1 - 11 Initial value problems (IVPs)
Solve the IVPs by the Laplace transform. If necessary, use partial fraction expansion as in example 4 of the text.

1. y' + 5.2 y = 19.4 Sin[2 t], y[0] = 0

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y'[t] + 5.2 y[t] == 19.4 Sin[2 t], t, s]
```

$$5.2\,\text{LaplaceTransform}[y[t], t, s] + s\,\text{LaplaceTransform}[y[t], t, s] - y[0] == \frac{38.8}{4 + s^2}$$

```
e2 = e1 /. {y[0] → 0, LaplaceTransform[y[t], t, s] → yoft}
```

$$5.2\,\text{yoft} + s\,\text{yoft} == \frac{38.8}{4 + s^2}$$

```
e3 = Solve[e2, yoft]
```

$$\left\{\left\{\text{yoft} \to \frac{38.8}{(5.2 + s)\left(4. + s^2\right)}\right\}\right\}$$

```
e4 = e3[[1, 1, 2]]
```

$$\frac{38.8}{(5.2 + s)\left(4. + s^2\right)}$$

```
e5 = FullSimplify[InverseLaplaceTransform[e4, s, t]]
```

```
-1.25 Cos[2. t] + 1.25 Cosh[5.2 t] + 3.25 Sin[2. t] - 1.25 Sinh[5.2 t]
```

```
e6 = TrigToExp[
  1.2499999999999998` Cosh[5.2` t] - 1.2499999999999998` Sinh[5.2` t]]
```

$$0. + 1.25\,e^{-5.2\,t}$$

```
e8 = e5 /.
  (1.2499999999999998` Cosh[5.2` t] - 1.2499999999999998` Sinh[5.2` t]) ->
   1.2499999999999998` e^(-5.2` t)
```

$$1.25\,e^{-5.2\,t} - 1.25\,\text{Cos}[2.\,t] + 3.25\,\text{Sin}[2.\,t]$$

The above answer matches the text’s.

3. y'' = y' - 6 y = 0, y[0] = 11, y'[0] = 28

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y''[t] - y'[t] - 6 y[t] == 0, t, s]
```

$-6\,\text{LaplaceTransform}[y[t],t,s] - s\,\text{LaplaceTransform}[y[t],t,s] + s^2\,\text{LaplaceTransform}[y[t],t,s] + y[0] - s\,y[0] - y'[0] == 0$

```
e2 = e1 /. {y[0] → 11, y'[0] → 28, LaplaceTransform[y[t], t, s] → yoft}
```

$-17 - 11 s - 6\,\text{yoft} - s\,\text{yoft} + s^2\,\text{yoft} == 0$

```
e3 = Solve[e2, yoft]
```

$\left\{\left\{\text{yoft} \to \frac{17 + 11 s}{-6 - s + s^2}\right\}\right\}$

```
e4 = e3[[1, 1, 2]]
```

$\frac{17 + 11 s}{-6 - s + s^2}$

```
e5 = InverseLaplaceTransform[e4, s, t]
```

$e^{-2 t} + 10\, e^{3 t}$

The above answer matches the text.

5. $y'' - \frac{1}{4} y = 0$, $y[0] = 12$, $y'[0] = 0$

```
Clear["Global`*"]
```

$\text{e1} = \text{LaplaceTransform}\left[y''[t] - \frac{1}{4} y[t] == 0, t, s\right]$

$-\frac{1}{4}\,\text{LaplaceTransform}[y[t],t,s] + s^2\,\text{LaplaceTransform}[y[t],t,s] - s\,y[0] - y'[0] == 0$

```
e2 = e1 /. {y[0] -> 12, y'[0] -> 0, LaplaceTransform[y[t], t, s] → yoft}
```

$-12 s - \frac{\text{yoft}}{4} + s^2\,\text{yoft} == 0$

```
e3 = Solve[e2, yoft]
```

$\left\{\left\{\text{yoft} \to \frac{48 s}{-1 + 4 s^2}\right\}\right\}$

```
e4 = e3[[1, 1, 2]]
```

$\frac{48 s}{-1 + 4 s^2}$

```
e5 = InverseLaplaceTransform[e4, s, t]
```

$6\, e^{-t/2} \left(1 + e^{t}\right)$

```
e6 = Simplify[ExpToTrig[e5]]
```

$$12\,\text{Cosh}\left[\frac{t}{2}\right]$$

The above answer matches the text.

7. $y'' + 7\,y' + 12\,y = 21\,e^{3\,t}$, $y[0] = 3.5$, $y'[0] = -10$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y''[t] + 7 y'[t] + 12 y[t] == 21 e^(3 t), t, s]
```

$$12\,\text{LaplaceTransform}[y[t], t, s] + s^2\,\text{LaplaceTransform}[y[t], t, s] + 7\,(s\,\text{LaplaceTransform}[y[t], t, s] - y[0]) - s\,y[0] - y'[0] == \frac{21}{-3 + s}$$

```
e2 = e1 /. {y[0] → 3.5, y'[0] → -10, LaplaceTransform[y[t], t, s] → yoft}
```

$$10 - 3.5\,s + 12\,\text{yoft} + s^2\,\text{yoft} + 7\,(-3.5 + s\,\text{yoft}) == \frac{21}{-3 + s}$$

```
e3 = Solve[e2, yoft]
```

$$\left\{\left\{\text{yoft} \to \frac{14.5 + \frac{21.}{-3.+s} + 3.5\,s}{12. + 7.\,s + s^2}\right\}\right\}$$

```
e4 = e3[[1, 1, 2]]
```

$$\frac{14.5 + \frac{21.}{-3.+s} + 3.5\,s}{12. + 7.\,s + s^2}$$

```
e5 = InverseLaplaceTransform[e4, s, t]
```

$$2.5\,e^{-4.\,t} + 0.5\,e^{-3.\,t} + 0.5\,e^{3.\,t}$$

The above answer matches the text.

9. $y'' - 4\,y' + 3\,y = 6\,t - 8$, $y[0] = 0$, $y'[0] = 0$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y''[t] - 4 y'[t] + 3 y[t] == 6 t - 8, t, s]
```

$$3\,\text{LaplaceTransform}[y[t], t, s] + s^2\,\text{LaplaceTransform}[y[t], t, s] - 4\,(s\,\text{LaplaceTransform}[y[t], t, s] - y[0]) - s\,y[0] - y'[0] == \frac{6}{s^2} - \frac{8}{s}$$

```
e2 = e1 /. {y[0] → 0, y'[0] → 0, LaplaceTransform[y[t], t, s] → yoft}
```

$$3\,\text{yoft} - 4\,s\,\text{yoft} + s^2\,\text{yoft} == \frac{6}{s^2} - \frac{8}{s}$$

```
e3 = Solve[e2, yoft]
```

$$\left\{\left\{yoft \to -\frac{2\,(-3+4\,s)}{s^2\,\left(3-4\,s+s^2\right)}\right\}\right\}$$

```
e4 = e3[[1, 1, 2]]
```

$$-\frac{2\,(-3+4\,s)}{s^2\,\left(3-4\,s+s^2\right)}$$

```
InverseLaplaceTransform[e4, s, t]
```

$$e^t - e^{3\,t} + 2\,t$$

The above answer matches the text.

11. y'' + 3 y' + 2.25 y = 9 t^3 + 64, y[0] = 1, y'[0] = 31.5

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y''[t] + 3 y'[t] + 2.25 y[t] == 9 t^3 + 64, t , s]
```

$$2.25\,\text{LaplaceTransform}[y[t], t, s] + s^2\,\text{LaplaceTransform}[y[t], t, s] + 3\,(s\,\text{LaplaceTransform}[y[t], t, s] - y[0]) - s\,y[0] - y'[0] == \frac{54}{s^4} + \frac{64}{s}$$

```
e2 = e1 /. {y[0] → 1, y'[0] → 31.5, LaplaceTransform[y[t], t, s] → yoft}
```

$$-31.5 - s + 2.25\,yoft + s^2\,yoft + 3\,(-1 + s\,yoft) == \frac{54}{s^4} + \frac{64}{s}$$

```
e3 = Solve[e2, yoft]
```

$$\left\{\left\{yoft \to \frac{34.5 + \frac{54.}{s^4} + \frac{64.}{s} + 1.\,s}{2.25 + 3.\,s + s^2}\right\}\right\}$$

```
e4 = e3[[1, 1, 2]]
```

$$\frac{34.5 + \frac{54.}{s^4} + \frac{64.}{s} + 1.\,s}{2.25 + 3.\,s + s^2}$$

```
e5 = InverseLaplaceTransform[e4, s, t] // Simplify
```

$$e^{-1.5\,t}\,(1. + 1.\,t) + t\,\left(32. - 16.\,t + 4.\,t^2\right)$$

The above answer matches the text.

12 - 15 Shifted data problems
Solve the shifted data IVPs by the Laplace transform.

13. y' - 6 y = 0, y[-1] = 4

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y'[t] - 6 y[t] == 0, t, s]
```

-6 LaplaceTransform[y[t], t, s] + s LaplaceTransform[y[t], t, s] - y[0] == 0

Below: the shift in t-space. Here $t + 1 = \tilde{t}$ takes care of the initial value. It is only necessary to remember to retract the tilde formula when back in t-space after solving.

```
e2 = e1 /. {y[0] → 4, LaplaceTransform[y[t], t, s] → yoft}
```

-4 - 6 yoft + s yoft == 0

```
e3 = Solve[e2, yoft]
```

$\left\{\left\{\text{yoft} \to \frac{4}{-6+s}\right\}\right\}$

```
e4 = e3[[1, 1, 2]]
```

$\frac{4}{-6+s}$

```
e5 = InverseLaplaceTransform[e4, s, t]
```

$4\,e^{6t}$

```
e6 = e5 /. t → t + 1
```

$4\,e^{6(1+t)}$

The above answer matches the text.

15. $y'' + 3y' - 4y = 6e^{2t-3}$, $y[1.5] = 4$, $y'[1.5] = 5$

The answer to this problem is incorrect. The problem the answer would go with is close to $y'[t] - 2y[t] = -7$, which is a much simpler problem.

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[y''[t] + 3 y'[t] - 4 y[t] == 6 e^(2 t-3), t, s]
```

$-4\,\text{LaplaceTransform}[y[t],t,s] + s^2\,\text{LaplaceTransform}[y[t],t,s] + 3\,(s\,\text{LaplaceTransform}[y[t],t,s] - y[0]) - s\,y[0] - y'[0] == \frac{6}{e^3\,(-2+s)}$

The shift will be $t - 1.5 = \tilde{t}$.

```
e2 = e1 /. {y[0] → 4, y'[0] → 5, LaplaceTransform[y[t], t, s] → yoft}
```

$-5 - 4s - 4\,\text{yoft} + s^2\,\text{yoft} + 3\,(-4 + s\,\text{yoft}) == \frac{6}{e^3\,(-2+s)}$

```
e3 = Solve[e2, yoft]
```

$$\left\{\left\{\text{yoft} \to \frac{6 - 34\, e^3 + 9\, e^3\, s + 4\, e^3\, s^2}{e^3\, (-2 + s)\, \left(-4 + 3\, s + s^2\right)}\right\}\right\}$$

```
e4 = e3[[1, 1, 2]]
```

$$\frac{6 - 34\, e^3 + 9\, e^3\, s + 4\, e^3\, s^2}{e^3\, (-2 + s)\, \left(-4 + 3\, s + s^2\right)}$$

```
e5 = FullSimplify[e4]
```

$$\frac{-34 + \frac{6}{e^3} + 9\, s + 4\, s^2}{8 - 10\, s + s^2 + s^3}$$

```
e6 = InverseLaplaceTransform[e5, s, t]
```

$$\frac{1}{5}\, e^{-3-4\,t}\, \left(1 - e^3 - 6\, e^{5\,t} + 5\, e^{6\,t} + 21\, e^{3+5\,t}\right)$$

```
e7 = Collect[e6, e]
```

$$\frac{1}{5}\, e^{-4\,t}\, \left(-1 + 21\, e^{5\,t}\right) + \frac{1}{5}\, e^{-3-4\,t}\, \left(1 - 6\, e^{5\,t} + 5\, e^{6\,t}\right)$$

```
e8 = e7 /. t → t - 1.5
```

$$\frac{1}{5}\, e^{-4\,(-1.5+t)}\, \left(-1 + 21\, e^{5\,(-1.5+t)}\right) + \frac{1}{5}\, e^{-3-4\,(-1.5+t)}\, \left(1 - 6\, e^{5\,(-1.5+t)} + 5\, e^{6\,(-1.5+t)}\right)$$

Since the text answer is wrong, I can't see doing anything further to simplify here although it obviously could use some. Below I use the Y(s) equation given in the answer to generate the text answer. This Y(s) is much different from the one created by the problem as printed. Compare yellow with pink cells. For one thing, in the pink there is no factor of e.

$$\text{e55} = (s - 1)\,(s + 4)\,\text{yt} == 4\, s + 17 + \frac{6}{(s - 2)}$$

$$(-1 + s)\,(4 + s)\,\text{yt} == 17 + \frac{6}{-2 + s} + 4\, s$$

```
e56 = Expand[e55]
```

$$-4\,\text{yt} + 3\, s\,\text{yt} + s^2\,\text{yt} == 17 + \frac{6}{-2 + s} + 4\, s$$

```
e57 = Solve[e56, yt]
```

$$\left\{\left\{yt \to \frac{-7 + 4 s}{(-2 + s)\ (-1 + s)}\right\}\right\}$$

```
e58 = e57[[1, 1, 2]]
```

$$\frac{-7 + 4 s}{(-2 + s)\ (-1 + s)}$$

```
e59 = InverseLaplaceTransform[e58, s, t]
```

$$e^{t} \left(3 + e^{t}\right)$$

```
e60 = e59 /. t → t - 1.5
```

$$e^{-1.5+t} \left(3 + e^{-1.5+t}\right)$$

The above answer matches the text.

16 - 21 Obtaining transforms by differentiation
Using numbered lines (1) or (2) on p. 211, find $\mathcal{L}$[f] if f[t] equals:

17. $t\, e^{-\alpha t}$

```
Clear["Global`*"]
```

```
LaplaceTransform[t e^(-a t), t, s]
```

$$\frac{1}{(a + s)^2}$$

The answer above matches the text.

19. $\text{Sin}[\omega\, t]^2$

```
Clear["Global`*"]
```

```
LaplaceTransform[Sin[ω t]^2, t, s]
```

$$\frac{2\,\omega^2}{s^3 + 4\, s\, \omega^2}$$

The answer above matches the text.

21. $\text{Cosh}[t]^2$

```
Clear["Global`*"]
```

```
LaplaceTransform[Cosh[t]^2, t, s]
```

$$\frac{-2+s^2}{s\left(-4+s^2\right)}$$

The answer above matches the text.

23 - 29 Inverse transforms by integration
Using theorem 3, p. 213, find f[t] if $\mathcal{L}$[F] equals:

23. $\frac{3}{s^2 + s/4}$

```
Clear["Global`*"]
```

```
InverseLaplaceTransform[3/(s^2 + s/4), s, t]
```

$$3\left(4-4\,e^{-t/4}\right)$$

The answer above matches the text.

25. $\frac{1}{s\left(s^2+\omega^2\right)}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[1/(s (s^2 + ω^2)), s, t]
```

$$\frac{1-\text{Cos}[t\,\omega]}{\omega^2}$$

The answer above matches the text.

27. $\frac{s+1}{s^4+9\,s^2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[(s + 1)/(s^4 + 9 s^2), s, t]
```

$$\frac{1}{27}(3 + 3\,t - 3\,\mathrm{Cos}[3\,t] - \mathrm{Sin}[3\,t])$$

The answer above matches the text.

29. $\frac{1}{s^3 + a\,s^2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[1/(s^3 + a s^2), s, t]
```

$$-\frac{1}{a^2} + \frac{e^{-a\,t}}{a^2} + \frac{t}{a}$$

The answer above matches the text.