1 - 10 Rank, row space, column space

Find the rank. Find a basis for the row space. Find a basis for the column space. Hint. Row-reduce the matrix and its transpose. (You may omit obvious factors from the vectors of these bases.)

1. $\begin{pmatrix} 4 & -2 & 6 \\ -2 & 1 & -3 \end{pmatrix}$

```
Clear["Global`*"]
```

e1 = $\begin{pmatrix} 4 & -2 & 6 \\ -2 & 1 & -3 \end{pmatrix}$

```
{{4, -2, 6}, {-2, 1, -3}}
```

```
e2 = RowReduce[e1]
```

$\left\{\left\{1, -\frac{1}{2}, \frac{3}{2}\right\}, \{0, 0, 0\}\right\}$

Above: The basis for the row space, in agreement with the text. The rank is 1.

e3 = $e1^T$

```
{{4, -2}, {-2, 1}, {6, -3}}
```

```
e4 = RowReduce[e3]
```

$\left\{\left\{1, -\frac{1}{2}\right\}, \{0, 0\}, \{0, 0\}\right\}$

Above: The basis for the column space, in agreement with the text. The rank is 1.

3. $\begin{pmatrix} 0 & 3 & 5 \\ 3 & 5 & 0 \\ 5 & 0 & 10 \end{pmatrix}$

```
Clear["Global`*"]
```

e1 = $\begin{pmatrix} 0 & 3 & 5 \\ 3 & 5 & 0 \\ 5 & 0 & 10 \end{pmatrix}$

```
{{0, 3, 5}, {3, 5, 0}, {5, 0, 10}}
```

```
e2 = RowReduce[e1]
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

The rank is 3. The three vectors shown are a basis for the row space.

```
e3 = e2ᵀ
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

RowReduce[e3]
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

Above: The rank is still three. The three vectors shown are a basis for the column space. The bases which are exposed by **RowReduce** are not too exciting, perhaps, but valid bases they remain. These are not the bases contained in the text answer.

5. $\begin{pmatrix} 0.2 & -0.1 & 0.4 \\ 0 & 1.1 & -0.3 \\ 0.1 & 0 & -2.1 \end{pmatrix}$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 0.2 & -0.1 & 0.4 \\ 0 & 1.1 & -0.3 \\ 0.1 & 0 & -2.1 \end{pmatrix}$$

```
{{0.2, -0.1, 0.4}, {0, 1.1, -0.3}, {0.1, 0, -2.1}}

e2 = RowReduce[e1]
{{1, 0., 0.}, {0, 1, 0.}, {0, 0, 1}}
```

Above: The row space rank is 3. The three vectors shown form a basis for the row space.

```
e3 = e2ᵀ
{{1, 0, 0}, {0., 1, 0}, {0., 0., 1}}

RowReduce[e3]
{{1, 0., 0.}, {0, 1, 0.}, {0, 0, 1}}
```

Above. The column space rank is 3. The three vectors shown form a basis for the column space. The text agrees on the ranks. However, different bases are shown in the text answer.

7. $\begin{pmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 4 & 0 & 2 & 0 \end{pmatrix}$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 4 & 0 & 2 & 0 \end{pmatrix}$$

```
{{8, 0, 4, 0}, {0, 2, 0, 4}, {4, 0, 2, 0}}
```

```
e2 = RowReduce[e1]
```

$\{\{1, 0, \frac{1}{2}, 0\}, \{0, 1, 0, 2\}, \{0, 0, 0, 0\}\}$

Above: The row space rank is 2. The two non-zero vectors constitute a basis. The text basis consists of a multiple of the basis above.

```
e3 = Transpose[e1]
{{8, 0, 4}, {0, 2, 0}, {4, 0, 2}, {0, 4, 0}}
```

```
RowReduce[e3]
```

$\{\{1, 0, \frac{1}{2}\}, \{0, 1, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

Above: The column space rank is 2. The two non-zero vectors constitute a basis. The text basis consists of a multiple of the basis above.

9. $\begin{pmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

```
{{9, 0, 1, 0}, {0, 0, 1, 0}, {1, 1, 1, 1}, {0, 0, 1, 0}}
```

```
e2 = RowReduce[e1]
{{1, 0, 0, 0}, {0, 1, 0, 1}, {0, 0, 1, 0}, {0, 0, 0, 0}}
```

Above: The row rank is 3. The basis in row space constitutes the first three vectors. This is a different basis than the one shown in the text.

$e3 = e1^{T}$

```
{{9, 0, 1, 0}, {0, 0, 1, 0}, {1, 1, 1, 1}, {0, 0, 1, 0}}
```

Above: e1 is symmetric, so the rank and basis info already calculated for rows also applies to column space.

17 - 25 Linear independence
Are the following sets of vectors linearly independent?

17. {3, 4, 0, 2}, {2, -1, 3, 7}, {1, 16, -12, -22}

```
Clear["Global`*"]
```

$$\text{e1} = \begin{pmatrix} 3 & 4 & 0 & 2 \\ 2 & -1 & 3 & 7 \\ 1 & 16 & -12 & -22 \end{pmatrix}$$

```
{{3, 4, 0, 2}, {2, -1, 3, 7}, {1, 16, -12, -22}}
```

```
MatrixRank[e1]
```

```
2
```

Above: The vectors are linearly dependent. So the answer to the problem question is no, they are not linearly independent.

19. {0, 1, 1}, {1, 1, 1}, {0, 0, 1}

```
Clear["Global`*"]
```

$$\text{e1} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

```
{{0, 1, 1}, {1, 1, 1}, {0, 0, 1}}
```

```
MatrixRank[e1]
```

```
3
```

Above: The vectors are linearly independent. So the answer is yes.

21. {2, 0, 0, 7}, {2, 0, 0, 8}, {2, 0, 0, 9}, {2, 0, 1, 0}

```
Clear["Global`*"]
```

$$\text{e1} = \begin{pmatrix} 2 & 0 & 0 & 7 \\ 2 & 0 & 0 & 8 \\ 2 & 0 & 0 & 9 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

```
{{2, 0, 0, 7}, {2, 0, 0, 8}, {2, 0, 0, 9}, {2, 0, 1, 0}}
```

```
MatrixRank[e1]
```

```
3
```

Above: The vectors are linearly dependent. So the answer is no, they are not linearly independent.

23. {9, 8, 7, 6, 5}, {9, 7, 5, 3, 1}

```
Clear["Global`*"]
```

$$\text{e1} = \begin{pmatrix} 9 & 8 & 7 & 6 & 5 \\ 9 & 7 & 5 & 3 & 1 \end{pmatrix}$$

```
{{9, 8, 7, 6, 5}, {9, 7, 5, 3, 1}}
```

```
MatrixRank[e1]
```

```
2
```

Above: Yes, the vectors are linearly independent.

25. {6, 0, -1, 3}, {2, 2, 5, 0}, {-4, -4, -4, -4}

```
Clear["Global`*"]
```

$$e1 = \begin{pmatrix} 6 & 0 & -1 & 3 \\ 2 & 2 & 5 & 0 \\ -4 & -4 & -4 & -4 \end{pmatrix}$$

```
{{6, 0, -1, 3}, {2, 2, 5, 0}, {-4, -4, -4, -4}}
```

```
MatrixRank[e1]
```

```
3
```

Above: Yes, the vectors are linearly independent. The answers above for nos 17 -- 25 agree with the text.