Laplace transforms is something that Mathematica is good at. Shifting, as far as I can tell, is something that can be routinely shined on.

1 - 16 Laplace transforms

Find the transform. Assume that a, b, ω, θ are constants.

1. $3t + 12$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[3 t + 12, t, s]
```

$$\frac{3}{s^2} + \frac{12}{s}$$

The correct answer.

3. $\text{Cos}[\pi t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Cos[π t], t, s]
```

$$\frac{s}{\pi^2 + s^2}$$

The correct answer.

5. $e^{2t}\,\text{Sinh}[t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[e^(2 t) Sinh[t], t, s]
```

$$\frac{1}{3 - 4s + s^2}$$

The correct answer.

7. $\text{Sin}[\omega t + \theta]$

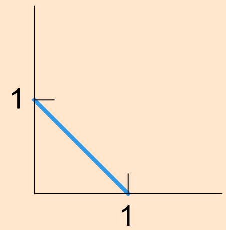
```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Sin[ω t + θ], t, s]
```

$$\frac{\omega\,\text{Cos}[\theta] + s\,\text{Sin}[\theta]}{s^2 + \omega^2}$$

The correct answer.

9.




```
Clear["Global`*"]
```

```
e1 = f[t_] = -t + 1
```

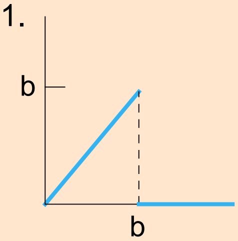
```
1 - t
```

```
e4 = LaplaceTransform[If[t > 0 && t < 1, 1 - t, 0], t, s]
```

$$\frac{-1 + e^{-s} + s}{s^2}$$

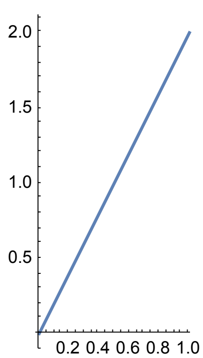
This is the right answer. There is a big difference on what comes out of the Laplace Transform based on whether the domain is restricted or not.

11.




```
Clear["Global`*"]
```

```
Plot[ 2 x, {x, 0, 1}, PlotRange → Automatic,
 ImageSize → 100, AspectRatio → Automatic]
```




```
e1 = t
```

```
t
```

```
e2 = Simplify[LaplaceTransform[If[t > 0 && t < b, t, 0], t, s]]
```

$$\begin{cases} \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2} & b>0 \\ 0 & \text{True} \end{cases}$$

$$e3 = \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

$$\frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

$$e4 = e3 \;/.\; \left(e^{-b s}\left(-1+e^{b s}-b s\right)\right) \to \text{Expand}\left[e^{-b s}\left(-1+e^{b s}-b s\right)\right]$$

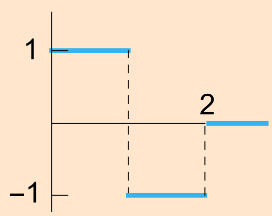
$$\frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2}$$

$$e5 = e4 \;/.\; \frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2} \text{->} \frac{1-e^{-b s}}{s^2}-\frac{b\, e^{-b s}\, s}{s^2}$$

$$\frac{1-e^{-b s}}{s^2}-\frac{b\, e^{-b s}}{s}$$

Above: This is the text answer.

13.




```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[
  Piecewise[{{1, t > 0 && t < 1}, {-1, t > 1 && t < 2}}], t, s]
```

$$-\frac{1}{s}e^{-2 s}\left(-1+e^{s}-e^{2 s}+e^{2 s}\,\text{Cosh}[s]-e^{2 s}\,\text{Sinh}[s]\right)$$

```
e2 = Simplify[e1]
```

$$\frac{e^{-2 s}\left(-1+e^{s}\right)^2}{s}$$

$$\text{PossibleZeroQ}\left[\frac{e^{-2 s}\left(-1+e^{s}\right)^2}{s}-\frac{\left(1-e^{-s}\right)^2}{s}\right]$$

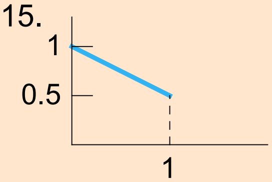
True

The above shows that Mathematica's answer and the text answer are equivalent. With a ‘sleight’ maneuver I could even do

```
e6 = e2 /. ⅇ^(-2 s) (-1 + ⅇ^s)^2 -> (1 - ⅇ^(-s))^2
```

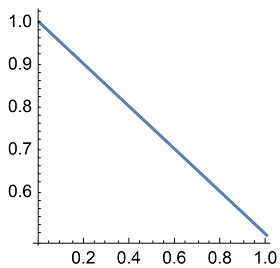
$$\frac{(1 - e^{-s})^2}{s}$$

15.




```
Clear["Global`*"]
```

$$\text{Plot}\left[-\frac{1}{2}x + 1, \{x, 0, 1\},\right.$$
$$\left. \text{PlotRange} \to \text{Automatic}, \text{ImageSize} \to 140, \text{AspectRatio} \to 1\right]$$



$$\text{e1} = \text{LaplaceTransform}\left[\text{Piecewise}\left[\left\{\left\{-\frac{1}{2}t + 1, t > 0 \,\&\&\, t < 1\right\}, \{0, t > 1 \,\&\&\, t < 0\}\right\}\right], t, s\right]$$

$$\frac{e^{-s}\,(1 - e^{s} - s + 2\,e^{s}\,s)}{2\,s^2}$$

```
e2 = TrigToExp[e1]
```

$$-\frac{1}{2\,s^2} + \frac{e^{-s}}{2\,s^2} + \frac{1}{s} - \frac{e^{-s}}{2\,s}$$

The above answer is correct.

21. Nonexistence. Give sample examples of functions (defined for all t ≥ 0) that have no Laplace transform.

If the following expression is true for some constants M and k it satisfies the “growth restriction”

$$|\, f(t) \le M\,e^{k\,t}\,|$$

The sol’n text gives e^{t^2} as an example. The expression above is a more general guide. The *k* and *M* are just constants, so the *lhs* can overwhelm them if it has an exponential or similar nature.

25 - 32 Inverse Laplace transforms

Given F(s) = L(f), find f(t). Here a, b, L, n, are constants.

25. $\frac{0.2\, s + 1.8}{s^2 + 3.24}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{0.2\, s + 1.8}{s^2 + 3.24}, s, t\right]$

$(0.5 + 0.1\, i)\, e^{(0.-1.8\, i)\, t} \left((0.384615 + 0.923077\, i) - (0. + 1.\, i)\, e^{(0.+3.6\, i)\, t}\right)$

```
e2 = FullSimplify[e1]
```

$e^{(0.-1.8\, i)\, t} \left((0.1 + 0.5\, i) + (0.1 - 0.5\, i)\, e^{(0.+3.6\, i)\, t}\right)$

```
e3 = ExpToTrig[e2]
```

```
(Cos[(1.8 + 0. i) t] - i Sin[(1.8 + 0. i) t])
 ((0.1 + 0.5 i) + (0.1 - 0.5 i) (Cos[(3.6 + 0. i) t] + i Sin[(3.6 + 0. i) t]))
```

```
e4 = FullSimplify[e3]
```

```
0.2 Cos[1.8 t] + 1. Sin[1.8 t]
```

The above answer matches the text. A bit of work to recast it.

27. $\frac{s}{L^2 s^2 + n^2 \pi^2}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{s}{L^2 s^2 + n^2 \pi^2}, s, t\right]$

$\frac{\text{Cos}\left[\frac{n \pi t}{L}\right]}{L^2}$

The above answer matches the text.

29. $\frac{12}{s^4} - \frac{228}{s^6}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[12/s^4 - 228/s^6, s, t]
```

$$2\,t^3 - \frac{19\,t^5}{10}$$

The above answer matches the text. Here is demonstrated the linear nature of L^{-1}: separate fractions can be calculated as a single operand.

31. $\frac{s+10}{s^2-s-2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[(s + 10)/(s^2 - s - 2), s, t]
```

$$-3\,e^{-t} + 4\,e^{2t}$$

The above answer matches the text.

33 - 45 Application of s-shifting

In problems 33 - 36 find the transform. In problems 37 - 45 find the inverse transform.

33. $t^2\,e^{-3t}$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[t^2 E^(-3 t), t, s]
```

$$\frac{2}{(3+s)^3}$$

The above answer matches the text.

35. $0.5\,e^{-4.5t}\,\mathrm{Sin}[2\,\pi\,t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[0.5 E^(-4.5 t) Sin[2 π t], t, s]
```

$$\frac{3.14159}{4\,\pi^2 + (4.5+s)^2}$$

The above answer matches the text.

37. $\frac{\pi}{(s+\pi)^2}$

```
Clear["Global`*"]
```

$$e1 = \text{InverseLaplaceTransform}\left[\frac{\pi}{(s+\pi)^2}, s, t\right]$$

$$e^{-\pi t}\,\pi\,t$$

The above answer matches the text.

39. $\dfrac{21}{\left(s+\sqrt{2}\right)^4}$

```
Clear["Global`*"]
```

$$e1 = \text{InverseLaplaceTransform}\left[\frac{21}{\left(s+\sqrt{2}\right)^4}, s, t\right]$$

$$\frac{7}{2}\,e^{-\sqrt{2}\,t}\,t^3$$

The above answer matches the text.

41. $\dfrac{\pi}{s^2+10\,\pi\,s+24\,\pi^2}$

```
Clear["Global`*"]
```

$$e1 = \text{InverseLaplaceTransform}\left[\frac{\pi}{s^2+10\,\pi\,s+24\,\pi^2}, s, t\right]$$

$$\frac{1}{2}\,e^{-6\,\pi\,t}\left(-1+e^{2\,\pi\,t}\right)$$

$$e2 = \text{FullSimplify}\left[\text{ExpToTrig}\left[\frac{\left(-1+e^{2\,\pi\,t}\right)}{2\,e^{\pi\,t}}\right]\right]$$

$$\text{Sinh}[\pi\,t]$$

$$e3 = e^{-5\,\pi\,t}\,e2$$

$$e^{-5\,\pi\,t}\,\text{Sinh}[\pi\,t]$$

The above answer matches the text.

43. $\dfrac{2\,s-1}{s^2-6\,s+18}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{2\,s-1}{s^2-6\,s+18}, s, t\right]$

$\frac{1}{6}\,e^{(3-3\,i)\,t}\left((6+5\,i)+(6-5\,i)\,e^{6\,i\,t}\right)$

e2 = FullSimplify[e1]

$\frac{1}{3}\,e^{3\,t}\,(6\,\text{Cos}[3\,t]+5\,\text{Sin}[3\,t])$

The above answer matches the text.

45. $\frac{k_0\,(s+a)+k_1}{(s+a)^2}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\frac{k_0\,(s+a)+k_1}{(s+a)^2}, s, t\right]$

$e^{-a\,t}\,(k_0+t\,k_1)$

The above answer matches the text.

The problems since No. 33 demonstrate that S-shifting doesn’t really exist for Mathematica user. Just put the expression in and turn the crank.