Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

4 - 8 Solve by Laplace transforms

5. w[x, 0] == 0 // x >= 0
w[0, t] == 0 // t >= 0
$x \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} = xt$

This problem is explained and worked out in the s.m. But before getting into those details, I thought I would just try it once with DSolve, without preliminaries.

```
Clear["Global`*"]
```

```
eqn = x D[w[x, t], x] + D[w[x, t], t] - x t == 0
```

$-t\,x + w^{(0,1)}[x, t] + x\, w^{(1,0)}[x, t] == 0$

Initial conditions with regard to x:

```
icx = {w[0, t] == 0}
{w[0, t] == 0}
```

Initial conditions with regard to t:

```
ict = {T[x, 0] == Sin[π x]}
{T[x, 0] == Sin[π x]}
```

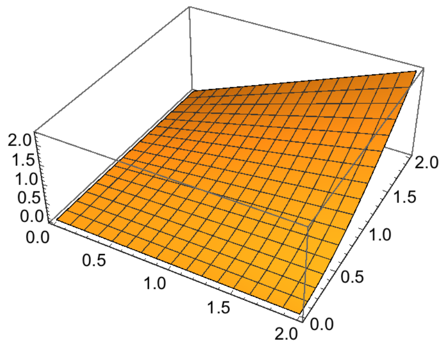
```
DSolve[{eqn, icx, ict}, w[x, t], {x, t}, Assumptions → {x ≥ 0, t ≥ 0}]
{{w[x, t] → (-1 + t) x}}
```

$\text{Simplify}\left[e^{-t}\left(1 - e^{t} + e^{t}\, t\right) x\right]$

$\left(-1 + e^{-t} + t\right) x$

The green cell above matches the text answer. This problem was simple enough to use **DSolve** without delving into Laplace transforms.

```
Plot3D[(-1 + e^-t + t) x, {x, 0, 2}, {t, 0, 2}]
```



7. Solve problem 5 by separating variables

I can’t see the relevancy of testing or tampering with DSolve’s operational style, so I’ll skip this problem.

****

Insert extra material: The following is a problem presented on MMAStack Exchange, #104385, answered by ‘march’. It shows some good procedural steps for the case when it may be necessary to use Laplace transforms with PDEs.

$$\frac{\partial T}{\partial t} + \frac{\partial^2 T}{\partial x^2}$$

T[0, t] = T[1, t] = 0

T[x, 0] = Sin[π x]

```
Clear["Global`*"]
```

```
eqn = D[T[x, t], t] - D[T[x, t], {x, 2}] == 0
```

$T^{(0,1)}[x, t] - T^{(2,0)}[x, t] == 0$

Note that the Laplace transform below is executed with an initial condition as a post-position substitution:

```
LaplaceTransform[eqn, t, s] /. T[x, 0] → Sin[π x]
```

$s\,\text{LaplaceTransform}[T[x, t], t, s] - \text{LaplaceTransform}\left[T^{(2,0)}[x, t], t, s\right] - \text{Sin}[\pi x] == 0$

Quoting: “In the second term [of above cell] we actually can interchange the order of integration and differentiation to see that it’s just D[LaplaceTransform[T[x,t], t, s], {x,2}]. Therefore we replace the transformed function with a dummy:”

```
eqn2 = s tT[x, s] - D[tT[x, s], {x, 2}] - Sin[π x] == 0;
```

“We can then solve this analytically:”

```
func =
 tT[x, s] /. First@DSolve[{eqn2, tT[0, s] == 0, tT[1, s] == 0}, tT[x, s], x]
(*Sin[π x]/(π^2+s)*)
```

$$\frac{\text{Sin}[\pi\, x]}{\pi^2 + s}$$

"Finally, then,"

```
InverseLaplaceTransform[func, s, t]
(*E^(-π^2 t) Sin[π x]*)
```

$e^{-\pi^2 t}\,\text{Sin}[\pi\, x]$

The above example is a good one. However, in this case the problem is simple enough that **DSolve** can handle it without reference to Laplace transforms.

```
Clear["Global`*"]
```

```
eqn = D[T[x, t], t] - D[T[x, t], {x, 2}] == 0
```

$T^{(0,1)}[x, t] - T^{(2,0)}[x, t] == 0$

```
icx = {T[0, t] == T[1, t] == 0}
```

{T[0, t] == T[1, t] == 0}

```
ict = {T[x, 0] == Sin[π x]}
```

{T[x, 0] == Sin[π x]}

```
DSolve[{eqn, icx, ict}, T[x, t], {x, t}]
```

$\{\{T[x, t] \to e^{-\pi^2 t}\,\text{Sin}[\pi\, x]\}\}$

It would be a good idea to keep either one of these approaches in the toolbox.