Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

3 - 10 Maclaurin Series

Find the Maclaurin series and its radius of convergence.

3. $\sin[2 z^2]$

```
Clear["Global`*"]
```

By asking for the series to be generated around the origin, I ask for the Maclaurin series,

```
ser = Series[Sin[2 z^2], {z, 0, 16}]
```

$$2 z^2 - \frac{4 z^6}{3} + \frac{4 z^{10}}{15} - \frac{8 z^{14}}{315} + O[z]^{17}$$

It seems that Mathematica cannot provide useful cooperation in the case of the problem function. **SumConvergence**, with or without **TrigReduce**, does not seem to work in establishing the radius of convergence. In fact, observe the following,

```
SumConvergence[Sin[2 z^2], z]
```

False

The false indication makes me wary altogether. So this alternative method. First I will copy from numbered line (14) on p. 695, the series expression of the sine function,

```
thi = HoldForm[Sum[(-1)^n z^(2 n+1)/(2 n + 1)!, {n, 0, ∞}]]
```

$$\sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$$

Mathematica identifies this with sine function immediately, thus the need for **HoldForm** to prevent undesired simplification. Then to modify the sine series to match the problem expression,

```
thi2 = thi /. z → 2 z^2
```

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(2 z^2\right)^{2n+1}}{(2n+1)!}$$

Now I will hand this term on to the **SumConvergence** function,

$$\texttt{SumConvergence}\left[\frac{(-1)^n\left(2\,z^2\right)^{2\,n+1}}{(2\,n+1)\,!},\ n\right]$$

True

The previous cell confirms convergence, but I want the radius of convergence. To use the Cauchy-Hadamard formula, I need to separate the *z* and *n* parts of the expression,

$$\texttt{Expand}\left[\left(2\,z^2\right)^{2\,n+1}\right]$$

$$2^{1+2\,n}\left(z^2\right)^{1+2\,n}$$

The above form reveals the center of the series, which is 0, no great surprise for a Maclaurin series. Note that z, when called on, will be raised to the 4th power of n. Okay, at this point I can use the Cauchy-Hadamard formula,

$$\texttt{Limit}\left[\texttt{Abs}\left[\frac{(-1)^n\,2^{1+2\,n}}{(2\,n+1)\,!}\left(\frac{(2\,n+2)\,!}{(-1)^{n+1}\,2^{2+2\,n}}\right)\right],\ n\to\infty\right]$$

∞

Applying the power of the power term,

$\infty^{1/4}$

∞

The above answer matches the radius of convergence shown in the text's answer.

5. $\dfrac{1}{2+z^4}$

```
Clear["Global`*"]
```

By asking for the series to be generated around the origin, I ask for the Maclaurin series,

$$\texttt{Series}\left[\frac{1}{2+z^4},\ \{z,\ 0,\ 16\}\right]$$

$$\frac{1}{2}-\frac{z^4}{4}+\frac{z^8}{8}-\frac{z^{12}}{16}+\frac{z^{16}}{32}+O[z]^{17}$$

Testing for convergence,

$$\texttt{SumConvergence}\left[\frac{1}{2+z^4},\ z\right]$$

True

in a verification of some radius of convergence. Then try to find the radius of convergence the easy way,

```
Sum[1/(2 + z^4), {z, 0, ∞}, GenerateConditions → True]
```

$$\frac{1}{8}\left(2+(-2)^{1/4}\pi\,\text{Cot}\left[(-2)^{1/4}\pi\right]+(-2)^{1/4}\pi\,\text{Coth}\left[(-2)^{1/4}\pi\right]\right)$$

The above cell is beyond my capability to simplify, and Mathematica won't alter it either. I can kludge the general term,

```
tri[n_] = (-1)^n z^(4 n)/2^(n+1)
```

$$(-1)^n\, 2^{-1-n}\, z^{4n}$$

and check it,

```
TableForm[Table[{n, tri[n]}, {n, 0, 4}]]
```

0	$\frac{1}{2}$
1	$-\frac{z^4}{4}$
2	$\frac{z^8}{8}$
3	$-\frac{z^{12}}{16}$
4	$\frac{z^{16}}{32}$

At this point it looks like I'm ready to try Cauchy-Hadamard,

```
Limit[Abs[(-1)^n/2^(n+1) (2^(n+2)/(-1)^(n+1))], n → ∞]
```

$$2$$

And the power of the power term is 4, thus

```
(2)^(1/4)
```

$$2^{1/4}$$

The above cell matches the answer in the text.

7. $\text{Cos}\left[\frac{z}{2}\right]^2$

```
Clear["Global`*"]
```

Calling for my Maclaurin,

```
Series[Cos[z/2]^2, {z, 0, 12}]
```

$$1-\frac{z^2}{4}+\frac{z^4}{48}-\frac{z^6}{1440}+\frac{z^8}{80\,640}-\frac{z^{10}}{7\,257\,600}+\frac{z^{12}}{958\,003\,200}+O[z]^{13}$$

Somehow I need to put together the general term,

```
FindSequenceFunction[{-4, 48, -1440, 80640, -7257600, 958003200}, n]
```

$(-1)^n\, 2^{1+2n}\, \text{Pochhammer}\left[\frac{1}{2}, n\right] \text{Pochhammer}[1, n]$

I couldn't get an answer out of Mathematica using the above function unless I dropped the first term of the series. Trying to simplify a bit,

```
FullSimplify[%]
```

$2\,(-1)^n\, \text{Gamma}[1 + 2\,n]$

The Gamma function is not difficult, although some of the others *Mathematica* throws out are daunting. In general in this situation though, the amount of knowledge necessary consists solely of knowing how to form the a_{n+1} term from the a_n term.

```
testg[n_] = (-1)^n 2 Gamma[1 + 2 n]
```

$2\,(-1)^n\, \text{Gamma}[1 + 2\,n]$

```
testh[n_] = (-1)^n 2 (2 n) !
```

$2\,(-1)^n\,(2\,n)\,!$

Checking, starting with n=1,

```
TableForm[Table[{n, testg[n], testh[n]}, {n, 1, 6}]]
```

1	-4	-4
2	48	48
3	-1440	-1440
4	80 640	80 640
5	-7 257 600	-7 257 600
6	958 003 200	958 003 200

testh[n] was lifted from the text answer. If I had had a brainwave and had seen the pattern on my own, the process would be more direct than using **FindSequenceFunction**.

Setting up the n part of the expression, not worrying about the z part at the moment,

$$\text{coef} = \frac{(-1)^n}{2\,\text{Gamma}[1 + 2\,n]}$$

$$\frac{(-1)^n}{2\,\text{Gamma}[1 + 2\,n]}$$

Ready to examine the limit governing the Cauchy-Hadamard. Not having the initial term should not be a handicap, as I have my sights set on much bigger terms now,

$$\text{Limit}\left[\text{Abs}\left[\frac{(-1)^n}{2\,\text{Gamma}[1 + 2\,n]}\left(\frac{2\,\text{Gamma}[1 + 2\,(n + 1)]}{(-1)^{n+1}}\right)\right], n \to \infty\right]$$

∞

Up until now I have ignored the power term. But it is of the form z^{2n}. So applying its effect amounts to

$\infty^{1/2}$

∞

The green cell above matches the answer in the text. (That a_0 term that I ignored, I continue to treat as inconsequential.)

9. Integrate$\left[\text{Exp}\left[\frac{-t^2}{2}\right], \{t, 0, z\}\right]$

```
Clear["Global`*"]
```

Calling for my Maclaurin,

Series$\left[\text{Exp}\left[\frac{-t^2}{2}\right], \{t, 0, 12\}\right]$

$$1-\frac{t^2}{2}+\frac{t^4}{8}-\frac{t^6}{48}+\frac{t^8}{384}-\frac{t^{10}}{3840}+\frac{t^{12}}{46\,080}+O[t]^{13}$$

At *https : // www.dummies.com/education/math/calculus/how - to - integrate - a - power - series/* I learned how to integrate a power series term by term. A key concept was that after the term by term integration you can turn the result back into another power series. So how about a short cut, would that work?

Integrate$\left[\text{Exp}\left[\frac{-t^2}{2}\right], \{t, 0, z\}\right]$

$$\sqrt{\frac{\pi}{2}}\ \text{Erf}\left[\frac{z}{\sqrt{2}}\right]$$

Series$\left[\sqrt{\frac{\pi}{2}}\ \text{Erf}\left[\frac{z}{\sqrt{2}}\right], \{z, 0, 15\}\right]$

$$z-\frac{z^3}{6}+\frac{z^5}{40}-\frac{z^7}{336}+\frac{z^9}{3456}-\frac{z^{11}}{42\,240}+\frac{z^{13}}{599\,040}-\frac{z^{15}}{9\,676\,800}+O[z]^{16}$$

The above green cell matches the text answer. The thing I haven't got yet is the radius of convergence.

Aiming to build the general term,

```
FindSequenceFunction[
 {1, -6, 40, -336, 3456, -42 240, 599 040, -9 676 800}, n]
```

$(-1)^{1+n}\ 2^{-1+n}\ (-1+2\,n)$ Pochhammer$[1, -1+n]$

```
FullSimplify[%]
```

$(-1)^{1+n}\, 2^{-1+n}\, (-1+2\, n)\, \text{Gamma}[n]$

```
testg[n_] = (-1)^(1+n) 2^(-1+n) (-1 + 2 n) Gamma[n]
```

$(-1)^{1+n}\, 2^{-1+n}\, (-1+2\, n)\, \text{Gamma}[n]$

```
testh[n_] = (-1)^(1+n) (2 n - 1) 2^(n-1) (n - 1) !
```

$(-1)^{1+n}\, 2^{-1+n}\, (-1+2\, n)\, (-1+n)\, !$

```
TableForm[Table[{n, testg[n], testh[n]}, {n, 1, 7}]]
```

1	1	1
2	-6	-6
3	40	40
4	-336	-336
5	3456	3456
6	-42 240	-42 240
7	599 040	599 040

testh[n], inspired from the text answer, and realized through the **FindSeqenceFunction**, begs the search, because I could not have done it entirely on my own.

Setting up the n part of the expression, not worrying about the z part at the moment,

$$\text{coef} = \frac{(-1)^{n+1}}{2^{-1+n}\, (-1+2\, n)\, \text{Gamma}[n]}$$

$$\frac{(-1)^{1+n}\, 2^{1-n}}{(-1+2\, n)\, \text{Gamma}[n]}$$

Ready to examine the limit governing the Cauchy-Hadamard.

$$\text{Limit}\left[\text{Abs}\left[\frac{(-1)^{n+1}}{2^{-1+n}\, (-1+2\, n)\, \text{Gamma}[n]} \Big/ \left((2^n\, (-1+2\, (n+1)))\, \text{Gamma}[n+1]\right) \Big/ (-1)^{n+2}\right)\right],\ n \to \infty\right]$$

∞

Up until now I have ignored the power term. But it is of the form z^{2n-1}. So applying its effect amounts to going with my old standard policy of regarding only the part of the term involving n, which makes it, in this case, 2 n. So,

$\infty^{1/2}$

∞

The green cell above matches the answer in the text.

11 - 14 Higher transcendental functions

Find the Maclaurin series by termwise integrating the integrand. (The integrals cannot be evaluated by the usual methods of calculus. They define the error function erf z, sine integral Si(z) and Fresnel integrals S(z) and C(z), which occur in statistics, heat conduction, optics, and other applications. These are special so-called higher transcendental functions.)

11. $S[z\_] = \text{Integrate}\left[\text{Sin}\left[t^2\right], \{t, 0, z\}\right]$

```
Clear["Global`*"]
```

Getting ready to work on a Fresnel integral. Working this experimentally. First, I use plain old Integrate applied to the problem function,

$$\text{Integrate}\left[\text{Sin}\left[t^2\right], \{t, 0, z\}\right]$$

$$\sqrt{\frac{\pi}{2}}\ \text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ z\right]$$

As the above cell shows, the answer to the integration is expressed in FresnelS, a function which is built in to Mathematica. So I can express this answer as a series, saving aside the leading coefficient for now,

$$\text{Series}\left[\text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ z\right], \{z, 0, 36\}\right]$$

$$\frac{1}{3}\sqrt{\frac{2}{\pi}}\ z^3 - \frac{z^7}{21\left(\sqrt{2}\ \sqrt{\pi}\right)} + \frac{z^{11}}{660\sqrt{2}\ \sqrt{\pi}} - \frac{z^{15}}{37\,800\left(\sqrt{2}\ \sqrt{\pi}\right)} + \frac{z^{19}}{3\,447\,360\sqrt{2}\ \sqrt{\pi}} - \frac{z^{23}}{459\,043\,200\left(\sqrt{2}\ \sqrt{\pi}\right)} + \frac{z^{27}}{84\,064\,780\,800\sqrt{2}\ \sqrt{\pi}} - \frac{z^{31}}{20\,268\,952\,704\,000\left(\sqrt{2}\ \sqrt{\pi}\right)} + \frac{z^{35}}{6\,224\,529\,991\,680\,000\sqrt{2}\ \sqrt{\pi}} + O[z]^{37}$$

Then, inserting the coefficient and simplifying, I get

$$\text{Simplify}\Big[\sqrt{\frac{\pi}{2}}\left(\frac{1}{3}\sqrt{\frac{2}{\pi}}\,z^3-\frac{z^7}{21\left(\sqrt{2}\,\sqrt{\pi}\right)}+\frac{z^{11}}{660\,\sqrt{2}\,\sqrt{\pi}}-\frac{z^{15}}{37\,800\left(\sqrt{2}\,\sqrt{\pi}\right)}+\frac{z^{19}}{3\,447\,360\,\sqrt{2}\,\sqrt{\pi}}-\frac{z^{23}}{459\,043\,200\left(\sqrt{2}\,\sqrt{\pi}\right)}+\frac{z^{27}}{84\,064\,780\,800\,\sqrt{2}\,\sqrt{\pi}}-\frac{z^{31}}{20\,268\,952\,704\,000\left(\sqrt{2}\,\sqrt{\pi}\right)}+\frac{z^{35}}{6\,224\,529\,991\,680\,000\,\sqrt{2}\,\sqrt{\pi}}+O[z]^{37}\right)\Big]$$

$$\frac{z^3}{3}-\frac{z^7}{42}+\frac{z^{11}}{1320}-\frac{z^{15}}{75\,600}+\frac{z^{19}}{6\,894\,720}-\frac{z^{23}}{918\,086\,400}+\frac{z^{27}}{168\,129\,561\,600}-\frac{z^{31}}{40\,537\,905\,408\,000}+\frac{z^{35}}{12\,449\,059\,983\,360\,000}+O[z]^{37}$$

And the above green cell happens to be the answer. There is still the radius of convergence to discover. It takes a little work to get the **FindSequenceFunction** enough numbers to consider, here, 8, in order to yield an answer. It wouldn't do it with 7. When it did return an answer, it was nearly instantaneous, in spite of the rather large numbers involved.

```
FindSequenceFunction[{3, -42, 1320, -75 600,
  6 894 720, -918 086 400, 168 129 561 600, -40 537 905 408 000}, n]
```

$$(-1)^{1+n}\,4^{-1+n}\,(-1+4\,n)\ \text{Pochhammer}[1,\,-1+n]\ \text{Pochhammer}\left[\frac{3}{2},\,-1+n\right]$$

```
FullSimplify[%]
```

$$(-1)^{1+n}\,(-1+4\,n)\ \text{Gamma}[2\,n]$$

Now, like before, I'll test to make sure the denominator coefficients spawn true.

```
testg[n_] = (-1)^(1+n) (-1 + 4 n) Gamma[2 n]
```

$$(-1)^{1+n}\,(-1+4\,n)\ \text{Gamma}[2\,n]$$

```
testh[n_] = (-1)^(1+n) (2 n - 1) ! (4 n - 1)
```

$$(-1)^{1+n}\,(-1+4\,n)\,(-1+2\,n)\,!$$

```
TableForm[Table[{n, testg[n], testh[n]}, {n, 1, 7}]]
```

1	3	3
2	-42	-42
3	1320	1320
4	-75 600	-75 600
5	6 894 720	6 894 720
6	-918 086 400	-918 086 400
7	168 129 561 600	168 129 561 600

testh[n] was lifted from the text answer. If I had had magic synapses and had seen the pattern on my own, the process might have been slightly simpler than using

FindSequenceFunction.

Setting up the n part of the Cauchy-Hadamard expression, not worrying about the z part at the moment,

```
coef = (-1)^(1+n) (-1 + 4 n) Gamma[2 n]
```

$(-1)^{1+n}(-1+4n)\,\text{Gamma}[2n]$

$$\text{Limit}\left[\text{Abs}\left[\frac{1}{(-1)^{1+n}(-1+4n)\,\text{Gamma}[2n]}\left(\frac{1}{1}(-1)^{2+n}(-1+4(n+1))\,\text{Gamma}[2(n+1)]\right)\right],\ n\to\infty\right]$$

∞

How about the power term? The power of the power term is 2n+1. Which from my method in previous problems, will have the effect of

$\infty^{1/2}$

∞

$$13.\ \text{erf}[z] = \frac{2}{\sqrt{\pi}}\,\text{Integrate}\left[e^{-t^2},\ \{t,\ 0,\ z\}\right]$$

```
Clear["Global`*"]
```

To match the text answer, it will help to set aside the leading coefficient for a moment,

$$\text{Integrate}\left[e^{-t^2},\ \{t,\ 0,\ z\}\right]$$

$$\frac{1}{2}\sqrt{\pi}\,\text{Erf}[z]$$

Mathematica immediately rolls it up into the eponymus function.

$$\text{Series}\left[\frac{1}{2}\sqrt{\pi}\,\text{Erf}[z],\ \{z,\ 0,\ 12\}\right]$$

$$z-\frac{z^3}{3}+\frac{z^5}{10}-\frac{z^7}{42}+\frac{z^9}{216}-\frac{z^{11}}{1320}+O[z]^{13}$$

Then restoring the coefficient,

$$\frac{2}{\sqrt{\pi}}\left(z-\frac{z^3}{3}+\frac{z^5}{10}-\frac{z^7}{42}+\frac{z^9}{216}-\frac{z^{11}}{1320}+O[z]^{13}\right)$$

gives me the text answer.

18 - 25 Taylor series

Find the Taylor series with center z_0 and its radius of convergence.

19. $\frac{1}{1-z}$, $z_0 = 1$

```
Clear["Global`*"]
```

First, a naïve try,

```
ard = Series[1/(1 - z), {z, 1, 8}]
```

$$-\frac{1}{z-1} + O[z-1]^9$$

is not successful. The s.m. advises to work this according to example 7, p. 696, using c=1 and $z_0 = i$, developing in powers of $z - z_0$, where $c - z_0 \neq 0$. It would look like

$$\frac{1}{c-z} == \frac{1}{1-z} == \frac{1}{1-z_0-(z-z_0)} ==$$
$$\frac{1}{1-i-(z-i)} == \frac{1}{(1-i)\left(1-\frac{z-i}{1-i}\right)} == \frac{1}{1-i}\sum_{n=0}^{\infty}\left(\frac{z-i}{1-i}\right)^n;$$

working from the analogous progression of terms in the example. To get across the last double-equals sign involves numbered lines (7), (7*), (8*), and (8) on p. 691.

Taking the expression to the right of the last double-equals sign, and multiplying by $\frac{1+i}{1+i}$ both inside and outside the sigma, gives

$$\frac{1+i}{2}\sum_{n=0}^{\infty}\left(\frac{1+i}{2}\right)^n (z-i)^n$$

as the general term. Putting it in a form that can be easily compared with the text answer, and asking for 7 terms instead of an infinite number,

```
Sum[(1 + i)/2 Sequence[((1 + i)/2)^n (z - i)^n], {n, 0, 6}]
```

$$\left(\frac{1}{2}+\frac{i}{2}\right) + \frac{1}{2} i (-i+z) - \left(\frac{1}{4}-\frac{i}{4}\right)(-i+z)^2 - \frac{1}{4}(-i+z)^3 - \left(\frac{1}{8}+\frac{i}{8}\right)(-i+z)^4 - \frac{1}{8} i (-i+z)^5 + \left(\frac{1}{16}-\frac{i}{16}\right)(-i+z)^6$$

The green cell above matches the answer in the text, for series form. As for the radius of convergence, also using example 7,

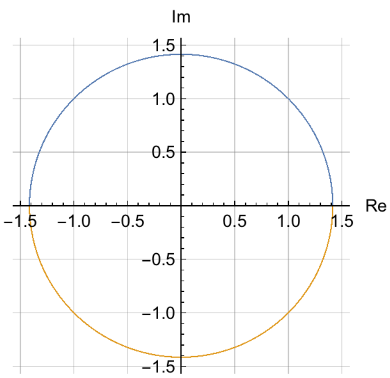
```
Reduce[Abs[(z - i)/(1 - i)] < 1, z]
```

$$-\sqrt{2} < \text{Re}[z] < \sqrt{2} \ \&\& \ 1 - \frac{\sqrt{4 - 2\,\text{Re}[z]^2}}{\sqrt{2}} < \text{Im}[z] < 1 + \frac{\sqrt{4 - 2\,\text{Re}[z]^2}}{\sqrt{2}}$$

The real part of the yellow cell matches the text answer, but the text answer is silent as to imaginary parts.

If the yellow cell is correct, a plot of the radius of curvature would resemble the one below,

```
ParametricPlot[{{z, Sqrt[4 - 2 z^2]/Sqrt[2]}, {z, -Sqrt[4 - 2 z^2]/Sqrt[2]}},
 {z, -Sqrt[2], Sqrt[2]}, ImageSize → 200, AxesLabel → {"Re", "Im"},
 PlotRange → All, AspectRatio → Automatic,
 GridLines -> Automatic, PlotStyle → {Thickness[0.004]}]
```



21. Sin[z], $z_0 = \frac{\pi}{2}$

```
Clear["Global`*"]
```

First, a naïve try,

```
Series[Sin[z], {z, π/2, 12}]
```

$$1 - \frac{1}{2}\left(z - \frac{\pi}{2}\right)^2 + \frac{1}{24}\left(z - \frac{\pi}{2}\right)^4 - \frac{1}{720}\left(z - \frac{\pi}{2}\right)^6 + \frac{\left(z - \frac{\pi}{2}\right)^8}{40\,320} - \frac{\left(z - \frac{\pi}{2}\right)^{10}}{3\,628\,800} + \frac{\left(z - \frac{\pi}{2}\right)^{12}}{479\,001\,600} + O\left[z - \frac{\pi}{2}\right]^{13}$$

Surprisingly, this one pops right out. Using the powers of the power terms, I think I can actually deduce the sequence of this one. On second thought, maybe I will use **FindSequenceFunction**, since it is so much easier,

```
FindSequenceFunction[{1, -2, 24, -720, 40 320, -3 628 800, 479 001 600}, n]
```

$$(-1)^{1+n}\, 4^{-1+n}\, \text{Pochhammer}\left[\frac{1}{2}, -1+n\right]\, \text{Pochhammer}[1, -1+n]$$

```
FullSimplify[%]
```

$(-1)^{1+n}\, \text{Gamma}[-1+2\,n]$

```
testg[n_] = (-1)^(1+n) Gamma[-1 + 2 n]
```

$(-1)^{1+n}\, \text{Gamma}[-1+2\,n]$

```
TableForm[Table[{n, testg[n]}, {n, 1, 7}]]
```

1	1
2	-2
3	24
4	-720
5	40 320
6	-3 628 800
7	479 001 600

Setting up the n part of the Cauchy-Hadamard expression, not worrying about the z part at the moment,

$$\text{Limit}\left[\text{Abs}\left[\frac{1}{(-1)^{1+n}\,\text{Gamma}[-1+2\,n]}\left(\frac{(-1)^{2+n}\,\text{Gamma}[-1+2\,(n+1)]}{1}\right)\right], n\to\infty\right]$$

∞

How about the power term? In this problem, the power term is determined in a simple way. (It's surprising how addictive that FSF can become.)

```
FindSequenceFunction[{0, 2, 4, 6, 8, 10, 12}, n]
```

$2\,(-1+n)$

And having the effect of

$\infty^{1/2}$

∞

Which agrees with the text answer.

23. $\dfrac{1}{(z+i)^2}$, $z_0 = i$

```
Clear["Global`*"]
```

First, a naïve try,

$$\text{Series}\left[\frac{1}{(z+i)^2}, \{z, i, 8\}\right]$$

$$-\frac{1}{4}-\frac{1}{4}i(z-i)+\frac{3}{16}(z-i)^2+\frac{1}{8}i(z-i)^3-\frac{5}{64}(z-i)^4-\frac{3}{64}i(z-i)^5+\frac{7}{256}(z-i)^6+\frac{1}{64}i(z-i)^7-\frac{9(z-i)^8}{1024}+O[z-i]^9$$

The green cell above matches the text answer. Now to address a strange looking sequence of term coefficients

$$\text{FindSequenceFunction}\left[\left\{-\frac{1}{4}, -\frac{i}{4}, \frac{3}{16}, \frac{i}{8}, -\frac{5}{64}, -\frac{3i}{64}, \frac{7}{256}, \frac{i}{64}, -\frac{9}{1024}\right\}, n\right]$$

$$\left(\frac{i}{2}\right)^{1+n} n$$

$$\text{testg}[n\_] = \left(\frac{i}{2}\right)^{1+n} n$$

$$\left(\frac{i}{2}\right)^{1+n} n$$

$$\text{TableForm}[\text{Table}[\{n, \text{testg}[n]\}, \{n, 1, 9\}]]$$

1	$-\frac{1}{4}$
2	$-\frac{i}{4}$
3	$\frac{3}{16}$
4	$\frac{i}{8}$
5	$-\frac{5}{64}$
6	$-\frac{3i}{64}$
7	$\frac{7}{256}$
8	$\frac{i}{64}$
9	$-\frac{9}{1024}$

Setting up the n part of the Cauchy - Hadamard expression, not worrying about the z part at the moment. Since I looked for the complete coefficient this time, not just the denominator part, the relation of n and n+1 parts is different,

$$\text{Limit}\left[\text{Abs}\left[\frac{\left(\frac{i}{2}\right)^{1+n} n}{1}\left(\frac{1}{\left(\frac{i}{2}\right)^{2+n}(n+1)}\right)\right], n \to \infty\right]$$

$$2$$

In this problem the power of the power term is simply n. Since I haven't been concerned about the presence of a non-n factor in the power term up to now, I won't let myself be bothered if it happens to be imaginary. So I give the final radius as

$2^{1/1}$

2

The green cell agrees with the text answer.

25. Sinh[2 z - i], $z_0 = \frac{i}{2}$

```
Clear["Global`*"]
```

First, a naïve try,

$$\text{Series}\left[\text{Sinh}[2 z - i], \left\{z, \frac{i}{2}, 15\right\}\right]$$

$$2\left(z-\frac{i}{2}\right)+\frac{4}{3}\left(z-\frac{i}{2}\right)^3+\frac{4}{15}\left(z-\frac{i}{2}\right)^5+\frac{8}{315}\left(z-\frac{i}{2}\right)^7+\frac{4\left(z-\frac{i}{2}\right)^9}{2835}+\frac{8\left(z-\frac{i}{2}\right)^{11}}{155\,925}+\frac{8\left(z-\frac{i}{2}\right)^{13}}{6\,081\,075}+\frac{16\left(z-\frac{i}{2}\right)^{15}}{638\,512\,875}+O\left[z-\frac{i}{2}\right]^{16}$$

The green cell above matches the text answer. Now to try to indentify the term coefficients

$$\text{FindSequenceFunction}\left[\left\{2, \frac{4}{3}, \frac{4}{15}, \frac{8}{315}, \frac{4}{2835}, \frac{8}{155\,925}, \frac{8}{6\,081\,075}\right\}, n\right]$$

$$2\Big/\left(\text{Pochhammer}[1, -1+n]\ \text{Pochhammer}\left[\frac{3}{2}, -1+n\right]\right)$$

```
FullSimplify[%]
```

$$\frac{2^{-1+2n}}{\text{Gamma}[2 n]}$$

$$\text{testg}[n\_] = \frac{2^{-1+2n}}{\text{Gamma}[2 n]}$$

$$\frac{2^{-1+2n}}{\text{Gamma}[2 n]}$$

```
TableForm[Table[{n, testg[n]}, {n, 1, 9}]]
```

1	2
2	$\frac{4}{3}$
3	$\frac{4}{15}$
4	$\frac{8}{315}$
5	$\frac{4}{2835}$
6	$\frac{8}{155\,925}$
7	$\frac{8}{6\,081\,075}$
8	$\frac{16}{638\,512\,875}$
9	$\frac{4}{10\,854\,718\,875}$

Setting up the n part of the Cauchy - Hadamard expression, not worrying about the z part yet. Looking a lot like the last problem at the moment.

$$\text{Limit}\left[\text{Abs}\left[\frac{2^{-1+2\,n}}{\text{Gamma}[2\,n]}\left(\frac{\text{Gamma}[2\,(n+1)]}{2^{-1+2\,(n+1)}}\right)\right],\ n\to\infty\right]$$

∞

Now to handle the power term part of the radius calculation. The power of the power term is 2n-1. So the effect of the power term will be

$\infty^{1/2}$

∞

The green cell matches the text answer.