```
Clear["Global`*"]
```

2. $y^3 y' + x^3 = 0$

```
eqn = y[x]^3 + x^3 == 0;
```

```
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -x]\},$
$\{y \to \text{Function}[\{x\}, (-1)^{1/3} x]\}, \{y \to \text{Function}[\{x\}, -(-1)^{2/3} x]\}\}$

```
eqn /. sol[[1]]
```

```
True
```

```
eqn /. sol[[2]]
```

```
True
```

```
eqn /. sol[[3]]
```

```
True
```

3. $y' = \sec^2 y$

```
Clear["Global`*"]
```

```
eqn = y'[x] == Sec[y[x]]^2;
eqn2 = y'[x] Cos[y[x]]^2 == 1;
```

$$eqn3 = y'[x] == \frac{2}{\text{Cos}[2\, y[x]] + 1};$$

$$eqn4 = y'[x] == \frac{4}{\left(e^{-i\, y[x]} + e^{i\, y[x]}\right)^2};$$

```
sol = DSolve[eqn3, y, x]
```

$\left\{\left\{y \to \text{Function}\left[\{x\}, \text{InverseFunction}\left[2 \left(\frac{1}{4} \text{Sin}[2\, \#1] + \frac{\#1}{2}\right) \&\right][2\, x + C[1]]\right]\right\}\right\}$

$$c_1 + 2 x = 2 \left(\frac{y[x]}{2} + \frac{1}{4} \text{Sin}[2\, y[x]]\right)$$

```
Simplify[eqn /. sol]
```

```
{True}
```

4. y' sin 2πx = πy cos 2πx

```
Clear["Global`*"]
```

```
eqn = y'[x] Sin[2 π x] == π y[x] Cos[2 π x];
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, C[1] \sqrt{Sin[2 \pi x]}\right]\right\}\right\}$$

```
eqn /. sol
```

```
{True}
```

5. y y' + 36 x = 0

```
Clear["Global`*"]
```

```
eqn = y[x] y'[x] + 36 x == 0;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -\sqrt{2} \sqrt{-18 x^2 + C[1]}\right]\right\}, \right.$$
$$\left.\left\{y \rightarrow \text{Function}\left[\{x\}, \sqrt{2} \sqrt{-18 x^2 + C[1]}\right]\right\}\right\}$$

```
eqn /. sol[[1]]
```
```
True
```

```
eqn /. sol[[2]]
```
```
True
```

6. $y' = e^{2x-1} y^2$

```
Clear["Global`*"]
```

```
eqn = y'[x] == e^{2 x-1} y[x]^2;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -\frac{2 e}{e^{2 x} + 2 e C[1]}\right]\right\}\right\}$$

```
eqn /. sol
```

```
{True}
```

7. $xy' = y + 2 x^3 \sin^2 \frac{y}{x}$ (Set y/x = u)

```
Clear["Global`*"]
```

```
eqn = x y'[x] == y[x] + 2 x^3 Sin[ y[x]/x ]^2;
sol = DSolve[eqn, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -x \text{ArcCot}\left[x^2 - 2 C[1]\right]\right]\right\}\right\}$$

`Simplify[eqn /. sol]`

`{True}`

8. $y' = (y + 4x)^2$ (Set $y + 4x = v$)

`Clear["Global`*"]`

`eqn = y'[x] == (y[x] + 4 x)²;`
`sol = DSolve[eqn, y, x]`

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -2\,\dot{\mathbb{i}} - 4\,x + \frac{1}{-\frac{\dot{\mathbb{i}}}{4} + \mathbb{e}^{4\,\dot{\mathbb{i}}\,x}\,C[1]}\right]\right\}\right\}$$

`Simplify[eqn /. sol]`

`{True}`

9. $xy' = y^2 + y$ (Set $y/x = u$)

`Clear["Global`*"]`

`eqn = x y'[x] == y[x]² + y[x];`
`sol = DSolve[eqn, y, x]`

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -\frac{\mathbb{e}^{C[1]}\,x}{-1 + \mathbb{e}^{C[1]}\,x}\right]\right\}\right\}$$

`Simplify[eqn /. sol]`

`{True}`

10. $xy' = x + y$ (Set $y/x = u$)

`Clear["Global`*"]`

`eqn = xy'[x] == x + y[x];`
`sol = DSolve[eqn, y, x]`

`{{y → Function[{x}, -x + xy′[x]]}}`

`eqn /. sol`

`{True}`

11-17 Initial Value Problems (IVPs). Solve the IVP. Show the steps of derivation, beginning with the general solution.

11. $xy' + y = 0$, $y(4) = 6$

`Clear["Global`*"]`

```
eqn = x y'[x] + y[x] == 0;
sol = DSolve[{eqn, y[4] == 6}, y, x]
```

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, \frac{24}{x}\right]\right\}\right\}$$

```
eqn /. sol
```

```
{True}
```

12. $y' = 1 + 4y^2, \ y(1) = 0$

```
Clear["Global`*"]
```

```
eqn = y'[x] == 1 + 4 y[x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, \frac{1}{2} \text{Tan}[2 \ (-1 + x)]\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

13. $y' \cosh^2 x = \sin^2 y, \ y(0) = \frac{1}{2}\pi$

```
Clear["Global`*"]
```

```
eqn = y'[x] Cosh[x]^2 == Sin[y[x]]^2;
sol = DSolve[{eqn, y[0] == 1/2 π}, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{y → Function[{x}, -ArcCot[Tanh[x]]]}}
```

```
Simplify[eqn /. sol]
```

```
{True}
```

14. $dr/dt = -2 \, tr, \ r(0) = r_0$

```
Clear["Global`*"]
```

```
eqn = r'[t] == -2 t r[t];
sol = DSolve[{eqn, r[0] == r0}, r[t], t]
```

$$\left\{\left\{r[t] \rightarrow e^{-t^2} r0\right\}\right\}$$

```
Simplify[eqn] /. Simplify[sol]
```

$$\left\{r'[t] == -2 e^{-t^2} r0 t\right\}$$

While Mathematica will not declare this equality to be true, it works when the substitutions are made.

15. $y' = -4 xy$, $y(2) = 3$

```
Clear["Global`*"]
```

```
eqn = y'[x] == -4 x y[x];
sol = DSolve[{eqn, y[2] == 3}, y, x]
```

$\left\{\left\{y \to \text{Function}\left[\{x\}, 3\, e^{8-2\,x^2}\right]\right\}\right\}$

```
eqn /. sol
```

```
{True}
```

16. $y' = (x + y - 2)^2$, $y(0) = 2$, (Set v = x + y - 2)

```
Clear["Global`*"]
```

```
eqn = y'[x] == (x + y[x] - 2)^2;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```

$\left\{\left\{y \to \text{Function}\left[\{x\}, -\dfrac{(-2 - \mathrm{i}) - (2 - \mathrm{i})\, e^{2\,\mathrm{i}\,x} + x + e^{2\,\mathrm{i}\,x}\, x}{1 + e^{2\,\mathrm{i}\,x}}\right]\right\}\right\}$

```
Simplify[eqn /. sol]
```

```
{True}
```

17. $xy' = y + 3 x^4 \cos^2\left(\frac{y}{x}\right)$, $y(1) = 0$, Set $\frac{y}{x} = u$

```
Clear["Global`*"]
```

```
eqn = x y'[x] == y[x] + 3 x^4 Cos[ y[x]/x ]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve:ifun:
  Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$\left\{\left\{y \to \text{Function}\left[\{x\}, -x\, \text{ArcTan}\left[1 - x^3\right]\right]\right\}\right\}$

```
Simplify[eqn /. sol]
```

```
{True}
```