

Example 2, p 83.

```
ClearAll["Global`*"]
```

```
hig = {y''[x] + 3 y'[x] + 2.25 y[x] == -10 e^{-1.5 x}, y[0] == 1, y'[0] == 0}
```

```
velm = DSolve[hig, y, x]
```

```
{2.25 y[x] + 3 y'[x] + y''[x] == -10 e^{-1.5 x}, y[0] == 1, y'[0] == 0}
```

```
{ {y -> Function[{x}, -5. e^{-1.5 x} (-0.2 - 0.3 x + 1. x^2)] } }
```

```
Expand[-5. e^{-1.5 x} (-0.2 - 0.3 x + 1. x^2)]
```

```
1. e^{-1.5 x} + 1.5 e^{-1.5 x} x - 5. e^{-1.5 x} x^2
```

```
Simplify[%]
```

```
e^{-1.5 x} (1. + 1.5 x - 5. x^2)
```

The answer to example 2 in the text is duplicated.

1 - 10 Nonhomogeneous linear ODEs: General solution

Find a (real) general solution. State which rule you are using.

1. $y'' + 5y' + 4y = 10e^{-3x}$

```
ClearAll["Global`*"]
```

```
xen = y''[x] + 5 y'[x] + 4 y[x] == 10 e^{-3 x}
```

```
jud = DSolve[xen, y, x]
```

```
4 y[x] + 5 y'[x] + y''[x] == 10 e^{-3 x}
```

```
{ {y -> Function[{x}, -5 e^{-3 x} + e^{-4 x} C[1] + e^{-x} C[2]] } }
```

1. The text answer is found.

3. $y'' + 3y' + 2y = 12x^2$

```
ClearAll["Global`*"]
```

```
oen = y''[x] + 3 y'[x] + 2 y[x] == 12 x^2
```

```
gas = DSolve[oen, y, x]
```

```
2 y[x] + 3 y'[x] + y''[x] == 12 x^2
```

```
{ {y -> Function[{x}, 3 (7 - 6 x + 2 x^2) + e^{-2 x} C[1] + e^{-x} C[2]] } }
```

```
Expand[3 (7 - 6 x + 2 x^2)]
```

```
21 - 18 x + 6 x^2
```

$$\text{gas} /. \left(3 \left(7 - 6 x + 2 x^2 \right) \right) \rightarrow 21 - 18 x + 6 x^2$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, \left(21 - 18 x + 6 x^2 \right) + e^{-2 x} C[1] + e^{-x} C[2] \right] \right\} \right\}$$

1. The text answer is found.

$$5. \quad y'' + 4 y' + 4 y = e^{-x} \cos[x]$$

```
ClearAll["Global`*"]
```

$$\text{up} = y''[x] + 4 y'[x] + 4 y[x] == e^{-x} \cos[x]$$

```
nap = DSolve[up, y, x]
```

$$4 y[x] + 4 y'[x] + y''[x] == e^{-x} \cos[x]$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, e^{-2 x} C[1] + e^{-2 x} x C[2] + \frac{1}{2} e^{-x} \sin[x] \right] \right\} \right\}$$

1. The text answer is found.

$$7. \quad \left(D^2 + 2 D + \frac{3}{4} I \right) y = 3 e^x + \frac{9}{2} x$$

```
ClearAll["Global`*"]
```

$$\text{mop} = y''[x] + 2 y'[x] + \frac{3}{4} y[x] == 3 e^x + \frac{9}{2} x$$

```
lam = DSolve[mop, y, x]
```

$$\frac{3 y[x]}{4} + 2 y'[x] + y''[x] == 3 e^x + \frac{9 x}{2}$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, \frac{2}{5} (-40 + 2 e^x + 15 x) + e^{-3 x/2} C[1] + e^{-x/2} C[2] \right] \right\} \right\}$$

$$\text{Expand} \left[\frac{2}{5} (-40 + 2 e^x + 15 x) \right]$$

$$-16 + \frac{4 e^x}{5} + 6 x$$

$$\text{lam} /. \left(\frac{2}{5} (-40 + 2 e^x + 15 x) \right) \rightarrow -16 + \frac{4 e^x}{5} + 6 x$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, \left(-16 + \frac{4 e^x}{5} + 6 x \right) + e^{-3 x/2} C[1] + e^{-x/2} C[2] \right] \right\} \right\}$$

1. The text answer is found.

$$9. \quad (D^2 - 16 I) y = 9.6 e^{4 x} + 30 e^x$$

```
ClearAll["Global`*"]
```

```

track = y''[x] - 16 y[x] == 9.6 e4 x + 30 ex
nard = DSolve[track, y, x]
-16 y[x] + y''[x] == 30 ex + 9.6 e4 x

{{y -> Function[{x}, 1.2 e-7. x (-1.666667 e8. x - 0.125 e11. x + 1. e11. x x) +
  e4. x C[1] + e-4. x C[2]]}}

Expand[1.2 e-7. x (-1.6666666666666667 e8. x - 0.125 e11. x + 1. e11. x x)]
-2. e1. x - 0.15 e4. x + 1.2 e4. x x

```

1. Above: altered format of a section prior to hand replacement.

```

scis = nard /.
  (1.2 e-7. x (-1.6666666666666667 e8. x - 0.125 e11. x + 1. e11. x x)) ->
  -2. e1. x - 0.15 e4. x + 1.2 e4. x x

{{y ->
  Function[{x}, (-2. e1. x - 0.15 e4. x + 1.2 e4. x x) + e4. x C[1] + e-4. x C[2]]}}

```

2. Above: hand replacement of a section.

```

yit = -2. e1. x - 0.15 e4. x + 1.2 e4. x x + e4. x C[1] + e-4. x C[2]

-2. e1. x - 0.15 e4. x + 1.2 e4. x x + e4. x C[1] + e-4. x C[2]

```

3. Above: removed parentheses from a section by hand.

```

bag = yit /. -0.15 e4. x + e4. x C[1] -> +e4. x C[3]

-2. e1. x + 1.2 e4. x x + e-4. x C[2] + e4. x C[3]

```

4. Above: consolidated constants in a factor's coefficient by hand, resulting in the text answer.

11 - 18 Nonhomogeneous linear ODEs: IVPs

Solve the initial value problem. State which rule you are using. Show each step of your calculation in detail.

11. $y'' + 3y = 18x^2$, $y[0] = -3$, $y'[0] = 0$

```
ClearAll["Global`*"]
```

```
nom = {y''[x] + 3 y[x] == 18 x^2, y[0] == -3, y'[0] == 0}
kla = DSolve[nom, y, x]
{3 y[x] + y''[x] == 18 x^2, y[0] == -3, y'[0] == 0}
```

```
{ {y -> Function[{x}, -4 + 6 x^2 + Cos[Sqrt[3] x]] } }
```

1. The answer matches the text.

13. $8y'' - 6y' + y = 6 \cosh[x]$, $y[0] = 0.2$, $y'[0] = 0.05$

```
ClearAll["Global`*"]
uil = {8 y''[x] - 6 y'[x] + y[x] == 6 Cosh[x], y[0] == 0.2, y'[0] == 0.05}
qwx = DSolve[uil, y[x], x]
{y[x] - 6 y'[x] + 8 y''[x] == 6 Cosh[x], y[0] == 0.2, y'[0] == 0.05}
{{y[x] -> e^-x (0.2 + 1. e^(5 x/4) - 2. e^(3 x/2) + e^2 x)}}
Expand[qwx]
```

```
{ {y[x] -> 0.2 e^-x + 1. e^(x/4) - 2. e^(x/2) + e^x} }
```

1. The answer matches the text.

15. $(x^2 D^2 - 3x D + 3I)y = 3 \log[x] - 4$, $y[1] = 0$, $y'(1) = 1$;
 $y_p = \log[x]$

```
ClearAll["Global`*"]
mil = {x^2 y''[x] - 3 x y'[x] + 3 y[x] == 3 Log[x] - 4, y[1] == 0, y'[1] == 1}
jyt = DSolve[mil, y[x], x]
{3 y[x] - 3 x y'[x] + x^2 y''[x] == -4 + 3 Log[x], y[1] == 0, y'[1] == 1}
```

```
{ {y[x] -> Log[x]} }
```

1. The answer matches the text.

17. $(D^2 + 0.2 D + 0.26 I)y = 1.22 e^{0.5 x}$, $y[0] = 3.5$, $y'[0] = 0.35$

```
ClearAll["Global`*"]
```

```

hal = {y''[x] + 0.2 y'[x] + 0.26 y[x] == 1.22 e0.5 x, y[0] == 3.5, y'[0] == 0.35}
xxa = DSolve[hal, y[x], x]
{0.26 y[x] + 0.2 y'[x] + y''[x] == 1.22 e0.5 x, y[0] == 3.5, y'[0] == 0.35}
{{y[x] → 2. e-0.1 x (0.75 Cos[0.5 x] +
  1. e0.6 x Cos[0.5 x]2 - 0.5 Sin[0.5 x] + 1. e0.6 x Sin[0.5 x]2)}},
bur = xxa /. (1. e0.6 x Cos[0.5 x]2 + 1. e0.6 x Sin[0.5 x]2) → 1. e0.6 x
{{y[x] → 2. e-0.1 x (1. e0.6 x + 0.75 Cos[0.5 x] - 0.5 Sin[0.5 x])}}

```

1. Above: altered with hand-inserted trig ident $\sin^2 x + \cos^2 x = 1$.

Expand[bur]

```

{{y[x] → 2. e0.5 x + 1.5 e-0.1 x Cos[0.5 x] - 1. e-0.1 x Sin[0.5 x]}}

```

2. The above answer matches the text.

19. CAS project. Structure of solutions of Initial Value Problems. Using the present method, find, graph, and discuss the solutions y of initial value problems of your own choice. Explore effects on solutions caused by changes of initial conditions. Graph y_p , y , $y - y_p$ separately, to see the separate effects. Find a problem in which (a) the part of y resulting from y_h decreases to zero, (b) increases, (c) is not present in the answer y . Study a problem with $y(0) = 0$, $y'(0) = 0$. Consider a problem in which you need the Modification Rule (a) for a simple root, (b) for a double root. Make sure that your problems cover all three Cases I, II, III (see section 2.2).