In [1]:

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Autosave disabled
```

Defining Parabolic PDE's

•The general form for a second order linear PDE with two independent variables (x, y) and one dependent variable (u) is

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

•Recall the criteria for an equation of this type to be considered parabolic

$$B^2 - 4AC = 0$$

•For example, examine the heat-conduction equation given by

$$\alpha\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where $A = \alpha, B = 0, C = 0$ and $D = -1$

then

$$B^2 - 4A\,C = 0 - 4(\alpha)(0) = 0$$

thus allowing us to classify the heat equation as parabolic.

With the finite difference implicit method solve the heat problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t} + x - t$$

with initial condition:

$$u(0, x) = \sin(x)$$

and boundary conditions:

$$u(t, 0) = e^t,\ u(t, 1) = e^t \sin 1$$

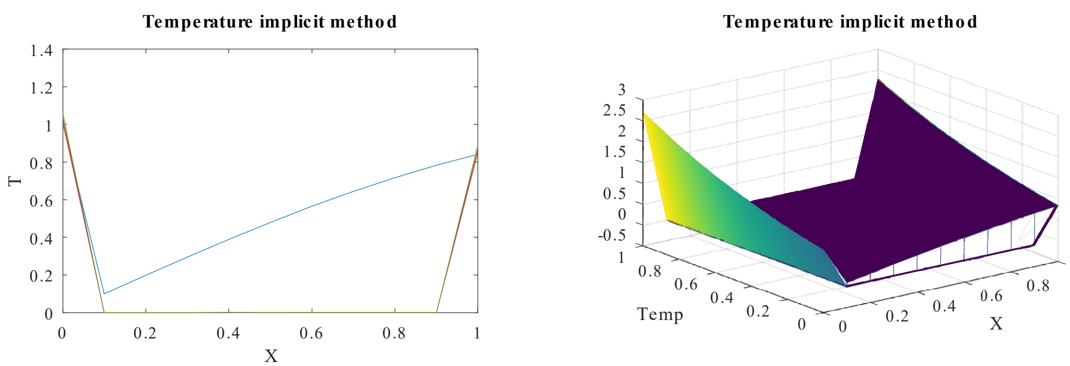
In [ ]:

```
clear;
L = 1.; % Lenth of the wire 0<x<L
T =1; % Number of space steps  0<t<T
% Parameters needed to solve the equation within the fully implicit method.
maxk = 1000; % Number of time steps
dt = T/maxk;
n = 10; % Number of space steps
dx = L/n;
a = 1;
b = (a^2)*dt/(dx*dx); % b Parameter of the method
% Initial temperature of the wire:
for i = 1:n+1
    x(i) =(i-1)*dx;
    u(i,1) =sin(x(i));
end
% Temperature at the boundary
for t=1:maxk+1
    time(t) = (t-1)*dt;
    u(1,t) = exp(time(t));
    u(n+1,t) = sin(1)*exp(time(t));

end
% Implicit Method
aa(1:n-1) = -b;
b1=-b;
```

```
bb(1:n-1) = 1.+2.*b;
a1=1.+2.*b;
cc(1:n-1) = -b;
c1=-b;
for t = 2:maxk      % Time loop
    uu = u(2:n,t) + dt*(x(2:n)-time(t)).';
    v = zeros(n-1,1);
    w = a1;
    u(2,t) = uu(1)/w;
    for i=2:(n-1)
        v(i-1) = c1/w;
        w = a1 - b1*v(i-1);
        u(i+1,t) = (uu(i) - b1*u(i,t))/w;
    end
    for j=(n-2):-1:1
        u(j+1,t) = u(j+1,t) - v(j)*u(j+2,t);
    end

end
% Graphical representation of the temperature at different selected times
subplot(2,2,3);
plot(x,u(:,1),'-',x,u(:,10),'-',x,u(:,45),'-',x,u(:,30),'-',x,u(:,60),'-')
title('Temperature implicit method')
xlabel('X')
ylabel('T')
subplot(2,2,4);
mesh(x,time,u')
title('Temperature implicit method')
xlabel('X')
```



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