

1 - 7 General solution

Solve the following ODEs, showing the details of your work.

$$1. \quad y'''' + 3y' + y = e^x - x - 1$$

```
ClearAll["Global`*"]
```

```
dapple = y''''[x] + 3 y'''[x] + 3 y''[x] + y[x] == e^x - x - 1
```

```
apple = DSolve[dapple, y[x], x]
```

```
y[x] + 3 y'[x] + 3 y''[x] + y'''[x] == -1 + e^x - x
```

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{8} (16 + e^x - 8x) + e^{-x} C[1] + e^{-x} x C[2] + e^{-x} x^2 C[3] \right\} \right\}$$

```
Collect[apple, e^-x]
```

$$\left\{ \left\{ y[x] \rightarrow 2 + \frac{e^x}{8} - x + e^{-x} (C[1] + x C[2] + x^2 C[3]) \right\} \right\}$$

1. Above: The expression matches the answer in the text.

$$3. \quad (D^4 + 10D^2 + 9I) y = 6.5 \sinh[2x]$$

```
ClearAll["Global`*"]
```

```
prank = y''''[x] + 10 y''[x] + 9 y[x] == 6.5 Sinh[2 x]
```

```
dank = DSolve[prank, y[x], x]
```

```
9 y[x] + 10 y''[x] + y'''[x] == 6.5 Sinh[2 x]
```

$$\left\{ \left\{ y[x] \rightarrow 1. C[3] \cos[1. x] + 1. C[1] \cos[3. x] + 1. C[4] \sin[1. x] + 1. C[2] \sin[3. x] + 0.1625 (0. + 1. \cos[1. x]^2 \sinh[2. x] - 0.384615 \cos[3. x]^2 \sinh[2. x] + 1. \sin[1. x]^2 \sinh[2. x] - (0.384615 + 2.31296 \times 10^{-17} i) \sin[3. x]^2 \sinh[2. x]) \right\} \right\}$$

```
bank = Chop[dank, 10^-16]
```

$$\left\{ \left\{ y[x] \rightarrow 1. C[3] \cos[1. x] + 1. C[1] \cos[3. x] + 1. C[4] \sin[1. x] + 1. C[2] \sin[3. x] + 0.1625 (1. \cos[1. x]^2 \sinh[2. x] - 0.384615 \cos[3. x]^2 \sinh[2. x] + 1. \sin[1. x]^2 \sinh[2. x] - 0.384615 \sin[3. x]^2 \sinh[2. x]) \right\} \right\}$$

```
sank = Simplify[bank]
```

$$\left\{ \left\{ y[x] \rightarrow 1. C[3] \cos[(1. + 0. i) x] + 1. C[1] \cos[(3. + 0. i) x] + 1. C[4] \sin[(1. + 0. i) x] + 1. C[2] \sin[(3. + 0. i) x] + 0.1 \sinh[2. x] \right\} \right\}$$

Chop[sank, 10^-15]

```
{ {Y[x] -> 1. C[3] Cos[1. x] + 1. C[1] Cos[3. x] +
  1. C[4] Sin[1. x] + 1. C[2] Sin[3. x] + 0.1 Sinh[2. x] } }
```

1. Above: The expression matches the text's answer.

$$5. \quad (x^3 D^3 + x^2 D^2 - 2x D + 2 I) y = x^{-2}$$

ClearAll["Global`*"]

```
plow = x^3 y'''[x] + x^2 y''[x] - 2 x y'[x] + 2 y[x] == x^-2
cow = DSolve[plow, y[x], x]
```

$$2 y[x] - 2 x y'[x] + x^2 y''[x] + x^3 y^{(3)}[x] == \frac{1}{x^2}$$

```
{ {Y[x] -> -\frac{1}{12 x^2} + \frac{C[1]}{x} + x C[2] + x^2 C[3] } }
```

1. Above: The answer matches the text's.

$$7. \quad (D^3 - 9 D^2 + 27 D - 27 I) y = 27 \sin[3 x]$$

```
boat = y'''[x] - 9 y''[x] + 27 y'[x] - 27 y[x] == 27 Sin[3 x]
coat = DSolve[boat, y[x], x]
```

$$-27 y[x] + 27 y'[x] - 9 y''[x] + y^{(3)}[x] == 27 \sin[3 x]$$

```
{ {Y[x] -> e^{3 x} C[1] + e^{3 x} x C[2] + e^{3 x} x^2 C[3] + \frac{1}{4} (-Cos[3 x] + Sin[3 x]) } }
```

```
goat = Collect[coat, e^{3 x}]
```

```
{ {Y[x] -> e^{3 x} (C[1] + x C[2] + x^2 C[3]) - \frac{1}{4} Cos[3 x] + \frac{1}{4} Sin[3 x] } }
```

1. Above: The answer matches the text's.

8 - 13 Initial value problem

Solve the given IVP.

$$9. \quad y^{iv} + 5 y''' + 4 y = 90 \sin[x], \quad y[0] = 1, \\ y'[0] = 2, \quad y''[0] = -1, \quad y'''[0] = -32$$

ClearAll["Global`*"]

```

sing = {y''''[x] + 5 y'''[x] + 4 y[x] == 90 Sin[4 x],
  y[0] == 1, y'[0] == 2, y''[0] == -1, y'''[0] == -32}
ring = DSolve[sing, y[x], x]
{4 y[x] + 5 y''[x] + y(4)[x] == 90 Sin[4 x],
  y[0] == 1, y'[0] == 2, y''[0] == -1, y(3)[0] == -32}
{ {y[x] →
  
$$\frac{1}{4} (4 \cos[x] + 80 \cos[x]^3 \sin[x] - 40 \cos[3 x] \sin[x] - 12 \cos[5 x] \sin[x] - 80 \cos[x] \sin[x]^3 + 15 \cos[2 x] \sin[2 x] + 20 \cos[2 x]^3 \sin[2 x] - 40 \cos[x] \sin[3 x] + 12 \cos[x] \sin[5 x] - 5 \cos[2 x] \sin[6 x]) }$$


```

Below I do some hammering to try to get the Mathematica solution into the same form as the text answer.

```

thing = Simplify[ring]
{{y[x] → Cos[x] (1 - Sin[x] + Sin[3 x]) }}

```

1. Below: To see what I need to make equal to $\frac{1}{2} \sin[4x]$.

```

TrigExpand[-Cos[x] Sin[x] + Cos[x] Sin[3 x]]

```

```

2 Cos[x]3 Sin[x] - 2 Cos[x] Sin[x]3

```

```

bling = thing /.
  (Cos[x] (1 - Sin[x] + Sin[3 x])) → (Cos[x] - Cos[x] Sin[x] + Cos[x] Sin[3 x])
{{y[x] → Cos[x] - Cos[x] Sin[x] + Cos[x] Sin[3 x] }}

```

2. Below: Put together some indents to use.

```

TrigExpand[Sin[2 x]]
2 Cos[x] Sin[x]

TrigExpand[Sin[3 x]]
3 Cos[x]2 Sin[x] - Sin[x]3

TrigExpand[Cos[2 x]]
Cos[x]2 - Sin[x]2

```

3. Therefore $\sin[4x] = 2 \cos[2x] \sin[2x] = 2 ((\cos[x]^2 - \sin[x]^2) (2 \cos[x] \sin[x]))$

```

sling = bling /. (Sin[3 x]) → (3 Cos[x]2 Sin[x] - Sin[x]3)
{{y[x] → Cos[x] - Cos[x] Sin[x] + Cos[x] (3 Cos[x]2 Sin[x] - Sin[x]3) }}

```

```

string = sling /. (Cos[x] (3 Cos[x]^2 Sin[x] - Sin[x]^3)) ->
  (Cos[x] Sin[x] (3 Cos[x]^2 - Sin[x]^2))
{{y[x] -> Cos[x] - Cos[x] Sin[x] + Cos[x] Sin[x] (3 Cos[x]^2 - Sin[x]^2)}}

zing = string /. (3 Cos[x]^2 - Sin[x]^2) -> (2 Cos[x]^2 + Cos[2 x])
{{y[x] -> Cos[x] - Cos[x] Sin[x] + Cos[x] (2 Cos[x]^2 + Cos[2 x]) Sin[x]}}

fling = zing /.
  (Cos[x] (2 Cos[x]^2 + Cos[2 x]) Sin[x]) -> (1/2 Sin[2 x] (Cos[2 x] + 2 Cos[x]^2))
{{y[x] -> Cos[x] - Cos[x] Sin[x] + 1/2 (2 Cos[x]^2 + Cos[2 x]) Sin[2 x]}}

ping = fling /.
  (1/2 (2 Cos[x]^2 + Cos[2 x]) Sin[2 x]) -> (1/2 (2 Cos[x]^2 Sin[2 x] + 1/2 Sin[4 x]))
{{y[x] -> Cos[x] - Cos[x] Sin[x] + 1/2 (2 Cos[x]^2 Sin[2 x] + 1/2 Sin[4 x])}}

```

4. Above: This is looking worse and worse. Time to swing for the fence.

```

p1 = Cos[x]
Cos[x]

p2 = Simplify[-Cos[x] Sin[x] + 1/2 (2 Cos[x]^2 Sin[2 x] + 1/2 Sin[4 x])]
1/2 Sin[4 x]

```

5. Above: A miraculous hit.

```
out = p1 + p2
```

$$\cos[x] + \frac{1}{2} \sin[4 x]$$

6. Above: The answer does match the text answer.

$$11. \quad (D^3 - 2D^2 - 3D) y = 74 e^{-3x} \sin[x], \\ y[0] = -1.4, \quad y'[0] = 3.2, \quad y''[0] = -5.2$$

```
ClearAll["Global`*"]
```

```
alt = {y'''[x] - 2 y''[x] - 3 y'[x] == 74 e-3 x Sin[x],
  y[0] == -1.4, y'[0] == 3.2, y''[0] == -5.2}
kalt = DSolve[alt, y[x], x]
{-3 y'[x] - 2 y''[x] + y(3)[x] == 74 e-3 x Sin[x],
  y[0] == -1.4, y'[0] == 3.2, y''[0] == -5.2}
{{y[x] -> -1/5 e-3 x (7 Cos[x] + 5 Sin[x])}}
```

$$\text{salt} = \text{kalt} /. \left(-\frac{1}{5} e^{-3x} (7 \cos[x] + 5 \sin[x]) \right) \rightarrow \left(e^{-3x} \left(-\frac{7}{5} \cos[x] - \frac{5}{5} \sin[x] \right) \right)$$

```
{{y[x] -> e-3 x \left( -\frac{7 \cos[x]}{5} - \sin[x] \right)}}
```

1. Above: Substitution by hand results in the text's answer.

13. $(D^3 - 4D)y = 10 \cos[x] + 5 \sin[x], y[0] = 3, y'[0] = -2, y''[0] = -1$

```
ClearAll["Global`*"]
rog = {y'''[x] - 4 y'[x] == 10 Cos[x] + 5 Sin[x],
  y[0] == 3, y'[0] == -2, y''[0] == -1}
dog = DSolve[rog, y[x], x]
{-4 y'[x] + y(3)[x] == 10 Cos[x] + 5 Sin[x], y[0] == 3, y'[0] == -2, y''[0] == -1}
{{y[x] -> 2 + Cos[x] - 2 Sin[x]}}
```

1. Above: The answer matches the text's.