In [4]:
```
%autosave 0
```

```
Autosave disabled
```

(Custom CSS files are not reliable for controlling Jupyter font style. To establish the same appearance as the original notebook, depend on the browser to control the font, by setting the desired font faces in the browser settings. For example, Chrome 135 or Firefox 134 can do this. In this notebook series, Bookerly font is for markdown and Monaco is for code.)

Chapter 22 Inverse Laplace Transform. Introduction to a the system for accomplishing the return from the frequency domain back to the time domain. Up to this point, and including this chapter, there are no problems which involve a complete round trip through both transforms.

22.1 Find $\mathscr{L}^{-1}\left\{\frac{1}{s}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/s] |!

Result
1

At first entry Wolfram Alpha interprets this as a *unit*, (i.e. *per second*), not as a *variable*. By toggling the highlighted label, the correct interpretation is reached, and the correct answer is dispensed.

22.2 Find $\mathscr{L}^{-1}\left\{\frac{1}{s-8}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/(s - 8)] |!

Result
e^{8t}

22.3 Find $\mathscr{L}^{-1}\left\{\frac{s}{s^2+6}\right\}$

This problem can be entered into Wolfram Alpha:

!| expand[inverselaplacetransform[s/(s^2 -+ 6)]] |!

Result

$\cos\left(\sqrt{6}\,t\right)$

22.4 Find $\mathscr{L}^{-1}\left\{\frac{5s}{(s^2+1)^2}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[5 s/(s^2 + 1)^2]] |!

Result

$\frac{5t}{2}\sin(t)$

22.5 Find $\mathscr{L}^{-1}\left\{\frac{1}{\sqrt{s}}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/sqrt(s)] |!

Result

$\frac{1}{\sqrt{\pi}\sqrt{t}}$

22.6 Find $\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-9}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 1)/(s^2 - 9)]] |!

Result

$$\frac{1}{3}\sinh(3t) + \cosh(3t)$$

22.7 Find $\mathscr{L}^{-1}\left\{\frac{s}{(s-2)^2+9}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[s/((s - 2)^2 + 9)]] |!

Result

$$\frac{1}{3}e^{2t}(2\sin(3t) + 3\cos(3t))$$

Using the keyword 'simplify' caused W|A to present the results in 3 different factor groupings, the simplest of which was chosen above.

22.8 Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2-2s+9}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/(s^2 - 2 s + 9)] |!

Result

$$\frac{1}{\sqrt{8}}e^{x}\sin\sqrt{8}\,x$$

22.9 Find $\mathscr{L}^{-1}\left\{\frac{s+4}{s^2+4s+8}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 4)/(s^2 + 4 s +8)]] |!

Result

$$e^{-2t}(\sin(2t) + \cos(2t))$$

22.10 Find $\mathscr{L}^{-1}\left\{\frac{s+2}{s^2-3s+4}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[(s + 2)/(s^2 -3 s + 4)]] |!

Result

$$e^{3t/2}\left(\sqrt{7}\sin\left(\frac{\sqrt{7}\,t}{2}\right)+\cos\left(\frac{\sqrt{7}\,t}{2}\right)\right)$$

22.11 Use partial fractions to decompose $\frac{1}{(s+1)(s^2+1)}$

This problem can be entered into Wolfram Alpha:

!| Apart[1/((s + 1) (s^2 +1))] |!

Result

$$\frac{1-s}{2(s^2+1)}+\frac{1}{2(s+1)}$$

22.12 Use partial fractions to decompose $\frac{1}{(s^2+1)(s^2+4s+8)}$

This problem can be entered into Wolfram Alpha:

!| Apart[1/((s^2 + 1) (s^2 +4 s + 8))] |!

Result

$$\frac{7-4s}{65(s^2+1)}+\frac{4s+9}{65(s^2+4s+8)}$$

22.13 Use partial fractions to decompose $\frac{s+3}{(s-2)(s+1)}$

This problem can be entered into Wolfram Alpha:

!| Apart[(s + 3)/((s - 2) (s + 1))] |!

Result

$$\frac{5}{3\,(s-2)}-\frac{2}{3\,(s+1}$$

22.14 Use partial fractions to decompose $\frac{8}{s^3\,(s^2-s-2)}$.

This problem can be entered into Wolfram Alpha:

!| Apart[8/(s^3 (s^2 - s - 2))] |!

Result

$$-\frac{4}{s^3}+\frac{2}{s^2}-\frac{3}{s}+\frac{8}{3\,(s+1)}+\frac{1}{3\,(s-2)}$$

22.15 Find $\mathscr{L}^{-1}\left\{\frac{s+3}{(s-2)\,(s+1)}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[(s + 3)/ ((s - 2)(s + 1))] |!

Result

$$\frac{1}{3}\,e^{-t}\,(5\,e^{3t}-2)$$

22.16 Find $\mathscr{L}^{-1}\left\{\frac{8}{s^3(s^2-s-2)}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[8/(s^3 (s^2 - s - 2))] |!

Result

$$8\left(-\frac{t^2}{4}+\frac{t}{4}+\frac{e^{-t}}{3}+\frac{e^{2t}}{24}-\frac{3}{8}\right)$$

22.17 Find $\mathscr{L}^{-1}\left\{\frac{1}{(s+1)\,(s^2+1)}\right\}$

This problem can be entered into Wolfram Alpha:

!| inverselaplacetransform[1/((s + 1) (s^2 + 1))] |!

Result

$$\frac{e^{-t}}{2} + \frac{1}{2}(\sin(t) - \cos(t))$$

22.18 Find $\mathscr{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4s+8)}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[1/((s^2 + 1) (s^2 + 4 s + 8))]] |!

Result

$$\frac{1}{130}(14\sin(t) - 8\cos(t) + e^{-2t}(\sin(2t) + 8\cos(2t)))$$

22.19 Find $\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$

This problem can be entered into Wolfram Alpha:

!| simplify[inverselaplacetransform[1/(s (s^2 + 4))]] |!

Result

$$\frac{1}{4}(1 - \cos(2t))$$

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