Comment: *Mathematica*'s Convolve function has an odd syntax. Note that in the orange cells below, the asterisk represents the convolve function.

1 - 7 Convolutions by integration
Find:

1. 1*1

```
Clear["Global`*"]
```

```
Integrate[1 × 1, {y, 0, t}]
```

```
t
```

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

3. $e^t * e^{-t}$

```
Clear["Global`*"]
```

Integrate[$e^y e^{-(t-y)}$, {y, 0, t}]

```
Sinh[t]
```

Below: Alternate method, this one using Convolve. But the syntax is strange.

Convolve[e^y UnitStep[y], e^{-y} UnitStep[y], y, t, Assumptions → t > 0]

```
Sinh[t]
```

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

5. (Sin[ω t])*(Cos[ω t])

```
Clear["Global`*"]
```

```
e1 = Integrate[Sin[ω y] Cos[ω (t - y)], {y, 0, t}]
```

$\frac{1}{2}$ t Sin[t ω]

With the alternate method,

```
Convolve[Sin[ω y] UnitStep[y],
 Cos[ω y] UnitStep[y], y, t, Assumptions → t > 0]
```

$\frac{1}{2} t \operatorname{Sin}[t\, \omega]$

Above: The answer matches the text. However, no domain interval was specified. The limited one used is okay, but not the whole real line, or even half of it.

7. $t * e^t$

```
Clear["Global`*"]
```

$e1 = \text{Integrate}\left[y\, e^{t-y}, \{y, 0, t\}\right]$

$-1 + e^t - t$

$\text{Convolve}[y\, \text{UnitStep}[y], e^y\, \text{UnitStep}[y], y, t, \text{Assumptions} \to t > 0]$

$-1 + e^t - t$

The above answer matches the text. This time only the **Convolve** method was used.

8 - 14 Integral equations
Solve by the Laplace transform, showing the details:

9. y[t] - Integrate[y[τ], {τ, 0, t}] = 1

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Integrate[y[τ], {τ, 0, t}], t, s]
```

$\frac{\text{LaplaceTransform}[y[t], t, s]}{s}$

```
e2 = LaplaceTransform[y[t], t, s]
```

LaplaceTransform[y[t], t, s]

```
e3 = LaplaceTransform[1, t, s]
```

$\frac{1}{s}$

```
e4 = e2 - e1 == e3
```

$\text{LaplaceTransform}[y[t], t, s] - \frac{\text{LaplaceTransform}[y[t], t, s]}{s} == \frac{1}{s}$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```

$\text{bigY} - \frac{\text{bigY}}{s} == \frac{1}{s}$

```
e6 = Solve[e5, bigY]
```

$$\left\{\left\{\text{bigY} \to \frac{1}{-1+s}\right\}\right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{1}{-1+s}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

$$e^t$$

Above: The answer matches the text answer.

11. y[t] + Integrate[(t - τ) y[τ], {τ, 0, t}] = 1

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Integrate[(t - τ) y[τ], {τ, 0, t}], t, s]
```

$$\frac{\text{LaplaceTransform}[y[t], t, s]}{s^2}$$

```
e2 = LaplaceTransform[y[t], t, s]
```

LaplaceTransform[y[t], t, s]

```
e3 = LaplaceTransform[1, t, s]
```

$$\frac{1}{s}$$

```
e4 = e2 + e1 == e3
```

$$\text{LaplaceTransform}[y[t], t, s] + \frac{\text{LaplaceTransform}[y[t], t, s]}{s^2} == \frac{1}{s}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```

$$\text{bigY} + \frac{\text{bigY}}{s^2} == \frac{1}{s}$$

```
e6 = Solve[e5, bigY]
```

$$\left\{\left\{\text{bigY} \to \frac{s}{1+s^2}\right\}\right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{s}{1+s^2}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

```
Cos[t]
```

Above: The answer matches the text answer.

```
13. y[t] + 2 e^t Integrate[y[τ] e^-τ = t e^t, {τ, 0, t}]
```

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[2 e^t Integrate[y[τ] e^-τ, {τ, 0, t}], t, s]
```

$$\frac{2\,\text{LaplaceTransform}[y[t], t, s]}{-1+s}$$

```
e2 = LaplaceTransform[y[t], t, s]
```

```
LaplaceTransform[y[t], t, s]
```

```
e3 = LaplaceTransform[t e^t, t, s]
```

$$\frac{1}{(-1+s)^2}$$

```
e4 = e2 + e1 == e3
```

$$\text{LaplaceTransform}[y[t], t, s] + \frac{2\,\text{LaplaceTransform}[y[t], t, s]}{-1+s} == \frac{1}{(-1+s)^2}$$

```
e5 = e4 /. LaplaceTransform[y[t], t, s] → bigY
```

$$\text{bigY} + \frac{2\,\text{bigY}}{-1+s} == \frac{1}{(-1+s)^2}$$

```
e6 = Solve[e5, bigY]
```

$$\left\{\left\{\text{bigY} \to \frac{1}{(-1+s)\ (1+s)}\right\}\right\}$$

```
e7 = e6[[1, 1, 2]]
```

$$\frac{1}{(-1+s)\ (1+s)}$$

```
e8 = InverseLaplaceTransform[e7, s, t]
```

$$\frac{1}{2} e^{-t} \left(-1 + e^{2t}\right)$$

```
e9 = ExpToTrig[e8]
```

$$\frac{1}{2} (\text{Cosh}[t] - \text{Sinh}[t])\ (-1 + \text{Cosh}[2t] + \text{Sinh}[2t])$$

```
e10 = FullSimplify[e9]
```

```
Sinh[t]
```

Above: This answer matches the text answer.

17 - 26 Inverse transforms by convolution
Showing details, find f[t] if $\mathcal{L}$[f] equals:

17. $\frac{5.5}{(s + 1.5)\ (s - 4)}$

Note: use of the **Convolve** function did not seem necessary.

```
Clear["Global`*"]
```

$$\text{e1} = \text{InverseLaplaceTransform}\left[\frac{5.5}{(s + 1.5)\ (s - 4)},\ s,\ t\right]$$

$$5.5\left(-0.181818\, e^{-1.5\, t} + 0.181818\, e^{4.\, t}\right)$$

```
e2 = ExpandAll[e1]
```

$$-1.\ e^{-1.5\, t} + 1.\ e^{4.\, t}$$

Above: This answer matches the text answer.

19. $\frac{2\,\pi\, s}{\left(s^2 + \pi^2\right)^2}$

```
Clear["Global`*"]
```

$$\text{e1} = \text{InverseLaplaceTransform}\left[\frac{2\,\pi\, s}{\left(s^2 + \pi^2\right)^2},\ s,\ t\right]$$

$$t\ \text{Sin}[\pi\, t]$$

Above: This answer matches the text answer.

21. $\frac{\omega}{s^2\left(s^2 + \omega^2\right)}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[ω/(s^2 (s^2 + ω^2)), s, t]
```

$$\frac{t\,\omega - \text{Sin}[t\,\omega]}{\omega^2}$$

Above: This answer matches the text answer.

23. $\frac{40.5}{s\left(s^2-9\right)}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[40.5/(s (s^2 - 9)), s, t]
```

$2.25\, e^{-3\,t}\left(-1+e^{3\,t}\right)^2$

```
e2 = ExpToTrig[e1]
```

$2.25\,(\text{Cosh}[3\,t]-\text{Sinh}[3\,t])\,(-1+\text{Cosh}[3\,t]+\text{Sinh}[3\,t])^2$

```
e3 = FullSimplify[e2]
```

```
-4.5 + 4.5 Cosh[3 t]
```

Above: This answer matches the text answer.

25. $\frac{18\,s}{\left(s^2+36\right)^2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[18 s/(s^2 + 36)^2, s, t]
```

$\frac{3}{2}\,t\,\text{Sin}[6\,t]$

Above: This answer matches the text answer.