Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

6 - 18 Radius of convergence

Find the center and the radius of convergence.

7. $\text{Sum}\left[\frac{(-1)^n}{(2\,n)\,!}\left(z-\frac{1}{2}\pi\right)^{2\,n},\ \{n,\ 0,\ \infty\}\right]$

```
Clear["Global`*"]
```

The s.m. does this problem, and informs me that the center of the series is $\frac{\pi}{2}$.

So to find it, I will look in that part of the series general term which is in the form (z - a). As for the radius of curvature itself, it is discovered through the ratio of a_n to a_{n+1}. Here I will boldly try it without refining,

$\text{Limit}\left[\text{Abs}\left[\frac{(-1)^n}{(2\,n)\,!}\left(\frac{(2\,n+2)\,!}{(-1)^{n+1}}\right)\right],\ n\to\infty\right]$

∞

Numbered line (6) on p. 683 has the form showing the absolute value, which I will try to remember. In this case including it made the difference in the sign in the answer, which otherwise would have been negative.

9. $\text{Sum}\left[\frac{n\,(n-1)}{3^n}\,(z-i)^{2\,n},\ \{n,\ 0,\ \infty\}\right]$

```
Clear["Global`*"]
```

Now that I have got my feet wet, here is a second one.

I know to go right to the characteristic form to find that the center is i.

And I will immediately try to format that which worked last time,

$\text{Limit}\left[\text{Abs}\left[\frac{n\,(n-1)}{3^n}\left(\frac{3^{n+1}}{(n+1)\,(n)}\right)\right],\ n\to\infty\right]$

3

Since the radius of convergence is finite, the power on the power term will have an effect. This power term is 2n, so the result will be raised to the $\frac{1}{2}$ power. Thus

$3^{1/2}$

$\sqrt{3}$

Gives the net radius of convergence. Problem 1 also had a power term with power of 2n. However, in that case the radius was infinity, and the square root of infinity is infinity.

11. $\text{Sum}\left[\left(\frac{(2-i)}{1+5\,i}\right) z^n, \{n, 0, \infty\}\right]$

Here the center of the series is 0.

```
Clear["Global`*"]
```

$\text{Limit}\left[\text{Abs}\left[\frac{(2-i)}{1+5\,i}\left(\frac{1+5\,i}{(2-i)}\right)\right], n \to \infty\right]$

1

The power of the power term being 1, the full determination should be

$(1)^{1/1}$

1

However, this does not match the text answer. Let me try an alternate route, from numbered line (6*) on p. 684,

$\tilde{L} = \text{Limit}\left[\sqrt[n]{\text{Abs}\left[\frac{(2-i)}{1+5\,i}\right]}, n \to \infty\right]$

1

$R = \frac{1}{\tilde{L}}$

1

That does not help either. There is also numbered line (6**) on p. 684, describing $\frac{1}{\tilde{l}}$, but that gives the same thing, 1. Either an error in answer or problem, or something that I don't see at this point.

13. $\text{Sum}\left[16^n (z+i)^{4n}, \{n, 0, \infty\}\right]$

```
Clear["Global`*"]
```

Here the center of the series is -i.

$$\text{Limit}\left[\text{Abs}\left[\frac{16^n}{16^{n+1}}\right], n \to \infty\right]$$

$$\frac{1}{16}$$

The power on the power term is 4n, so the radius of convergence will be affected as so,

$$\left(\frac{1}{16}\right)^{1/4}$$

$$\frac{1}{2}$$

$$15.\quad \text{Sum}\left[\left(\frac{(2\,n)\,!}{4^n\,(n\,!)^2}\right)(z - 2\,i)^n, \{n, 0, \infty\}\right]$$

```
Clear["Global`*"]
```

Here the center of the series is 2 *i*.

$$\text{Limit}\left[\text{Abs}\left[\frac{(2\,n)\,!}{4^n\,(n\,!)^2}\left(\frac{4^{n+1}\,((n+1)\,!)^2}{(2\,n+2)\,!}\right)\right], n \to \infty\right]$$

$$1$$

Since the power of the power term is 1, I will have

$$(1)^{1/1}$$

$$1$$

$$17.\quad \text{Sum}\left[\left(\frac{2^n}{n\,(n+1)}\right) z^{2\,n+1}, \{n, 1, \infty\}\right]$$

```
Clear["Global`*"]
```

Here the center of the series is 0. However, the text answer does not mention this, so I won’t green it.

$$\text{Limit}\left[\text{Abs}\left[\frac{2^n}{n\,(n+1)}\left(\frac{(n+1)\,(n+2)}{2^{n+1}}\right)\right], n \to \infty\right]$$

$$\frac{1}{2}$$

Now what to do about the power term power? It is not even. First I will treat it as if it were even.

$$\left(\frac{1}{2}\right)^{1/2}$$

$$\frac{1}{\sqrt{2}}$$

Hmm, that worked. So for the time being I will assume that only the part of the exponent which includes the ‘n’ variable is to be consulted.