Laplace transforms is something that Mathematica is good at. Shifting, as far as I can tell, is something that can be routinely shined on.

1 - 16 Laplace transforms
Find the transform. Assume that a, b, ω, θ are constants.

1. $3\,t + 12$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[3 t + 12, t, s]
```

$$\frac{3}{s^2} + \frac{12}{s}$$

The correct answer.

3. $\text{Cos}[\pi\, t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Cos[π t], t, s]
```

$$\frac{s}{\pi^2 + s^2}$$

The correct answer.

5. $e^{2\,t}\,\text{Sinh}[t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[e^(2 t) Sinh[t], t, s]
```

$$\frac{1}{3 - 4\,s + s^2}$$

The correct answer.

7. $\text{Sin}[\omega\, t + \theta]$

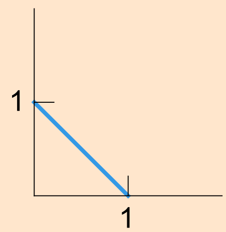
```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[Sin[ω t + θ], t, s]
```

$$\frac{\omega\,\text{Cos}[\theta] + s\,\text{Sin}[\theta]}{s^2 + \omega^2}$$

The correct answer.

9.




```
Clear["Global`*"]
```

```
e1 = f[t_] = -t + 1
```

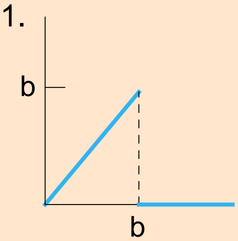
```
1 - t
```

```
e4 = LaplaceTransform[If[t > 0 && t < 1, 1 - t, 0], t, s]
```

$$\frac{-1 + e^{-s} + s}{s^2}$$

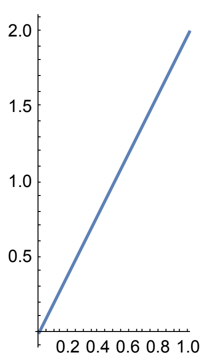
This is the right answer. There is a big difference on what comes out of the Laplace Transform based on whether the domain is restricted or not.

11.




```
Clear["Global`*"]
```

```
Plot[ 2 x, {x, 0, 1}, PlotRange → Automatic,
 ImageSize → 100, AspectRatio → Automatic]
```



```
e1 = t
t
```

```
e2 = Simplify[LaplaceTransform[If[t > 0 && t < b, t, 0], t, s]]
```

$$\begin{cases} \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2} & b>0 \\ 0 & \text{True} \end{cases}$$

$$e3 = \frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

$$\frac{e^{-b s}\left(-1+e^{b s}-b s\right)}{s^2}$$

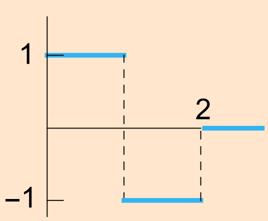
$$e4 = e3 \;/.\; \left(e^{-b s}\left(-1+e^{b s}-b s\right)\right) \to \text{Expand}\left[e^{-b s}\left(-1+e^{b s}-b s\right)\right]$$

$$\frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2}$$

$$e5 = e4 \;/.\; \frac{1-e^{-b s}-b\, e^{-b s}\, s}{s^2} \;\text{->}\; \frac{1-e^{-b s}}{s^2}-\frac{b\, e^{-b s}\, s}{s^2}$$

$$\frac{1-e^{-b s}}{s^2}-\frac{b\, e^{-b s}}{s}$$

Above: This is the text answer.

13. 


```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[
  Piecewise[{{1, t > 0 && t < 1}, {-1, t > 1 && t < 2}}], t, s]
```

$$-\frac{e^{-2 s}\left(-1+e^{s}-e^{2 s}+e^{2 s}\,\text{Cosh}[s]-e^{2 s}\,\text{Sinh}[s]\right)}{s}$$

```
e2 = Simplify[e1]
```

$$\frac{e^{-2 s}\left(-1+e^{s}\right)^2}{s}$$

$$\text{PossibleZeroQ}\left[\frac{e^{-2 s} (-1 + e^{s})^2}{s} - \frac{(1 - e^{-s})^2}{s}\right]$$

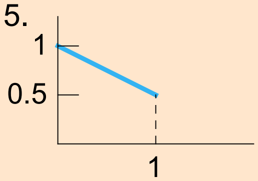
True

The above shows that Mathematica's answer and the text answer are equivalent. With a ‘sleight’ maneuver I could even do

$$\text{e6} = \text{e2} \;/.\; e^{-2 s} (-1 + e^{s})^2 \;\text{->}\; (1 - e^{-s})^2$$

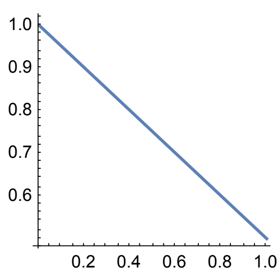
$$\frac{(1 - e^{-s})^2}{s}$$

15.




```
Clear["Global`*"]
```

$$\text{Plot}\left[-\frac{1}{2} x + 1, \{x, 0, 1\}, \text{PlotRange} \to \text{Automatic}, \text{ImageSize} \to 140, \text{AspectRatio} \to 1\right]$$



$$\text{e1} = \text{LaplaceTransform}\left[\text{Piecewise}\left[\left\{\left\{-\frac{1}{2} t + 1, t > 0 \,\&\&\, t < 1\right\}, \{0, t > 1 \,\&\&\, t < 0\}\right\}\right], t, s\right]$$

$$\frac{e^{-s} (1 - e^{s} - s + 2 e^{s} s)}{2 s^2}$$

$$\text{e2} = \text{TrigToExp}[\text{e1}]$$

$$-\frac{1}{2 s^2} + \frac{e^{-s}}{2 s^2} + \frac{1}{s} - \frac{e^{-s}}{2 s}$$

The above answer is correct.

21. Nonexistence. Give sample examples of functions (defined for all t ≥ 0) that have no Laplace transform.

If the following expression is true for some constants M and k it satisfies the “growth restriction”

$| f(t) \leq M e^{k t} |$

The sol’n text gives e^{t^2} as an example. The expression above is a more general guide. The *k* and *M* are just constants, so the *lhs* can overwhelm them if it has an exponential or similar nature.

25 - 32 Inverse Laplace transforms
Given F(s) = L(f), find f(t). Here a, b, L, n, are constants.

25. $\dfrac{0.2\, s + 1.8}{s^2 + 3.24}$

```
Clear["Global`*"]
```

$$\text{e1} = \text{InverseLaplaceTransform}\left[\frac{0.2\, s + 1.8}{s^2 + 3.24}, s, t\right]$$

$$(0.5 + 0.1\, i)\, e^{(0.-1.8\, i)\, t}\left((0.384615 + 0.923077\, i) - (0. + 1.\, i)\, e^{(0.+3.6\, i)\, t}\right)$$

```
e2 = FullSimplify[e1]
```

$$e^{(0.-1.8\, i)\, t}\left((0.1 + 0.5\, i) + (0.1 - 0.5\, i)\, e^{(0.+3.6\, i)\, t}\right)$$

```
e3 = ExpToTrig[e2]
```

```
(Cos[(1.8 + 0. i) t] - i Sin[(1.8 + 0. i) t])
 ((0.1 + 0.5 i) + (0.1 - 0.5 i) (Cos[(3.6 + 0. i) t] + i Sin[(3.6 + 0. i) t]))
```

```
e4 = FullSimplify[e3]
```

```
0.2 Cos[1.8 t] + 1. Sin[1.8 t]
```

The above answer matches the text. A bit of work to recast it.

27. $\dfrac{s}{L^2 s^2 + n^2 \pi^2}$

```
Clear["Global`*"]
```

$$\text{e1} = \text{InverseLaplaceTransform}\left[\frac{s}{L^2 s^2 + n^2 \pi^2}, s, t\right]$$

$$\frac{\text{Cos}\left[\frac{n \pi t}{L}\right]}{L^2}$$

The above answer matches the text.

29. $\frac{12}{s^4} - \frac{228}{s^6}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[12/s^4 - 228/s^6, s, t]
```

$2\,t^3 - \frac{19\,t^5}{10}$

The above answer matches the text. Here is demonstrated the linear nature of L^{-1}: separate fractions can be calculated as a single operand.

31. $\frac{s + 10}{s^2 - s - 2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[(s + 10)/(s^2 - s - 2), s, t]
```

$-3\,e^{-t} + 4\,e^{2\,t}$

The above answer matches the text.

33 - 45 Application of s-shifting
In problems 33 - 36 find the transform. In problems 37 - 45 find the inverse transform.

33. $t^2\,e^{-3\,t}$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[t^2 e^(-3 t), t, s]
```

$\frac{2}{(3 + s)^3}$

The above answer matches the text.

35. $0.5\,e^{-4.5\,t}\,\text{Sin}[2\,\pi\,t]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[0.5 e^(-4.5 t) Sin[2 π t], t, s]
```

$\frac{3.14159}{4\,\pi^2 + (4.5 + s)^2}$

The above answer matches the text.

37. $\dfrac{\pi}{(s+\pi)^2}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\dfrac{\pi}{(s+\pi)^2}, s, t\right]$

$e^{-\pi t}\,\pi\,t$

The above answer matches the text.

39. $\dfrac{21}{\left(s+\sqrt{2}\right)^4}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\dfrac{21}{\left(s+\sqrt{2}\right)^4}, s, t\right]$

$\dfrac{7}{2}\,e^{-\sqrt{2}\,t}\,t^3$

The above answer matches the text.

41. $\dfrac{\pi}{s^2+10\,\pi\,s+24\,\pi^2}$

```
Clear["Global`*"]
```

e1 = InverseLaplaceTransform$\left[\dfrac{\pi}{s^2+10\,\pi\,s+24\,\pi^2}, s, t\right]$

$\dfrac{1}{2}\,e^{-6\pi t}\left(-1+e^{2\pi t}\right)$

e2 = FullSimplify$\left[\text{ExpToTrig}\left[\dfrac{\left(-1+e^{2\pi t}\right)}{2\,e^{\pi t}}\right]\right]$

Sinh[π t]

e3 = $e^{-5\pi t}$ e2

$e^{-5\pi t}$ Sinh[π t]

The above answer matches the text.

43. $\dfrac{2s-1}{s^2-6s+18}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[(2 s - 1)/(s^2 - 6 s + 18), s, t]
```

$$\frac{1}{6}e^{(3-3i)t}\left((6+5i)+(6-5i)e^{6it}\right)$$

```
e2 = FullSimplify[e1]
```

$$\frac{1}{3}e^{3t}(6\,\mathrm{Cos}[3t]+5\,\mathrm{Sin}[3t])$$

The above answer matches the text.

45. $\dfrac{k_0(s+a)+k_1}{(s+a)^2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[(k0 (s + a) + k1)/(s + a)^2, s, t]
```

$$e^{-at}(k_0+t\,k_1)$$

The above answer matches the text.

The problems since No. 33 demonstrate that S-shifting doesn’t really exist for Mathematica user. Just put the expression in and turn the crank.