Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

4 - 8 Double Fourier Series

Represent f(x,y) by a series (15), where

$$u[x, y, 0] = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \operatorname{Sin}\left[\frac{m \pi x}{a}\right] \operatorname{Sin}\left[\frac{n \pi y}{b}\right] = f[x, y]$$

and

5. f(x,y)=y, a=b=1

```
Clear["Global`*"]
```

For this type of problem, numbered line (15) is shown above. After a little development, the text presents numbered line (18), p.582, which is the **generalized Euler formula**:

$$B_{mn} = \frac{4}{a\, b} \int_0^b \int_0^a f[x, y] \operatorname{Sin}\left[\frac{m \pi x}{a}\right] \operatorname{Sin}\left[\frac{n \pi y}{b}\right] dx\, dy \qquad n, m \to 1, 2, \ldots$$

in the case of this problem,

$$\mathbf{Bmn} = 4 \int_0^1 \int_0^1 y \operatorname{Sin}[m \pi x] \operatorname{Sin}[n \pi y]\, dx\, dy$$

$$\frac{4\, (-1 + \operatorname{Cos}[m \pi])\, (n \pi \operatorname{Cos}[n \pi] - \operatorname{Sin}[n \pi])}{m\, n^2\, \pi^3}$$

If m is even then Bmn is zero (because **Cos[m π]** would then equal 1), else if m is odd, then

$$\mathbf{Bmno} = \mathbf{Bmn} \;/.\; \{(-1 + \operatorname{Cos}[m \pi]) \to -2,\ \operatorname{Cos}[n \pi] \to (-1)^{n+1},\ \operatorname{Sin}[n \pi] \to 0\}$$

$$-\frac{8\, (-1)^{1+n}}{m\, n\, \pi^2}$$

The green cell above matches the text answer for B_{mn}. There is no text answer for u(x,y,0), but it would be the pattern shown above, in cyan, with the restriction that m be odd. The general token B_{1n} does not have a negative sign, which I guess is why $(-1)^{n+1}$ was chosen as the formula for the sign of Cos[n π].

7. f(x,y)=x y, a and b arbitrary

$$\mathbf{Bmn} = \frac{4}{a\, b} \int_0^b \int_0^a x\, y \operatorname{Sin}\left[\frac{m \pi x}{a}\right] \operatorname{Sin}\left[\frac{n \pi y}{b}\right] dx\, dy$$

$$\frac{1}{m^2\, n^2\, \pi^4} 4\, a\, b\, (m \pi \operatorname{Cos}[m \pi] - \operatorname{Sin}[m \pi])\, (n \pi \operatorname{Cos}[n \pi] - \operatorname{Sin}[n \pi])$$

The circumstances are not the same as in the last problem. No pattern, even or odd, makes

B_{mn} equal zero.

```
Bmno = Bmn /. {Sin[n π] → 0, Sin[m π] → 0}
```

$$\frac{4\,a\,b\,\text{Cos}[m\,\pi]\,\text{Cos}[n\,\pi]}{m\,n\,\pi^2}$$

I hope the above cell would do if required, because I had to cheat by looking at the answer to see the clever device for getting the sign:

```
Bmnf = Bmno /. Cos[m π] Cos[n π] → (-1)^(m+n)
```

$$\frac{4\,(-1)^{m+n}\,a\,b}{m\,n\,\pi^2}$$

The above green cell matches the text answer.

11 - 13 Square Membrane

Find the deflection u(x,y,t) of the square membrane of side π and $c^2 = 1$ for initial velocity 0 and initial deflection

11. 0.1 Sin[2 x] Sin[4 y]

To do this problem I need numbered line (9) on p. 580:

$$\lambda = \lambda_{mn} = c\,\pi\,\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \qquad m,\ n \to 1,\ 2,\ 3,\ \ldots$$

and numbered line (14) on p. 582 :

$$u[x,\ y,\ t] = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\text{Bmn Cos}[\lambda_{mn}\,t] + \text{Bast}_{mn}\,\text{Sin}[\lambda_{mn}\,t]\right)\text{Sin}\left[\frac{m\,\pi\,x}{a}\right]\,\text{Sin}\left[\frac{n\,\pi\,y}{b}\right]$$

and numbered line (18) on p. 582 :

$$B_{mn} = \frac{4}{a\,b}\int_0^b\int_0^a f[x,\ y]\,\text{Sin}\left[\frac{m\,\pi\,x}{a}\right]\,\text{Sin}\left[\frac{n\,\pi\,y}{b}\right]\,dx\,dy \qquad m,\ n \to 1,\ 2,\ 3,\ \ldots$$

as well as numbered line (19) on p. 583 :

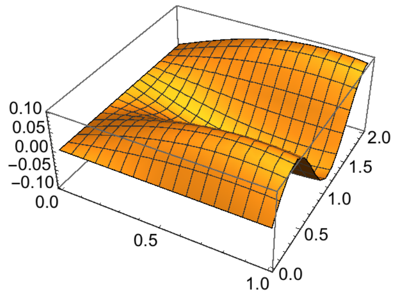
$$\text{Bast}_{mn} = \frac{4}{a\,b\,\lambda_{mn}}\int_0^b\int_0^a g[x,\ y]\,\text{Sin}\left[\frac{m\,\pi\,x}{a}\right]\,\text{Sin}\left[\frac{n\,\pi\,y}{a}\right]\,dx\,dy \qquad m,$$
$n \to 1,\ 2,\ 3\ \ldots$

```
Clear["Global`*"]
```

The initial displacement is defined.

```
f[x_, y_] = 0.1 Sin[2 x] Sin[4 y]
0.1 Sin[2 x] Sin[4 y]
```

```
Plot3D[f[x, y], {x, 0, 1}, {y, 0, 2}, ImageSize → 200]
```



The Fourier coefficients are computed:

```
a[n_, m_] = Integrate[
   Integrate[f[x, y] Sin[m π y / 3.14], {y, 0, 3.14}] Sin[n π x / 3.14], {x, 0, 3.14}];
Grid[Table[a[n, m], {n, 1, 10}, {m, 1, 10}]]
```

4.515⋱78 × 10^{-8} + 0. ⅈ	-1.1⋱29⋱25 × 10^{-7} + 0. ⅈ	2.9066 × 10^{-7} + 0. ⅈ	0.000⋱166⋱88 + 0. ⅈ	-3.7⋱52⋱32 × 10^{-7} + 0. ⅈ	2.028⋱26 × 10^{-7} + 0. ⅈ	-1.4⋱34⋱58 × 10^{-7} + 0. ⅈ	1.127⋱34 × 10^{-7} + 0. ⅈ	-9.3⋱66⋱46 × 10^{-8} + 0. ⅈ	8. 0
0.000⋱066⋱733⋱8 + 0. ⅈ	-0.0⋱00⋱16⋱68⋱8 + 0. ⅈ	0.000⋱429⋱534 + 0. ⅈ	0.246⋱613 + 0. ⅈ	-0.0⋱00⋱55⋱45⋱14 + 0. ⅈ	0.000⋱299⋱734 + 0. ⅈ	-0.0⋱00⋱21⋱20⋱01 + 0. ⅈ	0.000⋱166⋱598 + 0. ⅈ	-0.0⋱00⋱13⋱84⋱17 + 0. ⅈ	0. 0
-8.11⋱085 × 10^{-8} + 0. ⅈ	2.028⋱26 × 10^{-7} + 0. ⅈ	-5.22⋱05⋱8 × 10^{-7} + 0. ⅈ	-0.00⋱02⋱99⋱73⋱4 + 0. ⅈ	6.739⋱59 × 10^{-7} + 0. ⅈ	-3.64⋱29⋱8 × 10^{-7} + 0. ⅈ	2.576⋱67 × 10^{-7} + 0. ⅈ	-2.02⋱48⋱4 × 10^{-7} + 0. ⅈ	1.682⋱32 × 10^{-7} + 0. ⅈ	-1 0
4.508⋱16 × 10^{-8} + 0. ⅈ	-1.1⋱27⋱34 × 10^{-7} + 0. ⅈ	2.901⋱69 × 10^{-7} + 0. ⅈ	0.000⋱1665⋱98 + 0. ⅈ	-3.7⋱45⋱99 × 10^{-7} + 0. ⅈ	2.024⋱84 × 10^{-7} + 0. ⅈ	-1.4⋱32⋱16 × 10^{-7} + 0. ⅈ	1.125⋱44 × 10^{-7} + 0. ⅈ	-9.3⋱50⋱64 × 10^{-8} + 0. ⅈ	8. 0
-3.22⋱058 × 10^{-8} + 0. ⅈ	8.053⋱63 × 10^{-8} + 0. ⅈ	-2.07⋱29⋱4 × 10^{-7} + 0. ⅈ	-0.000⋱11⋱90⋱16 + 0. ⅈ	2.676⋱09 × 10^{-7} + 0. ⅈ	-1.44⋱65⋱2 × 10^{-7} + 0. ⅈ	1.023⋱12 × 10^{-7} + 0. ⅈ	-8.04⋱00⋱3 × 10^{-8} + 0. ⅈ	6.68 × 10^{-8} + 0. ⅈ	-5 0

2.536⋱37 × 10^{-8} + 0. ⅈ	-6.3⋱42⋱65 × 10^{-8} + 0. ⅈ	1.632⋱55 × 10^{-7} + 0. ⅈ	0.0000⋱93731 + 0. ⅈ	-2.1⋱07⋱56 × 10^{-7} + 0. ⅈ	1.139⋱21 × 10^{-7} + 0. ⅈ	-8.0⋱57⋱6 × 10^{-8} + 0. ⅈ	6.331⋱94 × 10^{-8} + 0. ⅈ	-5.2⋱60⋱85 × 10^{-8} + 0. ⅈ	4. 0
-2.10⋱433 × 10^{-8} + 0. ⅈ	5.262⋱24 × 10^{-8} + 0. ⅈ	-1.35⋱44⋱6 × 10^{-7} + 0. ⅈ	-0.0000⋱7776⋱49 + 0. ⅈ	1.748⋱56 × 10^{-7} + 0. ⅈ	-9.45⋱15⋱7 × 10^{-8} + 0. ⅈ	6.685⋱06 × 10^{-8} + 0. ⅈ	-5.25⋱33⋱6 × 10^{-8} + 0. ⅈ	4.364⋱71 × 10^{-8} + 0. ⅈ	-3 0
1.803⋱75 × 10^{-8} + 0. ⅈ	-4.5⋱10⋱6 × 10^{-8} + 0. ⅈ	1.160⋱99 × 10^{-7} + 0. ⅈ	0.00006⋱66571 + 0. ⅈ	-1.4⋱98⋱8 × 10^{-7} + 0. ⅈ	8.101⋱53 × 10^{-8} + 0. ⅈ	-5.7⋱30⋱18 × 10^{-8} + 0. ⅈ	4.502⋱98 × 10^{-8} + 0. ⅈ	-3.7⋱41⋱27 × 10^{-8} + 0. ⅈ	3. 0
-1.58⋱123 × 10^{-8} + 0. ⅈ	3.954⋱15 × 10^{-8} + 0. ⅈ	-1.01⋱77⋱7 × 10^{-7} + 0. ⅈ	-0.0000⋱5843⋱41 + 0. ⅈ	1.3139 × 10^{-7} + 0. ⅈ	-7.10⋱21 × 10^{-8} + 0. ⅈ	5.023⋱29 × 10^{-8} + 0. ⅈ	-3.94⋱74⋱8 × 10^{-8} + 0. ⅈ	3.279⋱73 × 10^{-8} + 0. ⅈ	-2 0
1.409⋱22 × 10^{-8} + 0. ⅈ	-3.5⋱23⋱99 × 10^{-8} + 0. ⅈ	9.070⋱47 × 10^{-8} + 0. ⅈ	0.00005⋱20772 + 0. ⅈ	-1.1⋱70⋱97 × 10^{-7} + 0. ⅈ	6.329⋱48 × 10^{-8} + 0. ⅈ	-4.4⋱76⋱82 × 10^{-8} + 0. ⅈ	3.518⋱04 × 10^{-8} + 0. ⅈ	-2.9⋱22⋱94 × 10^{-8} + 0. ⅈ	2. 0

```
(*Lambda[n_,m_]=(n π)^2 + (m π/2)^2;*)
```

The eigenvalues are computed:

$$\text{Lambda}[m\_,\ n\_] = \pi \sqrt{\frac{m^2}{(3.14)^2} + \frac{n^2}{(3.14)^2}}$$

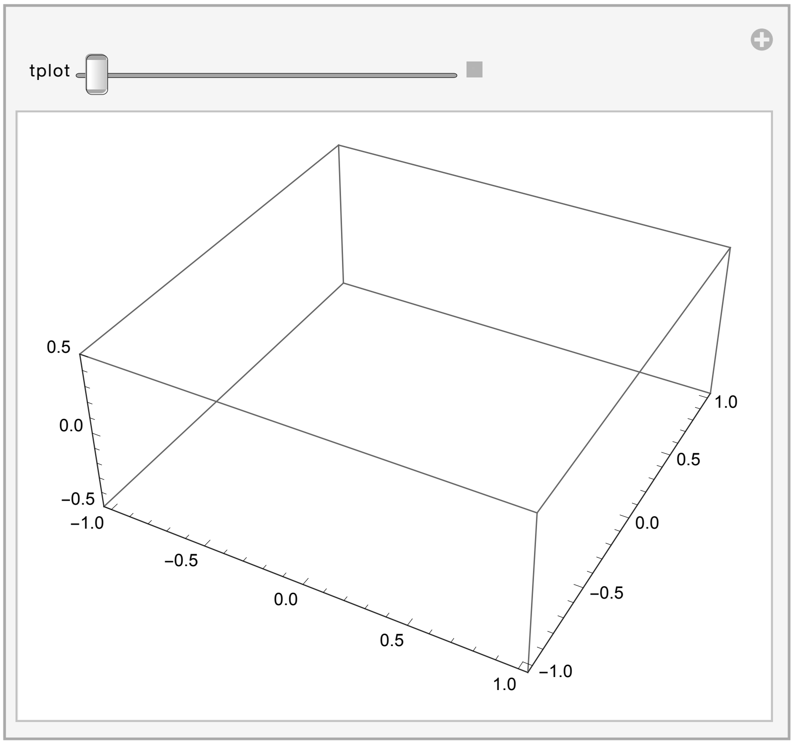
$$\sqrt{0.101424\, m^2 + 0.101424\, n^2}\ \pi$$

The solution, truncated at N and M, respectively, is given by

$$u[x\_,\ y\_,\ t\_,\ N\_,\ M\_] := \text{Sum}\Big[\text{Sum}\Big[a[n,\ m]\ \text{Cos}[\text{Lambda}[n,\ m]\ t]\ \text{Sin}\Big[\frac{n\,\pi\,x}{3.14}\Big]\ \text{Sin}\Big[\frac{m\,\pi\,y}{3.14}\Big],\ \{n,\ 1,\ N\}\Big],\ \{m,\ 1,\ M\}\Big]$$

```
uplot = u[x, y, t, 10, 20];
```

```
Manipulate[Plot3D[uplot /. t → tplot, {x, 0, 1}, {y, 0, 2},
  PlotRange → {All, All, {-1 / 2, 1 / 2}}], {tplot, 0, 5}]
```




```
u[x, y, t, 2, 4]
```

$$
\begin{aligned}
&\left(4.51578\times10^{-8} + 0.\,i\right) \mathrm{Cos}[1.41493\,t]\,\mathrm{Sin}[1.00051\,x]\,\mathrm{Sin}[1.00051\,y] + \\
&(0.0000667338 + 0.\,i)\,\mathrm{Cos}[2.2372\,t]\,\mathrm{Sin}[2.00101\,x]\,\mathrm{Sin}[1.00051\,y] - \\
&\left(1.12925\times10^{-7} + 0.\,i\right) \mathrm{Cos}[2.2372\,t]\,\mathrm{Sin}[1.00051\,x]\,\mathrm{Sin}[2.00101\,y] - \\
&(0.00016688 + 0.\,i)\,\mathrm{Cos}[2.82986\,t]\,\mathrm{Sin}[2.00101\,x]\,\mathrm{Sin}[2.00101\,y] + \\
&\left(2.9066\times10^{-7} + 0.\,i\right) \mathrm{Cos}[3.16388\,t]\,\mathrm{Sin}[1.00051\,x]\,\mathrm{Sin}[3.00152\,y] + \\
&(0.000429534 + 0.\,i)\,\mathrm{Cos}[3.60738\,t]\,\mathrm{Sin}[2.00101\,x]\,\mathrm{Sin}[3.00152\,y] + \\
&(0.00016688 + 0.\,i)\,\mathrm{Cos}[4.1252\,t]\,\mathrm{Sin}[1.00051\,x]\,\mathrm{Sin}[4.00203\,y] + \\
&(0.246613 + 0.\,i)\,\mathrm{Cos}[4.4744\,t]\,\mathrm{Sin}[2.00101\,x]\,\mathrm{Sin}[4.00203\,y]
\end{aligned}
$$

$(4.4744)^2$

20.0203

Although the above yellow cell is not exactly the text answer, it has the same form. There appears to be something wrong with this problem. Maybe a typo? (Except for the leading coefficient, the last yellow cell line is very close to the text answer.)

****

Following example 2 on p. 582,

c = 1

1

To test Mathematica's abilities, B_{mn} is calculted two ways:

$$B_{mn} = \frac{4}{\pi^2} \int_0^{\pi} \int_0^{\pi} .1 \, \text{Sin}[2\,x] \, \text{Sin}[4\,y] \, \text{Sin}[m\,x] \, \text{Sin}[n\,y] \, dy \, dx$$

$$\frac{0.324228 \, \text{Sin}[m\,\pi] \, \text{Sin}[n\,\pi]}{(-4. + m^2)\,(-16. + n^2)}$$

$$\text{Bmny} = \frac{.4}{\pi^2} \int_0^{\pi} \frac{1}{2} (\text{Cos}[(n-4)\,y] - \text{Cos}[(n+4)\,y]) \, dy$$

$$\frac{0.162114 \, \text{Sin}[n\,\pi]}{-16 + n^2}$$

$$\text{Bmnx} = \int_0^{\pi} \frac{1}{2} (\text{Cos}[(2-m)\,x] - \text{Cos}[(2+m)\,x]) \, dx$$

$$\frac{2 \, \text{Sin}[m\,\pi]}{-4 + m^2}$$

bmn = Bmny Bmnx

$$\frac{0.324228 \, \text{Sin}[m\,\pi] \, \text{Sin}[n\,\pi]}{(-4 + m^2)\,(-16 + n^2)}$$

$$\text{bmnc} = \left(0.3242277876554809\text{\`{}} \, (\text{Cos}[\pi\,(m-n)] - \text{Cos}[\pi\,(m+n)]) \, \frac{1}{2}\right) \Big/ \left((-4 + m^2)\,(-16 + n^2)\right)$$

$$(0.162114 \, (\text{Cos}[(m-n)\,\pi] - \text{Cos}[(m+n)\,\pi])) \Big/ \left((-4 + m^2)\,(-16 + n^2)\right)$$

bmnc makes use of a trig identity that I hoped would help out with the usability of bm.

$$\lambda_{mn} = \text{Simplify}\left[c\,\pi \sqrt{\frac{m^2}{\pi^2} + \frac{n^2}{\pi^2}}\right]$$

$$\sqrt{m^2 + n^2}$$

```
uxyt2 =
 Sum[Sum[bmnc (1/2 (Cos[m x - n y] - Cos[m x + n y])) Cos[λmn t], {m, 1, ∞, 2}],
  {n, 1, ∞, 2}]
```

```
Sum[Sum[(0.0810569 (Cos[(m - n) π] - Cos[(m + n) π])
      Cos[Sqrt[m^2 + n^2] t] (Cos[m x - n y] - Cos[m x + n y]))/
    ((-4 + m^2) (-16 + n^2)), {m, 1, ∞, 2}], {n, 1, ∞, 2}]
```

The trig identity is used again. But there are still problems.

I was not able to get the text answer, which looks very nice. The yellow cell above, which was assembled using the rules as best as I was able to apply them, looks very messy.

```
u[x_, y_, t_, m_, n_, j_, k_] := Sum[Sum[1/((-4 + m^2) (-16 + n^2))
    0.08105694691387022` (Cos[(m - n) π] - Cos[(m + n) π]) Cos[Sqrt[m^2 + n^2] t]
     (Cos[m x - n y] - Cos[m x + n y]), {m, 1, ∞, 2}], {n, 1, ∞, 2}]
```

It's not blowing up at the moment, but I have little hope of verifying it.

```
u[x, y, t, m, n, 1, 1]
```

```
Sum[Sum[(0.0810569 (Cos[(m - n) π] - Cos[(m + n) π])
      Cos[Sqrt[m^2 + n^2] t] (Cos[m x - n y] - Cos[m x + n y]))/
   ((-4 + m^2) (-16 + n^2)), {m, 1, ∞, 2}], {n, 1, ∞, 2}]
```

****

```
partialuo[j_, k_, x_, y_, t_] :=
 Sum[0.1 Sin[m x] Sin[n y] Cos[Sqrt[m^2 + n^2] t], {m, j, j, 2}, {n, k, k, 2}]
part1 = partialuo[2, 4, x, y, t]
```

```
0.1 Cos[2 Sqrt[5] t] Sin[2 x] Sin[4 y]
```

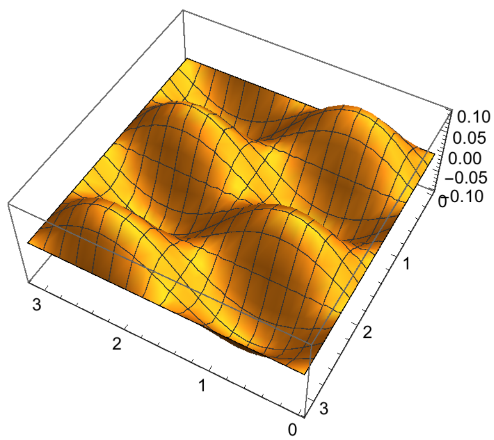
Just playing around, trying to get the text answer, I came up with the above yellow cell. The below yellow cell also gets there. No justification for this entertainment unfortunately.

```
f[x_, y_] = 0.1 Sin[2 x] Sin[4 y]
0.1 Sin[2 x] Sin[4 y]
```

```
partialuo2[j_, k_, x_, y_, t_] :=
 Sum[f[x, y] Cos[√(m² + n²) t], {m, j, j, 2}, {n, k, k, 2}]
part2 = partialuo[2, 4, x, y, t]
```

$0.1 \, \text{Cos}\left[2\sqrt{5}\, t\right] \text{Sin}[2\, x] \, \text{Sin}[4\, y]$

```
uti = 0.1 Sin[2 x] Sin[4 y];
Plot3D[Evaluate[uti], {x, 0, Pi}, {y, 0, Pi},
 PlotPoints → {20, 20}, ViewPoint → {-1.5, 3, 0.5}]
```



13. 0.1 x y(π - x)(π - y)

```
Clear["Global`*"]
```

$\text{c} = 1; \; \lambda_{\text{mn}} = \text{Simplify}\left[\text{c}\, \pi \sqrt{\frac{\text{m}^2}{\pi^2} + \frac{\text{n}^2}{\pi^2}}\right]$

$\sqrt{\text{m}^2 + \text{n}^2}$

$\text{f}[\text{x}\_, \text{y}\_] = 0.1\, \text{x}\, \text{y}\, (\pi - \text{x})\, (\pi - \text{y})$

$0.1\, (\pi - \text{x})\, \text{x}\, (\pi - \text{y})\, \text{y}$

The expression in the cell below is not complete, because I have pulled out a factor of $.4/\pi^2$ to save for later.

$\text{Bmn1} = \int_0^\pi \int_0^\pi \text{x}\, \text{y}\, (\pi - \text{x})\, (\pi - \text{y})\, \text{Sin}[\text{m}\, \text{x}]\, \text{Sin}[\text{n}\, \text{y}]\, d\text{x}\, d\text{y}$

$\frac{1}{\text{m}^3\, \text{n}^3} (-2 + 2\, \text{Cos}[\text{m}\, \pi] + \text{m}\, \pi\, \text{Sin}[\text{m}\, \pi])\, (-2 + 2\, \text{Cos}[\text{n}\, \pi] + \text{n}\, \pi\, \text{Sin}[\text{n}\, \pi])$

Now as for the cell above: I want to save the denominator, and to evaluate the numerator:

```
Sum[(-2 + 2 Cos[m π] + m π Sin[m π]) (-2 + 2 Cos[n π] + n π Sin[n π]),
 {m, 1, 1, 2}, {n, 1, 1, 2}]
```

16

```
6.4 / .4
```

16.

Combining the operations above means that I have a total leading factor now of $\frac{6.4}{\pi^2}$, and all that is left of Bmnl is $\frac{1}{m^3 n^3}$.

outeq = Simplify$\left[\text{partialu}[j_, k_, x_, y_, t_] := \sum_{n=1}^{k}\sum_{m=1}^{j} \text{Bmn Sin}[m\,x]\ \text{Sin}[n\,y]\ \text{Cos}\left[\sqrt{m^2+n^2}\ t\right], \text{Assumptions} \to \{m, n \in \text{OddQ}\}\right]$

outeq1 = Simplify$\left[\text{partialu}[j_, k_, x_, y_, t_] := \frac{6.4}{\pi^2}\sum_{n=1}^{k}\sum_{m=1}^{j} \frac{1}{m^3\, n^3} \text{Sin}[m\,x]\ \text{Sin}[n\,y]\ \text{Cos}\left[\sqrt{m^2+n^2}\ t\right], \text{Assumptions} \to \{m, n \in \text{OddQ}\}\right]$

The above green cell matches the answer of the text.

14 - 19 Rectangular Membrane

17. Find eigenvalues of the rectangular membrane of sides a = 2 and b = 1 to which there correspond two or more different (independent) eigenfunctions.

```
Clear["Global`*"]
```

$\lambda_{mn} = c\,\pi \sqrt{\frac{m^2}{4} + \frac{n^2}{1}}$

$c \sqrt{\frac{m^2}{4} + n^2}\ \pi$

$\text{eig}[m\_, n\_] = \frac{m^2}{4} + n^2$

$\frac{m^2}{4} + n^2$

$\text{Solve}\left[\frac{m^2}{4} == n^2, \{m, n\}\right]$

Solve::svars: Equations may not give solutions for all "solve" variables ≫

$\left\{\left\{n \to -\frac{m}{2}\right\}, \left\{n \to \frac{m}{2}\right\}\right\}$

```
Table[eig[m, n], {m, {4, 16}}, {n, {16, 14}}]
{{260, 200}, {320, 260}}

eig[10, 5]
50

Table[eig[m, n], {m, {2, 4, 6, 8}}, {n, {1, 2, 3, 4}}]
{{2, 5, 10, 17}, {5, 8, 13, 20}, {10, 13, 18, 25}, {17, 20, 25, 32}}
```

So for example $\lambda_{8,3} = \lambda_{6,4} = c\sqrt{25}\,\pi$. These are much smaller m,n than the text uses in its answer, but if I understand correctly, they are okay.

19. Deflection. Find the deflection of the membrane of sides a and b with $c^2 = 1$ for the initial deflection
$f(x, y) = \mathrm{Sin}\left[\frac{6\pi x}{a}\right]\mathrm{Sin}\left[\frac{2\pi y}{b}\right]$ and initial velocity 0.

```
Clear["Global`*"]
```

$$c = 1;\ \lambda_{mn} = \mathrm{Simplify}\left[c\,\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}\right]$$

$$\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}\,\pi$$

$$f[x\_, y\_] = \mathrm{Sin}\left[\frac{6\pi x}{a}\right]\mathrm{Sin}\left[\frac{2\pi y}{b}\right]$$

$$\mathrm{Sin}\left[\frac{6\pi x}{a}\right]\mathrm{Sin}\left[\frac{2\pi y}{b}\right]$$

$$B_{mn} = \frac{4}{a\,b}\int_0^b\int_0^a f[x, y]\,\mathrm{Sin}\left[\frac{m\pi x}{a}\right]\mathrm{Sin}\left[\frac{n\pi y}{b}\right]dx\,dy$$

$$\frac{48\,\mathrm{Sin}[m\pi]\,\mathrm{Sin}[n\pi]}{(-36+m^2)\,(-4+n^2)\,\pi^2}$$

$$u[x\_, y\_, t\_, j\_, k\_] = \sum_{m=1}^{j}\sum_{n=1}^{k}(B_{mn}\,\mathrm{Cos}[\lambda_{mn}\,t])\,\mathrm{Sin}\left[\frac{m\pi x}{a}\right]\mathrm{Sin}\left[\frac{n\pi y}{b}\right]$$

0

This problem has exactly the same problem as problem 11, and I have not yet figured out how to avoid it.