

```

ClearAll["Global`*"]

grr = {y''[x] + y'[x] - 2 y[x] == 0, y[0] == 4, y'[0] == -5}
out = DSolve[grr, y, x]
Simplify[grr /. out]
{-2 y[x] + y'[x] + y''[x] == 0, y[0] == 4, y'[0] == -5}

{{y -> Function[{x}, e-2 x (3 + e3 x)]}}
```

The above (example 2, p. 55) demonstrates that Mathematica can solve Homogeneous Linear ODEs with Constant Coefficients without the (manual) step of performing root solving.

```

ClearAll["Global`*"]

DSolve[y''[x] + 6 y'[x] + 9 y[x] == 0, y, x]
{{y -> Function[{x}, e-3 x C[1] + e-3 x x C[2]]}}
```

The above (example 3, p. 56) shows that Mathematica can find sol'n to another HLOCC without showing the characteristic equation, without constructing a basis.

```

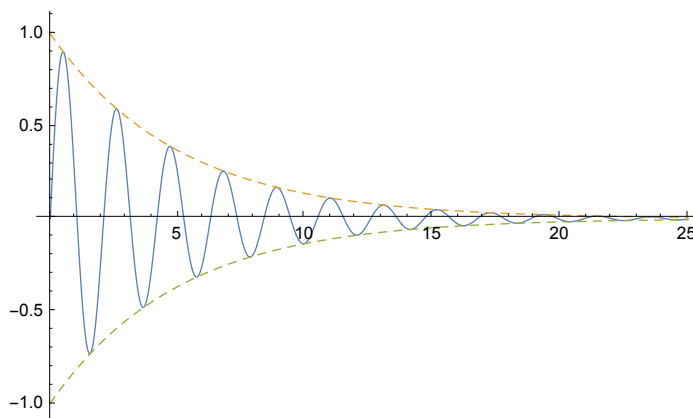
ClearAll["Global`*"]

DSolve[{y''[x] + 9.04 y[x] + 0.4 y'[x] == 0, y[0] == 0, y'[0] == 3}, y, x]
{{y -> Function[{x}, 1. e-0.2 x Sin[3. x]]}}
```

The above (example 5, p. 57) shows sol'n to eqn with complex roots, and with no special steps.

```

Plot[{e-0.2 x Sin[3. x], e-0.2 x, -e-0.2 x}, {x, 0, 25},
  PlotRange -> All, PlotStyle -> {{Thickness[0.002]},
    {Dashed, Thickness[0.002]}, {Dashed, Thickness[0.002]}}]
```



```

ClearAll["Global`*"]
```

**Series** $[e^{it}, \{t, 0, 10\}]$

$$1 + i \operatorname{Log}[e] t - \frac{1}{2} \operatorname{Log}[e]^2 t^2 - \frac{1}{6} i \operatorname{Log}[e]^3 t^3 + \frac{1}{24} \operatorname{Log}[e]^4 t^4 + \frac{1}{120} i \operatorname{Log}[e]^5 t^5 - \frac{1}{720} \operatorname{Log}[e]^6 t^6 - \frac{i \operatorname{Log}[e]^7 t^7}{5040} + \frac{\operatorname{Log}[e]^8 t^8}{40320} + \frac{i \operatorname{Log}[e]^9 t^9}{362880} - \frac{\operatorname{Log}[e]^{10} t^{10}}{3628800} + O[t]^{11}$$

% /. **Log** $[e] \rightarrow 1$

$$1 + i t - \frac{t^2}{2} - \frac{i t^3}{6} + \frac{t^4}{24} + \frac{i t^5}{120} - \frac{t^6}{720} - \frac{i t^7}{5040} + \frac{t^8}{40320} + \frac{i t^9}{362880} - \frac{t^{10}}{3628800} + O[t]^{11}$$

1 - 15 General solution

Find a general solution. Check your answer by substitution. ODEs of this kind have important applications to be discussed in sections 2.4, 2.7, and 2.9.

1.  $4y'' - 25y = 0$

**ClearAll** $["Global`*"]$

**DSolve** $[4 y''[x] - 25 y[x] == 0, y, x]$

$\{\{y \rightarrow \text{Function}[\{x\}, e^{5x/2} C[1] + e^{-5x/2} C[2]]\}\}$

Answer above is correct.

3.  $y'' + 6y' + 8.96y = 0$

**ClearAll** $["Global`*"]$

**DSolve** $[y''[x] + 6 y'[x] + 8.96 y[x] == 0, y, x]$

$\{\{y \rightarrow \text{Function}[\{x\}, e^{-3.2x} C[1] + e^{-2.8x} C[2]]\}\}$

Answer above is correct.

5.  $y'' + 2\pi y' + \pi^2 y = 0$

**ClearAll** $["Global`*"]$

**DSolve** $[y''[x] + 2 \pi y'[x] + \pi^2 y[x] == 0, y, x]$

$\{\{y \rightarrow \text{Function}[\{x\}, e^{-\pi x} C[1] + e^{-\pi x} x C[2]]\}\}$

Answer above is correct.

$$7. y'' + 4.5 y' = 0$$

```
ClearAll["Global`*"]
```

```
DSolve[y''[x] + 4.5 y'[x] == 0, y, x]
```

```
{ {y -> Function[{x}, -0.222222 e^{-4.5 x} C[1] + C[2]] } }
```

Answer above is correct, ( the constant coefficient C[1] was consolidated in the text answer, making the 2's factor disappear).

$$9. y'' + 1.8 y' - 2.08 y = 0$$

```
ClearAll["Global`*"]
```

```
DSolve[y''[x] + 1.8 y'[x] - 2.08 y[x] == 0, y, x]
```

```
{ {y -> Function[{x}, e^{-2.6 x} C[1] + e^{0.8 x} C[2]] } }
```

Above answer is correct.

$$11. 4 y'' - 4 y' - 3 y = 0$$

```
ClearAll["Global`*"]
```

```
DSolve[4 y''[x] - 4 y'[x] - 3 y[x] == 0, y, x]
```

```
{ {y -> Function[{x}, e^{-x/2} C[1] + e^{3 x/2} C[2]] } }
```

Answer above is correct.

$$13. 9 y'' - 30 y' + 25 y = 0$$

```
ClearAll["Global`*"]
```

```
DSolve[9 y''[x] - 30 y'[x] + 25 y[x] == 0, y, x]
```

```
{ {y -> Function[{x}, e^{5 x/3} C[1] + e^{5 x/3} x C[2]] } }
```

Answer above is correct.

$$15. y'' + 0.54 y' + (0.0729 + \pi) y = 0$$

```
ClearAll["Global`*"]
```

```
DSolve[y''[x] + 0.54 y'[x] + (0.0729 + \pi) y[x] == 0, y, x]
```

```
{ {y -> Function[{x}, e^{-0.27 x} C[2] Cos[1.77245 x] + e^{-0.27 x} C[1] Sin[1.77245 x]] } }
```

$(1.7724538509055159)^2$

3.14159

The above answer is correct.

16 - 20 Find an ODE

$y'' + a y' + b y = 0$  for the given basis.

17.  $e^{-\sqrt{5}x}, x e^{-\sqrt{5}x}$

```
ClearAll["Global`*"]
```

```
h[x_] := e-√5 x
```

```
h'[x]
```

```
-√5 e-√5 x
```

```
h''[x]
```

```
5 e-√5 x
```

```
j[x_] := x e-√5 x
```

```
j'[x]
```

```
e-√5 x - √5 e-√5 x x
```

```
j''[x]
```

```
-2 √5 e-√5 x + 5 e-√5 x x
```

```
Solve[h''[x] + a h'[x] + b h[x] == 0 && j''[x] + a j'[x] + b j[x] == 0, {a, b}]
```

```
{{a → 2 √5, b → 5}}
```

The above answer is correct.

19.  $e^{(-2+i)x}, e^{(-2-i)x}$

```
ClearAll["Global`*"]
```

```
k[x_] := e(-2+i)x
```

```
m[x_] := e(-2-i)x
```

```
Solve[k''[x] + a k'[x] + b k[x] == 0 && m''[x] + a m'[x] + b m[x] == 0, {a, b}]
```

```
{{a → 4, b → 5}}
```

The above answer is correct.

## 21 - 30 Initial value problems

Solve the IVP. Check that your answer satisfies the ODE as well as the initial conditions.

$$21. \ y'' + 25 y = 0, y(0) = 4.6, y'(0) = -1.2$$

```
ClearAll["Global`*"]

arkosa = {y''[x] + 25 y[x] == 0, y[0] == 4.6, y'[0] == -1.2}
hap = DSolve[arkosa, y, x]
{25 y[x] + y''[x] == 0, y[0] == 4.6, y'[0] == -1.2}
{{y -> Function[{x}, 4.6 Cos[5 x] - 0.24 Sin[5 x]]}}
```

Simplify[arkosa /. hap]

```
{Cos[5 x] == 0, True, True}
```

The above line indicates that Mathematica tested the initial value points and found that the sol'n worked correctly at those points.

$$r[x_] := 4.6 \cos[5 x] - 0.24 \sin[5 x]$$

```
Simplify[r''[x] + 25 r[x]]
-1.42109 × 10-14 Cos[5 x]
```

What the above line seems to indicate is that, within default working precision, the answer is correct.

$$23. \ y'' + y' - 6 y = 0, y(0) = 10, y'(0) = 0$$

```
ClearAll["Global`*"]

redondo = {y''[x] + y'[x] - 6 y[x] == 0, y[0] == 10, y'[0] == 0}
ghent = DSolve[redondo, y, x]
{-6 y[x] + y'[x] + y''[x] == 0, y[0] == 10, y'[0] == 0}
{{y -> Function[{x}, 2 e-3 x (2 + 3 e5 x)]}}
```

Simplify[redondo /. ghent]

```
{{True, True, True}}
```

Simplify[2 e<sup>-3 x</sup> (2 + 3 e<sup>5 x</sup>)]

$$e^{-3 x} (4 + 6 e^{5 x})$$

The above answer is correct; Mathematica tested both the initial value points as well as the general sol'n and found all sat.

25.  $y'' - y = 0, y(0) = 2, y'(0) = -2$

```
ClearAll["Global`*"]

treacle = {y''[x] - y[x] == 0, y[0] == 2, y'[0] == -2}
rhet = DSolve[treacle, y, x]
{-y[x] + y''[x] == 0, y[0] == 2, y'[0] == -2}
```

```
{y -> Function[{x}, 2 e-x]}
```

```
Simplify[treacle /. rhet]
{{True, True, True}}
```

The above answer is correct; Mathematica tested both the initial value points as well as the general sol'n and found all sat.

27. The ODE in problem 5,  $y(0) = 4.5, y'(0) = -4.5\pi - 1 = 13.137$

```
ClearAll["Global`*"]

jam = {y''[x] + 2 π y'[x] + π2 y[x] == 0, y[0] == 4.5, y'[0] == -4.5 π - 1}
blank = DSolve[jam, y, x]
{π2 y[x] + 2 π y'[x] + y''[x] == 0, y[0] == 4.5, y'[0] == -15.1372}
{{y -> Function[{x}, -1. e-π x (-4.5 + 1. x)]}}
```

```
rodz = blank[[1, 1, 2, 2]]
-1. e-π x (-4.5 + 1. x)
```

```
Expand[rodz]
```

```
4.5 e-π x - 1. e-π x x
```

```
Simplify[jam /. blank]
{{e-π x == 0, True, True}}
```

```
TableForm[Table[Evaluate[rodz[x], {x, 8}]]]
```

```
0.151249[1]
0.00466861[2]
0.000121049[3]
1.74367 × 10-6[4]
(-7.53509 × 10-8)[5]
(-9.76862 × 10-9)[6]
(-7.03567 × 10-10)[7]
(-4.25654 × 10-11)[8]
```

```
TableForm[Table[{x, rodz}, {x, 8}],
  TableHeadings -> {{}, {"x ", "rodz value"}}]
```

x	rodz value
1	0.151249
2	0.00466861
3	0.000121049
4	$1.74367 \times 10^{-6}$
5	$-7.53509 \times 10^{-8}$
6	$-9.76862 \times 10^{-9}$
7	$-7.03567 \times 10^{-10}$
8	$-4.25654 \times 10^{-11}$

The above expression I interpret to mean that if  $x > 5$ , then  $e^{-\pi x}$  is effectively zero, and thereafter the checking statement  $e^{-\pi x} = 0$  will be true. But this doesn't seem very satisfactory.

```
mn[x_] := 4.5` e-π x - 1.` e-π x x
mn'[x] + 2 π mn'[x] + π2 mn[x]
```

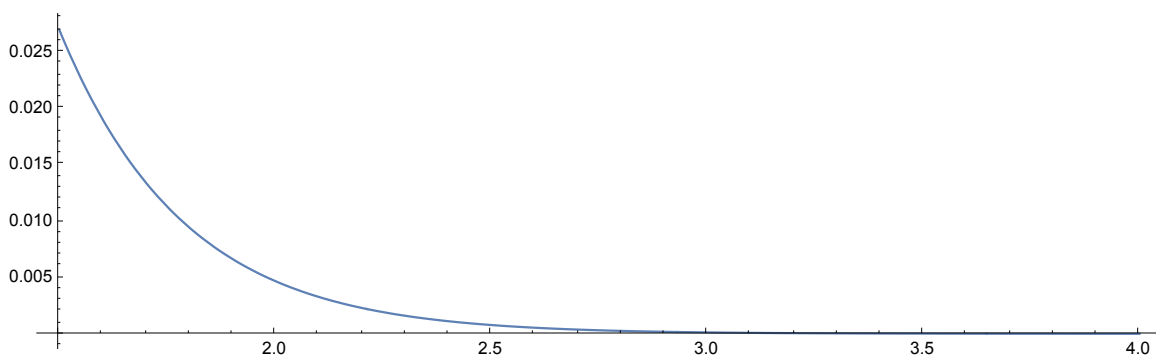
```
50.6964 e-π x - 9.8696 e-π x x +
π2 (4.5 e-π x - 1. e-π x x) + 2 π (-15.1372 e-π x + 3.14159 e-π x x)
```

```
Simplify[%]
```

```
0.
```

Okay, that's more like it. The text answer agrees. And incidentally, there is a slight mistake in the problem, because the 13.137 should have a minus sign.

```
Plot[4.5` e-π x - 1.` e-π x x, {x, 1.5, 4}, PlotRange -> All,
  ImageSize -> 600, AspectRatio -> 0.3, PlotStyle -> Thickness[0.002]]
```



29. The ODE in problem 15,  $y(0) = 0$ ,  $y'(0) = 1$

```
ClearAll["Global`*"]
```

```

quil = {y''[x] + 0.54 y'[x] + (0.0729 +  $\pi$ ) y[x] == 0, y[0] == 0, y'[0] == 1}
tig = DSolve[quil, y, x]
{3.21449 y[x] + 0.54 y'[x] + y''[x] == 0, y[0] == 0, y'[0] == 1}

```

```
{ {y -> Function[{x}, 0.56419 e-0.27 x Sin[1.77245 x]] } }
```

```
Simplify[quil /. tig]
```

```
{ {e-0.27 x (-1.11022 × 10-16 Cos[1.77245 x] + 3.88578 × 10-16 Sin[1.77245 x]) == 0,
  True, True} }
```

```
N[ $\sqrt{\pi}$ ]
```

```
1.77245
```

```
1 / %
```

```
0.56419
```

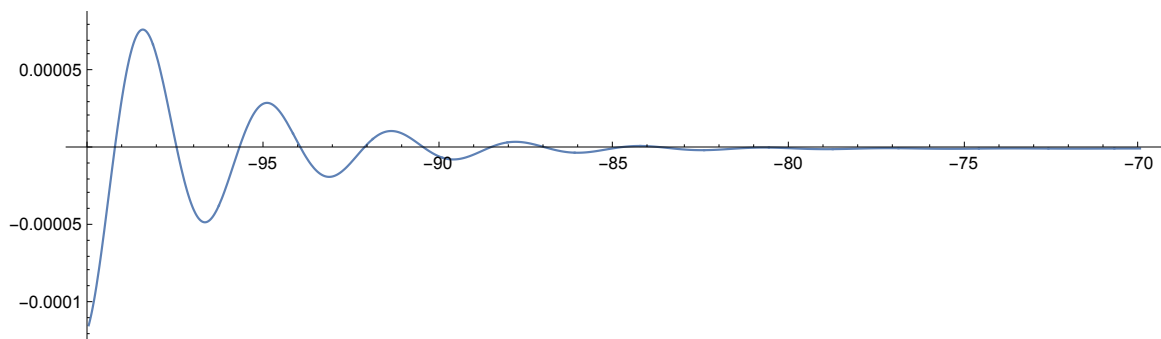
Checking these two quantities establishes that the answer agrees with the text's. As for actually checking the answer, Mathematica has declined to do so.

```
fes[x_] :=  $\frac{1}{\sqrt{\pi}}$  e-0.27 x Sin[ $\sqrt{\pi}$  x]
```

```
kres = Simplify[fes''[x] + 0.54 fes'[x] + (0.0729 +  $\pi$ ) fes[x]]
```

```
2.22045 × 10-16 e-0.27 x Sin[ $\sqrt{\pi}$  x]
```

```
Plot[2.220446049250313`*^-16 e-0.27` x Sin[ $\sqrt{\pi}$  x],
  {x, -100, -70}, PlotRange -> All, ImageSize -> 600,
  AspectRatio -> 0.3, PlotStyle -> Thickness[0.002]]
```





```
TableForm[Table[{x, fes[x]}, {x, -100, 100, 20}],
  TableHeadings -> {{}, {"x ", "func value"}}]
```

x	func value
-100	$-2.905 \times 10^{11}$
-80	$5.58566 \times 10^8$
-60	$2.75639 \times 10^6$
-40	-27 036.
-20	97.1904
0	0.
20	-0.00198264
40	0.0000112507
60	$-2.33991 \times 10^{-8}$
80	$-9.67281 \times 10^{-11}$
100	$1.02623 \times 10^{-12}$

31 - 36 Linear independence is of basic importance, in this chapter, in connection with general solutions, as explained in the text. Are the following functions linearly independent on the given interval?

31.  $e^{kx}$ ,  $x e^{kx}$ , any interval

```
ClearAll["Global`*"]
r = FullSimplify[Solve[a e^{k x} == x e^{k x}, a]]
{{a -> x}}
```

A sol'n for  $a$  has been found, but the sol'n says  $a$  is not a constant. Therefore the functions must be linearly independent. Answer agrees with text's.

33.  $x^2$ ,  $x^2 \text{Log}[x]$ ,  $x > 1$

```
ClearAll["Global`*"]
r = FullSimplify[Solve[a x^2 == x^2 Log[x], a]]
{{a -> Log[x]}}
```

Again, the functions must be independent, because the connecting factor is not constant. (Text agrees.)

35.  $\text{Sin}[2 x]$ ,  $\text{Cos}[x] \text{Sin}[x]$ ,  $x < 0$

```
ClearAll["Global`*"]
r = FullSimplify[Solve[a Sin[2 x] == Cos[x] Sin[x] && x < 0, a]]
{{a -> ConditionalExpression[1/2, x < 0]}}
```

Here Mathematica comes through by sniffing out a constant sol'n. This means that the two functions are linearly dependent. (And the text agrees.)

37. Instability. Solve  $y'' - y = 0$  for the initial conditions  $y(0) = 1, y'(0) = -1$ . Then change the initial conditions to  $y(0) = 1.001, y'(0) = -0.999$  and explain why this small change of 0.001 at  $t = 0$  causes a large change later, e.g. 22 at  $t = 10$ . This is instability: a small initial difference in setting a quantity (a current, for instance) becomes larger and larger with time  $t$ . This is undesirable.

```
ClearAll["Global`*"]
```

```
hak = {y''[t] - y[t] == 0, y[0] == 1, y'[0] == -1}
```

```
drak = DSolve[hak, y, t]
```

```
{-y[t] + y''[t] == 0, y[0] == 1, y'[0] == -1}
```

```
{ {y -> Function[{t}, e^-t] }
```

```
hur = drak[[1, 1, 2, 2]]
```

```
e^-t
```

```
hak1 = {y''[t] - y[t] == 0, y[0] == 1.001, y'[0] == -0.999}
```

```
drak1 = DSolve[hak1, y, t]
```

```
{-y[t] + y''[t] == 0, y[0] == 1.001, y'[0] == -0.999}
```

```
{ {y -> Function[{t}, 0.001 e^-t (1000. + 1. e^2 t) ] }
```

```
hur1 = drak1[[1, 1, 2, 2]]
```

```
0.001 e^-t (1000. + 1. e^2 t)
```

```
TableForm[Table[{t, N[hur, 4], N[hur1, 4]}, {t, 0, 30, 2}],
  TableHeadings → {{}, {"t ", "drak value ", "drak1 value"}}]
```

t	drak value	drak1 value
0	1.000	1.001
2	0.1353	0.142724
4	0.01832	0.0729138
6	0.002479	0.405908
8	0.0003355	2.98129
10	0.00004540	22.0265
12	$6.144 \times 10^{-6}$	162.755
14	$8.315 \times 10^{-7}$	1202.6
16	$1.125 \times 10^{-7}$	8886.11
18	$1.523 \times 10^{-8}$	65 660.
20	$2.061 \times 10^{-9}$	485 165.
22	$2.789 \times 10^{-10}$	$3.58491 \times 10^6$
24	$3.775 \times 10^{-11}$	$2.64891 \times 10^7$
26	$5.109 \times 10^{-12}$	$1.9573 \times 10^8$
28	$6.914 \times 10^{-13}$	$1.44626 \times 10^9$
30	$9.358 \times 10^{-14}$	$1.06865 \times 10^{10}$

The table verifies what the text says, i.e. that there is a difference of 22 at  $t = 10$ . And it gets much larger later.