Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1 - 10 Adams-Moulton method

Solve the initial value problem by Adams-Moulton (7a), (7b), 10 steps with 1 correction per step. Solve exactly and compute the error. Use RK where no starting values are given.

1. y'[x] == y, y[0] == 1, h = 0.1, (1.105171, 1.221403, 1.349858)

Using the same settings for NDSolve here as were used in section 21.1, problem 15. It is not Adams-Moulton, but the automatically adaptive calculation strategy which NDSolve performs by default. Green cells indicate that all equations solutions were retrieved as equal to the text answer.

```
Clear["Global`*"]

s1 = DSolve[{y'[x] == y[x], y[0] == 1}, y[x], x]
```

$\{\{y[x] \to e^x\}\}$

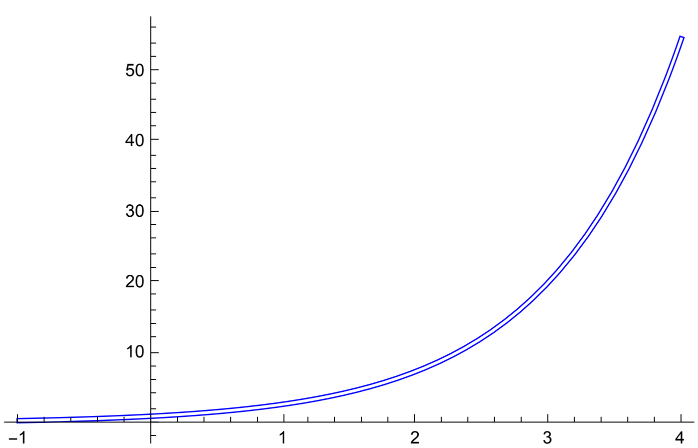
```
p1 = Plot[y[x] /. s1, {x, -1, 4}, PlotStyle → {Blue, Thickness[0.008]}];

s2 = NDSolve[{y'[x] == y[x], y[0] == 1}, y, {x, -1, 4},
  AccuracyGoal → ∞, PrecisionGoal → 20, WorkingPrecision → 35]
```

```
{{y → InterpolatingFunction[ Domain: {{-1.00000000000000000000000000000000, 4.00000000000000000000000000000000}}
                              Output: scalar ]}}
```

```
p2 = Plot[y[x] /. s2, {x, -1, 4}, PlotStyle → {White, Thickness[0.004]}];

Show[p1, p2]
```



The agreement between the two functions seems to be at least 9S. The enhancement options make a difference. For example, when Precision Goal was 10 and Working Precision was 15, then only 7S was achieved.

```
TableForm[
 Table[NumberForm[{y[x] /. s1, y[x] /. s2}, {8, 8}], {x, -1, 4, 0.4}]]
```

```
{{0.36787944}, {0.36787944}}
{{0.54881164}, {0.54881164}}
{{0.81873075}, {0.81873075}}
{{1.22140280}, {1.22140280}}
{{1.82211880}, {1.82211880}}
{{2.71828180}, {2.71828180}}
{{4.05520000}, {4.05520000}}
{{6.04964750}, {6.04964750}}
{{9.02501350}, {9.02501350}}
{{13.46373800}, {13.46373800}}
{{20.08553700}, {20.08553700}}
{{29.96410000}, {29.96410000}}
{{44.70118400}, {44.70118400}}
```

3. y'[x] == 1 + y[x]^2, y[0] == 0, h = 0.1,
(0.100335, 0.202710, 0.309336)

```
Clear["Global`*"]
```

```
s1 = DSolve[{y'[x] == 1 + y[x]^2, y[0] == 0}, y[x], x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

```
{{y[x] → Tan[x]}}
```

```
p1 = Plot[y[x] /. s1, {x, -1.4, 1.4}, PlotStyle → {Red, Thickness[0.008]}];
```

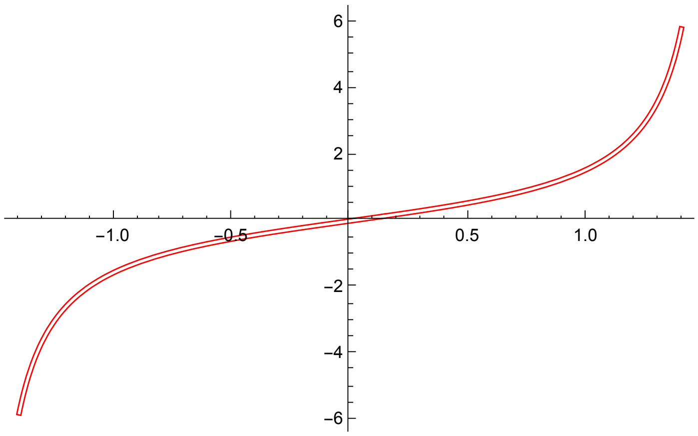
There are developments here with s2. In this case the AccuracyGoal cannot be ∞, because then Mathematica finds a $\frac{1}{0}$ condition. PrecisionGoal and WorkingPrecision cannot be sky high without error messages, but as they are set below, they are plenty high enough.

```
s2 = NDSolve[{y'[x] == 1 + y[x]^2, y[0] == 0}, y, {x, -1.4, 1.4},
  AccuracyGoal → 16, PrecisionGoal → 16, WorkingPrecision → 20]
```

```
{{y → InterpolatingFunction[ Domain: {{-1.39999999999999991, 1.39999999999999991}} 12
                             Output: scalar ]}}
```

```
p2 =
  Plot[y[x] /. s2, {x, -1.4, 1.4}, PlotStyle → {White, Thickness[0.004]}];
```

```
Show[p1, p2]
```



Agreement in the tables between the two solving methods seems to be at least S9.

```
TableForm[
 Table[NumberForm[{y[x] /. s1, y[x] /. s2}, {8, 8}], {x, -1.4, 1.4, 0.3}]]
{{-5.79788370}, {-5.79788370}}
{{-1.96475970}, {-1.96475970}}
{{-1.02963860}, {-1.02963860}}
{{-0.54630249}, {-0.54630249}}
{{-0.20271004}, {-0.20271004}}
{{0.10033467}, {0.10033467}}
{{0.42279322}, {0.42279322}}
{{0.84228838}, {0.84228838}}
{{1.55740770}, {1.55740770}}
{{3.60210240}, {3.60210240}}
```

5. Do problem 3 by RK

Problem 3 is already as RK as it's going to get.

7. y'[x] == 3 y[x] − 12 y^2, y[0] == 0.2, h = 0.1

```
Clear["Global`*"]
```

```
s1 = DSolve[{y'[x] == 3 y[x] - 12 y[x]^2, y[0] == 0.2}, y[x], x]
```

Solve::ifun :
  Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y[x] \to \frac{e^{3x}}{1 + 4 e^{3x}}\right\}\right\}$$

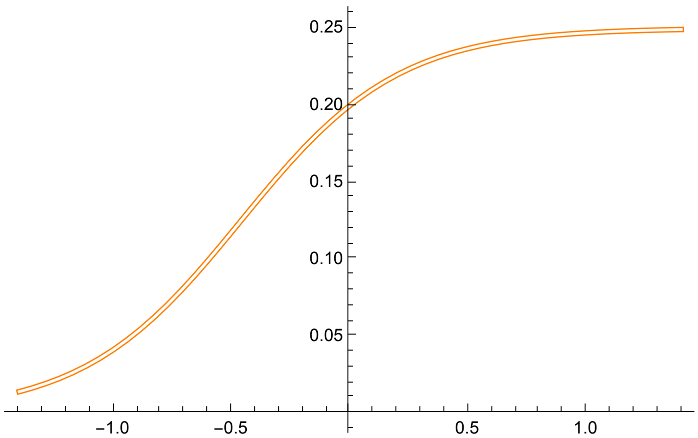
```
p1 = Plot[y[x] /. s1, {x, -1.4, 1.4},
   PlotStyle → {Orange, Thickness[0.008]}];
```

```
s2 = NDSolve[{y'[x] == 3 y[x] - 12 y[x]^2, y[0] == 0.2}, y, {x, -1.4, 1.4},
  AccuracyGoal → 16, PrecisionGoal → 16, WorkingPrecision → 20]
```

NDSolve::precw :
The precision of the differential equation ({{y′[x] == 3 y[x] − 12 y[x]^2, y[0] == 0.2}, {}, {}, {}, {}}) is less than WorkingPrecision (20.`). ≫

```
{{y → InterpolatingFunction[ Domain: {{-1.39999999999999991, 1.39999999999999991}}
                             Output: scalar ]}}
```

```
p2 =
  Plot[y[x] /. s2, {x, -1.4, 1.4}, PlotStyle → {White, Thickness[0.004]}];
Show[p1, p2]
```



Although Mathematica shows a note deprecating its WorkingPrecision, the results look good to me.

```
TableForm[
 Table[NumberForm[{y[x] /. s1, y[x] /. s2}, {8, 8}], {x, -1.4, 1.4, 0.3}]]
{{0.01414701}, {0.01414701}}
{{0.03214128}, {0.03214128}}
{{0.06656382}, {0.06656382}}
{{0.11790104}, {0.11790104}}
{{0.17175878}, {0.17175878}}
{{0.21093405}, {0.21093405}}
{{0.23249357}, {0.23249357}}
{{0.24257382}, {0.24257382}}
{{0.24692656}, {0.24692656}}
{{0.24874125}, {0.24874125}}
```

9. $y'[x] == 3 x^2 (1 + y[x]), \; y[0] == 0, \; h = 0.05$

```
Clear["Global`*"]
s1 = DSolve[{y'[x] == 3 x^2 (1 + y[x]), y[0] == 0}, y[x], x]
```

$\{\{y[x] \to -1 + e^{x^3}\}\}$

In the plot I try to capture all the parts of the function which are interesting.

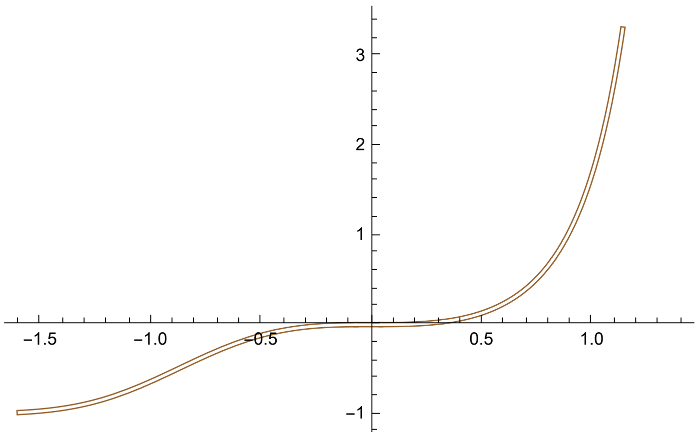
```
p1 =
  Plot[y[x] /. s1, {x, -1.6, 1.4}, PlotStyle → {Brown, Thickness[0.008]}];
```

```
s2 = NDSolve[{y'[x] == 3 x^2 (1 + y[x]), y[0] == 0}, y, {x, -1.6, 1.4},
  AccuracyGoal → 16, PrecisionGoal → 16, WorkingPrecision → 20]
```

```
{{y → InterpolatingFunction[ Domain {{-1.60000000000000000888, 1.39999999999999999911}}  Output: scalar ]}}
```

```
p2 =
  Plot[y[x] /. s2, {x, -1.6, 1.4}, PlotStyle → {White, Thickness[0.004]}];
```

```
Show[p1, p2]
```



The usual excellent agreement fills the table.

```
TableForm[
 Table[NumberForm[{y[x] /. s1, y[x] /. s2}, {8, 8}], {x, -1.6, 1.4, 0.3}]]
```

```
{{-0.98336090}, {-0.98336090}}
{{-0.88886393}, {-0.88886393}}
{{-0.63212056}, {-0.63212056}}
{{-0.29036179}, {-0.29036179}}
{{-0.06199500}, {-0.06199500}}
{{-0.00099950}, {-0.00099950}}
{{0.00803209}, {0.00803209}}
{{0.13314845}, {0.13314845}}
{{0.66862511}, {0.66862510}}
{{2.78482630}, {2.78482630}}
{{14.54905700}, {14.54905700}}
```