(Custom CSS files are not reliable for controlling Jupyter font style. To establish the same appearance as the original notebook, depend on the browser to control the font, by setting the desired font faces in the browser settings. For example, Chrome 135 or Firefox 134 can do this. In this notebook series, Bookerly font is for markdown and Monaco is for code.)

Chapter 11: The Method of Undetermined Coefficients

This method is for use on nonhomogeneous equations. In general terms, it removes the rhs, then initiates a search for constants after consulting the homogeneous version of the equation. When relying on Wolfram Alpha to do the heavy lifting, the method is basically skipped.

Cutting and pasting and Wolfram Alpha. Wolfram Alpha is amenable to accepting pasted entries. In this chapter pastable expressions are given a distinctive boundary fence, exemplified by the sample: !| abcdef |!

In the above pseudo-entry, only the alpha characters would be copied for transfer to Wolfram Alpha.

11.1 Solve $y'' - y' - 2y = 4x^2$

The entry is made into Wolfram Alpha:

!| y'' - y' - 2 * y = 4 * x^2 |!

and the answer is received:

$$y(x) = c_1 e^{-x} + c_2 e^{2x} - 2x^2 + 2x - 3$$

11.2 Solve $y'' - y' - 2y = e^{3x}$

The entry is made into Wolfram Alpha:

!| y'' - y' - 2 * y = e^(3 * x) |!

and the answer is received:

$$y(x) = c_1 e^{-x} + c_2 e^{2x} + \frac{e^{3x}}{4}$$

11.3 Solve $y'' - y' - 2y = \sin 2x$

The entry is made into Wolfram Alpha:

!| y'' - y' -2 * y = sin(2 * x) |!

and the answer is received:

$$y(x) = c_1 e^{-x} + c_2 e^{2x} - \frac{3}{20} \sin(2x) + \frac{1}{20} \cos(2x)$$

11.4 Solve $\ddot{y} - 6\dot{y} + 25y = 2\sin\frac{t}{2} - \cos\frac{t}{2}$

The entry is made into Wolfram Alpha:

!| d^2y/dt^2 - 6 * dy/dt + 25 * y = 2 * sin(t/2) - cos(t/2) |!

and the answer is received:

$$y(t) = c_1 e^{3t} \sin(4t) + c_2 e^{3t} \cos(4t) + \frac{56}{663} \sin\left(\frac{t}{2}\right) - \frac{20}{663} \cos\left(\frac{t}{2}\right)$$

11.5 Solve $\ddot{y} - 6\dot{y} + 25y = 64e^{-t}$

The entry is made into Wolfram Alpha:

!| y'' - 6 * dy/dt + 25 * y = 64 * e^(-t) |!

and the answer is received:

$$y(t) \ = \ c_1 \, e^{3t} \, \sin(4t) \ + \ c_2 \, e^{3t} \, \cos(4t) \ + \ 2e^{-t}$$

In this case Wolfram Alpha accepts the hint about the identity of the independent variable.

11.6 Solve $\ddot{y} \ - \ 6 \, \dot{y} \ + \ 25y \ = \ 50t^3 \ - \ 36t^2 \ - \ 63t \ + \ 18$

The entry is made into Wolfram Alpha:

!| y'' - 6 * y' + 25 * y = 50 * t^3 - 36 * t^2 - 63 * t + 18 |!

and the answer is received:

$$y(t) \ = \ c_1 \, e^{3t} \, \sin(4t) \ + \ c_2 \, e^{3t} \, \cos(4t) \ + \ 2e^3 \ - \ 3t$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.7 Solve $y''' \ - \ 6y'' \ + \ 11y' \ - \ 6y \ = \ 2xe^{-x}$

The entry is made into Wolfram Alpha:

!| y''' - 6 * y'' + 11 * y' - 6 * y = 2 * x * e^-x |!

and the answer is received:

$$y(x) \ = \ c_1 \, e^x \ + \ c_2 \, e^{2x} \ + \ c_3 \, e^{3x} \ - \ \frac{1}{12} \, e^{-x} \, x \ - \ \frac{13e^{-x}}{144}$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.9 Solve $y'' \ = \ 9x^2 \ + \ 2x \ - \ 1$

The entry is made into Wolfram Alpha:

!| y'' = 9 * x^2 + 2 * x - 1 |!

and the answer is received:

$$y(x) = c_1 x + c_2 + \frac{3x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2}$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.10 Solve $y' - 5y = 2e^{5x}$

The entry is made into Wolfram Alpha:

!| y' - 5 * y = 2e^(5 * x) |!

and the answer is received:

$$y(x) = c_1 e^{5x} + 2e^{5x} x$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.12 Solve $y' - 5y = (x - 1) \sin x + (x + 1) \cos x$

The entry is made into Wolfram Alpha:

!| y' - 5 * y = (x - 1) * sin(x) + (x + 1) * cos(x) |!

and the answer is received:

$$y(x) = c_1 e^{5x} - \frac{2}{13} x \sin(x) + \frac{71 \sin(x)}{338} - \frac{3}{13} x \cos(x) - \frac{69 \cos(x)}{338}$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.13 Solve $y' - 5y = 3e^x - 2x + 1$

The entry is made into Wolfram Alpha:

!| y' - 5 * y = 3 * e^x - 2 * x + 1 |!

and the answer is received:

$$y(x) = c_1 e^{5x} - \frac{2x}{5} - \frac{3\,e^x}{4} - \frac{3}{25}$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

11.14 Solve $y' - 5y = x^2 e^x - x e^{5x}$

The entry is made into Wolfram Alpha:

!| y' - 5 * y = x^2 * e^x - x * e^(5 * x) |!

and the answer is received:

$$y(x) = c_1 e^{5x} - \frac{1}{4} e^x x^2 - \frac{1}{2} e^{5x} x^2 - \frac{e^x\,x}{8} - \frac{e^x}{32}$$

In this case Wolfram Alpha can identify independent and dependent variables by local review.

In [ ]: