The answers to the following are in agreement with the text except for the yellow cells below, and for those particular cases the text answer has the suggestion of a typo.

11 - 20 Multiplication, addition, and transposition of matrices and vectors

```
A = {{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}} // MatrixForm
```

$$\begin{pmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{pmatrix}$$

```
{{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}}
```

```
B = {{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}} // MatrixForm
```

$$\begin{pmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

```
{{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}}
```

```
CC = {{0, 1}, {3, 2}, {-2, 0}} // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{pmatrix}$$

```
{{0, 1}, {3, 2}, {-2, 0}}
```

```
ar = {1, -2, 0}
```

```
{1, -2, 0}
```

```
bc = {{3}, {1}, {-1}} // MatrixForm
```

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

```
bcr = {3, 1, -1} // MatrixForm
```

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

Showing all intermediate results, calculate the following expressions or give reasons why they are undefined:

11. AB, AB^T, BA, B^TA

```
A.B // MatrixForm
```

$$\begin{pmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{pmatrix}$$

```
A.B^† // MatrixForm
```

$$\begin{pmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{pmatrix}$$

```
B.A // MatrixForm
```

$$\begin{pmatrix} 10 & -5 & -15 \\ -14 & 7 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

Above: The result shown is not in agreement with the text answer, which has -33 for a_{23} (instead of -3).

```
B^T.A // MatrixForm
```

$$\begin{pmatrix} 10 & -5 & -15 \\ -14 & 7 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

Above: Since the answer block lists 'same' for this result, the answer is again at odds (technically) with the text.

The operations above agree with the text answers.

13. CC^T, BC, CB, C^TB

```
CC.CC^T // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 13 & -6 \\ 0 & -6 & 4 \end{pmatrix}$$

```
B.CC // MatrixForm
```

$$\begin{pmatrix} -9 & -5 \\ 3 & -1 \\ 4 & 0 \end{pmatrix}$$

```
CC.B // MatrixForm
```

```
Dot::dotsh: Tensors {{0, 1}, {3, 2}, {-2, 0}} and {{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}} have incompatible shapes >>
```

```
{{0, 1}, {3, 2}, {-2, 0}}.{{1, -3, 0}, {-3, 1, 0}, {0, 0, -2}}
```

```
CC^†.B // MatrixForm
```

$$\begin{pmatrix} -9 & 3 & 4 \\ -5 & -1 & 0 \end{pmatrix}$$

The operations above agree with the text answers.

15. Aa, $Aa^\dagger$, $(Ab)^\dagger$, $b^\dagger A^\dagger$

I'm going to redefine the vectors so they are clearly visible.

```
ar = {{1, -2, 0}}
```

```
{{1, -2, 0}}
```

```
bc = {{3}, {1}, {-1}}
{{3}, {1}, {-1}}
```

Mathematica can do a dot product between matrix and vector. However, it does not distiguish between row vectors and column vectors.

```
Dot[A, ar]
```

Dot::dotsh: Tensors{{4, –2, 3}, {–2, 1, 6}, {1, 2, 2}} and{{1, –2, 0}} haveincompatibleshapes ≫

```
{{4, -2, 3}, {-2, 1, 6}, {1, 2, 2}}.{{1, -2, 0}}
```

Above: Mathematica returns the input to show it cannot perform the first operation, which agrees with the text answer assessment.

```
Dot[A, bc]
{{7}, {-11}, {3}}
```

Above: Mathematica formed a dot product with what were disguised as a row vector, then a column vector.

```
cja = ConjugateTranspose[ar]
{{1}, {-2}, {0}}
```

Above: Transposing a row vector in Mathematica does work.

```
cjb = ConjugateTranspose[bc]
{{3, 1, -1}}
```

Above:Transposing a column vector does work in Mathematica.

```
Dot[A, cja]
```

```
{{8}, {-4}, {-3}}
```

Above: This is the second listed operation, and agrees with the text.

```
inter = Dot[A, bc]
{{7}, {-11}, {3}}
```

```
fin = ConjugateTranspose[inter]
```

```
{{7, -11, 3}}
```

Above: This is the third-listed operation, and Mathematica produces an answer. The answer does not agree with the text answer, though it looks good to me. The text answer would be {{7, -1, 3}}.

```
Dot[cjb, ConjugateTranspose[A]]
```

```
{{7, -11, 3}}
```

Above: This is the 4th-listed operation. The text answer states that the result is the same as on the third operation, which, if true, means there is a disagreement with the Mathematica answer.

17. ABC, ABa, ABb, $Ca^{\dagger}$

```
A.B.CC // MatrixForm
```

$$\begin{pmatrix} -30 & -18 \\ 45 & 9 \\ 5 & -7 \end{pmatrix}$$

Above: The answer in green agrees with the text answer.

```
A.B.ar
```

```
Dot::dotsh: Tensors {{10, -14, -6}, {-5, 7, -12}, {-5, -1, -4}} and {{1, -2, 0}} have incompatible shapes >>
```

```
{{10, -14, -6}, {-5, 7, -12}, {-5, -1, -4}}.{{1, -2, 0}}
```

Above: Mathematica agrees with the text that the operation called for is undefined.

```
A.B.bc
```

```
{{22}, {4}, {-12}}
```

Above: Mathematica's answer agrees with the text answer.

```
Dot[CC, cja]
```

```
Dot::dotsh: Tensors {{0, 1}, {3, 2}, {-2, 0}} and {{1}, {-2}, {0}} have incompatible shapes >>
```

```
{{0, 1}, {3, 2}, {-2, 0}}.{{1}, {-2}, {0}}
```

Above: Undefined as stated in the text.

19. 1.5a+3.0b, $1.5a^{\dagger}$ + 3.0b, (A - B)b, Ab - Bb

```
1.5 ar + 3.0 bc // MatrixForm
```

```
Thread::tdlen: Objects of unequal length in {{1.5, -3., 0.}} + {{9.}, {3.}, {-3.}} cannot be combined >>
```

```
{{1.5, -3., 0.}} + {{9.}, {3.}, {-3.}}
```

```
1.5 cja + 3 bc // MatrixForm
```

$$\begin{pmatrix} 10.5 \\ 0. \\ -3. \end{pmatrix}$$

The answer in the green cell above matches the text answer.

```
(A - B).bc
```

```
{{7}, {-3}, {1}}
```

The answer in the green cell above matches the text answer.

```
A.bc - B.bc
```

```
{{7}, {-3}, {1}}
```

The answer in the green cell above matches the text answer.

Take home thoughts from this section. Mathematica is very compliant and free-wheeling with regard to vectors. In order to get expected results where it is necessary to distinguish between row vectors and column vectors, row vectors should be entered as {{a, b, c}}, and column vectors as {{a}, {b}, {c}}.