1 - 5 Components and length

Find the components of the vector v with initial point P and terminal point Q. Find **|v|**. Sketch **|v|**. Find the unit vector **u** in the direction of **v**.

1. P : (1, 1, 0), Q : (6, 2, 0)

```
Clear["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {1, 1, 0}; qQ = {6, 2, 0};
vec = qQ - pP
```

```
{5, 1, 0}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

$\sqrt{26}$

Below: find normalized version of vector.

```
euc2 = Normalize[vec]
```

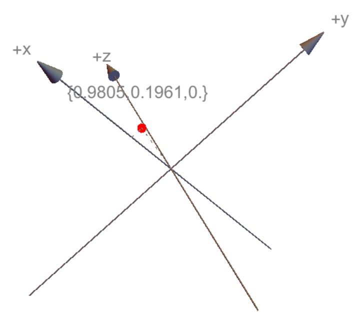
$\left\{\frac{5}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 0\right\}$

The green cells above match the answers in the text.

Below: find normalized version in decimal form.

```
euc3 = N[Normalize[vec], 4]
{0.9806, 0.1961, 0}
```

```
pad = N[euc3 + {.3, .3, .3}, 4]
{1.28058, 0.496116, 0.3}
```



3. P : (-3.0, 4.0, -0.5), Q : (5.5, 0, 1.2)

```
Clear["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {-3.0, 4.0, -0.5}; qQ = {5.5, 0, 1.2};
vec = qQ - pP
```

```
{8.5, -4., 1.7}
```

Below: calculate length of vector.

```
eucl = Norm[vec]
```

```
9.54673
```

```
PossibleZeroQ[Chop[eucl] - Chop[√91.14]]
True
```

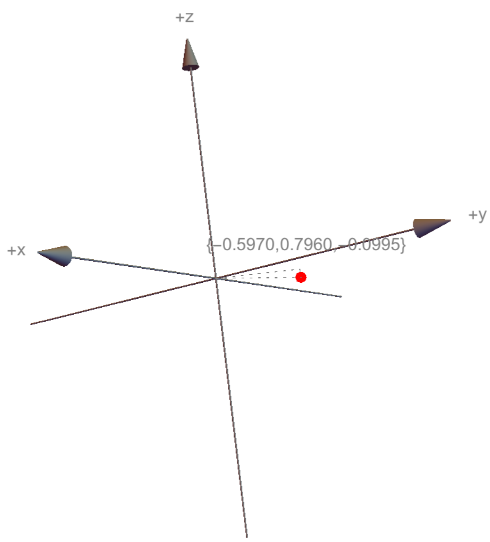
Below: find normalized version of vector.

```
euc2 = N[Normalize[vec], 4]
```

```
{0.890357, -0.418992, 0.178071}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
{1.19036, -0.118992, 0.478071}
```

The green cells above match the answers in the text.



5. P : (0, 0, 0), Q : (2, 1, -2)

```
Clear["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {0, 0, 0}; qQ = {2, 1, -2};
vec = qQ - pP
```

```
{2, 1, -2}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
3
```

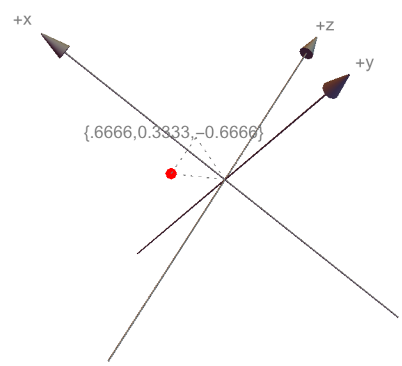
Below: find normalized version of vector.

```
euc2 = N[Chop[Normalize[vec], 10^-4]]
```

```
{0.666667, 0.333333, -0.666667}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
{0.966667, 0.633333, -0.366667}
```

The green cells above match the answers in the text (no entry for vector length).



6 - 10 Find the terminal point Q of the vector **v** with components as given and initial point P. Find $|\mathbf{v}|$.

7. $\frac{1}{2}$, 3, $-\frac{1}{4}$; P : $\left(\frac{7}{2}, -3, \frac{3}{4}\right)$

```
Clear["Global`*"]
```

$$vec = \left\{\frac{1}{2}, 3, -\frac{1}{4}\right\};\ po = \left\{\frac{7}{2}, -3, \frac{3}{4}\right\};$$

```
que = vec + po
```

$$\left\{4, 0, \frac{1}{2}\right\}$$

```
e1 = Norm[vec + po]
```

$$\frac{\sqrt{65}}{2}$$

$$\text{FullSimplify}\left[\frac{\sqrt{65}}{2} == \sqrt{16.25}\right]$$

```
True
```

The green cells above match the answers in the text.

9. 6, 1, -4; P : (-6, -1, -4)

```
Clear["Global`*"]
```

```
vec = {6, 1, -4}; po = {-6, -1, -4};
que = vec + po
```

```
{0, 0, -8}
```

```
e1 = Norm[que]
```

```
8
```

The green cells above match the answers in the text.

11 - 18 Addition, scalar multiplication

Let a = {3, 2, 0} = 3 i + 2 j; b = {-4, 6, 0} = 4 i + 6 j; c = {5, -1, 8} = 5 i - j + 8 k, d = {0, 0, 4} = 4 k

Find

11. 2 a, $\frac{1}{2}$ a, - a

```
Clear["Global`*"]
```

```
aa = {3, 2, 0}; bb = {-4, 6, 0}; cc = {5, -1, 8}; dd = {0, 0, 4}
{0, 0, 4}
```

```
2 aa
```

```
{6, 4, 0}
```

$\frac{1}{2}$ aa

$\{\frac{3}{2}, 1, 0\}$

```
- aa
```

```
{-3, -2, 0}
```

The green cells above match the answers in the text.

13. b + c, c + b

```
bb + cc
```

```
{1, 5, 8}
```

```
cc + bb
{1, 5, 8}
```

15. 7 (c - b), 7 c - 7 b

```
7 (cc + bb)
{7, 35, 56}
```

```
7 (cc - bb)
```

```
{63, -49, 56}
```

17. (7 - 3) a, 7 a - 3 a

```
(7 - 3) aa
{12, 8, 0}
```

```
7 aa - 3 aa
```

```
{12, 8, 0}
```

The green cells above match the answers in the text.

21 - 25 Forces, resultant

Find the resultant in terms of components and its magnitude.

21. p = {2, 3, 0}, q = {0, 6, 1}, u = {2, 0, -4}

```
res = {2 + 0 + 2, 3 + 6 + 0, 0 + 1 - 4}
```

```
{4, 9, -3}
```

```
Norm[res]
```

$\sqrt{106}$

$$\texttt{matt} = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 6 & 1 \\ 2 & 0 & -4 \end{pmatrix}$$

```
{{2, 3, 0}, {0, 6, 1}, {2, 0, -4}}
```

```
e2 = RowReduce[matt]
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

23. u = {18, -1, 0}, v = $\left\{\frac{1}{2}, 0, \frac{4}{3}\right\}$, w = $\left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$

res = $\left\{8+\frac{1}{2}-\frac{17}{2}, -1+0+1, 0+\frac{4}{3}+\frac{11}{3}\right\}$

{0, 0, 5}

Norm[res]

5

mat = $\begin{pmatrix} 8 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{4}{3} \\ -\frac{17}{2} & 1 & \frac{11}{3} \end{pmatrix}$

$\left\{\{8, -1, 0\}, \left\{\frac{1}{2}, 0, \frac{4}{3}\right\}, \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}\right\}$

e1 = RowReduce[mat]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

25. u = {3, 1, -6}, v = {0, 2, 5}, w = {3, -1, -13}

res = {3 + 0 + 3, 1 + 2 - 1, -6 + 5 - 13}

{6, 2, -14}

Norm[res]

$2\sqrt{59}$

mattt = $\begin{pmatrix} 3 & 1 & -6 \\ 0 & 2 & 5 \\ 3 & -1 & -13 \end{pmatrix}$

{{3, 1, -6}, {0, 2, 5}, {3, -1, -13}}

RowReduce[mattt]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Above: for this one, the text says the result is 2**u.** I don't think that is a correct statement. In fact, the reduced echelon form says that the three vectors are linearly independent. Therefore is it not doubtful that I can find factors which express one in terms of the other two?

26 - 37 Forces, velocities

27. Find **p** such that **u, v, w** in problem 23 and **p** are in equilibrium.

From problem 37, it is understood that vectors, considered as forces, are in equilibrium when they form a 'force polygon.' This polygon will have to be 4-sided. From the s.m., I find that ". . . "Equilibrium" means that the resultant of the given forces is the zero vector." Meaning I need to find **p** such that **u** + **v** + **w** + **p** = 0.

```
Clear["Global`*"]
```

```
uu = {8, -1, 0}; vv = {1/2, 0, 4/3}; ww = {-17/2, 1, 11/3}
```

$\{-\frac{17}{2}, 1, \frac{11}{3}\}$

```
pp = - (uu + vv + ww)
```

```
{0, 0, -5}
```

29. Restricted resultant. Find all **v** such that the resultant of **v, p, q, u** with **p, q, u** as in problem 21 is parallel to the *xy*-plane.

The resultant vector in problem 21 was $\{4, 9, -3\}$. So any vector of the form $\{x, y, -3\}$ will be parallel to the xy-plane when added to the resultant of problem 21.

31. For what *k* is the resultant of {2, 0, -7}, {1, 2, -3}, and {0, 3, *k*} parallel to the *xy*-plane?

```
res1 = {2 + 1, 0 + 2, -3 - 7}
{3, 2, -10}
```

```
k = 10
```

```
10
```

The green cells above match the answers in the text.

32. If |**p**| = 6 and |**q**| = 4, what can you say about the magnitude and direction of the resultant? Can you think of an application to robotics?

33. Same question as in problem 32 if |**p**| = 9, |**q**| = 6, |**u**| = 3.

Comprising three cloud spheres. Any resulting magnitude greater than or equal to zero and less than or equal to 18. Any octant.

34. Relative velocity. If airplanes A and B are moving southwest with speeds $|\mathbf{v}_A| = 550$ mph, and northwest with speed $|\mathbf{v}_B| = 450$ mph, respectively, what is the relative velocity $\mathbf{v} = \mathbf{v}_B - \mathbf{v}_A$ of B with respect to A?

```
Clear["Global`*"]
```

```
N[Solve[2 aa^2 == 550^2], 4]
```

```
{{aa → -388.9}, {aa → 388.9}}
```

```
aavec = {-388.9, -388.9}
```

```
{-388.9, -388.9}
```

```
N[Solve[2 bb^2 == 450^2], 4]
```

```
{{bb → -318.2}, {bb → 318.2}}
```

```
bbvec = {-318.2, 318.2}
```

```
{-318.2, 318.2}
```

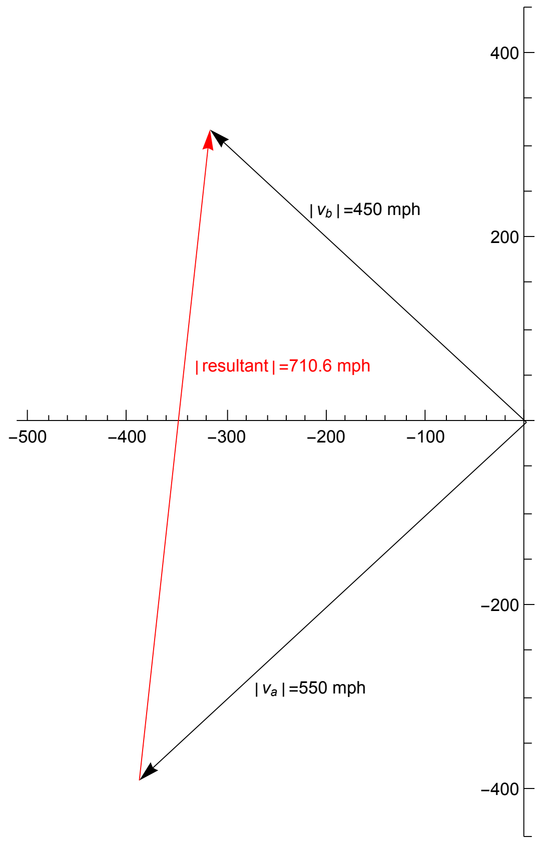
```
bbvec - aavec
```

```
{70.7, 707.1}
```

```
Norm[bbvec - aavec]
```

```
710.626
```

The relative velocity of bbvec with respect to aavec is 70.7 mph east, 707.1 mph north.



35. Same question as in problem 34 for two ships moving northeast with speed $|\boldsymbol{v}_A| = 22$ knots and west with speed $|\boldsymbol{v}_B| = 19$ knots.

```
Clear["Global`*"]
```

```
N[Solve[2 as^2 == 22^2], 4]
```

```
{{as → -15.56}, {as → 15.56}}
```

```
asvec = {15.56, 15.56}
```

```
{15.56, 15.56}
```

```
N[Solve[ bs^2 == 19^2], 4]
```

```
{{bs → -19.00}, {bs → 19.00}}
```

```
bsvec = {-19, 0}
```

```
{-19, 0}
```

The problem doesn't say what the perspective is, let's say I want velocity of ship b with respect to ship a.

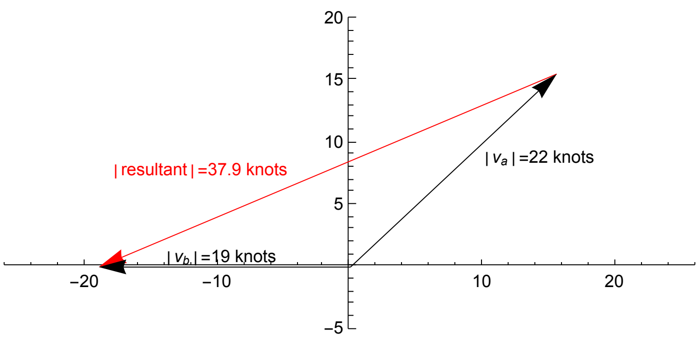
```
resulta = bsvec - asvec
```

```
{-34.56, -15.56}
```

The relative velocity of ship b with respect to ship a is 34.6 knots west, 15.6 knots south.

```
Norm[resulta]
```

```
37.9013
```



$$N\left[\left\{-19-22\Big/\sqrt{2},\ -22\Big/\sqrt{2}\right\}\right]$$

```
{-34.5563, -15.5563}
```

The green cells above match the answers in the text.

37.