1 - 10 Inner product

Let a = {1, -3, 5}, b={4, 0, 8}, c = {-2, 9, 1}

1. a.b, b.a, b.c

```
Clear["Global`*"]
```

```
aa = {1, -3, 5}; bb = {4, 0, 8}; cc = {-2, 9, 1}
```

```
{-2, 9, 1}
```

```
e1 = aa.bb
```

```
44
```

```
e2 = bb.aa
```

```
44
```

```
e3 = bb.cc
```

```
0
```

3. |a|, |2b|, |-c|

```
Norm[aa]
```

$\sqrt{35}$

```
Norm[2 bb]
```

$8 \sqrt{5}$

```
Norm[-cc]
```

$\sqrt{86}$

5. |b + c|, |b| + |c|

```
e7 = Norm[bb + cc]
```

$\sqrt{166}$

```
e8 = Norm[bb] + Norm[cc]
```

$4\sqrt{5} + \sqrt{86}$

```
e9 = FullSimplify[e7 == e8]
False
```

7. | a.c |, | a || c |

```
e10 = Norm[aa.cc]
```

24

```
e11 = Norm[aa] Norm[cc]
```

$\sqrt{3010}$

9. 15a.b + 15a.c, 15a.(b+c)

```
e12 = 15 aa.bb + 15 aa.cc
```

300

```
e13 = 15 aa.(bb + cc)
```

300

17 - 20 Work

Find the work done by a force **p** acting on a body if the body is displaced along the straight segment $\overline{AB}$ from A to B. Sketch $\overline{AB}$ and **p**.

17. p = {2, 5, 0}, A: {1, 3, 3}, B: {3, 5, 5}

```
Clear["Global`*"]

aA = {1, 3, 3}; bB = {3, 5, 5}
{3, 5, 5}

pP = {2, 5, 0}
{2, 5, 0}

dis = bB - aA
{2, 2, 2}
```

```
wW = dis.pP
```

```
14
```

$$\text{cosinealpha} = N\left[\frac{\text{wW}}{\text{Norm[dis] Norm[pP]}}\right]$$

```
0.750479
```

```
alpha = ArcCos[cosinealpha]
0.72201
```

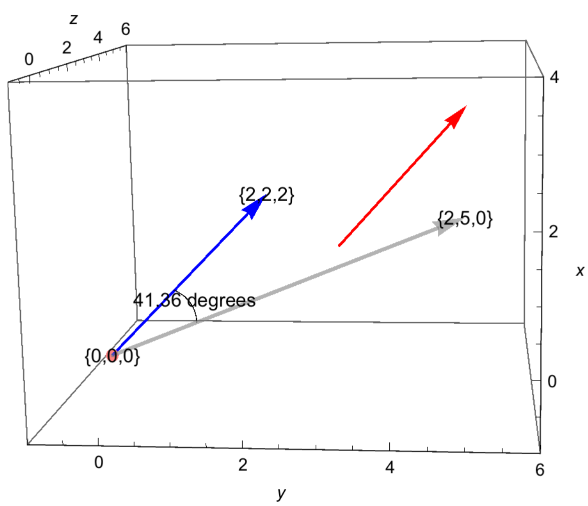
Mathematica doesn’t like to use degrees, but one way to get there is

$$\frac{1.}{\text{Degree}}$$

```
57.2958
```

```
% alpha
41.3681
```

The above way of calculating the work moves everything into the frame of reference of the origin. However, the problem description requested a view of $\overline{AB}$, so that is drawn in red.



Note: drawing arcs in Mathematica's 3D plot is not very easy. I found several recommended methods on line, but finally just flogged an approximated arc out of Blender.

19. p = {0, 4, 3}, A: {4, 5, -1}, B: (1, 3, 0}

```
In[1]:= Clear["Global`*"]

In[2]:= pP = {0, 4, 3}; aA = {4, 5, -1}; bB = {1, 3, 0}

Out[2]= {1, 3, 0}
```

In[3]:=
```
dis = bB - aA
```
Out[3]=
```
{-3, -2, 1}
```

In[4]:=
```
wW = dis.pP
```
Out[4]=
```
-5
```

In[5]:=
$$\text{cosinealpha} = \text{N}\left[\frac{\text{wW}}{\text{Norm[dis] Norm[pP]}}\right]$$
Out[5]=
```
-0.267261
```

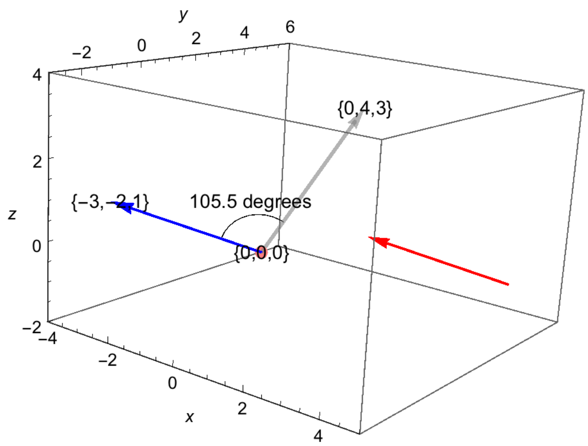
In[6]:=
```
alpha = ArcCos[cosinealpha]
```
Out[6]=
```
1.84135
```

In[7]:=
$$\frac{1.}{\text{Degree}}$$
Out[7]=
```
57.2958
```

In[8]:=
```
% alpha
```
Out[8]=
```
105.501
```

In[26]:=
```
Graphics3D[{{Blue, Thickness[0.005], Arrowheads[.04],
    Arrow[{{0, 0, 0}, {-3, -2, 1}}]}, {Red, Thickness[0.005],
    Arrowheads[.04], Arrow[{{4, 5, -1}, {1, 3, 0}}]},
   {Gray, Opacity[0.6], Thick, Arrow[{{0, 0, 0}, {0, 4, 3}}]},
   {Line[{{-.802, -.534, .265}, {-.831, -.384, .4},
      {-.827, -.225, .514}, {-.791, -.057, .608}, {-.724, .112, .679},
      {-.63, .278, .724}, {-.511, .439, .741}, {-.373, .572, .73},
      {-.22, .688, .69}, {-.059, .778, .624}, {-.002, .803, .594}}]},
   {Text[Style["105.5 degrees", 10], {-0.8, 0.5, 1}]},
   {Text[Style["{0,0,0}", 10], {0, 0, 0}]},
   {Text[Style["{-3,-2,1}", 10], {-3, -2, 1}]},
   {Text[Style["{0,4,3}", 10], {0, 4, 3}]},
   {PointSize[Large], Pink, Point[{0, 0, 0}]}},
 BoxRatios → Automatic, ImageSize → 300, Axes → True,
 PlotRangePadding → 1, AxesLabel → {x, y, z}]
```

Out[26]=



The requested sketch is shown.

22 - 30 Angle between vectors

Let aA = {1, 1, 0}; bB = {3, 2, 1}; cC = {1, 0, 2}

23. b, c

```
dotbc = bB.cC
```

5

$$e1 = \frac{dotbc}{Norm[bB]\ Norm[cC]}$$

$$\sqrt{\frac{5}{14}}\ //\ N$$

0.597614

```
e2 = ArcCos[e1]
```

```
ArcCos[Sqrt[5/14]] // N
```

0.930274

```
e3 = e2/Degree // N
```

53.3008

31 - 35 Orthogonality is particularly important, mainly because of orthogonal coordinates, such as Cartesian coordinates, whose natural basis consists of three orthogonal unit vectors.

31. For what values of a_1 are $\{a_1, 4, 3\}$ and $\{3, -2, 12\}$ orthogonal?

```
Clear["Global`*"]
```

```
e1 = {a1, 4, 3}
```

$\{a_1, 4, 3\}$

```
e2 = {3, -2, 12}
```

{3, -2, 12}

```
e3 = e1.e2
```

$28 + 3 a_1$

```
Solve[e3 == 0]
```

$\{\{a_1 \to -\frac{28}{3}\}\}$

33. Unit vectors. Find all unit vectors a = $\{a_1, a_2\}$ in the plane orthogonal to {4, 3}

```
Clear["Global`*"]
```

```
e1 = {4, 3}
```

{4, 3}

```
e2 = Norm[e1]
```

5

```
e3 = {a1, a2}
```

$\{a_1, a_2\}$

```
e4 = Norm[e3]
```

$$\sqrt{\text{Abs}[a_1]^2 + \text{Abs}[a_2]^2}$$

```
e5 = Solve[e1.e3 == 0 && Norm[e3] == 1]
```

$$\left\{\left\{a_1 \to \frac{3}{5},\ a_2 \to -\frac{4}{5}\right\},\ \left\{a_1 \to -\frac{3}{5},\ a_2 \to \frac{4}{5}\right\}\right\}$$

36 - 40 Component in the direction of a vector

Find the component of a in the direction of b. Make a sketch.

37. a = {3, 4, 0}, b = {4, -3, 2}

```
Clear["Global`*"]
```

To find the component of **a** in the direction of **b**, I first need to find the angle separating them.

```
e1 = {3, 4, 0}
{3, 4, 0}
```

```
e2 = {4, -3, 2}
{4, -3, 2}
```

$$e3 = \frac{e1.e2}{\text{Norm}[e1]\ \text{Norm}[e2]}$$

```
0
```

```
e4 = ArcCos[e3]
```

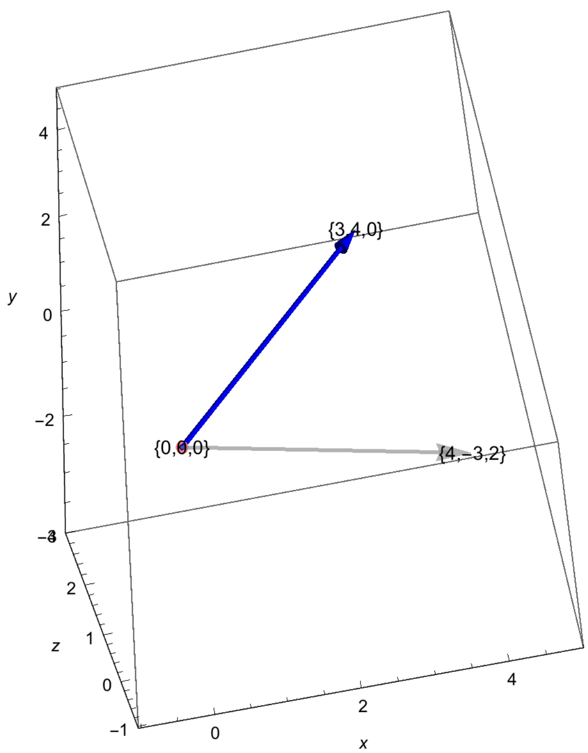
$$\frac{\pi}{2}$$

These two vectors are perpendicular; therefore there is no projection (=0).

```
e5 = Norm[e1] Cos[e4]
```

```
0
```

Out[48]=



In a case like this, the component of b in a would normally be the projection of b onto a. Here however, the two vectors are perpendicular, so the projection (and the component), are zero. This graphic shows the arrowhead bug in Mathematica, talked about at *https://community.wolfram.com/groups/-/m/t/1302365* and *https://mathematica.stackexchange.com/questions/81306/arrowhead-becomes-unattached-to-line-in-a-graphics3d-manipulate?noredirect=1* and probably other places. In this case if the blue **tube** is not used, the arrowhead becomes detached and floats around outside the display cube.