Chapter 10: nth Order Linear Homogeneous ODEs with Constant Coefficients

Discussing a procedure for solving these nth order equations using a "characteristic equation" built specifically for the problem at hand. Wolfram Alpha does not need to refer to these characteristic equations and therefore that aspect of the solving process is completely lost in this version of the solutions.

Cutting and pasting and Wolfram Alpha. Wolfram Alpha is amenable to accepting pasted entries. In this chapter pastable expressions are given a distinctive boundary fence, exemplified by the sample: !| abcdef |!

In the above pseudo-entry, only the alpha characters would be copied for transfer to Wolfram Alpha.

10.1 Solve $y''' - 6y'' + 11y' - 6y = 0$

The entry is made into Wolfram Alpha:

!| y''' - 6 * y'' + 11 * y' - 6 * y = 0 |!

and the answer is received:

$$y(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$$

In this case the solution was arrived at without generating, factoring, or finding roots of the characteristic equation. Which probably defeats the purpose of the question.

10.2 Solve $y^{(4)} - 9y'' + 20y = 0$

The entry is made into Wolfram Alpha:

!| y'''' - 9 * y'' + 20 * y = 0 |!

and the answer is received:

$$y(x) = c_1 e^{\sqrt{5}\,x} + c_2 e^{-\sqrt{5}\,x} + c_3 e^{-2x} + c_4 e^{2x}$$

(A successful way to enter derivatives of higher orders in W|A is as yet unknown.)

10.3 Solve $y' - 5y = 0$

The entry is made into Wolfram Alpha:

!| y' - 5 * y = 0 |!

and the answer is received:

$$y(x) = c_1 e^{5x}$$

10.4 Solve $y''' - 6y'' + 2y' + 36y = 0$

The entry is made into Wolfram Alpha:

!| y''' - 6 * y'' + 2 * y' + 36 * y = 0 |!

and the answer is received:

$$y(x) = c_3 e^{-2x} + c_1 e^{4x} \sin(\sqrt{2}\,x) + c_2 e^{4x} \cos(\sqrt{2}\,x)$$

10.5 Solve $\frac{d^4x}{dt^4} - 4\frac{d^3x}{dt^3} + 7\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 6x = 0$

The entry is made into Wolfram Alpha:

!| d^4x/dt^4 - 4 * d3x/dt^3 + 7 * d^2x/dt^2 - 4 * dx/dt + 6 * x = 0 |!

and the answer is received:

$$x(t) = c_1 e^{2t} \sin(\sqrt{2}\,t) + c_4 \sin(t) + c_2 e^{2t} \cos(\sqrt{2}\,t) + c_3 \cos(t)$$

In this case W|A refuses to take a hint on the independent variable and the whole thing must be written out.

10.6 Solve $y^{(4)} + 8y''' + 24y'' + 32y' + 16y = 0$

The entry is made into Wolfram Alpha:

!| y'''' + 8 * y''' + 24 * y'' + 32 * y' + 16 * y = 0 |!

and the answer is received:

$$y(x) = c_4 e^{-2x} x^3 + c_3 e^{-2x} x^2 + c_2 e^{-2x} + c_1 e^{-2x}$$

10.7 Solve $\frac{d^5 P}{dt^5} - \frac{d^4 P}{dt^4} - 2\frac{d^3 P}{dt^3} + 2\frac{d^2 P}{dt^2} + \frac{dP}{dt} - P = 0$

The entry is made into Wolfram Alpha:

!| d^5P/dt^5 - d^4P/dt^4 - 2 * d^3P/dt^3 + 2 * d^2P/dt^2 + dP/dt - P = 0 |!

and the answer is received:

$$P(t) = c_5 e^t t^2 + c_2 e^{-t} t + c_4 e^t t + c_1 e^{-t} + c_3 e^t$$

10.8 Solve $\frac{d^4 Q}{dx^4} - 8\frac{d^3 Q}{dx^3} + 32\frac{d^2 Q}{dx^2} - 64\frac{dQ}{dx} + 64Q = 0$

The entry is made into Wolfram Alpha:

!| d^4Q/dx^4 - 8 * d^3Q/dx^3 + 32 * d^2Q/dx^2 - 64 * dQ/dx + 64 * Q = 0 |!

and the answer is received:

$$Q(x) = c_1 e^{2x} \sin(2x) + c_2 e^{2x} x \sin(2x) + c_3 e^{2x} \cos(2x) + c_4 e^{2x} x \cos(2x)$$

10.15 Solve $\frac{d^4 y}{dx^4} - 4\frac{d^3 y}{dx^3} - 5\frac{d^2 y}{dx^2} + 36\frac{dy}{dx} - 36y = 0$ if one solution is $x e^{2x}$.

The entry is made into Wolfram Alpha:

!| d^4y/dx^4 - 4 * d^3y/dx^3 - 5 * d^2y/dx^2 + 36 * dy/dx - 36 * y = |!

and the answer is received:

$$y(x) = c_1 e^{-3x} + c_2 e^{2x} + c_3 e^{2x} x + c_4 e^{3x}$$

(without referring to the information about one previously known solution.)

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