Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1 - 8 Regions of practical interest

Determine and sketch or graph the sets in the complex plane given by

1. Abs[z + 1 - 5 i] ≤ $\frac{3}{2}$

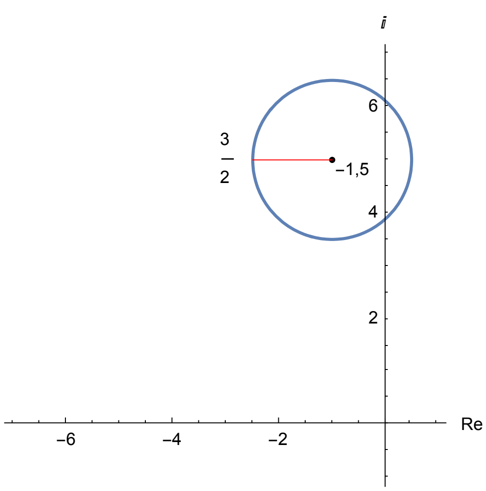
This problem refers to construction of a closed set in the complex plane, according to the description on p. 619 of the text. It is a “Closed Circular Disk” that I want to build of radius ρ and center **a**, with the formula $|z - \mathbf{a}| \leq \rho$, and in which **a** is a complex number with its real part describing the x-coordinate of the center, and the imaginary part describing the y-coordinate.

```
Abs[z - a] ≤ ρ
Abs[-a + z] ≤ ρ

Solve[1 - 5 i == -a, a]
{{a → -1 + 5 i}}
```

Giving me x=-1, y=5, and ρ=3/2.

```
RegionPlot[(x + 1)^2 + (y - 5)^2 ≤ (3 / 2)^2, {x, -7, 1}, {y, -1, 7},
 Axes → True, Frame → False, PlotStyle → White, AxesLabel → {"Re", "i"},
 ImageSize → 250, Epilog → {{PointSize[0.014], Point[{-1, 5}]},
   {Red, Line[{{-1, 5}, {-2.5, 5}}]},
   {Text["3/2", {-3, 5}]}, {Text["-1,5", {-0.6, 4.8}]}}]
```



3. π < Abs[z - 4 + 2 I] < 3 π

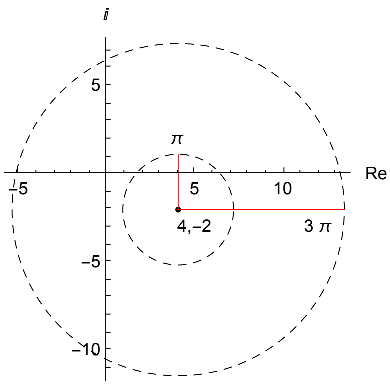
```
Clear["Global`*"]
```

```
Abs[z - a] ≤ ρ
```

```
Solve[-4 + 2 ⅈ == -a, a]
{{a → 4 - 2 ⅈ}}
```

Giving me x=4, y=-2, and an annulus between radius $\rho=\pi$ and $\rho=3\ \pi$

```
Graphics[{{Dashed, Circle[{4, -2}, 3 π]}, {Dashed, Circle[{4, -2}, π]},
   {Point[{4, -2}]}, {Red, Line[{{4, -2}, {4 + 3 π, -2}}]},
   {Text["3  π", {12, -3}]}, {Text["4,-2", {5, -3}]},
   {Red, Line[{{4, -2}, {4, -2 + π}}]}, {Text["π", {4, 2}]}},
 Axes → True,  ImageSize → 200, AxesLabel → {"Re", "ⅈ"}]
```



Plot construction using **Graphics** commands instead of **RegionPlot** is a little shorter, and also a little easier. The dashed circles represent open sets.

5. Abs[Arg[z]] < $\frac{1}{4}\pi$

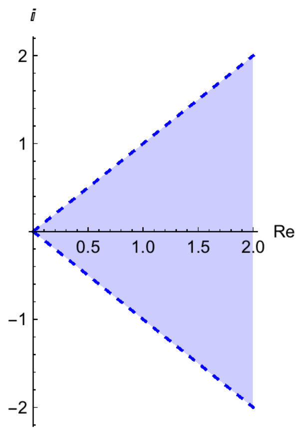
```
Clear["Global`*"]
```

```
Abs[z - a] ≤ ρ
Abs[-a + z] ≤ ρ
```

```
Solve[0 - Arg[z] == -a, a]
{{a → Arg[z]}}
```

Giving me x=0, y=Arg[z], and $\rho = \frac{\pi}{4}$

```
Plot[{x, -x}, {x, 0, 2}, ImageSize → 150, AspectRatio → 1.7,
 PlotStyle → {{Dashed, Blue}, {Dashed, Blue}},
 Filling → Axis, AxesLabel → {"Re", "i"}]
```



This plot should be considered as that of a quarter circle of infinite radius, centered on the origin, with open boundaries.

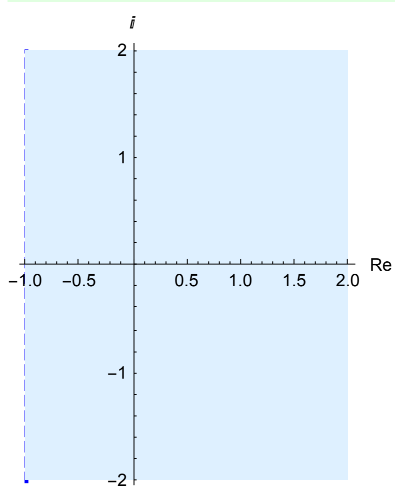
7. Re[z] >= -1

```
Clear["Global`*"]
```

```
Abs[z - a] ≤ ρ
```

```
Abs[-a + z] ≤ ρ
```

```
Graphics[{{Dashed, Blue, Thick, Line[{{-1, -2}, {-1, 2}}]},
   {LightBlue, Rectangle[{-1, -2}, {2, 2}]}},
 Axes → True,  ImageSize → 200, AxesLabel → {"Re", "i"}]
```



I'm not really sure how to do the above with an equation. I see it as an infinite open semi-circle centered at -1,0.

The real part of z is equal to or greater than -1, and Arg[z] is unrestricted.

10 - 12 Complex functions and their derivatives

Function values. Find Re[f] and Im[f] and their values at the given point z.

11. $f[z\_] = \frac{1}{1-z}$ at $1 - i$

```
Clear["Global`*"]
```

$$f[z\_] = \frac{1}{1-z}$$

$$\frac{1}{1-z}$$

```
dek = f[1 - ⅈ]
```

$-i$

Or expressed as 0 - 1 (i). The yellow cell is not given in the text answer, though I believe it satisfies the problem requirement.

14 - 17 Continuity. Find out, and give reason, whether f(z) is continuous at z=0 if f(0)=0 and for z≠0 the function is equal to:

15. $\text{Abs}[z]^2\ \text{Im}\left[\frac{1}{z}\right]$

```
Clear["Global`*"]
```

$$f[z\_] = \text{Abs}[z]^2\ \text{Im}\left[\frac{1}{z}\right]$$

$$\text{Abs}[z]^2\ \text{Im}\left[\frac{1}{z}\right]$$

```
Limit[f[z], z → 0]
```

0

Mathematica did not cite difficulties in performing the above limit, so I will take the result as positive. The answers in the text give the reasons.

17. $\frac{\text{Re}[z]}{1 - \text{Abs}[z]}$

```
Clear["Global`*"]
```

```
f[z] = Re[z] / (1 - Abs[z])
```

$$\frac{\mathrm{Re}[z]}{1 - \mathrm{Abs}[z]}$$

```
Limit[f[z], z → 0]
```

0

Again, the limit maneuver did not involve a snag. The answers in the text give the reasons.

18 - 23 Differentiation. Find the value of the derivative of

19. $(z - 4\,i)^8$ at $3 + 4\,i$

```
Clear["Global`*"]
f[z_] = (z - 4 i)^8
```

$(-4\,i + z)^8$

```
dif = D[f[z], z]
```

$8\,(-4\,i + z)^7$

```
dif1 = dif /. z → (3 + 4 i)
```

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21. $i\,(1 - z)^n$ at 0

```
Clear["Global`*"]
f[z] = i (1 - z)^n
```

$i\,(1 - z)^n$

```
der = D[f[z], z]
```

$-i\,n\,(1 - z)^{-1+n}$

```
der1 = der /. z → (0)
```

$-i\,n$

The above yellow cell does not agree with the text answer (n * *i*). However, I ran the problem in Symbolab, and Symbolab agreed with Mathematica's solution.

23. $\frac{z^3}{(z + i)^3}$ at i

```
Clear["Global`*"]
```

```
f[z] = z^3/(z + i)^3
```

$$\frac{z^3}{(i + z)^3}$$

```
der = Simplify[D[f[z], z]]
```

$$\frac{3\, i\, z^2}{(i + z)^4}$$

```
der1 = der /. z → i
```

$$-\frac{3\, i}{16}$$