Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

4 - 8 Calculation of curl

Find curl v for v given with respect to right-handed Cartesian coordinates.

5. v = x y z {x, y, z}

```
Clear["Global`*"]
```

```
e1 = Curl[{x^2 y z, x y^2 z, x y z^2}, {x, y, z}]
```

$\{-x y^2 + x z^2,\ x^2 y - y z^2,\ -x^2 z + y^2 z\}$

7. v = {0, 0, e^{-x} Sin[y]}

```
Clear["Global`*"]
```

```
e1 = Curl[{0, 0, e^-x Sin[y]}, {x, y, z}]
```

$\{e^{-x} \text{Cos}[y],\ e^{-x} \text{Sin}[y],\ 0\}$

9 - 13 Fluid flow

Let v be the velocity vector of a steay fluid flow. Is the flow irrotational? Incompressible? Find the streamlines (the paths of the particles.) Hint. See the answers to problems 9 and 11 for a determination of a path.

9. v = {0, 3 z^2, 0}

```
Clear["Global`*"]
```

```
e1 = Div[{0, 3 z^2, 0}, {x, y, z}]
```

0

The divergence being zero means that the flow is incompressible, by numbered line (7) on p. 405.

```
e2 = Curl[{0, 3 z^2, 0}, {x, y, z}]
```

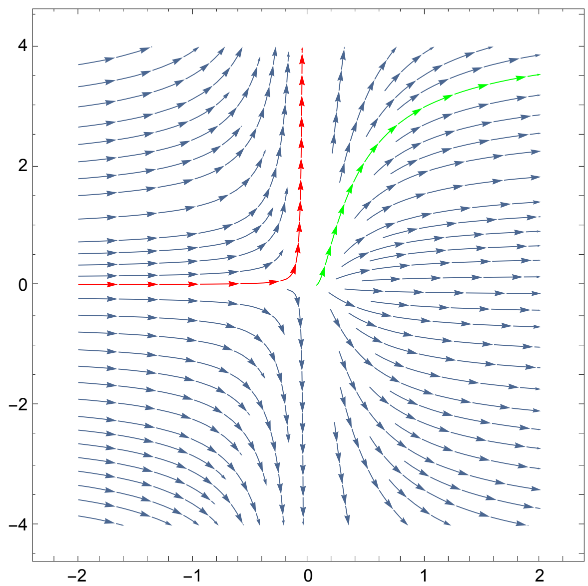
{-6 z, 0, 0}

Example 3, p. 408 says that if the flow is irrotational, the curl should be zero. The curl of the present function is not zero, so it is rotational.

```
e3 = DSolve[3 z² == y'[z], y, z]
{{y → Function[{z}, z³ + C[1]]}}
```

The solution to e3 is possibly the flow function, but I think direction fields and streamplots are about differential equations. The streamplot below gives an impression of bending flow, but is that rotational?

```
StreamPlot[{3 z², y}, {z, -2, 2}, {y, -4, 4}, StreamPoints →
  {{{{1, 3}, Green}, {{-.2, .12}, Red}, Automatic}}, ImageSize → 300]
```



11. v = {y, -2 x, 0}

```
Clear["Global`*"]
e1 = Div[{y, -2 x, 0}, {x, y, z}]
0
```

The divergence being zero means that the flow is incompressible, by (7) on p. 405.

```
e2 = Curl[{y, -2 x, 0}, {x, y, z}]
{0, 0, -3}
```

The curl not being zero implies it is rotational.

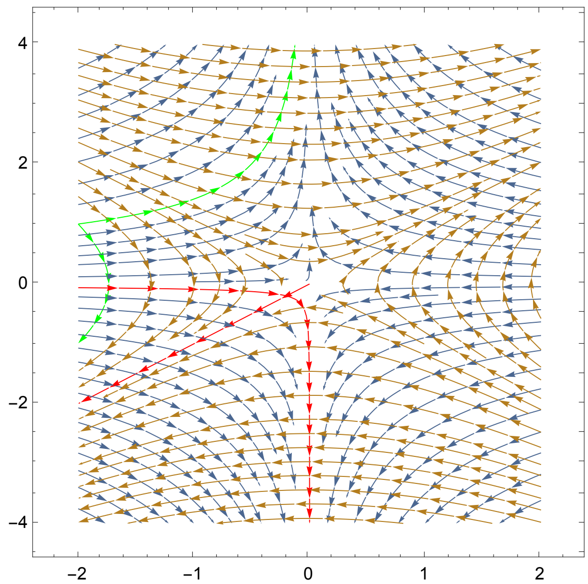
```
e3 = DSolve[-2 x == y'[x], y, x]
{{y → Function[{x}, -x² + C[1]]}}
```

```
e4 = DSolve[y == x'[y], x, y]
```

$$\left\{\left\{x \to \text{Function}\left[\{y\}, \frac{y^2}{2} + C[1]\right]\right\}\right\}$$

With an expression of x in the y slot and an expression of y in the x slot, it might make for a plot that is both shaken and stirred. Just as a speculation, I'll look at the following. I'm not sure this could be called rotational either.

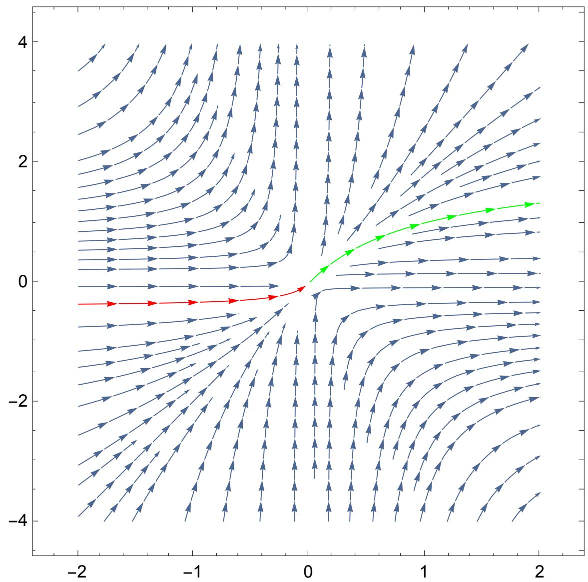
```
StreamPlot[{{-2 x, y}, {y, x}}, {x, -2, 2}, {y, -4, 4}, StreamPoints →
  {{{{-2, 1}, Green}, {{-.2, -.2}, Red}, Automatic}}, ImageSize → 300]
```



Looking at the text answer, I see that it may be possible to consolidate the equation. I have $x' = y$ and $y' = -2x \Rightarrow y' + 2x = 0 \Rightarrow y' y + 2 x' x = 0$

Integrating in hopscotch pattern, I can come up with $x^2 + \frac{1}{2} y^2 = c$, and though it's not the differential form, I can still try plotting.

```
StreamPlot[{x^2, y^2/2}, {x, -2, 2}, {y, -4, 4}, StreamPoints →
  {{{{1, 1}, Green}, {{-.2, -.2}, Red}, Automatic}}, ImageSize → 300]
```



13. v = {x, y, -z}

```
Clear["Global`*"]
e1 = Div[{x, y, -z}, {x, y, z}]
```

1

The divergence being nonzero means that the flow is compressible, by (7) on p. 405.

```
e2 = Curl[{x, y, -z}, {x, y, z}]
{0, 0, 0}
```

The curl being zero implies it is irrotational.

15 - 20 Div and curl

With respect to right-handed coordinates, let **u**={y, z, x}, **v** = {yz, zx, xy}, **f** = x y z, and **g** = x + y + z. Find the given expressions. Check your result by a formula in project 14 if applicable.

15. curl (u + v), curl v

```
Clear["Global`*"]
```

```
e1 = uu[x_, y_, z_] = {y, z, x}
```
```
{y, z, x}
```

```
e2 = vv[x_, y_, z_] = {y z, z x, x y}
```
```
{y z, x z, x y}
```

```
e3 = ff[x_, y_, z_] = x y z
```
```
x y z
```

```
e4 = gg[x_, y_, z_] = x + y + z
```
```
x + y + z
```

```
e5 = Curl[uu[x, y, z] + vv[x, y, z], {x, y, z}]
```
```
{-1, -1, -1}
```

```
e6 = Curl[vv[x, y, z], {x, y, z}]
```
```
{0, 0, 0}
```

Above: in the text answer, e5 and e6 were supposed to come out the same. Why didn't they?

```
e66 = Curl[uu[x, y, z], {x, y, z}]
```
```
{-1, -1, -1}
```

Above: Possible typo alert. Perhaps the problem description was meant to read "curl u" instead of "curl v".

17. v.curl u, u.curl v, u.curl u

```
e9 = vv[x, y, z].Curl[uu[x, y, z], {x, y, z}]
  (* text answer = -yz -zx -xy *)
```
```
-x y - x z - y z
```

The above answer, e9, does not match the text answer. However, I assume that x, y, and z are real numbers, and therefore due to real commutativity, they should be equal to the text answer.

```
e10 = uu[x, y, z].Curl[vv[x, y, z], {x, y, z}]
```
```
0
```

```
e11 =
 uu[x, y, z].Curl[uu[x, y, z], {x, y, z}] (* text answer = -y -z -x *)
```
```
-x - y - z
```

Above: Green invoked by commutativity principle for reals.

19. curl (gu + v), curl (gu)

```
e12 = Curl[gg[x, y, z] uu[x, y, z] + vv[x, y, z], {x, y, z}]
```

```
{-y - 2 z, -2 x - z, -x - 2 y}
```

```
e13 = Curl[gg[x, y, z] uu[x, y, z], {x, y, z}]
```

```
{-y - 2 z, -2 x - z, -x - 2 y}
```