Example 2, p 83.

```
Clear["Global`*"]
```

```
hig = {y''[x] + 3 y'[x] + 2.25 y[x] == -10 e^(-1.5 x), y[0] == 1, y'[0] == 0}
velm = DSolve[hig, y, x]
```

$\{2.25\, y[x] + 3\, y'[x] + y''[x] == -10\, e^{-1.5\, x},\ y[0] == 1,\ y'[0] == 0\}$

$\{\{y \to \text{Function}[\{x\},\ -5.\, e^{-1.5\, x} (-0.2 - 0.3\, x + 1.\, x^2)]\}\}$

```
Expand[-5.` e^(-1.5` x) (-0.2` - 0.3` x + 1.` x^2)]
```

$1.\, e^{-1.5\, x} + 1.5\, e^{-1.5\, x}\, x - 5.\, e^{-1.5\, x}\, x^2$

```
Simplify[%]
```

$e^{-1.5\, x} (1. + 1.5\, x - 5.\, x^2)$

The answer to example 2 in the text is duplicated.

1 - 10 Nonhomogeneous linear ODEs: General solution
Find a (real) general solution. State which rule you are using.

1. $y'' + 5\, y' + 4\, y = 10\, e^{-3\, x}$

```
Clear["Global`*"]
```

```
xen = y''[x] + 5 y'[x] + 4 y[x] == 10 e^(-3 x)
jud = DSolve[xen, y, x]
```

$4\, y[x] + 5\, y'[x] + y''[x] == 10\, e^{-3\, x}$

$\{\{y \to \text{Function}[\{x\},\ -5\, e^{-3\, x} + e^{-4\, x}\, C[1] + e^{-x}\, C[2]]\}\}$

1. The text answer is found.

3. $y'' + 3\, y' + 2\, y = 12\, x^2$

```
Clear["Global`*"]
```

```
oen = y''[x] + 3 y'[x] + 2 y[x] == 12 x^2
qas = DSolve[oen, y, x]
```

$2\, y[x] + 3\, y'[x] + y''[x] == 12\, x^2$

$\{\{y \to \text{Function}[\{x\},\ 3 (7 - 6\, x + 2\, x^2) + e^{-2\, x}\, C[1] + e^{-x}\, C[2]]\}\}$

```
Expand[3 (7 - 6 x + 2 x^2)]
```

$21 - 18\, x + 6\, x^2$

```
qas /. (3 (7 - 6 x + 2 x^2)) -> 21 - 18 x + 6 x^2
```

$\{\{y \to \text{Function}[\{x\}, (21 - 18x + 6x^2) + e^{-2x} C[1] + e^{-x} C[2]]\}\}$

1. The text answer is found.

5. $y'' + 4y' + 4y = e^{-x} \text{Cos}[x]$

```
Clear["Global`*"]
```

```
up = y''[x] + 4 y'[x] + 4 y[x] == e^-x Cos[x]
nap = DSolve[up, y, x]
```

$4y[x] + 4y'[x] + y''[x] == e^{-x} \text{Cos}[x]$

$\{\{y \to \text{Function}[\{x\}, e^{-2x} C[1] + e^{-2x} x C[2] + \frac{1}{2} e^{-x} \text{Sin}[x]]\}\}$

1. The text answer is found.

7. $\left(D^2 + 2D + \frac{3}{4} I\right) y = 3e^x + \frac{9}{2} x$

```
Clear["Global`*"]
```

```
mop = y''[x] + 2 y'[x] + 3/4 y[x] == 3 e^x + 9/2 x
lam = DSolve[mop, y, x]
```

$\frac{3y[x]}{4} + 2y'[x] + y''[x] == 3e^x + \frac{9x}{2}$

$\{\{y \to \text{Function}[\{x\}, \frac{2}{5}(-40 + 2e^x + 15x) + e^{-3x/2} C[1] + e^{-x/2} C[2]]\}\}$

```
Expand[2/5 (-40 + 2 e^x + 15 x)]
```

$-16 + \frac{4e^x}{5} + 6x$

```
lam /. (2/5 (-40 + 2 e^x + 15 x)) -> -16 + 4 e^x/5 + 6 x
```

$\{\{y \to \text{Function}[\{x\}, \left(-16 + \frac{4e^x}{5} + 6x\right) + e^{-3x/2} C[1] + e^{-x/2} C[2]]\}\}$

1. The text answer is found.

9. $(D^2 - 16 I) y = 9.6 e^{4x} + 30 e^x$

```
Clear["Global`*"]
```

```
track = y''[x] - 16 y[x] == 9.6 e^(4 x) + 30 e^x
nard = DSolve[track, y, x]
```

$-16\, y[x] + y''[x] == 30\, e^{x} + 9.6\, e^{4\, x}$

$\{\{y \to \text{Function}[\{x\}, 1.2\, e^{-7.\, x}\, (-1.66667\, e^{8.\, x} - 0.125\, e^{11.\, x} + 1.\, e^{11.\, x}\, x) + e^{4.\, x}\, C[1] + e^{-4.\, x}\, C[2]]\}\}$

```
Expand[1.2` e^(-7.` x) (-1.6666666666666667` e^(8.` x) - 0.125` e^(11.` x) + 1.` e^(11.` x) x)]
```

$-2.\, e^{1.\, x} - 0.15\, e^{4.\, x} + 1.2\, e^{4.\, x}\, x$

1. Above: altered format of a section prior to hand replacement.

```
scis = nard /.
  (1.2` e^(-7.` x) (-1.6666666666666667` e^(8.` x) - 0.125` e^(11.` x) + 1.` e^(11.` x) x)) ->
   -2.` e^(1.` x) - 0.15` e^(4.` x) + 1.2` e^(4.` x) x
```

$\{\{y \to \text{Function}[\{x\}, (-2.\, e^{1.\, x} - 0.15\, e^{4.\, x} + 1.2\, e^{4.\, x}\, x) + e^{4.\, x}\, C[1] + e^{-4.\, x}\, C[2]]\}\}$

2. Above: hand replacement of a section.

```
yit = -2.` e^(1.` x) - 0.15` e^(4.` x) + 1.2` e^(4.` x) x + e^(4.` x) C[1] + e^(-4.` x) C[2]
```

$-2.\, e^{1.\, x} - 0.15\, e^{4.\, x} + 1.2\, e^{4.\, x}\, x + e^{4.\, x}\, C[1] + e^{-4.\, x}\, C[2]$

3. Above: removed parentheses from a section by hand.

```
bag = yit /. -0.15` e^(4.` x) + e^(4.` x) C[1] -> +e^(4.` x) C[3]
```

$-2.\, e^{1.\, x} + 1.2\, e^{4.\, x}\, x + e^{-4.\, x}\, C[2] + e^{4.\, x}\, C[3]$

4. Above: consolidated constants in a factor's coefficient by hand, resulting in the text answer.

11 - 18 Nonhomogeneous linear ODEs: IVPs
Solve the initial value problem. State which rule you are using. Show each step of your calculation in detail.

11. $y'' + 3\, y = 18\, x^2,\ y[0] = -3,\ y'[0] = 0$

```
Clear["Global`*"]
```

```
nom = {y''[x] + 3 y[x] == 18 x^2, y[0] == -3, y'[0] == 0}
kla = DSolve[nom, y, x]
```

$\{3\, y[x] + y''[x] == 18\, x^2,\ y[0] == -3,\ y'[0] == 0\}$

$\{\{y \to \text{Function}[\{x\}, -4 + 6\, x^2 + \text{Cos}[\sqrt{3}\, x]]\}\}$

1. The answer matches the text.

13. 8 y" - 6 y' + y = 6 Cosh[x], y[0] = 0.2, y'[0] = 0.05

```
Clear["Global`*"]
```

```
uil = {8 y''[x] - 6 y'[x] + y[x] == 6 Cosh[x], y[0] == 0.2, y'[0] == 0.05}
qwx = DSolve[uil, y[x], x]
```

$\{y[x] - 6\, y'[x] + 8\, y''[x] == 6\, \text{Cosh}[x],\ y[0] == 0.2,\ y'[0] == 0.05\}$

$\{\{y[x] \to e^{-x} \left(0.2 + 1.\, e^{5x/4} - 2.\, e^{3x/2} + e^{2x}\right)\}\}$

```
Expand[qwx]
```

$\{\{y[x] \to 0.2\, e^{-x} + 1.\, e^{x/4} - 2.\, e^{x/2} + e^{x}\}\}$

1. The answer matches the text.

15. $\left(x^2 D^2 - 3\, x\, D + 3\, I\right) y = 3\, \text{Log}[x] - 4,\ y[1] = 0,\ y'(1) = 1;$
$y_p = \text{Log}[x]$

```
Clear["Global`*"]
```

```
mil = {x^2 y''[x] - 3 x y'[x] + 3 y[x] == 3 Log[x] - 4, y[1] == 0, y'[1] == 1}
jyt = DSolve[mil, y[x], x]
```

$\{3\, y[x] - 3\, x\, y'[x] + x^2\, y''[x] == -4 + 3\, \text{Log}[x],\ y[1] == 0,\ y'[1] == 1\}$

$\{\{y[x] \to \text{Log}[x]\}\}$

1. The answer matches the text.

17. $\left(D^2 + 0.2\, D + 0.26\, I\right) y = 1.22\, e^{0.5\, x},\ y[0] = 3.5,\ y'[0] = 0.35$

```
Clear["Global`*"]
```

```
hal = {y''[x] + 0.2 y'[x] + 0.26 y[x] == 1.22 ℯ^(0.5 x), y[0] == 3.5, y'[0] == 0.35}
xxa = DSolve[hal, y[x], x]
```

$\{0.26\, y[x] + 0.2\, y'[x] + y''[x] == 1.22\, e^{0.5\, x},\ y[0] == 3.5,\ y'[0] == 0.35\}$

$\{\{y[x] \to 2.\, e^{-0.1\, x}\, (0.75\, \mathrm{Cos}[0.5\, x] + 1.\, e^{0.6\, x}\, \mathrm{Cos}[0.5\, x]^2 - 0.5\, \mathrm{Sin}[0.5\, x] + 1.\, e^{0.6\, x}\, \mathrm{Sin}[0.5\, x]^2)\}\}$

```
bur = xxa /. (1.` ℯ^(0.6` x) Cos[0.5` x]^2 + 1.` ℯ^(0.6` x) Sin[0.5` x]^2) → 1.` ℯ^(0.6` x)
```

$\{\{y[x] \to 2.\, e^{-0.1\, x}\, (1.\, e^{0.6\, x} + 0.75\, \mathrm{Cos}[0.5\, x] - 0.5\, \mathrm{Sin}[0.5\, x])\}\}$

1. Above: altered with hand-inserted trig ident $\sin^2 x + \cos^2 x = 1$.

```
Expand[bur]
```

$\{\{y[x] \to 2.\, e^{0.5\, x} + 1.5\, e^{-0.1\, x}\, \mathrm{Cos}[0.5\, x] - 1.\, e^{-0.1\, x}\, \mathrm{Sin}[0.5\, x]\}\}$

2. The above answer matches the text.

19. CAS project. Structure of solutions of Initial Value Problems. Using the present method, find, graph, and discuss the solutions y of initial value problems of your own choice. Explore effects on solutions caused by changes of initial conditions. Graph y_p, y, $y - y_p$ separately, to see the separate effects. Find a problem in which (a) the part of y resulting from y_h decreases to zero, (b) increases, (c) is not present in the answer y. Study a problem with y(0) = 0, y'(0) = 0. Consider a problem in which you need the Modification Rule (a) for a simple root, (b) for a double root. Make sure that your problems cover all three Cases I, II, III (see section 2.2).