

2-10 General Solution. Find a general solution. Show the steps of derivation. Check your answer by substitution.

$$2. y^3 y' + x^3 = 0$$

```
ClearAll["Global`*"]
eqn = y[x]^3 + x^3 == 0;
sol = DSolve[eqn, y, x]
{{y -> Function[{x}, -x]},
 {y -> Function[{x}, (-1)^(1/3) x]}, {y -> Function[{x}, -(-1)^(2/3) x]}}
eqn /. sol[[1]]
True
eqn /. sol[[2]]
True
eqn /. sol[[3]]
True
```

$$3. y' = \sec^2 y$$

```
ClearAll["Global`*"]
eqn = y'[x] == Sec[y[x]]^2;
eqn2 = y'[x] Cos[y[x]]^2 == 1;
eqn3 = y'[x] ==  $\frac{2}{\cos[2 y[x]] + 1}$ ;
eqn4 = y'[x] ==  $\frac{4}{(e^{-i y[x]} + e^{i y[x]})^2}$ ;
sol = DSolve[eqn3, y, x]
{{y -> Function[{x}, InverseFunction[2 (1/4 Sin[2 #1] + #1/2) &][2 x + C[1]]]}}
```

The WolframAlpha solution is shown as

$$c_1 + 2x = 2 \left(\frac{y[x]}{2} + \frac{1}{4} \sin[2 y[x]] \right)$$

```
Simplify[eqn /. sol]
{True}
```

$$4. y' \sin 2\pi x = \pi y \cos 2\pi x$$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] Sin[2 π x] == π y[x] Cos[2 π x];
sol = DSolve[eqn, y, x]
{{y -> Function[{x}, C[1] Sqrt[Sin[2 π x]]]}}
```

```
eqn /. sol
{True}
```

$$5. y y' + 36 x = 0$$

```
ClearAll["Global`*"]

eqn = y[x] y'[x] + 36 x == 0;
sol = DSolve[eqn, y, x]
{{y -> Function[{x}, -Sqrt[2] Sqrt[-18 x^2 + C[1]]]},
 {y -> Function[{x}, Sqrt[2] Sqrt[-18 x^2 + C[1]]]}}
```

```
eqn /. sol[[1]]
True
```

```
eqn /. sol[[2]]
True
```

$$6. y' = e^{2x-1} y^2$$

```
ClearAll["Global`*"]

eqn = y'[x] == e^{2 x - 1} y[x]^2;
sol = DSolve[eqn, y, x]
{{y -> Function[{x}, -\frac{2 e}{e^{2 x} + 2 e C[1]}]}}
```

```
eqn /. sol
{True}
```

$$7. xy' = y + 2x^3 \sin^2 \frac{y}{x} \text{ (Set } y/x = u \text{)}$$

```
ClearAll["Global`*"]

eqn = x y'[x] == y[x] + 2 x^3 Sin[\frac{y[x]}{x}]^2;
sol = DSolve[eqn, y, x]
```

Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information>>

```
{{y -> Function[{x}, -x ArcCot[x^2 - 2 C[1]]]}}
```

```
Simplify[eqn /. sol]
{True}
```

8. $y' = (y + 4x)^2$ (Set $y + 4x = v$)

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == (y[x] + 4 x)^2;
sol = DSolve[eqn, y, x]
```

```
{ {y -> Function[{x}, -2 \[ImaginaryI] - 4 x + \frac{1}{-\frac{\[ImaginaryI]}{4} + e^{4 \[ImaginaryI] x} C[1]}] } }
```

```
Simplify[eqn /. sol]
{True}
```

9. $xy' = y^2 + y$ (Set $y/x = u$)

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] == y[x]^2 + y[x];
sol = DSolve[eqn, y, x]
```

```
{ {y -> Function[{x}, -\frac{e^{C[1]} x}{-1 + e^{C[1] x}}] } }
```

```
Simplify[eqn /. sol]
{True}
```

10. $xy' = x + y$ (Set $y/x = u$)

```
ClearAll["Global`*"]
```

```
eqn = xy'[x] == x + y[x];
sol = DSolve[eqn, y, x]
```

```
{ {y -> Function[{x}, -x + xy'[x]] } }
```

```
eqn /. sol
{True}
```

11-17 Initial Value Problems (IVPs). Solve the IVP. Show the steps of derivation, beginning with the general solution.

11. $xy' + y = 0$, $y(4) = 6$

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] + y[x] == 0;
sol = DSolve[{eqn, y[4] == 6}, y, x]
{{y -> Function[{x},  $\frac{24}{x}$ ]}}
```

```
eqn /. sol
{True}
```

12. $y' = 1 + 4y^2$, $y(1) = 0$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == 1 + 4 y[x]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve::fun:

Inverse functions are being used by Solve so some solutions may not be found use Reduce for complete solution information >>

```
{{y -> Function[{x},  $\frac{1}{2} \tan[2(-1 + x)]$ ]}}
```

```
Simplify[eqn /. sol]
{True}
```

13. $y' \cosh^2 x = \sin^2 y$, $y(0) = \frac{1}{2}\pi$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] Cosh[x]^2 == Sin[y[x]]^2;
sol = DSolve[{eqn, y[0] ==  $\frac{1}{2}\pi$ }, y, x]
```

Solve::fun:

Inverse functions are being used by Solve so some solutions may not be found use Reduce for complete solution information >>

```
{{y -> Function[{x}, -ArcCot[Tanh[x]]]}}
```

```
Simplify[eqn /. sol]
{True}
```

14. $dr/dt = -2tr$, $r(0) = r_0$

```
ClearAll["Global`*"]
```

```
eqn = r'[t] == -2 t r[t];
sol = DSolve[{eqn, r[0] == r0}, r[t], t]
{{r[t] ->  $e^{-t^2} r_0$ }}
```

```
Simplify[eqn] /. Simplify[sol]
{r'[t] == -2 e-t2 r0 t}
```

While Mathematica will not declare this equality to be true, it works when the substitutions are made.

$$15. y' = -4xy, y(2) = 3$$

```
ClearAll["Global`*"]

eqn = y'[x] == -4 x y[x];
sol = DSolve[{eqn, y[2] == 3}, y, x]
{{y -> Function[{x}, 3 e^{8-2 x^2}]}}
```

eqn /. sol

```
{True}
```

$$16. y' = (x + y - 2)^2, y(0) = 2, (\text{Set } v = x + y - 2)$$

```
ClearAll["Global`*"]

eqn = y'[x] == (x + y[x] - 2)^2;
sol = DSolve[{eqn, y[0] == 2}, y, x]

{{y -> Function[{x}, -\frac{(-2 - i) - (2 - i) e^{2 i x} + x + e^{2 i x} x}{1 + e^{2 i x}}]}}
```

Simplify[eqn /. sol]

```
{True}
```

$$17. xy' = y + 3x^4 \cos^2\left(\frac{y}{x}\right), y(1) = 0, \text{Set } \frac{y}{x} = u$$

```
ClearAll["Global`*"]

eqn = x y'[x] == y[x] + 3 x^4 Cos[\frac{y[x]}{x}]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
Solve::ifun: Inverse functions are being used by Solve so some solutions may not be found; use Reduce for complete solution information >>
{{y -> Function[{x}, -x ArcTan[1 - x^3]]}}
```

Simplify[eqn /. sol]

```
{True}
```