Comment: *Mathematica*'s Convolve function has an odd syntax. Note that in the orange cells below, the asterisk represents the convolve function.

1 - 7 Convolutions by integration
Find:

1. 1*1

```
ClearAll["Global`*"]

Integrate[1 × 1, {y, 0, t}]
```

```
t
```

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

3. $e^t * e^{-t}$

```
ClearAll["Global`*"]

Integrate[e^y e^{-(t-y)}, {y, 0, t}]
```

```
Sinh[t]
```

Below: Alternate method, this one using Convolve. But the syntax is strange.

```
Convolve[ e^y UnitStep[y], e^{-y} UnitStep[y], y, t, Assumptions → t > 0]
```

```
Sinh[t]
```

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

5. $(Sin[\omega\ t])*(Cos[\omega\ t])$

```
ClearAll["Global`*"]

e1 = Integrate[Sin[ω y] Cos[ω (t - y)], {y, 0, t}]
```

$$\frac{1}{2} t \, Sin[t\ \omega]$$

With the alternate method,

```
Convolve[Sin[ω y] UnitStep[y],
 Cos[ω y] UnitStep[y], y, t, Assumptions → t > 0]
```

$$\frac{1}{2} t \, Sin[t \, ω]$$

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

7. $t * e^t$

```
ClearAll["Global`*"]
```

```
e1 = Integrate[y e^{t-y}, {y, 0, t}]
```
$-1 + e^t - t$

```
Convolve[y UnitStep[y], e^y UnitStep[y], y, t, Assumptions → t > 0]
```

$-1 + e^t - t$

The above answer matches the text. This time only the **Convolve** method was used.

8 - 14 Integral equations
Solve by the Laplace transform, showing the details:

9. y[t] - Integrate[y[τ], {τ, 0, t}] = 1

```
ClearAll["Global`*"]
```

```
e1 = LaplaceTransform[Integrate[y[τ], {τ, 0, t}], t, s]
```
$$\frac{LaplaceTransform[y[t], t, s]}{s}$$

```
e2 = LaplaceTransform[y[t], t, s]
```
```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[1, t, s]
```
$$\frac{1}{s}$$

```
e4 = e2 - e1 == e3
```
$$LaplaceTransform[y[t], t, s] - \frac{LaplaceTransform[y[t], t, s]}{s} == \frac{1}{s}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```
$$bigY - \frac{bigY}{s} == \frac{1}{s}$$

e6 = Solve[e5, bigY]

$\left\{\left\{\text{bigY} \rightarrow \dfrac{1}{-1 + s}\right\}\right\}$

e7 = e6[[1, 1, 2]]

$\dfrac{1}{-1 + s}$

e8 = InverseLaplaceTransform[e7, s, t]

$\mathbb{e}^t$

Above: The answer matches the text answer.

11. y[t] + Integrate[(t - τ) y[τ], {τ, 0, t}] = 1

ClearAll["Global`*"]

e1 = LaplaceTransform[Integrate[(t - τ) y[τ], {τ, 0, t}], t, s]

$\dfrac{\text{LaplaceTransform[y[t], t, s]}}{s^2}$

e2 = LaplaceTransform[y[t], t, s]

LaplaceTransform[y[t], t, s]

e3 = LaplaceTransform[1, t, s]

$\dfrac{1}{s}$

e4 = e2 + e1 == e3

$\text{LaplaceTransform[y[t], t, s]} + \dfrac{\text{LaplaceTransform[y[t], t, s]}}{s^2} == \dfrac{1}{s}$

e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY

$\text{bigY} + \dfrac{\text{bigY}}{s^2} == \dfrac{1}{s}$

e6 = Solve[e5, bigY]

$\left\{\left\{\text{bigY} \rightarrow \dfrac{s}{1 + s^2}\right\}\right\}$

e7 = e6[[1, 1, 2]]

$\dfrac{s}{1 + s^2}$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

```
Cos[t]
```

Above: The answer matches the text answer.

13. $y[t] + 2 e^t \text{Integrate}\left[y[\tau] e^{-\tau} = t e^t, \{\tau, 0, t\}\right]$

```
ClearAll["Global`*"]
```

```
e1 = LaplaceTransform[2 e^t Integrate[y[τ] e^{-τ}, {τ, 0, t}], t, s]
```

$$\frac{2 \text{ LaplaceTransform}[y[t], t, s]}{-1 + s}$$

```
e2 = LaplaceTransform[y[t], t, s]
```

```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[t e^t, t, s]
```

$$\frac{1}{(-1 + s)^2}$$

```
e4 = e2 + e1 == e3
```

$$\text{LaplaceTransform}[y[t], t, s] + \frac{2 \text{ LaplaceTransform}[y[t], t, s]}{-1 + s} == \frac{1}{(-1 + s)^2}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```

$$\text{bigY} + \frac{2 \text{ bigY}}{-1 + s} == \frac{1}{(-1 + s)^2}$$

```
e6 = Solve[e5, bigY]
```

$$\left\{\left\{\text{bigY} \rightarrow \frac{1}{(-1 + s) (1 + s)}\right\}\right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{1}{(-1 + s) (1 + s)}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

$$\frac{1}{2} e^{-t} \left(-1 + e^{2 t}\right)$$

```
e9 = ExpToTrig[e8]
```

$$\frac{1}{2} (\text{Cosh}[t] - \text{Sinh}[t]) (-1 + \text{Cosh}[2 t] + \text{Sinh}[2 t])$$

e10 = FullSimplify[e9]

Sinh[t]

Above: This answer matches the text answer.

17 - 26 Inverse transforms by convolution
Showing details, find f[t] if $\mathcal{L}$[f] equals:

17. $\dfrac{5.5}{(s + 1.5)\ (s - 4)}$

Note: use of the **Convolve** function did not seem necessary.

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\dfrac{5.5}{(s + 1.5)\ (s - 4)},\ s,\ t\right]$

$5.5\ \left(-0.181818\ e^{-1.5\,t} + 0.181818\ e^{4.\,t}\right)$

e2 = ExpandAll[e1]

$-1.\ e^{-1.5\,t} + 1.\ e^{4.\,t}$

Above: This answer matches the text answer.

19. $\dfrac{2\,\pi\,s}{\left(s^2 + \pi^2\right)^2}$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\dfrac{2\,\pi\,s}{\left(s^2 + \pi^2\right)^2},\ s,\ t\right]$

t Sin[π t]

Above: This answer matches the text answer.

21. $\dfrac{\omega}{s^2\ \left(s^2 + \omega^2\right)}$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\dfrac{\omega}{s^2\left(s^2+\omega^2\right)}, s, t\right]$

$\dfrac{t\,\omega - \text{Sin}[t\,\omega]}{\omega^2}$

Above: This answer matches the text answer.

23. $\dfrac{40.5}{s\left(s^2-9\right)}$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\dfrac{40.5}{s\left(s^2-9\right)}, s, t\right]$

2.25 e$^{-3\,t}$ $\left(-1+e^{3\,t}\right)^2$

e2 = ExpToTrig[e1]

2.25 (Cosh[3 t] - Sinh[3 t]) (-1 + Cosh[3 t] + Sinh[3 t])^2

e3 = FullSimplify[e2]

- 4.5 + 4.5 Cosh[3 t]

Above: This answer matches the text answer.

25. $\dfrac{18\,s}{\left(s^2+36\right)^2}$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\dfrac{18\,s}{\left(s^2+36\right)^2}, s, t\right]$

$\dfrac{3}{2}$ t Sin[6 t]

Above: This answer matches the text answer.