Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

The s.m. has problem 1.

1 - 6 Verifications

1. Harmonic functions. Verify theorem 1, p. 460, for $f = 2z^2 - x^2 - y^2$ and S the surface of the box $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

This deals with the divergence theorem in potential theory. Get a bunch of normals to a surface, and dot them with the grad.

```
Clear["Global`*"]
```

```
innen = 2 z^2 - x^2 - y^2
```

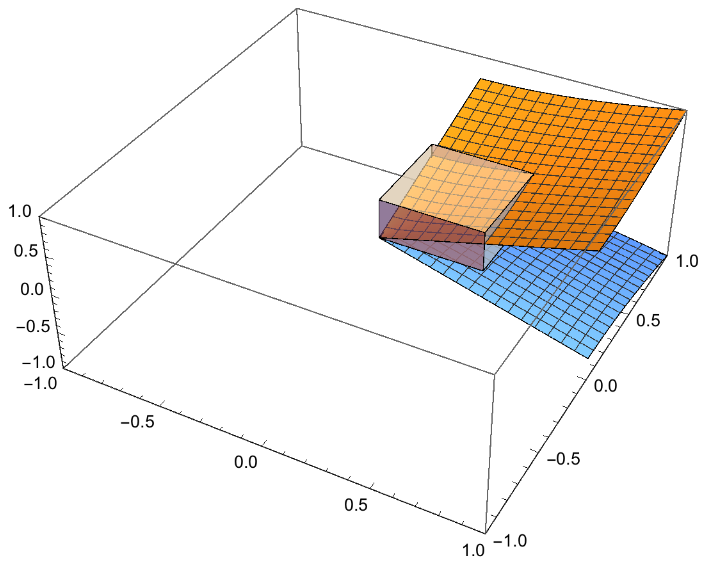
```
inne = Solve[2 z^2 - x^2 - y^2 == 0, z]
```

$$\left\{\left\{z \to -\frac{\sqrt{x^2+y^2}}{\sqrt{2}}\right\}, \left\{z \to \frac{\sqrt{x^2+y^2}}{\sqrt{2}}\right\}\right\}$$

```
fir = Plot3D[{Sqrt[x^2 + y^2]/Sqrt[2], -Sqrt[x^2 + y^2]/Sqrt[2]}, {x, 0, 1},
   {y, 0, 1}, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}];
```

```
sec = Graphics3D[
   {Opacity[.6], Cuboid[{0.5, 0.5, 0.5}, {0, 0, 0}]}, Axes → True];
```

```
Show[fir, sec]
```




```
s1 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{0, 0, 1}
4 z

s2 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{1, 0, 0}
-2 x

s3 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{0, 1, 0}
-2 y

s4 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{0, -1, 0}
2 y

s5 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{-1, 0, 0}
2 x

s6 = Grad[2 z^2 - x^2 - y^2, {x, y, z}].{0, 0, -1}
-4 z

tot = s1 + s2 + s3 + s4 + s5 + s6
0
```

The above answer agrees with the text's.

3. Green's first identity. Verify numbered line (8), p. 461, for f = $4y^2$, g = x^2, S the surface of the "unit cube", $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. What are the assumptions on f and g in numbered line (8)? Must f and g be harmonic?

5. Green's second identity. Verify numbered line (9), p. 461, for f = $6y^2$, g = $2x^2$, S the unit cube in problem 3.