```
SetOptions[EvaluationNotebook[], StyleHints → {"CodeFont" → "Courier"}]
```

Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

Fourier-Legendre Series

Showing the details, develop:

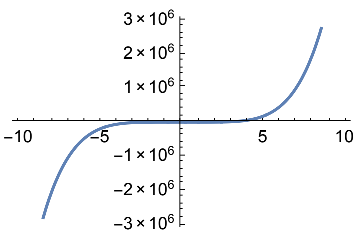
1. $63x^5 - 90x^3 + 35x$

```
Clear["Global`*"]

fp[x_] = 63 x^5 - 90 x^3 + 35 x
35 x - 90 x^3 + 63 x^5

FourierLegendreA[f_, x_, n_] :=
 (2 n + 1) / 2 Integrate[LegendreP[n, x] f, {x, -1, 1}]

Plot[fp[x], {x, -10, 10}]
```




```
Factor[Table[FourierLegendreA[63 x^5 - 90 x^3 + 35 x , x, n], {n, 0, 7}]]
```

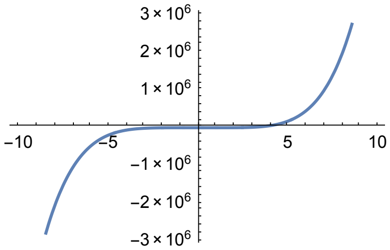
```
{0, 8, 0, -8, 0, 8, 0, 0}
```

The green cell above agrees with the answer in the text (showing non-zero coefficients at P_1, P_3, and P_5). The FLA function was found on Eric Weisstein's Math World. The s.m. points out that the reason the odd coefficients are non-zero is that the function is odd.

3. $1 - x^4$

```
fg[x] = 1 - x^4
1 - x^4
```

```
Plot[fp[x], {x, -10, 10}, ImageSize → 200
]
```




```
Factor[Table[FourierLegendreA[1 - x^4 , x, n], {n, 0, 7}]]
```

$$\left\{\frac{4}{5}, 0, -\frac{4}{7}, 0, -\frac{8}{35}, 0, 0, 0\right\}$$

The answer above matches that of the text.

8 -- 13 Fourier-Legendre Series

Find and graph (on common axes) the partial sums up to S_{m_0} whose graph practically coincides with that of f(x) within graphical accuracy. State m_0. On what does the size of m_0 seem to depend?

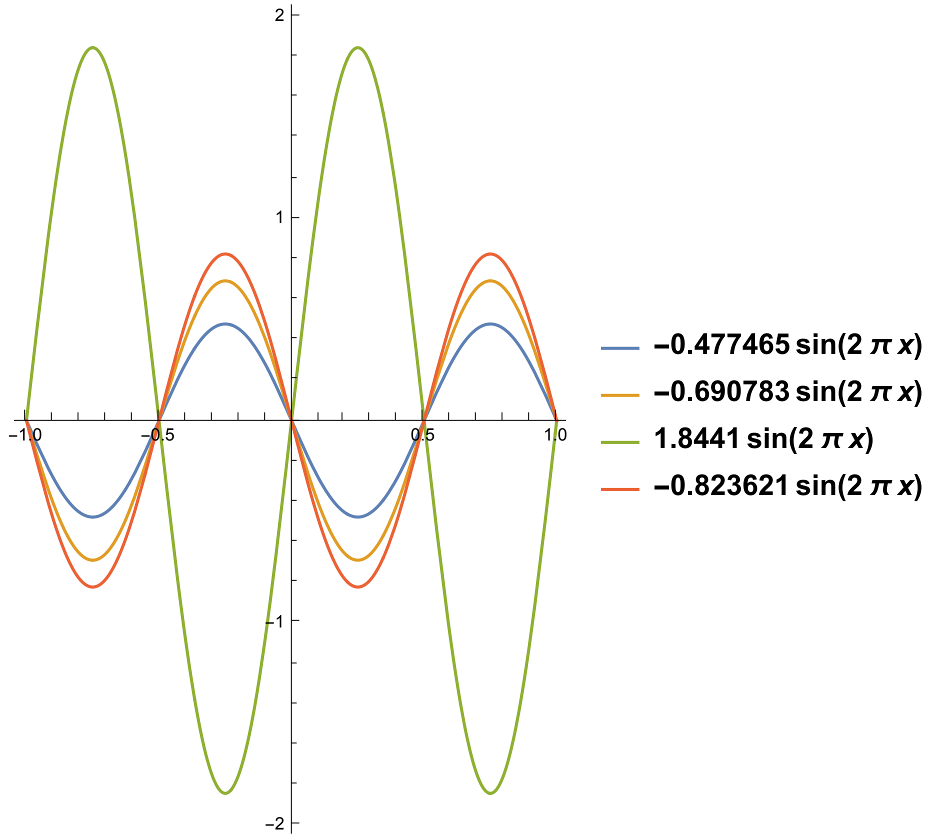
9. $f(x) = \sin 2\pi x$

```
N[Factor[Table[FourierLegendreA[Sin[2 π x] , x, n], {n, 0, 7}]]]
```

```
{0., -0.477465, 0., -0.690783, 0., 1.8441, 0., -0.823621}
```

The above coefficients match the values in the text.

```
Plot[{{-0.477464829275686` Sin[2 π x]}, {-0.6907832122075241` Sin[2 π x]},
  {1.8440983138592046` Sin[2 π x]}, {-0.8236205222550245` Sin[2 π x]}},
 {x, -1, 1}, AspectRatio → 3 / 2, PlotLegends → "Expressions"]
```



The above plot resembles that in the s.m., but not closely. As far as m_0 goes, I don’t know how to establish it. Logically, even functions would have m_0 of zero, and odd functions have m_0 of one. But the question above implies it could be sizable. I couldn’t find a place in either text or s.m. which gave the answer; they *both* coyly asked the student what it was. In the above functions, the order is: teal, orange, green, red.

11. $f(x) = (1 + x^2)^{-1}$

```
Clear["Global`*"]
```

```
FourierLegendreA[f_, x_, n_] :=
 (2 n + 1) / 2 Integrate[LegendreP[n, x] f, {x, -1, 1}]
```

```
fh[x_] = (1 + x^2)^-1
```

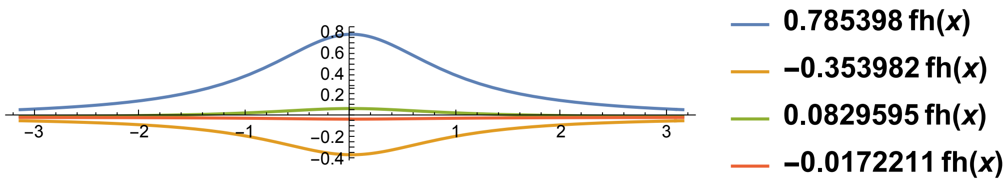
$$\frac{1}{1 + x^2}$$

```
N[Factor[Table[FourierLegendreA[fh[x] , x, n], {n, 0, 7}]]]
```

```
{0.785398, 0., -0.353982, 0., 0.0829595, 0., -0.0172211, 0.}
```

The above coefficients match the values in the text.

```
Plot[{{0.7853981633974483` fh[x]}, {-0.3539816339744828` fh[x]},
  {0.08295949990479201` fh[x]}, {-0.017221090839916544` fh[x]}},
 {x, -π, π}, AspectRatio → Automatic, PlotRange → Full,
 PlotLegends → "Expressions"]
```



The color order is the same as the last problem.

13. f(x) = Subscript[J, 0] $(\alpha_{0,2}\, x)$, $\quad a_{0,2}$ = the second positive zero of $J_0(x)$

I don't understand this problem.