4 - 10 Orthogonal trajectories (OTs)
Sketch or graph some of the given curves. Guess what their OTs may look like. Find these OTs.

4. y = x^2 +c

```
Clear["Global`*"]
```

```
y' = D[C x^2 + c, x]
```

2 C x

$$\tilde{y}'[x\_] = \frac{-1}{2\,C\,x}$$

$$-\frac{1}{2\,C\,x}$$

$$\text{inter}[x\_] = \int \tilde{y}'[x]\, dx$$

$$-\frac{\text{Log}[x]}{2\,C}$$

```
inter[x] = inter[x] + c
```

$$c - \frac{\text{Log}[x]}{2\,C}$$

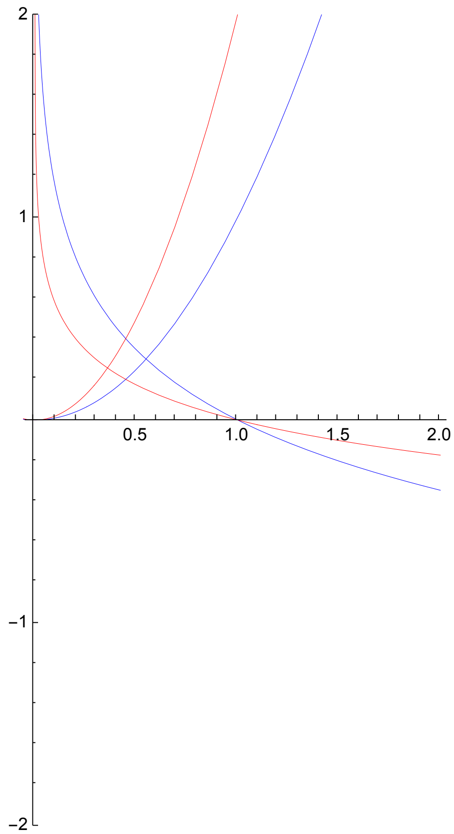
```
(*tab[x_]=Table[inter[x]/.c→j,{j,-2,2,0.5}/.C→p,{p,1.5}];*)
```

```
(*ytab[x_]=Table[C x^2+c1/.c1→k,{k,-2,2,0.5}/. C → r, {r, 1.5}];*)
tab[x_] = Table[inter[x] /. {c → 0, C → 1}];
tabgr[x_] = Table[inter[x] /. {c → 0, C → 2}];
```

```
ytab[x_] = Table[C x^2 + c1 /. {c1 → 0, C → 1}];
ytabgr[x_] = Table[C x^2 + c1 /. {c1 → 0, C → 2}];
```

```
Show[Plot[tab[x], {x, -2, 2}, PlotRange → {-2, 2},
   PlotStyle → {Blue, Thin}, AspectRatio → Automatic],
 Plot[ytab[x], {x, -2, 2}, PlotRange → {-2, 2},
   PlotStyle → {Blue, Thin}, AspectRatio → Automatic],

 Plot[tabgr[x], {x, -2, 2}, PlotRange → {-2, 2},
   PlotStyle → {Red, Thin}, AspectRatio → Automatic],
 Plot[ytabgr[x], {x, -2, 2}, PlotRange → {-2, 2},
   PlotStyle → {Red, Thin}, AspectRatio → Automatic]]
```



The integration constant is not meaningful here, the big C, relating to the independent variable, is what makes the orthogonality apparent.

5. y = c x

```
Clear["Global`*"]

y[x_] = c x
c x

y' = D[y[x], x]
c
```

```
ỹ'[x_] = -1/c
```

$$-\frac{1}{c}$$

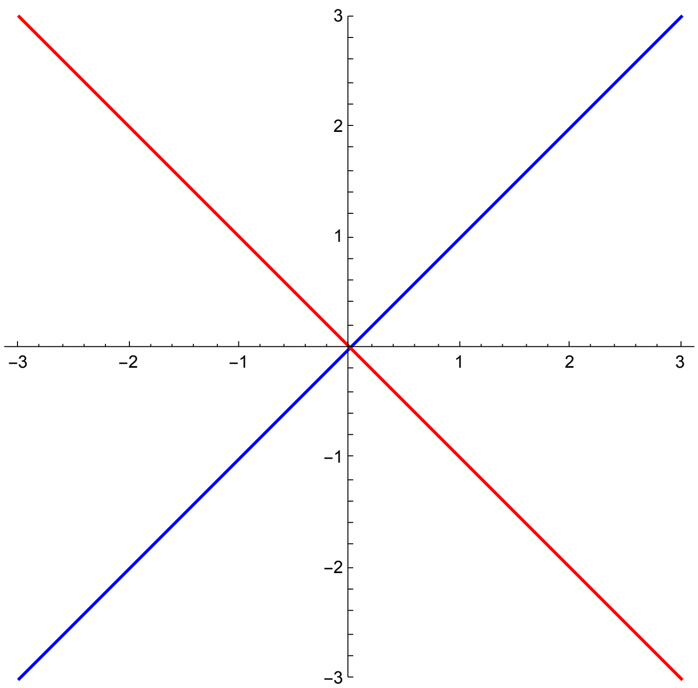
```
inter[x_] = ∫ỹ'[x] dx
```

$$-\frac{x}{c}$$

```
tab[x_] = Table[inter[x] /. c → j, {j, -1, -0.001, 1.5}];

ytab[x_] = Table[c1 x /. c1 → k, {k, -1, 0, 1.5}];

Show[Plot[tab[x], {x, -3, 3}, PlotRange → {-3, 3},
  PlotStyle → {Blue, Medium}, AspectRatio → Automatic],
 Plot[ytab[x], {x, -3, 3}, PlotRange → {-3, 3},
  PlotStyle → {Red, Medium}, AspectRatio → Automatic]]
```



6. x y = c

```
Clear["Global`*"]
```

```
y[x_] = c/x
```

$$\frac{c}{x}$$

```
y ' = D[y[x], x]
```

$$-\frac{c}{x^2}$$

```
ỹ '[x_] = x^2/c
```

$$\frac{x^2}{c}$$

```
inter[x_] := ∫ỹ '[x] dx
```

$$\frac{x^3}{3\,c}$$

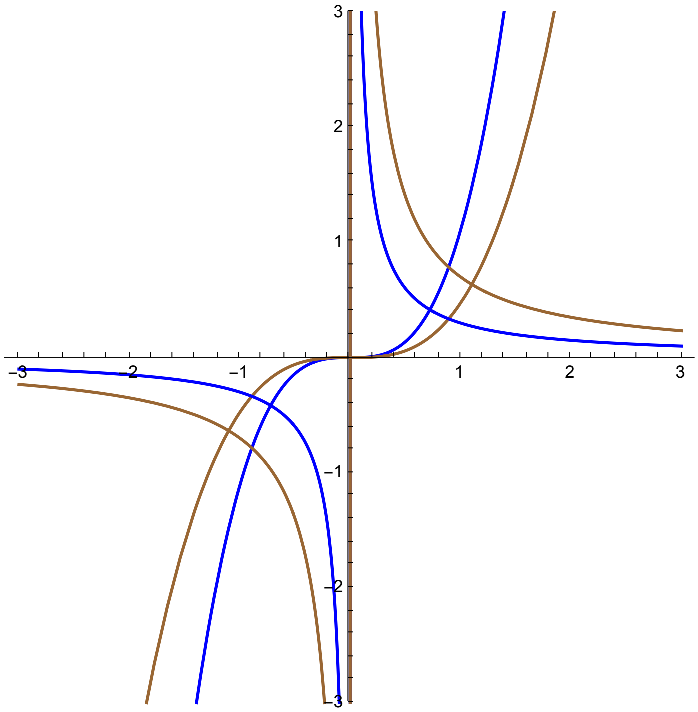
```
(*inter[x] = x^3/(3 c)*)
```

$$\frac{x^3}{3\,c}$$

```
tab[x_] = inter[x] /. c → .3;
tab2[x_] = inter[x] /. c → .7;
```

```
ytab[x_] = c/x /. c → .3;
ytab2[x_] = Table[c/x /. c → .7];
```

```
Show[Plot[tab[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Blue, Medium}, AspectRatio → 1 / 1],
 Plot[tab2[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Brown, Medium}, AspectRatio → 1 / 1],
 Plot[ytab[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Blue, Medium}, AspectRatio → 1 / 1],
 Plot[ytab2[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Brown, Medium}, AspectRatio → 1 / 1]]
```



7. $y = \frac{c}{x^2}$

```
Clear["Global`*"]
```

$y[x\_] = \frac{c}{x^2}$

$\frac{c}{x^2}$

$y' = D[y[x], x]$

$-\frac{2c}{x^3}$

$\tilde{y}'[x\_] = \frac{x^3}{2c}$

$\frac{x^3}{2c}$

```
inter[x_] = ∫ỹ'[x] ⅆx
```

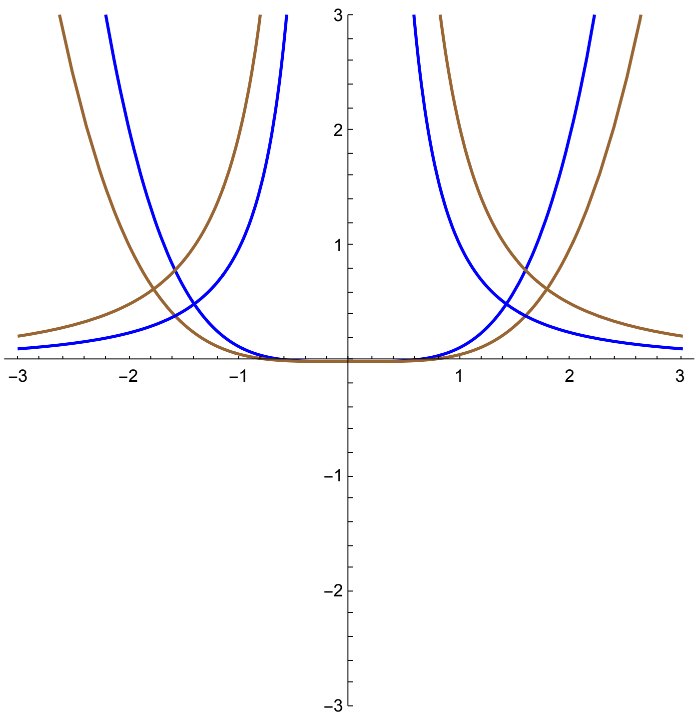
$$\frac{x^4}{8\,c}$$

```
tab[x_] = inter[x] /. c → 1;

tab2[x_] = inter[x] /. c → 2;

ytab[x_] = c/x^2 /. c → 1;
ytab2[x_] = c/x^2 /. c → 2;

Show[Plot[tab[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Blue, Medium}, AspectRatio → 1 / 1],
 Plot[tab2[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Brown, Medium}, AspectRatio → 1 / 1],
 Plot[ytab[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Blue, Medium}, AspectRatio → 1 / 1],
 Plot[ytab2[x], {x, -3, 3}, PlotRange → {-3, 3},
   PlotStyle → {Brown, Medium}, AspectRatio → 1 / 1]]
```



8. $y = \sqrt{x+c}$

```
Clear["Global`*"]

y[x_] := √(C x + c)
```

```
D[y[x], x]
```

$$\frac{C}{2\sqrt{c + C\,x}}$$

$$\tilde{y}'[x\_] := \frac{-2\sqrt{c + C\,x}}{C}$$

```
(*inter[x_]:=∫ỹ'[x]ⅆx*)
Integrate[ỹ'[x], x]
```

$$-\frac{4\,(c + C\,x)^{3/2}}{3\,C^2}$$

$$\text{thisx}[x\_] := -\frac{4\,(c + C\,x)^{3/2}}{3\,C^2}$$

```
thisx1[x_] = thisx[x] /. {c → -1.5, C → -.5}
```

$$-5.33333\,(-1.5 - 0.5\,x)^{3/2}$$

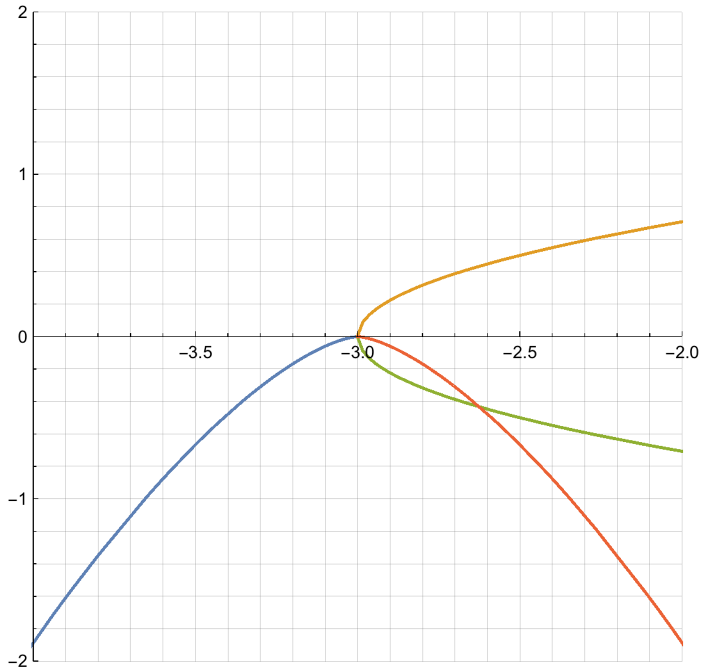
```
thisx2[x_] := thisx[x] /. {c → 1.5, C → .5}
thisx2[1]
-15.0849
```

$$\text{ytab}[x\_] := \sqrt{C\,x + c}\ /.\ \{c \to 1.5,\ C \to .5\};$$

$$\text{ytabm}[x\_] = -\sqrt{C\,x + c}\ /.\ \{c \to 1.5,\ C \to .5\};$$

```
ytabm[-1]
-1.
```

```
Plot[{thisx1[x], ytab[x], ytabm[x], thisx2[x]}, {x, -30, 30},
 PlotRange → {{-4, -2}, {-2, 2}}, AspectRatio → 1 / 1, GridLines → All]
```



To me, it looks like these display orthogonality, in pairs.

9. $y = ce^{-x^2}$

```
Clear["Global`*"]
```

$y[x\_] := c\, e^{-C\, x^2}$

```
D[y[x], x]
```

$-2\, c\, C\, e^{-C\, x^2}\, x$

$\tilde{y}'[x\_] := \dfrac{1}{2\, c\, C\, x\, e^{-C\, x^2}}$

(*inter:=∫ỹ'[x]ⅆx*)

```
Integrate[ỹ'[x], x]
```

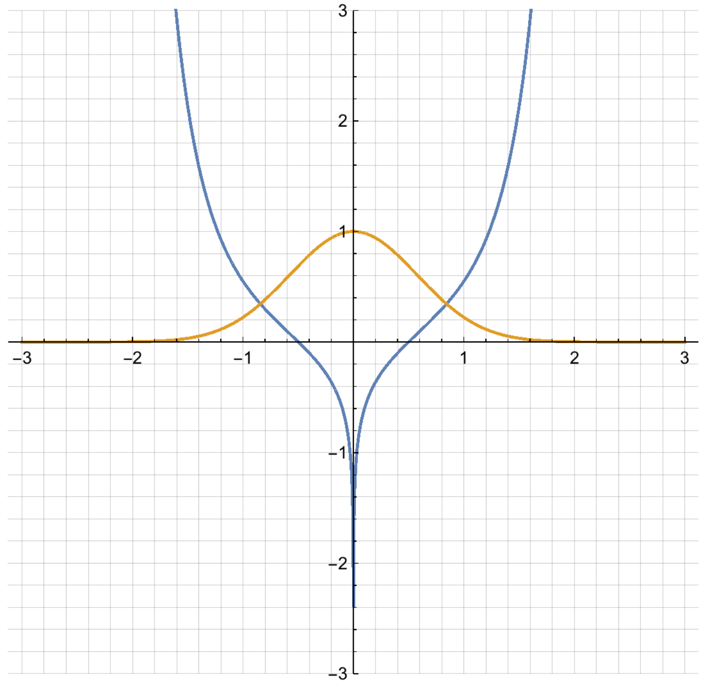
$\dfrac{\text{ExpIntegralEi}[C\, x^2]}{4\, c\, C}$

$\text{perx}[x\_] := \dfrac{\text{ExpIntegralEi}[C\, x^2]}{4\, c\, C}$

```
tab[x_] := perx[x] /. {c → 1, C → 1.5};
tab2[x_] := Table[inter /. c → o, {o, 0.001, 2, .5}];
```

$\text{ytab}[x\_] := c\, e^{-C\, x^2}$ /. {c → 1, C → 1.5};

```
Plot[{tab[x], ytab[x]}, {x, -3, 3}, PlotRange → {-3, 3},
 AspectRatio → Automatic, GridLines → Full]
```



10. $x^2 + (y - c)^2 = c^2$

```
Clear["Global`*"]
```

$$\texttt{Solve}\left[C\,x^2 + (y - c)^2 == c^2,\ y\right]$$

$$\left\{\left\{y \to c - \sqrt{c^2 - C\,x^2}\right\},\ \left\{y \to c + \sqrt{c^2 - C\,x^2}\right\}\right\}$$

$$y[x\_] := c + \sqrt{c^2 - C\,x^2}$$

```
D[y[x], x]
```

$$-\frac{C\,x}{\sqrt{c^2 - C\,x^2}}$$

$$\tilde{y}'[x\_] := \frac{\sqrt{c^2 - C\,x^2}}{C\,x}$$

$$(*\text{inter}=\int \tilde{y}'[x]\,dx*)$$

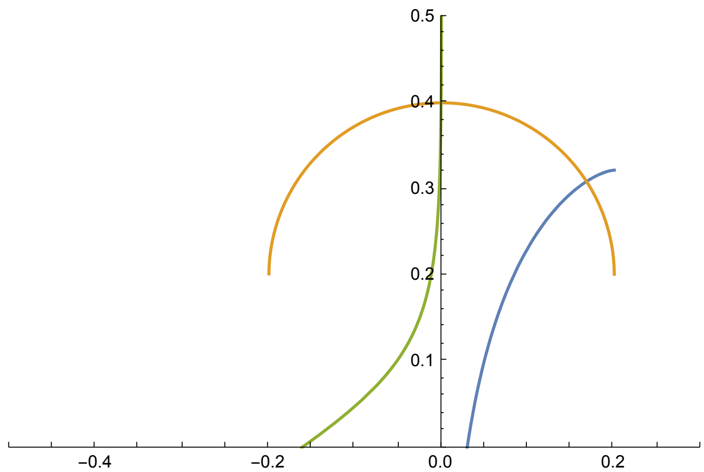
```
Integrate[ỹ'[x], x]
```

$$\frac{\sqrt{c^2 - C\,x^2} + c\,\mathrm{Log}[x] - c\,\mathrm{Log}\left[c^2 + c\,\sqrt{c^2 - C\,x^2}\right]}{C}$$

$$\mathrm{cras}[x\_] := \frac{\sqrt{c^2 - C\,x^2} + c\,\mathrm{Log}[x] - c\,\mathrm{Log}\left[c^2 + c\,\sqrt{c^2 - C\,x^2}\right]}{C}$$

```
fab[x_] := cras[x] /. {c → .2, C → 1};
faby[x_] := y[x] /. {c → .2, C → 1};
fab2[x_] := cras[x] /. {c → -.2, C → -3};
```

```
Plot[{fab[x], faby[x], fab2[x]}, {x, -0.5, .3},
 PlotRange → {{-0.5, .3}, {0, 0.5}}, AspectRatio → Automatic]
```



For this one, the third curve (green) is just ad hoc.