ODEs. Integrating Factors. Test for exactness. If exact, solve. If not, use an integrating factor as given or obtained by inspection or by the theorems in the text. Also, if an initial condition is given, find the corresponding particular solution.

1. $2\,xy\,dx + x^2\,dy = 0$

```
ClearAll["Global`*"]
```

```
eqn = 2 x y[x] + x^2 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \frac{C[1]}{x^2}]\}\}$

```
eqn /. sol
```

```
{True}
```

```
ClearAll["Global`*"]
```

2. $x^3 + y[x]^3\, y'[x] = 0$

```
eqn = x^3 + y[x]^3 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, -i\,(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, i\,(-x^4 + 4\,C[1])^{1/4}]\},$
$\{y \to \text{Function}[\{x\}, (-x^4 + 4\,C[1])^{1/4}]\}\}$

```
eqn /. sol[[1]]
```

```
True
```

```
eqn /. sol[[2]]
```

```
True
```

```
eqn /. sol[[3]]
```

```
True
```

```
eqn /. sol[[4]]
```

```
True
```

3. $\sin x \cos y + \cos x \sin y y' = 0$

```
ClearAll["Global`*"]
```

```
eqn = Sin[x] Cos[y[x]] + Cos[x] Sin[y[x]] y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{y \to \text{Function}[\{x\}, -\text{ArcCos}[\tfrac{1}{2} C[1] \text{Sec}[x]]]\}, \{y \to \text{Function}[\{x\}, \text{ArcCos}[\tfrac{1}{2} C[1] \text{Sec}[x]]]\}\}$$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True
```

4. $e^{3\theta}(r'[\theta] + 3 r[\theta]) = 0$

```
ClearAll["Global`*"]

eqn = e^(3 θ) (r'[θ] + 3 r[θ]) == 0;
sol = DSolve[eqn, r, θ]
```

$$\{\{r \to \text{Function}[\{\theta\}, e^{-3\theta} C[1]]\}\}$$

```
eqn /. sol
{True}
```

5. $(x^2 + y^2) - 2\,xyy' = 0$

```
ClearAll["Global`*"]

eqn = x^2 + y[x]^2 - 2 x y[x] y'[x] == 0;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\sqrt{x}\sqrt{x + C[1]}]\}, \{y \to \text{Function}[\{x\}, \sqrt{x}\sqrt{x + C[1]}]\}\}$$

```
Simplify[eqn /. sol[[1]]]
True

Simplify[eqn /. sol[[2]]]
True
```

6. $3(y+1) = 2\,xy', \ (y+1)x^{-4}$

```
ClearAll["Global`*"]
```

```
eqn = 3 (y[x] + 1) == 2 x y'[x];
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -1 + x^{3/2}\, C[1]]\}\}$

```
eqn /. sol
```

```
{True}
```

7. $2x\tan y + \sec^2 y\, y' = 0$

```
ClearAll["Global`*"]
```

```
eqn = 2 x Tan[y[x]] + Sec[y[x]]^2 y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, \text{ArcCot}[e^{x^2 - 2\, C[1]}]]\}\}$

```
Simplify[eqn /. sol]
```

```
{True}
```

8. $e^x(\cos y - \sin y\, y') = 0$

```
ClearAll["Global`*"]
```

```
eqn = e^x (Cos[y[x]] - Sin[y[x]] y'[x]) == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, -\text{ArcCos}[e^{-x-C[1]}]]\},$
$\{y \to \text{Function}[\{x\}, \text{ArcCos}[e^{-x-C[1]}]]\}\}$

```
Simplify[eqn /. sol[[1]]]
```

```
True
```

```
Simplify[eqn /. sol[[2]]]
```

```
True
```

9. $e^{2x}(2\cos y - \sin y\, y') = 0,\ y(0) = 0$

```
ClearAll["Global`*"]
```

```
eqn = ⅇ^(2 x) (2 Cos[y[x]] - Sin[y[x]] y'[x]) == 0;
sol = DSolve[{eqn, y[0] == 0}, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

General::stop: Further output of Solve::ifun will be suppressed during this calculation. >>

```
{{y → Function[{x}, -ArcCos[ⅇ^(-2 x)]]}, {y → Function[{x}, ArcCos[ⅇ^(-2 x)]]}}
```

```
Simplify[eqn /. sol[[1]]]
True
```

```
Simplify[eqn /. sol[[2]]]
True
```

10. $y + (y + \tan(x+y))\, y' = 0$, $\cos(x + y)$ [or 2(cos x cos y)]

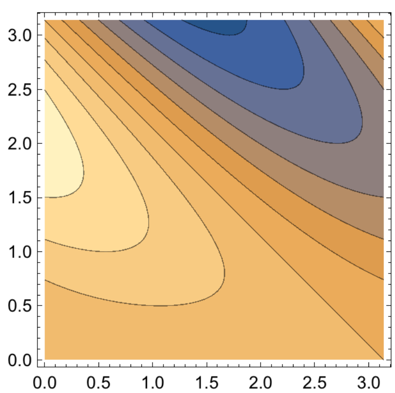
```
ClearAll["Global`*"]
```

```
eqn = y[x] + (y[x] + Tan[x + y[x]]) y'[x] == 0;
```

```
sol = DSolve[eqn, y, x] // Simplify
Solve[C[1] == Sin[x + y[x]] y[x], y[x]]
```

WolframAlpha comes up with the same thing. I don’t know how to untangle it. I don’t think the following plot is correct, but I stick it in anyway.

```
ContourPlot[Sin[x + y] y, {x, 0, π}, {y, 0, π}, ImageSize → 200]
```



11. $2 \cosh x \cos y = \sinh x \sin y\, y'$

```
ClearAll["Global`*"]
```

```
eqn = 2 Cosh[x] Cos[y[x]] == Sinh[x] Sin[y[x]] y'[x];
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, -\text{ArcCos}[-\frac{1}{2} i\, C[1]\, \text{Csch}[x]^2]]\},$
$\{y \to \text{Function}[\{x\}, \text{ArcCos}[-\frac{1}{2} i\, C[1]\, \text{Csch}[x]^2]]\}\}$

```
Simplify[eqn /. sol[[1]]]
True

Simplify[eqn /. sol[[2]]]
True
```

12. $(2\,xy + y')\,e^{x^2} = 0,\; y(0) = 2$

```
ClearAll["Global`*"]

eqn = (2 x y[x] + y'[x]) e^(x^2) == 0;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```

$\{\{y \to \text{Function}[\{x\}, 2\, e^{-x^2}]\}\}$

```
eqn /. sol
{True}
```

13. $e^{-y[x]} + e^{-x}\left(-e^{-y[x]} + 1\right) y'[x] = 0,\; F = e^{x+y[x]}$

```
ClearAll["Global`*"]

eqn = e^-y[x] + e^-x (-e^-y[x] + 1) y'[x] == 0;
sol = DSolve[eqn, y, x]
```

Solve::ifun:
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$\{\{y \to \text{Function}[\{x\}, e^x - C[1] - \text{ProductLog}[-e^{e^x - C[1]}]]\}\}$

```
Simplify[eqn /. sol]
{True}
```

14. $(a+1)\,y + (b+1)\,x\,y' = 0,\; y(1) = 1,\; F = x^a\,y^b$

```
ClearAll["Global`*"]

eqn = (a + 1) y[x] + (b + 1) x y'[x] == 0;
sol = DSolve[{eqn, y[1] == 1}, y, x]
```

$\{\{y \to \text{Function}[\{x\}, (1+b)^{\frac{1}{1+b}+\frac{a}{1+b}}\,(x + b\,x)^{-\frac{1}{1+b}-\frac{a}{1+b}}]\}\}$

```
Simplify[eqn /. sol]
{True}
```

15. Exactness. Under what conditions for the constants a, b, k, l is (a x + b y)dx + (k x + l y)dy = 0 exact? Solve the exact ODE.

```
ClearAll["Global`*"]
```

According to the exactness test, b = k. The text answer also has the relationship a*x^2 + 2*k*x*y + l*y^2 = c, but I haven't been able to track this down yet. As for the exact equation, (and substituting b for k)

$$\text{eqn} = y'[x] == -\frac{(a\,x + b\,y[x])}{(b\,x + l\,y[x])}$$

$$y'[x] == -\frac{a\,x + b\,y[x]}{b\,x + l\,y[x]}$$

```
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{-b\,x - \sqrt{e^{2\,C[1]}\,l + b^2\,x^2 - a\,l\,x^2}}{l}\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, \frac{-b\,x + \sqrt{e^{2\,C[1]}\,l + b^2\,x^2 - a\,l\,x^2}}{l}\right]\right\}\right\}$$

```
FullSimplify[eqn /. sol[[1]]]
True
```

```
FullSimplify[eqn /. sol[[2]]]
True
```