Evidently I put in some extracurricular practice junk up at the top of this section. For the first actual problem, see the orange cell below, numbered 1.

```
grating = DSolve[x^2 y''[x] + 1.5 x y'[x] - 0.5 y[x] == 0, y[x], x]
```

$$\{\{y[x] \to x^{0.5+0.\,i}\, C[1] + x^{-1.+0.\,i}\, C[2]\}\}$$

```
grid = Simplify[grating]
```

$$\left\{\left\{y[x] \to \frac{x^{1.5}\, C[1] + C[2]}{x^{1.}}\right\}\right\}$$

```
Apart[grid]
```

$$\left\{\left\{y[x] \to x^{0.5}\, C[1] + \frac{C[2]}{x^{1.}}\right\}\right\}$$

Cauchy-Euler equation. The auxilary equation is $m^2 + 0.5\, m - 0.5 == 0$

```
Solve[m^2 + 0.5 m - 0.5 == 0, m]
```

$$\{\{m \to -1.\}, \{m \to 0.5\}\}$$

Numbered line (1) on p. 71 goes like this: $x^2\, y'' + a\, x\, y' + b\, y = 0$

Numbered line (2) on p. 71 goes like this: $m^2 + (a - 1)\, m + b = 0$

1. Double root. Verify directly by substitution that $x^{(1-1)/2}$Log[x] is a solution of (1) if (2) has a double root, but x^{m_1}Log[x] and x^{m_2}Log[x] are not solutions of (1) if the roots m_1 and m_2 of (2) are different.

```
Clear["Global`*"]
```

Maybe I should solve (2).

```
Solve[m^2 + (a - 1) m + b == 0, m]
```

$$\left\{\left\{m \to \frac{1}{2}\left(1 - a - \sqrt{1 - 2\,a + a^2 - 4\,b}\right)\right\}, \left\{m \to \frac{1}{2}\left(1 - a + \sqrt{1 - 2\,a + a^2 - 4\,b}\right)\right\}\right\}$$

The above lists the two possible roots to (2). A double root would mean they're equal.

$$\text{Solve}\left[\left(-\sqrt{(1 - a)^2 - 4\,b}\right) == \left(\sqrt{(1 - a)^2 - 4\,b}\right), a\right]$$

$$\left\{\left\{a \to 1 - 2\sqrt{b}\right\}, \left\{a \to 1 + 2\sqrt{b}\right\}\right\}$$

So it looks like (2) has a double root under two circumstances. And the relationship between a and b has also been established (call it 'Rel'). Now it is time to look at the problem's proposed sol'n.

$$y[x\_] := x^{\frac{(1-a)}{2}}\, \text{Log}[x]$$

```
cran = FullSimplify[x^2 y''[x] + a x y'[x] + b y[x]]
```

$$-\frac{1}{4}\left((-1+a)^2-4b\right)x^{\frac{1}{2}-\frac{a}{2}}\text{Log}[x]$$

```
Simplify[cran /. a → (1 - 2 √b)]
```

0

```
Simplify[cran /. a -> (1 + 2 √b)]
```

0

I think I have done what the first part of the problem asked: if (2) has a double root, a certain relationship 'Rel' exists between the coefficients a and b in the double root. The proposed sol'n only incorporates one of the coefficients, but the candidate equation does have both, and after substituting the proposed sol'n into the candidate equation and analyzing the resulting wreckage, the relationship 'Rel' is found to be present. In other words, invoking 'Rel' proves the proposed sol'n to be an actual sol'n.

For the second part of the problem. If the roots of (2) are not equal, it would imply that $m_1 = \frac{1}{2}\left(1-a-\sqrt{(1-a)^2-4b}\right) \mathrel{!}= m_2 = \frac{1}{2}\left(1-a+\sqrt{(1-a)^2-4b}\right)$. It seems obvious that the exclamation point cannot be removed by squaring both sides! However, I don't see what is wrong with multiplying both sides (of the pertinent sub-expressions) by the same quantity, e.g. $\sqrt{(1-a)^2-4b}$.

$$-\left((1-a)^2-4b\right) \mathrel{!}= \left((1-a)^2-4b\right)$$

$$-(1-a)^2+4b \neq (1-a)^2-4b$$

$$8b \neq 2(1-a)^2$$

$$4b \neq (1-a)^2$$

But in the expansion of **cran** something very much like this came up. What if I look at

```
Simplify[(1 - a)^2 == (a - 1)^2]
```

True

Let me bring **cran** back.

```
cran
```

$$-\frac{1}{4}\left((-1+a)^2-4b\right)x^{\frac{1}{2}-\frac{a}{2}}\text{Log}[x]$$

I contend that having the expression inside parentheses above, i.e. $\left((-1+a)^2-4b\right)$, equal to zero is necessary for **cran** to go to zero, and, backtracking a few steps, this only happens if the roots are equal. This line of reasoning does not manipulate the specific proposed sol'ns, however. (I think it covers the same ground, though.)

3. $5\,x^2\,y'' + 23\,x\,y' + 16.2\;y = 0$

```
Clear["Global`*"]
```

```
doxy = {5 x^2 y''[x] + 23 x y'[x] + 16.2 y[x] == 0}
plat = DSolve[doxy, y, x]
```

$\{16.2\,y[x] + 23\,x\,y'[x] + 5\,x^2\,y''[x] == 0\}$

$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{C[1]}{x^{1.8}} + \frac{1.8\,C[2]\,\text{Log}[x]}{x^{1.8}}\right]\right\}\right\}$

```
gerz = plat[[1, 1, 2, 2]]
```

$\frac{C[1]}{x^{1.8}} + \frac{1.8\,C[2]\,\text{Log}[x]}{x^{1.8}}$

```
derz = gerz /. {C[1] → c1, C[2] → c2}
```

$\frac{c1}{x^{1.8}} + \frac{1.8\,c2\,\text{Log}[x]}{x^{1.8}}$

```
lint[x_, c1_, c2_] := derz
```

```
TableForm[Table[{x, c1, c2, derz}, {x, 4}, {c1, -1, 1}, {c2, -3, 3, .25}],
 TableHeadings -> {{}, {"c1=-1", "c1=0 ", "c1=1"}}]
```

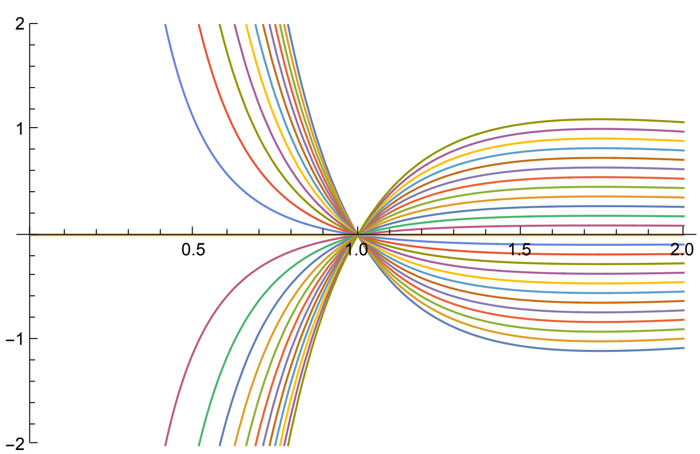
c1=-1	c1=0	c1=1
1 -1 -3. -1.	1 0 -3. 0.	1 1 -3. 1.
1 -1 -2.75 -1.	1 0 -2.75 0.	1 1 -2.75 1.
1 -1 -2.5 -1.	1 0 -2.5 0.	1 1 -2.5 1.
1 -1 -2.25 -1.	1 0 -2.25 0.	1 1 -2.25 1.
1 -1 -2. -1.	1 0 -2. 0.	1 1 -2. 1.
1 -1 -1.75 -1.	1 0 -1.75 0.	1 1 -1.75 1.
1 -1 -1.5 -1.	1 0 -1.5 0.	1 1 -1.5 1.
1 -1 -1.25 -1.	1 0 -1.25 0.	1 1 -1.25 1.
1 -1 -1. -1.	1 0 -1. 0.	1 1 -1. 1.
1 -1 -0.75 -1.	1 0 -0.75 0.	1 1 -0.75 1.
1 -1 -0.5 -1.	1 0 -0.5 0.	1 1 -0.5 1.
1 -1 -0.25 -1.	1 0 -0.25 0.	1 1 -0.25 1.
1 -1 0. -1.	1 0 0. 0.	1 1 0. 1.
1 -1 0.25 -1.	1 0 0.25 0.	1 1 0.25 1.
1 -1 0.5 -1.	1 0 0.5 0.	1 1 0.5 1.
1 -1 0.75 -1.	1 0 0.75 0.	1 1 0.75 1.
1 -1 1. -1.	1 0 1. 0.	1 1 1. 1.
1 -1 1.25 -1.	1 0 1.25 0.	1 1 1.25 1.
1 -1 1.5 -1.	1 0 1.5 0.	1 1 1.5 1.
1 -1 1.75 -1.	1 0 1.75 0.	1 1 1.75 1.
1 -1 2. -1.	1 0 2. 0.	1 1 2. 1.
1 -1 2.25 -1.	1 0 2.25 0.	1 1 2.25 1.
1 -1 2.5 -1.	1 0 2.5 0.	1 1 2.5 1.
1 -1 2.75 -1.	1 0 2.75 0.	1 1 2.75 1.
1 -1 3. -1.	1 0 3. 0.	1 1 3. 1.

2	-1	-3.	-1.36207	2	0	-3.	-1.07489	2	1	-3.	-0.787
2	-1	-2.75	-1.27249	2	0	-2.75	-0.985319	2	1	-2.75	-0.698
2	-1	-2.5	-1.18292	2	0	-2.5	-0.895744	2	1	-2.5	-0.608
2	-1	-2.25	-1.09334	2	0	-2.25	-0.80617	2	1	-2.25	-0.518
2	-1	-2.	-1.00377	2	0	-2.	-0.716595	2	1	-2.	-0.429
2	-1	-1.75	-0.914195	2	0	-1.75	-0.627021	2	1	-1.75	-0.339
2	-1	-1.5	-0.824621	2	0	-1.5	-0.537446	2	1	-1.5	-0.250
2	-1	-1.25	-0.735047	2	0	-1.25	-0.447872	2	1	-1.25	-0.160
2	-1	-1.	-0.645472	2	0	-1.	-0.358298	2	1	-1.	-0.071
2	-1	-0.75	-0.555898	2	0	-0.75	-0.268723	2	1	-0.75	0.0184
2	-1	-0.5	-0.466323	2	0	-0.5	-0.179149	2	1	-0.5	0.1080
2	-1	-0.25	-0.376749	2	0	-0.25	-0.0895744	2	1	-0.25	0.1976
2	-1	0.	-0.287175	2	0	0.	0.	2	1	0.	0.2871
2	-1	0.25	-0.1976	2	0	0.25	0.0895744	2	1	0.25	0.3767
2	-1	0.5	-0.108026	2	0	0.5	0.179149	2	1	0.5	0.4663
2	-1	0.75	-0.0184513	2	0	0.75	0.268723	2	1	0.75	0.5558
2	-1	1.	0.0711231	2	0	1.	0.358298	2	1	1.	0.6454
2	-1	1.25	0.160697	2	0	1.25	0.447872	2	1	1.25	0.7350
2	-1	1.5	0.250272	2	0	1.5	0.537446	2	1	1.5	0.8246
2	-1	1.75	0.339846	2	0	1.75	0.627021	2	1	1.75	0.9141
2	-1	2.	0.429421	2	0	2.	0.716595	2	1	2.	1.0037
2	-1	2.25	0.518995	2	0	2.25	0.80617	2	1	2.25	1.0933
2	-1	2.5	0.60857	2	0	2.5	0.895744	2	1	2.5	1.1829
2	-1	2.75	0.698144	2	0	2.75	0.985319	2	1	2.75	1.2724
2	-1	3.	0.787718	2	0	3.	1.07489	2	1	3.	1.3620
3	-1	-3.	-0.95956	3	0	-3.	-0.821145	3	1	-3.	-0.682
3	-1	-2.75	-0.891131	3	0	-2.75	-0.752716	3	1	-2.75	-0.614
3	-1	-2.5	-0.822702	3	0	-2.5	-0.684288	3	1	-2.5	-0.545
3	-1	-2.25	-0.754273	3	0	-2.25	-0.615859	3	1	-2.25	-0.477
3	-1	-2.	-0.685845	3	0	-2.	-0.54743	3	1	-2.	-0.409
3	-1	-1.75	-0.617416	3	0	-1.75	-0.479001	3	1	-1.75	-0.340
3	-1	-1.5	-0.548987	3	0	-1.5	-0.410573	3	1	-1.5	-0.272
3	-1	-1.25	-0.480558	3	0	-1.25	-0.342144	3	1	-1.25	-0.203
3	-1	-1.	-0.41213	3	0	-1.	-0.273715	3	1	-1.	-0.135
3	-1	-0.75	-0.343701	3	0	-0.75	-0.205286	3	1	-0.75	-0.066
3	-1	-0.5	-0.275272	3	0	-0.5	-0.136858	3	1	-0.5	0.0015
3	-1	-0.25	-0.206843	3	0	-0.25	-0.0684288	3	1	-0.25	0.0699
3	-1	0.	-0.138415	3	0	0.	0.	3	1	0.	0.1384
3	-1	0.25	-0.0699858	3	0	0.25	0.0684288	3	1	0.25	0.2068
3	-1	0.5	-0.00155702	3	0	0.5	0.136858	3	1	0.5	0.2752
3	-1	0.75	0.0668717	3	0	0.75	0.205286	3	1	0.75	0.3437
3	-1	1.	0.135301	3	0	1.	0.273715	3	1	1.	0.4121
3	-1	1.25	0.203729	3	0	1.25	0.342144	3	1	1.25	0.4805
3	-1	1.5	0.272158	3	0	1.5	0.410573	3	1	1.5	0.5489
3	-1	1.75	0.340587	3	0	1.75	0.479001	3	1	1.75	0.6174
3	-1	2.	0.409016	3	0	2.	0.54743	3	1	2.	0.6858
3	-1	2.25	0.477444	3	0	2.25	0.615859	3	1	2.25	0.7542
3	-1	2.5	0.545873	3	0	2.5	0.684288	3	1	2.5	0.8227
3	-1	2.75	0.614302	3	0	2.75	0.752716	3	1	2.75	0.8911
3	-1	3.	0.682731	3	0	3.	0.821145	3	1	3.	0.9595

4	-1	-3.	-0.699833	4	0	-3.	-0.617364	4	1	-3.	-0.534
4	-1	-2.75	-0.648386	4	0	-2.75	-0.565917	4	1	-2.75	-0.483
4	-1	-2.5	-0.596939	4	0	-2.5	-0.51447	4	1	-2.5	-0.432
4	-1	-2.25	-0.545492	4	0	-2.25	-0.463023	4	1	-2.25	-0.380
4	-1	-2.	-0.494045	4	0	-2.	-0.411576	4	1	-2.	-0.329
4	-1	-1.75	-0.442598	4	0	-1.75	-0.360129	4	1	-1.75	-0.277
4	-1	-1.5	-0.391151	4	0	-1.5	-0.308682	4	1	-1.5	-0.226
4	-1	-1.25	-0.339704	4	0	-1.25	-0.257235	4	1	-1.25	-0.174
4	-1	-1.	-0.288257	4	0	-1.	-0.205788	4	1	-1.	-0.123
4	-1	-0.75	-0.23681	4	0	-0.75	-0.154341	4	1	-0.75	-0.071
4	-1	-0.5	-0.185363	4	0	-0.5	-0.102894	4	1	-0.5	-0.020
4	-1	-0.25	-0.133916	4	0	-0.25	-0.051447	4	1	-0.25	0.0310
4	-1	0.	-0.0824692	4	0	0.	0.	4	1	0.	0.0824
4	-1	0.25	-0.0310223	4	0	0.25	0.051447	4	1	0.25	0.1339
4	-1	0.5	0.0204247	4	0	0.5	0.102894	4	1	0.5	0.1853
4	-1	0.75	0.0718717	4	0	0.75	0.154341	4	1	0.75	0.2368
4	-1	1.	0.123319	4	0	1.	0.205788	4	1	1.	0.2882
4	-1	1.25	0.174766	4	0	1.25	0.257235	4	1	1.25	0.3397
4	-1	1.5	0.226213	4	0	1.5	0.308682	4	1	1.5	0.3911
4	-1	1.75	0.27766	4	0	1.75	0.360129	4	1	1.75	0.4425
4	-1	2.	0.329107	4	0	2.	0.411576	4	1	2.	0.4940
4	-1	2.25	0.380554	4	0	2.25	0.463023	4	1	2.25	0.5454
4	-1	2.5	0.432001	4	0	2.5	0.51447	4	1	2.5	0.5969
4	-1	2.75	0.483448	4	0	2.75	0.565917	4	1	2.75	0.6483
4	-1	3.	0.534895	4	0	3.	0.617364	4	1	3.	0.6998

According to above table, the general sol'n is a specific sol'n only when c1 = 0. There are a number of instances shown for various values of c2. The plot below shows these functions, each of which is a particular sol'n (when x =1).

```
plot3 =
 Plot[Evaluate[Table[lint[x, c1, c2], {c1, 0, 0}, {c2, -3, 3, .25}]],
  {x, 0, 2}, PlotRange → {-2, 2}, PlotStyle → Thickness[0.003]]
```



What happened? The sol'n found by DSolve matches the text's answer. However, the original DE only vanishes when $x = 1$ && c1 = 0.

5. $4 x^2 y'' + 5 y = 0$

```
Clear["Global`*"]
```

```
tier = {4 x^2 y''[x] + 5 y[x] == 0}
```

$\{5\,y[x] + 4\,x^2\,y''[x] == 0\}$

```
cins = DSolve[tier, y, x]
```

$\{\{y \to \text{Function}[\{x\}, \sqrt{x}\ C[2]\ \text{Cos}[\text{Log}[x]] + \sqrt{x}\ C[1]\ \text{Sin}[\text{Log}[x]]]\}\}$

```
Simplify[tier /. cins]
```

{{True}}

In the above case the sol'n checks out with Mathematica as well as matching the text's answer.

7. $(x^2\,D^2 - 4\,x\,D + 6\,I)\ y = C$

```
Clear["Global`*"]
```

```
hol = {x^2 y''[x] - 4 x y'[x] + 6 y[x] == c}
dea = DSolve[hol, y, x]
```

$\{6\,y[x] - 4\,x\,y'[x] + x^2\,y''[x] == c\}$

$\{\{y \to \text{Function}[\{x\}, \frac{c}{6} + x^2\,C[1] + x^3\,C[2]]\}\}$

```
Simplify[hol /. dea]
```

{{True}}

The above answer matches the text's, I think. The constant in the original rendition of the equation shows up in the solution, but not in the text. I suppose it doesn't matter.

9. $(x^2 D\ =\ 0.2\,x\,D\ +\ 0.36\,I)\,y\ =\ 0$

```
Clear["Global`*"]
```

```
reg = {x^2 y''[x] - 0.2 x y'[x] + 0.36 y[x] == 0}
dirk = DSolve[reg, y, x]
```

$\{0.36\,y[x] - 0.2\,x\,y'[x] + x^2\,y''[x] == 0\}$

$\{\{y \to \text{Function}[\{x\}, x^{0.6}\,C[1] + 0.6\,x^{0.6}\,C[2]\,\text{Log}[x]]\}\}$

```
Chop[Simplify[reg /. dirk], 10^-16]
```

{{True}}

The above answer matches the text's. However, the checking step does not work cleanly. The default tolerance for Chop is 10^{-10}, and in this case 10 ^ –16 will work. But an exact answer would be preferred.

11. $\left(x^2 D^2 - 3 x D + 10 I\right) y = 0$

```
Clear["Global`*"]
```

```
deam = {x^2 y''[x] - 3 x y'[x] + 10 y[x] == 0}
vre = DSolve[deam, y, x]
```

$\{10\, y[x] - 3\, x\, y'[x] + x^2\, y''[x] == 0\}$

$\{\{y \to \text{Function}[\{x\}, x^2\, C[2] \cos[\sqrt{6}\, \text{Log}[x]] + x^2\, C[1] \sin[\sqrt{6}\, \text{Log}[x]]]\}\}$

```
Simplify[deam /. vre]
```

```
{{True}}
```

In this case the substitution worked cleanly. The answer also matches the text's.

12 - 19 Initial value problem
Solve and graph the solution.

13. $x^2 y'' + 3 x y' + 0.75 y = 0,\ y[1] = 1,\ y'[1] = -1.5$

```
Clear["Global`*"]
```

```
non = {x^2 y''[x] + 3 x y'[x] + 0.75 y[x] == 0, y[1] == 1, y'[1] == -1.5}
by = DSolve[non, y, x]
```

$\{0.75\, y[x] + 3\, x\, y'[x] + x^2\, y''[x] == 0,\ y[1] == 1,\ y'[1] == -1.5\}$

$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{1.\, x^{0.5} + 8.88178 \times 10^{-16}\, x^{1.5}}{x^{2.}}\right]\right\}\right\}$

```
rek = Chop[by, 10^-15]
```

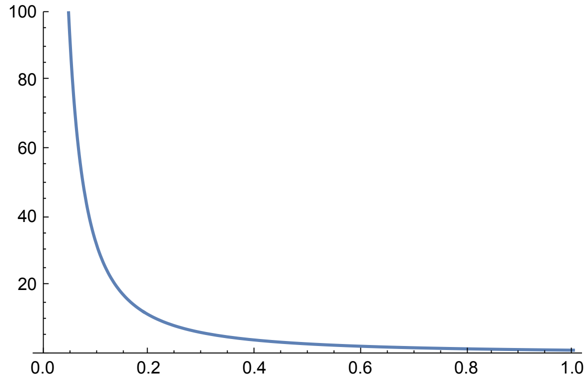
$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{1.\, x^{0.5} + 0\, x^{1.5}}{x^{2.}}\right]\right\}\right\}$

The above step is necessary for the below step to work. Otherwise Mathematica only signs off on the initial value points. Is this method compromised? Depends on the standards applied. Except for numerical versus rational, the answer matches the text's.

```
Simplify[non /. rek]
```

```
{{True, True, True}}
```

```
Plot[y[x] /. rek, {x, 0, 1}, PlotRange → {0, 100}, ImageSize → 300]
```



15. $x^2 y'' + 3 x y + y = 0$, $y[1] = 3.6$, $y'[1] = 0.4$

```
Clear["Global`*"]
```

```
hote = {x^2 y''[x] + 3 x y'[x] + y[x] == 0, y[1] == 3.6, y'[1] == 0.4}
deli = DSolve[hote, y, x]
```

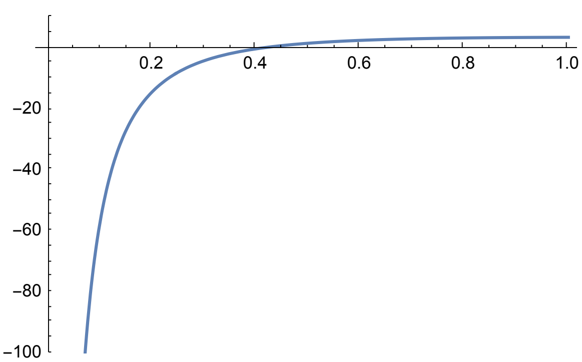
$\{y[x] + 3\,x\,y'[x] + x^2\,y''[x] == 0,\ y[1] == 3.6,\ y'[1] == 0.4\}$

$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{4.\ (0.9 + 1.\ \text{Log}[x])}{x}\right]\right\}\right\}$

```
Simplify[hote /. deli]
```

{{True, True, True}}

```
Plot[y[x] /. deli, {x, 0, 1}, PlotRange → {-100, 10}, ImageSize → 300]
```



The answer matches the text's.

17. $\left(x^2 D^2 + x D + I\right) y = 0$, $y[1] = 1$, $y'[1] = 1$

```
Clear["Global`*"]
```

```
zog = {x^2 y''[x] + x y'[x] + y[x] == 0, y[1] == 1, y'[1] == 1}
nilt = DSolve[zog, y, x]
```

$\{y[x] + x\, y'[x] + x^2\, y''[x] == 0,\ y[1] == 1,\ y'[1] == 1\}$

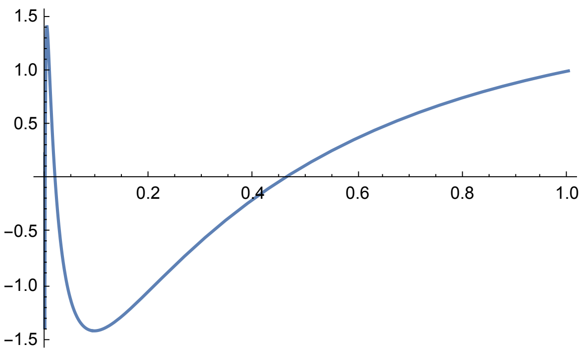
$\{\{y \to \text{Function}[\{x\}, \text{Cos}[\text{Log}[x]] + \text{Sin}[\text{Log}[x]]]\}\}$

```
Simplify[zog /. nilt]
```

{{True, True, True}}

The answer matches the text's.

```
Plot[y[x] /. nilt, {x, -1, 1}, PlotRange → All, ImageSize → 300]
```



19. $\left(x^2 D^2 - x D - 15 I\right) y = 0,\ y[1] = 0.1,\ y'[1] = -4.5$

```
Clear["Global`*"]
```

```
wer = {x^2 y''[x] - x y'[x] - 15 y[x] == 0, y[1] == 0.1, y'[1] == -4.5}
jip = DSolve[wer, y, x]
```

$\{-15\, y[x] - x\, y'[x] + x^2\, y''[x] == 0,\ y[1] == 0.1,\ y'[1] == -4.5\}$

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{0.525\left(-1.19048 + 1.\, x^8\right)}{x^3}\right]\right\}\right\}$$

```
Simplify[wer /. jip]
```

{{x == 0, True, True}}

The initial value points are verified. However, the general sol'n is still under a shadow.

```
ard = jip[[1, 1, 2, 2]]
```

$$-\frac{0.525\left(-1.19048 + 1.\, x^8\right)}{x^3}$$

```
poi = ExpandNumerator[ard]
```

$$\frac{0.625 - 0.525\, x^8}{x^3}$$

```
deke = Expand[poi]
```

$$\frac{0.625}{x^3} - 0.525\, x^5$$

The above form matches the text's answer. Below is a series of steps to check the sol'n 'by hand'.

```
mot[x_] := 0.625`/x^3 - 0.5249999999999999` x^5
```

```
mot1 = D[mot[x], x]
```

$$-\frac{1.875}{x^4} - 2.625\, x^4$$

```
mot2 = D[mot[x], {x, 2}]
```

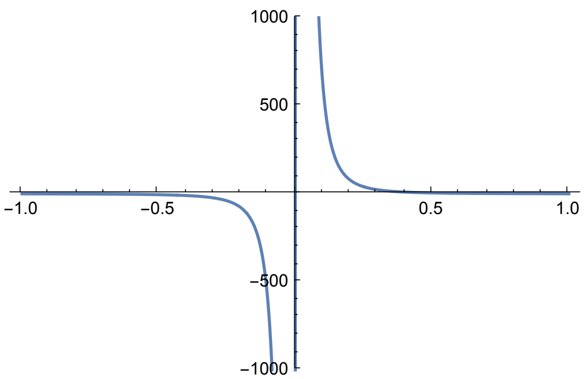
$$\frac{7.5}{x^5} - 10.5\, x^3$$

```
Simplify[-15 mot[x] - x mot′[x] + x^2 mot″[x]]
```

0.

The sol'n is checked 'by hand' and found to be correct.

```
Plot[poi, {x, -1, 1}, PlotRange → {-1000, 1000}, ImageSize → 300]
```



The vertical line in the plot is an artifact, as the function is undefined at $x = 0$.