11 - 14 Linear transformations
Find the inverse transformation.

11. $y_1 = 0.5\, x_1 - 0.5\, x_2$
$y_2 = 1.5\, x_1 - 2.5\, x_2$

```
Clear["Global`*"]
```

```
e1 = ( 0.5  -0.5 )
     ( 1.5  -2.5 )
```

```
{{0.5, -0.5}, {1.5, -2.5}}
```

```
e3 = {y1, y2}
```

```
{y1, y2}
```

```
e4 = {x1, x2}
```

```
{x1, x2}
```

```
e5 = Thread[Inverse[e1].e3 == e4]
```

```
{5. y1 - 1. y2 == x1, 3. y1 - 1. y2 == x2}
```

Above: The expressions match the text.

13. $y_1 = 5\, x_1 + 3\, x_2 - 3\, x_3$
$y_2 = 3\, x_1 + 2\, x_2 - 2\, x_3$
$y_3 = 2\, x_1 - x_2 + 2\, x_3$

```
Clear["Global`*"]
```

```
e1 = ( 5   3  -3 )
     ( 3   2  -2 )
     ( 2  -1   2 )
```

```
{{5, 3, -3}, {3, 2, -2}, {2, -1, 2}}
```

```
e2 = {y1, y2, y3}
```

```
{y1, y2, y3}
```

```
e3 = {x1, x2, x3}
```

```
{x1, x2, x3}
```

```
e4 = Thread[Inverse[e1].e2 == e3]
```

```
{2 y1 - 3 y2 == x1, -10 y1 + 16 y2 + y3 == x2, -7 y1 + 11 y2 + y3 == x3}
```

Above: The answer matches the text.

15. $\{\{3, 1, -4\}\}^\dagger$

```
Clear["Global`*"]
```

```
e1 = {3, 1, -4}
```

```
{3, 1, -4}
```

```
e2 = Norm[e1]
```

$\sqrt{26}$

Above: The answer matches the text. If I do the problem with literal interpretation of the given vector

```
Norm[{{3, 1, -4}}†]
```

$\sqrt{26}$

it still comes out right.

17. $\{\{1, 0, 0, 1, -1, 0, -1, 1\}\}^\dagger$

```
Clear["Global`*"]
```

```
e1 = {1, 0, 0, 1, -1, 0, -1, 1}
```

```
{1, 0, 0, 1, -1, 0, -1, 1}
```

```
e2 = Norm[e1]
```

$\sqrt{5}$

Above: The answer matches the text.

19. $\left\{\left\{\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, 0\right\}\right\}^\dagger$

```
Clear["Global`*"]
```

```
e1 = {2/3, 2/3, 1/3, 0}
```

$\left\{\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, 0\right\}$

```
e2 = Norm[e1]
```

1

Above: The answer matches the text.

21 - 25 Inner product. Orthogonality.

21. Orthogonality. For what value(s) of k are the vectors
$\{\{2, \frac{1}{2}, -4, 0\}\}^{\dagger}$ and $\{\{5, k, 0, \frac{1}{4}\}\}^{\dagger}$ orthogonal?

```
Clear["Global`*"]
```

```
e1 = {2, 1/2, -4, 0}
```

$$\left\{2, \frac{1}{2}, -4, 0\right\}$$

```
e2 = {5, k, 0, 1/4}
```

$$\left\{5, k, 0, \frac{1}{4}\right\}$$

```
e3 = Dot[e1, e2]
```

$$10 + \frac{k}{2}$$

```
e4 = Solve[e3 == 0, k]
```

```
{{k → -20}}
```

Above: The answer matches the text.