1 - 14 Gauss elimination
Solve the linear system given explicitly or by its augmented matrix.

1. $4x - 6y = -11$
$-3x + 8y = 10$

```
ClearAll["Global`*"]
```

```
Solve[4 x - 6 y == -11 && -3 x + 8 y == 10]
```

$\{\{x \to -2,\ y \to \frac{1}{2}\}\}$

The green cell above agrees with the text answer.

3. $x + y - z = 9$, $8y + 6z = -6$, $-2x + 4y - 6z = 40$

```
ClearAll["Global`*"]
```

```
Solve[x + y - z == 9 && 8 y + 6 z == -6 && -2 x + 4 y - 6 z == 40]
```

$\{\{x \to 1,\ y \to 3,\ z \to -5\}\}$

The green cell above agrees with the text answer.

5. $\begin{pmatrix} 13 & 12 & -6 \\ -4 & 7 & -73 \\ 11 & -13 & 157 \end{pmatrix}$

```
{{65., 60., -30.}, {-20., 35., -365.}, {55., -65., 785.}}
```

```
ClearAll["Global`*"]
```

```
Solve[
 13 x + 12 y + -6 z == 0 && -4 x + 7 y - 73 z == 0 && 11 x - 13 y + 157 z == 0, {x, y}]
```

$\{\{x \to -6 z,\ y \to 7 z\}\}$

$A = \begin{pmatrix} 13 & 12 & -6 \\ -4 & 7 & -73 \\ 11 & -13 & 157 \end{pmatrix}$

```
{{13, 12, -6}, {-4, 7, -73}, {11, -13, 157}}
```

```
RowReduce[A]
```

```
{{1, 0, 6}, {0, 1, -7}, {0, 0, 0}}
```

Above: It is seen from inspection that $x = 6$ and $y = -7$, the same values found in the text.

7. $\begin{pmatrix} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{pmatrix}$

```
{{14., 28., 7., 0.}, {-7., 7., -14., 0.}, {28., 0., 42., 0.}}

ClearAll["Global`*"]
```

$A = \begin{pmatrix} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{pmatrix}$

```
{{2, 4, 1, 0}, {-1, 1, -2, 0}, {4, 0, 6, 0}}

e1 = RowReduce[A] // MatrixForm
```

$\begin{pmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\text{Solve}\left[x + \frac{3}{2} z == 0 \,\&\&\, y - \frac{1}{2} z == 0,\ \{x, z\}\right]$

```
{{x → -3 y, z → 2 y}}
```

Above: The text sets $y = t$, but this is not necessary. Either one is arbitrary in value. To accord with text, $x = -3\,t$, $z = 2\,t$.

9. $\begin{pmatrix} 0 & -2 & -2 & -8 \\ 3 & 4 & -5 & 13 \end{pmatrix}$

```
{{0., -18., -18., -72.}, {27., 36., -45., 117.}}

ClearAll["Global`*"]

Solve[-2 y - 2 z == -8 && 3 x + 4 y - 5 z == 13, {x, y}]

{{x → -1 + 3 z, y → 4 - z}}
```

Above: The answer agrees with text, provided a parameter t is invented such that $z = t$.

11. $\begin{pmatrix} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{pmatrix}$

```
{{0., 55., 55., -110., 0.},
 {22., -33., -33., 66., 22.}, {44., 11., 11., -22., 44.}}

Clear["Global`*"]
```

$$\mathbf{A} = \begin{pmatrix} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{pmatrix}$$

```
{{0, 5, 5, -10, 0}, {2, -3, -3, 6, 2}, {4, 1, 1, -2, 4}}
```

Below: The row reduction comes up with a null row. Is that an ominous sign?

```
e1 = RowReduce[A]
{{1, 0, 0, 0, 1}, {0, 1, 1, -2, 0}, {0, 0, 0, 0, 0}}
```

The dot product has fewer equation seeds that there are rows in the A matrix. I play around with the assigned positions and signs of t1 and t2 to try to come up with the text answer.

```
e2 = e1.{x, y, z, t2, -t1}
{-t1 + x, -2 t2 + y + z, 0}

Solve[-t1 + x == 0 && -2 t2 + y + z == 0, {x, y}]
```

```
{{x → t1, y → 2 t2 - z}}
```

Let me trot out the text answer: x = t_1 arb.; y = 2 t_2 - t_1; z = t_2 arb. Rearranging the text answer results in the equation y = 2z - x. If I adopt the stance that z = t_2, then I can make the same equation out of the results above, that is, y = 2z - x.

13. $\begin{pmatrix} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{pmatrix}$

```
{{0., 130., 52., -26., -52.}, {-39., -221., 13., 26., 26.},
 {13., 13., 13., 0., 78.}, {104., -442., 208., -130., 52.}}

ClearAll["Global`*"]
```

$$\mathbf{e1} = \begin{pmatrix} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{pmatrix}$$

```
{{0, 10, 4, -2, -4}, {-3, -17, 1, 2, 2},
 {1, 1, 1, 0, 6}, {8, -34, 16, -10, 4}}

e2 = RowReduce[e1]
{{1, 0, 0, 0, 4}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 2}, {0, 0, 0, 1, 6}}

e3 = e2.{w, x, y, z, t1}
{4 t1 + w, x, 2 t1 + y, 6 t1 + z}

e4 = Solve[4 t1 + w == 0 && 2 t1 + y == 0 && 6 t1 + z == 0 && x == 0]
{{w → -4 t1, x → 0, y → -2 t1, z → -6 t1}}
```

```
e5 = e4 /. t1 → -1
```

```
{{w → 4, x → 0, y → 2, z → 6}}
```

Above: The answer matches the text.