Example 1, p 100

```
Clear["Global`*"]
```

```
eqn = y''[x] + y[x] == Sec[x]
```

```
y[x] + y″[x] == Sec[x]
```

```
sol = DSolve[eqn, y, x]
```

```
{{y → Function[{x},
    C[1] Cos[x] + Cos[x] Log[Cos[x]] + x Sin[x] + C[2] Sin[x]]}}
```

```
eqn /. sol // Simplify
```

```
{True}
```

```
frx = Collect[sol, {Cos[x], Sin[x]}]
```

```
{{y → Function[{x},
    C[1] Cos[x] + Cos[x] Log[Cos[x]] + x Sin[x] + C[2] Sin[x]]}}
```

Above: The answer is correct, except for the Cos[x] which is the argument of Log. This argument should be an absolute value. Putting in the Abs[] makes for a nice-looking solution, but I have no clue how to diagnose its desirability.

```
grx = frx /. {C[1] → 1, C[2] → 1}
```

```
{{y → Function[{x}, 1 Cos[x] + Cos[x] Log[Cos[x]] + x Sin[x] + 1 Sin[x]]}}
```

```
xrx = Cos[x] (1 + Log[Abs[Cos[x]]]) + (x + 1) Sin[x]
```

```
Cos[x] (1 + Log[Abs[Cos[x]]]) + (1 + x) Sin[x]
```

Here I get a couple of initial values for use for NDSolve. The accuracy of location of the first can be seen on the colored plot below.

```
(Cos[x] + Cos[x] Log[Cos[x]] + x Sin[x] + 1 Sin[x]) /. x → 0
```

```
1
```

```
D[(Cos[x] + Cos[x] Log[Cos[x]] + x Sin[x] + 1 Sin[x]), x]
```

```
Cos[x] + x Cos[x] - Sin[x] - Log[Cos[x]] Sin[x]
```

```
(Cos[x] + x Cos[x] - Sin[x] - Log[Cos[x]] Sin[x]) /. x → 0
```

```
1
```

```
s2 = NDSolve[{eqn, y[0] == 1, y'[0] == 1},
  y, {x, -6 - 6}, Method -> "ImplicitRungeKutta"]
```

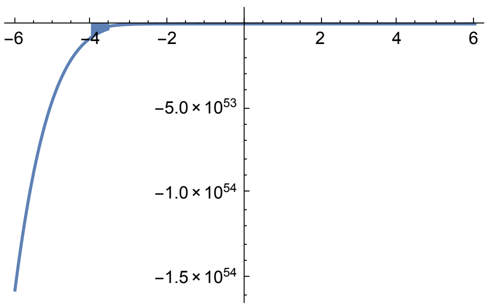
NDSolve::ndsz: At x == −1.5708, step size is effectively zero; singularity or stiff system suspected. ≫

```
{{y → InterpolatingFunction[ Domain: {{-1.57, 0.}} Output: scalar ]}}
```

Using NDSolve does me no good at all. I was hoping it would fix the problem with the gaps in the plot, but it looks meaningless.

```
Plot[Evaluate[y[x] /. s2], {x, -6, 6}, PlotRange → All, ImageSize → 250]
```

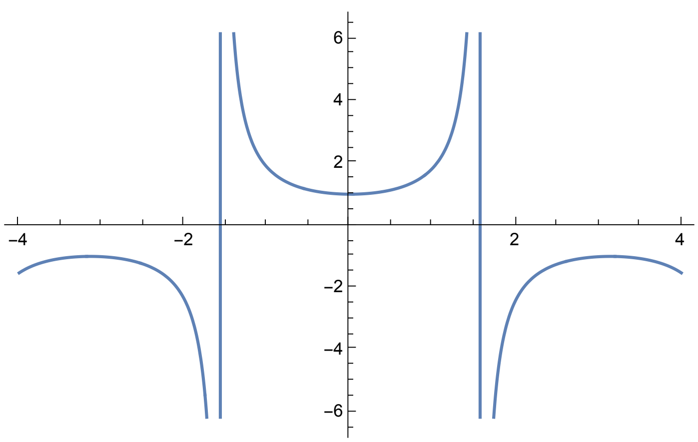
InterpolatingFunction::dmval:
Input value {−5.99975} lies outside the range of data in the interpolating function. Extrapolation will be used. ≫



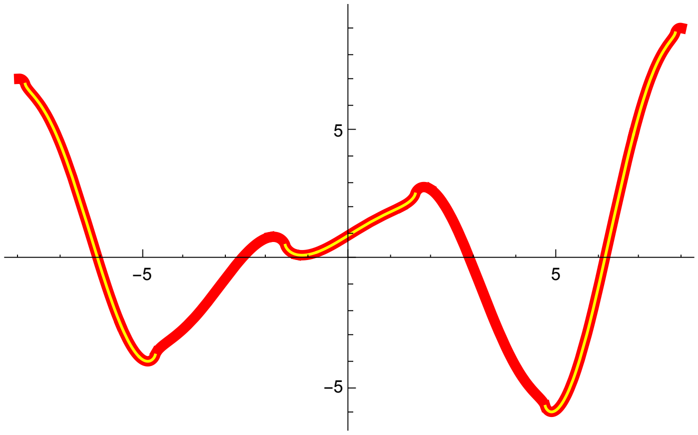

```
plot1 = Plot[y[x] /. grx, {x, -8, 8},
   PlotRange → Automatic, PlotStyle → {Yellow, Thickness[0.004]}];
plot2 = Plot[xrx, {x, -8, 8}, PlotRange → Automatic,
   PlotStyle → {Red, Thickness[0.015]}];
```

Looking at the plot of the RHS of the original problem might have given a clue to trouble ahead.

```
Plot[Sec[x], {x, -4, 4}]
```



```
Show[plot2, plot1]
```




The Mathematica function is in yellow, and the text version, with Abs, is in red. The question is, how is it to be detected that the solution by Mathematica is flawed and how is it to be understood what correction needs to be made?

1 - 13 General solution

Solve the given nonhomogeneous linear ODE by variation of parameters or undetermined coefficients.

1. y" + 9 y = Sec[3 x]

```
Clear["Global`*"]

has = y''[x] + 9 y[x] == Sec[3 x]
bas = DSolve[has, y[x], x]
```

$$9\, y[x] + y''[x] == \text{Sec}[3\, x]$$

$$\{\{y[x] \to C[1]\, \text{Cos}[3\, x] + C[2]\, \text{Sin}[3\, x] + \frac{1}{9}\,(\text{Cos}[3\, x]\, \text{Log}[\text{Cos}[3\, x]] + 3\, x\, \text{Sin}[3\, x])\}\}$$

1. Above: This is one case where Mathematica comes up with the text answer without significant simplification.

```
nas = bas /. {C[1] → A, C[2] → B}
```

$$\{\{y[x] \to A\, \text{Cos}[3\, x] + B\, \text{Sin}[3\, x] + \frac{1}{9}\,(\text{Cos}[3\, x]\, \text{Log}[\text{Cos}[3\, x]] + 3\, x\, \text{Sin}[3\, x])\}\}$$

2. Above: The slightly altered form matches the text answer.

3. $x^2\, y'' - 2\, x\, y' + 2\, y = x^3\, \text{Sin}[x]$

```
Clear["Global`*"]
```

```
din = x^2 y''[x] - 2 x y'[x] + 2 y[x] == x^3 Sin[x]
fin = DSolve[din, y[x], x]
```

$2\,y[x] - 2\,x\,y'[x] + x^2\,y''[x] == x^3\,\text{Sin}[x]$

$\{\{y[x] \to x\,C[1] + x^2\,C[2] - x\,\text{Sin}[x]\}\}$

1. Above: Mathematica produces the text answer on the first try.

5. y" + y = Cos[x] - Sin[x]

```
Clear["Global`*"]
```

```
sar = y''[x] + y[x] == Cos[x] - Sin[x]
zar = DSolve[sar, y[x], x]
```

$y[x] + y''[x] == \text{Cos}[x] - \text{Sin}[x]$

$$\left\{\left\{y[x] \to C[1]\,\text{Cos}[x] + C[2]\,\text{Sin}[x] + \frac{1}{4}\left(2\,x\,\text{Cos}[x] + 2\,\text{Cos}[x]^3 + 2\,x\,\text{Sin}[x] + 2\,\text{Cos}[x]^2\,\text{Sin}[x] - \text{Cos}[x]\,\text{Sin}[2\,x] + \text{Sin}[x]\,\text{Sin}[2\,x]\right)\right\}\right\}$$

```
mar = Expand[zar]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\,x\,\text{Cos}[x] + C[1]\,\text{Cos}[x] + \frac{\text{Cos}[x]^3}{2} + \frac{1}{2}\,x\,\text{Sin}[x] + C[2]\,\text{Sin}[x] + \frac{1}{2}\,\text{Cos}[x]^2\,\text{Sin}[x] - \frac{1}{4}\,\text{Cos}[x]\,\text{Sin}[2\,x] + \frac{1}{4}\,\text{Sin}[x]\,\text{Sin}[2\,x]\right\}\right\}$$

```
yar = TrigReduce[mar]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\,(\text{Cos}[x] + x\,\text{Cos}[x] + 2\,C[1]\,\text{Cos}[x] + x\,\text{Sin}[x] + 2\,C[2]\,\text{Sin}[x])\right\}\right\}$$

```
Collect[yar, x]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\,x\,(\text{Cos}[x] + \text{Sin}[x]) + \frac{1}{2}\,(\text{Cos}[x] + 2\,C[1]\,\text{Cos}[x] + 2\,C[2]\,\text{Sin}[x])\right\}\right\}$$

1. Above: The expression ‘yar’ looks pretty good. However, getting Mathematica to refine it further is a challenge.

$$\text{yar1} = \frac{1}{2}\,x\,(\text{Cos}[x] + \text{Sin}[x])$$

$$\frac{1}{2}\,x\,(\text{Cos}[x] + \text{Sin}[x])$$

2. Above: I resorted to splitting the expression into two pieces. ‘yar1’ is the first piece.

```
yar2 = 1/2 (Cos[x] + 2 C[1] Cos[x] + 2 C[2] Sin[x])
```

$$\frac{1}{2}\,(\text{Cos}[x] + 2\,C[1]\,\text{Cos}[x] + 2\,C[2]\,\text{Sin}[x])$$

3. Above: ‘yar2’ is the second piece.

```
ear2 = Simplify[yar2]
```

$$\left(\frac{1}{2} + C[1]\right)\text{Cos}[x] + C[2]\,\text{Sin}[x]$$

```
war2 = ear2 /. {(1/2 + C[1]) → A, C[2] → B}
A Cos[x] + B Sin[x]
```

4. Above: I was finally able to replace constant names.

```
out = yar1 + war2
```

$$A\,\text{Cos}[x] + B\,\text{Sin}[x] + \frac{1}{2}\,x\,(\text{Cos}[x] + \text{Sin}[x])$$

5. The above form matches the text answer.

7. $(D^2 - 4\,D + 4\,I)\; y = 6\,e^{2\,x/x^4}$

```
Clear["Global`*"]
```

```
sop = y''[x] - 4 y'[x] + 4 y[x] == 6 e^(2 x)/x^4
drop = DSolve[sop, y[x], x]
```

$$4\,y[x] - 4\,y'[x] + y''[x] == \frac{6\,e^{2\,x}}{x^4}$$

$$\left\{\left\{y[x] \to \frac{e^{2\,x}}{x^2} + e^{2\,x}\,C[1] + e^{2\,x}\,x\,C[2]\right\}\right\}$$

1. Above: Except for minor difference in form, the answer matches the text’s.

9. $(D^2 - 2\,D + I)\; y = 35\,x^{3/2}\,e^x$

```
Clear["Global`*"]
```

```
gat = y''[x] - 2 y'[x] + y[x] == 35 x^(3/2) E^x
fat = DSolve[gat, y[x], x]
```

$y[x] - 2 y'[x] + y''[x] == 35\, e^x x^{3/2}$

$\{\{y[x] \to 4\, e^x x^{7/2} + e^x C[1] + e^x x\, C[2]\}\}$

1. Above: Except for minor difference in form, the answer matches the text's.

11. $(x^2 D^2 - 4 x D + 6 I)\; y = 21\, x^{-4}$

```
Clear["Global`*"]
```

```
stek = x^2 y''[x] - 4 x y'[x] + 6 y[x] == 21 x^-4
peck = DSolve[stek, y[x], x]
```

$6 y[x] - 4 x y'[x] + x^2 y''[x] == \frac{21}{x^4}$

$\{\{y[x] \to \frac{1}{2 x^4} + x^2 C[1] + x^3 C[2]\}\}$

1. Above: The answer matches the text's.

13. $(x^2 D^2 + x D - 9 I)\; y = 48\, x^5$

```
Clear["Global`*"]
```

```
naru = x^2 y''[x] + x y'[x] - 9 y[x] == 48 x^5
garu = DSolve[naru, y[x], x]
```

$-9 y[x] + x y'[x] + x^2 y''[x] == 48 x^5$

$\{\{y[x] \to 4 x^2 \mathrm{Cosh}[3 \mathrm{Log}[x]] - x^8 \mathrm{Cosh}[3 \mathrm{Log}[x]] + C[1] \mathrm{Cosh}[3 \mathrm{Log}[x]] + 4 x^2 \mathrm{Sinh}[3 \mathrm{Log}[x]] + x^8 \mathrm{Sinh}[3 \mathrm{Log}[x]] + i\, C[2] \mathrm{Sinh}[3 \mathrm{Log}[x]]\}\}$

```
haru = FullSimplify[TrigReduce[garu]]
```

$\{\{y[x] \to \frac{1}{2 x^3}\left(6 x^8 + C[1] + x^6 C[1] + i\,(-1 + x^6)\, C[2]\right)\}\}$

```
baru = haru /. (-1 + x^6) -> 0
```

$\{\{y[x] \to \frac{6 x^8 + C[1] + x^6 C[1]}{2 x^3}\}\}$

```
paru = Apart[baru]
```

$\{\{y[x] \to 3 x^5 + \frac{(1 + x^6)\, C[1]}{2 x^3}\}\}$

```
maru = ExpandNumerator[paru]
```

$$\left\{\left\{y[x] \to 3\, x^5 + \frac{C[1] + x^6\, C[1]}{2\, x^3}\right\}\right\}$$

$$\text{yaru} = \text{maru} \;/.\; \left(\frac{C[1] + x^6\, C[1]}{2\, x^3}\right) \to \left(\frac{C[1]}{2\, x^3} + \frac{x^6\, C[1]}{2\, x^3}\right)$$

$$\left\{\left\{y[x] \to 3\, x^5 + \frac{C[1]}{2\, x^3} + \frac{1}{2}\, x^3\, C[1]\right\}\right\}$$

$$\text{saru} = \text{yaru} \;/.\; \left\{\frac{C[1]}{2\, x^3} \to \frac{C[3]}{x^3},\ \frac{1}{2}\, x^3\, C[1] \to C[3]\, x^3\right\}$$

$$\left\{\left\{y[x] \to 3\, x^5 + \frac{C[3]}{x^3} + x^3\, C[3]\right\}\right\}$$

1. Above: The expression matches (approximately) the form of the text answer. I was a little surprised I did not have to split up the whole expression somewhere in the simplification process. There is the matter of having only one distinct constant. I don't know if that detracts in some way.