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**ICE, CLOUD, and Land Elevation Satellite-2**

**(ICESat-2) Project**

**Algorithm Theoretical Basis Document (ATBD)**

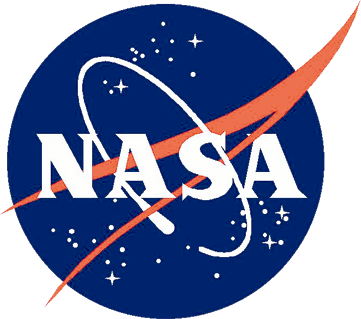
**For**

**Land-Ice Along-Track Products**

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****

**National Aeronautics and Space Administration**

Abstract

CM Foreword

This document is an Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2) Project Science Office controlled document. Changes to this document require prior approval of the Science Development Team ATBD Lead or designee. Proposed changes shall be submitted in the ICESat-II Management Information System (MIS) via a Signature Controlled Request (SCoRe), along with supportive material justifying the proposed change.

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Questions or comments concerning this document should be addressed to:

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Preface

This document is the Algorithm Theoretical Basis Document for the TBD processing to be implemented at the ICESat-2 Science Investigator-led Processing System (SIPS). The SIPS supports the ATLAS (Advance Topographic Laser Altimeter System) instrument on the ICESat-2 Spacecraft and encompasses the ATLAS Science Algorithm Software (ASAS) and the Scheduling and Data Management System (SDMS). The science algorithm software will produce Level 0 through Level 4 standard data products as well as the associated product quality assessments and metadata information.

The ICESat-2 Science Development Team, in support of the ICESat-2 Project Science Office (PSO), assumes responsibility for this document and updates it, as required, as algorithms are refined or to meet the needs of the ICESat-2 SIPS. Reviews of this document are performed when appropriate and as needed updates to this document are made. Changes to this document will be made by complete revision.

Changes to this document require prior approval of the Change Authority listed on the signature page. Proposed changes shall be submitted to the ICESat-2 PSO, along with supportive material justifying the proposed change.

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Review/Approval Page

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Change History Log

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| --- | --- | --- | --- |
| Revision  Level | Description of Change | SCoRe  No. | Date  Approved |
| 1.0 | Initial Release |  |  |

List of TBDs/TBRs

| Item No. | Location | Summary | Ind./Org. | Due Date |
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# INTRODUCTION

This document describes the theoretical basis and implementation of the level-3b land-ice processing algorithms. It describes ATL11, which provides time series of surface heights. The higher-level products, providing mapped height, and mapped height change will be described in supplements to this document available in late 2016.

ATL11 is based on the ICESat-2 ATL06 product, which is described in a separate ATBD. ATL11 provides heights corrected for displacements between the reference tracks and the location of the ATLAS measurements. It is intended as an input for the ATL15 and ATL16 products, which will provide gridded estimates of ice-sheet height and height change, and as a spatially-organized product that allows easy access to height-change information derived from ICESat-2.

Techniques similar to those used in generating this product have been used to measure short-term elevation changes using ICESat data. Some studies have subtracted the mean from a collection of height measurements from the same repeat track to leave the rapidly-changing components associated with subglacial water motion (Fricker and others, 2007) or tidal flexure (Brunt and others, 2011), which gives reliable results on ice plains and ice shelves where surface slopes are small. Subtracting the mean height and an estimate of the surface slope (Smith and others, 2009) allows recovery of height changes in areas with larger slopes, although the degree to which the surface slope estimate and the elevation-change pattern are independent is not easy to quantify.

ICESat-2, like the scanning altimeters that have flown on aircraft in Greenland and Antarctica for the last two decades, provides both surface height and surface-slope information each time it overflies its reference tracks. This makes techniques originally developed for scanning laser altimeters appropriate for use in interpreting ATLAS data. The SERAC (Surface Elevation Reconstruction and Change Detection) algorithm (Schenk & Csatho, 2012) provides an integrated framework for the derivation of elevation change from altimetry data. In this algorithm, polynomial surfaces are fit to collections of altimetry data in small (< 1 km) patches, and these surfaces are used to correct the data for short-scale surface topography. The residuals to the surface then give the pattern of elevation change, and polynomial fits to the residuals as a function of time give the long-term pattern of elevation change. In generating ATL11, we use methods very similar to SERAC, except that (1) polynomial fit correction is formulated somewhat differently, so that the ATL11 correction gives the surface height at the fit center, not the height residual, and (2) ATL11 does not include a polynomial fit with respect to time.

# BACKGROUND INFORMATION and OVERVIEW

This section provides a conceptual description of ICESat-2’s ice-sheet height measurements and gives a brief description of the derived products.

## Background

A major goal of the ICESat-2 mission is to estimate mass-balance rates for the Earth’s ice sheets. An important step in this process is the calculation of change at specific locations on the ice sheets. Previous missions, such as the ERS radar altimeters, have accomplished a similar task by estimating surface height differences as crossover points, where ascending and descending groundtracks cross. This strategy was motivated by the large footprints of the radar, because the effects of geolocation errors on the height-difference measurements are canceled in the crossover-height-difference calculation. With the first ICESat mission, height-difference measurements on repeat tracks became feasible, because ICESat’s relatively precise (50-150-m) pointing accuracy, precise (4-15 m) geolocation accuracy, and small (35-70-m) footprint made elevation-change estimates based on collections of points on repeat tracks much more accurate than those of radar altimeters. However, because ICESat had a single-beam instrument, its repeat-track measurements were reliable only for measuring the mean rate of elevation change, because shorter-term height differences could be influenced by the horizontal distribution of tracks on a sloping surface.

ICESat-2 improves on ICESat repeat-track differencing by making its measurements in pairs, so that each repeat measurement can determine the surface slope independently, and a height difference can be derived from any two repeat measurements of a reference track (**Figure 2‑1**). The spacing between the laser beams in each pair is 90 m, equal to the RMS accuracy with which ICESat-2 can be pointed at its reference tracks. This means that for most, but not all, repeat measurements of a given RPT, the pairs of beams will overlap one another. To obtain a record of elevation change from the collection of paired measurements on each RPT, some correction is still necessary to account for the effects of small-scale surface topography around the RPT in the ATL06 surface heights. ATL11 is the lowest-level land-ice product that brings together data from multiple passes over the same points, avoiding the need for users to collect the individual ATL06 files for this task. In addition, further processing of ATL06 heights will produce heights corrected for surface slope and curvature that give the estimated time-varying height for selected points on the RPTs.

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| **Figure 2‑1**. ICESat-2 repeat-track schematic |
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| Schematic drawing showing the pattern made by ATLAS’s 6-beam configuration on the ground, for a track running from lower left to upper right. The RPTs (Reference Pair Tracks, dashed lines) are defined in advance of launch; the central RPT follows the RGT (Reference Ground Track, matching the nadir track of the predicted orbit). The Ground Tracks are the tracks actually measured by ATLAS (GT1L, GT1R, etc., shown by green footprints). Measured Pair Tracks (PTs) are defined by the centers of the pairs of GTs, and deviate slightly from the RPTs because of inaccuracies in repeat-track pointing. The separation of GTs in each pair in this figure is greatly exaggerated relative to the separation of the PTs. |

## Physical Basis of Measurements

Surface slopes on the Antarctic and Greenland ice sheets are generally small, with surface slopes less than two degrees over 99% of Antarctica’s area. Smaller-scale (3-5 km) undulations, generated by ice flow over hilly or mountainous terrain may have amplitudes of up to a few degrees. Although we expect that the surface height will change over time, the surface shape is likely to remain essentially constant. This allows us to use estimates of ice-sheet surface shape derived from data spanning the full mission to correct for small (<100-m) differences in measurement locations between repeat measurements of the same RPT, to produce records of height change for specific locations. Further, we can use the surface slope estimates in ATL06 to determine whether different sets of measurements for the same fit center are self-consistent: We can assume that if an ATL06 segment shows a slope significantly different from others measured around the same fit center, it likely is in error.

## Description of the ATL 11: Land Ice H (t) product

The ATL11 product takes advantage of the pre-defined reference point geometry developed in ATL06, combining ATL06 segment heights from repeat measurements made close to a set of *fit points*, spaced every 60 m along the RPTs, to synthesize a height time series for each fit point. A polynomial surface is used to correct the segment measurements for their displacement relative to the RPT and for local (100-200 m-scale) surface slope and curvature. Misfit, reflectance, cloud, and signal strength information is retained from ATL06 to allow users to decide whether to use each repeat’s measurement in further processing. An additional group within the product provides historical laser-altimetry data, and gives crossover information from crossing ICESat-2 RPTs.

# ALGORITHM THEORY: Derivation of Land Ice H (t)/ATL11 (L3B)

In this section, we describe in detail the algorithms used in calculating the ATL11 land-ice parameters. This product is intended to provide time series of surface heights for specific land-ice and ice-shelf locations, along with parameters useful in determining whether each height estimate is valid or a result of a variety of potential errors (see ATL06 ATBD, section 1).

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| Figure ‑. ATL11 fitting schematic |
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| Schematic of the ATL11 fitting strategy. Both plots at left contain the same diagram, rendered in three dimensions (top) and as seen along the y axis (bottom). Lines show simulated ATL11 profiles; symbols show segment endpoints for segments within 50 m of the fit center (at x=y=0). Red lines and symbols indicate strong beams, blue indicate weak beams. ‘o’ markers indicate valid data segments, ‘x’ markers indicate invalid data segments. The white (transparent) surface shows the unperturbed, true surface height. The gray surface shows the fit correction surface, offset vertically to match the true surface. At right, the uncorrected heights (top) and the corrected heights (bottom) are plotted for each repeat. |

ATL11 height estimates are generated by correcting ATL06 height measurements for the combined effects of short-scale (100-200-m) surface topography around the fit centers, and small (up to 180-m) horizontal offsets between repeat measurements. We fit a reference surface to height measurements from different passes that is a polynomial in horizontal coordinates around the fit centers, and use this polynomial surface to correct height measurements to the fit center. The resulting values reflect the time history of surface heights at the reference points, with near-zero contributions from small-scale local topography.

In our version of this algorithm, for a set of fit centers spaced every 60 meters along each RPT (centered on every third segment center), all ATL06 segments with centers within 50 m along-track and 110 m across-track of the fit center are considered, so that each ATL11 fit contains as many as five distinct along-track segments from each laser beam and pass. A subset of these segments with consistent ATL06 slope estimates and small error estimates are selected, and these segments are fit with a time-variable surface height and a polynomial surface-shape model. The surface model is then used to calculate corrected heights for the segments from passes not included in the fit and from laser altimetry data from other altimetry missions. Error propagation for each of these steps gives formal errors estimates that take into account the sampling error from ATL06. Propagation of the geolocation errors with the slope of the surface-shape model gives an estimate of systematic errors in the height estimates.

Figure 3-1 shows a schematic diagram of the fitting process. In this example, height measurements are collected for six passes over a smooth ice-sheet surface (transparent grid). Between repeats 3 and 4, the surface height has risen by 2 m. Two of the segments contain errors: The weak beam for one segment from repeat 3 is displaced downward and has an abnormal apparent slope in the *x* direction, and one segment from repeat 5 is displaced upwards, so that its pair has an abnormal apparent slope in the *y* direction. Segments falling within the across and along-track windows of the fit center (at *x*=*y*=0 in this plot) are selected, and fit with a polynomial reference surface (shown in gray). When plotted as a function of repeat number, the measured heights show considerable scatter (top right-hand plot) but when corrected to the reference surface, each repeat shows a consistent height, and the segments with errors are clearly distinct from the accurate measurements.

## Elevation-correction Coordinate Systems

ATL13 calculations are carried out in the along-track coordinate system described in the level-3 ATL06 ATBD. Briefly, the along-track coordinate is measured parallel to the RGT, starting at each RGT’s origin at the equator. The across-track coordinate is measured to the left of the RGT, so that the two horizontal basis vectors and the local vertical vector form a right-handed coordinate system. Because the along-track coordinates are calculated for each photon event (PE) in ATL03, and the coordinates for the segment centers are known in ATL06, no conversion into this coordinate system is needed in ATL11 for comparisons between data from the same RGT. However, in processing ATLAS data at crossovers, location data must be expressed in the along-track coordinate system. Details of this conversion are provided in 3.7.

## Input data editing

Each ATL06 measurement includes location estimates, along- and across-track slope estimates, and PE (Photon-Event)-height misfit estimates.  We seek to calculate the reference surface using the most reliable subset of available data, so we perform tests on the surface-slope estimates and error statistics from each ATL06-pair to select a self-consistent set of data. These tests determine whether each pair of measurements is *valid* and can be used in the reference-shape calculation or *invalid*. Segments from invalid pairs may be used in elevation-change calculations, but not in the reference-shape calculation.

The parameters used to make these selections and their values are listed in Table 3‑1.

Table 3‑1 Parameter Filters to determine the validity of segments for ATL11 estimates

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| **Segment parameter** | **Filter strategy** |
| *ATL06\_quality\_summary* | *ATL06\_quality\_summary =*0 (indicates high-quality segments) |
| *sigma\_h\_li* | *sigma\_h\_li* < 3 max(0.1, median(*sigma\_h\_li*)) |
| Along-track slope | |*r\_slope\_x*|< 3 *slope\_tolerance\_x* |
| Across-track slope | |*r\_slope\_y*| < 3 *slope\_tolerance\_y* |
| Segment location | *|x\_atc-x0| < L\_search\_XT*  *|y\_atc-y0|* < *L\_search\_AT* |

### Input data editing by ATL06 parameters

We first check the *ATL06\_quality\_summary* parameter. Segments with nonzero values for this parameter are marked as invalid. We next check the magnitude of *sigma\_h\_li* for each segment. If this value is larger than three times the maximum of 0.1 m and the median *sigma\_h\_li* value for all segments for the current fit center, it is marked as invalid.

### Input data editing by slope

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On the segments that remain, we perform a linear regression between the *x* slopes of the segments and their *x*  and *y* positions, and a regression between the *y* slopes of the pairs and the pair-center *x* and *y* positions. For each pair, this yields two *x-*slope residuals and one y-slope residual. For each slope component we compute a tolerance, *slope\_tolerance*, that allows us to test for consistency between the slope values among segments:

A pair is accepted if both of its *x* residuals are smaller than 3 *slope\_tolerance\_x* and its *y* residual is smaller than 3 *slope\_tolerance\_y.* Because rejecting outlier slopes may reduce *slope\_tolerance,* these selection steps are repeated twice for the *y* slope and twice for the *x* slope, and *slope\_tolerance* is recalculated after each iteration.

### Spatial data editing

Because the assumption that ice-sheet surface can be approximated by a low-degree polynomial becomes untenable as data from larger and larger areas are included in the calculation, data from the smallest feasible area are used. This area is chosen for each along-track point to include as many valid beam pairs as possible, while minimizing the across-track spread of points included in the fit. For each point along track, the center of this window is chosen to include the maximum number of pairs. This offset is calculated by searching a range of offset values,  around the fit center to maximize the metric:

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Maximizing this metric allows the maximum number of pairs with two valid beams to be included in the fit, while, as a second priority, maximizing the number of segments included close to the center of the fit. If multiple values of have the same M value, the with the minimum magnitude is chosen.

We have chosen *Lsearch* so that, for a Gaussian distribution of across-track offsets with a standard deviation of 45 m, for 95% of all RPTs in a 12-cycle mission, at least 10 repeats will be identified as valid. Monte-Carlo calculations based on this calculation give an *Lsearch* value of 110 m. The across-track offset is then selected that allows the maximum number of pairs to be included within a 2 *Lsearch* window. The location of the height-change measurement is reported as *lat0, lon0* with corresponding local coordinates *x0, y0*.

## Reference-shape Correction

The reference shape correction is based on the pairs selected in 3.2.

Once the segments for the reference surface calculation have been selected, the reference surface and their residuals from that surface are calculated using a least-squares inversion. This inversion solves for a reference surface and a set of height-difference values relative to the first epoch in the time series. The polynomial surface shape matrix, S, describes the functional basis for the spatial part of the inversion:

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**S** has one column for each combination of *p* and *q* between zero and the degree of the surface polynomial in each dimension, but does not include a *p=q=0* term. The scaling factor, *l0,* ensures that the components of S are on the order of 1, which improves the numerical accuracy of the computation. We set *l0*=100 m, to approximately match the intra-pair beam spacing.

The height-change component of the inversion is calculated using a matrix that encodes the repeat structure of the data:

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Here 𝛿 is the delta function, equal to 1 when its arguments are equal, zero otherwise, and *i* is an index that increments by one for each distinct repeat of the satellite over the fit center.

A third matrix describes the linear rate of change in the surface slope over the course of the mission:

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Here *t0* is equal to the mid-point of the mission at the time that ATL11 is generated, halfway between start of the mission and either the end of the mission or the processing time. This implies that on average, (*t-t0*) will have a zero mean. The time-scaling factor, τ, is set equal to one year.

The surface shape, slope change, and height time series are estimated by forming a composite design matrix, **G**, where

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| **G**=[**S S**t **D**], |  |

and a covariance matrix, **C**, containing the squares of the segment-height error estimates on its diagonal. The surface-shape polynomial and the height changes are found:

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| where |  |

The notation []-p designates the Moore-Penrose pseudo inverse of the quantity in brackets, and **z** is the vector of segment heights. The parameters derived in this fit are **s**, a vector of surface-shape polynomial coefficients, **s**t, the mean rate of surface-slope change, and **z**c, a vector of corrected height values, giving the height at (*lat0, lon0*) as inferred from the height measurements and the surface polynomial. The matrix **G-g** is the generalized inverse of **G**. The values of **s** are reported in the *ref\_surface/poly\_ref\_surf* parameter, as they are calculated from (6), with no correction made for the scaling in (3). The values for the slope-change rates are reported in *ref\_surface/slope\_change\_rate*, after rescaling to units of *years-1*.

The chi-squared misfit between the data and the fit surface is calculated based on the data covariance matrix and the residual vector, *r*:

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Whether this misfit statistic indicates consistency between the polynomial surface and the data can be evaluated with a P statistic, which gives the probability that the given 2 value would be obtained from a random Gaussian distribution of data points with a covariance matrix **C**. If the probability is less than 0.025, we calculate the RDE of the scaled residuals, and eliminate any segment whose scaled residual magnitude is larger than three times that RDE, and repeat the fit with the remaining segments. After each iteration, any column of **G** that has a uniform value (i.e. has all the same value) is eliminated from the calculation, and the corresponding value of the left-hand side of equation 6 is set to zero. This fitting procedure is continued until no further segments are eliminated. If more than three complete passes that passed the initial editing steps are eliminated in this way, the surface is assumed to be too complex for a simple polynomial approximation, and the fit and its statistics are reported for the complete set of pairs that passed the initial editing steps, using a first-degree polynomial (in *x* and *y*) for the fit. In this case, the *complex\_surface\_flag* is set to 1.

## Reference-shape Correction Error Estimates

We first calculate the errors in the corrected surface heights for segments included in the reference-surface fit. We form a second covariance matrix, **C**1, whose diagonal elements are the maximum of the square of the segment errors and <r2>. We estimate the covariance matrix for the height estimates:

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The square roots of the diagonal values of **C**m give the estimated errors in the surface-polynomial and height estimates due to short-spatial-scale errors in the segment heights. If there are *Ncoeff* coefficients in the surface-shape polynomial, and *Nshape-passes* passes included in the surface-shape fit, then the first *Ncoeff* diagonal elements of **C**m give the square of the errors in the surface-shape polynomial and the last *Nshape-passes* give the errors in the surface heights for the passes included in the fit. The portion of **C**m that refers only to the surface shape and surface-shape change components is **C**m,s.

## Calculating corrected height values for repeats with no selected pairs

Once the surface polynomial has been established from the edited data set, corrected heights are calculated for the unselected passes (*i.e.* those from which all pairs were removed in the editing steps): For the segments from each of these passes, we form a new surface and slope-change design matrix, [**S**k, **S**t,k] and multiply it by [**s**, **s**t] to give the surface-shape correction:

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Here ***S*** is the surface-shape polynomial, and **S**t is the slope-change-rate estimate. This gives up to ten corrected-height values per unselected cycle. The corrected height for each such cycle is equal to the corrected height value with the smallest error estimate, as calculated in the next step.

The height errors for segments from passes not included in the surface-shape fit are calculated:

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Here 𝝈*zk* is the error in the segment height, and 𝝈*zk,c* is the error in the corrected height. The results of these calculations give a height and a height error for each unselected segment. To obtain a corrected elevation for each repeat that contains no selected pairs, we identify the segment from that repeat that has the smallest error estimate, and report the value *zkc* as that repeat’s *pass\_h\_shapecorr*, and use 𝝈*zk,c* as its error (*pulse\_h\_shapecorr\_sigma)*.

## Calculating systematic error estimates

The errors that have been calculated up to this point are due to errors in fitting segments to photon-counting data and due to inaccuracies in the polynomial fitting model. Additional error components can result from more systematic errors, such as errors in the position of ICESat-2 as derived from POD, and pointing errors from PPD. These are estimated in the ATL06 *sigma\_g\_x, sigma\_g\_y*, and *sigma\_g\_h* parameters, and their average for each repeat is reported in ATL11 using the same parameter names. The geolocation component of the total height is the product of the geolocation error and the surface slope, added in quadrature with the vertical height error:

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The error for a single segment’s corrected height is:

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This represents the total error in the surface height for a single corrected height. In most cases, error estimates for ice-sheet quantities will depend on errors from many segments from different fit centers, and the spatial scale of the different error components will need to be taken into account in error propagation models. To allow users to separate these effects, we report both the total error including both effects, *pass\_h\_shapecorr\_sigma,* and the component due only to systematic errors, *pass\_h\_shapecorr\_sigma\_systematic.*

## Calculating shape-corrected heights for non-repeat-track data

To calculate surface heights for pre-ICESat-2 data sets, and for crossing ICESat-2 orbits, we first select all data within a distance *L\_search\_XT* of the updated fit point. We then calculate the coordinates of these points in along-track and across-track coordinates, and based on these coordinates, generate **S**k and **S**kt matrices. To prevent the extrapolation of surface-shape change rates beyond the duration of the ICESat-2 mission, we limit the value of *t* in equation 5 to, at minimum, the time of the start of the ICESat-2 mission. We then follow the procedure in 3.5 and 3.6 to identify the data point with the smallest error for each data set. We report the time, error estimate, corrected height, and data source for each point, as well as the location, pair, and track number corresponding to the fit point in the *non\_repeat\_track\_data* group.

# LAND ICE PRODUCTS: Land Ice H (t)(ATL 11/L3B)

The ATL11 consists of two main parameter groups: *corrected\_h*, which gives the corrected heights for each pass, and *non\_repeat\_track\_data*, which gives altimetry measurements for data from other missions and for crossover measurements. Other groups provide a set of data-quality parameters, and ancillary data describing the fitting process.

## *Corrected\_h* group.

Table 4‑1 shows the parameters in the *corrected\_h* groups. This group gives the principal output parameters of the ATL11. A corrected repeat measurement is given by *pass\_h\_shapecorr*, which gives the corrected height for coordinates (*ref\_pt\_lat, ref\_pt\_lon*) at time *mean\_pass\_time*. These coordinates give the adjusted fit center based on the selected segments, which is the location to which the surface-shape correction corrects the height measurements. We give mean latitudes and longitudes for the valid segments in each repeat in (*mean\_pass\_lat, mean\_pass\_lon*). Two error metrics are given in *pass\_h\_shapecorr\_sigma* and *pass\_h\_shapecorr\_sigma\_systematic*. The first gives the total error including all systematic and uncorrelated errors; the second gives only that part of the error that is correlated at scales larger than one fit center; subtracting the squares of these error terms and taking the square root can recover the uncorrelated part of the error.

Table 4‑1 *corrected\_h* group

| **D** | **Units** | **Dimensions** | **Description** |
| --- | --- | --- | --- |
| *ref\_pt\_lat* | degrees North | *Npts*×*1* | center latitude based on selected segments |
| *ref\_pt\_lon* | degrees East | *Npts*×*1* | center longitude based on selected segments |
| *ref\_pt\_number* | unitless | *Npts*×*1* | The reference point number, *m*, counted from the equator crossing of the RGT. |
| *mean\_pass\_time* | seconds | *Npts*× *Npasses* | mean GPS time for the segments for each pass |
| *pass\_h\_shapecorr* | meters | *Npts*× *Npasses* | the mean corrected height |
| *pass\_h\_shapecorr\_sigma* | meters | *Npts*× *Npasses* | the formal error in the corrected height |
| *pass\_h\_shapecorr\_sigma\_systematic* | meters | *Npts*× *Npasses* | the magnitude of all errors that might be correlated at scales larger than a single fit center (e.g. pointing errors, GPS errors, etc) |
| *quality\_summary* | counts | *Npts*× *Npasses* | Summary flag: zero indicates high-quality passes: min\_signal\_selection\_source <=1 or min\_SNR\_significance < 0.02, or ATL06\_summary\_zero\_count >0. |

## *Pass\_quality\_stats group*

The *pass\_quality\_stats* group provides information based on which the quality of the underlying ATL06 data can be assessed. The included variables give a picture of the signal strength for the best segments among those that define the corrected heights in ATL11 for each pass: For example, we provide the maximum uncorrected reflectance for each pass. If the maximum uncorrected reflectance for a pass small (i.e. 0.1) for a surface we expect to be bright, we can assume that the pass was affected by clouds. Likewise, a pass with a small *min\_signal\_selection\_source* can be presumed to contain at least one high-quality segment.

|  |
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| Table 4‑2 *pass\_quality\_stats* group |
| |  |  |  |  | | --- | --- | --- | --- | | Parameter | Units | Dimensions | Description | | *ATL06\_summary\_zero\_count* | counts | *Npts*× *Npasses* | Number of segments with *ATL06\_quality\_summary*=0 (0 indicates the best-quality data) | | *min\_SNR\_significance* | counts | *Npts*× *Npasses* | Minmum of SNR\_significance (indicates the quality of the best segment in the pass) | | *mean\_uncorr\_reflectance* | counts | *Npts*× *Npasses* | mean uncorrected reflectance (indicates the approximate signal strength) | | *min\_signal\_selection\_source* | counts | *Npts*× *Npasses* | Minimum of the ATL06 signal\_selection\_source value (indicates the highest-quality segment in the pass) | | *pass\_seg\_count* | counts | *Npts*× *Npasses* | Number of segments marked as valid for each pass. Equal to 1 for those passes not included in the reference-surface shape fit that contain at least one valid segment. | | *pass\_included\_in\_fit* | counts | *Npts*× *Npasses* | One value per pass, indicating whether pairs from the pass were included in the reference-surface fit. 1=yes, 0=no. | |

## *Reference\_point* group

Table 4‑3 describes the reference\_point group. This group contains information about the location of the reference point, and the pair track reference ground track to which it corresponds.

Table 4‑3 *reference\_point* group

| **Parameter** | **Units** | **Dimensions** | **Description** |
| --- | --- | --- | --- |
| *ref\_pt\_x\_ATC* | meters | *Npts*×*1* | Along-track coordinate of the reference point, measured along the RGT from its first equator crossing |
| *ref\_pt\_y\_ATC* | meters | *Npts*×*1* | Across-track coordinate of the reference point, measured perpendicular to the RGT, positive to the left. |
| *rgt\_azimuth* | degrees | *Npts*×*1* | Reference track azimuth, in degrees east of local north |
| *pairTrack* | counts | *Npts*×*1* | pair track, numbered from left to right in along-track coordinates |
| *ReferenceGroundTrack* | counts | *Npts*×*1* | reference ground-track number |

## *Ref\_surf* group

Table 4‑4 describes the *ref\_surf* group. This group includes parameters describing the reference surface fit at each fit center. The polynomial coefficients are given in *poly\_ref\_surf*, sorted first by total degree, then by x-component degree. Because the polynomial degree is chosen separately for each reference point, enough columns are provided in the *poly\_ref\_surf* and *poly\_ref\_surf\_sigma* to accommodate all possible components up to 3rd degree in *y* and 4th degree in *x*, and absent values are filled in with zeros. The degrees for the polynomial are given in the group attributes *poly\_exponent\_x* and *poly\_exponent\_y*. The time origin for the slope change is given in the group attribute *slope\_change\_t0.*

|  |
| --- |
| Table 4‑4 *ref\_surf* group |
| |  |  |  |  | | --- | --- | --- | --- | | Parameter | Units | Dimensions | Description | | *Complex\_surface\_flag* | unitless | *Npts*×*1* | 0 indicates that normal fitting was attempted, 1 indicates that the signal selection algorithm rejected too many repeats, and only a linear fit was attempted | | *fit\_curvature* | unitless | *Npts*×*1* | the RMS of the slope of the fit polynomial within 100 m of the fit center | | *fit\_E\_slope* | unitless | *Npts*×*1* | the mean East-component slope for the reference surface within 100 m of the fit center | | *fit\_N\_slope* | unitless | *Npts*×*1* | the mean North-component slope for the reference surface within 100 m of the fit center | | *n\_deg\_x* | counts | *Npts*×*1* | Maximum degree of non-zero polynomial components in x | | *n\_deg\_y* | counts | *Npts*×*1* | Maximum degree of non-zero polynomial components in y | | *N\_pass\_avail* | counts | *Npts*×*1* | total number of passes available | | *N\_pass\_used* | counts | *Npts*×*1* | number of passes available after editing | | *poly\_ref\_surf* | unitless | *Npts*×*12* | polynomial coefficients (up to degree 4), for polynomial components scaled by 100 m | | *poly\_ref\_surf\_sigma* | unitless | *Npts*×*12* | formal errors for the polynomial coefficients | | *ref\_pt\_number* | unitless | *Npts*×*1* | Ref point number, counted from the equator crossing along the RGT. | | *Slope\_change\_rate\_x* | years-1 | *Npts*×*1* | rate of change of the x component of the surface slope | | *Slope\_change\_rate\_y* | years-1 | *Npts*×*1* | rate of change of the y component of the surface slope | | *Slope\_change\_rate\_x\_sigma* | years-1 | *Npts*×*1* | Formal error in the rate of change of the x component of the surface slope | | *Slope\_change\_rate\_y\_sigma* | years-1 | *Npts*×*1* | Formal error in the rate of change of the y component of the surface slope | | *surf\_fit\_misfit\_chi2* | meters | *Npts*×*1* | misfit chi square | | *surf\_fit\_misfit\_RMS* | meters | *Npts*×*1* | the RMS misfit for the surface-polynomial fit | | *surf\_fit\_quality\_summary* | counts | *Npts*×*1* | Indicates quality of the fit: 0: no problem identified, 1: *n\_pass\_used* < 2 or any slope component > 0.1 | |

The availability and usage of data is described in parameters *N\_pass\_avail* and *N\_pass\_used*. The first gives the number of passes with at least one valid segment. The second gives the number of passes used to define the surface after editing. These parameters are useful in determining potential quality of the correction and the data availability.

The slope of the fit surface is given in the *fit\_N\_slope* and *fit\_E\_slope* parameters. For the along-track points, the surface slope is calculated by evaluating the correction-surface polynomial for a 10-m spaced grid of points extending ±50 m in x and y around the fit center, and calculating the mean slopes of these points. The calculation is performed in along-track coordinates and then projected onto the local north and east vectors. The surface curvature is derived from the same set of points, and is calculated as the RMS of the standard deviations of the slopes calculated from adjacent grid points, in *x* and *y*.

## *pass\_stats* group

The segment\_stats group gives summary information about the segments present for each fit center. Except where noted otherwise, these quantities are weighted averages of the corresponding ATL06 values. For selected pairs (i.e. those included in the reference-surface fit), the parameters are averaged over the selected segments from each pass, using weights derived from their formal errors, *sigma\_h\_LI*.*.* The parameter weighted average for the *Nk* segments from repeat *k* is then:

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Here *qi* are the parameter values for the segments. For repeats with no selected pairs, recall that the corrected height for only one segment is reported in *pass\_h\_shapecorr*; for these, we simply report the corresponding parameter values for that selected segment.

Table 4‑5 lists the parameters in this group. Most parameters are averaged according to equation 14, but for others (e.g. *signal\_selection\_flag\_best*, which is the minimum of the signal selection flags for the pass) table 4-4 describes how the summary statistics are derived.

Table 4‑5 *pass\_stats* group

| **Parameter** | **Units** | **Dimensions** | **Description** |
| --- | --- | --- | --- |
| *h\_robust\_spread\_mean* | meters | *Npts*× *Npasses* | Weighted-average robust spread in PE residuals to segments |
| *h\_li\_rms\_mean* | meters | *Npts*× *Npasses* | Weighted-average RMS misfit between PE heights and along-track land-ice segment fit |
| *r\_eff\_mean* | unitless | *Npts*× *Npasses* | Weighted-average effective, uncorrected reflectance for each pass. |
| *tide\_ocean* | meters | *Npts*× *Npasses* | Weighted-average ocean tide for each pass |
| *cloud\_flg\_best* | counts | *Npts*× *Npasses* | Minimum cloud flag from ATL06: Flag indicates confidence that clouds with OT > 0.2 are present in the lower 3 km of the atmosphere based on ATL09 |
| *bsl\_h\_mean* | meters | *Npts*× *Npasses* | Averaged blowing snow layer height |
| *bslh\_conf\_best* | counts | *Npts*× *Npasses* | BSL\_flag from ATL06: indicates the greatest (among segments) confidence flag for presence of blowing snow |
| *y\_atc* | meters | *Npts*× *Npasses* | Unweighted mean of pair-center RGT y coordinates for each pass |
| *ref\_pt\_number* | unitless | *Npts*× *Npasses* | Ref point number, counted from the equator crossing along the RGT. |
| *strong\_beam\_number* | unitless | *Npts*× *Npasses* | The beam number of the strong laser beam in the pair |
| *mean\_pass\_lat* | degrees\_north | *Npts*× *Npasses* | Unweighted mean Latitude, WGS84, North=+, Lat of valid segment centers |
| *mean\_pass\_lon* | degrees\_east | *Npts*× *Npasses* | Unweighted mean Longitude, WGS84, East=+,Lon of valid segment centers |
| *sigma\_g\_h* | meters | *Npts*× *Npasses* | Average total vertical geolocation error due to PPD and POD |
| *sigma\_g\_x* | meters | *Npts*× *Npasses* | Average local-coordinate x horizontal geolocation error for each pass due to PPD and POD |
| *sigma\_g\_y* | meters | *Npts*× *Npasses* | Average local-coordinate y horizontal geolocation error for each pass due to PPD and POD |
| *snr\_mean* | counts | *Npts*× *Npasses* | Weighted-average signal-to-noise ratio |

## *Crossing\_track\_data* group

The *crossing\_track\_data* group contains elevation data from crossover tracks within the ICESat-2 mission. The data in this group represent the elevations and times from a set of crossing tracks, corrected using the reference surface for a set of datum tracks. Each set of values gives the data from a single segment on the crossing track, that was selected as having the minimum error among all segments on the crossing track within the 2 *L\_search\_XT* –by-2 *L\_search\_AT* window around the reference point on the datum track. The systematic errors are evaluated based on the mean slope of the reference surface.

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| Table 4‑6 crossing\_track\_data group |
| |  |  |  |  | | --- | --- | --- | --- | | Parameter | Units | Dimensions | Description | | *ref\_pt\_number* | counts | *NXO*× *1* | The reference-point number of the fit center for the datum track | | *RGT* | counts | *NXO*× *1* | The track number of the datum track | | *PT* | counts | *NXO*× *1* | The pair-track number of the datum track | | *delta\_time* | years | *NXO*× *1* | Time relative to the ICESat-2 reference epoch | | *h\_shape\_corr* | m | *NXO*× *1* | WGS-84 height, corrected for the ATL11 surface shape | | *h\_shape\_corr\_sigma* | m | *NXO*× *1* | Error in the height estimate | | *h\_shape\_corr\_sigma\_systematic* | m | *NXO*× *1* | Error in the height estimate | | *RGT\_crossing* | counts | *NXO*× *1* | The RGT number for the crossing orbit | | *PT\_crossing* | counts | *NXO*× *1* | The pair track number for the crossing orbit | |

# ALGORITHM IMPLEMENTATION

Figure 5‑1 Flow Chart for ATL11 Surface-shape Corrections

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The following steps are performed for each along-track fit-center point.

1. Segments with *x\_ATC* within *L\_search\_AT* of the initial fit-center point are selected.
2. Valid segments are identified based on estimated errors, the *ATL06\_quality\_summary* parameter, and the along- and across-track segment slopes. Valid pairs, containing measurements from two different beams, are also identified.
3. The location of the fit center point is adjusted to allow the maximum number of repeats with at least one valid pair to fall within the across-track search distance of the fit center.
4. The reference surface is fit to pairs with two valid measurements within the search distance of the adjusted center point. This calculation also produces corrected heights for the selected pairs and the errors in the correction polynomial coefficients.
5. The correction surface is used to derive corrected heights for segments not selected in steps 1-3, and the height for the segment with the smallest error is selected for each
6. The reference surface is used to calculate heights for external (pre-ICESat-2) laser altimetry data sets and crossover ICESat-2 data.

A schematic of this calculation is shown in Figure 5‑1.

### Select ATL06 data for the current center point

**Inputs:**

*x\_fc*: Along-track coordinate of the current center point

*track\_num:* The track number for current point

*pair\_num:* The pair number for the current point

**Outputs:**

*D\_ATL06:* ATL06 data structure

**Parameters:**

*L\_search\_AT*: The along-track distance to search around each center point

**Algorithm:**

For each along-track point, load all ATL06 data from track *track\_num* and pair *pair\_num* that are within *L\_search\_AT* of *x\_fc:* These segments have *x\_fc - L\_search\_AT-5 < x\_ATC < x\_fc + L\_search\_AT+5.*

### Select pairs for the reference-surface calculation

**Inputs:**

*seg\_x\_center*: Along-track coordinate of the current fit-center point

*D\_ATL06*: ATL06 data structure

**Outputs:**

*valid\_pairs:* Pairs selected for the reference-surface calculation

*valid\_segs.x\_slope:* Segments identified as valid based on x-slope consistency

*valid\_segs.data:* Segments identified as valid based on ATL06 parameter values.

*valid\_pairs.y\_slope:* Pairs identified as valid based on y-slope consistency

**Parameters:**

*L\_search\_XT*: The across-track search distance.

**Algorithm:**

1. Flag valid segments based on ATL06 values.

1a. Define *valid\_segs.data:* Set *valid\_segs.data* to 1 for segments with *ATL06\_quality\_summary* equal to 0.

1b. Define *seg\_sigma\_threshold* as the maximum of0.05 or three times the median of *sigma\_h\_li* for segments with *valid\_segs.data* equal to 1. Set *valid\_segs.data* to 1 for segments with *h\_sigma\_li* less than this threshold and *ATL06\_quality\_summary* equal to 0.

1c. Define *valid\_pairs.data:* For each pair of segments, set *valid\_pairs.data* to 1 when both segments are marked as valid in *valid\_segs.data*.

2. Calculate representative values for the *x* and *y* coordinate for each pair, and filter by distance.

2a. For each pair containing two defined values, set *pair\_data.x* to the segments’ *x\_atc* value, and *pair\_data.y* to the mean of the segments’ *y\_atc* values.

2b. Calculate *y\_polyfit\_ctr*, equal to the median of *pair\_data.y* for pairs marked valid in *valid\_pairs.data.*

2c. Set *valid\_pairs.ysearch* to 1 for pairs with |*pair\_data.y - y\_polyfit\_ctr|* < *L\_search\_XT*.

3. Select pairs based on across-track slope consistency

3a. Define *pairs\_valid\_for\_y\_fit*, for the across-track slope regressionif they are marked as valid in *valid\_pairs.data,* and *valid\_pairs.ysearch,* not otherwise.

3b. Choose the degree of the regression for across-track slope

-If the valid pairs contain at least two different *x\_atc* values set the along-track degree to 1, 0 otherwise: *my\_regression\_x\_degree*.

-If valid pairs contain at least two different *y\_atc* values, set the across-track degree to 1, 0 otherwise: *my\_regression\_y\_degree*.

3c. Calculate *my\_regression\_tol*, equal to the maximum of 0.01 or three times the median of *y\_slope\_sigma* for valid segments. *y\_slope\_sigma* is the RSS of the *h\_li\_sigma* values for the two beams in the pair divided by the difference in their *y\_atc* values.

3d. Calculate the regression of *dh\_fit\_dy* against *pair\_data.x* and *pair\_data.y* for valid pairs (*pairs\_valid\_for\_y\_fit*). The result is *y\_slope\_model*, which gives the variation of *dh\_fit\_dy* as a function of *x\_atc* and *y\_atc.* Calculate *y\_slope\_resid,* the residuals between the *dh\_fit\_dy* and *y*\_*slope\_model* for all segments for this fit-center point, *seg\_x\_center* and *y\_polyfit\_ctr*.

3e. Calculate *y\_slope\_threshold*, equal to the maximum of either three times the RDE of *y\_slope\_resid* for valid segments or *my\_regresion\_tol*. (SD. I can’t come up with the best way to say the maximum. We could say the maximum of either A or B. We could say the maximum between A and B. We could say something else entirely. ☺ )

3f. Mark all valid pairs with |*y\_slope\_resid|* > y\_*slope\_threshold* as invalid. Re-establish *pairs\_valid\_for\_y\_fit*. Return to step 3d (allow two iterations total).

(SD. necessary?) 3g. After the second repetition of 3d-f, mark all pairs with |*y\_slope\_resid|* less than y\_*slope\_threshold* with 1 in *valid\_pairs.y\_slope*, 0 otherwise.

4. Select segments based on along-track slope consistency for both in the pair

4a. Define *pairs\_valid\_for\_x\_fit*, valid segments for the along-track slope regression*:* segments are valid if they come from pairs marked as valid in *valid\_pairs.data* and *valid\_pairs.ysearch,* not otherwise.

4b. Choose the degree of the regression for along-track slope

-If valid segments contain at least two different *x\_atc* values set the along-track degree to 1, 0 otherwise: *mx\_regression\_x\_degree*.

-If valid segments contain at least two different *y\_atc* values, set the across-track degree to 1, 0 otherwise: *mx\_regression\_y\_degree*.

4c. Calculate along-track slope regression tolerance, *mx\_regression\_tol*, equal to the maximum of either 0.01 or three times the median of the *dh\_fit\_dx\_sigma* values for the valid pairs.

4d. Calculate the regression of *dh\_fit\_dx* against *pair\_data.x* and *pair\_data.y* for valid segments (*pairs\_valid\_for\_x\_fit*). The result is *x\_slope\_model*, which gives the variation of *dh\_fit\_dx* as a function of *pair\_data.x* and *pair\_data.y*. Calculate *x\_slope\_resid*, the residuals between the *dh\_fit\_dx* and *x\_slope\_resid* for all segments for this fit-center point, *seg\_x\_center* and *y\_polyfit\_ctr*.

4e. Calculate *x\_slope\_threshold*, equal to the maximum of either *mx\_regression\_tol* or three times the RDE of *x\_slope\_resid* for valid segments.

4f. Mark *valid\_segs.x\_slope* with |*x\_slope\_resid*| > *x\_slope\_threshold* as invalid. Re-establish *valid\_pairs.x\_slope* when both *valid\_segs.x\_slope* equal 1. Re-establish *pairs\_valid\_for\_x\_fit*. Return to step 4d (allow two iterations total).

(SD. necessary?) 4g. After the second repetition of 4d-f, mark all segments with |*x\_slope\_resid|* less than x\_*slope\_threshold* with 1 in *seg\_valid\_xslope*, 0 otherwise. Define *valid\_pairs.x\_slope* as 1 for pairs that contain two segments with *valid\_segs.x\_slope=1*, 0 otherwise.

5. Re-establish *valid\_pairs.all.* Set equal to 1 if *valid\_pairs.x\_slope, valid\_pairs.y\_slope,* and *valid\_pairs.data* are all valid.

5a. Identify *unselected\_cycle\_segs*, as those *D6.cycles* where *valid\_pairs.all* are False.

### Adjust the fit-point center to include the maximum number of passes

**Inputs:**

*D\_ATL06*: ATL06 structure for the current center point.

*valid\_pairs:*Pairs selected based on parameter values and along- and across-track slopes.

(SD. not used): *xc0, yc0:* Coordinates of the fit point in (x,y)

**Outputs:**

*y\_best*: Adjusted fit-point center *y.*

# needs changing: *pair\_valid\_search:* Flag marking pairs identified as valid based on the spatial search to y\_best.

**Parameters**:

*L\_search\_XT*: Across-track search length.

**Algorithm:**

1. Define *y0* as the mean of the minimum and maximum *y\_atc* for *valid\_pairs.all*, Set a range of y values, *y0\_shifts*, as round(*y0)* +/- 100 meters in 2-meter increments.

2. For each value of *y0\_shifts* (*y0\_shift*), set a counter, *selected\_seg\_cycle\_count,* to the number of distinct cycles for which both segments of the pair are contained entirely within the *y* interval [*y0\_shift- L\_search\_XT*, *y0\_shift+ L\_search\_XT*]. Add to this, the number of distinct cycles represented by unpaired segments contained within that interval, weighted by 0.01. The sum is called *score*.

3. Search for an optimal y-center value (with the most distinct cycles). Set *y\_best* to the value of *y0\_shift* that maximizes *score*. If there are multiple *y0\_shift* values with the same, maximum *score*, set *y\_best* to the median of the *y0\_shift* values with the maximum *score*.

### Calculate the reference surface and corrected heights for selected pairs

**Inputs:**

*D\_ATL06*: ATL06 structure for the current center point.

*Selected\_pairs:*Pairs selected based on parameter values and along- and across-track slope.

**Outputs:**

*m\_surf\_poly*: Surface polynomial fit to the selected segments.

*m\_surf\_poly\_sigma*: Formal error in the components of *m\_surf\_poly.*

*r\_surf\_poly:*segment residuals to *m\_surf\_poly*.

*pass\_h\_shapecorr*: corrected height for each pass

*pass\_h\_shapecorr\_sigma:* Formal error in corrected height for each pass

*G\_full\_surf:* A matrix used in fitting the reference surface to the data.

*C\_m\_surf*: Covariance matrix for the reference surface model.

*h\_poly\_seg:* Reference-surface heights for all segments

*degree\_list\_x:* The x degrees corresponding to the columns *of G\_full\_poly.*

*degree\_list\_y:* The y degrees corresponding to the columns *of G\_full\_poly.*

**Parameters:**

*poly\_max\_degree\_AT:* Maximum polynomial degree for the along-track fit

*poly\_max\_degree:* Maximum polynomial degree for the polynomial fit.

*t0:* Half the duration of the mission (equal to the time of the last-possible elevation value minus the time of the start of data collection, divided by two).

*Selected\_segments:*  A set of arrays (one per pass) listing the selected segments after all editing steps are complete.

**Algorithm**:

1. Build the pass design matrix: ***G\_full\_z0*** is a matrix that has one column for each distinct pass in *selected\_pairs* and one row for each segment whose pair is in *selected pairs*. For each segment, the corresponding row of ***G\_full\_z0*** is 1 for the pass of that segment and zero otherwise.

2. Select the polynomial degree.

The degree of the *x* polynomial, *poly\_deg\_x,* is: *min*(*poly\_max\_degree\_AT,*  number of distinct values of *x\_ATC* among the selected segments -1), and the degree of the *y* polynomial, *poly\_deg\_y,* is : *min*(*poly\_max\_degree\_XT,*  number of distinct values of *y\_ATC* among the selected segments -1)

3. Perform an iterative fit for the across-track polynomial.

3a. Define *degree\_list\_x* and *degree\_list\_y*: These array defines the *x* and *y* degree of the polynomial coefficients in the polynomial surface model. There is one component for each unique degree combination of *x* degrees between 0 and *poly\_deg\_x* and for *y* degree between 0 and *poly\_deg\_y* that has *x\_degree* *+ y\_degree* <= *max(poly\_deg\_x, poly\_deg\_y)*, except that there is no *x\_degree=0* and *y\_degree=0* combination. They are sorted first by the sum of the *x* and *y* degrees, then by *x* degree, then by *y* degree.

3b. Define the polynomial fit matrix. ***G\_fit\_poly***  has one column for each element of the polynomial degree arrays, with values equal to ((*x-x\_c*)/100)x\_degree((*y-y\_c*)/100)y\_degree. There is one row in the matrix for every segment marked as valid.

3c. If the time span is longer than 1.5 years, define slope-change matrices, ***G\_fit\_slope\_change***. The first column of the matrix gives the rate of slope change in the x component, equal to *(x-x\_c)*/100 m\*(*time-t\_c*)/(1 year). The second column gives the rate of slope change in the y component, equal to (*y-y\_c*)/100 m\*(*time-t\_c*)/(1 year).

3d. Build the fitting matrix, **G\_full**: The full fitting matrix is equal to the horizontal catenation of **G\_full\_z0**, **G\_fit\_poly**, and, if defined, **G\_fit\_slope\_change.**

3e. Subset the fitting matrix. Subset ***G\_full***by row to include only rows corresponding to selected segments to produce ***G*** (on the first iteration, all are selected). Next, subset ***G*** by column to include only columns that are not of uniform value. Identify the selected columns in matrix *fit\_columns*.

3f. Generate the data-covariance matrix, **Cd**. The data-covariance matrix is a square matrix whose diagonal elements are the squares of the *h\_li\_sigma* values for the selected segments.

3g. Calculate the polynomial fit. Initialize *m\_ref*, the reference model, to a vector of zero values, with one value for each column of ***G\_full***. Calculate the generalized inverse (equation 7) of ***G***, and multiply it by the subset of *h\_li* corresponding to the selected segments. Fill in the components of *m\_ref* flagged in *fit\_coumns* with the resulting values.

3h. Calculate model residuals all segments, *r\_seg\_all*, equal to *h\_li-****G\_full*** *\* m\_ref.* The subset of *r\_seg* corresponding to valid segments is *r\_fit.*

3i. Calculate the fitting tolerance, *r\_tol*, equal to three times the RDE of the *r\_fit/sigma\_h\_li* for all valid segments. Calculate the reduced chi-squared value for these residuals, *surf\_fit\_misfit\_chi2*, equal to *r\_fitT***Cd-1***r\_fit.* Calculate the *P* value for the misfit, equal to one minus the CDF of a chi-squared distribution with *m-n* degrees of freedom for *surf\_fit\_misfit\_chi2*, where *m* is the number of rows in **G**, and *n* is the number of columns.

3j. If the *P* value is less than 0.025 and fewer than *max\_fit\_iterations* have taken place, mark all segments for which |*r\_seg/sigma\_h\_li|* < *r\_tol* as valid, and return to 3a. Otherwise, continue.

3k. Propagate the errors. Based on the last value of **Cd**, generate a revised data-covariance matrix, **Cd’**, whose diagonals values are the maximum of *sigma\_h\_LI2* and RDE(*r\_fit)2*. Calculate the model covariance matrix, **Cm,** using equation 10. For elements of *m* marked as valid in *fit\_columns* fill in the model error estimate, *sigma\_m*, with the square roots of the corresponding diagonal elements of **Cm**.

3l. Report values. For passes that have columns in **G** (i.e. those that contain a valid pair, for which the steps 3e and 3j did not eliminate the degree of freedom) fill in the values of *h\_shapecorr* and *h\_shapecorr\_sigma* from *m* and *sigma\_m*. Fill in the values for *surf\_fit\_poly,* and *surf\_fit\_poly\_sigma*, *slope\_change\_rate\_x, slope\_change\_rate\_y, slope\_change\_rate\_x\_sigma, slope\_change\_rate-y\_sigma*.

3m. Propagate the errors in **G.** Take a subset of **G** including only columns corresponding to the polynomial fit and the rate of slope change to form **G\_full\_surf**. Take a subset of the rows and columns of **Cm** to match the columns of **G\_full\_surf**, Report this subset of **Cm** as *C\_m\_surf*.

### Calculate corrected heights for repeats with no selected pairs.

**Inputs:**

*G\_full\_surf:* A matrix used in fitting the reference surface to the data.

*C\_m\_surf*: Covariance matrix for the reference surface model.

*xc, yc*: Center point for the surface fit

*h\_poly\_seg:* Reference-surface heights for all segments

*Unselected\_pairs:* Pairs not selected for the reference-surface calculation

*Seg\_valid\_xslope:* Segments identified as valid based on x-slope consistency

*Seg\_valid\_data:* Segments identified as valid based on ATL06 parameter values.

*pair\_number:* pair number for each segment

*h\_li:* land-ice height for each segment

*h\_li\_sigma*: formal error in *h\_LI.*

*h\_shapecorr:* Partially filled-in per-pass corrected height

*h\_shapecorr\_sigma:* Partially filled-in per-pass corrected height error

*selected\_segments:* A partially filled in set of arrays listing the selected segments for each pass.

**Outputs:**

*h\_shapecorr:* Per-pass corrected height

*h\_shapecorr\_sigma:* Per-pass corrected height error

*selected\_segments:* A set of arrays listing the selected segments for each pass.

**Algorithm:**

1. Propagate the polynomial surface errors and surface-height errors the based on **G\_m\_surf**, **C\_m\_surf,** and *h\_li\_sigma* using equation 11. These errors are *sigma\_hcorr\_seg*.

2. For each pass that includes no valid pairs, identify the valid segments that pass the along-track slope test and the data-quality test, and from among these, select the one with the smallest *sigma\_hcorr\_seg.* For this pass, fill in the corresponding values of *h\_shapecorr* and *h\_shapecorr\_sigma*. For passes containing no valid segments, report invalid data*.* Provide the number of the selected segment in the *selected\_segments* array for each pass.

### Calculate corrected heights for crossover data points

**Inputs:**

*G\_full\_surf:* A matrix used in fitting the reference surface to the data.

*C\_m\_surf*: Covariance matrix for the reference surface model.

*xc, yc*: Center point for the surface fit, in along-track coordinates

*lat\_c, lon\_c:* latitude and longitude for the surface fit

*PT:* Pair track for the surface fit

*RGT:* RGT for the surface fit

*RGT\_azimuth:* The azimuth of the RGT, relative to local north

*lat\_d, lon\_d:* Location for crossover data

*time\_d:* Time for crossover data

*h\_d:* Elevations for crossover data

*sigma\_h\_d:* estimated errors for crossover data

**Outputs:**

*PT*: pair track for the surface fit

*RGT:* Reference ground track for the surface fit

*Lat\_c, Lon\_c*: location for the fit center

*x\_atc\_c:* along-track coordinate for the fit center

*time\_dc*: time for the selected altimetry point

*h\_dc*: corrected elevation for the selected altimetry points

*sigma\_h\_dc*: error in the corrected elevation for the selected altimetry points

*C\_m*: The covariance matrix for the model fit.

**Parameters:**

*L\_search\_XT*: Across-track search distance

**Algorithm (executed independently for the data from each cycle of the mission):**

1. Project data points into the along-track coordinate system:

1a: Calculate along-track and across-track vectors:

x\_hat=[cos(RGT\_azimuth), sin(RGT\_azimuth)]

y\_hat=[sin(RGT\_azimuth), -cos(RGT\_azimuth)]

1b. Calculate the R\_earth, the WGS84 radius at lat\_c.

1c: The x and y coordinates for the data points, relative to the fit-center point are found:

dx\_d=<x\_hat, R\_earth[cos(lat\_c) (lat\_d-lat\_c), (lat\_d-lat\_c)]>

dy\_d=<y\_hat, R\_earth[cos(lat\_c) (lat\_d-lat\_c), (lat\_d-lat\_c)]>

2. Calculate the fitting matrix using equation 6.

3. Calculate the errors at each point using the fitting matrix and *C\_m,* using on equation 11.

4. Select the minimum-error data point and report the values in Table 4‑6.

### Provide error-averaged values for selected ATL06 parameters

**Inputs**:

*ATL06 data structure:* ATL06 data to be averaged

*Selected\_segments:* A set of arrays listing the selected segments for each pass.

*Paramteter\_list:* A list of parameters to be averaged

**Outputs:**

*Parameter\_averages:* One value for each parameter and each pass

**Algorithm:**

1. For each pass, select the values of *h\_li\_sigma* based on the values within *selected\_segments*. Calculate a set of weights, *w\_i*, such that the sum of the weights is equal to 1 and each weight is proportional to the inverse square of *h\_li\_sigma*. If only one value is present in *selected\_segments*, *w\_1=*1.

2. For each parameter, multiply the weights for each pass by the parameter values, report the averaged value in *parameter\_averages*.

### Provide miscellaneous ATL06 parameters

**Inputs:**

*ATL06 data structure:* ATL06 data to be averaged

*Selected\_segments:* A set of arrays listing the selected segments for each pass.

**Outputs:**

Weighted-averaged parameter values, with one value per pass, filled in with NaN for passes with no selected segments

*delta\_time\_mean*

*h\_robust\_spread\_mean*

*h\_li\_rms\_mean*

*h\_eff\_mean*

*tide\_ocean\_mean*

*bsl\_h\_mean*

*sigma\_g\_h\_mean*

*sigma\_g\_x\_mean*

*sigma\_g\_y\_mean*

*y\_ATC\_mean*

Parameter minimum values, with one value per pass, filled in NaN for passes with no selected segments:

*min\_signal\_selection\_source*

*cloud\_flg\_min*

*bslh\_conf\_min*

Other parameters:

*Strong\_beam\_number:* The laser beam number for the strong beam in the pair

**Algorithm:**

Perform the following steps for each pass:

1. Select the segments for the pass indicated in *selected\_segments* from the *ATL06\_data\_structure.*

2: Based on *sigma\_h\_li,* calculate the segment weights using equation 14.

3. For parameters *delta\_time, h\_robust\_spread, h\_li\_rms, r\_eff, tide\_ocean, bsl\_h, sigma\_g\_h, sigma\_g\_x, sigma\_g\_y,*  and *SNR,* calculate the weighted average of the parameter based on the segment weights. The output parameters names are the same as the input parameter names with “*\_mean”* appended.

4. For parameters *signal\_selection\_source*, *cloud\_flg, bslh\_conf*, report the best (minimum) value from among the selected values.

5. For the *strong\_beam\_number* parameter, report the laser beam number for the strong beam in the pair.

6. For *y\_ATC\_pair*, calculate the mean of all segment values (regardless of whether the segment is used or not) for the strong beam, and the mean for the weak beam, and average these means to give the mean *y\_ATC* for the pair.

# Appendix A: Glossary

This appendix defines terms that are used in ATLAS ATBDs, as derived from a document circulated to the SDT, written by Tom Neumann. Some naming conventions are borrowed from **Spots, Channels and Redundancy Assignments** (ICESat-2-ATSYS-TN-0910) by P. Luers. Some conventions are different than those used by the ATLAS team for the purposes of making the data processing and interpretation simpler.

**Spots.** The ATLAS instrument creates six spots on the ground, three that are weak and three that are strong, where strong is defined as approximately four times brighter than weak. These designations apply to both the laser-illuminated spots and the instrument fields of view. The spots are numbered as shown in Figure 1. At times, the weak spots are leading (when the direction of travel is in the ATLAS +x direction) and at times the strong spots are leading. However, the spot number does not change based on the orientation of ATLAS. The spots are always numbered with 1L on the far left and 3R on the far right of the pattern. Not: beams, footprints.

**Laser pulse (pulse for short).** Individual pulses of light emitted from the ATLAS laser are called laser pulses. As the pulse passes through the ATLAS transmit optics, this single pulse is split into 6 individual transmit pulses by the diffractive optical element. The 6 pulses travel to the earth’s surface (assuming ATLAS is pointed to the earth’s surface). Some attributes of a laser pulse are the wavelength, pulse shape and duration. Not: transmit pulse, laser shot, laser fire.

**Laser Beam.** The sequential laser pulses emitted from the ATLAS instrument that illuminate spots on the earth’s surface are called laser beams. ATLAS generates 6 laser beams. The laser beam numbering convention follows the ATLAS instrument convention with strong beams numbered 1, 3, and 5 and weak beams numbered 2, 4, and 6 as shown in the figures. Not: beamlet.

**Transmit Pulse.** Individual pulses of light emitted from the ICESat-2 observatory are called transmit pulses. The ATLAS instrument generates 6 transmit pulses of light from a single laser pulse. The transmit pulses generate 6 spots where the laser light illuminates the surface of the earth. Some attributes of a given transmit pulse are the wavelength, the shape, and the energy. Some attributes of the 6 transmit pulses may be different. Not: laser fire, shot, laser shot, laser pulse.

**Reflected Pulse.** Individual transmit pulses reflected off the surface of the earth and viewed by the ATLAS telescope are called reflected pulses. For a given transmit pulse, there may or may not be a reflected pulse. Not: received pulse, returned pulse.

**Photon Event.** Some of the energy in a reflected pulse passes through the ATLAS receiver optics and electronics. ATLAS detects and time tags some fraction of the photons that make up the reflected pulse, as well as background photons due to sunlight or instrument noise. Any photon that is time tagged by the ATLAS instrument is called a photon event, regardless of source. Not: received photon, detected photon.

**Reference Ground Track (RGT).** The reference ground track (RGT) is the track on the earth at which a specified unit vector within the observatory is pointed. Under nominal operating conditions, there will be no data collected along the RGT, as the RGT is spanned by GT2L and GT2R (which are not shown in the figures, but are similar to the GTs that are shown). During spacecraft slews or off pointing, it is possible that ground tracks may intersect the RGT. The precise unit vector has not yet been defined. The ICESat-2 mission has 1387 RGTs, numbered from 0001xx to 1387xx. The last two digits refer to the cycle number. Not: ground tracks, paths, sub-satellite track.

**Cycle Number.** Over 91 days, each of the 1387 RGTs will be targeted in the Polar Regions once. In subsequent 91-day periods, these RGTs will be targeted again. The cycle number tracks the number of 91-day periods that have elapsed since the ICESat-2 observatory entered the science orbit. The first 91-day cycle is numbered 01; the second 91-day cycle is 02, and so on. At the end of the first 3 years of operations, we expect the cycle number to be 12. The cycle number will be carried in the mid-latitudes, though the same RGTs will (in general) not be targeted more than once.

**Sub-satellite Track (SST).** The sub-satellite track (SST) is the time-ordered series of latitude and longitude points at the geodetic nadir of the ICESat-2 observatory. In order to protect the ATLAS detectors from damage due to specular returns, and the natural variation of the position of the observatory with respect to the RGT throughout the orbit, the SST is generally not the same as the RGT. Not: reference ground track, ground track.

**Ground Tracks (GT).** As ICESat-2 orbits the earths, sequential transmit pulses illuminate six ground tracks on the surface of the earth. The track width is approximately 10m wide. Each ground track is numbered, according to the laser spot number that generates a given ground track. Ground tracks are therefore always numbered with 1L on the far left of the spot pattern and 3R on the far right of the spot pattern. Not: tracks, paths, reference ground tracks, footpaths.

**Reference Pair Track (RPT).** The reference pair track is the imaginary line halfway between the planned locations of the strong and weak ground tracks that make up a pair. There are three RPTs: RPT1 is spanned by GT1L and GT1R, RPT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and RPT3 is spanned by GT3L and GT3R. Note that this is the planned location of the midway point between GTs. We will not know this location very precisely prior to launch. Not: tracks, paths, reference ground tracks, footpaths, pair tracks.

**Pair Track (PT).** The pair track is the imaginary line half way between the actual locations of the strong and weak ground tracks that make up a pair. There are three PTs: PT1 is spanned by GT1L and GT1R, PT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and PT3 is spanned by GT3L and GT3R. Note that this is the actual location of the midway point between GTs, and will be defined by the actual location of the GTs. Not: tracks, paths, reference ground tracks, footpaths, reference pair tracks.

**Pairs.** When considered together, individual strong and weak ground tracks form a pair. For example, GT2L and GT2R form the central pair of the array. The pairs are numbered 1 through 3: Pair 1 is comprised of GT1L and GT1R, pair 2 is comprised of GT2L and GT2R, and pair 3 is comprised of GT3L and 3R.

**Along-track.** The direction of travel of the ICESat-2 observatory in the orbit frame is defined as the along-track coordinate, and is denoted as the +x direction. The positive x direction is therefore along the Earth-Centered Earth-Fixed velocity vector of the observatory. Each pair has a unique coordinate system, with the +x direction aligned with the Reference Pair Tracks.

**Across-track.** The across-track coordinate is y and is positive to the left, with the origins at the Reference Pair Tracks.

**Segment.** An along-track span (or aggregation) of PE data from a single ground track or other defined track is called a segment. A segment can be measured as a time duration (e.g. from the time of the first PE to the time of the last PE), as a distance (e.g. the distance between the location of the first and last PEs), or as an accumulation of a desired number of photons. Segments can be as short or as long as desired.

**Signal Photon.** Any photon event that an algorithm determines to be part of the reflected pulse.

**Background Photon.** Any photon event that is not classified as a signal photon is classified as a background photon. Background photons could be due to noise in the ATLAS instrument (e.g. stray light, or detector dark counts), sunlight, or mis-classified signal photons. Not: noise photon.

**h\_\*\*.** Signal photons will be used by higher-level products to determine height above the WGS-84 reference ellipsoid, using a semi-major axis (equatorial radius) of 6378137m and a flattening of 1/298.257223563. This can be abbreviated as ‘ellipsoidal height’ or ‘height above ellipsoid’. These heights are denoted by h; the subscript \*\* will refer to the specific algorithm used to determine that elevation (e.g. is = ice sheet algorithm, si = sea ice algorithm, etc…). Not: elevation.

**Photon Cloud.** The collection of all telemetered photon time tags in a given segment is the (or a) photon cloud. Not: point cloud.

**Background Count Rate.** The number of background photons in a given time span is the background count rate. Therefore a value of the background count rate requires a segment of PEs and an algorithm to distinguish signal and background photons. Not: Noise rate, background rate.

**Noise Count Rate.** The rate at which the ATLAS instrument receives photons in the absence of any light entering the ATLAS telescope or receiver optics. The noise count rate includes PEs due to detector dark counts or stray light from within the instrument. Not: noise rate, background rate, and background count rate.

**Telemetry band.** The subset of PEs selected by the science algorithm on board ATLAS to be telemetered to the ground is called the telemetry band. The width of the telemetry band is a function of the signal to noise ratio of the data (calculated by the science algorithm onboard ATLAS), the location on the earth (e.g. ocean, land, sea ice, etc…), and the roughness of the terrain, among other parameters. The widths of telemetry bands are adjustable on-orbit. The telemetry bandwidth is described in Section 7 or the ATLAS Flight Science Receiver Algorithms document. The total volume of telemetered photon events must meet the data volume constraint (currently 577 GBits/day).

**Window, Window Width, Window Duration.** A subset of the telemetry band of PEs is called a window. If the vertical extent of a window is defined in terms of distance, the window is said to have a width. If the vertical extent of a window is defined in terms of time, the window is said to have a duration. The window width is always less than or equal to the telemetry band.

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| Figure ‑. Spots and tracks, forward flight |
| ATBD_defs_figure_170413_forward.jpg |
| Spot and track naming convention with ATLAS oriented in the forward (instrument coordinate +x) direction. |

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| Figure ‑. Spots and tracks, forward flight |
| ATBD_defs_figure_170413_backward.jpg |
| Spot and track naming convention with ATLAS oriented in the backward (instrument coordinate -x) direction. |

Glossary/Acronyms

|  |  |
| --- | --- |
| ASAS | ATLAS Science Algorithm Software |
| ATBD | Algorithm Theoretical Basis Document |
| ATLAS | ATLAS Advance Topographic Laser Altimeter System |
| CDF | Cumulative Distribution Function |
| DEM | Digital Elevation Model |
| GSFC | Goddard Space Flight Center |
| GTs | Ground Tracks |
| ICESat-2 | Ice, Cloud, and Land Elevation Satellite-2 |
| IKR | I Know, Right? |
| MABEL | Multiple altimeter Beam Experimental Lidar |
| MIS | Management Information System |
| NASA | National Aeronautics and Space Administration |
| PE | Photon Event |
| POD | Precision Orbit Determination |
| PPD | Precision Pointing Determination |
| PRD | Precise Range Determination |
| PSO | ICESat-2 Project Science Office |
| PTs | Pair Tracks |
| RDE | Robust Dispersion Estimate |
| RGT | Reference Ground Track |
| RMS | Root Mean Square |
| RPTs | Reference Pair Tracks |
| RT | Real Time |
| SCoRe | Signature Controlled Request |
| SIPS | ICESat-2 Science Investigator-led Processing System |
| TLDR | Too Long, Didn’t Read |
| TBD | To Be Determined |

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