Note: Cells in green match the answers in the text.

Calculus.
1 to 8. Solve the ODE by integration or by remembering a differentiation formula.

1. $y' + 2 \sin 2\pi x = 0$

```
Clear["Global`*"]
```

```
DSolve[y'[x] + 2 Sin[2 π x] == 0, y[x], x]
```

$\{\{y[x] \to C[1] + \frac{\text{Cos}[2 \pi x]}{\pi}\}\}$

```
Clear["Global`*"]
```

2. $y' + xe^{-x^2/2} = 0$

$\text{DSolve}[y[x] + x\, e^{-x^2/2} == 0, y[x], x]$

$\{\{y[x] \to -e^{-\frac{x^2}{2}} x\}\}$

```
Clear["Global`*"]
```

3. $y' = y$

```
DSolve[y'[x] == y[x], y[x], x]
```

$\{\{y[x] \to e^x C[1]\}\}$

4. $y' = -1.5y$

```
DSolve[y'[x] == -1.5 y[x], y[x], x]
```

$\{\{y[x] \to e^{-1.5 x} C[1]\}\}$

5. $y' = 4e^{-x} \cos x$

$\text{DSolve}[y'[x] == 4\, e^{-x} \text{Cos}[x], y[x], x]$

$\{\{y[x] \to C[1] + 2\, e^{-x} (-\text{Cos}[x] + \text{Sin}[x])\}\}$

6. $y'' = -y$

```
DSolve[y''[x] == -y[x], y[x], x]
```

```
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

7. $y' = \cosh 5.13x$

```
DSolve[y'[x] == Cosh[5.13 x], y[x], x]
```

```
{{y[x] → C[1] + 0.194932 Sinh[5.13 x]}}
```

```
1 / 5.13
0.194932
```

8. $y''' = e^{-0.2x}$

$$\text{DSolve}[y'''[x] == e^{-0.2\,x},\ y[x],\ x]$$

$$\{\{y[x] \to -125.\ e^{-0.2\,x} + C[1] + x\,C[2] + x^2\,C[3]\}\}$$

9 to 15. Verification. Initial value problem (IVP)
(a) Verify that y is a solution of the ODE. (b) Determine from y the particular solution of the IVP. (c) Graph the solution of the IVP.

9. $y' + 4y = 1.4$, $y = c\,e^{-4x} + 0.35$, $y(0) = 2$

```
eqn = y'[x] + 4 y[x] == 1.4;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\},\ 0.35 + e^{-4.\,x}\,C[1]]\}\}$$

(a)

The means of checking is as follows

```
Simplify[eqn /. sol]
{True}
```

Of course if I want to go to a lot of trouble to make sure the solution in the text is really correct, I could do

$$u[x\_] = (e^{-4\,x} + .35)$$

$$0.35 + e^{-4\,x}$$

$$\text{uprime} = D[e^{-4\,x} + .35,\ x]$$

$$-4\,e^{-4\,x}$$

$$-4\,e^{-4\,x}$$

$$-4\,e^{-4\,x}$$

```
rsec = 4 u[x] + uprime
```

$$-4\,e^{-4\,x} + 4\,(0.35 + e^{-4\,x})$$

```
Simplify[rsec]
1.4
```

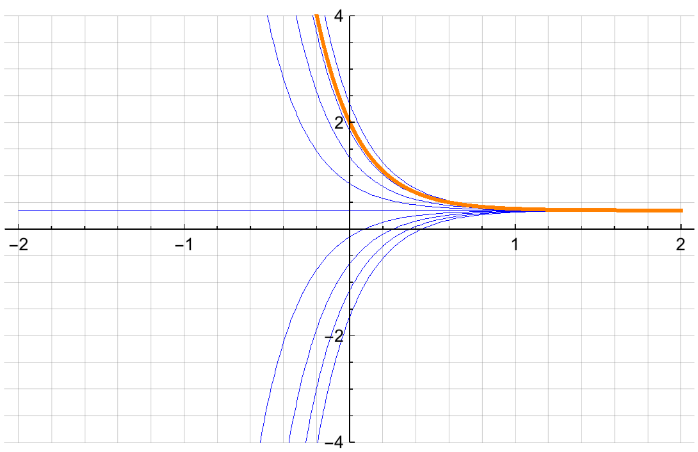
The 1.4 above is the rhs of the original equation. The book solution checks.

(b)

```
Solve[0.35 + c1 == 2, c1]
{{c1 → 1.65}}
```

(c) In the plot below, the IVP point is seen in orange. The lower, left-most is the -0.35 line.

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 2, 0.5}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 1.65, 1.65}];
Show[Plot[tab[x], {x, -2, 2}, PlotRange → {-4, 4},
   PlotStyle → {Blue, Thin}, GridLines → All],
 Plot[tabl[x], {x, -2, 2}, PlotRange → {-4, 4},
   PlotStyle → {Orange, Thick}]]
```




```
Clear["Global`*"]
```

10. $y' + 5xy = 0, \quad y = c e^{-2.5 x^2}, \quad y(0) = \pi$

```
eqn = y'[x] + 5 x y[x] == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-\frac{5 x^2}{2}} C[1]]\}\}$

(a) The DSolve answer is seen to equal the book's proposed solution.

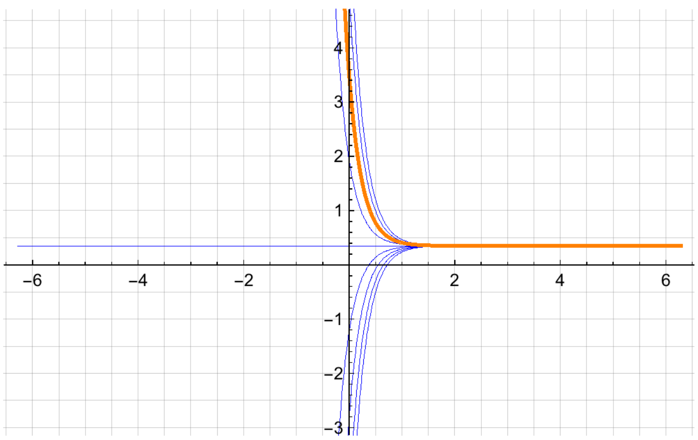
```
Simplify[eqn /. sol]
{True}
```

(b)

```
Solve[c1 == π, c1]
{{c1 → π}}
```

(c)

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2 π, 2 π, π/2}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, π, π}];
Show[Plot[tab[x], {x, -2 π, 2 π}, PlotRange → {-π, 3 π/2},
   PlotStyle → {Blue, Thin}, GridLines → All],
  Plot[tabl[x], {x, -2 π, 2 π}, PlotRange → {-π, 3 π/2},
   PlotStyle → {Orange, Thick}]]
```




```
Clear["Global`*"]
```

11. y' = y + e^x, $y = (x + c)\, e^x$, $y(0) = \frac{1}{2}$

```
eqn = y'[x] - y[x] - e^x == 0;
sol = DSolve[eqn, y, x]
{{y → Function[{x}, e^x x + e^x C[1]]}}
```

(a) The DSolve answer is seen to equal the book's proposed solution.

```
eqn /. sol
{True}
```

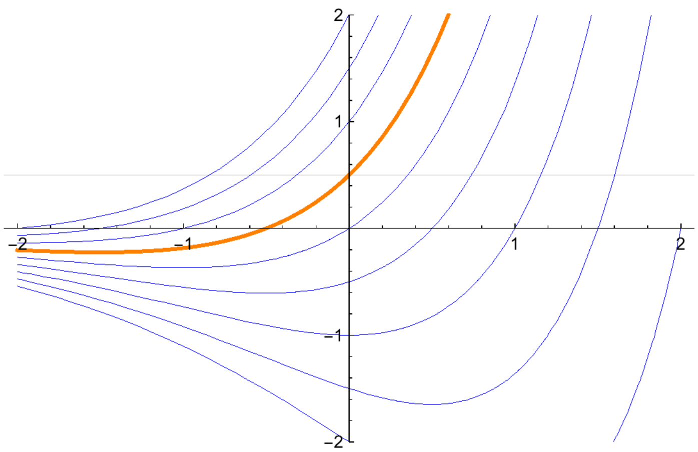
(b)

```
Solve[0 + c1 == 1/2, c1]
```

```
{{c1 → 1/2}}
```

(c)

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 2, 1/2}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 1/2, 1/2}];
Show[Plot[tab[x], {x, -2, 2}, PlotRange → {-2, 2},
  PlotStyle → {Blue, Thin}, GridLines → {{0}, {1/2}}],
 Plot[tabl[x], {x, -2, 2}, PlotRange → {-2, 2},
  PlotStyle → {Orange, Thick}]]
```




```
Clear["Global`*"]
```

12. yy’ = 4x, $y^2 - 4x^2 = C\,(y > 0),\ y(1) = 4$

```
eqn = y[x] y'[x] - 4 x == 0;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\sqrt{2}\sqrt{2x^2 + C[1]}]\},\ \{y \to \text{Function}[\{x\}, \sqrt{2}\sqrt{2x^2 + C[1]}]\}\}$$

(a) The DSolve answer [2] is seen to equal the book’s proposed solution.

i.e., for y > 0, $y^2 = 4x^2 + C$ −>y = $\sqrt{4x\,\hat{}\,2 + C}$ -> $\sqrt{2}\sqrt{2x\,\hat{}\,2 + C/\text{Sqrt}[2]}$

```
eqn /. sol[[2]]
```

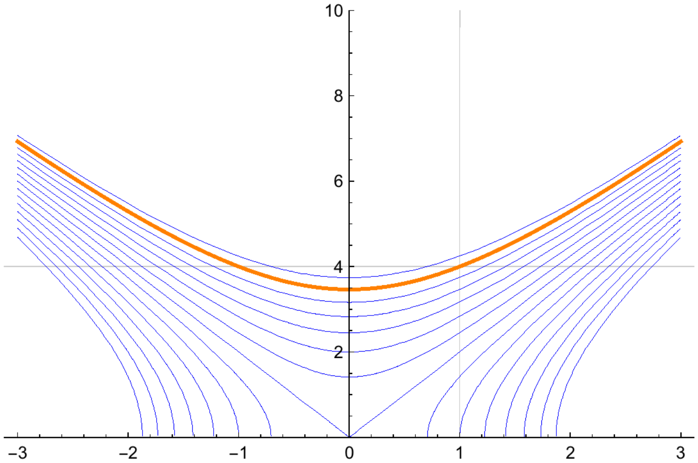
```
True
```

(b)

```
Solve[√2 √(2 + C[1]) == 4, C[1]]
{{C[1] → 6}}
```

(c)

```
g[x_] = y[x] /. sol[[2]];
tab[x_] = Table[g[x] /. C[1] → j, {j, -7, 7, 1}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 6, 6}];
Show[Plot[tab[x], {x, -3, 3}, PlotRange → {0, 10},
  PlotStyle → {Blue, Thin}, GridLines → {{1}, {4}}],
 Plot[tabl[x], {x, -3, 3}, PlotRange → {0, 10},
  PlotStyle → {Orange, Thick}]]
```




```
Clear["Global`*"]
```

13. $y' = y - y^2$, $y = \frac{1}{1+ce^{-x}}$, $y(0) = 0.25$

```
eqn = y'[x] - y[x] + (y[x])^2 == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \frac{e^x}{e^x + e^{C[1]}}]\}\}$

```
Simplify[eqn] /. Simplify[sol]
{True}
```

(a)

The question is whether the following is true:

$\frac{e^{\wedge}x}{e^{\wedge}x + e^{\wedge}C[1]} =^{?} \frac{1}{1 + C[2]\, e^{\wedge} -x}$

Multiplying the rhs by $\frac{e^{\wedge}x}{e^{\wedge}x}$ I get:

$\frac{e^{\wedge}x}{e^{\wedge}x + C[2]}$

So all I have to do is set C[2] = $e^{\wedge}$C[1].

Since this is legal (constants are real, not necessarily rational), I conclude that the expressions are equivalent.

(b)

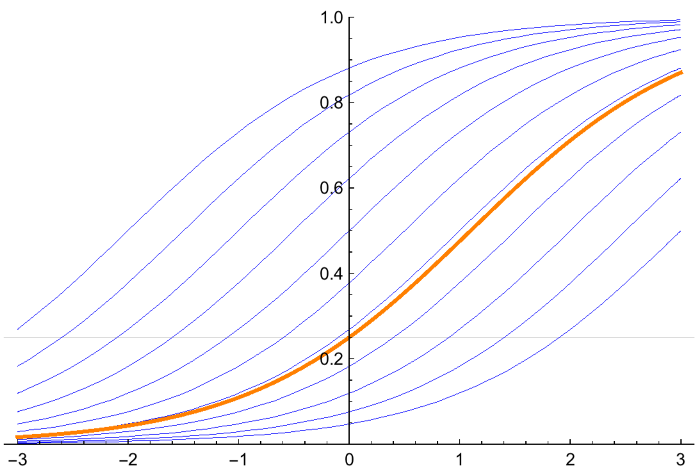
```
Solve[1/(1 + ⅇ^C[1]) == 0.25, C[1]]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

```
{{C[1] → 1.09861}}
```

(c)

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 3, 1/2}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 1.09861, 1.09861}];
Show[Plot[tab[x], {x, -3, 3}, PlotRange → {0, 1},
  PlotStyle → {Blue, Thin}, GridLines → {{0}, {0.25}}],
 Plot[tabl[x], {x, -3, 3}, PlotRange → {0, 1},
  PlotStyle → {Orange, Thick}]]
```




```
Clear["Global`*"]
```

14. y' tan x = 2y + 8, y = c $\sin^2 x + 4$, $y\left(\frac{1}{2}\pi\right) = 0$

```
eqn = y'[x] Tan[x] - 2 y[x] == 8;
sol = DSolve[eqn, y, x]
```

```
{{y → Function[{x}, -4 Cos[x]^2 + C[1] Sin[x]^2]}}
```

```
Simplify[eqn /. sol]
```

```
{True}
```

(a) So the decision is whether

$-4\cos^2 x + C[1]\sin^2 x =^? c\sin^2 x + 4.$ *I* can get to
$-4 + C[1]\sin^2 x =^? c\sin^2 x + 4.$ However, *I*
don'*t* really know the answer to that
proposed equality. So *I* guess *I* better do it the long way.

```
prop1 = Sin[x]^2 + 4
4 + Sin[x]^2
```

```
D[prop1, x]
2 Cos[x] Sin[x]
```

```
Simplify[(2 Cos[x] Sin[x]) Tan[x] - 2 (Sin[x]^2 + 4) - 8]
-16
```

It looks like Mathematica is disagreeing with the text answer.

```
Simplify[(2 Cos[x] Sin[x]) Tan[x] - 2 (Sin[x]^2 - 4) - 8]
0
```

(b) I don’t think this first ‘solve’ is necessary, but

```
Solve[-4 Cos[π/2]^2 + C[1] Sin[π/2]^2 == 0, C[1]]
{{C[1] → 0}}
```

```
sol2 = DSolve[{eqn, y[π/2] == 0}, y, x]
{{y → Function[{x}, -4 Cos[x]^2]}}
```

(c)

```
g[x_] = y[x] /. sol2;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 3, 1/2}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 0, 0}];
Show[Plot[tab[x], {x, -2 π, 2 π}, PlotRange → {-2 π, π},
   PlotStyle → {Blue, Thin}, GridLines → {{π/2}, {0}},
   Ticks → {{-2 Pi, -Pi, 0, Pi/2, Pi, 2 Pi}, {-4, -2, 0, 2}}],
 Plot[tabl[x], {x, -π, π}, PlotRange → {-2 π, 2 π},
   PlotStyle → {Orange, Thick}]]
```