

Comment: *Mathematica's* Convolve function has an odd syntax. Note that in the orange cells below, the asterisk represents the convolve function.

1 - 7 Convolutions by integration

Find:

$$1. 1 * 1$$

```
Clear["Global`*"]
```

```
Integrate[1 * 1, {y, 0, t}]
```

$t$

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

$$3. e^t * e^{-t}$$

```
Clear["Global`*"]
```

```
Integrate[e^y e^{-(t-y)}, {y, 0, t}]
```

$\text{Sinh}[t]$

Below: Alternate method, this one using Convolve. But the syntax is strange.

```
Convolve[e^y UnitStep[y], e^{-y} UnitStep[y], y, t, Assumptions -> t > 0]
```

$\text{Sinh}[t]$

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

$$5. (\sin[\omega t]) * (\cos[\omega t])$$

```
Clear["Global`*"]
```

```
e1 = Integrate[Sin[ω y] Cos[ω (t - y)], {y, 0, t}]
```

$$\frac{1}{2} t \sin[t \omega]$$

With the alternate method,

```
Convolve[Sin[ω y] UnitStep[y],
Cos[ω y] UnitStep[y], y, t, Assumptions → t > 0]
```

$$\frac{1}{2} t \sin[t \omega]$$

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

$$7. t * e^t$$

```
Clear["Global`*"]
```

```
e1 = Integrate[y e^{t-y}, {y, 0, t}]
-1 + e^t - t
```

```
Convolve[y UnitStep[y], e^y UnitStep[y], y, t, Assumptions → t > 0]
```

$$-1 + e^t - t$$

The above answer matches the text. This time only the **Convolve** method was used.

## 8 - 14 Integral equations

Solve by the Laplace transform, showing the details:

$$9. y[t] - \text{Integrate}[y[\tau], \{\tau, 0, t\}] = 1$$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Integrate[y[τ], {τ, 0, t}], t, s]
```

$$\frac{\text{LaplaceTransform}[y[t], t, s]}{s}$$

```
e2 = LaplaceTransform[y[t], t, s]
```

```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[1, t, s]
```

$$\frac{1}{s}$$

```
e4 = e2 - e1 == e3
```

$$\text{LaplaceTransform}[y[t], t, s] - \frac{\text{LaplaceTransform}[y[t], t, s]}{s} == \frac{1}{s}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```

$$\text{bigY} - \frac{\text{bigY}}{s} == \frac{1}{s}$$

```
e6 = Solve[e5, bigY]
```

$$\left\{ \left\{ \text{bigY} \rightarrow \frac{1}{-1 + s} \right\} \right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{1}{-1 + s}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

$$e^t$$

Above: The answer matches the text answer.

$$11. y[t] + \text{Integrate}[(t - \tau) y[\tau], \{\tau, 0, t\}] = 1$$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Integrate[(t - \tau) y[\tau], {\tau, 0, t}], t, s]
```

$$\frac{\text{LaplaceTransform}[y[t], t, s]}{s^2}$$

```
e2 = LaplaceTransform[y[t], t, s]
```

```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[1, t, s]
```

$$\frac{1}{s}$$

```
e4 = e2 + e1 == e3
```

$$\text{LaplaceTransform}[y[t], t, s] + \frac{\text{LaplaceTransform}[y[t], t, s]}{s^2} == \frac{1}{s}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] -> bigY
```

$$\text{bigY} + \frac{\text{bigY}}{s^2} == \frac{1}{s}$$

```
e6 = Solve[e5, bigY]
```

$$\left\{ \left\{ \text{bigY} \rightarrow \frac{s}{1 + s^2} \right\} \right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{s}{1 + s^2}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

```
Cos[t]
```

Above: The answer matches the text answer.

13.  $y[t] + 2 e^t \text{Integrate}[y[\tau] e^{-\tau}, \{\tau, 0, t\}]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[2 e^t Integrate[y[\tau] e^{-\tau}, {\tau, 0, t}], t, s]
2 LaplaceTransform[y[t], t, s]
-----
-1 + s
```

```
e2 = LaplaceTransform[y[t], t, s]
```

```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[t e^t, t, s]
1
-----
(-1 + s)^2
```

```
e4 = e2 + e1 == e3
```

```
LaplaceTransform[y[t], t, s] +
1
----- 2 LaplaceTransform[y[t], t, s] == 1
-1 + s (-1 + s)^2
```

```
e5 = e4 /. LaplaceTransform[y[t], t, s] -> bigY
```

```
bigY + 2 bigY
----- == 1
-1 + s (-1 + s)^2
```

```
e6 = Solve[e5, bigY]
```

```
{{bigY -> 1
-----}}
(-1 + s) (1 + s)
```

```
e7 = e6[[1, 1, 2]]
```

```
1
-----
(-1 + s) (1 + s)
```

```
e8 = InverseLaplaceTransform[e7, s, t]
```

```
1
-- e^{-t} (-1 + e^{2 t})
2
```

```
e9 = ExpToTrig[e8]
```

```
1
-- (Cosh[t] - Sinh[t]) (-1 + Cosh[2 t] + Sinh[2 t])
2
```

```
e10 = FullSimplify[e9]
```

```
Sinh[t]
```

Above: This answer matches the text answer.

17 - 26 Inverse transforms by convolution

Showing details, find  $f[t]$  if  $\mathcal{L}[f]$  equals:

$$17. \quad \frac{5.5}{(s + 1.5)(s - 4)}$$

Note: use of the **Convolve** function did not seem necessary.

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ $\frac{5.5}{(s + 1.5)(s - 4)}$ , s, t]
```

```
5.5 (-0.181818 e-1.5 t + 0.181818 e4. t)
```

```
e2 = ExpandAll[e1]
```

```
-1. e-1.5 t + 1. e4. t
```

Above: This answer matches the text answer.

$$19. \quad \frac{2 \pi s}{(s^2 + \pi^2)^2}$$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ $\frac{2 \pi s}{(s^2 + \pi^2)^2}$ , s, t]
```

```
t Sin[ $\pi t$ ]
```

Above: This answer matches the text answer.

$$21. \quad \frac{\omega}{s^2 (s^2 + \omega^2)}$$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ $\frac{\omega}{s^2 (s^2 + \omega^2)}$ , s, t]
```

```
 $\frac{t \omega - \text{Sin}[t \omega]}{\omega^2}$ 
```

Above: This answer matches the text answer.

$$23. \quad \frac{40.5}{s (s^2 - 9)}$$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ $\frac{40.5}{s (s^2 - 9)}$ , s, t]
```

```
2.25 e-3 t (-1 + e3 t)2
```

```
e2 = ExpToTrig[e1]
```

```
2.25 (Cosh[3 t] - Sinh[3 t]) (-1 + Cosh[3 t] + Sinh[3 t])2
```

```
e3 = FullSimplify[e2]
```

```
-4.5 + 4.5 Cosh[3 t]
```

Above: This answer matches the text answer.

$$25. \quad \frac{18 s}{(s^2 + 36)^2}$$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ $\frac{18 s}{(s^2 + 36)^2}$ , s, t]
```

```
 $\frac{3}{2} t \sin[6 t]$ 
```

Above: This answer matches the text answer.