

7 - 15 Evaluation of determinants

Showing the details, evaluate:

$$7. \begin{vmatrix} \cos[\alpha] & \sin[\alpha] \\ \sin[\beta] & \cos[\beta] \end{vmatrix}$$

```
Clear["Global`*"]
```

```
e1 = Det[{{Cos[α] Sin[α]  
Sin[β] Cos[β]}]}
```

```
Cos[α] Cos[β] - Sin[α] Sin[β]
```

```
e2 = TrigReduce[e1]
```

```
Cos[α + β]
```

The above cell matches the answer in the text.

$$9. \begin{vmatrix} \cos[n\theta] & \sin[n\theta] \\ -\sin[n\theta] & \cos[n\theta] \end{vmatrix}$$

```
Clear["Global`*"]
```

```
e1 = Det[{{Cos[n θ] Sin[n θ]  
-Sin[n θ] Cos[n θ]}]}
```

```
Cos[n θ]^2 + Sin[n θ]^2
```

```
e2 = TrigReduce[e1]
```

```
1
```

The above cell matches the answer in the text.

$$11. \begin{vmatrix} 4 & -1 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{vmatrix}$$

```
Clear["Global`*"]
```

```
Det[{{4 -1 8  
0 2 3  
0 0 5}}]
```

```
40
```

The above cell matches the answer in the text.

$$13. \begin{vmatrix} 0 & 4 & -1 & 5 \\ -4 & 0 & 3 & -2 \\ 1 & -3 & 0 & 1 \\ -5 & 2 & -1 & 0 \end{vmatrix}$$

```
Clear["Global`*"]
```

$$e1 = \text{Det} \left[\begin{pmatrix} 0 & 4 & -1 & 5 \\ -4 & 0 & 3 & -2 \\ 1 & -3 & 0 & 1 \\ -5 & 2 & -1 & 0 \end{pmatrix} \right]$$

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The above cell matches the answer in the text.

$$15. \begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{vmatrix}$$

```
Clear["Global`*"]
```

$$e1 = \text{Det} \left[\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{pmatrix} \right]$$

-64

The above cell matches the answer in the text.

17 - 19 Rank by determinants

Find the rank by theorem 3, p. 297, (which is not very practical) and check by row reduction.

$$17. \begin{vmatrix} 4 & 9 \\ -8 & -6 \\ 16 & 12 \end{vmatrix}$$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 4 & 9 \\ -8 & -6 \\ 16 & 12 \end{pmatrix}$$

```
{{4, 9}, {-8, -6}, {16, 12}}
```

```
e1 = MatrixRank[e1]
```

```
2
```

The above cell matches the answer in the text.

$$19. \left| \begin{array}{cccc} 1 & 5 & 2 & 2 \\ 1 & 3 & 2 & 6 \\ 4 & 0 & 8 & 48 \end{array} \right|$$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 1 & 5 & 2 & 2 \\ 1 & 3 & 2 & 6 \\ 4 & 0 & 8 & 48 \end{pmatrix}$$

```
{{1, 5, 2, 2}, {1, 3, 2, 6}, {4, 0, 8, 48}}
```

```
e2 = MatrixRank[e1]
```

```
2
```

The above cell matches the answer in the text.

21 - 25 Cramer's rule

Solve by Cramer's rule. Check by Gauss elimination and back substitution.

$$21. \quad 3x - 5y = 15.5$$

$$6x + 16y = 5.0$$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 3 & -5 \\ 6 & 16 \end{pmatrix}$$

```
{{3, -5}, {6, 16}}
```

```
e2 = {15.5, 5.0}
```

```
{15.5, 5.}
```

```
e3 = {x, y}
```

```
{x, y}
```

```
e4 = Thread[e1.e3 == e2]
```

```
{3 x - 5 y == 15.5, 6 x + 16 y == 5.}
```

```
e5 = Solve[e4, e3]
```

```
{{x -> 3.5, y -> -1.}}
```

```
e6 = e4 /. e5
{{True, True}}
```

The above cell matches the answer in the text.

$$\begin{aligned} 23. \quad & 3y - 4z = 16 \\ & 2x - 5y + 7z = -27 \\ & -x - 9z = 9 \end{aligned}$$

```
Clear["Global`*"]

e1 =  $\begin{pmatrix} 0 & 3 & -4 \\ 2 & -5 & 7 \\ -1 & 0 & -9 \end{pmatrix}$ 
{{0, 3, -4}, {2, -5, 7}, {-1, 0, -9}}

e2 = {16, -27, 9}
{16, -27, 9}

e3 = {x, y, z}
{x, y, z}

e4 = Thread[e1.e3 == e2]
{3 y - 4 z == 16, 2 x - 5 y + 7 z == -27, -x - 9 z == 9}

e5 = Solve[e4, e3]
```

```
{{x -> 0, y -> 4, z -> -1}}
```

```
e6 = e4 /. e5
{{True, True, True, True}}
```

The above cell matches the answer in the text.

$$\begin{aligned} 25. \quad & -4w + x + y = -10 \\ & w - 4x + z = 1 \\ & w - 4y + z = -7 \\ & x + y - 4z = 10 \end{aligned}$$

```
Clear["Global`*"]

e1 =  $\begin{pmatrix} -4 & 1 & 1 & 0 \\ 1 & -4 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -4 \end{pmatrix}$ 
{{-4, 1, 1, 0}, {1, -4, 0, 1}, {1, 0, -4, 1}, {0, 1, 1, -4}}
```

```

e2 = {-10, 1, -7, 10}
{-10, 1, -7, 10}

e3 = {w, x, y, z}
{w, x, y, z}

e4 = Thread[e1.e3 == e2]
{-4 w + x + y == -10, w - 4 x + z == 1, w - 4 y + z == -7, x + y - 4 z == 10}

e5 = Solve[e4, e3]
{{w -> 3, x -> 0, y -> 2, z -> -2}}

e6 = e4 /. e5
{{True, True, True, True}}

```

The above cell matches the answer in the text.