

(Custom CSS files are not reliable for controlling Jupyter font style. To establish the same appearance as the original notebook, depend on the browser to control the font, by setting the desired font faces in the browser settings. For example, Chrome 135 or Firefox 134 can do this. In this notebook series, Bookerly font is for markdown and Monaco is for code.)

Chapter 17 Reduction of Linear Differential Equations to a System of First-Order Equations.
Symbolic substitutions for achieving a system of tractable 1st order equations. The Problems do not ask for a solution; they merely require that the subject equations be broken down into a system of 1st order equations according to the system described. The problem treatment in this notebook is simply to see which equations can be solved outright by Wolfram Alpha.

17.1 Put the initial value problem

$$\ddot{x} + 2\dot{x} - 8x = e^t; \quad x(0) = 1, \quad \dot{x}(0) = -4$$

into the form of System (17.7).

The differential equation can be entered into Wolfram Alpha:

!! x'' + 2 * x' - 8 * x = e^t, x(0) = 1, x'(0) = -4 !!

and a solution equation received:

$$x(t) = \frac{1}{30} e^{-4t} (-6e^{5t} + 5 e^{6t} + 31)$$

17.2 Put the initial value problem

$$\ddot{x} + 2\dot{x} - 8x = 0; \quad x(1) = 2, \quad \dot{x}(1) = 3$$

into the form of System (17.7).

The differential equation can be entered into Wolfram Alpha:

!! x'' + 2 * x' - 8 * x = 0, x(1) = 2, x'(1) = 3 !!

and a solution equation received:

$$x(t) = \frac{1}{6} e^{-4t-2} (11e^{6t} + e^6)$$

17.3 Put the initial value problem

$$\ddot{x} + x = 3; \quad x(\pi) = 1, \quad \dot{x}(\pi) = 2$$

into the form of System (17.7).

The differential equation can be entered into Wolfram Alpha:

!! x'' + x = 3, x(pi) = 1, x'(pi) = 2 !!

and a solution equation received:

$$x(t) = -2 \sin(t) + 2 \cos(t) + 3$$

17.6 Put the initial value problem

$$e^{-t} \frac{d^4 x}{dt^4} - \frac{d^2 x}{dt^2} + e^t t^2 \frac{dx}{dt} = 5e^{-t}; \quad x(1) = 2, \quad \dot{x}(1) = 3, \quad \ddot{x}(1) = 4, \quad \dddot{x}(1) = 5$$

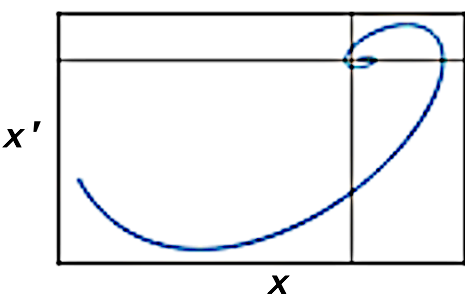
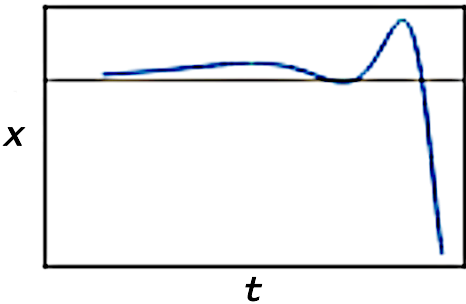
into the form of System (17.7).

The differential equation can be entered into Wolfram Alpha:

!! e^(-t) * d^4x/dt^4 - d^2x/dt^2 + e^t * t^2 * dx/dt = 5 * e^(-t), x(1)=2, x'(1)=3, x''(1)=4, x'''(1)=5 !!

but no solution equation is received, presumably because of computation time exceeded. (W|A does give a small example of the solution plot, shown below.) Maxima also fails to solve it, with the message "Not a proper differential equation. False". However, MMA10 also would not solve the problem, so maybe it is a difficult one.

Plots of the solution



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