Note: Cells in green match the answers in the text.

Calculus.

1 to 8. Solve the ODE by integration or by remembering a differentiation formula.

1. $y' + 2 \sin 2\pi x = 0$

```
Clear["Global`*"]
```

```
DSolve[y'[x] + 2 Sin[2 π x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \rightarrow C[1] + \frac{\text{Cos}[2 \pi x]}{\pi}\right\}\right\}$$

```
Clear["Global`*"]
```

2. $y' + xe^{-x^2/2} = 0$

$$\text{DSolve}\left[y[x] + x\, e^{-x^2/2} == 0, y[x], x\right]$$

$$\left\{\left\{y[x] \rightarrow -e^{-\frac{x^2}{2}} x\right\}\right\}$$

```
Clear["Global`*"]
```

3. $y' = y$

```
DSolve[y'[x] == y[x], y[x], x]
```

$$\{\{y[x] \rightarrow e^x C[1]\}\}$$

4. $y' = -1.5y$

```
DSolve[y'[x] == -1.5 y[x], y[x], x]
```

$$\left\{\left\{y[x] \rightarrow e^{-1.5\,x} C[1]\right\}\right\}$$

5. $y' = 4e^{-x} \cos x$

```
DSolve[y'[x] == 4 e^-x Cos[x], y[x], x]
```

$$\{\{y[x] \rightarrow C[1] + 2 e^{-x} (-\text{Cos}[x] + \text{Sin}[x])\}\}$$

6. $y'' = -y$

```
DSolve[y''[x] == -y[x], y[x], x]
```

$$\{\{y[x] \rightarrow C[1] \text{Cos}[x] + C[2] \text{Sin}[x]\}\}$$

7. $y' = \cosh 5.13x$

```
DSolve[y'[x] == Cosh[5.13 x], y[x], x]
```

$\{\{y[x] \to C[1] + 0.194932 \, \text{Sinh}[5.13 \, x]\}\}$

```
1 / 5.13
0.194932
```

8. $y''' = e^{-0.2x}$

```
DSolve[y'''[x] == e^{-0.2 x}, y[x], x]
```

$\{\{y[x] \to -125. \, e^{-0.2 \, x} + C[1] + x \, C[2] + x^2 \, C[3]\}\}$

9 to 15. Verification. Initial value problem (IVP)
(a) Verify that y is a solution of the ODE. (b) Determine from y the particular solution of the IVP. (c) Graph the solution of the IVP.

9. $y' + 4y = 1.4$, $y = c \, e^{-4x} + 0.35$, $y(0) = 2$

```
eqn = y'[x] + 4 y[x] == 1.4;
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \, 0.35 + e^{-4. \, x} \, C[1]]\}\}$

(a)

The means of checking is as follows

```
Simplify[eqn /. sol]
```

```
{True}
```

Of course if I want to go to a lot of trouble to make sure the solution in the text is really correct, I could do

```
u[x_] = (e^{-4 x} + .35)
```

$0.35 + e^{-4 \, x}$

```
uprime = D[e^{-4 x} + .35, x]
```

$-4 \, e^{-4 \, x}$

$-4 \, e^{-4 \, x}$

$-4 \, e^{-4 \, x}$

```
rsec = 4 u[x] + uprime
```

$-4 \, e^{-4 \, x} + 4 \, (0.35 + e^{-4 \, x})$

```
Simplify[rsec]
```

1.4

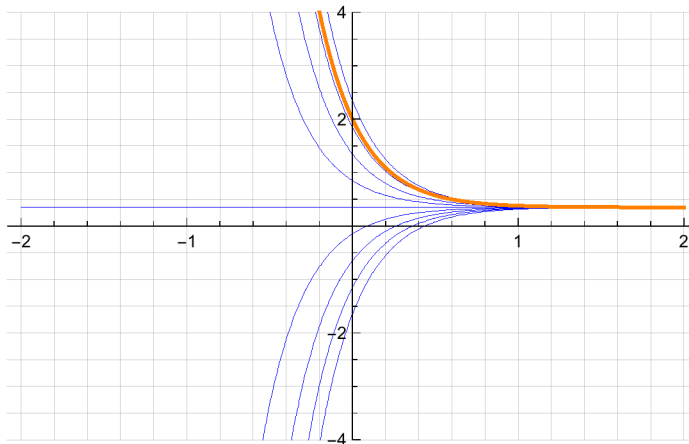The 1.4 above is the rhs of the original equation. The book solution checks.

(b)

```
Solve[0.35 + c1 == 2, c1]
```

{{c1 → 1.65}}

(c) In the plot below, the IVP point is seen in orange. The lower, left-most is the -0.35 line.

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 2, 0.5}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 1.65, 1.65}];
Show[Plot[tab[x], {x, -2, 2}, PlotRange → {-4, 4},
   PlotStyle → {Blue, Thin}, GridLines → All],
 Plot[tabl[x], {x, -2, 2}, PlotRange → {-4, 4},
   PlotStyle → {Orange, Thick}]]
```



```
Clear["Global`*"]
```

10. $y' + 5xy = 0$, $y = ce^{-2.5x^2}$, $y(0) = \pi$

```
eqn = y'[x] + 5 x y[x] == 0;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, e^{-\frac{5x^2}{2}} C[1]\right]\right\}\right\}$$

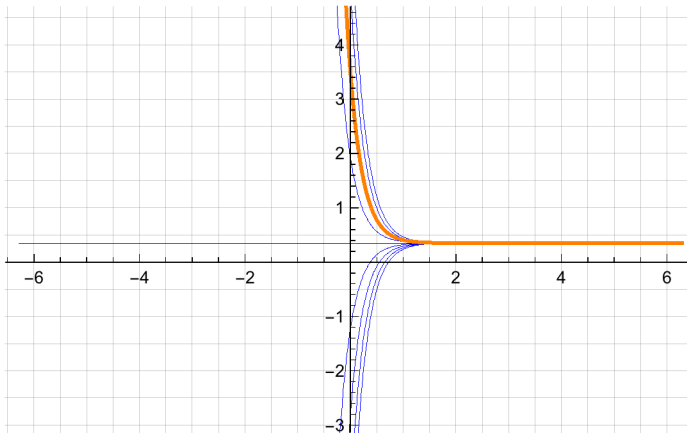(a) The DSolve answer is seen to equal the book's proposed solution.

```
Simplify[eqn /. sol]
```

{True}

(b)

```
Solve[c1 == π, c1]
```

```
{{c1 → π}}
```

(c)

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2 π, 2 π, π/2}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, π, π}];
Show[Plot[tab[x], {x, -2 π, 2 π}, PlotRange → {-π, 3π/2},
   PlotStyle → {Blue, Thin}, GridLines → All],
  Plot[tabl[x], {x, -2 π, 2 π}, PlotRange → {-π, 3π/2},
   PlotStyle → {Orange, Thick}]]
```



```
Clear["Global`*"]
```

11. $y' = y + e^x$, $y = (x + c)\, e^x$, $y(0) = \frac{1}{2}$

```
eqn = y'[x] - y[x] - e^x == 0;
sol = DSolve[eqn, y, x]
```

```
{{y → Function[{x}, e^x x + e^x C[1]]}}
```

(a) The DSolve answer is seen to equal the book's proposed solution.

```
eqn /. sol
```

```
{True}
```

(b)

```
Solve[0 + c1 == 1/2, c1]
```

$$\left\{\left\{c1 \to \frac{1}{2}\right\}\right\}$$
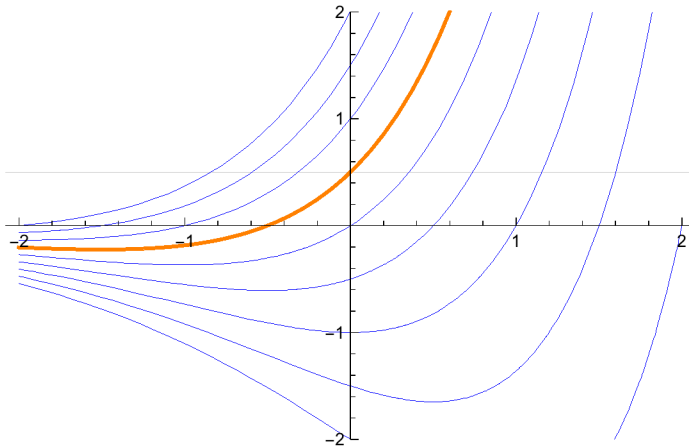
(c)

```
g[x_] = y[x] /. sol;
tab[x_] = Table[g[x] /. C[1] → j, {j, -2, 2, 1/2}];

tabl[x_] = Table[g[x] /. C[1] → v, {v, 1/2, 1/2}];
Show[Plot[tab[x], {x, -2, 2}, PlotRange → {-2, 2},

   PlotStyle → {Blue, Thin}, GridLines → {{0}, {1/2}}],

  Plot[tabl[x], {x, -2, 2}, PlotRange → {-2, 2},
   PlotStyle → {Orange, Thick}]]
```



```
Clear["Global`*"]
```

12. $yy' = 4x$, $y^2 - 4x^2 = C$ $(y > 0)$, $y(1) = 4$

```
eqn = y[x] y'[x] - 4 x == 0;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\sqrt{2}\sqrt{2 x^2 + C[1]}\right]\right\},\right.$$
$$\left.\left\{y \to \text{Function}\left[\{x\}, \sqrt{2}\sqrt{2 x^2 + C[1]}\right]\right\}\right\}$$

(a) The DSolve answer [2] is seen to equal the book's proposed solution.

i.e., for $y > 0$, $y^2 = 4x^2 + C \to y = \sqrt{4x^2 + C} \to \sqrt{2}\sqrt{2x^2 + C/\text{Sqrt}[2]}$
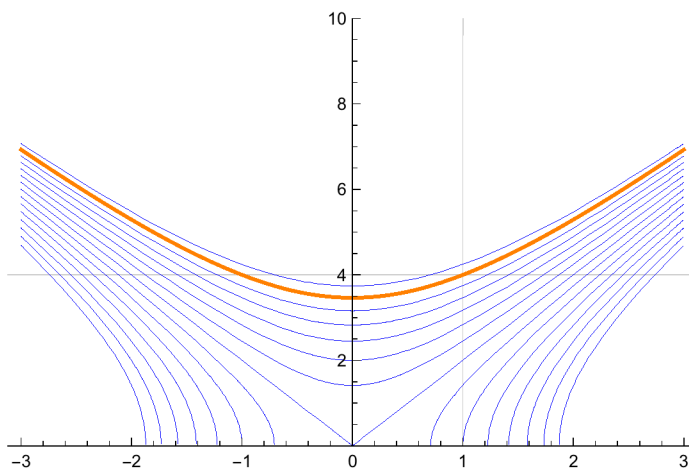
```
eqn /. sol[[2]]
True
```

(b)

```
Solve[√2 √2 + C[1] == 4, C[1]]

{{C[1] → 6}}
```

(c)

```
g[x_] = y[x] /. sol[[2]];
tab[x_] = Table[g[x] /. C[1] → j, {j, -7, 7, 1}];
tabl[x_] = Table[g[x] /. C[1] → v, {v, 6, 6}];
Show[Plot[tab[x], {x, -3, 3}, PlotRange → {0, 10},
   PlotStyle → {Blue, Thin}, GridLines → {{1}, {4}}],
  Plot[tabl[x], {x, -3, 3}, PlotRange → {0, 10},
   PlotStyle → {Orange, Thick}]]
```



```
Clear["Global`*"]
```

13. $y' = y - y^2$, $y = \frac{1}{1+ce^{-x}}$, $y(0) = 0.25$

```
eqn = y'[x] - y[x] + (y[x])² == 0;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{e^x}{e^x + e^{C[1]}}\right]\right\}\right\}$$

```
Simplify[eqn] /. Simplify[sol]

{True}
```

(a)

The question is whether the following is true:
$$\frac{e\,\hat{}\,x}{e\,\hat{}\,x + e\,\hat{}\,C[1]} =? \frac{1}{1 + C[2]\,e\,\hat{}\,{-x}}$$
Multiplying the rhs by $\frac{e\,\hat{}\,x}{e\,\hat{}\,x}$ I get:
$$\frac{e\,\hat{}\,x}{e\,\hat{}\,x + C[2]}$$
So all I have to do is set C[2] = $e\,\hat{}\,$C[1].

Since this is legal (constants are real, not necessarily rational), I conclude that the expressions are equivalent.

(b)

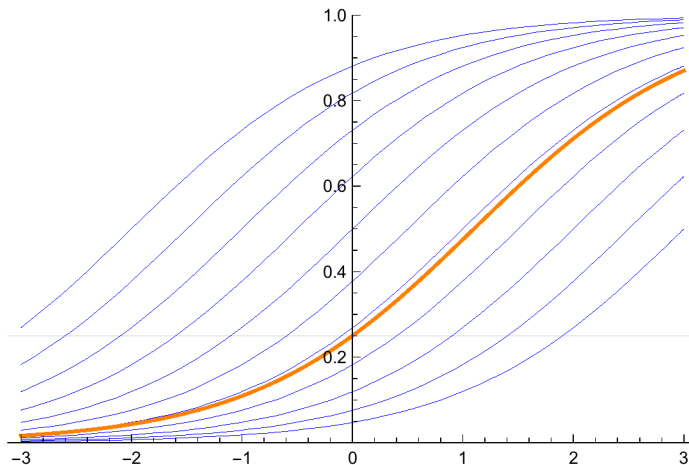$$\texttt{Solve}\left[\frac{1}{1 + e^{C[1]}} == 0.25, \ C[1]\right]$$

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information »

```
{{C[1] → 1.09861}}
```

(c)

```
g[x_] = y[x] /. sol;
```

$$\texttt{tab[x\_]} = \texttt{Table}\left[\texttt{g[x] /. C[1]} \rightarrow \texttt{j}, \ \left\{\texttt{j}, \ -2, \ 3, \ \frac{1}{2}\right\}\right];$$

```
tabl[x_] = Table[g[x] /. C[1] → v, {v, 1.09861, 1.09861}];
Show[Plot[tab[x], {x, -3, 3}, PlotRange → {0, 1},
  PlotStyle → {Blue, Thin}, GridLines → {{0}, {0.25}}],
 Plot[tabl[x], {x, -3, 3}, PlotRange → {0, 1},
  PlotStyle → {Orange, Thick}]]
```



```
Clear["Global`*"]
```

14. $y' \tan x = 2y + 8$, $y = c \sin^2 x + 4$, $y\left(\frac{1}{2}\pi\right) = 0$

```
eqn = y'[x] Tan[x] - 2 y[x] == 8;
sol = DSolve[eqn, y, x]
```

$$\left\{\left\{y \rightarrow \texttt{Function}\left[\{x\}, \ -4 \, \texttt{Cos}[x]^2 + C[1] \, \texttt{Sin}[x]^2\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

(a) So the decision is whether

-4 $\cos^2 x + C[1] \sin^2 x =^? c \sin^2 x + 4$. *I* can get to
$-4 + C[1] \sin^2 x =^? c \sin^2 x + 4$. However, *I*
don'*t* really know the answer to that
proposed equality. So *I* guess *I* better do it the long way.

```
prop1 = Sin[x]² + 4
```

$4 + \text{Sin[x]}^2$

```
D[prop1, x]
```

$2 \, \text{Cos[x] Sin[x]}$

```
Simplify[(2 Cos[x] Sin[x]) Tan[x] - 2 (Sin[x]² + 4) - 8]
```

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It looks like Mathematica is disagreeing with the text answer.

```
Simplify[(2 Cos[x] Sin[x]) Tan[x] - 2 (Sin[x]² - 4) - 8]
```

0

(b) I don't think this first 'solve' is necessary, but

```
Solve[- 4 Cos[π/2]² + C[1] Sin[π/2]² == 0, C[1]]
```

$\{\{C[1] \rightarrow 0\}\}$

```
sol2 = DSolve[{eqn, y[π/2] == 0}, y, x]
```

$\{\{y \rightarrow \text{Function}[\{x\}, -4 \, \text{Cos[x]}^2]\}\}$

(c)

```
g[x_] = y[x] /. sol2;
```

$$tab[x\_] = Table\left[g[x] /. C[1] \to j, \left\{j, -2, 3, \frac{1}{2}\right\}\right];$$

```
tabl[x_] = Table[g[x] /. C[1] → v, {v, 0, 0}];
```

$$Show\left[Plot\left[tab[x], \{x, -2\pi, 2\pi\}, PlotRange \to \{-2\pi, \pi\},\right.\right.$$

$$\left.PlotStyle \to \{Blue, Thin\}, GridLines \to \left\{\left\{\frac{\pi}{2}\right\}, \{0\}\right\},\right.$$

$$\left.Ticks \to \left\{\{-2\,Pi, -Pi, 0, Pi/2, Pi, 2\,Pi\}, \{-4, -2, 0, 2\}\right\}\right],$$

$$Plot[tabl[x], \{x, -\pi, \pi\}, PlotRange \to \{-2\pi, 2\pi\},$$

$$\left.PlotStyle \to \{Orange, Thick\}\right]\right]$$