

```
Clear["Global`*"]
```

3 - 9 Steepest descent

Do steepest descent steps when:

$$3. \quad f[x] = 2x_1^2 + x_2^2 - 4x_1 + 4x_2, \quad x_0 = 0, \quad 3 \text{ steps}$$

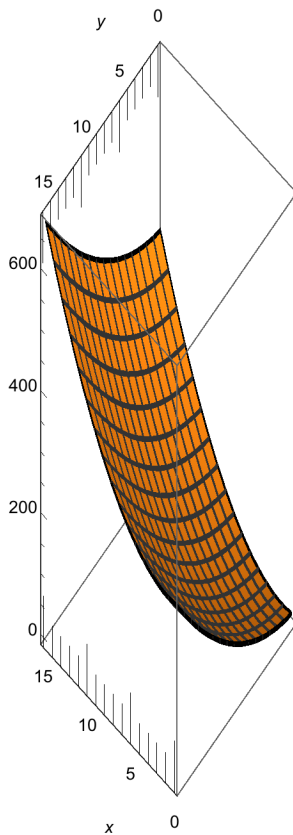
I replace the subscripted forms in the problem expression with x and y, which is easier for me to see. I need to mention the Wolfram demonstration by Michael Waters, <https://demonstrations.wolfram.com/CurvesOfSteepestDescentFor3DFunctions/>, which is appealing and gives the right answer. It is also accommodating about accepting an alien function into its central module.

```
Clear["Global`*"]
```

```
f[x_, y_] = 2 x^2 + y^2 - 4 x + 4 y  
- 4 x + 2 x^2 + 4 y + y^2
```

A plot may be useful in reconnoitering the district.

```
Plot3D[f[x, y], {x, 0, 15}, {y, 0, 15}, AxesLabel -> Automatic,  
ImageSize -> 150, AspectRatio -> Full, ViewPoint -> {-4000, 2000, 1000}]
```



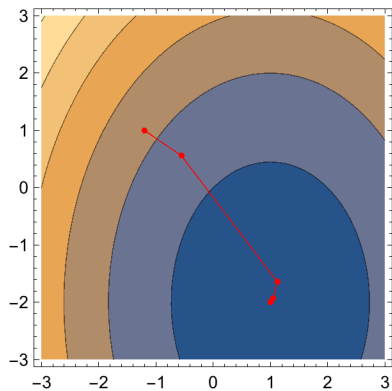
First objective is to establish the answer.

```
FindMinimum[f[x, y], {x, y}]
{-6., {x → 1., y → -2.}}
```

I see below that step 3 by Mathematica is very close to step 3 by the text answer procedure, whatever that may entail. The text answer for step 3 is {1.037, -1.926}.

```
pts = Reap[FindMinimum[f[x, y],
  {{x, -1.2}, {y, 1}}, StepMonitor :> Sow[{x, y}]]][[2, 1]]
pts = Join[{{-1.2, 1}}, pts];
{{-0.554696, 0.56002}, {1.12183, -1.64264}, {1.04891, -1.92555},
  {1.00106, -2.00055}, {1.00003, -2.00003}, {1., -2.}, {1., -2.}}
```

```
ContourPlot[f[x, y], {x, -3, 3}, {y, -3, 3},
  Epilog -> {Red, Line[pts], Point[pts]}, ImageSize -> 200]
```

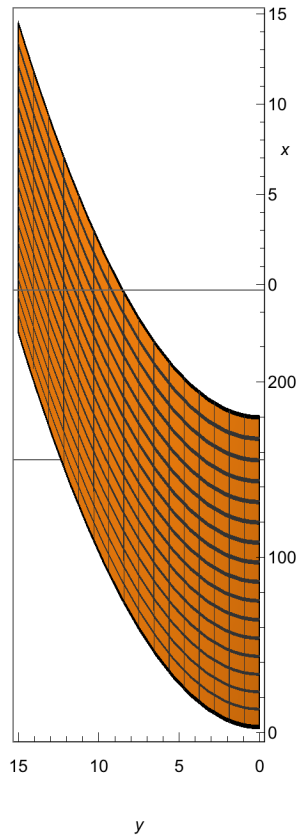


9. $f[x] = 0.1 x_1^2 + x_2^2 - 0.02 x_1$, $x_0 = (3, 3)$, 5 steps

```
Clear["Global`*"]
```

```
f[x_, y_] = 0.1 x^2 + y^2 - 0.02 x
-0.02 x + 0.1 x^2 + y^2
```

```
Plot3D[f[x, y], {x, 0, 15}, {y, 0, 15}, AxesLabel → Automatic,
ImageSize → 150, AspectRatio → Full, ViewPoint → {-4000, 0, 1000}]
```



```
FindMinimum[f[x, y], {{x, 3}, {y, 3}}]
{-0.001, {x → 0.1, y → 5.96629 × 10-11}}
```

The plot below makes me wonder if it is better not to give a guess about the preferred starting position. The text is interested in step 5, which in its search produced the coordinate {0.11247, -0.00012}.

```
pts =
  Reap[FindMinimum[f[x, y], {{x, 3}, {y, 3}}, StepMonitor ⇒ Sow[{x, y}]]][[
    2, 1]]
pts = Join[{{-1.2, 1}}, pts];
{{2.79589, 0.888522}, {2.44241, -0.0226433},
 {2.16341, -0.0800494}, {0.368991, -0.13073}, {0.111153, -0.0394899},
 {0.0942656, -0.0016205}, {0.0995506, -0.0000200394},
 {0.0999956, 1.46532 × 10-6}, {0.1, 2.66175 × 10-8}, {0.1, 5.96629 × 10-11}}
```

```
ContourPlot[f[x, y], {x, -3, 3}, {y, -3, 3},  
  Epilog -> {Red, Line[pts], Point[pts]}, ImageSize -> 200]
```

