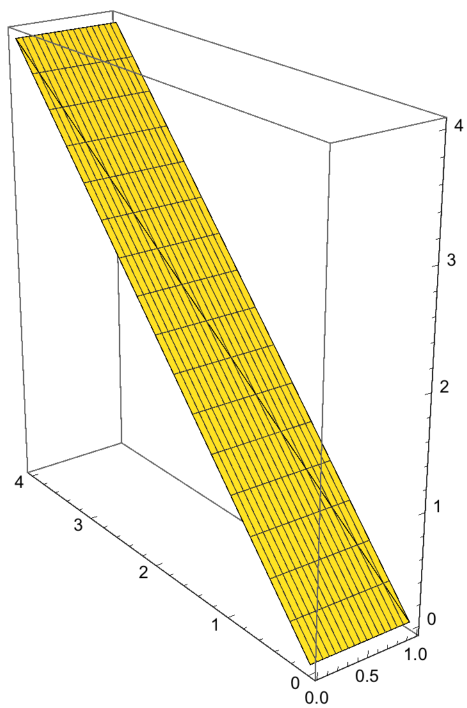
The s.m. has problems 3, 13.

1 - 10 Direct integration of surface integrals

Evaluate the surface integral $\int_S\int$ (curl F).n dA directly for the given F and S.

1. F = $\{z^2, -x^2, 0\}$, S the rectangle with vertices {0, 0, 0}, {1, 0, 0}, {0, 4, 4}, {1, 4, 4}

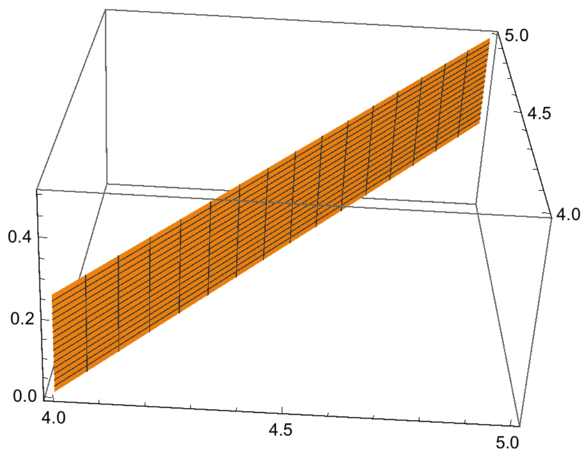
```
Clear["Global`*"]

r1 =
  ListPlot3D[{{0, 0, 0}, {1, 0, 0}, {0, 4, 4}}, AspectRatio → Automatic];
r2 = ListPlot3D[{{1, 0, 0}, {0, 4, 4}, {1, 4, 4}}, AspectRatio → Automatic];

Show[r1, r2, BoxRatios → Automatic]
```



For some reason I can't get the plots in Mathematica to come out right unless I do them separately, as above.

{0,4,4} +u{1,0,0}+v{1,4,4} implies x=u+v, y=4+4v, z=4+4v

```
ParametricPlot3D[{u + v, 4 + 4 v, 4 + 4 v},
 {u, 0, .25}, {v, 0, .25}, BoxRatios → Automatic]
```



Two vectors in the plane are {1,-4,-4} and {0,-4,-4}

```
planenormal = Cross[{1, -4, -4}, {0, -4, -4}]
{0, 4, -4}

{x, y, z} - {1, 0, 0}
{-1 + x, y, z}

planeeq = {0, 4, -4}.{-1 + x, y, z}
4 y - 4 z
```

So the equation of the plane is 4y -4z = 0. I'm going to suppose the following is what I want:

```
surf = {x, 4 y, -4 z}
{x, 4 y, -4 z}
```

$F = \{z^2, -x^2, 0\}$

$\{z^2, -x^2, 0\}$

```
dogm = Curl[F, {x, y, z}]
{0, 2 z, -2 x}

fir = D[surf, {x}]
{1, 0, 0}
```

```
sec = D[surf, {y}]
{0, 4, 0}
```

```
thi = D[planeeq, {z}]
0
```

```
crs = Cross[fir, sec]
{0, 0, 4}
```

```
arg = dogm.crs
-8 x
```

$$\text{inte} = \int_0^1 \int_0^{4x} \int_0^y (-8x)\, dz\, dy\, dx$$

-16

```
{0, 2 z, -2 z}.{0, -1, 1}
```

```
-4 z
```

$$\text{inte1} = \int_0^1 \int_0^4 \int_0^y (-4z)\, dz\, dy\, dx$$

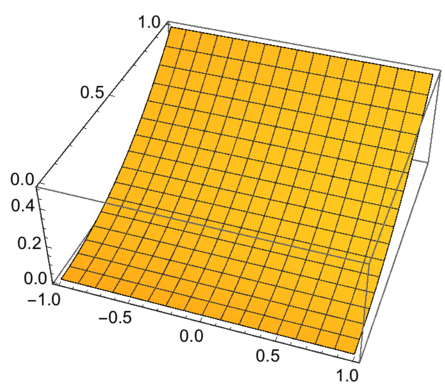
$$-\frac{128}{3}$$

Not surprising that it did not come out right.

$$3.\ F = \{e^{-z},\ e^{-z} \text{Cos}[y],\ e^{-z} \text{Sin}[y]\},\quad S: z = \frac{y^2}{2},\quad -1 \le x \le 1,\quad 0 \le y \le 1$$

```
Clear["Global`*"]
```

$$\text{surf} = \text{Plot3D}\left[\frac{y^2}{2}, \{x, -1, 1\}, \{y, 0, 1\}\right]$$



The solution manual calls attention to the surface as a parabolic cylinder.

fF = {e^{-z}, e^{-z} Cos[y], e^{-z} Sin[y]}

{e^{-z}, e^{-z} Cos[y], e^{-z} Sin[y]}

Finding the Curl is easy, unless something goes wrong. I had some doubts, as this didn't work smoothly the first time.

curly = Curl[{e^{-z}, e^{-z} Cos[y], e^{-z} Sin[y]}, {x, y, z}]

{e^{-z} Cos[y] + e^{-z} Cos[y], -e^{-z}, 0}

Below, eliminating the z expression.

curlyz = curly /. z → $\frac{y^2}{2}$

{$e^{-\frac{y^2}{2}}$ Cos[y] + $e^{-\frac{y^2}{2}}$ Cos[y], -$e^{-\frac{y^2}{2}}$, 0}

Below, *Mathematica* was hesitant to combine the two instances of $e^{-\frac{y^2}{2}}$ Cos[y].

curlyzz = curlyz /. $e^{-\frac{y^2}{2}}$ Cos[y] + $e^{-\frac{y^2}{2}}$ Cos[y] → 2 $e^{-\frac{y^2}{2}}$ Cos[y]

{2 $e^{-\frac{y^2}{2}}$ Cos[y], -$e^{-\frac{y^2}{2}}$, 0}

Below, writing the surface equation as the solution manual recommended.

surf = {x, y, $\frac{y^2}{2}$}

{x, y, $\frac{y^2}{2}$}

Below, finding the partials in preparation for crossing.

surfofx = D[surf, {x}]

{1, 0, 0}

surfofy = D[surf, {y}]

{0, 1, y}

Below, crossing gives the normal vector needed.

norm = Cross[surfofx, surfofy]

{0, -y, 1}

Below, the dot product will be the core of the integrand.

integr = curlyzz.norm

$e^{-\frac{y^2}{2}}$ y

Below, the limits are given explicitly.

$$\int_0^1 \int_{-1}^1 \left(e^{-\frac{y^2}{2}} y\right) dx\, dy$$

$$2 - \frac{2}{\sqrt{e}}$$

The above line matches the text's answer, except that the text has +/- on its answer.

5. $F = \left\{z^2, \frac{3}{2} x, 0\right\}$, S : $0 \le x \le a$, $0 \le y \le a$, $z = 1$

```
Clear["Global`*"]
```

7. $F = \{e^y, e^z, e^x\}$, S : $z = x^2$ $(0 \le x \le 2,\ 0 \le y \le 1)$

11. Stoke's theorem not applicable. Evaluate
$\oint F.r'\, ds$, $F = \left(x^2 + y^2\right)^{-1} \{-y, x\}$, C : $x^2 + y^2 = 1$, $z = 0$, oriented clockwise.
Why can Stoke's theorem not be applied? What (false) result would it give?

13 - 20 Evaluation of $\oint_C F.r'\, ds$

Calculate this line integral by Stoke's theorem for the given F and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative.

13. $F = \{-5 y, 4 x, z\}$, C the circle $x^2 + y^2 = 16$, $z = 4$

15. $F = \left\{y^2, x^2, z + x\right\}$ around the triangle with vertices $\{0, 0, 0\}$, $\{1, 0, 0\}$, $\{1, 1, 0\}$

17. $F = \left\{0, z^3, 0\right\}$, C the boundary curve of the cylinder $x^2 + y^2 = 1$, $x \ge 0$, $y \ge 0$, $0 \le z \le 1$

19. $F = \{z, e^z, 0\}$,
C the boundary curve of the portion of the cone $z = \sqrt{x^2 + y^2}$,
$x \ge 0$, $y \ge 0$, $0 \le z \le 1$