```
Clear["Global`*"]
```

1 - 12 Sample spaces, events

Graph a sample space for the experiments:

1. Drawing 3 screws from a lot of right-handed and left-handed screws.

Assume there are 24 in the lot, 12 of each.

```
d = MultivariateHypergeometricDistribution[3, {12, 12}]
MultivariateHypergeometricDistribution[3, {12, 12}]
```

In a draw of three, the probability of getting 1, 2 or 2 left-handed.

```
Table[Probability[x > n, {x, y} ≈ d], {n, 0, 2}]
```

$\left\{\frac{41}{46}, \frac{1}{2}, \frac{5}{46}\right\}$

But as for the entire probability sample space,

```
arr = Table[R, {n, 12}]
{R, R, R, R, R, R, R, R, R, R, R, R}
```

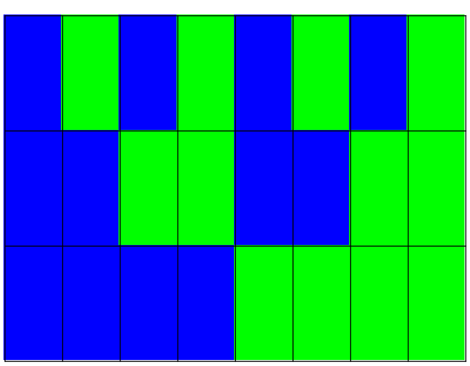
```
all = Table[L, {n, 12}]
{L, L, L, L, L, L, L, L, L, L, L, L}
```

```
jj = Join[arr, all]
{R, R, R, R, R, R, R, R, R, R, R, R, L, L, L, L, L, L, L, L, L, L, L, L}
```

Assuming the probabilities are vertically stacked rectangles three high, each stack of three a three-screw draw.

```
let = Permutations[jj, {3}]
```

```
{{R, R, R}, {R, R, L}, {R, L, R}, {R, L, L},
 {L, R, R}, {L, R, L}, {L, L, R}, {L, L, L}}
```



3. Rolling 2 dice.

A pair of fair dice can be modeled using a DiscreteUniformDistribution.

```
dice = DiscreteUniformDistribution[{2, 12}];
```

Generate a simulation of ten throws.

```
RandomVariate[dice, 10]
{3, 5, 11, 6, 7, 3, 5, 6, 2, 4}
```

Compute the probability that the sum of three dice throws is less than 12:

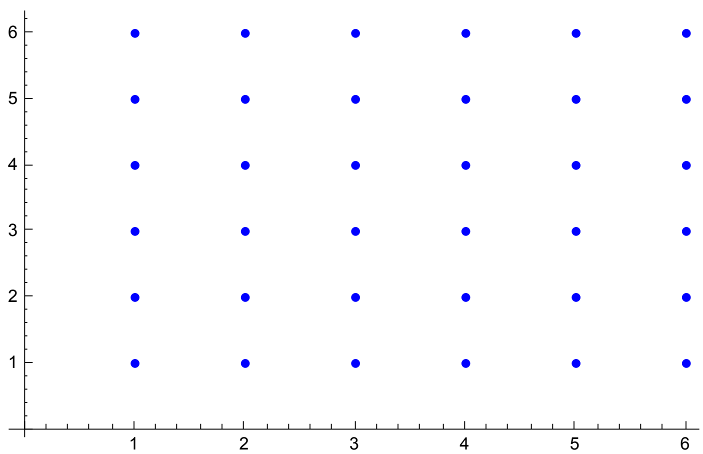
```
NProbability[Sum[x[i], {i, 3}] ≤ 12, {x[1] ≈ dice, x[2] ≈ dice, x[3] ≈ dice}]
0.0631104
```

As for the sample space, I am kludging it. A serious deficiency of the cell below is that dealing with a single list, the appearance of a double is not possible.

```
gr = Permutations[{1, 2, 3, 4, 5, 6}, {2}]
{{1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {2, 1}, {2, 3},
 {2, 4}, {2, 5}, {2, 6}, {3, 1}, {3, 2}, {3, 4}, {3, 5},
 {3, 6}, {4, 1}, {4, 2}, {4, 3}, {4, 5}, {4, 6}, {5, 1}, {5, 2},
 {5, 3}, {5, 4}, {5, 6}, {6, 1}, {6, 2}, {6, 3}, {6, 4}, {6, 5}}
```

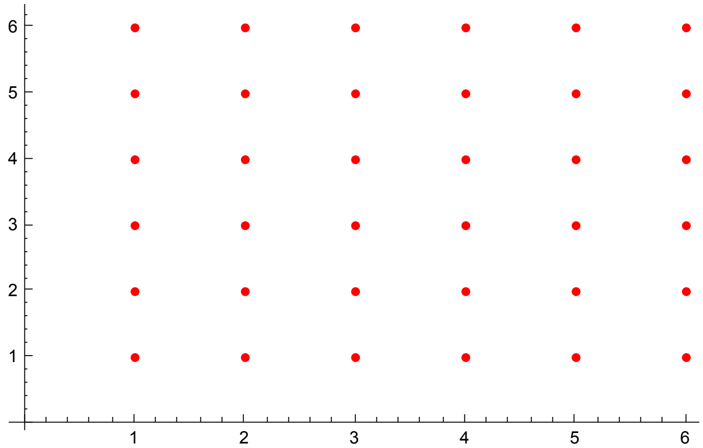
Since the problem is asking for a plot, maybe the deficiency is minor, since the diagonal can be easily filled in.

```
ListPlot[{gr, Range[6]}, PlotStyle → Blue]
```



or

```
ListPlot[Table[Table[j, {n, 1, 6}], {j, 1, 6}], PlotStyle → Red]
```



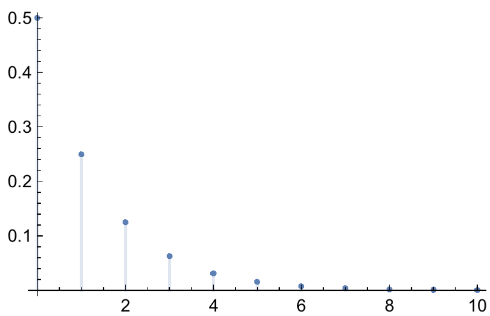
5. Tossing a coin until the first Head appears.

```
heads1 = NegativeBinomialDistribution[1, 1 / 2]
```

$$\text{NegativeBinomialDistribution}\left[1, \frac{1}{2}\right]$$

Plot the distribution of tail counts:

```
DiscretePlot[PDF[heads1, k], {k, 0, 10}, ImageSize → 250]
```



Compute the probability of getting at least 4 tails before getting 1 heads :

```
NProbability[tails ≥ 4, tails ≈ heads1]
```

```
0.0625
```

Compute the expected number of tails before getting the first heads :

```
Mean[heads1]
```

```
1
```

Now to draw something concerning the sample space.

```
s1 = Range[4]
```

```
{1, 2, 3, 4}
```

```
ex = Table[Table[0, {n, m}], {m, 0, 8}]
```

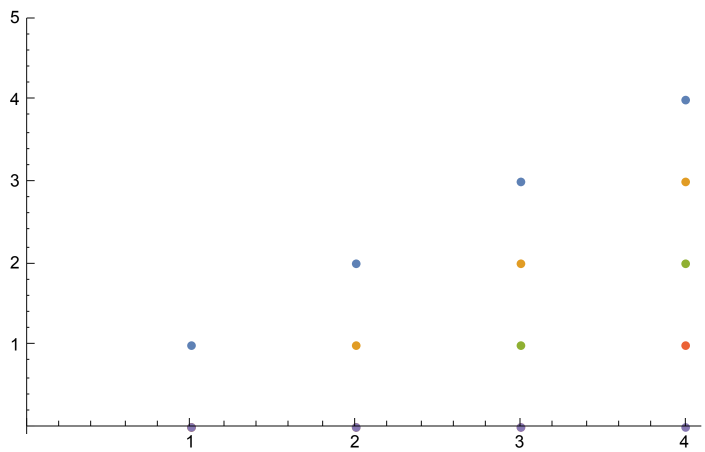
```
{{}, {0}, {0, 0}, {0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
ty = Table[Join[ex[[n]], s1], {n, 6}]
```

```
{{1, 2, 3, 4}, {0, 1, 2, 3, 4}, {0, 0, 1, 2, 3, 4}, {0, 0, 0, 1, 2, 3, 4},
 {0, 0, 0, 0, 1, 2, 3, 4}, {0, 0, 0, 0, 0, 1, 2, 3, 4}}
```

The sample space expands without bound while those coin tails keep showing up. Meanwhile the PDF is hugging the x-axis. I would rather that this plot expanded parallel to the y-axis, right now it seems something of a cheat.

```
ListPlot[Table[ty[[n]], {n, 1, 5}], PlotRange → {{0, 4.1}, {-0.1, 5}}]
```



7. Recording the daily maximum temperature X and the daily maximum air pressure Y at Times Square in New York.

Trying to imitate some summer month.

```
Clear["Global`*"]
```

```
ap = NumberForm[RandomReal[{29.0, 30.9}, 30], {4, 2}];
```

```
apo = {"29.18", "30.65", "29.94", "29.11", "30.60", "29.08",
   "29.99", "30.46", "29.70", "29.45", "30.79", "30.38",
   "30.04", "30.01", "30.07", "29.44", "29.79", "29.98",
   "29.20", "30.69", "30.10", "30.83", "30.09", "29.38",
   "30.43", "29.36", "29.92", "29.91", "29.89", "30.34"};
```

```
tem = NumberForm[RandomReal[{80, 95}, 30], {4, 1}];
```

```
temo = {"86.0", "81.9", "87.7", "85.0", "85.6", "90.3", "87.5",
   "90.6", "81.0", "94.2", "92.0", "90.5", "82.7", "83.3", "81.4",
   "83.6", "81.6", "82.1", "85.3", "91.8", "92.9", "81.5", "83.3",
   "83.2", "88.6", "91.4", "94.8", "83.2", "89.2", "87.6"};

rn = Range[30];

rno = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
   16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30};

gr1 =
  Grid[Table[{rno[[n]], apo[[n]], temo[[n]]}, {n, 1, 30}], Frame → All];

gr2 = Grid[{{"D ", " Barom ", " Temp  "}}, Frame → All];

Column[{gr2, gr1}]
```

D	Barom	Temp
1	29.1823	86.0422
2	30.6537	81.8799
3	29.9417	87.6654
4	29.1096	84.9861
5	30.602	85.6174
6	29.0788	90.2995
7	29.9908	87.516
8	30.4619	90.6359
9	29.7044	80.9539
10	29.4492	94.1649
11	30.7925	92.0376
12	30.3816	90.5361
13	30.0417	82.7174
14	30.0127	83.2556
15	30.0736	81.4441
16	29.4426	83.6029
17	29.7918	81.5751
18	29.9754	82.0692
19	29.1969	85.3386
20	30.6868	91.8394
21	30.1033	92.8817
22	30.8257	81.4626
23	30.0897	83.3201
24	29.383	83.166
25	30.4293	88.5701
26	29.3594	91.4326
27	29.9168	94.8236
28	29.9136	83.238
29	29.892	89.1724
30	30.3383	87.5693

I don’t know what the plot of a legitimate sample space for this problem would be. Maybe something along the lines of brackets, with random values inside, would do. How about this: A plot based on the record high (and low) temperature for the date, bracketed with some sufficient cushion to catch the possible heat/cold wave. And then another dataset for the humidity percentage, similarly bracketed. The two plotted datasets could be shown together using the transparent filling style shown in the 5th example under basic plotting in the Mathematica documentation. And any point with double-valued coordinate inside the combined area would be in the sample universe.

9. Drawing gaskets from a lot of 10, containing one defective D, until D is drawn, one at a time, and assuming sampling without replacement, that is, gaskets drawn are not returned to the lot. (More about this in section 24.6).

I'm going to pull out one gasket from a box with changing number of good gaskets and one remaining bad gasket.

```
di[n_] = MultivariateHypergeometricDistribution[1, {n, 1}]
MultivariateHypergeometricDistribution[1, {n, 1}]
```

I want to get a table of probabilities for my chances of not getting the bad gasket on the next pull.

```
Table[Probability[x == 1 && y == 0, {x, y} ≈ di[n]], {n, 9, 1, -1}]
```

$$\left\{\frac{9}{10}, \frac{8}{9}, \frac{7}{8}, \frac{6}{7}, \frac{5}{6}, \frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}\right\}$$

I notice I have to paste the Out line to get a list I can calculate with.

$$\text{dip} = \left\{\frac{9}{10}, \frac{8}{9}, \frac{7}{8}, \frac{6}{7}, \frac{5}{6}, \frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}\right\};$$

I assemble a list of accumulating probabilities for avoiding the bad gasket. I feel sure I could save some space by using a loop, but not smart enough yet.

```
dip2 = dip[[1]] dip[[2]];

dip3 = dip2 dip[[3]];

dip4 = dip3 dip[[4]];

dip5 = dip4 dip[[5]];

dip6  = dip5 dip[[6]];

dip7 = dip6 dip[[7]];

dip8 = dip7 dip[[8]];

dip9 = dip8 dip[[9]];
```

The accumulated probabilities look familiar.

```
ndip = N[{9/10, dip2, dip3, dip4, dip5, dip6, dip7, dip8, dip9}]
```

```
{0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1}
```

10. In rolling 3 dice, are the events A: Sum Divisible by 3 and B: Sum divisible by 5 mutually exclusive?

11. Answer the questions in problem 10 for rolling 2 dice.

Yes. There are only 11 number results, therefore no common multiples of 3 and 5.

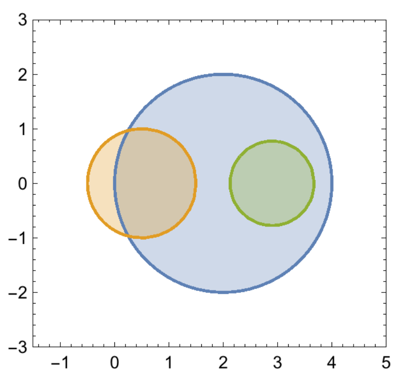
17. Using a Venn diagram, show that A ⊂ B if and only if A ⋃ B = B.

```
Clear["Global`*"]
```

```
disk1 = ImplicitRegion[(x - 2)^2 + (y)^2 < 4, {x, y}];
disk2 = ImplicitRegion[(x - 0.5)^2 + y^2 < 1, {x, y}];
disk3 = ImplicitRegion[(x - 2.9)^2 + y^2 < 0.6,  {x, y}];
```

I make three disks for the attempt at explaining. Left an orange, middle a blue, and right a green.

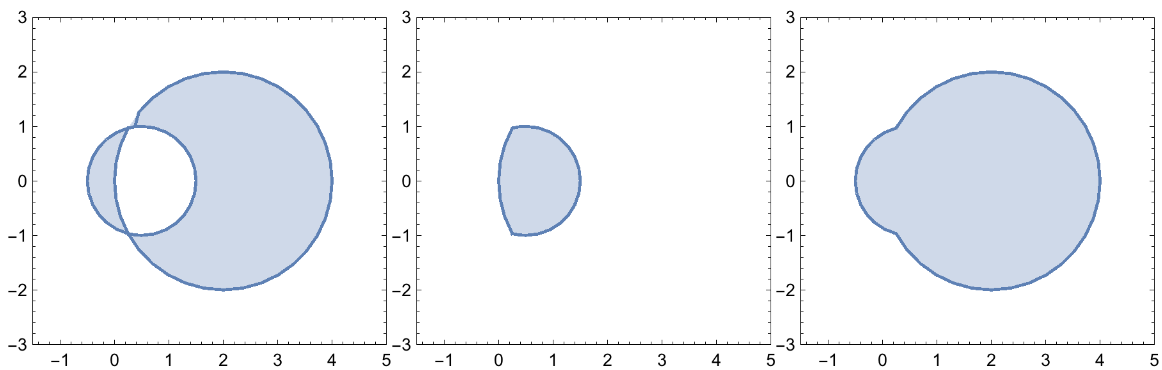
```
RegionPlot[{(x - 2)^2 + (y)^2 < 4, (x - 0.5)^2 + y^2 < 1, (x - 2.9)^2 + y^2 < 0.6},
 {x, -3, 6}, {y, -6, 6}, AspectRatio → Automatic,
 ImageSize → 200, PlotRange → {{-1.5, 5}, {-3, 3}},
 Epilog → {Blue, Text[Style["A", Bold, Large], {6, 0}]}]
```



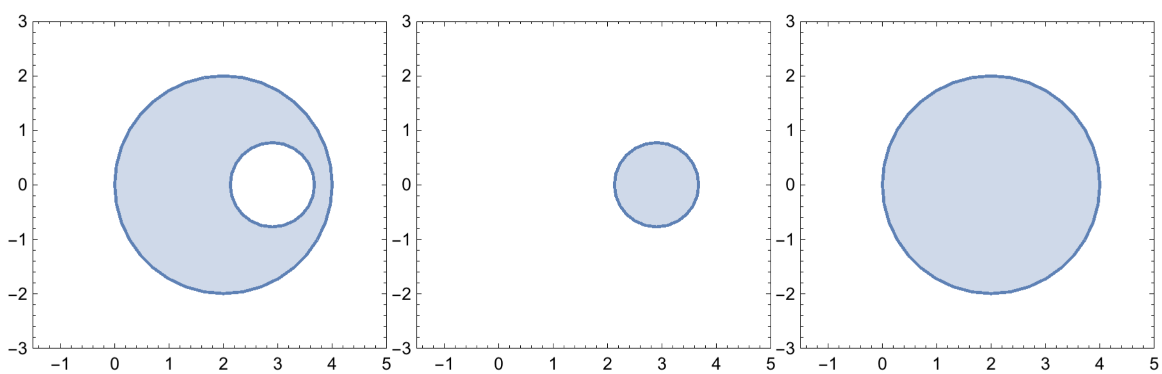

```
r2 = BooleanRegion[Xor, {disk1, disk2}];
r3 = BooleanRegion[And, {disk1, disk2}];
r4 = BooleanRegion[Or, {disk1, disk2}];
r5 = BooleanRegion[Xor, {disk1, disk3}];
r6 = BooleanRegion[And, {disk1, disk3}];
r7 = BooleanRegion[Or, {disk1, disk3}];
```

```
p1 = RegionPlot[r2, AspectRatio → Automatic,
    ImageSize → 200, PlotRange → {{-1.5, 5}, {-3, 3}}];
p2 = RegionPlot[r3, AspectRatio → Automatic, ImageSize → 200,
    PlotRange → {{-1.5, 5}, {-3, 3}}];
p3 = RegionPlot[r4, AspectRatio → Automatic, ImageSize → 200,
    PlotRange → {{-1.5, 5}, {-3, 3}}];
p4 = RegionPlot[r5, AspectRatio → Automatic, ImageSize → 200,
    PlotRange → {{-1.5, 5}, {-3, 3}}];
p5 = RegionPlot[r6, AspectRatio → Automatic, ImageSize → 200,
    PlotRange → {{-1.5, 5}, {-3, 3}}];
p6 = RegionPlot[r7, AspectRatio → Automatic,
    ImageSize → 200, PlotRange → {{-1.5, 5}, {-3, 3}}];
```

```
Row[{p1, p2, p3}]
```




```
Row[{p4, p5, p6}]
```



The if and only if goes two ways. Boolean operations appeal to logic. XOR between a parent set and a subset should be total destruction of the subset, since its reliance on the parent for points is total. Left, below is displayed such total destruction, whereas in left, above a surviving crescent exists. AND between a parent set and a subset should deflate the parent to the exact boundaries of the subset, demonstrating that all points inventoried in the subset are also in the parent. Middle, below is seen the total extent of the candidate, whereas middle, above shows deficits. OR between a parent set and a subset should reveal the parent totally, with total dissolution of the subset. This is basically the right-pointed arrow of the iff, and seen below right. The bump seen above right is an unauthorized augmentation of the candidate parent, which refutes a subset relation. To get the left-pointed iff arrow seems a little slippery, and I think of it as a combination of the two left panels.