Chapter 22 Inverse Laplace Transform. Introduction to a the system for accomplishing the return from the frequency domain back to the time domain. Up to this point, and including this chapter, there are no problems which involve a complete round trip through both transforms.

22.1 Find $\mathscr{L}^{-1}\left\{\frac{1}{s}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/s] |!

Result
1

At first entry Wolfram Alpha interprets this as a *unit*, (i.e. *per second*), not as a *variable*. By toggling the highlighted label, the correct interpretation is reached, and the correct answer is dispensed.

22.2 Find $\mathscr{L}^{-1}\left\{\frac{1}{s-8}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/(s - 8)] |!

Result
e^{8t}

22.3 Find $\mathscr{L}^{-1}\left\{\frac{s}{s^2+6}\right\}$

This problem can be entered into Wolfram Alpha:

!| expand[inverselaplacetransform[s/(s^2 -+ 6)]] |!

Result
$\cos\left(\sqrt{6}\,t\right)$

22.4 Find $\mathscr{L}^{-1}\left\{\frac{5s}{(s^2+1)^2}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[5 s/(s^2 + 1)^2]] |!

Result
$\frac{5t}{2}\sin(t)$

22.5 Find $\mathscr{L}^{-1}\left\{\frac{1}{\sqrt{s}}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/sqrt(s)] |!

Result
$\frac{1}{\sqrt{\pi}\sqrt{t}}$

22.6 Find $\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-9}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 1)/(s^2 - 9)]] |!

Result
$\frac{1}{3}\sinh(3t) + \cosh(3t)$

22.7 Find $\mathscr{L}^{-1}\left\{\frac{s}{(s-2)^2+9}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[s/((s - 2)^2 + 9)]] |!

Result
$\frac{1}{3}e^{2t}(2\sin(3t) + 3\cos(3t))$

Using the keyword 'simplify' caused W|A to present the results in 3 different factor groupings, the simplest of which was chosen above.

22.8 Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2-2s+9}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/(s^2 - 2 s + 9)] |!

Result

$\frac{1}{\sqrt{8}}\, e^x \sin\sqrt{8}\, x$

22.9 Find $\mathscr{L}^{-1}\left\{\frac{s+4}{s^2+4s+8}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 4)/(s^2 + 4 s +8)]] |!

Result

$e^{-2t}(\sin(2t) + \cos(2t))$

22.10 Find $\mathscr{L}^{-1}\left\{\frac{s+2}{s^2-3s+4}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 2)/(s^2 -3 s + 4)]] |!

Result

$e^{3t/2}\left(\sqrt{7}\sin\left(\frac{\sqrt{7}\,t}{2}\right) + \cos\left(\frac{\sqrt{7}\,t}{2}\right)\right)$

22.11 Use partial fractions to decompose $\frac{1}{(s+1)(s^2+1)}$

This problem can be entered into Wolfram Alpha:

!| Apart[1/((s + 1) (s^2 +1))] |!

Result

$\frac{1-s}{2(s^2+1)} + \frac{1}{2(s+1)}$

22.12 Use partial fractions to decompose $\frac{1}{(s^2+1)(s^2+4s+8)}$

This problem can be entered into Wolfram Alpha:

!| Apart[1/((s^2 + 1) (s^2 +4 s + 8))] |!

Result

$\frac{7-4s}{65(s^2+1)} + \frac{4s+9}{65(s^2+4s+8)}$

22.13 Use partial fractions to decompose $\frac{s+3}{(s-2)(s+1)}$

This problem can be entered into Wolfram Alpha:

!| Apart[(s + 3)/((s - 2) (s + 1))] |!

Result

$\frac{5}{3\,(s-2)} - \frac{2}{3\,(s+1}$

22.14 Use partial fractions to decompose $\frac{8}{s^3\,(s^2-s-2)}$.

This problem can be entered into Wolfram Alpha:

!| Apart[8/(s^3 (s^2 - s - 2))] |!

Result

$-\frac{4}{s^3} + \frac{2}{s^2} - \frac{3}{s} + \frac{8}{3\,(s+1)} + \frac{1}{3\,(s-2)}$

22.15 Find $\mathscr{L}^{-1}\left\{\frac{s+3}{(s-2)\,(s+1)}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[(s + 3)/ ((s - 2)(s + 1))] |!

Result

$$\frac{1}{3}\, e^{-t}\, (5\, e^{3t} \, - \, 2)$$

22.16 Find $\mathscr{L}^{-1}\left\{ \frac{8}{s^3(s^2 - s - 2)} \right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[8/(s^3 (s^2 - s - 2))] |!

Result

$$8\left(-\frac{t^2}{4} \, + \, \frac{t}{4} \, + \, \frac{e^{-t}}{3} \, + \, \frac{e^{2t}}{24} \, - \, \frac{3}{8}\right)$$

22.17 Find $\mathscr{L}^{-1}\left\{ \frac{1}{(s + 1)\,(s^2 + 1\,)} \right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/((s + 1) (s^2 + 1))] |!

Result

$$\frac{e^{-t}}{2} \, + \, \frac{1}{2}\, (\sin\,(t) \, - \, \cos\,(t))$$

22.18 Find $\mathscr{L}^{-1}\left\{ \frac{1}{(s^2 + 1)\,(s^2 + 4s + 8)} \right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[1/((s^2 + 1) (s^2 + 4 s + 8))]] |!

Result

$$\frac{1}{130}\, (14\, \sin\,(t) \, - \, 8\, \cos\,(t) \, + \, e^{-2t}\, (\sin\,(2t) \, + \, 8\, \cos\,(2t)))$$

22.19 Find $\mathscr{L}^{-1}\left\{ \frac{1}{s\,(s^2 + 4)} \right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[1/(s (s^2 + 4))]] |!

Result

$$\frac{1}{4}\, (1 \, - \, \cos\,(2t))$$

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