```
Clear["Global`*"]
```

1 - 6 Mixing problems.

1. Find out, without calculation, whether doubling the flow rate in example 1 has the same effect as halfing the tank sizes. (Give a reason.)

I see the answer to this problem is yes, which surprised me.

3. Derive the eigenvectors in example 1 without consulting this book.

$$A = \begin{pmatrix} -0.02 & 0.02 \\ 0.02 & -0.02 \end{pmatrix}$$

```
{{-0.02, 0.02}, {0.02, -0.02}}
```

```
Eigensystem[A]
```

```
{{-0.04, 0.}, {{0.707107, -0.707107}, {0.707107, 0.707107}}}
```

As there is no text answer to this problem, I can’t determine whether my guess is right or wrong.

5. If you extend example 1 by a tank T_3 of the same size as the others, and connected to T_2 by two tubes with flow rates as between T_1 and T_2, what system of ODEs will you get?

7 - 9 Electrical network
In example 2, find the currents:

7. If the initial currents are 0 A and -3 A (minus meaning the $I_2(0)$ flows against the direction of the arrow).

```
Clear["Global`*"]
```

In example 2 the applicable matrix is found as

$$\begin{pmatrix} -4 & 4 \\ -1.6 & 1.2 \end{pmatrix}$$

```
{{-4, 4}, {-1.6, 1.2}}
```

Mathematica , in calculating eigenvectors, always normalizes any which have any entries, in the parent matrix, which are floats. In this case I can pull the following into agreement with the text (which does not normalize the eigenvectors here) by rationalizing.

```
Rationalize[-1.6]
```

$$-\frac{8}{5}$$

```
Rationalize[1.2]
```

$\frac{6}{5}$

$A = \begin{pmatrix} -4 & 4 \\ -\frac{8}{5} & \frac{6}{5} \end{pmatrix}$

$\{\{-4, 4\}, \{-\frac{8}{5}, \frac{6}{5}\}\}$

For which the applicable eigenvalues and eigenvectors can be found as

```
{vals, vecs} = Eigensystem[A]
```

$\{\{-2, -\frac{4}{5}\}, \{\{2, 1\}, \{\frac{5}{4}, 1\}\}\}$

which I can then decimalize

```
NumberForm[N[{vals, vecs}], 3]
{{-2., -0.8}, {{2., 1.}, {1.25, 1.}}}
```

Scooping up at a later stage in the example, there will be two equations for the two circuit loops.

$I_1 = 2\, c_1\, e^{-2t} + c_2\, e^{-0.8t} + 3$ and $I_2 = c_1\, e^{-2t} + 0.8\, c_2\, e^{-0.8t}$

For the case where t=0, the example, at top of p. 134, states these as

$I_1[0] = 2\, c_1 + c_2 + 3 = 0$ and $I_2[0] = c_1 + 0.8\, c_2 = -3$

The alteration, from example 2, for this problem is that at t=0 the two current values are 0 and -3 Amp respectively, so the above equations can be solved by

$\text{Solve}[2\, c_1 + c_2 + 3 == 0\ \&\&\ c_1 + 0.8\, c_2 == -3,\ \{c_1, c_2\}]$

$\{\{c_1 \to 1.,\ c_2 \to -5.\}\}$

Then I will have

$I_1[t] = (2\, c_1\, e^{-2t} + c_2\, e^{-0.8t} + 3)\ /.$
$\{c_1 \to 0.9999999999999997`,\ c_2 \to -4.999999999999999`\}$

$3 + 2.\, e^{-2t} - 5.\, e^{-0.8t}$

and

$I_2[t] = c_1\, e^{-2t} + 0.8\, c_2\, e^{-0.8t}\ /.$
$\{c_1 \to 0.9999999999999997`,\ c_2 \to -4.999999999999999`\}$

$1.\, e^{-2t} - 4.\, e^{-0.8t}$

The text answer only encompasses the constant values in green above, not the actual resulting current equations.

9. If the initial currents in example 2 are 28 A and 14 A.

The use of example 2 on p. 132 is not finished, there is this additional problem concerning it. Using the last problem, and jumping down to the pertinent expressions

```
Solve[2 c1 + c2 + 3 == 28 && c1 + 0.8 c2 == 14, {c1, c2}]
```

```
{{c1 → 10., c2 → 5.}}
```

The above green cell matches the text answer. The text answer skips the final equations, so I will also.

10 - 13 Conversion to systems
Find a general solution of the given ODE (a) by first converting it to a system, (b), as given.

11. 4 y'' - 15 y' - 4 y = 0

(a) Convert to a system. Conversion to a system seems like it would be useful in some cases. However, as long as DSolve can get it done with such conversion, it is a little difficult to get motivated about it.

(b) As given

```
eqn = 4 y''[x] - 15 y'[x] - 4 y[x] == 0
-4 y[x] - 15 y′[x] + 4 y′′[x] == 0
```

```
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-x/4} C[1] + e^{4x} C[2]]\}\}$

```
eqn /. sol // Simplify
{True}
```

The answer in yellow above is correct, but not listed in the text answer. Instead, the test answer includes a vector of constants, which I think are ultimately absorbed by the constants shown above.

13. y'' + 2 y' - 24 y = 0

```
Clear["Global`*"]
```

(b) As given

```
eqn = y''[x] + 2 y'[x] - 24 y[x] == 0
-24 y[x] + 2 y′[x] + y′′[x] == 0
```

```
sol = DSolve[eqn, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-6x} C[1] + e^{4x} C[2]]\}\}$

```
eqn /. sol // Simplify
```

```
{True}
```

The answer in green above matches the answer in the text.

15. CAS experiment. Electrical network.
(a) In Example 2, p. 132, choose a sequence of values of *C* that increases beyond bound, and compare the corresponding sequences of eigenvalues of **A**. What limits of these sequences do your numeric values (approximately) suggest?
(b) Find these limits analytically.
(c) Explain your result physically.
(d) Below what value (approximately) must you decrease *C* to get vibrations?