Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

The s.m. has problem 1.

1 - 6 Verifications

1. Harmonic functions. Verify theorem 1, p. 460, for $f = 2z^2 - x^2 - y^2$ and S the surface of the box $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

This deals with the divergence theorem in potential theory. Get a bunch of normals to a surface, and dot them with the grad.

```
Clear["Global`*"]
```

```
innen = 2 z^2 - x^2 - y^2
```

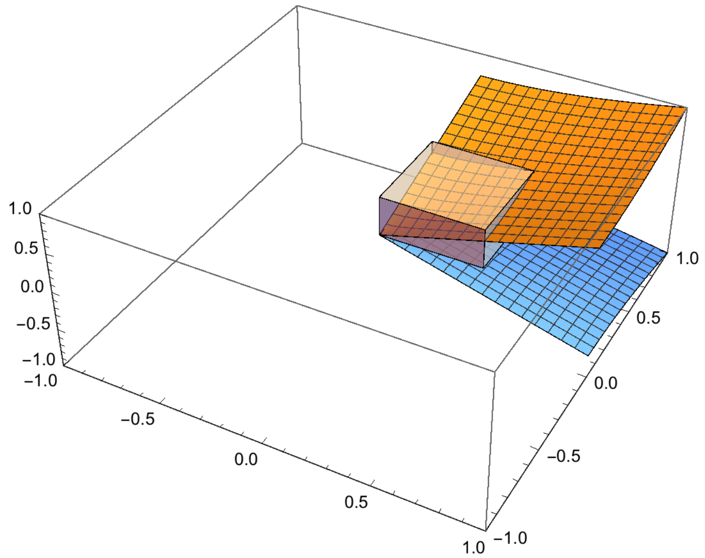
```
inne = Solve[2 z^2 - x^2 - y^2 == 0, z]
```

$$\left\{\left\{z \to -\frac{\sqrt{x^2 + y^2}}{\sqrt{2}}\right\}, \left\{z \to \frac{\sqrt{x^2 + y^2}}{\sqrt{2}}\right\}\right\}$$

$$\text{fir} = \text{Plot3D}\left[\left\{\frac{\sqrt{x^2 + y^2}}{\sqrt{2}}, \frac{-\sqrt{x^2 + y^2}}{\sqrt{2}}\right\}, \{x, 0, 1\}, \{y, 0, 1\}, \text{PlotRange} \to \{\{-1, 1\}, \{-1, 1\}, \{-1, 1\}\}\right];$$

```
sec = Graphics3D[
   {Opacity[.6], Cuboid[{0.5, 0.5, 0.5}, {0, 0, 0}]}, Axes → True];
```

```
Show[fir, sec]
```



s1 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{0, 0, 1\}$

4 z

s2 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{1, 0, 0\}$

-2 x

s3 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{0, 1, 0\}$

-2 y

s4 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{0, -1, 0\}$

2 y

s5 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{-1, 0, 0\}$

2 x

s6 = Grad$\left[2 z^2 - x^2 - y^2, \{x, y, z\}\right].\{0, 0, -1\}$

-4 z

tot = s1 + s2 + s3 + s4 + s5 + s6

0

The above answer agrees with the text’s.

3. Green’s first identity. Verify numbered line (8), p. 461, for f = $4y^2$, g = x^2, S the surface of the “unit cube”, 0 ≤ x ≤ 1, 0 ≤ y ≤ 1, 0 ≤ z ≤ 1. What are the assumptions on f and g in numbered line (8)? Must f and g be harmonic?

5. Green's second identity. Verify numbered line (9), p. 461, for f = $6y^2$, g = $2x^2$, S the unit cube in problem 3.