Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

5 - 15 Radius of Convergence by Differentiation or Integration
Find the radius of convergence in two ways: (a) directly by the Cauchy-Hadamard formula in Sec. 15.2, and (b) from a series of simpler terms by using Theorem 3, p. 687, or Theorem 4, p. 688.

5. $\text{Sum}\left[\frac{n\,(n-1)}{2^n}\,(z-2\,i)^n,\ \{n, 2, \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is 2 i.

$\text{Limit}\left[\text{Abs}\left[\frac{n\,(n-1)}{2^n}\left(\frac{2^{n+1}}{n\,(n+1)}\right)\right],\ n\to\infty\right]$

2

The power of the power term is 1, so the answer should be

$(2)^{1/1}$

2

The above green agrees with the text answer. This is the half of the problem worked with Cauchy-Hadamard. The other way, which is developed by the s.m., is the series method. However, using Mathematica, it is only a matter of invoking the command **SumConvergence**, which in this case works well with the original expression,

$\text{SumConvergence}\left[\frac{n\,(n-1)}{2^n}\,(z-2\,i)^n,\ n\right]$

$\text{Abs}[-2\,i+z]<2$

7. $\text{Sum}\left[\frac{n}{3^n}\,(z+2\,i)^{2\,n},\ \{n, 1, \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is -2 i. In part (a) I will look at a Cauchy-Hadamard solution,

$\text{Limit}\left[\text{Abs}\left[\frac{n}{3^n}\left(\frac{3^{n+1}}{(n+1)}\right)\right],\ n\to\infty\right]$

3

The power of the power term is 2, which implies the radius of convergence is,

$(3)^{1/2}$

$\sqrt{3}$

For part (b) I will look at differentiation of series terms, or in this case, the **SumConvergence** command,

$\text{SumConvergence}\left[\frac{n}{3^n}(z+2\,i)^{2\,n},\ n\right]$

$\text{Abs}[2\,i+z]^2<3$

Due to Mathematica's indolence, the square root symbol has to be applied by hand. Since the lhs is positive in sign, the resulting square root will be also.

9. $\text{Sum}\left[\frac{-2^n}{n\,(n+1)\,(n+2)}z^{2\,n},\ \{n,\ 1,\ \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is 0. For part (a),

$\text{Limit}\left[\text{Abs}\left[\frac{-2^n}{n\,(n+1)\,(n+2)}\left(\frac{(n+3)\,(n+1)\,(n+2)}{-2^{n+1}}\right)\right],\ n\to\infty\right]$

$\frac{1}{2}$

The power of the power term being 2 n, the resulting radius of convergence is

$\left(\frac{1}{2}\right)^{1/2}$

$\frac{1}{\sqrt{2}}$

As for part (b), the **SumConvergence** command again,

$\text{SumConvergence}\left[\frac{-2^n}{n\,(n+1)\,(n+2)}z^{2\,n},\ n\right]$

$\text{Abs}[z]<\frac{1}{\sqrt{2}}$

11. $\text{Sum}\left[\frac{3^n\,n\,(n+1)}{7^n}(z+2)^{2\,n},\ \{n,\ 1,\ \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is -2. For part (a),

```
Limit[Abs[3^n n (n + 1)/7^n (7^(n+1)/(3^(n+1) (n + 2) (n + 1)))], n → ∞]
```

$$\frac{7}{3}$$

The power of the power term being 2 n, the resulting radius of convergence is

$$\left(\frac{7}{3}\right)^{1/2}$$

$$\sqrt{\frac{7}{3}}$$

As for part (b), the SumConvergence command again,

```
SumConvergence[3^n n (n + 1)/7^n (z + 2)^(2 n), n]
```

```
3 Abs[2 + z]^2 < 7
```

Or, to spell it all out,

$$3\,\mathrm{Abs}[2+z]^2 < 7 \Longrightarrow \mathrm{Abs}[2+z]^2 < \frac{7}{3} \Longrightarrow \mathrm{Abs}[2+z] < \sqrt{\frac{7}{3}};$$

13. $\mathrm{Sum}\left[\binom{n+k}{k}^{-1} z^{n+k}, \{n, 0, \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is 0. For part (a),

```
Limit[Abs[(Binomial[n + k, k])^-1/(Binomial[n + 1 + k, k])^-1],
  n → ∞, Assumptions → {k ∈ Integers, k < 100}]
```

1

In the above cell some assumption has to be made about k, or else Mathematica will go into a trance and not come back.

The power of the power term is n+k. As I did before I will ignore anything that does not modify the n particle. This would result in

$$(1)^{1/1}$$

1

For part (b),

```
SumConvergence[(Binomial[n + k, k]^-1) z^(n+k), n]
```

```
Abs[z] < 1
```

What was interesting about this is how quickly Mathematica came back with this answer, without demanding a description of k.

15. Sum$\left[\frac{4^n\, n\, (n-1)}{3^n}\, (z - i)^n, \{n, 2, \infty\}\right]$

```
Clear["Global`*"]
```

The center of the series is i. For part (a),

$$\text{Limit}\left[\text{Abs}\left[\frac{4^n\, n\, (n-1)}{3^n}\left(\frac{3^{n+1}}{4^{n+1}\, n\, (n+1)}\right)\right],\ n \to \infty\right]$$

$$\frac{3}{4}$$

The power of the power term is n.

$$\left(\frac{3}{4}\right)^{1/1}$$

$$\frac{3}{4}$$

For part (b),

$$\text{SumConvergence}\left[\frac{4^n\, n\, (n-1)}{3^n}\, (z - i)^n,\ n\right]$$

```
4 Abs[-i + z] < 3
```

As before, some gathering is necessary to match the text answer.