KdV equation using FFT

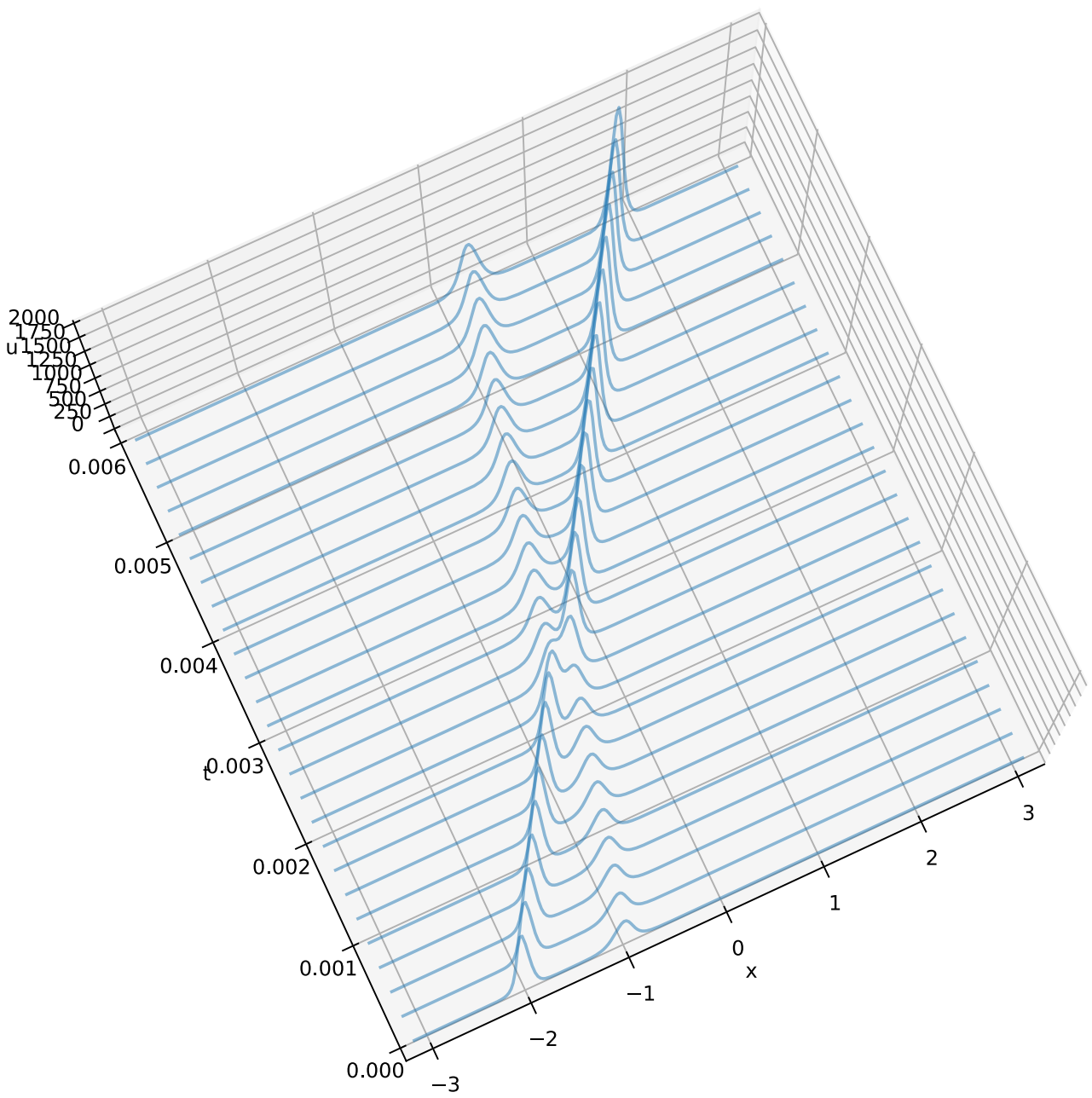
$$u_t + uu_x + u_{xxx} = 0, \qquad x \in [-\pi, \pi]$$

In [82]:

```
%matplotlib inline
%config InlineBackend.figure_format='svg'
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.collections import LineCollection
from numpy import pi,cosh,exp,round,zeros,arange,real
from numpy.fft import fft,ifft
from matplotlib.pyplot import figure

```

In [83]:

```
# Set up grid and differentiation matrix:
N = 256; dt = 0.4/N**2; x = (2*pi/N)*arange(-N/2,N/2);
A, B = 25.0, 16.0
u = 3*A**2/cosh(0.5*A*(x+2))**2 + 3*B**2/cosh(0.5*B*(x+1))**2
v = fft(u);
k = zeros(N); k[0:N//2] = arange(0,N/2); k[N//2+1:] = arange(-N/2+1,0,1)
ik3 = 1j*k**3

# Time-stepping by Runge-Kutta
tmax = 0.006; nplt = int(round((tmax/25)/dt))
nmax = int(round(tmax/dt))
udata = []; udata.append(list(zip(x, u)))
tdata = [0.0]
for n in range(1,nmax+1):
    t = n*dt; g = -0.5j*dt*k
    E = exp(dt*ik3/2); E2 = E**2
    a = g * fft(real(ifft( v         ))**2)
    b = g * fft(real(ifft( E*(v+a/2) ))**2)
    c = g * fft(real(ifft( E*v+b/2   ))**2)
    d = g * fft(real(ifft( E2*v+E*c  ))**2)
    v = E2*v + (E2*a + 2*E*(b+c) + d)/6
    if n%nplt == 0:
        u = real(ifft(v))
        udata.append(list(zip(x, u)))
        tdata.append(t);

fig = figure(figsize=(12,10))
ax = fig.add_subplot(111,projection='3d')
poly = LineCollection(udata)
poly.set_alpha(0.5)
ax.add_collection3d(poly, zs=tdata, zdir='y')
ax.set_xlabel('x')
ax.set_xlim3d(-pi, pi)
ax.set_ylabel('t')
ax.set_ylim3d(0, tmax)
ax.set_zlabel('u')
ax.set_zlim3d(0, 2000)
ax.view_init(80,-115);
```




Program 27b : Solve KdV equation, with animation

Solve the KdV equation using FFT

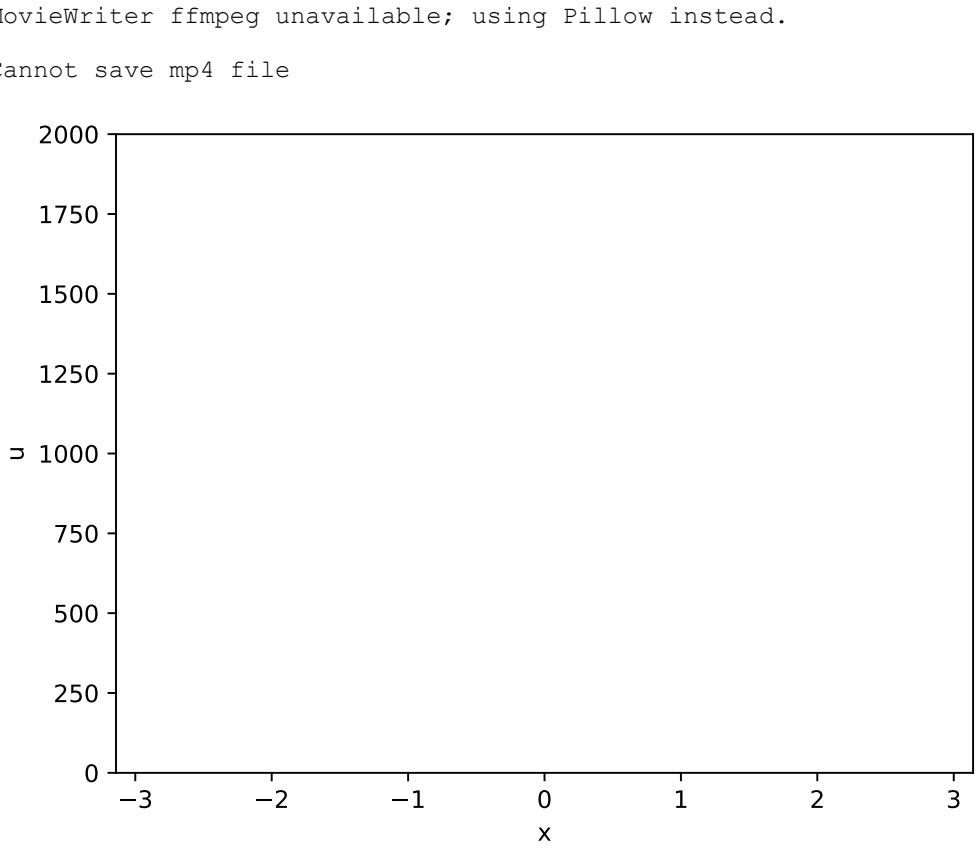
$$u_t + uu_x + u_{xxx} = 0, \qquad x \in [-\pi, \pi]$$

In [14]:
```
%matplotlib inline
%config InlineBackend.figure_format='svg'
from matplotlib import rc
rc('animation', html='jshtml')
from numpy import pi,cosh,exp,round,zeros,arange,real,mod
from numpy.fft import fft,ifft
import matplotlib.pyplot as plt
%config InlineBackend.figure_formats = ['svg']

```

In [15]:
```
# Set up grid and differentiation matrix:
N = 256; dt = 0.4/N**2; x = (2*pi/N)*arange(-N/2,N/2);
A, B = 25.0, 16.0
u = 3*A**2/cosh(0.5*A*(x+2))**2 + 3*B**2/cosh(0.5*B*(x+1))**2
v = fft(u);
k = zeros(N); k[0:N//2] = arange(0,N/2); k[N//2+1:] = arange(-N/2+1,0,1)
ik3 = 1j*k**3

# Time-stepping by Runge-Kutta
tmax = 0.006; nplt = 5 #int(round((tmax/25)/dt))
nmax = int(round(tmax/dt))
udata = []; udata.append(u)
tdata = [0.0]
for n in range(1,nmax+1):
    t = n*dt; g = -0.5j*dt*k
    E = exp(dt*ik3/2); E2 = E**2
    a = g * fft(real(ifft( v         ))**2)
    b = g * fft(real(ifft( E*(v+a/2) ))**2)
    c = g * fft(real(ifft( E*v+b/2   ))**2)
    d = g * fft(real(ifft( E2*v+E*c  ))**2)
    v = E2*v + (E2*a + 2*E*(b+c) + d)/6
    if mod(n,nplt) == 0:
        u = real(ifft(v))
        udata.append(u)
        tdata.append(t);

```

In [16]:
```
from matplotlib import animation

# First set up the figure, the axis, and the plot element we want to animate
fig = plt.figure()
ax = plt.axes(xlim=(-pi, pi), ylim=(0, 2000))
line, = ax.plot([], [], lw=2)
plt.xlabel('x'); plt.ylabel('u')

# initialization function: plot the background of each frame
def init():
    line.set_data([], [])
    return line,

# animation function.  This is called sequentially
def animate(i):
    line.set_data(x, udata[i])
    return line,

# call the animator.  blit=True means only re-draw the parts that have changed.
anim = animation.FuncAnimation(fig, animate, init_func=init,
                               frames=len(udata), interval=50, blit=True)
# Save to file
try:
    anim.save('p27.mp4', fps=20, extra_args=['-vcodec', 'libx264'])
except:
    print("Cannot save mp4 file")

```

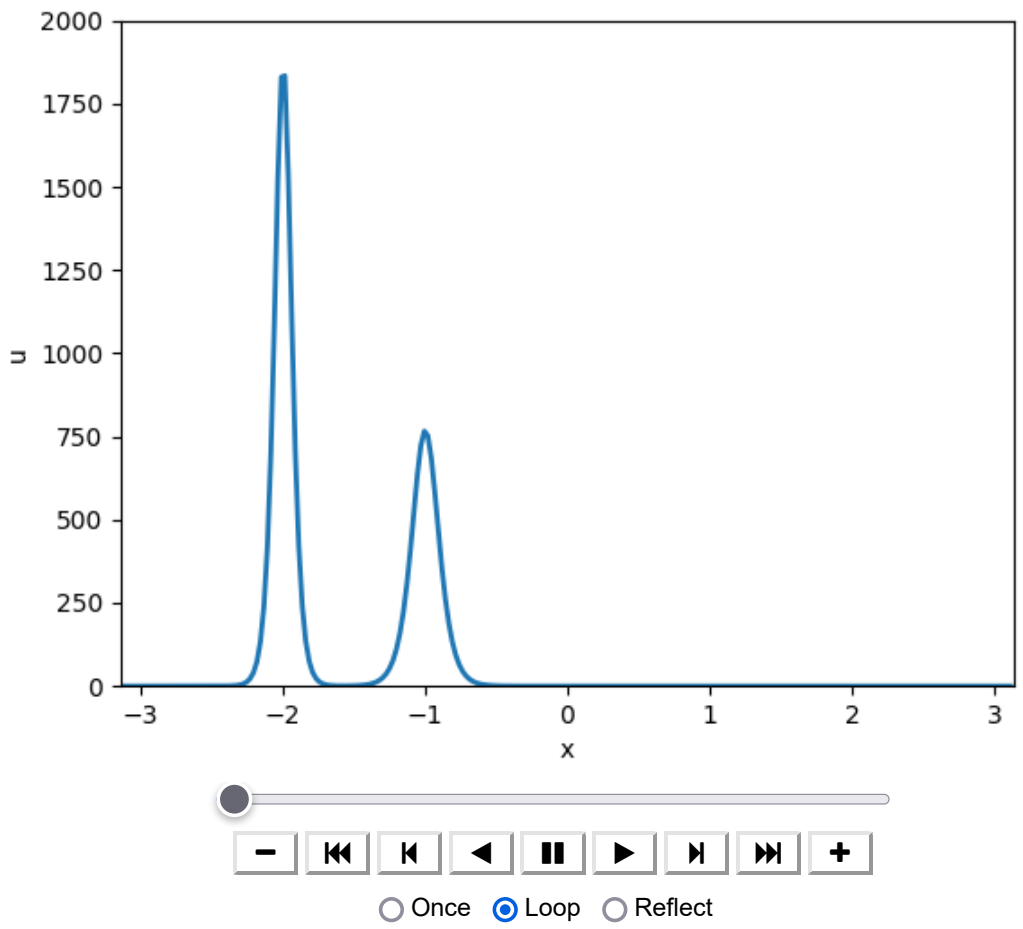
```
MovieWriter ffmpeg unavailable; using Pillow instead.
Cannot save mp4 file
```




In [19]:
```
# Use this for inline display with controls
anim

# Use this for inline display of movie
```

```
#from IPython.display import HTML
#HTML(anim.to_html5_video())
```

Out[19]:




In [ ]: