1 - 5 Application of differential operators

Apply the given operator to the given functions. Show all steps in detail.

1. $D^2 + 2\,D$; Cosh[2 x], $e^{-x} + e^{2x}$, Cos[x]

```
Clear["Global`*"]
```

```
r[x_] := Cosh[2 x]
```

```
r''[x] + 2 r'[x]
```

```
4 Cosh[2 x] + 4 Sinh[2 x]
```

```
FullSimplify[%]
```

```
4 e^(2 x)
```

```
s[x_] := e^-x + e^(2 x)
```

```
s''[x] + 2 s'[x]
```

```
e^-x + 4 e^(2 x) + 2 (-e^-x + 2 e^(2 x))
```

```
Simplify[%]
```

```
e^-x (-1 + 8 e^(3 x)) (* text answer is -e^-x+8e^2x *)
```

```
u[x_] := Cos[x]
```

```
u''[x] + 2 u'[x]
```

```
-Cos[x] - 2 Sin[x]
```

The above answers agree with the text (except yellow).

3. $(D - 2\,I)^2$; e^{2x}, x e^{-2x}

```
Clear["Global`*"]
```

```
v[x_] := e^(2 x)
```

```
v''[x] - 4 v'[x] + 4 v[x]
```

```
0
```

```
w[x_] := x e^(2 x)
```

```
w''[x] - 4 w'[x] + 4 w[x]
```

```
4 e^(2 x) + 8 e^(2 x) x - 4 (e^(2 x) + 2 e^(2 x) x)
```

```
Simplify[%]
```

```
0
```

```
z[x_] := e^-2 x
```

```
z''[x] - 4 z'[x] + 4 z[x]
```

```
16 e^-2 x
```

The above answers agree with the text.

5. (D - 2 I)(D + 3 I); e^{2x}, x e^{2x}, e^{-3x}

```
Clear["Global`*"]
```

```
r[x_] := e^2 x
```

```
r''[x] + r'[x] - 6 r[x]
```

```
0
```

```
s[x_] := x e^2 x
```

```
s''[x] + s'[x] - 6 s[x]
```

```
5 e^2 x
```

```
u[x_] := e^-3 x
```

```
u''[x] + u'[x] - 6 u[x]
```

```
0
```

The above answers agree with the text.

6 - 12 General solution

Facto as in the text and solve.

7. $(4\,D^2 - I)y = 0$

```
Clear["Global`*"]
```

```
r = 4 y''[x] - y[x] == 0
```

```
-y[x] + 4 y''[x] == 0
```

```
DSolve[r, y, x]
```

```
{{y → Function[{x}, e^x/2 C[1] + e^-x/2 C[2]]}}
```

The above answer matches the text's.

9. $(D^2 - 4.20\ D + 4.41\ I)y = 0$

```
Clear["Global`*"]
```

```
s = y''[x] - 4.20 y'[x] + 4.41 y[x] == 0
DSolve[s, y, x]
```

$$4.41\ y[x] - 4.2\ y'[x] + y''[x] == 0$$

$$\{\{y \to \text{Function}[\{x\},\ e^{2.1\ x}\ C[1] + e^{2.1\ x}\ x\ C[2]]\}\}$$

The above answer matches the text's.

11. $(D^2 - 4.00\ D + 3.84\ I)y = 0$

```
Clear["Global`*"]
```

```
u = y''[x] - 4.00 y'[x] + 3.84 y[x] == 0
DSolve[u, y, x]
```

$$3.84\ y[x] - 4.\ y'[x] + y''[x] == 0$$

$$\{\{y \to \text{Function}[\{x\},\ e^{1.6\ x}\ C[1] + e^{2.4\ x}\ C[2]]\}\}$$

The above answer matches the text's.