```
Clear["Global`*"]
```

2-10 General Solution. Find a general solution. Show the steps of derivation. Check your answer by substitution.

2. $y^3 y' + x^3 = 0$

```
eqn = y[x]^3 + x^3 == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -x]\},\ \{y \to \text{Function}[\{x\}, (-1)^{1/3}\, x]\},\ \{y \to \text{Function}[\{x\}, -(-1)^{2/3}\, x]\}\}$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True

eqn /. sol[[3]]
True
```

3. $y' = \sec^2 y$

```
Clear["Global`*"]
```

$\text{eqn} = y'[x] == \text{Sec}[y[x]]^2;$
$\text{eqn2} = y'[x]\ \text{Cos}[y[x]]^2 == 1;$
$$\text{eqn3} = y'[x] == \frac{2}{\text{Cos}[2\, y[x]] + 1};$$
$$\text{eqn4} = y'[x] == \frac{4}{\left(e^{-i\, y[x]} + e^{i\, y[x]}\right)^2};$$

```
sol = DSolve[eqn3, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \text{InverseFunction}\left[2\left(\frac{1}{4}\text{Sin}[2\,\#1] + \frac{\#1}{2}\right)\&\right][2\,x + C[1]]\right]\right\}\right\}$$

The WolframAlpha solution is shown as

$$c_1 + 2\,x = 2\left(\frac{y[x]}{2} + \frac{1}{4}\text{Sin}[2\,y[x]]\right)$$

```
Simplify[eqn /. sol]
{True}
```

4. $y' \sin 2\pi x = \pi y \cos 2\pi x$

```
Clear["Global`*"]
```

```
eqn = y'[x] Sin[2 π x] == π y[x] Cos[2 π x];
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, C[1] \sqrt{\text{Sin}[2 \pi x]}]\}\}$

```
eqn /. sol
```

{True}

5. y y' + 36 x = 0

```
Clear["Global`*"]
```

```
eqn = y[x] y'[x] + 36 x == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -\sqrt{2} \sqrt{-18 x^2 + C[1]}]\},$
$\{y \to \text{Function}[\{x\}, \sqrt{2} \sqrt{-18 x^2 + C[1]}]\}\}$

```
eqn /. sol[[1]]
```

True

```
eqn /. sol[[2]]
```

True

6. y' = $e^{2x-1} y^2$

```
Clear["Global`*"]
```

$\text{eqn} = y'[x] == e^{2x-1} y[x]^2;$
```
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -\frac{2 e}{e^{2x} + 2 e C[1]}]\}\}$

```
eqn /. sol
```

{True}

7. xy' = y + 2 $x^3 \sin^2 \frac{y}{x}$ (Set y/x = u)

```
Clear["Global`*"]
```

$\text{eqn} = x\, y'[x] == y[x] + 2 x^3 \text{Sin}[\frac{y[x]}{x}]^2;$
```
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, -x\, \text{ArcCot}[x^2 - 2 C[1]]]\}\}$

```
Simplify[eqn /. sol]
```

{True}

8. y' = $(y + 4x)^2$ (Set $y + 4x = v$)

```
Clear["Global`*"]

eqn = y'[x] == (y[x] + 4 x)^2;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -2\, i - 4\, x + \frac{1}{-\frac{i}{4} + e^{4\, i\, x}\, C[1]}\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

{True}

9. xy' = $y^2 + y$ (Set $y/x = u$)

```
Clear["Global`*"]

eqn = x y'[x] == y[x]^2 + y[x];
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{e^{C[1]}\, x}{-1 + e^{C[1]}\, x}\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

{True}

10. xy' = x + y (Set y/x = u)

```
Clear["Global`*"]

eqn = xy'[x] == x + y[x];
sol = DSolve[eqn, y, x]
```

{{y → Function[{x}, -x + xy′[x]]}}

```
eqn /. sol
```

{True}

11-17 Initial Value Problems (IVPs). Solve the IVP. Show the steps of derivation, beginning with the general solution.

11. xy' + y = 0, y(4) = 6

```
Clear["Global`*"]
```

```
eqn = x y'[x] + y[x] == 0;
sol = DSolve[{eqn, y[4] == 6}, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \frac{24}{x}]\}\}$

```
eqn /. sol
```

{True}

12. $y' = 1 + 4y^2, \;\; y(1) = 0$

```
Clear["Global`*"]
```

```
eqn = y'[x] == 1 + 4 y[x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, \frac{1}{2}\text{Tan}[2\,(-1 + x)]]\}\}$

```
Simplify[eqn /. sol]
```

{True}

13. $y' \cosh^2 x = \sin^2 y, \;\; y(0) = \frac{1}{2}\pi$

```
Clear["Global`*"]
```

```
eqn = y'[x] Cosh[x]^2 == Sin[y[x]]^2;
```

$\text{sol} = \text{DSolve}[\{\text{eqn}, y[0] == \frac{1}{2}\pi\}, y, x]$

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{{y → Function[{x}, -ArcCot[Tanh[x]]]}}

```
Simplify[eqn /. sol]
```

{True}

14. $dr/dt = -2\,tr, \;\; r(0) = r_0$

```
Clear["Global`*"]
```

```
eqn = r'[t] == -2 t r[t];
sol = DSolve[{eqn, r[0] == r0}, r[t], t]
```

$\{\{r[t] \to e^{-t^2}\,r0\}\}$

```
Simplify[eqn] /. Simplify[sol]
```

$\{r'[t] == -2\,e^{-t^2}\,r0\;t\}$

While Mathematica will not declare this equality to be true, it works when the substitutions are made.

15. $y' = -4\,xy,\ \ y(2) = 3$

```
Clear["Global`*"]
```

```
eqn = y'[x] == -4 x y[x];
sol = DSolve[{eqn, y[2] == 3}, y, x]
```

$\{\{y \to \text{Function}[\{x\},\ 3\,e^{8-2x^2}]\}\}$

```
eqn /. sol
```

{True}

16. $y' = (x + y - 2)^2,\ y(0) = 2$, (Set v = x + y - 2)

```
Clear["Global`*"]
```

```
eqn = y'[x] == (x + y[x] - 2)^2;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```

$\{\{y \to \text{Function}[\{x\},\ -\frac{(-2-i) - (2-i)\,e^{2ix} + x + e^{2ix}\,x}{1 + e^{2ix}}]\}\}$

```
Simplify[eqn /. sol]
```

{True}

17. $xy' = y + 3x^4\cos^2\left(\frac{y}{x}\right),\ \ y(1) = 0$, Set $\frac{y}{x} = u$

```
Clear["Global`*"]
```

```
eqn = x y'[x] == y[x] + 3 x^4 Cos[y[x]/x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\},\ -x\,\text{ArcTan}[1 - x^3]]\}\}$

```
Simplify[eqn /. sol]
```

{True}