Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1 - 6 Fitting a straight line
Fit a straight line to the given points (x,y) by least squares. Show the details. Check your result by sketching the points and the line. Judge the goodness of fit.

1. {0, 2}, {2, 0}, {3, -2}, {5, -3}

I like it that the following fit-finding functions provide the equations for the functions which are plotted. Fit is a least-squares function, which jibes with the imperative of the problem. As for IP, I couldn't resist using it also. Goodness of fit seems acceptable.

```
Clear["Global`*"]

lis = {{0, 2}, {2, 0}, {3, -2}, {5, -3}}
{{0, 2}, {2, 0}, {3, -2}, {5, -3}}

ip = InterpolatingPolynomial[lis, x]
```

$$2 + \left(-1 + \left(-\frac{1}{3} + \frac{1}{6}(-3 + x)\right)(-2 + x)\right) x$$

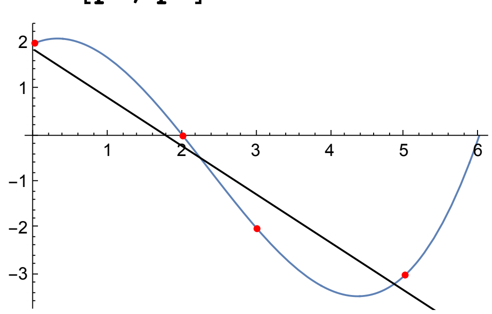
```
p1 = Plot[ip, {x, 0, 6}, PlotStyle → Thickness[0.004],
   Epilog → {Red, PointSize[0.015], Point /@ lis}, ImageSize → 250];
```

Demonstrating that the Fit function is actually using least squares,

```
line = Fit[lis, {1, x}, x]
```

```
1.84615 - 1.03846 x
```

```
p2 = Plot[line, {x, 0, 6},
   PlotStyle → {Black, Thickness[0.004]}, ImageSize → 250];

Show[p1, p2]
```

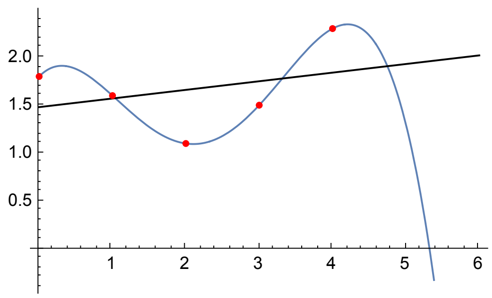


3. {0, 1.8}, {1, 1.6}, {2, 1.1}, {3, 1.5}, {4, 2.3}

```
Clear["Global`*"]

lis = {{0, 1.8}, {1, 1.6}, {2, 1.1}, {3, 1.5}, {4, 2.3}}
{{0, 1.8}, {1, 1.6}, {2, 1.1}, {3, 1.5}, {4, 2.3}}

ip = InterpolatingPolynomial[lis, x]
2.3 + (-4 + x) (0.125 + (0.2375 + (0.129167 - 0.0708333 (-1 + x)) (-2 + x)) x)

p1 = Plot[ip, {x, 0, 6}, PlotStyle → Thickness[0.004],
   Epilog → {Red, PointSize[0.015], Point /@ lis}, ImageSize → 250];

line = Fit[lis, {1, x}, x]
```

1.48 + 0.09 x

```
p2 = Plot[line, {x, 0, 6},
   PlotStyle → {Black, Thickness[0.004]}, ImageSize → 250];

Show[p1, p2]
```



5. Average Speed. Estimate the average speed v_{av} of a car traveling according to s = v · t [km] (s = distance traveled, t [hr] = time) from {t,s} = {9,140}, {10,220}, {11,310}, {12,410}.

```
Clear["Global`*"]

lin = {{9, 140}, {10, 220}, {11, 310}, {12, 410}}
{{9, 140}, {10, 220}, {11, 310}, {12, 410}}
```

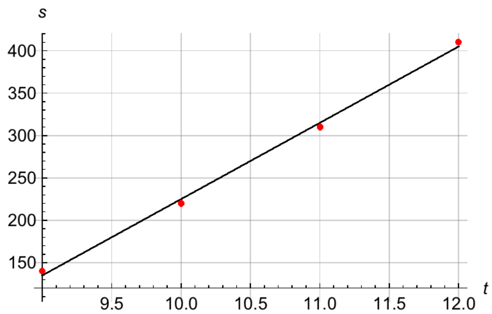
Mathematica has a easy-to-use arc length function. I'm not using it this time, but it's nice to know it's there.

```
N[ArcLength[Line[lin]]]
270.017

lins = Fit[lin, {1, t}, t]
```

-675. + 90. t

From the points list it is clear that the vehicle traveled 80 km in first hr, then 90 km in second hr, then 100 km in third hr, obviously having an average speed of 90 km/hr.

```
p1 = Plot[lins, {t, 9, 12}, PlotStyle → {Black, Thickness[0.004]},
  ImageSize → 250, AxesLabel → {t, s}, PlotRange → All,
  GridLines → Automatic, Epilog → {Red, PointSize[0.015], Point /@ lin}]
```



8 - 11 Fitting a quadratic parabola
Fit a parabola (7) to the points (x,y). Check by sketching.

9. {2, -3}, {3, 0}, {5, 1}, {6, 0}, {7, -2}

```
Clear["Global`*"]
```

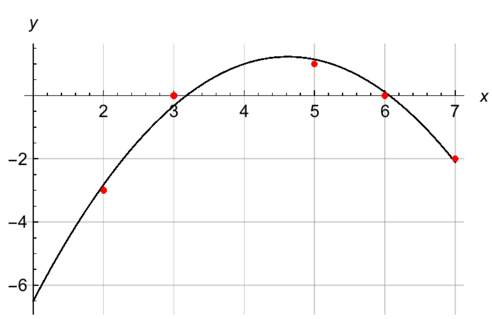
```
dat = {{2, -3}, {3, 0}, {5, 1}, {6, 0}, {7, -2}}
{{2, -3}, {3, 0}, {5, 1}, {6, 0}, {7, -2}}
```

The Fit function is checked out for performance in creating quadratic curves.

```
parabola = Fit[dat, {1, x, x^2}, x]
```

$-11.3571 + 5.44643\, x - 0.589286\, x^2$

```
p1 = Plot[parabola, {x, 1, 7}, PlotStyle → {Black, Thickness[0.004]},
  ImageSize → 250, AxesLabel → {x, y}, PlotRange → All,
  GridLines → Automatic, Epilog → {Red, PointSize[0.015], Point /@ dat}]
```



11. The data in problem 3. Plot the points, the line, and the parabola jointly. Compare and comment.

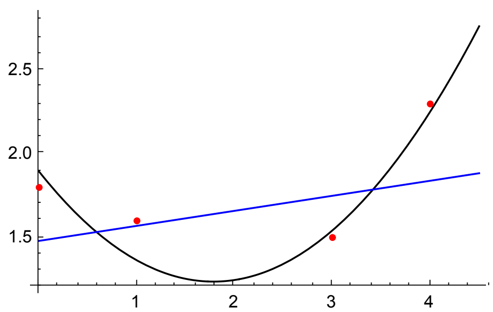
Interesting how little change is necessary to expand the plot to include both curves.

```
Clear["Global`*"]

lis = {{0, 1.8}, {1, 1.6}, {2, 1.1}, {3, 1.5}, {4, 2.3}}
{{0, 1.8}, {1, 1.6}, {2, 1.1}, {3, 1.5}, {4, 2.3}}

line = Fit[lis, {1, x}, x]
1.48 + 0.09 x
```

```
par = Fit[lis, {1, x, x^2}, x]
```

$1.89429 - 0.738571\,x + 0.207143\,x^2$

```
p2 = Plot[{par, line}, {x, 0, 4.5},
  PlotStyle → {{Black, Thickness[0.004]}, {Blue, Thickness[0.004]}},
  Epilog → {Red, PointSize[0.015], Point /@ lis}, ImageSize → 250]
```



13. Fit curves (2) and (7) and a cubic parabola by least squares to (x,y) = {-2,-30}, {-1,-4}, {0,4}, {1,4}, {2,22}, {3,68}. Graph these curves and the points on common axes. Comment on the goodness of fit.

```
Clear["Global`*"]

dat = {{-2, -30}, {-1, -4}, {0, 4}, {1, 4}, {2, 22}, {3, 68}}
{{-2, -30}, {-1, -4}, {0, 4}, {1, 4}, {2, 22}, {3, 68}}

line = Fit[dat, {1, x}, x]
```

$2.55238 + 16.2286\,x$

```
parab = Fit[dat, {1, x, x^2}, x]
```

$-4.11429 + 13.7286\,x + 2.5\,x^2$

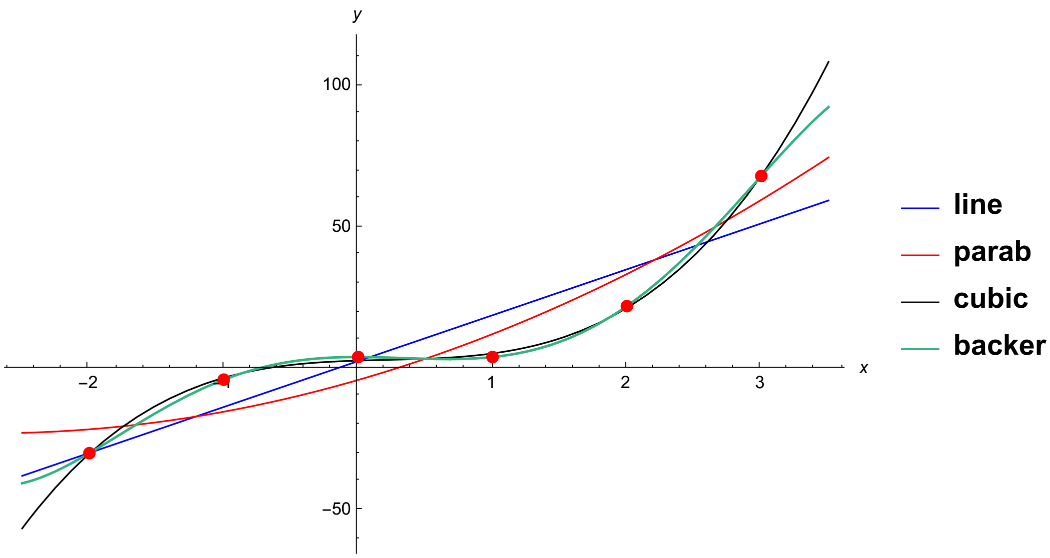
```
cubic = Fit[dat, {1, x, x^2, x^3}, x]
```

$2.73016 + 1.46561\,x - 1.77778\,x^2 + 2.85185\,x^3$

```
backer = Fit[dat, {1, x, x^2, x^3, x^4, x^5}, x]
```

$4. - 0.0666667\, x - 4.66667\, x^2 + 4.33333\, x^3 + 0.666667\, x^4 - 0.266667\, x^5$

```
p1 = Plot[{line, parab, cubic, backer}, {x, -2.5, 3.5},
  PlotStyle → {{Blue, Thickness[0.002]}, {Red, Thickness[0.002]}, {Black,
     Thickness[0.002]}, {RGBColor[0.2, 0.7, 0.5], Thickness[0.003]}},
  Epilog → {Red, PointSize[0.015], Point /@ dat}, ImageSize → 450,
  AxesLabel → {x, y}, PlotLegends → {"line", "parab", "cubic", "backer"}]
```



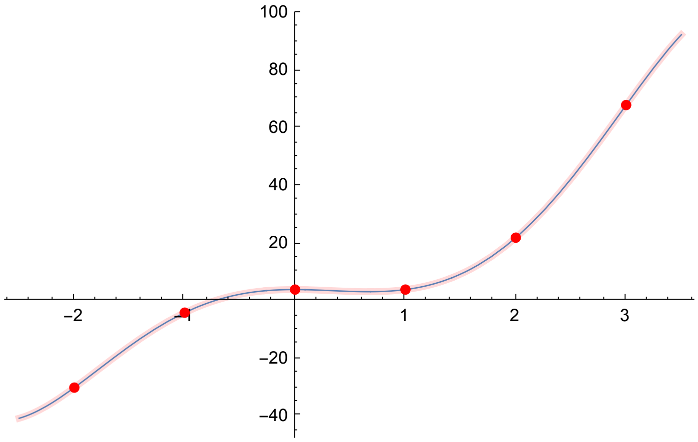
As I learned when reading about splines, it takes a cubic to conform well to an arbitrary set of points. However, it seems that performance is limited. Here, x=0, x=1 and even x=2 do not quite fall on the curve. I tried to remedy by adding the function backer, which improves the conformance quite a bit, though at the cost of fifth order.

```
ip = InterpolatingPolynomial[dat, x]
```

$$-30 + (2 + x) \left(26 + (1 + x) \left(-9 + \left(\frac{5}{3} + \left(\frac{2}{3} - \frac{4}{15} (-2 + x)\right) (-1 + x)\right) x\right)\right)$$

```
p3 = Plot[ip, {x, -2.5, 3.5}, PlotStyle → {LightRed, Thickness[0.01]},
   Epilog → {Red, PointSize[0.015], Point /@ dat}];
p4 = Plot[backer, {x, -2.5, 3.5}, PlotStyle → {Thickness[0.002]}];
```

```
Show[p3, p4]
```



InterpolatingPolynomial also deems it necessary to use 5th order. And it looks like the same curve that Fit used for making ‘backer’ conform to this set of points.