Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

2 - 14 Simplex method
Write in normal form and solve by the simplex method, assuming all x_j to be nonnegative.

3. Maximize $f = 3x_1 + 2x_2$ subject to $3x_1 + 4x_2 \le 60$, $4x_1 + 3x_2 \le 60$, $10x_1 + 2x_2 \le 120$

Having heard of the simplex method, I wouldn't mind trying to find out about it. However, if regular Maximize works, I prefer to stick with it.

```
Maximize[{3 x + 2 y, 3 x + 4 y ≤ 60, 4 x + 3 y ≤ 60, 10 x + 2 y ≤ 120}, {x, y}]
```

$$\left\{\frac{480}{11}, \left\{x \to \frac{120}{11}, y \to \frac{60}{11}\right\}\right\}$$

Reversing my position, I decide to try the simplex method, with no pivoting.

$$aa = \begin{pmatrix} 1 & -3 & -2 & 0 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 0 & 10 & 2 & 0 & 0 & 1 \end{pmatrix}$$

```
{{1, -3, -2, 0, 0, 0}, {0, 3, 4, 1, 0, 0},
 {0, 4, 3, 0, 1, 0}, {0, 10, 2, 0, 0, 1}}
```

$$bb = \begin{pmatrix} 0 \\ 60 \\ 60 \\ 120 \end{pmatrix}$$

```
{{0}, {60}, {60}, {120}}
```

In this case the simplex method comes up with the right answer.

```
LinearSolve[aa, bb]
```

$$\left\{\left\{\frac{480}{11}\right\}, \left\{\frac{120}{11}\right\}, \left\{\frac{60}{11}\right\}, \left\{\frac{60}{11}\right\}, \{0\}, \{0\}\right\}$$

5. Minimize $f = 5x_1 - 20x_2$ subject to $-2x_1 + 10x_2 \le 5$, $2x_1 + 5x_2 \le 10$

```
Clear["Global`*"]
```

When the problem is naïvely entered, it is not solved by Mathematica. What it needs is some slack relations.

```
Minimize[{5 x - 20 y, -2 x + 10 y ≤ 5, 2 x + 5 y ≤ 10}, {x, y}]
```

Minimize::natt : The minimum is not attained at any point satisfying the given constraints. >>

```
{-∞, {x → Indeterminate, y → Indeterminate}}
```

First I will try a simplex method approach, no pivoting.

$$aa = \begin{pmatrix} 1 & -5 & 20 & 0 & 0 \\ 0 & -2 & 10 & 1 & 0 \\ 0 & 2 & 5 & 0 & 1 \end{pmatrix}$$

```
{{1, -5, 20, 0, 0}, {0, -2, 10, 1, 0}, {0, 2, 5, 0, 1}}
```

$$bb = \begin{pmatrix} 0 \\ 5 \\ 10 \end{pmatrix}$$

```
{{0}, {5}, {10}}
```

The solution by the raw simplex method works, but does not produce the minimum, which apparently is -10. I do not take the time to understand how to manipulate the simplex matrix, though the text answer hints that doing so would get to the correct answer.

```
LinearSolve[aa, bb]
```

$$\left\{\left\{-\frac{15}{2}\right\}, \left\{\frac{5}{2}\right\}, \{1\}, \{0\}, \{0\}\right\}$$

I haven't had much luck with **`LinearProgramming`**. From the many failures I have experienced I believe that it simply will not accept two sets of constraints. One set works fine, and I did not notice any examples in the documentation that showed it solving two standard constraint relations. For instance, this works

```
LinearProgramming[{5, -20}, {{-2, 10}}, {{5, -1}}, {10, 10}]
```

$$\left\{\frac{95}{2}, 10\right\}$$

but this does not.

```
LinearProgramming[{5, -20}, {{-2, 10}},
 {{5, -1}}, {{2, 5}}, {{10, -1}}, {10, 10}]
```

LinearProgramming::nonopt : Options expected (instead of {10, 10}) beyond position 5 in
LinearProgramming[{5, -20}, {{-2, 10}}, {{5, -1}}, {{2, 5}}, {{10, -1}}, {10, 10}]. An option must be a rule or a list of rules. >>

```
LinearProgramming[{5, -20}, {{-2, 10}},
 {{5, -1}}, {{2, 5}}, {{10, -1}}, {10, 10}]
```

The way to get Minimize to work is to add some slack relations. I play with the four possibilities around the origin.

```
Minimize[{5 x - 20 y, -2 x + 10 y ≤ 5, 2 x + 5 y ≤ 10, x ≥ 0, y ≥ 0}, {x, y}]
```

$$\left\{-10, \left\{x \to 0, y \to \frac{1}{2}\right\}\right\}$$

The above cell matches the answer in the text.

```
Minimize[{5 x - 20 y, -2 x + 10 y ≤ 5, 2 x + 5 y ≤ 10, x ≤ 0, y ≥ 0}, {x, y}]
```

$$\left\{-\frac{25}{2}, \left\{x \to -\frac{5}{2}, y \to 0\right\}\right\}$$

The above cell reports an answer which is less than the text answer. Belatedly I see the restriction on x_j in the problem description to be nonnegative, so the answer is merely a curiosity.

$$5\,x - 20\,y \;/.\; \left\{x \to -\frac{5}{2}, y \to 0\right\}$$

$$-\frac{25}{2}$$

Another possibility, a somewhat desperate one, would have been to use **FindInstance** to generate a big table, use the table's list, cranking the values through the starting expression, then use **Min** on the output list. This would not be definitive, but then neither is the way I did it.

```
intab = Table[FindInstance[{5 x - 20 y ≤ n && -2 x + 10 y ≤ 5 && 2 x + 5 y ≤ 10},
    {x, y}], {n, 5, -5, -0.2}];
```

7. Suppose we produce x_1 AA batteries by process P1 and x_2 by process P2, furthermore x_3 A batteries by process P3 and x_4 by process P4. Let the profit for 100 batteries be \$10 for AA and \$20 for A. Maximize the total profit subject to the constraints
$12\,x_1 + 8\,x_2 + 6\,x_3 + 4\,x_4 \le 120$ (Material)
$3\,x_1 + 6\,x_2 + 12\,x_3 + 24\,x_4 \le 180$ (Labor)

Some battery manufacturing processes yield no profit but have to be present in constraint conditions. If x is profit (for 100 batteries) for process P1 and y is profit (for 100 batteries) for process P2 and z is profit (for 100 batteries) for process P3 and w is profit (for 100 batteries) for process P4, then

```
Clear["Global`*"]
```

```
Maximize[{10 x + 10 y + 20 z + 20 w, 12 x + 8 y + 6 z + 4 w ≤ 120 ,
  3 x + 6 y + 12 z + 24 w ≤ 180, x ≥ 0, y ≥ 0 , z ≥ 0, w ≥ 0}, {x, y, z, w}]
```

$$\left\{\frac{2200}{7}, \left\{x \to \frac{20}{7}, y \to 0, z \to \frac{100}{7}, w \to 0\right\}\right\}$$

The above cell matches the answer in the text. It looks like processes P2 and P4 do not accrue net profits.

9. Maximize $f = 2 x_1 + x_2 + 3 x_3$ subject to $4 x_1 + 3 x_2 + 6 x_3 = 12$

The answer I got here is the same answer as text's for the maximum, but the text answer has more info on the starting point.

```
Clear["Global`*"]
```

```
Maximize[{2 x + y + 3 z, 4 x + 3 y + 6 z ≤ 12, x ≥ 0, y ≥ 0, z ≥ 0} , {x, y, z}]
```

```
{6, {x → 0, y → 0, z → 2}}
```

```
(2 x + y + 3 z) /. {x → 0, y → 0, z → 2}
```

```
6
```

11. Problem 22 in problem set 22.2.

22. Nutrition. Foods A and B have 600 and 500 calories, contain 15 g and 30 g of protein, and cost \$1.80 and \$2.10 per unit, respectively. Find the minimum cost diet of at least 3900 calories containing at least 150 g protein.

Here I guess x will be the amount of food A and y will be the amount of food B that will fill the requirements stated in the problem.

```
Clear["Global`*"]
```

```
Minimize[{d x + d y, a x + b y ≥ 3900 c, a x + b y ≥ 150 g, a == 600 c, b == 500 c,
   a == 15 g, b = 30 g, a == 1.8 d, b = 2.1 d, x ≥ 0, y ≥ 0}, {x, y}];
```

```
Minimize[{1.8 x + 2.1 y, 600 x + 500 y ≥ 3900 , 15 x + 30 y ≥ 150}, {x, y}]
```

```
{13.5, {x → 4., y → 3.}}
```

The above cell matches the answer in the text. I will buy \$13.50 of food, consisting of 4 units of A and 3 units of B. The first formulation, unexecuted, allows me to look at the pieces. I consolidate into the second formulation, which runs successfully. It's nice that completely different units do not interfere with each other. It doesn't matter what they are, only that they are constraints.

13. Maximize $f = 34 x_1 + 29 x_2 + 32 x_3$ subject to $8 x_1 + 2 x_2 + x_3 \leq 54$, $3 x_1 + 8 x_2 + 2 x_3 \leq 59$, $x_1 + x_2 + 5 x_3 \leq 39$

```
Clear["Global`*"]
```

The problem takes on a complicated look. I have to sort it out.

```
Maximize[{34 x + 29 y + 32 z, 8 x + 2 y + z ≤ 54 ,
  3 x + 8 y + 2 z ≤ 59, x + y + 5 z ≤ 39}, {x, y}]
```

$$\left\{\begin{cases} \frac{1}{2}(2222 - 211\, z) & z > 6 \\ \frac{1}{58}(19666 + 1343\, z) & \text{True} \end{cases},\ \left\{x \to \begin{cases} \frac{1}{29}(157 - 2\, z) & z \le 6 \\ \frac{1}{2}(-8 + 3\, z) & \text{True} \end{cases},\ y \to \begin{cases} \frac{1}{2}(86 - 13\, z) & z > 6 \\ \frac{1}{58}(310 - 13\, z) & \text{True} \end{cases}\right\}\right\}$$

$$\text{Limit}\left[\frac{1}{2}(2222 - 211\, z),\ z \to 6,\ \text{Direction} \to -1\right]$$

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$$\text{Limit}\left[\frac{1}{58}(19666 + 1343\, z),\ z \to 6\right]$$

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So from the above it looks like I want $z = 6$. And

$$x = \text{N}\left[\frac{1}{29}(157 - 12)\right]$$

5.

and likewise

$$y = \text{N}\left[\frac{1}{58}(310 - 13 \times 6)\right]$$

4.