In [1]:

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Defining Parabolic PDE's

•The general form for a second order linear PDE with two independent variables (x, y) and one dependent variable (u) is

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

•Recall the criteria for an equation of this type to be considered parabolic

$$B^2 - 4AC = 0$$

•For example, examine the heat-conduction equation given by

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where $A = \alpha$, B = 0, C = 0 and D = -1

then

$$B^2 - 4AC = 0 - 4(\alpha)(0) = 0$$

thus allowing us to classify the heat equation as parabolic.

With the finite difference implicit method solve the heat problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t} + x - t$$

with initial condition:

$$u(0, x) = \sin(x)$$

and boundary conditions:

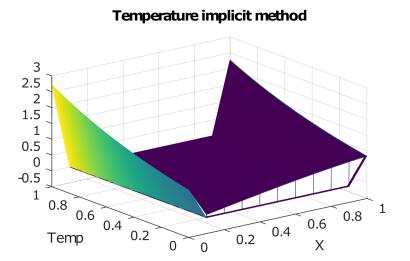
$$u(t, 0) = e^t, u(t, 1) = e^t \sin 1$$

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In [ ]:
         1 clear;
          2 L = 1.; \% Lenth of the wire 0 < x < L
          3 \mid T = 1; % Number of space steps 0 < t < T
          4 | % Parameters needed to solve the equation within the fully implicit method.
          5 maxk = 1000; % Number of time steps
          6 dt = T/maxk;
          7 \mid n = 10; % Number of space steps
          8 dx = L/n;
         10 b = (a^2)*dt/(dx*dx); \% b Parameter of the method
         11 % Initial temperature of the wire:
         12 for i = 1:n+1
         13
                x(i) = (i-1)*dx;
                 u(i,1) = sin(x(i));
         15 end
        16 % Temperature at the boundary
        17 | for t=1:maxk+1
                 time(t) = (t-1)*dt;
         18
         19
                 u(1,t) = exp(time(t));
         20
                 u(n+1,t) = \sin(1)*\exp(time(t));
         21
         22 end
         23 | % Implicit Method
         24 | aa(1:n-1) = -b;
         25 b1=-b;
         26 |bb(1:n-1) = 1.+2.*b;
         27 a1=1.+2.*b;
         28 | cc(1:n-1) = -b;
         29 c1=-b;
         30 for t = 2:maxk
                                 % Time loop
         31
                 uu = u(2:n,t) + dt*(x(2:n)-time(t)).';
         32
                 v = zeros(n-1,1);
         33
                w = a1;
         34
                 u(2,t) = uu(1)/w;
         35
                 for i=2:(n-1)
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36
            v(i-1) = c1/w;
            w = a1 - b1*v(i-1);
37
            u(i+1,t) = (uu(i) - b1*u(i,t))/w;
38
39
        end
40
        for j=(n-2):-1:1
41
            u(j+1,t) = u(j+1,t) - v(j)*u(j+2,t);
42
        end
43
44 end
45 % Graphical representation of the temperature at different selected times
46 subplot(2,2,3);
47 plot(x,u(:,1),'-',x,u(:,10),'-',x,u(:,45),'-',x,u(:,30),'-',x,u(:,60),'-')
48 title('Temperature implicit method')
49 xlabel('X')
50 ylabel('T')
51 subplot(2,2,4);
52 mesh(x,time,u')
53 title('Temperature implicit method')
54 xlabel('X')
55 ylabel('Temp')
<u>56</u>
```

1.4 1.2 1 0.8 0.6 0.4 0.2 0 0 0.2 0.4 0.6 0.8 1 X



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