

In [1]:

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Defining Parabolic PDE's

•The general form for a second order linear PDE with two independent variables (x, y) and one dependent variable (u) is

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D = 0$$

•Recall the criteria for an equation of this type to be considered parabolic

$$B^2 - 4AC = 0$$

•For example, examine the heat-conduction equation given by

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where $A = \alpha$, $B = 0$, $C = 0$ and $D = -1$

then

$$B^2 - 4AC = 0 - 4(\alpha)(0) = 0$$

thus allowing us to classify the heat equation as parabolic.

With the finite difference implicit method solve the heat problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t} + x - t$$

with initial condition:

$$u(0, x) = \sin(x)$$

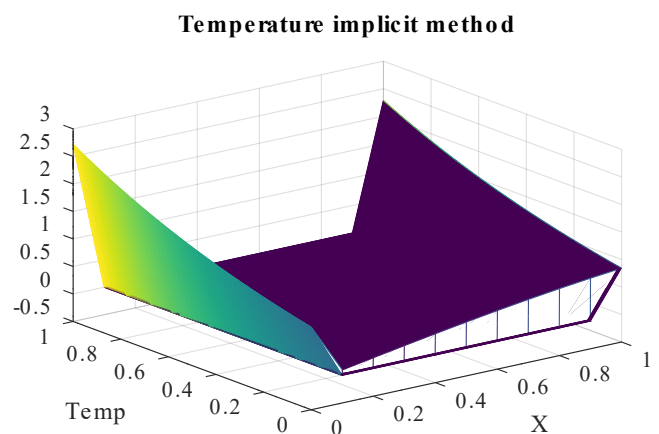
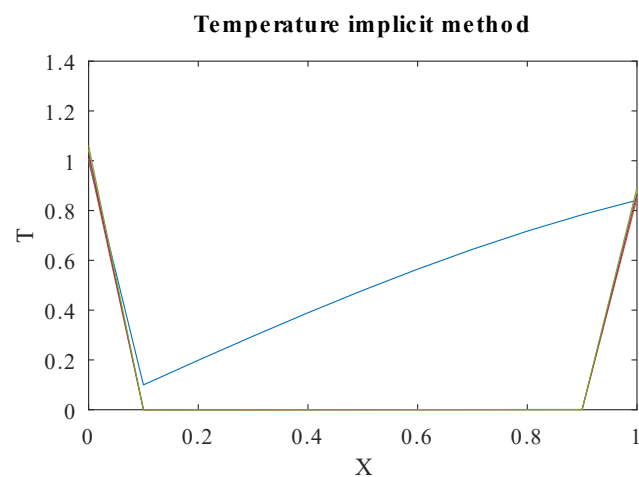
and boundary conditions:

$$u(t, 0) = e^t, u(t, 1) = e^t \sin 1$$

In []:

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1 clear;
2 L = 1.; % Lenth of the wire 0<x<L
3 T =1; % Number of space steps 0<t<T
4 % Parameters needed to solve the equation within the fully implicit method.
5 maxk = 1000; % Number of time steps
6 dt = T/maxk;
7 n = 10; % Number of space steps
8 dx = L/n;
9 a = 1;
10 b = (a^2)*dt/(dx*dx); % b Parameter of the method
11 % Initial temperature of the wire:
12 for i = 1:n+1
13     x(i) =(i-1)*dx;
14     u(i,1) =sin(x(i));
15 end
16 % Temperature at the boundary
17 for t=1:maxk+1
18     time(t) = (t-1)*dt;
19     u(1,t) = exp(time(t));
20     u(n+1,t) = sin(1)*exp(time(t));
21 end
22 % Implicit Method
23 aa(1:n-1) = -b;
24 b1=-b;
25 bb(1:n-1) = 1.+2.*b;
26 a1=1.+2.*b;
27 cc(1:n-1) = -b;
28 c1=-b;
29 for t = 2:maxk % Time loop
30     uu = u(2:n,t) + dt*(x(2:n)-time(t)).';
31     v = zeros(n-1,1);
32     w = a1;
33     u(2,t) = uu(1)/w;
34     for i=2:(n-1)
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36         v(i-1) = c1/w;
37         w = a1 - b1*v(i-1);
38         u(i+1,t) = (uu(i) - b1*u(i,t))/w;
39     end
40     for j=(n-2):-1:1
41         u(j+1,t) = u(j+1,t) - v(j)*u(j+2,t);
42     end
43
44 end
45 % Graphical representation of the temperature at different selected times
46 subplot(2,2,3);
47 plot(x,u(:,1),'-',x,u(:,10),'-',x,u(:,45),'-',x,u(:,30),'-',x,u(:,60),'-')
48 title('Temperature implicit method')
49 xlabel('X')
50 ylabel('T')
51 subplot(2,2,4);
52 mesh(x,time,u')
53 title('Temperature implicit method')
54 xlabel('X')
55 ylabel('Temp')
56
57
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