

1 - 9 General solution

Find a real general solution of the following systems.

$$\begin{aligned} 1. \quad y_1' &= y_1 + y_2 \\ y_2' &= 3y_1 - y_2 \end{aligned}$$

```
Clear["Global`*"]
```

Mathematica solves the system, but to knock the solutions into a framework which can be directly compared with the text answer, some wrangling, rearranging, and substituting must be done.

```
rit = {y1'[t] == y1[t] + y2[t], y2'[t] == 3 y1[t] - y2[t]}
git = DSolve[rit, {y1, y2}, t]
{y1'[t] == y1[t] + y2[t], y2'[t] == 3 y1[t] - y2[t]}
{{y1 -> Function[{t},  $\frac{1}{4} e^{-2t} (1 + 3 e^{4t}) C[1] + \frac{1}{4} e^{-2t} (-1 + e^{4t}) C[2]$ ],
  y2 -> Function[{t},  $\frac{3}{4} e^{-2t} (-1 + e^{4t}) C[1] + \frac{1}{4} e^{-2t} (3 + e^{4t}) C[2]$ ]}}
```

```
fit = Expand[git[[1, 1, 2, 2]]]
 $\frac{1}{4} e^{-2t} C[1] + \frac{3}{4} e^{2t} C[1] - \frac{1}{4} e^{-2t} C[2] + \frac{1}{4} e^{2t} C[2]$ 
```

```
vit = Expand[4 fit]
 $e^{-2t} C[1] + 3 e^{2t} C[1] - e^{-2t} C[2] + e^{2t} C[2]$ 
```

```
bit = Collect[vit, e^{-2t}]
 $e^{-2t} (C[1] - C[2]) + e^{2t} (3 C[1] + C[2])$ 
```

Having reconciled the form of the constants of integration, a recognizable variant emerges.

```
mit = bit /. {(C[1] - C[2]) -> c1, (3 C[1] + C[2]) -> c2}
```

```
 $c1 e^{-2t} + c2 e^{2t}$ 
```

```
wit = Expand[git[[1, 2, 2, 2]]]
 $-\frac{3}{4} e^{-2t} C[1] + \frac{3}{4} e^{2t} C[1] + \frac{3}{4} e^{-2t} C[2] + \frac{1}{4} e^{2t} C[2]$ 
pit = Expand[4 wit]
 $-3 e^{-2t} C[1] + 3 e^{2t} C[1] + 3 e^{-2t} C[2] + e^{2t} C[2]$ 
```

```

sit = Collect[pit, e-2 t]
e2 t (3 C[1] + C[2]) + e-2 t (-3 C[1] + 3 C[2])

kit = sit /. (-3 C[1] + 3 C[2]) → (-3 (C[1] - C[2]))
-3 e-2 t (C[1] - C[2]) + e2 t (3 C[1] + C[2])

```

```
lit = kit /. {(C[1] - C[2]) → c1, (3 C[1] + C[2]) → c2}
```

```
-3 c1 e-2 t + c2 e2 t
```

1. Above: The top green cell 'mit' is y1, the bottom green cell 'lit' is y2. They both match the text expressions, even to the constants. Care was taken to make sure equal constant substitutions were made in both cases (yellow).

```

3. y1' = y1 + 2 y2
y2' = y1 + 2 y2

```

```
Clear["Global`*"]
```

```
nar = {y1'[t] == y1[t] + 2 y2[t], y2'[t] ==  $\frac{1}{2}$  y1[t] + y2[t]}
```

```
bar = DSolve[nar, {y1, y2}, t]
```

```
{y1'[t] == y1[t] + 2 y2[t], y2'[t] ==  $\frac{y1[t]}{2}$  + y2[t]}
```

```

{{y1 → Function[{t},  $\frac{1}{2} (1 + e^{2 t}) C[1] + (-1 + e^{2 t}) C[2]$ ],
  y2 → Function[{t},  $\frac{1}{4} (-1 + e^{2 t}) C[1] + \frac{1}{2} (1 + e^{2 t}) C[2]$ ]}}

```

```
mar = Expand[bar[[1, 1, 2, 2]]]
```

```
 $\frac{C[1]}{2} + \frac{1}{2} e^{2 t} C[1] - C[2] + e^{2 t} C[2]$ 
```

```
uar = Expand[2 mar]
```

```
C[1] + e2 t C[1] - 2 C[2] + 2 e2 t C[2]
```

```
sar = uar /. (C[1] - 2 C[2]) → (c2)
```

```
c2 + e2 t C[1] + 2 e2 t C[2]
```

```
tar = Collect[sar, e2 t]
```

```
c2 + e2 t (C[1] + 2 C[2])
```

```
var = tar /. (C[1] + 2 C[2]) -> c1
```

```
c2 + c1 e2 t
```

```
jar = Expand[2 var]
```

```
2 c2 + 2 c1 e2 t
```

```
par = Expand[bar[[1, 2, 2, 2]]]
```

```
-  $\frac{C[1]}{4} + \frac{1}{4} e^{2t} C[1] + \frac{C[2]}{2} + \frac{1}{2} e^{2t} C[2]$ 
```

```
har = Expand[4 par]
```

```
-C[1] + e2 t C[1] + 2 C[2] + 2 e2 t C[2]
```

```
dar = har /. (-C[1] + 2 C[2]) -> (-c2)
```

```
-c2 + e2 t C[1] + 2 e2 t C[2]
```

```
qar = Collect[dar, e2 t]
```

```
-c2 + e2 t (C[1] + 2 C[2])
```

```
xar = qar /. (C[1] + 2 C[2]) -> c1
```

```
-c2 + c1 e2 t
```

1. Above: The functions 'jar' and 'xar', (upper and lower green cells respectively), are y_1 and y_2 , and match the text answer. Note that in assembling the functions, each was multiplied by 4. However, care was taken so that the proportions and signs of the constants match those of the text.

```
5.  $y_1' = 2 y_1 + 5 y_2$ 
```

```
 $y_2' = 5 y_1 + 12.5 y_2$ 
```

```
Clear["Global`*"]
```

```
e1 = {y1'[t] == 2 y1[t] + 5 y2[t], y2'[t] == 5 y1[t] + 12.5 y2[t]}
```

```
e2 = DSolve[e1, {y1, y2}, t]
```

```
{y1'[t] == 2 y1[t] + 5 y2[t], y2'[t] == 5 y1[t] + 12.5 y2[t]}
```

```
{ {y1 -> Function[{t}, 0.137931 e-2.22045×10-16 t (6.25 + 1. e14.5 t) C[1] +  
0.344828 e-2.22045×10-16 t (-1. + 1. e14.5 t) C[2]],  
y2 -> Function[{t}, 0.344828 e-2.22045×10-16 t (-1. + 1. e14.5 t) C[1] +  
0.862069 e-2.22045×10-16 t (0.16 + 1. e14.5 t) C[2]] }
```

```
e3 = e2[[1, 1, 2, 2]]
```

```
0.137931 e-2.22045×10-16 t (6.25 + 1. e14.5 t) C[1] +  
0.344828 e-2.22045×10-16 t (-1. + 1. e14.5 t) C[2]
```

```
e16 = e3 /. {C[1] → C1, C[2] → C2}
```

```
0.344828 C2 e-2.22045×10-16 t (-1. + 1. e14.5 t) +  
0.137931 C1 e-2.22045×10-16 t (6.25 + 1. e14.5 t)
```

```
e4 = Chop[e16, 10-15]
```

```
0.344828 C2 (-1. + 1. e14.5 t) + 0.137931 C1 (6.25 + 1. e14.5 t)
```

```
e5 = Expand[e4]
```

```
0.862069 C1 - 0.344828 C2 + 0.137931 C1 e14.5 t + 0.344828 C2 e14.5 t
```

```
e6 = Collect[e5, e14.5 t]
```

```
0.862069 C1 - 0.344828 C2 + (0.137931 C1 + 0.344828 C2) e14.5 t
```

```
e7 = e6 /. (0.13793103448275862` C1 + 0.3448275862068965` C2) → 2 c2
```

```
0.862069 C1 - 0.344828 C2 + 2 c2 e14.5 t
```

```
e8 = e7 /. (0.8620689655172413` C1 - 0.3448275862068966` C2) → 5 c1
```

```
5 c1 + 2 c2 e14.5 t
```

```
Solve[0.13793103448275862` + 0.3448275862068965` == 2 c21, c21]
```

```
{{c21 → 0.241379}}
```

```
Solve[0.8620689655172413` - 0.3448275862068966` == 5 c11, c11]
```

```
{{c11 → 0.103448}}
```

```
e10 = e2[[1, 2, 2, 2]]
```

```
0.344828 e-2.22045×10-16 t (-1. + 1. e14.5 t) C[1] +  
0.862069 e-2.22045×10-16 t (0.16 + 1. e14.5 t) C[2]
```

```
e17 = e10 /. {C[1] → C1, C[2] → C2}
```

```
0.344828 C1 e-2.22045×10-16 t (-1. + 1. e14.5 t) +  
0.862069 C2 e-2.22045×10-16 t (0.16 + 1. e14.5 t)
```

```
e11 = Chop[e17, 10-15]
```

```
0.344828 C1 (-1. + 1. e14.5 t) + 0.862069 C2 (0.16 + 1. e14.5 t)
```

```
e12 = Expand[e11]
```

```
-0.344828 C1 + 0.137931 C2 + 0.344828 C1 e14.5 t + 0.862069 C2 e14.5 t
```

```
e13 = Collect[e12, e14.5 t]
```

```
-0.344828 C1 + 0.137931 C2 + (0.344828 C1 + 0.862069 C2) e14.5 t
```

```
e14 = e13 /. (0.3448275862068966` C1 + 0.8620689655172414` C2) → 5 c2
```

```
-0.344828 C1 + 0.137931 C2 + 5 c2 e14.5 t
```

```
Solve[0.3448275862068966` + 0.8620689655172414` == 5 c22, c22]
```

```
{{c22 → 0.241379}}
```

```
e15 = e14 /. (-0.3448275862068965` C1 + 0.13793103448275862` C2) → (-2 c1)
```

```
-2 c1 + 5 c2 e14.5 t
```

```
Solve[-0.3448275862068965` + 0.13793103448275862` == -2 c21, c21]
```

```
{{c21 → 0.103448}}
```

1. Above: y_1 is given by e8; y_2 is given by e15. These expressions match the text answers. Green cells are for function formulas, yellow cells for sites of assignment of values of constants, pink cells for constant value verification. The equality of c11,c12; and c21,c22 shows that due consideration was given to preserving the values and proportions of the constants. (In calculating the numerical value of constants for comparison, the values of Mathematica's constants was taken as all 1.)

```
7.  $y_1' = y_2$ 
```

```
 $y_2' = -y_1 + y_3$ 
```

```
 $y_3' = -y_2$ 
```

```
Clear["Global`*"]
```

```

e1 = {y1'[t] == y2[t], y2'[t] == -y1[t] + y3[t], y3'[t] == -y2[t]}
e2 = DSolve[e1, {y1, y2, y3}, t]
{y1'[t] == y2[t], y2'[t] == -y1[t] + y3[t], y3'[t] == -y2[t]}
{ {y1 → Function[{t},
  
$$\frac{1}{2} C[3] (1 - \cos[\sqrt{2} t]) + \frac{1}{2} C[1] (1 + \cos[\sqrt{2} t]) + \frac{C[2] \sin[\sqrt{2} t]}{\sqrt{2}},$$

  y2 → Function[{t},  $C[2] \cos[\sqrt{2} t] - \frac{C[1] \sin[\sqrt{2} t]}{\sqrt{2}} + \frac{C[3] \sin[\sqrt{2} t]}{\sqrt{2}}]$ },
  y3 → Function[{t},
  
$$\frac{1}{2} C[1] (1 - \cos[\sqrt{2} t]) + \frac{1}{2} C[3] (1 + \cos[\sqrt{2} t]) - \frac{C[2] \sin[\sqrt{2} t]}{\sqrt{2}}]}}$$


```

```
e3 = e2[[1, 1, 2, 2]]
```

$$\frac{1}{2} C[3] (1 - \cos[\sqrt{2} t]) + \frac{1}{2} C[1] (1 + \cos[\sqrt{2} t]) + \frac{C[2] \sin[\sqrt{2} t]}{\sqrt{2}}$$

```
e4 = Expand[2 e3]
```

$$C[1] + C[3] + C[1] \cos[\sqrt{2} t] - C[3] \cos[\sqrt{2} t] + \sqrt{2} C[2] \sin[\sqrt{2} t]$$

```
e5 = Collect[e4, Cos[\sqrt{2} t]]
```

$$C[1] + C[3] + (C[1] - C[3]) \cos[\sqrt{2} t] + \sqrt{2} C[2] \sin[\sqrt{2} t]$$

```
e6 = e5 /. (C[1] + C[3]) → c1
```

$$c1 + (C[1] - C[3]) \cos[\sqrt{2} t] + \sqrt{2} C[2] \sin[\sqrt{2} t]$$

```
e7 = e6 /. (C[1] - C[3]) → -c2
```

$$c1 - c2 \cos[\sqrt{2} t] + \sqrt{2} C[2] \sin[\sqrt{2} t]$$

```
e8 = e7 /. (\sqrt{2} C[2]) → c3
```

$$c1 - c2 \cos[\sqrt{2} t] + c3 \sin[\sqrt{2} t]$$

```
e9 = e2[[1, 2, 2, 2]]
```

$$C[2] \cos[\sqrt{2} t] - \frac{C[1] \sin[\sqrt{2} t]}{\sqrt{2}} + \frac{C[3] \sin[\sqrt{2} t]}{\sqrt{2}}$$

e10 = Expand[2 e9]

$$2 C[2] \cos[\sqrt{2} t] - \sqrt{2} C[1] \sin[\sqrt{2} t] + \sqrt{2} C[3] \sin[\sqrt{2} t]$$

$$e11 = e10 /. C[2] \rightarrow \frac{c3}{\sqrt{2}}$$

$$\sqrt{2} c3 \cos[\sqrt{2} t] - \sqrt{2} C[1] \sin[\sqrt{2} t] + \sqrt{2} C[3] \sin[\sqrt{2} t]$$

$$e12 = \text{Collect}[e11, \sqrt{2} \sin[\sqrt{2} t]]$$

$$\sqrt{2} c3 \cos[\sqrt{2} t] + \sqrt{2} (-C[1] + C[3]) \sin[\sqrt{2} t]$$

$$e13 = e12 /. (-C[1] + C[3]) \rightarrow c2$$

$$\sqrt{2} c3 \cos[\sqrt{2} t] + \sqrt{2} c2 \sin[\sqrt{2} t]$$

$$e14 = e2[[1, 3, 2, 2]]$$

$$\frac{1}{2} C[1] (1 - \cos[\sqrt{2} t]) + \frac{1}{2} C[3] (1 + \cos[\sqrt{2} t]) - \frac{C[2] \sin[\sqrt{2} t]}{\sqrt{2}}$$

$$e15 = \text{Expand}[2 e14]$$

$$C[1] + C[3] - C[1] \cos[\sqrt{2} t] + C[3] \cos[\sqrt{2} t] - \sqrt{2} C[2] \sin[\sqrt{2} t]$$

$$e16 = e15 /. (C[1] + C[3]) \rightarrow c1$$

$$c1 - C[1] \cos[\sqrt{2} t] + C[3] \cos[\sqrt{2} t] - \sqrt{2} C[2] \sin[\sqrt{2} t]$$

$$e17 = \text{Collect}[e16, \cos[\sqrt{2} t]]$$

$$c1 + (-C[1] + C[3]) \cos[\sqrt{2} t] - \sqrt{2} C[2] \sin[\sqrt{2} t]$$

$$e18 = e17 /. (-C[1] + C[3]) \rightarrow c2$$

$$c1 + c2 \cos[\sqrt{2} t] - \sqrt{2} C[2] \sin[\sqrt{2} t]$$

$$e19 = e18 /. (\sqrt{2} C[2]) \rightarrow c3$$

$$c1 + c2 \cos[\sqrt{2} t] - c3 \sin[\sqrt{2} t]$$

1. Above: The function forms for y_1 , y_2 , and y_3 match the green cells above in order, conforming to the text function forms. The system of symbolic constant conversion established

for y_1 was carried forward and used for the remaining two functions. It was found that this system matched the text symbolic constant assignments exactly. Each function was upscaled by 2 during assembly.

$$\begin{aligned} 9. \quad y_1' &= 10 y_1 - 10 y_2 - 4 y_3 \\ y_2' &= -10 y_1 + y_2 - 14 y_3 \\ y_3' &= -4 y_1 - 14 y_2 - 2 y_3 \end{aligned}$$

```
Clear["Global`*"]
```

```
e1 = {y1'[t] == 10 y1[t] - 10 y2[t] - 4 y3[t],
      y2'[t] == -10 y1[t] + y2[t] - 14 y3[t],
      y3'[t] == -4 y1[t] - 14 y2[t] - 2 y3[t]}
```

```
e2 = DSolve[e1, {y1, y2, y3}, t]
```

```
{y1'[t] == 10 y1[t] - 10 y2[t] - 4 y3[t],
 y2'[t] == -10 y1[t] + y2[t] - 14 y3[t], y3'[t] == -4 y1[t] - 14 y2[t] - 2 y3[t]}
```

```
{ {y1 → Function[{t},  $\frac{1}{9} e^{-18 t} (1 + 4 e^{27 t} + 4 e^{36 t}) C[1] -$ 
 $\frac{2}{9} e^{-18 t} (-1 - e^{27 t} + 2 e^{36 t}) C[2] + \frac{2}{9} e^{-18 t} (1 - 2 e^{27 t} + e^{36 t}) C[3]$  ],
  y2 → Function[{t},  $-\frac{2}{9} e^{-18 t} (-1 - e^{27 t} + 2 e^{36 t}) C[1] +$ 
 $\frac{1}{9} e^{-18 t} (4 + e^{27 t} + 4 e^{36 t}) C[2] - \frac{2}{9} e^{-18 t} (-2 + e^{27 t} + e^{36 t}) C[3]$  ],
  y3 → Function[{t},  $\frac{2}{9} e^{-18 t} (1 - 2 e^{27 t} + e^{36 t}) C[1] -$ 
 $\frac{2}{9} e^{-18 t} (-2 + e^{27 t} + e^{36 t}) C[2] + \frac{1}{9} e^{-18 t} (4 + 4 e^{27 t} + e^{36 t}) C[3]$  ] } }
```

```
e3 = e2[[1, 1, 2, 2]]
```

```
 $\frac{1}{9} e^{-18 t} (1 + 4 e^{27 t} + 4 e^{36 t}) C[1] -$ 
 $\frac{2}{9} e^{-18 t} (-1 - e^{27 t} + 2 e^{36 t}) C[2] + \frac{2}{9} e^{-18 t} (1 - 2 e^{27 t} + e^{36 t}) C[3]$ 
```

```
Expand[e3]
```

```
 $\frac{1}{9} e^{-18 t} C[1] + \frac{4}{9} e^{9 t} C[1] + \frac{4}{9} e^{18 t} C[1] + \frac{2}{9} e^{-18 t} C[2] +$ 
 $\frac{2}{9} e^{9 t} C[2] - \frac{4}{9} e^{18 t} C[2] + \frac{2}{9} e^{-18 t} C[3] - \frac{4}{9} e^{9 t} C[3] + \frac{2}{9} e^{18 t} C[3]$ 
```

```
e4 = Expand[9 e3]
```

```
 $e^{-18 t} C[1] + 4 e^{9 t} C[1] + 4 e^{18 t} C[1] + 2 e^{-18 t} C[2] +$ 
 $2 e^{9 t} C[2] - 4 e^{18 t} C[2] + 2 e^{-18 t} C[3] - 4 e^{9 t} C[3] + 2 e^{18 t} C[3]$ 
```


$$e5 = \text{Collect}[e4, e^{-18t}]$$

$$e^{9t} (4 C[1] + 2 C[2] - 4 C[3]) + e^{18t} (4 C[1] - 4 C[2] + 2 C[3]) + e^{-18t} (C[1] + 2 C[2] + 2 C[3])$$

$$e6 = e5 /. (C[1] + 2 C[2] + 2 C[3]) \rightarrow \frac{1}{2} c1$$

$$\frac{1}{2} c1 e^{-18t} + e^{9t} (4 C[1] + 2 C[2] - 4 C[3]) + e^{18t} (4 C[1] - 4 C[2] + 2 C[3])$$

$$e7 = e6 /. (4 C[1] + 2 C[2] - 4 C[3]) \rightarrow 2 c2$$

$$\frac{1}{2} c1 e^{-18t} + 2 c2 e^{9t} + e^{18t} (4 C[1] - 4 C[2] + 2 C[3])$$

$$e8 = e7 /. (4 C[1] - 4 C[2] + 2 C[3]) \rightarrow -c3$$

$$\frac{1}{2} c1 e^{-18t} + 2 c2 e^{9t} - c3 e^{18t}$$

$$e9 = e2[[1, 2, 2, 2]]$$

$$-\frac{2}{9} e^{-18t} (-1 - e^{27t} + 2 e^{36t}) C[1] + \frac{1}{9} e^{-18t} (4 + e^{27t} + 4 e^{36t}) C[2] - \frac{2}{9} e^{-18t} (-2 + e^{27t} + e^{36t}) C[3]$$

$$e10 = \text{Expand}[9 e9]$$

$$2 e^{-18t} C[1] + 2 e^{9t} C[1] - 4 e^{18t} C[1] + 4 e^{-18t} C[2] + e^{9t} C[2] + 4 e^{18t} C[2] + 4 e^{-18t} C[3] - 2 e^{9t} C[3] - 2 e^{18t} C[3]$$

$$e11 = \text{Collect}[e10, e^{-18t}]$$

$$e^{9t} (2 C[1] + C[2] - 2 C[3]) + e^{18t} (-4 C[1] + 4 C[2] - 2 C[3]) + e^{-18t} (2 C[1] + 4 C[2] + 4 C[3])$$

$$e12 = e11 /. (2 C[1] + C[2] - 2 C[3]) \rightarrow c2$$

$$c2 e^{9t} + e^{18t} (-4 C[1] + 4 C[2] - 2 C[3]) + e^{-18t} (2 C[1] + 4 C[2] + 4 C[3])$$

$$e13 = e12 /. (-4 C[1] + 4 C[2] - 2 C[3]) \rightarrow c3$$

$$c2 e^{9t} + c3 e^{18t} + e^{-18t} (2 C[1] + 4 C[2] + 4 C[3])$$

$$\mathbf{e14} = \mathbf{e13} /. (2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3]) \rightarrow \mathbf{c1}$$

$$\mathbf{c1} e^{-18t} + \mathbf{c2} e^{9t} + \mathbf{c3} e^{18t}$$

$$\mathbf{e15} = \mathbf{e2}[[1, 2, 2, 2]]$$

$$-\frac{2}{9} e^{-18t} (-1 - e^{27t} + 2 e^{36t}) \mathbf{C}[1] + \\ \frac{1}{9} e^{-18t} (4 + e^{27t} + 4 e^{36t}) \mathbf{C}[2] - \frac{2}{9} e^{-18t} (-2 + e^{27t} + e^{36t}) \mathbf{C}[3]$$

$$\mathbf{e16} = \text{Expand}[9 \mathbf{e15}]$$

$$2 e^{-18t} \mathbf{C}[1] + 2 e^{9t} \mathbf{C}[1] - 4 e^{18t} \mathbf{C}[1] + 4 e^{-18t} \mathbf{C}[2] + \\ e^{9t} \mathbf{C}[2] + 4 e^{18t} \mathbf{C}[2] + 4 e^{-18t} \mathbf{C}[3] - 2 e^{9t} \mathbf{C}[3] - 2 e^{18t} \mathbf{C}[3]$$

$$\mathbf{e17} = \text{Collect}[\mathbf{e16}, e^{-18t}]$$

$$e^{9t} (2 \mathbf{C}[1] + \mathbf{C}[2] - 2 \mathbf{C}[3]) + \\ e^{18t} (-4 \mathbf{C}[1] + 4 \mathbf{C}[2] - 2 \mathbf{C}[3]) + e^{-18t} (2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3])$$

$$\mathbf{e18} = \mathbf{e17} /. (2 \mathbf{C}[1] + \mathbf{C}[2] - 2 \mathbf{C}[3]) \rightarrow -2 \mathbf{c2}$$

$$-2 \mathbf{c2} e^{9t} + e^{18t} (-4 \mathbf{C}[1] + 4 \mathbf{C}[2] - 2 \mathbf{C}[3]) + e^{-18t} (2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3])$$

$$\mathbf{e19} = \mathbf{e18} /. (-4 \mathbf{C}[1] + 4 \mathbf{C}[2] - 2 \mathbf{C}[3]) \rightarrow -\frac{1}{2} \mathbf{c3}$$

$$-2 \mathbf{c2} e^{9t} - \frac{1}{2} \mathbf{c3} e^{18t} + e^{-18t} (2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3])$$

$$\mathbf{e20} = \mathbf{e19} /. (2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3]) \rightarrow \mathbf{c1}$$

$$\mathbf{c1} e^{-18t} - 2 \mathbf{c2} e^{9t} - \frac{1}{2} \mathbf{c3} e^{18t}$$

$$\text{Solve}[(2 \mathbf{C}[1] + 4 \mathbf{C}[2] + 4 \mathbf{C}[3]) == \mathbf{c1} \&\& (-4 \mathbf{C}[1] + 4 \mathbf{C}[2] - 2 \mathbf{C}[3]) == -\frac{1}{2} \mathbf{c3} \&\& \\ (2 \mathbf{C}[1] + \mathbf{C}[2] - 2 \mathbf{C}[3]) == -2 \mathbf{c2}, \{\mathbf{c1}, \mathbf{c2}, \mathbf{c3}\}]$$

$$\left\{ \left\{ \mathbf{c1} \rightarrow 2 (\mathbf{C}[1] + 2 \mathbf{C}[2] + 2 \mathbf{C}[3]), \right. \right. \\ \left. \left. \mathbf{c2} \rightarrow \frac{1}{2} (-2 \mathbf{C}[1] - \mathbf{C}[2] + 2 \mathbf{C}[3]), \mathbf{c3} \rightarrow 4 (2 \mathbf{C}[1] - 2 \mathbf{C}[2] + \mathbf{C}[3]) \right\} \right\}$$

```
Solve[(2 C[1] + 4 C[2] + 4 C[3]) == c1 && (-4 C[1] + 4 C[2] - 2 C[3]) == c3 &&
(2 C[1] + C[2] - 2 C[3]) == c2, {c1, c2, c3}]
```

```
{ {c1 → 2 (C[1] + 2 C[2] + 2 C[3]),
c2 → 2 C[1] + C[2] - 2 C[3], c3 → -2 (2 C[1] - 2 C[2] + C[3]) } }
```

```
Solve[(4 C[1] - 4 C[2] + 2 C[3]) == -c3 && (4 C[1] + 2 C[2] - 4 C[3]) == 2 c2 &&
(C[1] + 2 C[2] + 2 C[3]) ==  $\frac{1}{2}$  c1, {c1, c2, c3}]
```

```
{ {c1 → 2 (C[1] + 2 C[2] + 2 C[3]),
c2 → 2 C[1] + C[2] - 2 C[3], c3 → -2 (2 C[1] - 2 C[2] + C[3]) } }
```

1. Above: Referring to green cells top to bottom, the function expressions match those of the text, y_1 , y_2 , y_3 , respectively. The constant coefficients were substituted as required to match those of the text; the three Solve jobs just above (pink cells) verify the coefficient assignment system as consistent and equivalent to the text.

10 - 15 IVPs

Solve the following initial value problems.

$$\begin{aligned} 11. \quad y_1' &= 2 y_1 + 5 y_2 \\ y_2' &= -\frac{1}{2} y_1 - \frac{3}{2} y_2 \\ y_1[0] &= -12, \quad y_2[0] = 0 \end{aligned}$$

```
Clear["Global`*"]
```

```
e1 = {y1'[t] == 2 y1[t] + 5 y2[t],
      y2'[t] == - $\frac{1}{2}$  y1[t] -  $\frac{3}{2}$  y2[t], y1[0] == -12, y2[0] == 0}
```

```
e2 = DSolve[e1, {y1, y2}, t]
```

```
{y1'[t] == 2 y1[t] + 5 y2[t],
 y2'[t] == - $\frac{y1[t]}{2}$  -  $\frac{3 y2[t]}{2}$ , y1[0] == -12, y2[0] == 0}
```

```
{ {y1 → Function[{t}, -4 e-t/2 (-2 + 5 e3 t/2)],
  y2 → Function[{t}, 4 e-t/2 (-1 + e3 t/2) ] } }
```

```
e3 = e2[[1, 1, 2, 2]]
```

```
-4 e-t/2 (-2 + 5 e3 t/2)
```

```
e4 = Expand[e3]
```

```
8 e-t/2 - 20 et
```

```
e5 = e2[[1, 2, 2, 2]]
```

```
4 e-t/2 (-1 + e3 t/2)
```

```
e6 = Expand[e5]
```

```
-4 e-t/2 + 4 et
```

1. Above: The expressions match the text answer for y1 (top green cell) and y2 (bottom green cell).

```
13. y1' = y2
```

```
y2' = y1
```

```
y1[0] = 0, y2[0] = 2
```

```
Clear["Global`*"]
```

```
e1 = {y1'[t] == y2[t], y2'[t] == y1[t], y1[0] == 0, y2[0] == 2}
```

```
e2 = DSolve[e1, {y1, y2}, t]
```

```
{y1'[t] == y2[t], y2'[t] == y1[t], y1[0] == 0, y2[0] == 2}
```

```
{{y1 -> Function[{t}, e-t (-1 + e2 t)], y2 -> Function[{t}, e-t (1 + e2 t)]}}
```

```
e3 = e2[[1, 1, 2, 2]]
```

```
e-t (-1 + e2 t)
```

```
e4 = ExpToTrig[e3]
```

```
(Cosh[t] - Sinh[t]) (-1 + Cosh[2 t] + Sinh[2 t])
```

```
e5 = Expand[e4]
```

```
-Cosh[t] + Cosh[t] Cosh[2 t] + Sinh[t] -
```

```
Cosh[2 t] Sinh[t] + Cosh[t] Sinh[2 t] - Sinh[t] Sinh[2 t]
```

```
e6 = Simplify[e5]
```

```
2 Sinh[t]
```

```
e7 = e2[[1, 2, 2, 2]]
```

```
e-t (1 + e2 t)
```

```
e8 = ExpToTrig[e7]
```

```
(Cosh[t] - Sinh[t]) (1 + Cosh[2 t] + Sinh[2 t])
```

```
e9 = Expand[e8]
```

```
Cosh[t] + Cosh[t] Cosh[2 t] - Sinh[t] -
```

```
Cosh[2 t] Sinh[t] + Cosh[t] Sinh[2 t] - Sinh[t] Sinh[2 t]
```

```
e10 = Simplify[e9]
```

```
2 Cosh[t]
```

1. Above: The expressions for y1 (top green) and y2 (bottom green) match the text.

```
15. y1' = 3 y1 + 2 y2
y2' = 2 y1 + 3 y2
y1[0] = 0.5, y2[0] = -0.5
```

```
Clear["Global`*"]
```

```
e1 = {y1'[t] == 3 y1[t] + 2 y2[t],
      y2'[t] == 2 y1[t] + 3 y2[t], y1[0] == 0.5, y2[0] == -0.5}
e2 = DSolve[e1, {y1, y2}, t]
{y1'[t] == 3 y1[t] + 2 y2[t],
 y2'[t] == 2 y1[t] + 3 y2[t], y1[0] == 0.5, y2[0] == -0.5}
```

```
{ {y1 -> Function[{t}, 0.5 e^t], y2 -> Function[{t}, -0.5 e^t]} }
```

```
Simplify[e1 /. e2]
```

```
{{True, True, True, True}}
```

1. Above: The expression for y1 matches the text. The expression for y2 differs by sign. However, I think the text may be wrong in this case, as the Mathematica answer checks.