Chapter 19 Further Numerical Methods for Solving 1st Order Differential Equations. For this section further numerical methods simply refers to a certain procedure for entering successive problems into Wolfram Alpha.

19.1 Use the modified Euler's method to solve $y' = y - x; \quad y(0) = 2$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y - x; y(0) = 2} from 0 to 1 using Dormand-Prince method |!

Of ten numerical methods available to W|F, the D-P method is the most accurate, with a global error on the present problem listed as -6.34×10^{-9}. For this problem Wolfram Alpha happens to have the exact solution available. The idea is that by entering the D-P method as the preferred option, the most accurate result will be obtained in case the exact solution is not available for some reason.

Exact solution: $y(x) = x + e^x + 1$

19.2 Use the modified Euler's method to solve $y = y^2 + 1; \quad y(0) = 0$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y^2 + 1; y(0) = 0} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of 1.62×10^{-8} if the exact solution had not been available.

Exact solution: $y(x) = \tan(x)$

19.3 Find $y(1.6)$ for $y, = 2x; \quad y(1) = 1$ using the modified Euler's method with $h = 0.2$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = 2 * x; y(1) = 1} using Dormand-Prince method |!

The Dormand-Prince method, along with a few others, would have produced a global error of zero if the exact solution had not been available.

Exact solution: $y(x) = x^2$

$(1.6)^2 = 2.56$

19.4 Use the Runge-Kutta method to solve $y' = y - x; \quad y(0) = 2$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y - x; y(0) = 2} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of -6.34×10^{-9} if the exact solution had not been available.

Exact solution: $y(x) = x + e^x + 1$

19.5 Use the Runge-Kutta method to solve $y' = y; \quad y(0) = 1$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y; y(0) = 1} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of -5.48×10^{-9} if the exact solution had not been available.

Exact solution: $y(x) = e^x$

19.6 Use the Runge-Kutta method to solve $y' = y^2 + 1; \quad y(0) = 0$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y^2 + 1; y(0) = 0} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of 1.62×10^{-8} if the exact solution had not been available.

Exact solution: $y(x) = \tan(x)$

19.7 Use the Adams-Bashforth-Moulton method to solve $y' = y - x; \quad y(0) = 2$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y - x; y(0) = 2} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of -6.34×10^{-9} if the exact solution had not been available.

Exact solution: $y(x) = x + e^x + 1$

19.8 Use the Adams-Bashforth-Moulton method to solve $y' = y^2 + 1; \quad y(0) = 0$ on the interval [0, 1] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = y^2 + 1; y(0) = 0} from 0 to 1 using Dormand-Prince method |!

The Dormand-Prince method would have produced a global error of 1.62×10^{-8} if the exact solution had not been available.

Exact solution: $y(x) = \tan(x)$

19.9 Use the Adams-Bashforth-Moulton method to solve $y' = 2xy/(x^2 - y^2); \quad y(1) = 3$ on the interval [1, 2] with $h = 0.1$.

This problem can be fed to Wolfram Alpha:

!| solve {y' = (2 * x * y)/(x^2 - y^2); y(1) = 3} from 1 to 2 using Dormand-Prince method |!

The potential error ranges for various numerical methods are not available for this problem.

Exact solution: $y(x) \; = \; \frac{1}{3}\left(\sqrt{25 \; - 9x^2} \; + \; 5\right)$

The text shows a direction field plot for this function, so to match that, a plot is shown below. Two points about the plot: (a) runtime warnings have been suppressed, (b) strange squiggly lines herald something (?).

In [77]:

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import numpy as np
import matplotlib.pyplot as plt

%config InlineBackend.figure_formats = ['svg']

import warnings
with warnings.catch_warnings():
    warnings.simplefilter('ignore')

# Creating dataset
w = 4
Y, X = np.mgrid[-w:w:100j, -w:w:100j]
U = np.ones_like(X) #dxdt = 1
V = 2*X*Y/(X**2 - Y**2)
speed = np.sqrt(U**2 + V**2)

seek_points = np.array( [[-2,2.5,-3,3, 2,  -2,  -1,  -.5, 0, 0, 0,0],
                        [0,  0,   2,2,-.75,-.75,-1.3,-.75,-3, 1, 2,3]])

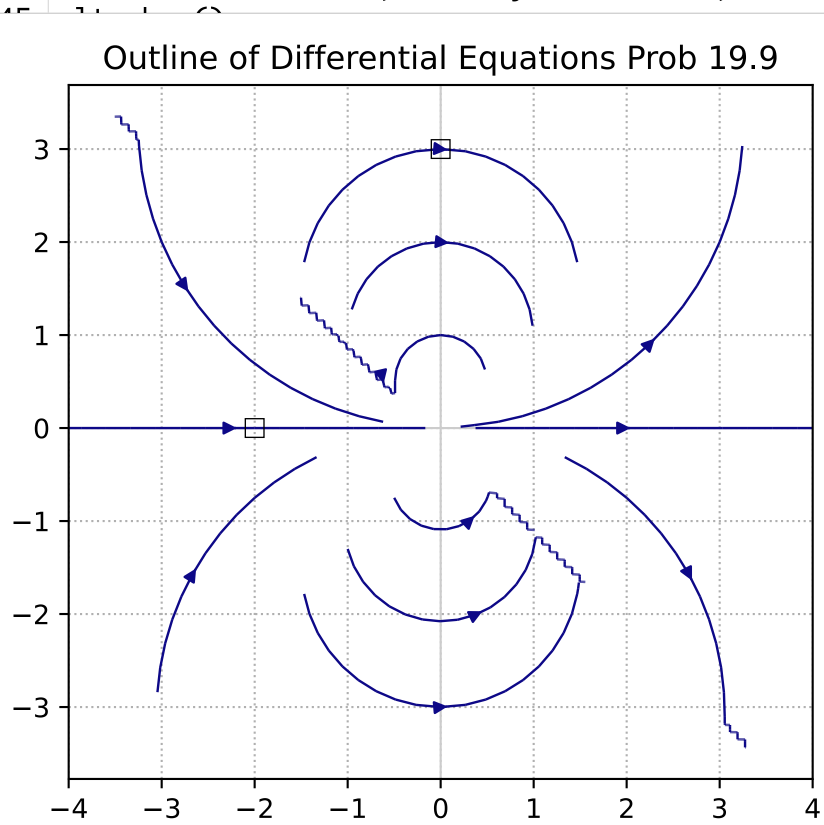

fig, ax = plt.subplots()
ax.grid(True, which='both', linestyle='dotted')
ax.axhline(y=0, color='0.8', linewidth=0.8)
ax.axvline(x=0, color='0.8', linewidth=0.8)

ratio = 1.0
#ratio is adjusted by eye to get squareness of x and y spacing
xleft, xright = ax.get_xlim()
ybottom, ytop = ax.get_ylim()
ax.set_aspect(abs((xright-xleft)/(ybottom-ytop))*ratio)


strm = ax.streamplot(X, Y, U, V, color = U,
                     linewidth = 0.9,
                     cmap ='plasma',
                     start_points = seek_points.T)

plt.title("Outline of Differential Equations Prob 19.9")
plt.rcParams['figure.figsize'] = [5, 5]

xpts = np.array([-2, 0])
ypts = np.array([0, 3])
plt.plot(xpts, ypts, markersize=7, color='k', marker='s', \
         mfc='none', linestyle = 'none', markeredgewidth=0.5)
```



In [ ]:

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In [ ]:

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