Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

3. If a steel wire 2 m in length weighs 0.9 nt (about 0.20 lb) and is stretched by a tensile force of 300 nt (about 67.4 lb), what is the corresponding speed of transverse waves?

The formula for the speed of transverse waves is just the sqrt of the tension divided by the density.

$$\texttt{density} = \frac{.9}{(2*9.8)} \quad (*\ \texttt{nts}/(\texttt{m}/\texttt{sec}^2)\ *)$$

0.0459184

```
tensile = 300 (* nts *)
```

300

$$\texttt{spd} = \sqrt{\frac{\texttt{tensile}}{\texttt{density}}}$$

80.829

```
80.82903768654761`    (* sqrt(m/sec²) *)
```

80.829

Obtaining text answer. Source: *https://www3.nd.edu/~apaul2/The_Jungle/PHYS31210L_-files/E%2010-Standing%20Waves.pdf*

5 - 8 Graphing Solutions

Using numbered line (13), p. 555, sketch or graph a figure (similar to Fig. 291 in Sec. 12.3) of the deflection u(x,t) of a vibrating string (length L=1, ends fixed, c=1) starting with initial velocity 0 and initial deflection (k small, say, k=0.01).

5. $f(x)=k \sin \pi x$

The Manipulate figure seems impossible, in view of the phantom u(x,t).

```
In[28]:= Clear["Global`*"]

Quit[]

In[29]:= c = 1; L = 1; k = 0.01;
f[x_] = k Sin[π x]

Out[30]= 0.01 Sin[π x]
```

In[31]:=
```
A[n_] =
  (2 / L) Integrate[f[x] * Sin[n π x / L], {x, 0, L}] // Assumptions → n > 1
```

Out[31]= $(\text{Assumptions} \to n > 1)\left[-\dfrac{0.0063662\ \text{Sin}[n\,\pi]}{-1. + n^2}\right]$

In[32]:=
```
Lambda[n_] = (c n π / L) ^2
```

Out[32]= $n^2\ \pi^2$

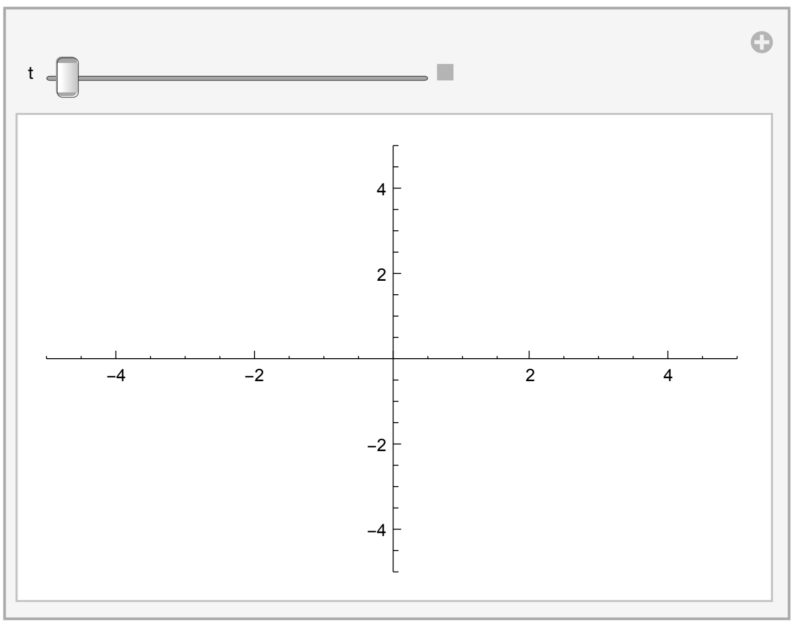
In[33]:=
```
u[x_, t_, N_] := Sum[A[n] Cos[Sqrt[Lambda[n]] t] Sin[n π x / L], {n, 2, N}]
```

In[35]:=
```
u[x, 1, 4]
```

Out[35]=
```
Sin[2 π x] (Assumptions → True) [0.] -
  Sin[3 π x] (Assumptions → True) [0.] + Sin[4 π x] (Assumptions → True) [0.]
```

In[39]:=
```
Manipulate[
  Plot[u[x, 1, 20], {x, 0, 5}, PlotRange → {{-5, 5}, {-5, 5}}], {t, 0, 20}]
```

Out[39]=



Directly using (13) from p. 555, below, does not seem to improve the situation.

In[40]:=
$$u[x\_,\ t\_] = \frac{1}{2}[f[x + c\,t] + f[x - c\,t]]$$

Out[40]=
$$\frac{1}{2}[0.01\ \text{Sin}[\pi\ (-t + x)] + 0.01\ \text{Sin}[\pi\ (t + x)]]$$

In[43]:=
```
Manipulate[Plot[u[x, t, 20], {x, 0, L}, PlotRange → {-1, 1}], {t, 0, 20}];
```

7. f(x)=k sin 2πx

For this problem I tried the trick of splitting f into a piecewise function, but to no avail. The microscopic plot range was also no help. I tried different values of k, but could not make the function show a significant value.

```
Clear["Global`*"]

c = 1; L = 1; k = 0.01;

f[x_] = Piecewise[{{k Sin[2 π x], 0 < x < L/2}, {k Sin[2 π x], L/2 < x < 1}}]
```

$$\begin{cases} 0.01\,\text{Sin}[2\pi x] & 0 < x < \frac{1}{2}\ ||\ \frac{1}{2} < x < 1 \\ 0 & \text{True} \end{cases}$$

```
A[n_] = (2 / L) Integrate[f[x] Sin[n π x / L], {x, 0, L}] // Assumptions → n > 2
```

$$(\text{Assumptions} \to n > 2)\left[\frac{0.0127324\,\text{Sin}[3.14159\,n]}{-4. + n^2}\right]$$

```
Lambda[n_] = (c n π / L) ^2
```

$n^2\,\pi^2$

```
u[x_, t_, N_] := Sum[A[n] Cos[Sqrt[Lambda[n]] t] Sin[n π x / L], {n, 3, N}]

u[x, t, 6]
```

$$\text{Cos}[4\pi t]\,\text{Sin}[4\pi x]\,(\text{Assumptions}\to\text{True})\left[-5.19756\times10^{-19}\right] +$$
$$\text{Cos}[6\pi t]\,\text{Sin}[6\pi x]\,(\text{Assumptions}\to\text{True})\left[-2.92363\times10^{-19}\right] +$$
$$\text{Cos}[5\pi t]\,\text{Sin}[5\pi x]\,(\text{Assumptions}\to\text{True})\left[3.71254\times10^{-19}\right] +$$
$$\text{Cos}[3\pi t]\,\text{Sin}[3\pi x]\,(\text{Assumptions}\to\text{True})\left[9.35561\times10^{-19}\right]$$

```
Manipulate[
  Plot[u[x, t, 20], {x, 0, L}, PlotRange → {-2*^-18, 2*^-18}], {t, 0, 20}];
```

9 - 18 Normal Forms

Find the type, transform to normal form, and solve.

Finding the type means checking numbered line (14) and the table on p. 555.

9. $u_{xx} + 4\,u_{yy} = 0$

Elliptic. As far as normal form is concerned, the form presented below is normal for **DSolve**.

```
Clear["Global`*"]

a = 1; b = 0; c = 4;
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
```

$4\,u^{(0,2)}[x, y] + u^{(2,0)}[x, y] == 0$

```
sol = DSolve[eqn, u, {x, y}]
```

{{u → Function[{x, y}, C[1][2 ⅈ x + y] + C[2][-2 ⅈ x + y]]}}

The green cell above matches the text answer, with the understanding that C[1] and C[2] are considered functions by the text.

From this point to the end of this problem section, I deviate from the text and s.m. in procedure, because I could not catch on to the s.m. or text in terms of details. For source, I use a YouTube video, *https://www.youtube.com/watch?v=rO-f3zh3_kw,* time 0:00-5:12 and 7:21-11:15. The differences are as follows: (1) the coefficient of the u_{xy} factor is b, not b/2; (2) the middle coefficient of the characteristic equation has positive sign, and (3) the defining equations go like so:
Hyperbolic: u(x,y) $= f(m_1 x + y) + g(m_2 x + y)$ [yielding two unequal real roots]
Elliptic: u(x,y) $= f(m_1 x + y) + g(m_2 x + y)$ [yielding two complex roots]
Parabolic: u(x,y) $= f(m_1 x + y) + x\, g(m_2 x + y)$ [yielding two equal real roots]
(Note the extra x in Parabolic.)

11. $u_{xx} + 2\, u_{xy} + u_{yy} = 0$

```
Clear["Global`*"]
```

```
a = 1; b = 2; c = 1; b^2 - 4 a c
0
```

Parabolic.

For solns of form u(x,y) = f(mx +y) + g(mx +y) the characteristic equation is am^2 +bm + c = 0 = $m^2 + 2\,m + 1 = 0$.

```
vi = Factor[m^2 + 2 m + 1 == 0]
(1 + m)^2 == 0
```

```
Solve[vi, m]
{{m → -1}, {m → -1}}
```

Therefore the two functions sought will be:

```
funcf = (x - y)
x - y
```

```
funcg = x (x - y) (* with extra x added for parabolic*)
x (x - y)
```

The green cells above match the answer in the text.

13. $u_{xx} + 5\, u_{xy} + 4\, u_{yy} = 0$

Hyperbolic.

```
Clear["Global`*"]

a = 1; b = 5; c = 4; b^2 - 4 a c
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
9

4 u^(0,2)[x, y] + 5 u^(1,1)[x, y] + u^(2,0)[x, y] == 0

characteqn = m^2 + 5 m + 4 == 0
4 + 5 m + m^2 == 0

vi = Factor[characteqn]
(1 + m) (4 + m) == 0

Solve[vi, m]
{{m → -4}, {m → -1}}
```

Therefore the eqns sought will be:

```
f[x_, y_] = y - 4 x
g[x_, y_] = y - x

-4 x + y

-x + y
```

The green cells above match the answer in the text.

15. $xu_{xx} - yu_{xy} = 0$

Hyperbolic. This one does not match the template, because the coefficients are not constants.

```
In[54]:= Clear["Global`*"]

In[55]:= a = 1; b = -1; c = 0; b^2 - 4 a c
         eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
Out[55]= 1

Out[56]= -u^(1,1)[x, y] + u^(2,0)[x, y] == 0

In[57]:= characteqn = m^2 - m == 0
Out[57]= -m + m^2 == 0

In[58]:= vi = Factor[characteqn]
Out[58]= (-1 + m) m == 0

In[59]:= characteqn = m^2 - m == 0
Out[59]= -m + m^2 == 0
```

In[60]:=
```
eqn2 = x * D[u[x, y], {x, 2}] - y * D[u[x, y], x, y] == 0
```
Out[60]= $-y\, u^{(1,1)}[x, y] + x\, u^{(2,0)}[x, y] == 0$

In[61]:=
```
DSolve[x * D[u[x, y], {x, 2}] - y * D[u[x, y], x, y] == 0, u, {x, y}]
```
Out[61]= $\text{DSolve}\left[-y\, u^{(1,1)}[x, y] + x\, u^{(2,0)}[x, y] == 0, u, \{x, y\}\right]$

Maybe at a later date I can try something here. The text answer provides the needed substitutions to perform the method of characteristics by hand. It is interesting that, as it stands, it is unsolvable by Mathematica 10.3, 11.1, or WolframAlpha.

17. $u_{xx} - 4\, u_{xy} + 5\, u_{yy} = 0$

Elliptic.

```
Clear["Global`*"]
```

```
a = 1; b = -4; c = 5; b^2 - 4 a c
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
```
-4

$5\, u^{(0,2)}[x, y] - 4\, u^{(1,1)}[x, y] + u^{(2,0)}[x, y] == 0$

```
characteqn = m^2 - 4 m + 5 == 0
```
$5 - 4\, m + m^2 == 0$

```
vi = Factor[characteqn]
```
$5 - 4\, m + m^2 == 0$

```
Solve[vi, m]
```
$\{\{m \to 2 - i\}, \{m \to 2 + i\}\}$

Therefore the two sought functions will be:

```
f[x_, y_] = x (2 - i) + y
g[x_, y_] = x (2 + i) + y
```
$(2 - i)\, x + y$

$(2 + i)\, x + y$

The functions shown in the green cell above match the text answer (after digesting signs).