1 - 16 Eigenvalues, eigenvectors

Find the eigenvalues. Find the corresponding eigenvectors. Use the given λ or factor in problems 11 and 15.

1. $\begin{pmatrix} 3.0 & 0 \\ 0 & -0.6 \end{pmatrix}$

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 3.0 & 0 \\ 0 & -0.6 \end{pmatrix}$

```
{{3., 0}, {0, -0.6}}
```

e1 = {{λ_1, λ_2}, {v1, v2}} = Eigensystem[aA]

```
{{3., -0.6}, {{-1., 0.}, {0., -1.}}}
```

Above: The eigenvalues match the text. The eigenvectors do not. If my checking process is accurate, Mathematica's eigenvectors fare better than the text's.

aA.v1 == λ_1 v1

```
True
```

aA.v2 == λ_2 v2

```
True
```

```
aA.v1 == 3 {1, 0}
False
```

```
aA.v2 == -0.6 {0, 1}
False
```

3. $\begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}$

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}$

```
{{5, -2}, {9, -6}}
```

```
e1 = {vals, vecs} = Eigensystem[aA]
```

```
{{-4, 3}, {{2, 9}, {1, 1}}}
```

Above: Both eigenvalues and eigenvectors match the text.

```
e2 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True

e3 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True
```

5. $\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$

```
{{0, 3}, {-3, 0}}

e1 = {vals, vecs} = Eigensystem[aA]

{{3 i, -3 i}, {{-i, 1}, {i, 1}}}
```

Above: The eigenvalues match the text, but the eigenvector entries are flipped. Mathematica's eigenvectors show better than the text's when tested.

```
e2 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True

e3 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True

e4 = aA.vecs[[1]] == vals[[1]] {{1, -i}}
False

e5 = aA.vecs[[2]] == vals[[2]] {{1, i}}
False
```

7. $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

```
{{0, 1}, {0, 0}}

e1 = {vals, vecs} = Eigensystem[aA]

{{0, 0}, {{1, 0}, {0, 0}}}
```

Above: the eigenvalues match the text. The text does not mention the null eigenvector, leaving me to suppose it means to have the first one used twice. However, the null vector is seen to succeed in testing below.

```
e2 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True
```

```
e3 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True
```

9. $\begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$

```
{{0.8, -0.6}, {0.6, 0.8}}
```

```
e1 = {vals, vecs} = Eigensystem[aA]
```

```
{{0.8 + 0.6 i, 0.8 - 0.6 i},
 {{0. - 0.707107 i, -0.707107 + 0. i}, {0. + 0.707107 i, -0.707107 + 0. i}}}
```

Above: The eigenvalues match the text, but the eigenvectors do not.

```
e2 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True
```

```
e3 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True
```

To allow the greatest opportunity for success, I try using both **Transpose** and **ConjugateTranspose**.

$\{\{1, i\}\}^{\dagger}$

```
{{1}, {-i}}
```

$\{\{1, i\}\}^{T}$

```
{{1}, {i}}
```

The text's eigenvector will not play with double curlies, so I took out one set.

```
e4 = aA.vecs[[2]] == vals[[2]] {1, -i}
False
```

```
e4 = aA.vecs[[2]] == vals[[2]] {1, i}
False
```

```
{{1, -ⅈ}}†
{{1}, {ⅈ}}

{{1, -ⅈ}}ᵀ
{{1}, {-ⅈ}}

e5 = aA.vecs[[1]] == vals[[1]] {1, ⅈ}
False

e5 = aA.vecs[[1]] == vals[[1]] {1, -ⅈ}
False
```

Above: Mathematica's eigenvectors look better when tested than those of the text.

11. $\begin{pmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{pmatrix}$, $\lambda = 3$

For Mathematica computing three eigenvectors with eigenvalues is just as easy as doing one, so I ignore the λ provided.

```
Clear["Global`*"]
```

aA = $\begin{pmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{pmatrix}$

```
{{6, 2, -2}, {2, 5, 0}, {-2, 0, 7}}

e1 = {vals, vecs} = Eigensystem[aA]

{{9, 6, 3}, {{-2, -1, 2}, {1, 2, 2}, {2, -2, 1}}}
```

Above: The eigenvalues match the text, but one eigenvector does not.

```
e3 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True

e4 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True

e5 = aA.vecs[[3]] == vals[[3]] vecs[[3]]
True

{{2, 1, -2}}†
{{2}, {1}, {-2}}

{{2, 1, -2}}ᵀ
{{2}, {1}, {-2}}
```

```
e9 = aA.vecs[[1]] == vals[[1]] {2, 1, -2}
False
```

Above: Mathematica's eigenvectors check out, but the one disagreeable one in the text answer does not check.

13. $\begin{pmatrix} 13 & 5 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{pmatrix}$

```
Clear["Global`*"]
```

$$aA = \begin{pmatrix} 13 & 5 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{pmatrix}$$

```
{{13, 5, 2}, {2, 7, -8}, {5, 4, 7}}

e1 = {vals, vecs} = Eigensystem[aA]

{{9, 9, 9}, {{2, -2, 1}, {0, 0, 0}, {0, 0, 0}}}
```

The eigenvalues and eigenvectors match the text.

```
e2 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True

e3 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True
```

15. $\begin{pmatrix} -1 & 0 & 12 & 0 \\ 0 & -1 & 0 & 12 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & -4 & -1 \end{pmatrix}$, $(\lambda + 1)^2$

```
Clear["Global`*"]
```

$$aA = \begin{pmatrix} -1 & 0 & 12 & 0 \\ 0 & -1 & 0 & 12 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & -4 & -1 \end{pmatrix}$$

```
{{-1, 0, 12, 0}, {0, -1, 0, 12}, {0, 0, -1, -4}, {0, 0, -4, -1}}

e1 = {vals, vecs} = Eigensystem[aA]

{{-5, 3, -1, -1},
 {{-3, -3, 1, 1}, {3, -3, 1, -1}, {0, 1, 0, 0}, {1, 0, 0, 0}}}
```

Above: The eigenvalues and eigenvectors match the text answer.

```
e2 = aA.vecs[[1]] == vals[[1]] vecs[[1]]
True

e3 = aA.vecs[[2]] == vals[[2]] vecs[[2]]
True

e4 = aA.vecs[[3]] == vals[[3]] vecs[[3]]
True

e5 = aA.vecs[[4]] == vals[[4]] vecs[[4]]
True
```