11 - 23 Vector and scalar triple products

With respect to right-handed Cartesian coordinates, let a = {2, 1, 0} , b = {-3, 2, 0}, c = {1, 4, -2}, and d = {5, -1, 3}. Showing details, find:

11. a × b, b × a, a.b

```
Clear["Global`*"]
```

```
e1 = a = {2, 1, 0}
{2, 1, 0}
```

```
e2 = b = {-3, 2, 0}
{-3, 2, 0}
```

```
e3 = c = {1, 4, -2}
{1, 4, -2}
```

```
e4 = d = {5, -1, 3}
{5, -1, 3}
```

```
e5 = e1 × e2
```

```
{0, 0, 7}
```

```
e6 = e2 × e1
```

```
{0, 0, -7}
```

```
e7 = e1.e2
```

```
-4
```

13. c × (a+b), a × c + b × c

```
e8 = e3 × (e1 + e2)
```

```
{6, 2, 7}
```

```
e85 = e1 × e3 + e2 × e3
```

```
{-6, -2, -7}
```

15. (a + d) × (d + a)

```
e9 = (e1 + e4) × (e4 + e1)
```

```
{0, 0, 0}
```

17. (b × c) × d, b × (c × d)

```
e10 = (e2 × e3) × e4
```

```
{-32, -58, 34}
```

```
e11 = e2 × (e3 × e4)
```

```
{-42, -63, 19}
```

19. (i j k), (i k j)

```
i1 = {1, 0, 0}; j1 = {0, 1, 0}; k1 = {0, 0, 1}
{0, 0, 1}
```

```
e12 = (i1 × j1.k1)
```

```
1
```

```
e13 = i1.k1 × j1
```

```
-1
```

Above: the text did not show any operator symbols, so I took a guess, experimenting a little to get the text answer.

21. 4b × 3c, 12|b × c|, 12|c × b|

```
e14 = (4 e2) × (3 e3)
```

```
{-48, -72, -168}
```

```
e15 = 12 Norm[e2 × e3]
```

$24\sqrt{62}$

```
e16 = 12 Norm[e3 × e2]
```

$24\sqrt{62}$

23. b × b, (b - c) × (c - b), b.b

```
e17 = e2 × e2
```

```
{0, 0, 0}
```

```
e18 = (e2 - e3) × (e3 - e2)
```

```
{0, 0, 0}
```

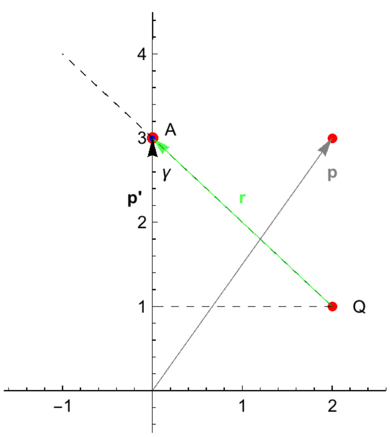
```
e19 = e2.e2
```

```
13
```

25 - 35 Applications

25. Moment **m** of a force **p**. Find the moment vector **m** and *m* of **p** = {2, 3, 0} about *Q*: (2, 1, 0) acting on a line through *A*: {0, 3, 0}. Make a sketch.

Since all the coordinates for the z-axis are zero, this problem can be considered in two dimensions. However, if I need to do any cross products, I will need to include all three coordinates.



In example 3 on p. 371, the line of action of **p**' goes through A. I think that needs to be maintained. The vector **p** does not actually go through A, but there would be a component. The length of this component would be the norm of **p** times the cosine of the angle γ between. I could call the vector with this length and A's direction, **p**'.

```
Clear["Global`*"]
```

```
e2 = {2, 3}
```

```
{2, 3}
```

```
e3 = Norm[e2]
```

$\sqrt{13}$

```
e4 = ArcTan[2/3]
```

$\text{ArcTan}\left[\frac{2}{3}\right]$

```
e5 = Cos[e4]
```

$\frac{3}{\sqrt{13}}$

```
e6 = e3 e5
```

3

Here is something remarkable. If I haven't miscalculated, the vector **p**' is a vector terminating at A.

The length of **p**' is 3. The quantity mlight is the norm of mbold. Not written in the text or problem description as a norm, though just light face, not bold.

Because of the length of its sides being equal, the angle γ is seen to be $\frac{\pi}{4}$.

```
e7 = rbold = {0, 3} - {2, 1}
```

{-2, 2}

```
e9 = mbold = e7 × e2
```

Cross::nonn1: The arguments are expected to be vectors of equal length, and the number of arguments is expected to be 1 less than their length ≫

{-2, 2} × {2, 3}

Above: here is where I have to put the third coordinate back in.

```
e10 = mbold3 = {-2, 2, 0} × {2, 3, 0}
```

{0, 0, -10}

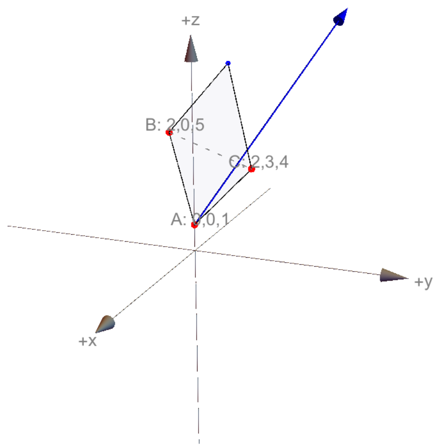
```
e11 = mlight = Norm[e10]
```

10

Looking down the z-axis toward the sketch of the problem, with positive x to the right, the moment mbold would tend to exert a clockwise motion around Q.

29. Triangle. Find the area if the vertices are {0, 0, 1}, {2, 0, 5}, and {2, 3, 4}.

```
Clear["Global`*"]
```



In the sketch, I need to find the area of the triangle, ABC. Following the s.m. pretty closely, I make two vectors out of points B and C, using the common point A as their origin.

```
bbold = {2 - 0, 0 - 0, 5 - 1}
{2, 0, 4}
```

```
cbold = {2 - 0, 3 - 0, 4 - 1}
{2, 3, 3}
```

Then I cross these two,

```
vbold = bbold × cbold
{-12, 2, 6}
```

and find the norm of the cross vbold,

```
e1 = Norm[vbold]
```

$2\sqrt{46}$

The s.m. reminds me that the cross product is defined in such a way that its length is equal to the area of the base parallelogram (see sketch). Since the area of the triangle I want is exactly half the area of the parallelogram, I have,

$$e2 = \frac{e1}{2}$$

$\sqrt{46}$

I added vbold to the sketch.

Note: Green cells in this problem set agree with the corresponding answers in the text.

31. Plane. Find the plane through (1, 3, 4), (1, -2, 6) and (4, 0, 7).

```
Clear["Global`*"]
```

In[49]:= $v_1 = \{1, 3, 4\}$

Out[49]= $\{1, 3, 4\}$

In[50]:= $v_2 = \{1, -2, 6\}$

Out[50]= $\{1, -2, 6\}$

In[51]:= $v_3 = \{4, 0, 7\}$

Out[51]= $\{4, 0, 7\}$

In[52]:= perp = Cross[$v_2 - v_1$, $v_2 - v_3$]

Out[52]= $\{9, -6, -15\}$

In[55]:= `Simplify[9 (x - 1) - 6 (y + 2) - 15 (z - 6) == 0]`

Out[55]= $23 + 3x == 2y + 5z$

The equation in the yellow cell does not match the answer in the text. However, it does check with the answer put forth by WolframAlpha.

33. Tetrahedron. Find the volume if the vertices are (1, 1, 1), (5, -7, 3), (7, 4, 8), and (10, 7, 4).

In[56]:= `Clear["Global`*"]`

Taking a tip from Weisstein's *MathWorld*, the volume should be

In[57]:= $$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 5 & -7 & 3 & 1 \\ 7 & 4 & 8 & 1 \\ 10 & 7 & 4 & 1 \end{pmatrix}$$

Out[57]= $\{\{1, 1, 1, 1\}, \{5, -7, 3, 1\}, \{7, 4, 8, 1\}, \{10, 7, 4, 1\}\}$

In[58]:= $$\text{volume} = \frac{1}{3!}\text{Det}[A]$$

Out[58]= 79

The answer in the green cell above matches that of the text.