

Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

Clear["Global`*"]

5 - 11 Principal value $\text{Ln } z$. Find $\text{Ln } z$ when z equals

5. - 11

Log[-11]

$i \pi + \text{Log}[11]$

7. $4 - 4i$

ComplexExpand[Log[4 - 4 i]]

$-\frac{i \pi}{4} + \frac{\text{Log}[32]}{2}$

N[%]

$1.73287 - 0.785398 i$

9. $0.6 + 0.8i$

Log[0.6 + 0.8 i]

$0. + 0.927295 i$

11. $e i$

Log[e i]

$1 + \frac{i \pi}{2}$

12 - 16 All Values of $\text{Log } x$. Find all values and graph some of them in the complex plane.

13. $\text{Log}[1]$

ComplexExpand[Log[1]]

0

According to numbered line (3) on p. 637, $\text{Log}[z] = \text{Log}[z] \pm 2n\pi i$. The extra factor is ignored by Mathematica however, since the software only concerns itself with the principal value.

15. $\text{Log}[e^i]$ **Log**[e^i] i **Log**[**Abs**[e^i]] + i **Arg**[e^i] i **ComplexExpand**[**Log**[**Abs**[e^i]] + i **Arg**[e^i]] i

Mathematica's system of presenting only the principal value of the complex number works against me here. I would have liked to have found a way to get Mathematica to tack on a $2n\pi i$ factor, which would make it match the text answer.

17. Show that the set of values of $\text{Log}[i^2]$ differs from the set of values of $2 \text{Log}[i]$.**Log**[**ComplexExpand**[i^2]] $i \pi$ **2 ComplexExpand**[**Log**[i]] $i \pi$

Mathematica is not responsive.

18 - 21 Equations.Solve for z.

19. $\text{Log}[z] = 4 - 3i$ **Solve**[**Log**[z] == $4 - 3i$, z] $\{ \{ z \rightarrow e^{4-3i} \} \}$ **dis** = $e^4 e^{-3i}$ $e^4 e^{-3i}$ **ComplexExpand**[**dis**] $e^4 \text{Cos}[3] - i e^4 \text{Sin}[3]$ 21. $\text{Log}[z] = 0.6 + 0.4i$ **Clear**["Global`*"]**Solve**[**Log**[z] == $0.6 + 0.4i$, z] $\{ \{ z \rightarrow 1.67828 + 0.709566i \} \}$

$$\mathbf{dib} = e^{0.6} e^{0.4 i}$$

$$1.67828 + 0.709566 i$$

I don't know how to make Mathematica cough up Euler's identity, so the symbolic version of the answer is not here.

22 - 28 General Powers. Find the principal value.

$$23. (1 + i)^{1-i}$$

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Clear["Global`*"]
```

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ComplexExpand[(1 + i)^(1-i)]
```

$$\sqrt{2} e^{\pi/4} \cos\left[\frac{\pi}{4} - \frac{\text{Log}[2]}{2}\right] + i \sqrt{2} e^{\pi/4} \sin\left[\frac{\pi}{4} - \frac{\text{Log}[2]}{2}\right]$$

```
Simplify[%]
```

$$(1 + i) e^{\pi/4} \left(\cos\left[\frac{\text{Log}[2]}{2}\right] - i \sin\left[\frac{\text{Log}[2]}{2}\right] \right)$$

The above cell contains Euler's identity but the text answer does not.

$$(1 + i) e^{\frac{\pi}{4}} e^{-\frac{\text{Log}[2]}{2} i}$$

$$(1 + i) 2^{-\frac{i}{2}} e^{\pi/4}$$

```
N[%]
```

$$2.80788 + 1.31787 i$$

$$25. (-3)^{3-i}$$

```
Clear["Global`*"]
```

```
ComplexExpand[(-3)^(3-i)]
```

$$-27 e^{\pi} \cos[\text{Log}[3]] + 27 i e^{\pi} \sin[\text{Log}[3]]$$

```
Simplify[%]
```

$$-27 e^{\pi} (\cos[\text{Log}[3]] - i \sin[\text{Log}[3]])$$

```
N[%]
```

$$-284.179 + 556.431 i$$

The numeric equivalent scores, but the text symbolic answer is not what Mathematica comes up with.

$$27. (-1)^{2-i}$$

```
Clear["Global`*"]
```

```
ComplexExpand[ $(-1)^{2-i}$ ]
```

```
 $e^{\pi}$ 
```

```
N[%]
```

```
23.1407
```