11 - 23 Vector and scalar triple products

With respect to right-handed Cartesian coordinates, let a = {2, 1, 0} , b = {-3, 2, 0}, c = {1, 4, -2}, and d = {5, -1, 3}. Showing details, find:

11. a × b, b × a, a.b

```
Clear["Global`*"]
```

```
e1 = a = {2, 1, 0}
```

```
{2, 1, 0}
```

```
e2 = b = {-3, 2, 0}
```

```
{-3, 2, 0}
```

```
e3 = c = {1, 4, -2}
```

```
{1, 4, -2}
```

```
e4 = d = {5, -1, 3}
```

```
{5, -1, 3}
```

```
e5 = e1 × e2
```

```
{0, 0, 7}
```

```
e6 = e2 × e1
```

```
{0, 0, -7}
```

```
e7 = e1.e2
```

```
-4
```

13. c × (a+b), a × c + b × c

```
e8 = e3 × (e1 + e2)
```

```
{6, 2, 7}
```

```
e85 = e1 × e3 + e2 × e3
```

```
{-6, -2, -7}
```

15. (a + d) × (d + a)

```
e9 = (e1 + e4) × (e4 + e1)
```

```
{0, 0, 0}
```

17. (b × c) × d, b × (c × d)

```
e10 = (e2 × e3) × e4
```

```
{-32, -58, 34}
```

```
e11 = e2 × (e3 × e4)
```

```
{-42, -63, 19}
```

19. (i j k), (i k j)

```
i1 = {1, 0, 0}; j1 = {0, 1, 0}; k1 = {0, 0, 1}
{0, 0, 1}
```

```
e12 = (i1 × j1.k1)
```

```
1
```

```
e13 = i1.k1 × j1
```

```
-1
```

Above: the text did not show any operator symbols, so I took a guess, experimenting a little to get the text answer.

21. 4b × 3c, 12|b × c|, 12|c × b|

```
e14 = (4 e2) × (3 e3)
```

```
{-48, -72, -168}
```

```
e15 = 12 Norm[e2 × e3]
```

$24\sqrt{62}$

```
e16 = 12 Norm[e3 × e2]
```

$24\sqrt{62}$

23. b × b, (b - c) × (c - b), b.b

```
e17 = e2 × e2
```

```
{0, 0, 0}
```

```
e18 = (e2 - e3) × (e3 - e2)
```

```
{0, 0, 0}
```

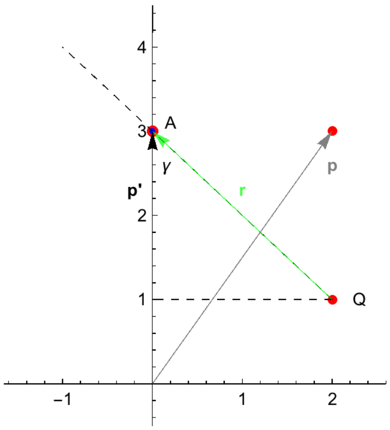
```
e19 = e2.e2
```

```
13
```

25 - 35 Applications

25. Moment **m** of a force **p**. Find the moment vector **m** and m of **p** = {2, 3, 0} about Q: (2, 1, 0) acting on a line through A: {0, 3, 0}. Make a sketch.

Since all the coordinates for the z-axis are zero, this problem can be considered in two dimensions. However, if I need to do any cross products, I will need to include all three coordinates.



In example 3 on p. 371, the line of action of **p**' goes through A. I think that needs to be maintained. The vector **p** does not actually go through A, but there would be a component. The length of this component would be the norm of **p** times the cosine of the angle γ between. I could call the vector with this length and A's direction, **p**'.

```
Clear["Global`*"]
```

```
e2 = {2, 3}
```

```
{2, 3}
```

```
e3 = Norm[e2]
```

$\sqrt{13}$

```
e4 = ArcTan[2/3]
```

$\text{ArcTan}\left[\frac{2}{3}\right]$

```
e5 = Cos[e4]
```

$\frac{3}{\sqrt{13}}$

```
e6 = e3 e5
```

3

Here is something remarkable. If I haven’t miscalculated, the vector **p**' is a vector terminating at A.

The length of **p**' is 3. The quantity mlight is the norm of mbold. Not written in the text or problem description as a norm, though just light face, not bold.

Because of the length of its sides being equal, the angle γ is seen to be $\frac{\pi}{4}$.

```
e7 = rbold = {0, 3} - {2, 1}
```

{-2, 2}

```
e9 = mbold = e7 × e2
```

Cross::nonn1 : The arguments are expected to be vectors of equal length, and the number of arguments is expected to be 1 less than their length ≫

{-2, 2} × {2, 3}

Above: here is where I have to put the third coordinate back in.

```
e10 = mbold3 = {-2, 2, 0} × {2, 3, 0}
```

{0, 0, -10}

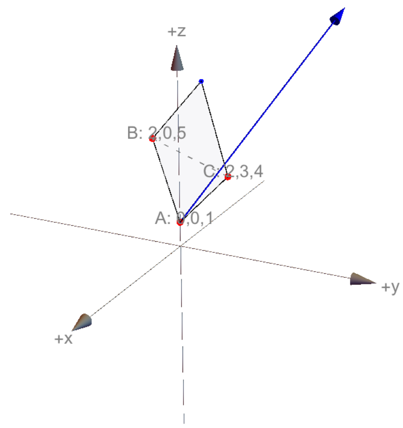
```
e11 = mlight = Norm[e10]
```

10

Looking down the z-axis toward the sketch of the problem, with positive x to the right, the moment mbold would tend to exert a clockwise motion around Q.

29. Triangle. Find the area if the vertices are {0, 0, 1}, {2, 0, 5}, and {2, 3, 4}.

```
Clear["Global`*"]
```



In the sketch, I need to find the area of the triangle, ABC. Following the s.m. pretty closely, I make two vectors out of points B and C, using the common point A as their origin.

```
bbold = {2 - 0, 0 - 0, 5 - 1}
```

```
{2, 0, 4}
```

```
cbold = {2 - 0, 3 - 0, 4 - 1}
```

```
{2, 3, 3}
```

Then I cross these two,

```
vbold = bbold × cbold
```

```
{-12, 2, 6}
```

and find the norm of the cross vbold,

```
e1 = Norm[vbold]
```

$2\sqrt{46}$

The s.m. reminds me that the cross product is defined in such a way that its length is equal to the area of the base parallelogram (see sketch). Since the area of the triangle I want is exactly half the area of the parallelogram, I have,

$$e2 = \frac{e1}{2}$$

$$\sqrt{46}$$

I added vbold to the sketch.

Note: Green cells in this problem set agree with the corresponding answers in the text.

31.

33.