```
Clear["Global`*"]
```

2-10 General Solution. Find a general solution. Show the steps of derivation. Check your answer by substitution.

2. $y^3 y' + x^3 = 0$

```
eqn = y[x]^3 + x^3 == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -x]\},$
$\{y \to \text{Function}[\{x\}, (-1)^{1/3}\, x]\}, \{y \to \text{Function}[\{x\}, -(-1)^{2/3}\, x]\}\}$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True

eqn /. sol[[3]]
True

Clear["Global`*"]
```

3. $y' = \sec^2 y$

```
eqn = y'[x] == Sec[y[x]]^2;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \text{InverseFunction}[2 \left(\frac{1}{4} \text{Sin}[2\,\#1] + \frac{\#1}{2}\right) \&][2\,x + C[1]]]\}\}$

```
Simplify[eqn /. sol]
{True}

Clear["Global`*"]
```

4. y’ sin 2πx = πy cos 2πx

```
eqn = y'[x] Sin[2 π x] == π y[x] Cos[2 π x];
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, C[1]\sqrt{\text{Sin}[2\,\pi\,x]}]\}\}$

```
eqn /. sol
{True}

Clear["Global`*"]
```

5. y y’ + 36 x = 0

```
eqn = y[x] y'[x] + 36 x == 0;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\sqrt{2}\sqrt{-18 x^2 + C[1]}]\}, \{y \to \text{Function}[\{x\}, \sqrt{2}\sqrt{-18 x^2 + C[1]}]\}\}$$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True

Clear["Global`*"]
```

6. $y' = e^{2x-1} y^2$

```
eqn = y'[x] == e^(2 x-1) y[x]^2;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\frac{2 e}{e^{2x} + 2 e C[1]}]\}\}$$

```
eqn /. sol
{True}

Clear["Global`*"]
```

7. $xy' = y + 2x^3 \sin^2 \frac{y}{x}$ (Set y/x = u)

```
eqn = x y'[x] == y[x] + 2 x^3 Sin[y[x]/x]^2;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{y \to \text{Function}[\{x\}, -x \,\text{ArcCot}[x^2 - 2 C[1]]]\}\}$$

```
Simplify[eqn /. sol]
{True}

Clear["Global`*"]
```

8. $y' = (y + 4x)^2$ (Set $y + 4x = v$)

```
eqn = y'[x] == (y[x] + 4 x)^2;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -2 i - 4 x + \frac{1}{-\frac{i}{4} + e^{4 i x} C[1]}]\}\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

```
Clear["Global`*"]
```

9. xy’ $= y^2 + y$ (Set $y/x = u$)

```
eqn = x y'[x] == y[x]^2 + y[x];
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{e^{C[1]}\, x}{-1 + e^{C[1]}\, x}\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

```
Clear["Global`*"]
```

10. xy’ = x + y (Set y/x = u)

```
eqn = xy'[x] == x + y[x];
sol = DSolve[eqn, y, x]
```

```
{{y → Function[{x}, -x + xy′[x]]}}
```

```
eqn /. sol
```

```
{True}
```

11-17 Initial Value Problems (IVPs). Solve the IVP. Show the steps of derivation, beginning with the general solution.

11. xy’ + y = 0, y(4) = 6

```
Clear["Global`*"]
```

```
eqn = x y'[x] + y[x] == 0;
sol = DSolve[{eqn, y[4] == 6}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{24}{x}\right]\right\}\right\}$$

```
eqn /. sol
```

```
{True}
```

```
Clear["Global`*"]
```

12. y’ $= 1 + 4y^2$, $y(1) = 0$

```
eqn = y'[x] == 1 + 4 y[x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{1}{2}\text{Tan}[2(-1+x)]\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

{True}

```
Clear["Global`*"]
```

13. $y' \cosh^2 x = \sin^2 y,\ \ y(0) = \frac{1}{2}\pi$

```
eqn = y'[x] Cosh[x]^2 == Sin[y[x]]^2;
sol = DSolve[{eqn, y[0] == 1/2 π}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{{y → Function[{x}, -ArcCot[Tanh[x]]]}}

```
Simplify[eqn /. sol]
```

{True}

```
Clear["Global`*"]
```

14. $dr/dt = -2\,tr,\ \ r(0) = r_0$

```
eqn = r'[t] == -2 t r[t];
sol = DSolve[{eqn, r[0] == r0}, r[t], t]
```

$$\{\{r[t] \to e^{-t^2}\, r0\}\}$$

```
Simplify[eqn] /. Simplify[sol]
```

$$\{r'[t] == -2\, e^{-t^2}\, r0\, t\}$$

While Mathematica will not declare this equality to be true, it works when the substitutions are made.

```
Clear["Global`*"]
```

15. $y' = -4\,xy,\ \ y(2) = 3$

```
eqn = y'[x] == -4 x y[x];
sol = DSolve[{eqn, y[2] == 3}, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, 3\, e^{8-2x^2}]\}\}$$

```
eqn /. sol
```

```
{True}
```

```
Clear["Global`*"]
```

16. $y' = (x + y - 2)^2$, $y(0) = 2$, (Set v = x + y - 2)

```
eqn = y'[x] == (x + y[x] - 2)^2;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{(-2 - i) - (2 - i)\, e^{2 i x} + x + e^{2 i x} x}{1 + e^{2 i x}}\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

```
Clear["Global`*"]
```

17. $xy' = y + 3x^4 \cos^2\left(\frac{y}{x}\right)$, $y(1) = 0$, Set $\frac{y}{x} = u$

```
eqn = x y'[x] == y[x] + 3 x^4 Cos[y[x]/x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -x\, \text{ArcTan}\left[1 - x^3\right]\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```