

1 - 5 Components and length

Find the components of the vector \mathbf{v} with initial point P and terminal point Q. Find $|\mathbf{v}|$. Sketch $|\mathbf{v}|$. Find the unit vector \mathbf{u} in the direction of \mathbf{v} .

1. P : (1, 1, 0), Q : (6, 2, 0)

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {1, 1, 0}; qQ = {6, 2, 0};  
vec = qQ - pP
```

```
{5, 1, 0}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
 $\sqrt{26}$ 
```

Below: find normalized version of vector.

```
euc2 = Normalize[vec]
```

```
 $\left\{ \frac{5}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 0 \right\}$ 
```

The green cells above match the answers in the text.

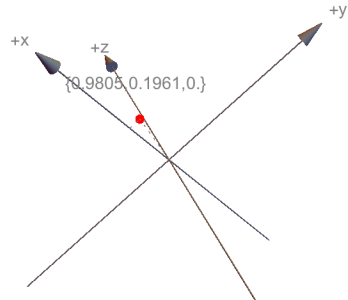
Below: find normalized version in decimal form.

```
euc3 = N[Normalize[vec], 4]
```

```
{0.9806, 0.1961, 0}
```

```
pad = N[euc3 + {.3, .3, .3}, 4]
```

```
{1.28058, 0.496116, 0.3}
```



3. $P : (-3.0, 4.0, -0.5), Q : (5.5, 0, 1.2)$

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {-3.0, 4.0, -0.5}; qQ = {5.5, 0, 1.2};
vec = qQ - pP
```

```
{8.5, -4., 1.7}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
9.54673
```

```
PossibleZeroQ[Chop[euc1] - Chop[Sqrt[91.14]]]
```

```
True
```

Below: find normalized version of vector.

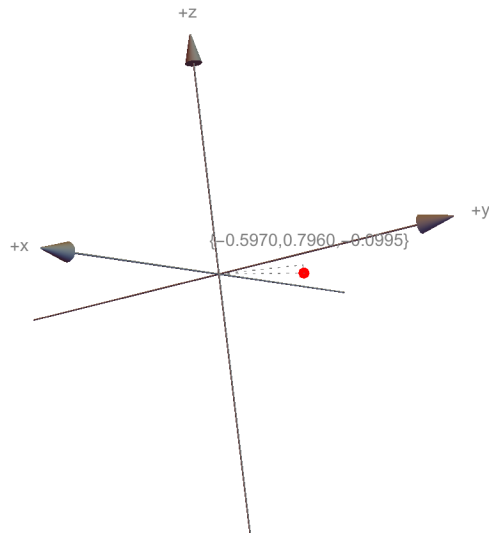
```
euc2 = N[Normalize[vec], 4]
```

```
{0.890357, -0.418992, 0.178071}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
```

```
{1.19036, -0.118992, 0.478071}
```

The green cells above match the answers in the text.



5. $P : (0, 0, 0)$, $Q : (2, 1, -2)$

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {0, 0, 0}; qQ = {2, 1, -2};
vec = qQ - pP
```

```
{2, 1, -2}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
3
```

Below: find normalized version of vector.

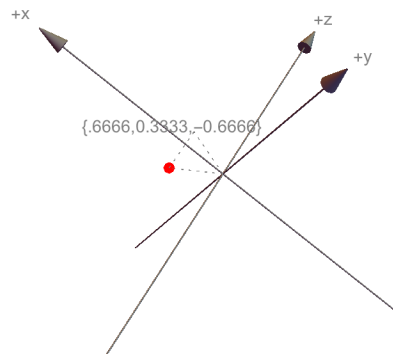
```
euc2 = N[Chop[Normalize[vec], 10^-4]]
```

```
{0.666667, 0.333333, -0.666667}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
```

```
{0.966667, 0.633333, -0.366667}
```

The green cells above match the answers in the text (no entry for vector length).



6 - 10 Find the terminal point Q of the vector \mathbf{v} with components as given and initial point P . Find $|\mathbf{v}|$.

$$7. \quad \frac{1}{2}, \quad 3, \quad -\frac{1}{4}; \quad P : \left(\frac{7}{2}, \quad -3, \quad \frac{3}{4} \right)$$

```
ClearAll["Global`*"]
```

```
vec = {1/2, 3, -1/4}; po = {7/2, -3, 3/4};
```

```
que = vec + po
```

$$\left\{ 4, 0, \frac{1}{2} \right\}$$

```
e1 = Norm[vec + po]
```

$$\frac{\sqrt{65}}{2}$$

```
FullSimplify[ $\frac{\sqrt{65}}{2} == \sqrt{16.25}$ ]
```

True

The green cells above match the answers in the text.

9. 6, 1, -4; P : (-6, -1, -4)

```
ClearAll["Global`*"]
```

```
vec = {6, 1, -4}; po = {-6, -1, -4};  
que = vec + po
```

```
{0, 0, -8}
```

```
e1 = Norm[que]
```

```
8
```

The green cells above match the answers in the text.

11 - 18 Addition, scalar multiplication

Let $a = \{3, 2, 0\} = 3i + 2j$; $b = \{-4, 6, 0\} = 4i + 6j$; $c = \{5, -1, 8\} = 5i - j + 8k$, $d = \{0, 0, 4\} = 4k$

Find

11. $2a$, $\frac{1}{2}a$, $-a$

```
ClearAll["Global`*"]
```

```
aa = {3, 2, 0}; bb = {-4, 6, 0}; cc = {5, -1, 8}; dd = {0, 0, 4}  
{0, 0, 4}
```

2 aa

```
{6, 4, 0}
```

$\frac{1}{2}aa$

```
{ $\frac{3}{2}$ , 1, 0}
```

- aa $\{-3, -2, 0\}$

The green cells above match the answers in the text.

13. $b + c, c + b$ **bb + cc** $\{1, 5, 8\}$ **cc + bb** $\{1, 5, 8\}$ 15. $7(c - b), 7c - 7b$ **7 (cc + bb)** $\{7, 35, 56\}$ **7 (cc - bb)** $\{63, -49, 56\}$ 17. $(7 - 3) a, 7a - 3a$ **(7 - 3) aa** $\{12, 8, 0\}$ **7 aa - 3 aa** $\{12, 8, 0\}$

The green cells above match the answers in the text.

21 - 25 Forces, resultant

Find the resultant in terms of components and its magnitude.

21. $p = \{2, 3, 0\}, q = \{0, 6, 1\}, u = \{2, 0, -4\}$ **res = $\{2 + 0 + 2, 3 + 6 + 0, 0 + 1 - 4\}$** $\{4, 9, -3\}$

Norm[res]

$$\sqrt{106}$$

$$\text{matt} = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 6 & 1 \\ 2 & 0 & -4 \end{pmatrix}$$

$$\{\{2, 3, 0\}, \{0, 6, 1\}, \{2, 0, -4\}\}$$

e2 = RowReduce[matt]

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

$$23. \mathbf{u} = \{18, -1, 0\}, \mathbf{v} = \left\{\frac{1}{2}, 0, \frac{4}{3}\right\}, \mathbf{w} = \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$$

$$\text{res} = \left\{8 + \frac{1}{2} - \frac{17}{2}, -1 + 0 + 1, 0 + \frac{4}{3} + \frac{11}{3}\right\}$$

$$\{0, 0, 5\}$$

Norm[res]

$$5$$

$$\text{mat} = \begin{pmatrix} 8 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{4}{3} \\ -\frac{17}{2} & 1 & \frac{11}{3} \end{pmatrix}$$

$$\{\{8, -1, 0\}, \left\{\frac{1}{2}, 0, \frac{4}{3}\right\}, \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}\}$$

e1 = RowReduce[mat]

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

$$25. \mathbf{u} = \{3, 1, -6\}, \mathbf{v} = \{0, 2, 5\}, \mathbf{w} = \{3, -1, -13\}$$

$$\text{res} = \{3 + 0 + 3, 1 + 2 - 1, -6 + 5 - 13\}$$

$$\{6, 2, -14\}$$

Norm[res]

$$2\sqrt{59}$$

$$\text{mattt} = \begin{pmatrix} 3 & 1 & -6 \\ 0 & 2 & 5 \\ 3 & -1 & -13 \end{pmatrix}$$

$$\{\{3, 1, -6\}, \{0, 2, 5\}, \{3, -1, -13\}\}$$

RowReduce[mattt]

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

Above: for this one, the text says the result is $2\mathbf{u}$. I don't think that is a correct statement. In fact, the reduced echelon form says that the three vectors are linearly independent. Therefore is it not doubtful that I can find factors which express one in terms of the other two?

26 - 37 Forces, velocities

27. Find \mathbf{p} such that \mathbf{u} , \mathbf{v} , \mathbf{w} in problem 23 and \mathbf{p} are in equilibrium.

From problem 37, it is understood that vectors, considered as forces, are in equilibrium when they form a 'force polygon.' This polygon will have to be 4-sided. From the s.m., I find that "... "Equilibrium" means that the resultant of the given forces is the zero vector." Meaning I need to find \mathbf{p} such that $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{p} = \mathbf{0}$.

ClearAll["Global`*"]

$$\mathbf{uu} = \{8, -1, 0\}; \mathbf{vv} = \left\{\frac{1}{2}, 0, \frac{4}{3}\right\}; \mathbf{ww} = \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$$

$$\left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$$

$$\mathbf{pp} = -(\mathbf{uu} + \mathbf{vv} + \mathbf{ww})$$

$$\{0, 0, -5\}$$

29. Restricted resultant. Find all \mathbf{v} such that the resultant of \mathbf{v} , \mathbf{p} , \mathbf{q} , \mathbf{u} with \mathbf{p} , \mathbf{q} , \mathbf{u} as in problem 21 is parallel to the xy -plane.

The resultant vector in problem 21 was $\{4, 9, -3\}$. So any vector of the form $\{x, y, -3\}$ will be parallel to the xy -plane when added to the resultant of problem 21.

31. For what k is the resultant of $\{2, 0, -7\}$, $\{1, 2, -3\}$, and $\{0, 3, k\}$ parallel to the xy -plane?


```
res1 = {2 + 1, 0 + 2, -3 - 7}
{3, 2, -10}
```

```
k = 10
```

```
10
```

The green cells above match the answers in the text.

32. If $|\mathbf{p}| = 6$ and $|\mathbf{q}| = 4$, what can you say about the magnitude and direction of the resultant? Can you think of an application to robotics?

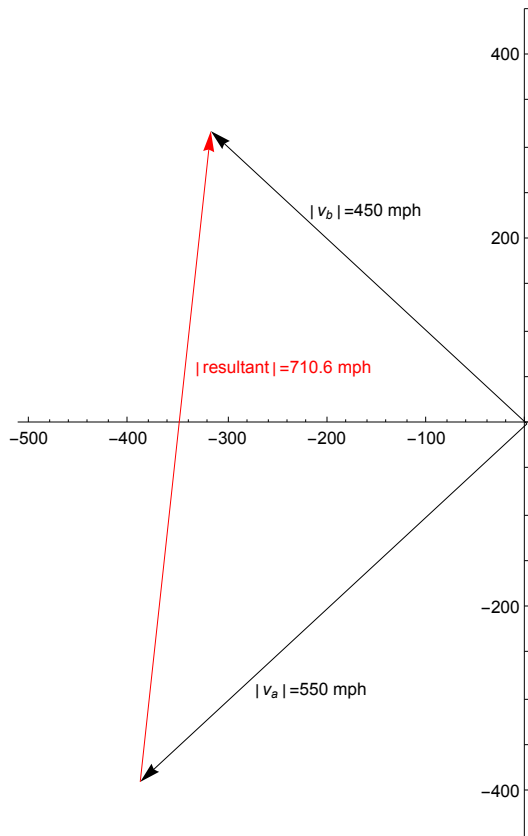
33. Same question as in problem 32 if $|\mathbf{p}| = 9$, $|\mathbf{q}| = 6$, $|\mathbf{u}| = 3$.

Comprising three cloud spheres. Any resulting magnitude greater than or equal to zero and less than or equal to 18. Any octant.

34. Relative velocity. If airplanes A and B are moving southwest with speeds $|\mathbf{v}_A| = 550$ mph, and northwest with speed $|\mathbf{v}_B| = 450$ mph, respectively, what is the relative velocity $\mathbf{v} = \mathbf{v}_B - \mathbf{v}_A$ of B with respect to A ?

```
ClearAll["Global`*"]
N[Solve[2 aa^2 == 550^2], 4]
{{aa -> -388.9}, {aa -> 388.9}}
aavec = {-388.9, -388.9}
{-388.9, -388.9}
N[Solve[2 bb^2 == 450^2], 4]
{{bb -> -318.2}, {bb -> 318.2}}
bbvec = {-318.2, 318.2}
{-318.2, 318.2}
bbvec - aavec
{70.7, 707.1}
Norm[bbvec - aavec]
710.626
```

The relative velocity of \mathbf{bbvec} with respect to \mathbf{aavec} is 70.7 mph east, 707.1 mph north.



35. Same question as in problem 34 for two ships moving northeast with speed $|v_A| = 22$ knots and west with speed $|v_B| = 19$ knots.

```
ClearAll["Global`*"]
N[Solve[2 as^2 == 22^2], 4]
{{as -> -15.56}, {as -> 15.56}}

asvec = {15.56, 15.56}
{15.56, 15.56}

N[Solve[bs^2 == 19^2], 4]
{{bs -> -19.00}, {bs -> 19.00}}

bsvec = {-19, 0}
```

```
{-19, 0}
```

The problem doesn't say what the perspective is, let's say I want velocity of ship b with respect to ship a.

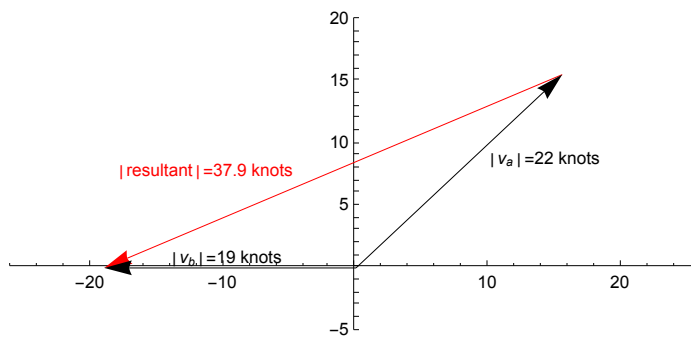
```
resulta = bsvec - asvec
```

```
{-34.56, -15.56}
```

The relative velocity of ship b with respect to ship a is 34.6 knots west, 15.6 knots south.

```
Norm[resulta]
```

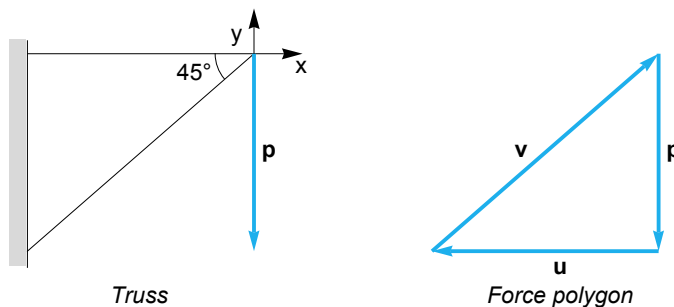
```
37.9013
```



```
N[{-19 - 22 / Sqrt[2], -22 / Sqrt[2]}]
{-34.5563, -15.5563}
```

The green cells above match the answers in the text.

37. Force polygon. Truss. Find the forces in the system of two rods (truss) in the figure, where $|\mathbf{p}| = 1000$ nt. *Hint.* Forces in equilibrium form a polygon, the *force polygon*.



```
ClearAll["Global`*"]
```