

The s.m. has problems 3, 13.

1 - 10 Direct integration of surface integrals

Evaluate the surface integral $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA$ directly for the given \mathbf{F} and S .

1. $\mathbf{F} = \{z^2, -x^2, 0\}$, S the rectangle with vertices $\{0, 0, 0\}$, $\{1, 0, 0\}$, $\{0, 4, 4\}$, $\{1, 4, 4\}$

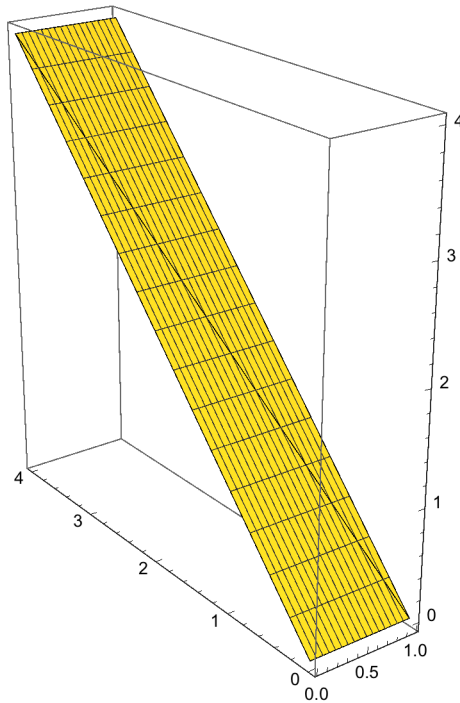
```
Clear["Global`*"]
```

```
r1 =
```

```
ListPlot3D[{{0, 0, 0}, {1, 0, 0}, {0, 4, 4}}, AspectRatio → Automatic];
```

```
r2 = ListPlot3D[{{1, 0, 0}, {0, 4, 4}, {1, 4, 4}}, AspectRatio → Automatic];
```

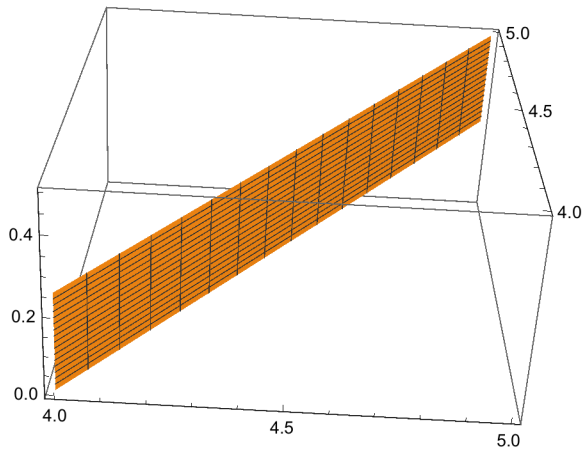
```
Show[r1, r2, BoxRatios → Automatic]
```



For some reason I can't get the plots in Mathematica to come out right unless I do them separately, as above.

$\{0,4,4\} + u\{1,0,0\} + v\{1,4,4\}$ implies $x=u+v$, $y=4+4v$, $z=4+4v$

```
ParametricPlot3D[{u + v, 4 + 4 v, 4 + 4 v},
  {u, 0, .25}, {v, 0, .25}, BoxRatios -> Automatic]
```



Two vectors in the plane are $\{1, -4, -4\}$ and $\{0, -4, -4\}$

```
planenormal = Cross[{1, -4, -4}, {0, -4, -4}]
{0, 4, -4}
```

```
{x, y, z} - {1, 0, 0}
{-1 + x, y, z}
```

```
planeeq = {0, 4, -4} . {-1 + x, y, z}
4 y - 4 z
```

So the equation of the plane is $4y - 4z = 0$. I'm going to suppose the following is what I want:

```
surf = {x, 4 y, -4 z}
{x, 4 y, -4 z}
```

```
F = {z^2, -x^2, 0}
{z^2, -x^2, 0}
```

```
dogm = Curl[F, {x, y, z}]
{0, 2 z, -2 x}
```

```
fir = D[surf, {x}]
{1, 0, 0}
```

```
sec = D[surf, {y}]
{0, 4, 0}

thi = D[planeeq, {z}]
0

crs = Cross[fir, sec]
{0, 0, 4}

arg = dogm.crs
-8 x

inte =  $\int_0^1 \int_0^{4x} \int_0^y (-8x) \, dz \, dy \, dx$ 
-16
```

```
{0, 2 z, -2 z} . {0, -1, 1}
```

```
-4 z
```

```
intel =  $\int_0^1 \int_0^{4x} \int_0^y (-4z) \, dz \, dy \, dx$ 
```

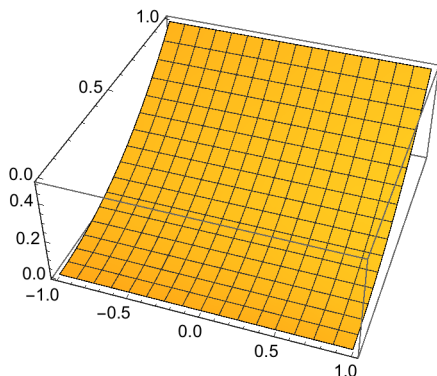
```
-  $\frac{128}{3}$ 
```

Not surprising that it did not come out right.

```
3. F = {e-z, e-z Cos[y], e-z Sin[y]}, S : z =  $\frac{y^2}{2}$ , -1 ≤ x ≤ 1, 0 ≤ y ≤ 1
```

```
Clear["Global`*"]
```

```
surf = Plot3D[ $\frac{y^2}{2}$ , {x, -1, 1}, {y, 0, 1}]
```



The solution manual calls attention to the surface as a parabolic cylinder.

$$\mathbf{F} = \{e^{-z}, e^{-z} \cos[y], e^{-z} \sin[y]\}$$

$$\{e^{-z}, e^{-z} \cos[y], e^{-z} \sin[y]\}$$

Finding the Curl is easy, unless something goes wrong. I had some doubts, as this didn't work smoothly the first time.

$$\text{curly} = \text{Curl}[\{e^{-z}, e^{-z} \cos[y], e^{-z} \sin[y]\}, \{x, y, z\}]$$

$$\{e^{-z} \cos[y] + e^{-z} \cos[y], -e^{-z}, 0\}$$

Below, eliminating the z expression.

$$\text{curlyz} = \text{curly} /. z \rightarrow \frac{y^2}{2}$$

$$\{e^{-\frac{y^2}{2}} \cos[y] + e^{-\frac{y^2}{2}} \cos[y], -e^{-\frac{y^2}{2}}, 0\}$$

Below, *Mathematica* was hesitant

to combine the two instances of $e^{-\frac{y^2}{2}} \cos[y]$.

$$\text{curlyzz} = \text{curlyz} /. e^{-\frac{y^2}{2}} \cos[y] + e^{-\frac{y^2}{2}} \cos[y] \rightarrow 2 e^{-\frac{y^2}{2}} \cos[y]$$

$$\{2 e^{-\frac{y^2}{2}} \cos[y], -e^{-\frac{y^2}{2}}, 0\}$$

Below, writing the surface equation as the solution manual recommended.

$$\text{surf} = \{x, y, \frac{y^2}{2}\}$$

$$\{x, y, \frac{y^2}{2}\}$$

Below, finding the partials in preparation for crossing.

$$\text{surfofx} = \text{D}[\text{surf}, \{x\}]$$

$$\{1, 0, 0\}$$

$$\text{surfofy} = \text{D}[\text{surf}, \{y\}]$$

$$\{0, 1, y\}$$

Below, crossing gives the normal vector needed.

$$\text{norm} = \text{Cross}[\text{surfofx}, \text{surfofy}]$$

$$\{0, -y, 1\}$$

Below, the dot product will be the core of the integrand.

$$\text{integr} = \text{curlyzz} \cdot \text{norm}$$

$$e^{-\frac{y^2}{2}} y$$

Below, the limits are given explicitly.

$$\int_0^1 \int_{-1}^1 \left(e^{-\frac{y^2}{2}} \mathbf{y} \right) d\mathbf{x} d\mathbf{y}$$

$$2 - \frac{2}{\sqrt{e}}$$

The above line matches the text's answer, except that the text has +/- on its answer.

$$5. \mathbf{F} = \left\{ z^2, \frac{3}{2}x, 0 \right\}, \quad S : 0 \leq x \leq a, \quad 0 \leq y \leq a, \quad z = 1$$

Clear["Global`*"]

$$7. \mathbf{F} = \{e^y, e^z, e^x\}, \quad S : z = x^2 \quad (0 \leq x \leq 2, \quad 0 \leq y \leq 1)$$

11. Stoke's theorem not applicable. Evaluate

$\oint \mathbf{F} \cdot \mathbf{r}' \, ds$, $\mathbf{F} = (x^2 + y^2)^{-1} \{-y, x\}$, $C : x^2 + y^2 = 1, \quad z = 0$, oriented clockwise.
Why can Stoke's theorem not be applied? What (false) result would it give?

13 - 20 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' \, ds$

Calculate this line integral by Stoke's theorem for the given \mathbf{F} and C . Assume the Cartesian coordinates to be right-handed and the z -component of the surface normal to be nonnegative.

$$13. \mathbf{F} = \{-5y, 4x, z\}, \quad C \text{ the circle } x^2 + y^2 = 16, \quad z = 4$$

$$15. \mathbf{F} = \{y^2, x^2, z + x\} \text{ around the triangle with vertices } \{0, 0, 0\}, \{1, 0, 0\}, \{1, 1, 0\}$$

$$17. \mathbf{F} = \{0, z^3, 0\}, \quad C \text{ the boundary curve of the cylinder } x^2 + y^2 = 1, \quad x \geq 0, \quad y \geq 0, \quad 0 \leq z \leq 1$$

$$19. \mathbf{F} = \{z, e^z, 0\},$$

C the boundary curve of the portion of the cone $z = \sqrt{x^2 + y^2}$,
 $x \geq 0, \quad y \geq 0, \quad 0 \leq z \leq 1$