```
Clear["Global`*"]
```

```
grr = {y''[x] + y'[x] - 2 y[x] == 0, y[0] == 4, y'[0] == -5}
out = DSolve[grr, y, x]
Simplify[grr /. out]
```

$\{-2\, y[x] + y'[x] + y''[x] == 0,\ y[0] == 4,\ y'[0] == -5\}$

$\{\{y \to \text{Function}[\{x\},\ e^{-2x}\,(3 + e^{3x})]\}\}$

```
{{True, True, True}}
```

The above (example 2, p. 55) demonstrates that Mathematica can solve Homogeneous Linear ODEs with Constant Coefficients without the (manual) step of performing root solving.

```
Clear["Global`*"]
```

```
DSolve[y''[x] + 6 y'[x] + 9 y[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\},\ e^{-3x}\, C[1] + e^{-3x}\, x\, C[2]]\}\}$

The above (example 3, p. 56) shows that Mathematica can find sol'n to another HLOCC without showing the characteristic equation, without constructing a basis.

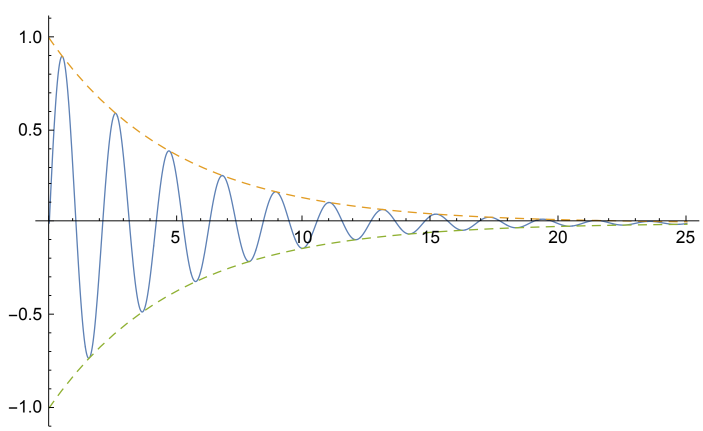
```
Clear["Global`*"]
```

```
DSolve[{y''[x] + 9.04 y[x] + 0.4 y'[x] == 0, y[0] == 0, y'[0] == 3}, y, x]
```

$\{\{y \to \text{Function}[\{x\},\ 1.\, e^{-0.2x}\, \text{Sin}[3.\, x]]\}\}$

The above (example 5, p. 57) shows sol'n to eqn with complex roots, and with no special steps.

```
Plot[{e^(-0.2 x) Sin[3. x], e^(-0.2 x), -e^(-0.2 x)}, {x, 0, 25},
 PlotRange → All, PlotStyle → {{Thickness[0.002]},
   {Dashed, Thickness[0.002]}, {Dashed, Thickness[0.002]}}]
```




```
Clear["Global`*"]
```

```
Series[e^(i t), {t, 0, 10}]
```

$$1 + i\,\text{Log}[e]\,t - \frac{1}{2}\text{Log}[e]^2 t^2 - \frac{1}{6} i\,\text{Log}[e]^3 t^3 + \frac{1}{24}\text{Log}[e]^4 t^4 + \frac{1}{120} i\,\text{Log}[e]^5 t^5 - \frac{1}{720}\text{Log}[e]^6 t^6 - \frac{i\,\text{Log}[e]^7 t^7}{5040} + \frac{\text{Log}[e]^8 t^8}{40\,320} + \frac{i\,\text{Log}[e]^9 t^9}{362\,880} - \frac{\text{Log}[e]^{10} t^{10}}{3\,628\,800} + O[t]^{11}$$

```
% /. Log[e] → 1
```

$$1 + i\,t - \frac{t^2}{2} - \frac{i\,t^3}{6} + \frac{t^4}{24} + \frac{i\,t^5}{120} - \frac{t^6}{720} - \frac{i\,t^7}{5040} + \frac{t^8}{40\,320} + \frac{i\,t^9}{362\,880} - \frac{t^{10}}{3\,628\,800} + O[t]^{11}$$

1 - 15 General solution

Find a general solution. Check your answer by substitution. ODEs of this kind have important applications to be discussed in sections 2.4, 2.7, and 2.9.

1. $4\,y'' - 25\,y = 0$

```
Clear["Global`*"]
DSolve[4 y''[x] - 25 y[x] == 0, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, e^{5x/2} C[1] + e^{-5x/2} C[2]]\}\}$$

Answer above is correct.

3. $y'' + 6\,y' + 8.96\,y = 0$

```
Clear["Global`*"]
DSolve[y''[x] + 6 y'[x] + 8.96 y[x] == 0, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, e^{-3.2x} C[1] + e^{-2.8x} C[2]]\}\}$$

Answer above is correct.

5. $y'' + 2\pi\,y' + \pi^2\,y = 0$

```
Clear["Global`*"]
DSolve[y''[x] + 2 π y'[x] + π^2 y[x] == 0, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, e^{-\pi x} C[1] + e^{-\pi x} x\, C[2]]\}\}$$

Answer above is correct.

7. $y'' + 4.5\, y' = 0$

```
Clear["Global`*"]
DSolve[y''[x] + 4.5 y'[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\}, -0.222222\, e^{-4.5\, x}\, C[1] + C[2]]\}\}$

Answer above is correct, (the constant coefficient C[1] was consolidated in the text answer, making the 2's factor disappear).

9. $y'' + 1.8\, y' - 2.08\, y = 0$

```
Clear["Global`*"]
DSolve[y''[x] + 1.8 y'[x] - 2.08 y[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-2.6\, x}\, C[1] + e^{0.8\, x}\, C[2]]\}\}$

Above answer is correct.

11. $4\, y'' - 4\, y' - 3\, y = 0$

```
Clear["Global`*"]
DSolve[4 y''[x] - 4 y'[x] - 3 y[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-x/2}\, C[1] + e^{3\, x/2}\, C[2]]\}\}$

Answer above is correct.

13. $9\, y'' - 30\, y' + 25\, y = 0$

```
Clear["Global`*"]
DSolve[9 y''[x] - 30 y'[x] + 25 y[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{5\, x/3}\, C[1] + e^{5\, x/3}\, x\, C[2]]\}\}$

Answer above is correct.

15. $y'' + 0.54\, y' + (0.0729 + \pi)\, y = 0$

```
Clear["Global`*"]
DSolve[y''[x] + 0.54 y'[x] + (0.0729 + π) y[x] == 0, y, x]
```

$\{\{y \to \text{Function}[\{x\}, e^{-0.27\, x}\, C[2]\, \text{Cos}[1.77245\, x] + e^{-0.27\, x}\, C[1]\, \text{Sin}[1.77245\, x]]\}\}$

```
(1.7724538509055159`)^2
```

```
3.14159
```

The above answer is correct.

16 - 20 Find an ODE
$y'' + a\, y' + b\, y = 0$ for the given basis.

17. $e^{-\sqrt{5}x}$, x $e^{-\sqrt{5}x}$

```
Clear["Global`*"]
```

$h[x\_] := e^{-\sqrt{5}\, x}$
h'[x]

$-\sqrt{5}\, e^{-\sqrt{5}\, x}$

h''[x]

$5\, e^{-\sqrt{5}\, x}$

$j[x\_] := x\, e^{-\sqrt{5}\, x}$

j'[x]

$e^{-\sqrt{5}\, x} - \sqrt{5}\, e^{-\sqrt{5}\, x}\, x$

j''[x]

$-2\sqrt{5}\, e^{-\sqrt{5}\, x} + 5\, e^{-\sqrt{5}\, x}\, x$

```
Solve[h''[x] + a h'[x] + b h[x] == 0 && j''[x] + a j'[x] + b j[x] == 0, {a, b}]
```

$\{\{a \to 2\sqrt{5},\ b \to 5\}\}$

The above answer is correct.

19. $e^{(-2+i)x}$, $e^{(-2-i)x}$

```
Clear["Global`*"]
```

$k[x\_] := e^{(-2+i)\, x}$

$m[x\_] := e^{(-2-i)\, x}$

```
Solve[k''[x] + a k'[x] + b k[x] == 0 && m''[x] + a m'[x] + b m[x] == 0, {a, b}]
```

```
{{a → 4, b → 5}}
```

The above answer is correct.

21 - 30 Initial value problems
Solve the IVP. Check that your answer satisfies the ODE as well as the initial conditions.

21. $y'' + 25\, y = 0$, $y(0) = 4.6$, $y'(0) = -1.2$

```
Clear["Global`*"]

arkosa = {y''[x] + 25 y[x] == 0, y[0] == 4.6, y'[0] == -1.2}
hap = DSolve[arkosa, y, x]
{25 y[x] + y''[x] == 0, y[0] == 4.6, y'[0] == -1.2}

{{y → Function[{x}, 4.6 Cos[5 x] - 0.24 Sin[5 x]]}}

Simplify[arkosa /. hap]

{{Cos[5 x] == 0, True, True}}
```

The above line indicates that Mathematica tested the initial value points and found that the sol’n worked correctly at those points.

```
r[x_] := 4.6 Cos[5 x] - 0.24 Sin[5 x]

Simplify[r''[x] + 25 r[x]]
-1.42109 × 10^-14 Cos[5 x]
```

What the above line seems to indicate is that, within default working precision, the answer is correct.

23. $y'' + y' - 6\, y = 0$, $y(0) = 10$, $y'(0) = 0$

```
Clear["Global`*"]

redondo = {y''[x] + y'[x] - 6 y[x] == 0, y[0] == 10, y'[0] == 0}
ghent = DSolve[redondo, y, x]
{-6 y[x] + y'[x] + y''[x] == 0, y[0] == 10, y'[0] == 0}

{{y → Function[{x}, 2 e^(-3 x) (2 + 3 e^(5 x))]}}

Simplify[redondo /. ghent]
{{True, True, True}}

Simplify[2 e^(-3 x) (2 + 3 e^(5 x))]

e^(-3 x) (4 + 6 e^(5 x))
```

The above answer is correct; Mathematica tested both the initial value points as well as the general sol’n and found all sat.

25. $y'' - y = 0$, $y(0) = 2$, $y'(0) = -2$

```
Clear["Global`*"]

treacle = {y''[x] - y[x] == 0, y[0] == 2, y'[0] == -2}
rhet = DSolve[treacle, y, x]
{-y[x] + y''[x] == 0, y[0] == 2, y'[0] == -2}
```

$\{\{y \to \text{Function}[\{x\}, 2\, e^{-x}]\}\}$

```
Simplify[treacle /. rhet]
{{True, True, True}}
```

The above answer is correct; Mathematica tested both the initial value points as well as the general sol’n and found all sat.

27. The ODE in problem 5, $y(0) = 4.5$, $y'(0) = -4.5\,\pi - 1 = 13.137$

```
Clear["Global`*"]

jam = {y''[x] + 2 π y'[x] + π^2 y[x] == 0, y[0] == 4.5, y'[0] == -4.5 π - 1}
blank = DSolve[jam, y, x]
{π^2 y[x] + 2 π y'[x] + y''[x] == 0, y[0] == 4.5, y'[0] == -15.1372}
```

$\{\{y \to \text{Function}[\{x\}, -1.\, e^{-\pi x} (-4.5 + 1.\, x)]\}\}$

```
rodz = blank[[1, 1, 2, 2]]
```

$-1.\, e^{-\pi x} (-4.5 + 1.\, x)$

```
Expand[rodz]
```

$4.5\, e^{-\pi x} - 1.\, e^{-\pi x} x$

```
Simplify[jam /. blank]
```

$\{\{e^{-\pi x} == 0, \text{True}, \text{True}\}\}$

```
TableForm[Table[Evaluate[rodz[x], {x, 8}]]]
0.151249[1]
0.00466861[2]
0.000121049[3]
```

$1.74367 \times 10^{-6}[4]$

$(-7.53509 \times 10^{-8})[5]$

$(-9.76862 \times 10^{-9})[6]$

$(-7.03567 \times 10^{-10})[7]$

$(-4.25654 \times 10^{-11})[8]$

```
TableForm[Table[{x, rodz}, {x, 8}],
 TableHeadings -> {{}, {"x ", "rodz value"}}]
```

x	rodz value
1	0.151249
2	0.00466861
3	0.000121049
4	1.74367×10^{-6}
5	-7.53509×10^{-8}
6	-9.76862×10^{-9}
7	-7.03567×10^{-10}
8	-4.25654×10^{-11}

The above expression I interpret to mean that if $x > 5$, then $e^{-\pi x}$ is effectively zero, and thereafter the checking statement $e^{-\pi x} == 0$ will be true. But this doesn't seem very satisfactory.

```
mn[x_] := 4.5` ⅇ^(-π x) - 1.` ⅇ^(-π x) x
mn''[x] + 2 π mn'[x] + π^2 mn[x]
```

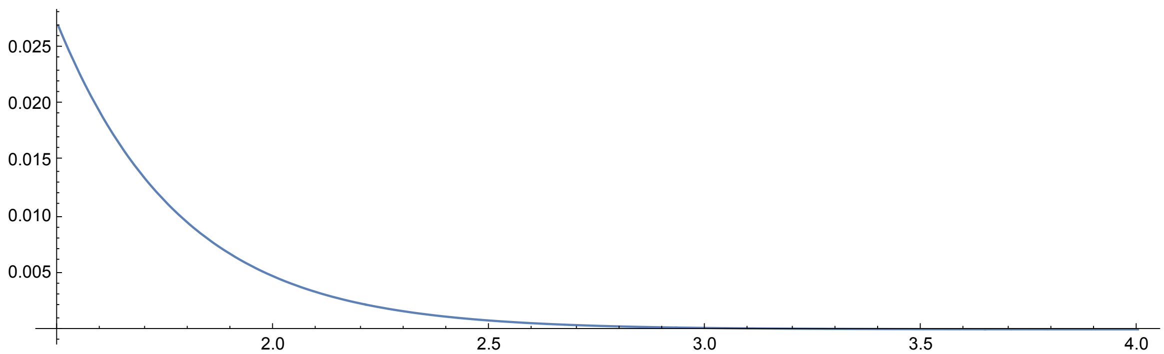
$$50.6964\, e^{-\pi x} - 9.8696\, e^{-\pi x}\, x + \pi^2 \left(4.5\, e^{-\pi x} - 1.\, e^{-\pi x}\, x\right) + 2\pi \left(-15.1372\, e^{-\pi x} + 3.14159\, e^{-\pi x}\, x\right)$$

```
Simplify[%]
```

0.

Okay, that's more like it. The text answer agrees. And incidentally, there is a slight mistake in the problem, because the 13.137 should have a minus sign.

```
Plot[4.5` ⅇ^(-π x) - 1.` ⅇ^(-π x) x, {x, 1.5, 4}, PlotRange → All,
 ImageSize → 600, AspectRatio → 0.3, PlotStyle → Thickness[0.002]]
```



29. The ODE in problem 15, $y(0) = 0$, $y'(0) = 1$

```
Clear["Global`*"]
```

```
quil = {y''[x] + 0.54 y'[x] + (0.0729 + π) y[x] == 0, y[0] == 0, y'[0] == 1}
tig = DSolve[quil, y, x]
{3.21449 y[x] + 0.54 y′[x] + y″[x] == 0, y[0] == 0, y′[0] == 1}
```

$\{\{y \to \text{Function}[\{x\}, 0.56419\, e^{-0.27 x} \text{Sin}[1.77245\, x]]\}\}$

```
Simplify[quil /. tig]
```

$\{\{e^{-0.27 x} (-1.11022 \times 10^{-16} \text{Cos}[1.77245 x] + 3.88578 \times 10^{-16} \text{Sin}[1.77245 x]) == 0,$ True, True$\}\}$

$N[\sqrt{\pi}]$

1.77245

```
1 / %
```

0.56419

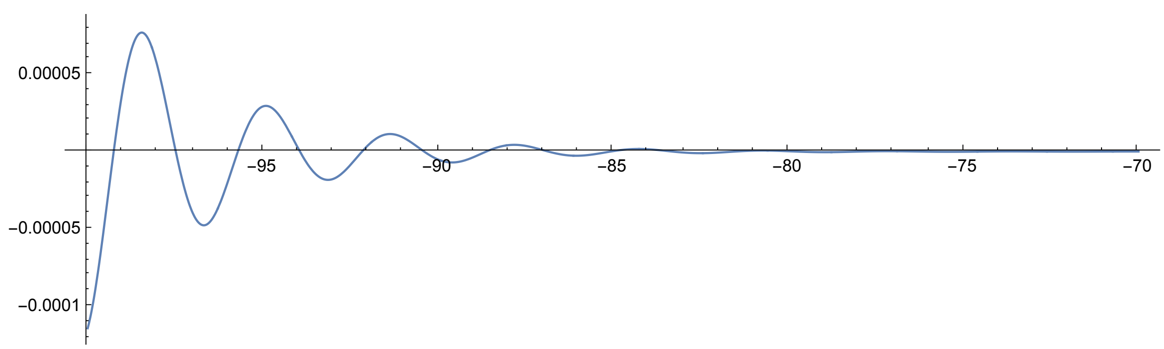
Checking these two quantities establishes that the answer agrees with the text's. As for actually checking the answer, Mathematica has declined to do so.

$\text{fes}[x\_] := \frac{1}{\sqrt{\pi}} e^{-0.27 x} \text{Sin}[\sqrt{\pi}\, x]$

```
kres = Simplify[fes''[x] + 0.54 fes'[x] + (0.0729 + π) fes[x]]
```

$2.22045 \times 10^{-16} e^{-0.27 x} \text{Sin}[\sqrt{\pi}\, x]$

```
Plot[2.220446049250313`*^-16 ⅇ^(-0.27` x) Sin[Sqrt[π] x],
 {x, -100, -70}, PlotRange → All, ImageSize → 600,
 AspectRatio → 0.3, PlotStyle → Thickness[0.002]]
```



```
TableForm[Table[{x, fes[x]}, {x, -100, 100, 20}],
 TableHeadings → {{}, {"x  ", "func value"}}]
```

x	func value
-100	-2.905×10^{11}
-80	5.58566×10^{8}
-60	2.75639×10^{6}
-40	-27036.
-20	97.1904
0	0.
20	-0.00198264
40	0.0000112507
60	-2.33991×10^{-8}
80	-9.67281×10^{-11}
100	1.02623×10^{-12}

31 - 36 Linear independence is of basic importance, in this chapter, in connection with general solutions, as explained in the text. Are the following functions linearly independent on the given interval?

31. e^{kx}, x e^{kx}, any interval

```
Clear["Global`*"]
r = FullSimplify[Solve[a ⅇ^(k x) == x ⅇ^(k x), a]]
{{a → x}}
```

A sol'n for a has been found, but the sol'n says a is not a constant. Therefore the functions must be linearly independent. Answer agrees with text's.

33. x^2, x^2 Log[x], x > 1

```
Clear["Global`*"]
r = FullSimplify[Solve[a x^2 == x^2 Log[x], a]]
{{a → Log[x]}}
```

Again, the functions must be independent, because the connecting factor is not constant. (Text agrees.)

35. Sin[2 x], Cos[x] Sin[x], x < 0

```
Clear["Global`*"]
r = FullSimplify[Solve[a Sin[2 x] == Cos[x] Sin[x] && x < 0, a]]
{{a → ConditionalExpression[1/2, x < 0]}}
```

Here Mathematica comes through by sniffing out a constant sol'n. This means that the two functions are linearly dependent. (And the text agrees.)

37. Instability. Solve $y'' - y = 0$ for the initial conditions $y(0) = 1$, $y'(0) = -1$. Then change the initial conditions to $y(0) = 1.001$, $y'(0) = -0.999$ and explain why this small change of 0.001 at t = 0 causes a large change later, e.g. 22 at t = 10. This is instability: a small initial difference in setting a quantity (a current, for instance) becomes larger and larger with time t. This is undesirable.

```
Clear["Global`*"]
```

```
hak = {y''[t] - y[t] == 0, y[0] == 1, y'[0] == -1}
drak = DSolve[hak, y, t]
```

```
{-y[t] + y''[t] == 0, y[0] == 1, y'[0] == -1}
```

$\{\{y \to \text{Function}[\{t\}, e^{-t}]\}\}$

```
hur = drak[[1, 1, 2, 2]]
```

e^{-t}

```
hak1 = {y''[t] - y[t] == 0, y[0] == 1.001, y'[0] == -0.999}
drak1 = DSolve[hak1, y, t]
```

```
{-y[t] + y''[t] == 0, y[0] == 1.001, y'[0] == -0.999}
```

$\{\{y \to \text{Function}[\{t\}, 0.001\, e^{-t}\, (1000. + 1.\, e^{2t})]\}\}$

```
hur1 = drak1[[1, 1, 2, 2]]
```

$0.001\, e^{-t}\, (1000. + 1.\, e^{2t})$

```
TableForm[Table[{t, N[hur, 4], N[hur1, 4]}, {t, 0, 30, 2}],
 TableHeadings → {{}, {"t ", "drak value ", "drak1 value"}}]
```

t	drak value	drak1 value
0	1.000	1.001
2	0.1353	0.142724
4	0.01832	0.0729138
6	0.002479	0.405908
8	0.0003355	2.98129
10	0.00004540	22.0265
12	6.144×10^{-6}	162.755
14	8.315×10^{-7}	1202.6
16	1.125×10^{-7}	8886.11
18	1.523×10^{-8}	65 660.
20	2.061×10^{-9}	485 165.
22	2.789×10^{-10}	3.58491×10^{6}
24	3.775×10^{-11}	2.64891×10^{7}
26	5.109×10^{-12}	1.9573×10^{8}
28	6.914×10^{-13}	1.44626×10^{9}
30	9.358×10^{-14}	1.06865×10^{10}

The table verifies what the text says, i.e. that there is a difference of 22 at $t = 10$. And it gets much larger later.