2 - 11 Transforms by differentiation.
Showing the details of your work, find $\mathcal{L}[f]$ if f[t] equals:

3. $\frac{1}{2} t e^{-3t}$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[1/2 t e^(-3 t), t, s]
```

$$\frac{1}{2 (3 + s)^2}$$

Above: The answer matches the text.

5. t Cos[$\omega$ t]

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[t Cos[ω t], t, s]
```

$$\frac{s^2 - \omega^2}{\left(s^2 + \omega^2\right)^2}$$

Above: The answer matches the text.

7. $t^2$ Cosh[2 t]

```
Clear["Global`*"]
```

```
LaplaceTransform[t^2 Cosh[2 t], t, s]
```

$$\frac{2 s \left(12 + s^2\right)}{\left(-4 + s^2\right)^3}$$

Above: The answer matches the text.

9. $\frac{1}{2} t^2$ Sin[$\pi$ t]

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[1/2 t^2 Sin[π t], t, s]
```

$$-\frac{\pi\left(\pi^2-3\,s^2\right)}{\left(\pi^2+s^2\right)^3}$$

Above: The answer matches the text.

11. $4\,t\,\text{Cos}\left[\frac{1}{2}\pi\,t\right]$

```
Clear["Global`*"]
```

```
e1 = LaplaceTransform[4 t Cos[1/2 π t], t, s]
```

$$\frac{16\left(-\pi^2+4\,s^2\right)}{\left(\pi^2+4\,s^2\right)^2}$$

```
e2 = (-π^2 + 4 s^2)/(1/4 π^2 + s^2)^2
```

$$\frac{-\pi^2+4\,s^2}{\left(\frac{\pi^2}{4}+s^2\right)^2}$$

```
e3 = FullSimplify[e1 == e2]
True
```

Above: The answer matches the text.

14 - 20 Inverse transforms
Using differentiation, integration, s-shifting, or convolution, and showing the details, find f[t] if $\mathcal{L}[f]$ equals:

15. $\frac{s}{\left(s^2-9\right)^2}$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[s/(s^2 - 9)^2, s, t]
```

$$\frac{1}{12}e^{-3t}\left(-1+e^{6t}\right)t$$

```
e2 = ExpToTrig[e1]
```

$$\frac{1}{12} t \, (\text{Cosh}[3\,t] - \text{Sinh}[3\,t]) \, (-1 + \text{Cosh}[6\,t] + \text{Sinh}[6\,t])$$

```
e3 = FullSimplify[e2]
```

$$\frac{1}{6} t \, \text{Sinh}[3\,t]$$

Above: The answer matches the text.

17. $\text{Log}\left[\frac{s}{s-1}\right]$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[Log[s/(s - 1)], s, t]
```

$$-\frac{1 - e^t}{t}$$

Above: The answer matches the text.

19. $\text{Log}\left[\frac{s^2+1}{(s-1)^2}\right]$

```
Clear["Global`*"]
```

```
e1 = InverseLaplaceTransform[Log[(s^2 + 1)/(s - 1)^2], s, t]
```

$$\frac{2\left(e^t - \text{Cos}[t]\right)}{t}$$

Above: The answer matches the text.