

Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

7. Location of maximum. Could we find a profit  $f[x_1, x_2] = a_1 x_1 + a_2 x_2$  whose maximum is at an interior point of the quadrangle in figure 474? Give a reason for your answer.

```
d1 = ImplicitRegion[
  {x > 0, y > 0, y < - $\frac{5}{2}$  x + 30, y < (2.5 / -10) x + 7.75}, {x, y}]
ImplicitRegion[x > 0 && y > 0 && y < 30 -  $\frac{5 x}{2}$  && y < 7.75 - 0.25 x, {x, y}]
ziz = Plot[{- $\frac{5 x}{2}$  + 30, - $\frac{2.5 x}{10}$  + 7.5}, {x, 0, 20}, AspectRatio -> Automatic];
dip = Graphics[Polygon[{{0, 0}, {12, 0}, {10, 5}, {0, 7.5}, {0, 0}}]];
Show[ziz, dip];
```

The answer to the question is no, as demonstrated below. I changed the less-than-or-equals signs for strictly less-than, and it blew up. So no solution is available which is interior to the boundary lines.

```
Clear["Global`*"]
```

```
Maximize[{40 x + 88 y, 2 x + 8 y ≤ 60, 5 x + 2 y ≤ 60}, {x, y}]
{840, {x -> 10, y -> 5}}
```

```
Maximize[{40 x + 88 y, 2 x + 8 y < 60, 5 x + 2 y < 60}, {x, y}]
```

Maximize::wksol: Warning: there is no maximum in the region in which the objective function is defined and the constraints are satisfied; a result on the boundary will be returned >>

```
{840, {x -> 10, y -> 5}}
```

9. What is the meaning of the slack variables  $x_3, x_4$  in example 2 in terms of the problem in example 1?

Looking at example 1.

```
Clear["Global`*"]
```

```
Maximize[{40 x + 88 y, 2 x + 8 y ≤ 60, 5 x + 2 y ≤ 60}, {x, y}]
{840, {x -> 10, y -> 5}}
```

The above expression shows that the slack variables are unnecessary here, unlike problem 17, where I found them to be necessary. I guess if I knew that the problem would find  $x$  and  $y$  positive, I could leave out the slacks. And if it wouldn't solve, or the signs came out wrong, I could put them in.

## 11 - 16 Maximization, minimization

Maximize or minimize the given objective function  $f$  subject to the give constraints.

11. Maximize  $f = 30x_1 + 10x_2$  in the region in problem 5.

The region of problem 5:  $-x_1 + x_2 \geq 0$ ;  $x_1 + x_2 \leq 5$ ;  $-2x_1 + x_2 \leq 16$

**Maximize**[{30 x + 10 y, -x + y ≥ 0, x + y ≤ 5, -2 x + y ≤ 16}, {x, y}]

{100, {x →  $\frac{5}{2}$ , y →  $\frac{5}{2}$ }}

13. Maximize  $f = 5x_1 + 25x_2$  in the region in problem 5.

**Maximize**[{5 x + 25 y, -x + y ≥ 0, x + y ≤ 5, -2 x + y ≤ 16}, {x, y}]

{ $\frac{595}{3}$ , {x →  $-\frac{11}{3}$ , y →  $\frac{26}{3}$ }}

15. Maximize  $f = 20x_1 + 30x_2$  subject to  $4x_1 + 3x_2 \geq 12$ ,  $x_1 - x_2 \geq -3$ ,  $x_2 \leq 6$ ,  $2x_1 - 3x_2 \leq 0$ .

**Maximize**[{20 x + 30 y, 4 x + 3 y ≥ 12, x - y ≥ -3, y ≤ 6, 2 x - 3 y ≤ 0}, {x, y}]

{360, {x → 9, y → 6}}

17. Maximum profit. United Metal, Inc., produces alloys  $B_1$  (special brass) and  $B_2$  (yellow tombac).  $B_1$  contains 50% copper and 50% zinc. (Ordinary brass contains about 65% copper and 35% zinc.)  $B_2$  contains 75% copper and 25% zinc. Net profits are \$120 per ton of  $B_1$  and \$100 per ton of  $B_2$ . The daily copper supply is 45 tons. The daily zinc supply is 30 tons. Maximize the net profit of the daily production.

**Clear**["Global`\*"]

This took longer than it should have due to my befuddlement. In this analysis, I have  $x$  as the weight per day of  $B_1$ , and  $y$  as weight per day of  $B_2$ . The 120 and 100 profit is also on per weight basis, so those weight units cancel. The tons units don't need to show in the raw ore weight either, since they are included in the  $x$  and  $y$ , which stand on the other side of the equals sign. The Maximize expression was judged as unbounded by Mathematica until I put in the constraints for both  $x$  and  $y$  to be greater than or equal to zero (the slacks). The other thing that I should point out is that what I am trying to maximize is actually a function of two variables.

**Maximize**[{120 x + 100 y, 0.5 x + 0.75 y ≤ 45 ,  
0.5 x + 0.25 y ≤ 30, x ≥ 0, y ≥ 0}, {x, y}]

{8400., {x → 45., y → 30.}}

19. Maximum output. Giant Ladders, Inc., wants to maximize its daily total output of large step ladders by producing  $x_1$  of them by a process  $P_1$  and  $x_2$  by a process  $P_2$ , where  $P_1$  requires 2 hours of labor and 4 machine hours per ladder, and  $P_2$  requires 3 hours of labor and 2 machine hours. For this kind of work, 1200 hours of labor and 1600 hours on the machines are, at most, available per day. Find the optimal  $x_1$  and  $x_2$ .

Here x and y will both be ladders per day.  $\frac{1}{D} P1 = x$  and  $\frac{1}{D} P2 = y$ .

**Maximize**[{x + y, 2 x + 3 y ≤ 1200, 4 x + 2 y ≤ 1600, x ≥ 0, y ≥ 0}, {x, y}]

{500, {x → 300, y → 200}}

21. Maximum profit. Universal Electric, Inc., manufactures and sells two models of lamps,  $L_1$  and  $L_2$ , the profit being \$150 and \$100 respectively. The process involves two workers  $W_1$  and  $W_2$  who are available for this kind of work 100 and 80 hours per month, respectively.  $W_1$  assembles  $L_1$  in 20 min and  $L_2$  in 30 min.  $W_2$  paints  $L_1$  in 20 min and  $L_2$  in 10 min. Assuming that all lamps made can be sold without difficulty, determine production figures that maximize the profit.

The profit from lamps. Where x represents the profit from L1 lamp and y the profit from L2 lamp. Remembering to convert hours to minutes.

**Maximize**[  
{150 x + 100 y, 20 x + 30 y ≤ 6000, 20 x + 10 y ≤ 4800, x ≥ 0, y ≥ 0}, {x, y}]

{37 500, {x → 210, y → 60}}

Showing agreement with the text answer is the monthly profit in dollars, and the number of lamps produced and sold of the two types.