```
Clear["Global`*"]
```

ODEs. Integrating Factors. Test for exactness. If exact, solve. If not, use an integrating factor as given or obtained by inspection or by the theorems in the text. Also, if an initial condition is given, find the corresponding particular solution.

1. $2\,xy\,dx + x^2\,dy = 0$

```
eqn = 2 x y[x] + x^2 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \frac{C[1]}{x^2}]\}\}$

```
eqn /. sol
```

```
{True}
```

```
Clear["Global`*"]
```

2. $x^3 + y[x]^3\, y'[x] = 0$

```
eqn = x^3 + y[x]^3 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, -i\,(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, i\,(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, (-x^4 + 4\,C[1])^{1/4}]\}\}$

```
eqn /. sol[[1]]
```

```
True
```

```
eqn /. sol[[2]]
```

```
True
```

```
eqn /. sol[[3]]
```

```
True
```

```
eqn /. sol[[4]]
```

```
True
```

```
Clear["Global`*"]
```

3. $\sin x \cos y + \cos x \sin y y' = 0$

```
eqn = Sin[x] Cos[y[x]] + Cos[x] Sin[y[x]] y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\text{ArcCos}\left[\frac{1}{2} C[1] \text{Sec}[x]\right]\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, \text{ArcCos}\left[\frac{1}{2} C[1] \text{Sec}[x]\right]\right]\right\}\right\}$$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True

Clear["Global`*"]
```

4. $e^{3\theta}(r'[\theta] + 3\, r[\theta]) = 0$

```
eqn = ⅇ^(3 θ) (r'[θ] + 3 r[θ]) == 0;
sol = DSolve[eqn, r, θ]
```

$$\{\{r \to \text{Function}[\{\theta\}, e^{-3\theta} C[1]]\}\}$$

```
eqn /. sol
{True}

Clear["Global`*"]
```

5. $(x^2 + y^2) - 2\,xyy' = 0$

```
eqn = x^2 + y[x]^2 - 2 x y[x] y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\sqrt{x}\sqrt{x + C[1]}\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, \sqrt{x}\sqrt{x + C[1]}\right]\right\}\right\}$$

```
Simplify[eqn /. sol[[1]]]
True

Simplify[eqn /. sol[[2]]]
True

Clear["Global`*"]
```

6. $3\,(y + 1) = 2\,xy', \ (y + 1)\,x^{-4}$

```
eqn = 3 (y[x] + 1) == 2 x y'[x];
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -1 + x^{3/2}\, C[1]]\}\}$

```
eqn /. sol
```

{True}

```
Clear["Global`*"]
```

7. $2 x \tan y + \sec^2 y\, y' = 0$

```
eqn = 2 x Tan[y[x]] + Sec[y[x]]^2 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
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$\{\{y \to \text{Function}[\{x\}, \text{ArcCot}[e^{x^2 - 2\, C[1]}]]\}\}$

```
Simplify[eqn /. sol]
```

{True}

```
Clear["Global`*"]
```

8. $e^x(\cos y - \sin y\, y') = 0$

```
eqn = e^x (Cos[y[x]] - Sin[y[x]] y'[x]) == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, -\text{ArcCos}[e^{-x-C[1]}]]\},$
$\{y \to \text{Function}[\{x\}, \text{ArcCos}[e^{-x-C[1]}]]\}\}$

```
Simplify[eqn /. sol[[1]]]
```

True

```
Simplify[eqn /. sol[[2]]]
```

True

```
Clear["Global`*"]
```

9. $e^{2x}(2 \cos y - \sin y\, y') = 0, \;\; y(0) = 0$

```
eqn = e^(2 x) (2 Cos[y[x]] - Sin[y[x]] y'[x]) == 0;
sol = DSolve[{eqn, y[0] == 0}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

General::stop: Further output of Solve::ifun will be suppressed during this calculation. >>

```
{{y → Function[{x}, -ArcCos[e^(-2 x)]]}, {y → Function[{x}, ArcCos[e^(-2 x)]]}}
```

```
Simplify[eqn /. sol[[1]]]
True
```

```
Simplify[eqn /. sol[[2]]]
True
```

```
Clear["Global`*"]
```

10. $y+(y + \tan(x+y))\, y' = 0$, $\cos(x + y)$ [or 2(cos x cos y)]

```
eqn = y[x] + (y[x] + Tan[x + y[x]]) y'[x] == 0;
sol = FullSimplify[DSolve[eqn, y, x]]
Solve[C[1] == Sin[x + y[x]] y[x], y[x]]
```

Here is one that Mathematica can't solve. I wonder what kind of manipulations would be necessary in order to get it to solve this implicit form.
The original equation was:
$y\, dx + (y + \tan(x+y))\, dy = 0$
Clear[t,y];
t0 = 0; y0 = 2; f[t_] = t/(1 + t^2); g[y_] = 1/y; (* define the initial values and the slope functions *)
F[t_] = Integrate[f[t], t]; G[y_] = Integrate[g[y], y];
gensol = Solve[G[y] == F[t] + c, y];

```
FullSimplify[y[x] (2 Cos[x] Cos[y[x]])]
2 Cos[x] Cos[y[x]] y[x]
```

```
Clear["Global`*"]
```

11. $2 \cosh x \cos y = \sinh x \sin y\, y'$

```
eqn = 2 Cosh[x] Cos[y[x]] == Sinh[x] Sin[y[x]] y'[x];
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\text{ArcCos}\left[-\frac{1}{2}\, i\, C[1]\, \text{Csch}[x]^2\right]\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, \text{ArcCos}\left[-\frac{1}{2}\, i\, C[1]\, \text{Csch}[x]^2\right]\right]\right\}\right\}$$

```
Simplify[eqn /. sol[[1]]]
True

Simplify[eqn /. sol[[2]]]
True

Clear["Global`*"]
```

12. $(2\,xy + y')\, e^{x^2} = 0,\ y(0) = 2$

```
eqn = (2 x y[x] + y'[x]) e^(x^2) == 0;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, 2\, e^{-x^2}]\}\}$$

```
eqn /. sol
{True}

Clear["Global`*"]
```

13. $e^{-y[x]} + e^{-x}\left(-e^{-y[x]} + 1\right) y'[x] = 0,\ F = e^{x+y[x]}$

```
eqn = e^(-y[x]) + e^(-x) (-e^(-y[x]) + 1) y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{y \to \text{Function}[\{x\}, e^x - C[1] - \text{ProductLog}[-e^{e^x - C[1]}]]\}\}$$

```
Simplify[eqn /. sol]
{True}

Clear["Global`*"]
```

14. $(a+1)\,y + (b+1)\,x\,y' = 0,\ y(1) = 1,\ F = x^a\, y^b$

```
eqn = (a + 1) y[x] + (b + 1) x y'[x] == 0;
sol = DSolve[{eqn, y[1] == 1}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, (1 + b)^{\frac{1}{1+b} + \frac{a}{1+b}}\, (x + b\, x)^{-\frac{1}{1+b} - \frac{a}{1+b}}\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
{True}
```

15. Exactness. Under what conditions for the constants a, b, k, l is (a x + b y)dx + (k x + l y)dy = 0 exact? Solve the exact ODE.

```
Clear["Global`*"]
```

According to the exactness test, b = k. The text answer also has the relationship a*x^2 + 2*k*x*y + l*y^2 = c, but I haven't been able to track this down yet. As for the exact equation, (and substituting b for k)

```
eqn = y'[x] == - (a x + b y[x]) / (b x + l y[x])
```

$$y'[x] == -\frac{a\,x + b\,y[x]}{b\,x + l\,y[x]}$$

```
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\},\ \frac{1}{l}\left(-b\,x - \sqrt{e^{2\,C[1]}\,l + b^2\,x^2 - a\,l\,x^2}\right)\right]\right\},\right.$$
$$\left.\left\{y \to \text{Function}\left[\{x\},\ \frac{1}{l}\left(-b\,x + \sqrt{e^{2\,C[1]}\,l + b^2\,x^2 - a\,l\,x^2}\right)\right]\right\}\right\}$$

```
FullSimplify[eqn /. sol[[1]]]
True
```

```
FullSimplify[eqn /. sol[[2]]]
True
```