1 - 10 Inner product

Let a = {1, -3, 5}, b={4, 0, 8}, c = {-2, 9, 1}

1. a.b, b.a, b.c

```
Clear["Global`*"]
```

```
aa = {1, -3, 5}; bb = {4, 0, 8}; cc = {-2, 9, 1}
{-2, 9, 1}
```

```
e1 = aa.bb
```

```
44
```

```
e2 = bb.aa
```

```
44
```

```
e3 = bb.cc
```

```
0
```

3. |a|, |2b|, |-c|

```
Norm[aa]
```

$\sqrt{35}$

```
Norm[2 bb]
```

$8\sqrt{5}$

```
Norm[-cc]
```

$\sqrt{86}$

5. |b + c|, |b| + |c|

```
e7 = Norm[bb + cc]
```

$\sqrt{166}$

```
e8 = Norm[bb] + Norm[cc]
```

$4\sqrt{5} + \sqrt{86}$

```
e9 = FullSimplify[e7 == e8]
False
```

7. | a.c |, | a || c |

```
e10 = Norm[aa.cc]
```

```
24
```

```
e11 = Norm[aa] Norm[cc]
```

$\sqrt{3010}$

9. 15a.b + 15a.c, 15a.(b+c)

```
e12 = 15 aa.bb + 15 aa.cc
```

```
300
```

```
e13 = 15 aa.(bb + cc)
```

```
300
```

17 - 20 Work

Find the work done by a force **p** acting on a body if the body is displaced along the straight segment $\overline{AB}$ from A to B. Sketch $\overline{AB}$ and **p**.

17. p = {2, 5, 0}, A: {1, 3, 3}, B: {3, 5, 5}

```
Clear["Global`*"]
aA = {1, 3, 3}; bB = {3, 5, 5}
{3, 5, 5}
pP = {2, 5, 0}
{2, 5, 0}
dis = bB - aA
{2, 2, 2}
wW = dis.pP
```

```
14
```

19. p = {0, 4, 3}, A: {4, 5, -1}, B: (1, 3, 0}

```
Clear["Global`*"]

pP = {0, 4, 3}; aA = {4, 5, -1}; bB = {1, 3, 0}
{1, 3, 0}

dis = bB - aA
{-3, -2, 1}

wW = dis.pP
-5
```

22 - 30 Angle between vectors

Let aA = {1, 1, 0}; bB = {3, 2, 1}; cC = {1, 0, 2}

23. b, c

```
dotbc = bB.cC
5
```

$$e1 = \frac{dotbc}{Norm[bB]\ Norm[cC]}$$

$$\sqrt{\frac{5}{14}}\ //\ N$$

```
0.597614

e2 = ArcCos[e1]
```

$$ArcCos\left[\sqrt{\frac{5}{14}}\right]\ //\ N$$

```
0.930274
```

$$e3 = \frac{e2}{Degree}\ //\ N$$

```
53.3008
```

31 - 35 Orthogonality is particularly important, mainly because of orthogonal coordinates, such as Cartesian coordinates, whose natural basis consists of three orthogonal unit vectors.

31. For what values of a_1 are $\{a_1, 4, 3\}$ and {3, -2, 12} orthogonal?

```
Clear["Global`*"]

e1 = {a1, 4, 3}
{a1, 4, 3}
```

```
e2 = {3, -2, 12}
```

{3, -2, 12}

```
e3 = e1.e2
```

$28 + 3\, a_1$

```
Solve[e3 == 0]
```

$\{\{a_1 \to -\frac{28}{3}\}\}$

33. Unit vectors. Find all unit vectors a = $\{a_1, a_2\}$ in the plane orthogonal to {4, 3}

```
Clear["Global`*"]
```

```
e1 = {4, 3}
```

{4, 3}

```
e2 = Norm[e1]
```

5

```
e3 = {a1, a2}
```

$\{a_1, a_2\}$

```
e4 = Norm[e3]
```

$\sqrt{\text{Abs}[a_1]^2 + \text{Abs}[a_2]^2}$

```
e5 = Solve[e1.e3 == 0 && Norm[e3] == 1]
```

$\{\{a_1 \to \frac{3}{5}, a_2 \to -\frac{4}{5}\}, \{a_1 \to -\frac{3}{5}, a_2 \to \frac{4}{5}\}\}$

36 - 40 Component in the direction of a vector

Find the component of a in the direction of b. Make a sketch.

37. a = {3, 4, 0}, b = {4, -3, 2}

```
Clear["Global`*"]
```

To find the component of **a** in the direction of **b**, I first need to find the angle separating them.

```
e1 = {3, 4, 0}
```

{3, 4, 0}

```
e2 = {4, -3, 2}
```

{4, -3, 2}

```
e3 = e1.e2 / (Norm[e1] Norm[e2])
```

$$e3 = \frac{e1.e2}{\text{Norm}[e1]\ \text{Norm}[e2]}$$

0

```
e4 = ArcCos[e3]
```

$$\frac{\pi}{2}$$

These two vectors are perpendicular; therefore there is not projection (=0).

```
e5 = Norm[e1] Cos[e4]
```

0

Green cells in this problem set denote agreement with the text answers.