

Example 2 on p 113

```
Clear["Global`*"]
```

```
hank = {y'''[x] - y''[x] + 100 y'[x] - 100 y[x] == 0,  
        y[0] == 4, y'[0] == 11, y''[0] == -299}
```

```
dank = DSolve[hank, y[x], x]
```

```
{-100 y[x] + 100 y'[x] - y''[x] + y(3)[x] == 0, y[0] == 4, y'[0] == 11, y''[0] == -299}
```

```
{ {y[x] → ex + 3 Cos[10 x] + Sin[10 x]} }
```

Above: This answer agrees with the text.

1 - 6 General solution

Solve the given ODE.

$$1. \ y''' + 25 y' = 0$$

```
Clear["Global`*"]
```

```
jav = y'''[x] + 25 y'[x] == 0
```

```
nav = DSolve[jav, y[x], x]
```

```
25 y'[x] + y(3)[x] == 0
```

$$\left\{ \left\{ y[x] \rightarrow C[3] - \frac{1}{5} C[2] \cos[5 x] + \frac{1}{5} C[1] \sin[5 x] \right\} \right\}$$

1. Above: This answer agrees with the text.

$$3. \ y^{iv} + 4 y'' = 0$$

```
Clear["Global`*"]
```

```
har = y''''[x] + 4 y''[x] == 0
```

```
mar = DSolve[har, y[x], x]
```

```
4 y''[x] + y(4)[x] == 0
```

$$\left\{ \left\{ y[x] \rightarrow C[3] + x C[4] - \frac{1}{4} C[1] \cos[2 x] - \frac{1}{4} C[2] \sin[2 x] \right\} \right\}$$

1. Above: This answer agrees with the text.

$$5. \ (D^4 + 10 D^2 + 9 I) y = 0$$

```
Clear["Global`*"]
```

```
yip = y''''[x] + 10 y'''[x] + 9 y''[x] == 0
nip = DSolve[yip, y[x], x]
9 y[x] + 10 y''[x] + y(4)[x] == 0
```

```
{ {y[x] → C[3] Cos[x] + C[1] Cos[3 x] + C[4] Sin[x] + C[2] Sin[3 x]} }
```

1. Above: This answer agrees with the text.

7 - 13 Initial value problem

Solve the IVP by a CAS, giving a general solution and the particular solution and its graph.

7. $y''' + 3.2 y'' + 4.81 y' = 0$, $y[0] = 3.4$, $y'[0] = -4.6$, $y''[0] = 9.91$

```
Clear["Global`*"]

de = y''''[x] + 3.2 y'''[x] + 4.81 y''[x] == 0
gs = DSolve[de, y[x], x]
4.81 y'[x] + 3.2 y''[x] + y(3)[x] == 0

{ {y[x] → C[3] + e-1.6 x ((-0.31185 C[1] - 0.33264 C[2]) Cos[1.5 x] +
  (-0.33264 C[1] + 0.31185 C[2]) Sin[1.5 x]) } } }

gsf = gs /. {C[1] → 1, C[2] → 1, C[3] → 1}
{ {y[x] → 1 + e-1.6 x (-0.644491 Cos[1.5 x] - 0.02079 Sin[1.5 x]) } }

de2 = {y''''[x] + 3.2 y'''[x] + 4.81 y''[x] == 0,
  y[0] == 3.4, y'[0] == -4.6, y''[0] == 9.91}
{4.81 y'[x] + 3.2 y''[x] + y(3)[x] == 0, y[0] == 3.4, y'[0] == -4.6, y''[0] == 9.91}

ps = DSolve[de2, y[x], x]
{ {y[x] → 2.4 e-1.6 x (1. e1.6 x + 0.416667 Cos[1.5 x] - 0.833333 Sin[1.5 x]) } }

trim = Expand[ps]
```

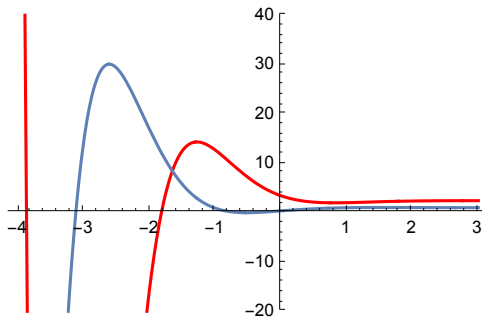
```
{ {y[x] → 2.4 + 1. e-1.6 x Cos[1.5 x] - 2. e-1.6 x Sin[1.5 x]} }
```

1. Above: The answer agrees with that of the text to 2S.

```

plot1 = Plot[y[x] /. ps, {x, -4, 3},
  PlotRange → {-20, 40}, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gsf, {x, -4, 3}, PlotRange → {-20, 40}];
Show[plot1, plot2]

```



2. Above: There was an odd gap at the max of gsf the first time it was plotted. Then the constant value of C[1] was jiggled and afterwards the gap disappeared.

9. $4y''' + 8y'' + 41y' + 37y = 0$, $y[0] = 9$, $y'[0] = -6.5$, $y''[0] = -39.75$

```
Clear["Global`*"]
```

```
gie = 4 y'''[x] + 8 y''[x] + 41 y'[x] + 37 y[x] == 0
```

```
gs = DSolve[gie, y[x], x]
```

```
37 y[x] + 41 y'[x] + 8 y''[x] + 4 y'''[x] == 0
```

```
{ {y[x] → e-x C[3] + e-x/2 C[2] Cos[3 x] + e-x/2 C[1] Sin[3 x]} }
```

```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1}
```

```
{ {y[x] → e-x + e-x/2 Cos[3 x] + e-x/2 Sin[3 x]} }
```

```
pie = {4 y'''[x] + 8 y''[x] + 41 y'[x] + 37 y[x] == 0,
```

```
  y[0] == 9, y'[0] == -6.5, y''[0] == -39.75}
```

```
ps = DSolve[pie, y[x], x]
```

```
{37 y[x] + 41 y'[x] + 8 y''[x] + 4 y'''[x] == 0,
```

```
  y[0] == 9, y'[0] == -6.5, y''[0] == -39.75}
```

```
{ {y[x] → 5. e-x (0.8 + 1. ex/2 Cos[3 x] + 6.09497 × 10-18 ex/2 Sin[3 x]) } }
```

```
pse = Expand[ps]
```

```
{ {y[x] → 4. e-x + 5. e-x/2 Cos[3 x] + 3.04749 × 10-17 e-x/2 Sin[3 x]} }
```

```
Chop[pse, 10-16]
```

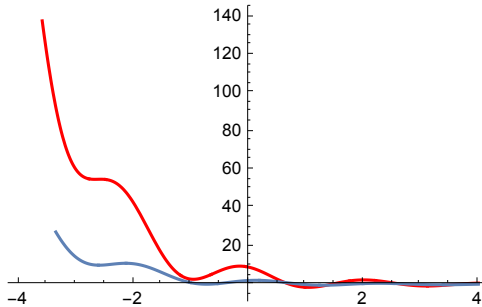
```
{ {y[x] → 4. e-x + 5. e-x/2 Cos[3 x]} }
```

1. Above: The answer agrees with the text's.

```

plot1 = Plot[y[x] /. pse, {x, -4, 4},
  PlotRange → Automatic, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gse, {x, -4, 4}, PlotRange → Automatic];
Show[plot1, plot2]

```



11. $y^{iv} - 9y'' - 400y = 0$, $y[0] = 0$, $y'[0] = 0$, $y''[0] = 41$, $y'''[0] = 0$

```
Clear["Global`*"]
```

```
nom = y''''[x] - 9 y''[x] - 400 y[x] == 0
```

```
gs = DSolve[nom, y[x], x]
```

```
-400 y[x] - 9 y''[x] + y(4)[x] == 0
```

```
{ {y[x] → e-5 x C[3] + e5 x C[4] + C[1] Cos[4 x] + C[2] Sin[4 x] } }
```

```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1, C[4] → 1}
```

```
{ {y[x] → e-5 x + e5 x + Cos[4 x] + Sin[4 x] } }
```

```
nomp = {y''''[x] - 9 y''[x] - 400 y[x] == 0,
  y[0] == 0, y'[0] == 0, y''[0] == 41, y'''[0] == 0}
```

```
ps = DSolve[nomp, y[x], x]
```

```
{ -400 y[x] - 9 y''[x] + y(4)[x] == 0, y[0] == 0, y'[0] == 0, y''[0] == 41, y(3)[0] == 0 }
```

```
{ {y[x] →  $\frac{1}{2} e^{-5 x} (1 + e^{10 x} - 2 e^{5 x} \cos[4 x])$  } }
```

```
ps1 = ExpToTrig[ps]
```

```
{ {y[x] →  $\frac{1}{2} (\cosh[5 x] - \sinh[5 x]) (1 - 2 \cos[4 x] \cosh[5 x] + \cosh[10 x] - 2 \cos[4 x] \sinh[5 x] + \sinh[10 x])$  } }
```

```
ps2 = Expand[ps1]
```

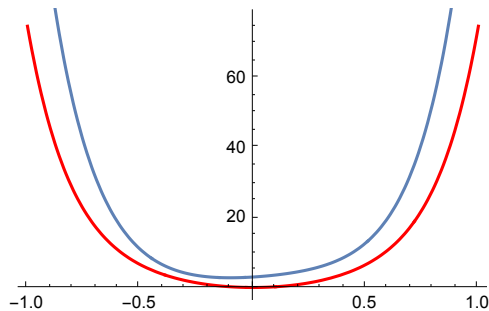
```
{ {y[x] →  $\frac{1}{2} \cosh[5 x] - \cos[4 x] \cosh[5 x]^2 + \frac{1}{2} \cosh[5 x] \cosh[10 x] - \frac{1}{2} \sinh[5 x] - \frac{1}{2} \cosh[10 x] \sinh[5 x] + \cos[4 x] \sinh[5 x]^2 + \frac{1}{2} \cosh[5 x] \sinh[10 x] - \frac{1}{2} \sinh[5 x] \sinh[10 x]$  } }
```

```
ps3 = Simplify[ps2]
```

```
{ {y[x] → -Cos[4 x] + Cosh[5 x]} }
```

1. Above: The answer matches the text's.

```
plot1 = Plot[y[x] /. ps3, {x, -1, 1},
  PlotRange → Automatic, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gse, {x, -1, 1}, PlotRange → Automatic];
Show[plot1, plot2]
```



```
13. yiv + 0.45 y''' - 0.165 y'' + 0.0045 y' - 0.00175 y = 0,
y[0] = 17.4, y'[0] = -2.82, y''[0] = 2.0485, y'''[0] = -1.458675
```

```
Clear["Global`*"]
```

```
bi = y''''[x] + 0.45 y'''[x] - 0.165 y''[x] + 0.0045 y'[x] - 0.00175 y[x] == 0
gs = DSolve[bi, y[x], x]
```

```
-0.00175 y[x] + 0.0045 y'[x] - 0.165 y''[x] + 0.45 y(3)[x] + y(4)[x] == 0
```

```
{ {y[x] → e-0.7 x C[1] + e0.25 x C[4] + 1. C[3] Cos[0.1 x] + 1. C[2] Sin[0.1 x]} }
```

```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1, C[4] → 1}
```

```
{ {y[x] → e-0.7 x + e0.25 x + 1. Cos[0.1 x] + 1. Sin[0.1 x]} }
```

```
bip =
```

```
{y''''[x] + 0.45 y'''[x] - 0.165 y''[x] + 0.0045 y'[x] - 0.00175 y[x] == 0,
y[0] == 17.4, y'[0] == -2.82, y''[0] == 2.0485, y'''[0] == -1.458675}
```

```
ps = DSolve[bip, y[x], x]
```

```
{-0.00175 y[x] + 0.0045 y'[x] - 0.165 y''[x] + 0.45 y(3)[x] + y(4)[x] == 0,
y[0] == 17.4, y'[0] == -2.82, y''[0] == 2.0485, y(3)[0] == -1.45868}
```

```
{ {y[x] →
  1. e-0.7 x (4.3 + 1. e0.95 x + 12.1 e0.7 x Cos[0.1 x] - 0.6 e0.7 x Sin[0.1 x]) } }
```

```
droop = Expand[ps]
```

```
{ {y[x] → 4.3 e-0.7 x + 1. e0.25 x + 12.1 Cos[0.1 x] - 0.6 Sin[0.1 x]} }
```

1. Above: The answer matches the text's.

```
plot1 = Plot[y[x] /. droop, {x, -5, 5},  
  PlotRange → {-100, 100}, PlotStyle → Red, ImageSize → 250];  
plot2 = Plot[y[x] /. gse, {x, -5, 5}, PlotRange → Automatic];  
Show[plot1, plot2]
```

