Chapter 15 Matrices: Matrix functions and operations.

15.1 Show that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Show that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ for the two matrices in the description.

The Wolfram Alpha entry lines go like:
!| {{5, 6}, {7, 8}} + {{1, 2}, {3, 4}} |!
and
!| {{1, 2}, {3, 4}} + {{5, 6}, {7, 8}} |!

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

15.2 Find $\mathbf{A} - \frac{1}{2}\,\mathbf{B}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

A subtraction example.

The Wolfram Alpha entry line goes like:
!| {{1, 2}, {3, 4}} - (1/2) * {{5, 6}, {7, 8}} |!

$$\begin{bmatrix} -\frac{3}{2} & -1 \\ -\frac{1}{2} & 0 \end{bmatrix}$$

15.3 Find $\mathbf{AB}$ and $\mathbf{BA}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Commutation of multiplication trial.

The Wolfram Alpha entry line goes like:
!| {{1, 2}, {3, 4}} * {{5, 6}, {7, 8}} |!
and
!| {{5, 6}, {7, 8}} * {{1, 2}, {3, 4}} |!

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \qquad \mathbf{B} \times \mathbf{A} = \begin{bmatrix} 23 & 24 \\ 31 & 46 \end{bmatrix}$$

15.4 Find $(2\mathbf{A} - \mathbf{B})^2$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Polynomial squared trial.

For some reason, the Wolfram Alpha entry line needs to be split up into two parts:
!| (2 * {{1, 2}, {3, 4}} - {{5, 6}, {7, 8}}) |!

to get the intermediate result of {{-3, -2}, {-1, 0}}, then

!| {{-3, -2}, {-1, 0}} ^2 |! to get the final answer, which is shown.

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 11 & 6 \\ 3 & 2 \end{bmatrix}$$

15.5 Find $\mathbf{AB}$ and $\mathbf{BA}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 7 & 0 \\ 8 & -1 \end{bmatrix}$$

The operation of multiplication is not defined for 3-column matrix times a 2-row matrix. However, looking at $\mathbf{BA}$, below is the outcome.

The entry into Wolfram Alpha goes like:
{{7, 0}, {8, -1}} * {{1, 2, 3}, {4, 5, 6}}

$$\mathbf{B} \times \mathbf{A} = \begin{bmatrix} 7 & 14 & 21 \\ 4 & 11 & 18 \end{bmatrix}$$

15.6 Find $\mathbf{AB}$ and $\mathbf{AC}$ if

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -2 & -2 \\ -1 & 2 & 1 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 3 & 1 & -3 \\ 0 & 2 & 6 \\ -1 & 2 & 1 \end{bmatrix}$$

In [48]:
```python
from scipy import linalg
import numpy as np

a = np.array([[4, 2, 0], [2, 1, 0], [-2, -1, 1]])
b = np.array([[2, 3, 1], [2, -2, -2], [-1, 2, 1]])
c = np.array([[3, 1, -3], [0, 2, 6], [-1, 2, 1]])

#of the 3 ways to multiply arrays in python,
#"matmul" gives the "matrix product"
#"multipy" returns element-wise multiplication
#"dot" returns dot product


res1 = np.matmul(a, b)
res2 = np.matmul(a, c)


#for line in res:
    #print (' '.join(map(str, line)))
print(res1)
print()
print(res2)
```
```
[[12  8  0]
 [ 6  4  0]
 [-7 -2  1]]

[[12  8  0]
 [ 6  4  0]
 [-7 -2  1]]
```

15.7 Find $\mathbf{Ax}$ if

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} 9 \\ -1 \\ -2 \\ 0 \end{bmatrix}$$

In [51]:
```python
from scipy import linalg
import numpy as np

a = np.array([[1, 2, 3, 4], [5, 6, 7, 8]])
x = np.array([[9], [-1], [-2], [0]])

#of the 3 ways to multiply arrays in python,
#"matmul" gives the "matrix product"
#"multipy" returns element-wise multiplication
#"dot" returns dot product


res1 = np.matmul(a, x)

#for line in res:
    #print (' '.join(map(str, line)))
print(res1)
print()
```
```
[[ 1]
 [25]]
```

15.8 Find $\dfrac{d\mathbf{A}}{dt}$ if

$$\mathbf{A} = \begin{bmatrix} t^2 + 1 & e^{2t} \\ \sin t & 45 \end{bmatrix}$$

The derivative of the matrix can be found with Wolfram Alpha. Using the entry

!| d[{{t^2 + 1, e^(2 * t)}, {sin(t), 45}}]/dt |!

the result is returned:

$$\mathbf{A} = \begin{bmatrix} 2t & 2e^{2t} \\ \cos t & 0 \end{bmatrix}$$

Also offered are expressions for the eigenvalues and eigenvectors, for which there is no present need.

15.9 Find $\dfrac{d\mathbf{x}}{dt}$ if

$$\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

The derivative of the matrix can be found with Wolfram Alpha. Using the entry

!| differentiate {{x1(t)}, {x2(t)}, {x3(t)}} with respect to t |!

the result is returned:

$$\text{d/dt } \{\{x1(t)\}, \{x2(t)\}, \{x3(t)\}\} = \begin{bmatrix} x1'(t) \\ x2'(t) \\ x3'(t) \end{bmatrix}$$

Here W|A is prone to ignore the underbar or subscript form, but will cooperate if the condensed form shown above is used.

15.10 Find $\int \mathbf{A}\, dt$ for $\mathbf{A}$ as given in Problem 15.8.

Wolfram Alpha flatly refuses to integrate over the matrix. Luckily it is an easy matter to integrate component-wise by hand. The matrix that emerges is:

$$\int \mathbf{A} dt = \begin{bmatrix} t^3/3 + t + c_1 & (e^{2t})/2 + c_2 \\ \text{-cos } t + c_3 & 45t + c_4 \end{bmatrix}$$

15.11   Find $\int_0^1 \mathbf{x}\, dt$ if

$$\mathbf{x} = \begin{bmatrix} 1 \\ e^t \\ 0 \end{bmatrix}$$

Again W|A refuses to do the integration. But again it is an easy process and results in:

$$\int \mathbf{x}\, dt = \begin{bmatrix} t + c_1 \\ e^t + c_2 \\ c_3 \end{bmatrix}$$

The above needs to be evaluated at the upper bound of 1 and at the lower bound of 0, with the answer being the difference between the two. That turns out to be:

$$\mathbf{evaluated} = \begin{bmatrix} 1 \\ e - 1 \\ 0 \end{bmatrix}$$

15.12   Find the eigenvalues of

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

```
In [7]:  import numpy as np

         a = np.array([[1, 3], [4, 2]])
         a = np.array([[2*t, 5*t], [-1*t, -2*t]])
         w, v = np.linalg.eig(a)

         print(w)
         print(v)

         [-2.  5.]
         [[-0.70710678 -0.6        ]
          [ 0.70710678 -0.8        ]]
```

15.13   Find the eigenvalues of $\mathbf{A}t$ if

$$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$$

The eigenvalues can be found by Wolfram Alpha. If the entry is made:
!| eigenvalues {{2 * t, 5 * t}, {-1 * t, -2 * t}} |!

then the response is

Results: $\lambda = it$ and $\lambda = -it$
Corresponding eigenvectors: $v_1 = (-2 - i,\ 1)$ and $(-2 + i,\ 1)$

The t factor in the problem expression makes getting an answer more difficult when Python is used than when W|A is used.

15.14   Find the eigenvalues of $\mathbf{A}$ if

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & 1 & -3 \end{bmatrix}$$

The eigenvalues can be found by Wolfram Alpha. If the entry is made:
!| eigenvalues {{4, 1, 0}, {-1, 2 , 0}, {2, 1, -3}} |!

then the response is

Results: $\lambda_1 = -3$ and $\lambda_2 = 3$
Corresponding eigenvectors: $v_1 = (0,\ 0,\ 1)$ and $v_2 = (6,\ -6,\ 1)$

15.15   Find the eigenvalues of $\mathbf{A}$ if

$$\mathbf{A} = \begin{bmatrix} 5 & 7 & 0 & 0 \\ -3 & -5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The eigenvalues can be found by Wolfram Alpha. If the entry is made:
!| eigenvalues {{5, 7, 0, 0}, {-3, 5, 0, 0}, {0, 0, 2, 1}, {0, 0, 0, 2}} |!

then the response is

Results: $\lambda_1 = -2$ and $\lambda_2 = 2$ and $\lambda_3 = 2$

Corresponding eigenvectors: $v_1 = (-1,\ 1,\ 0,\ 0)$ and $v_2 = (0,\ 0,\ 1,\ 0)$ and $v_3 = (-7,\ 3,\ 0,\ 0)$

```
In [ ]:
```

```
In [ ]:
```