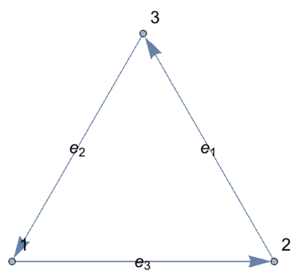
Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

8 - 13 Find the adjacency matrix of the given graph or digraph.

9.

```
g1 = Graph[{Labeled[1 ↔ 2, "e₃"], Labeled[2 ↔ 3, "e₁"],
   Labeled[3 ↔ 1, "e₂"]}, VertexLabels → "Name", ImageSize → 150]
```

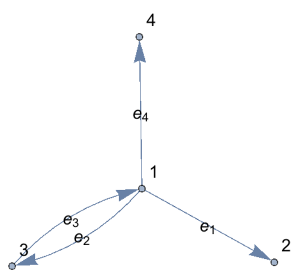



```
ceg = AdjacencyMatrix[g1] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

11.

```
g2 =
 Graph[{Labeled[1 ↔ 2, "e₁"], Labeled[1 ↔ 3, "e₂"], Labeled[3 ↔ 1, "e₃"],
   Labeled[1 ↔ 4, "e₄"]}, VertexLabels → "Name", ImageSize → 150]
```

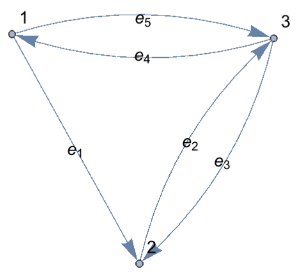



```
ceh = AdjacencyMatrix[g2] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

13.

```
g3 = Graph[{Labeled[1 ⟷ 2, "e₁"], Labeled[1 ⟷ 3, "e₅"],
   Labeled[3 ⟷ 1, "e₄"], Labeled[2 ⟷ 3, "e₂"], Labeled[3 ⟷ 2, "e₃"]},
  VertexLabels → "Name", ImageSize → 150]
```




```
cei = AdjacencyMatrix[g3] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

14 - 15 Sketch the graph for the given adjacency matrix.

15.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{cej} = \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{matrix}$$

```
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
```

```
AdjacencyGraph[cej, ImageSize → 200, VertexLabels → "Name"]
```

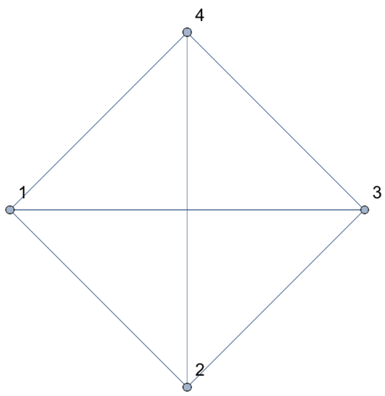
The relationship between vertices in the above sketch is comparable to that of the text, except mirrored.

17. In what case are all the off-diagonal entries of the adjacency matrix of a graph G equal to one?

$$\textbf{cek} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

```
{{0, 1, 1, 1}, {1, 0, 1, 1}, {1, 1, 0, 1}, {1, 1, 1, 0}}
```

```
AdjacencyGraph[cek, ImageSize → 200, VertexLabels → "Name"]
```



The example I made suggests that each pair of vertices is joined by an edge, i.e. it is a ***complete*** graph.

19. Incidence matrix $\tilde{B}$ of a digraph. The definition is $\tilde{B} = [b_{jk}]$,
where

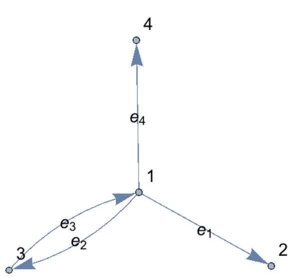
$$\tilde{b}_{jk} = \begin{cases} -1 & \text{if edge } e_k \text{ leaves vertex } j, \\ 1 & \text{if edge } e_k \text{ enters vertex } j, \\ 0 & \text{otherwise} \end{cases}$$

Find the incidence matrix of the digraph in problem 11.

I find that getting the neat table of edges and vertices requires imitating the exact steps shown in the documentation for IncidenceMatrix. Specifically, if the MatrixForm of g2 is not shielded by a buffer cell from the call for TableForm, then TableForm will just repeat the MatrixForm, as illogical as that seems.

```
Clear["Global`*"]
```

```
g2 =
 Graph[{Labeled[1 → 2, "e1"], Labeled[1 → 3, "e2"], Labeled[3 → 1, "e3"],
   Labeled[1 → 4, "e4"]}, VertexLabels → "Name", ImageSize → 150]
```



```
im1 = IncidenceMatrix[g2] // MatrixForm
```

$$\begin{pmatrix} -1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
im1 = IncidenceMatrix[
  g2 = Graph[{Labeled[1 ⟶ 2, "e₁"], Labeled[1 ⟶ 3, "e₂"],
     Labeled[3 ⟶ 1, "e₃"], Labeled[1 ⟶ 4, "e₄"]},
    VertexLabels → "Name", ImageSize → 150]]
```

```
SparseArray[ Specified elements: 8
             Dimensions: {4, 4} ]
```

Getting a table which looks a little like the text answer.

```
tek = TableForm[Normal[im1],
  TableHeadings → {VertexList[g2], EdgeList[g2]}]
```

	1 ⟶ 2	1 ⟶ 3	3 ⟶ 1	1 ⟶ 4
1	-1	-1	1	-1
2	1	0	0	0
3	0	1	-1	0
4	0	0	0	1