

ANN Lab 4

Hopfield Network

5.1 Convergence and attractors

Training patterns: X_1 : 0 0 1 0 1 0 0 1

X_2 : 0 0 0 0 0 1 0 0

X_3 : 0 1 1 0 0 1 0 1

Input: All combinations of 8 bit binary string

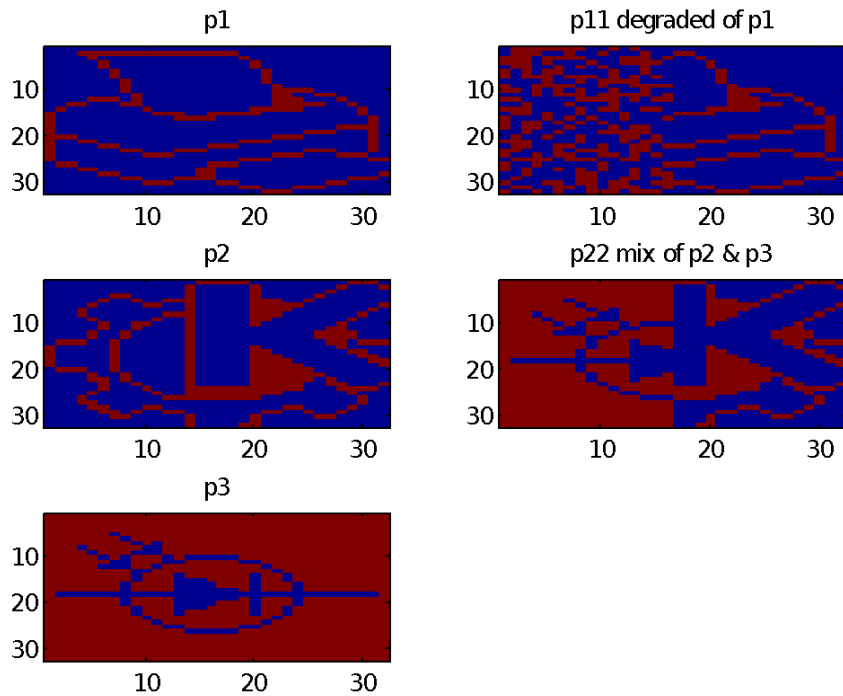
Results: Below are the attractors in this network (14 attractors)

0	0	0	0	0	1	0	0	x_2
0	0	1	0	0	1	0	1	
0	1	1	0	0	1	0	1	x_3
1	0	0	1	1	0	1	0	inverted x_3
1	1	0	1	0	1	1	0	inverted x_1
1	1	0	1	0	0	1	0	
0	0	0	0	1	0	0	0	
0	0	1	0	1	0	0	1	x_1
1	1	1	1	1	0	1	1	inverted x_2
1	0	0	1	0	1	1	0	
1	1	0	1	1	0	1	0	
0	0	1	0	0	0	0	1	
1	0	1	1	1	0	1	1	
1	1	1	1	0	1	1	1	

Max with 3 bit errors we can find attractors but with 4 bit errors (half of the bits) any attractor couldn't be found and no convergence.

Also by changing X from [x_1 ; x_2 ; x_3 ;] to [x_1 ; x_2 ; x_3 ; x_1 ; x_1] we get 8 attractors (changing the input pattern, the number of attractors are changed).

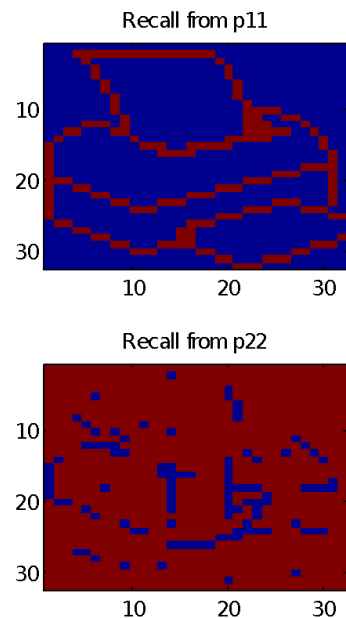
5.2 Sequential update



Synchronous update (Little network):

(In each iteration updating all units)

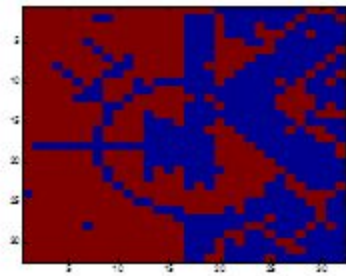
With 2 iterations p1 is recovered but in this method p22 is never completed and none of the p2 or p3 is not recovered.



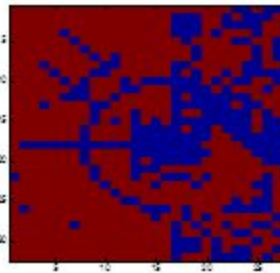
Asynchronous update: (Select one unit randomly each iteration)

This method needs more iteration to recover pictures.

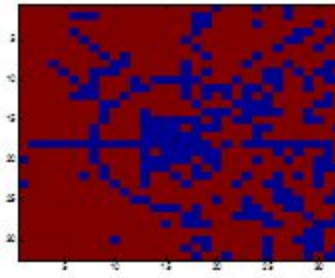
Iteration=100



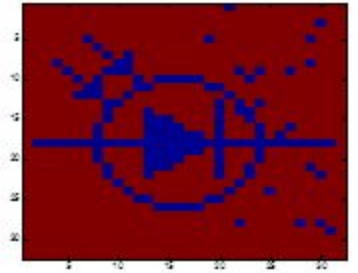
Iteration=500



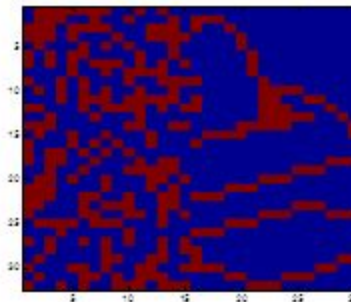
Iteration=1000



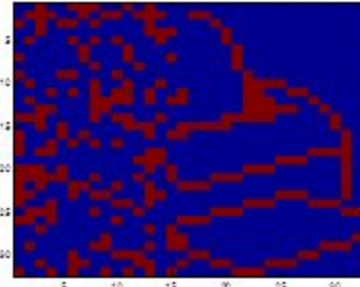
Iteration=3000



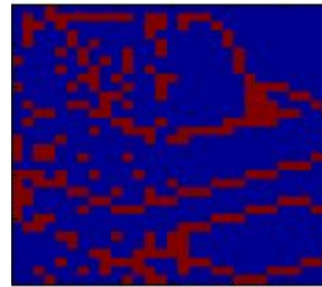
Iteration=100



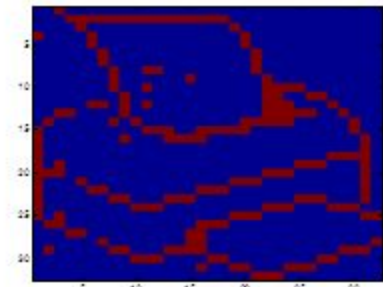
Iteration=500



Iteration=1000



Iteration=3000

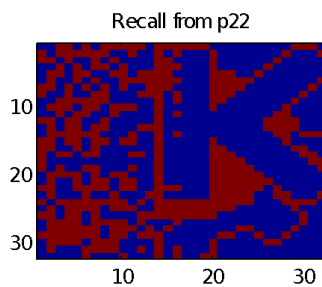
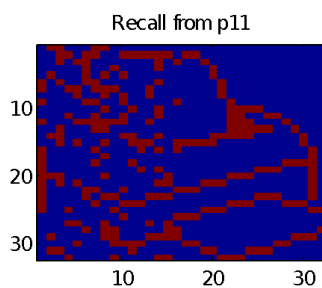


Asynchronous update (Sequential update):

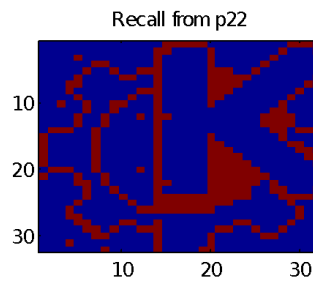
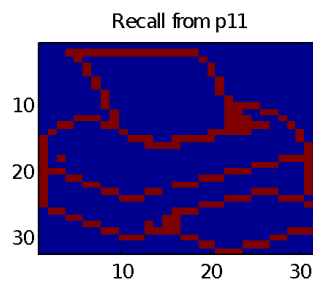
(Each iteration updating 10% randomly selected units)

With 40 iterations the p1 and p2 are recovered. This method is very fast and need less iteration for recovering the pictures.

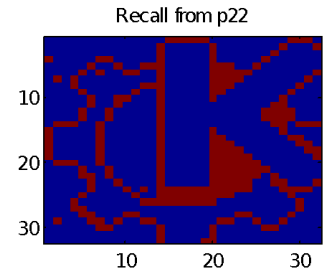
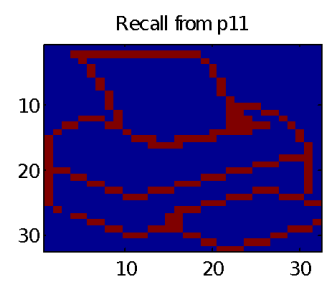
Iteration=10



Iteration=40



Iteration=50



5.3 Energy

- How do you express this calculation in Matlab? (Note: you do not need to use any loops!)

$$e = - \text{diag}(x * w * x')$$

- What is the energy at the different attractors?

Test on T5.1: e = -68 -68 -72	Test on picture: e = -1473936 (p1) -1398416 (p2) -1497344 (p3)
--	---

- What is the energy at the points of the distorted patterns?

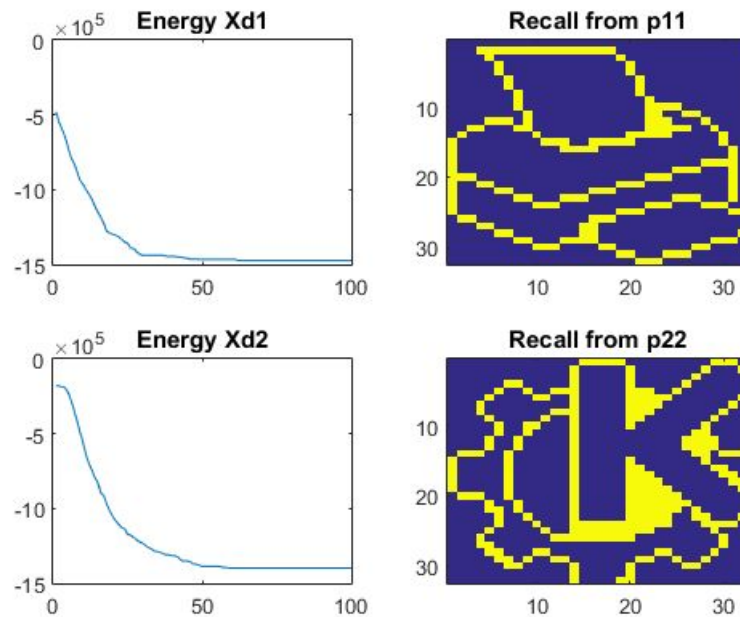
Test on T5.1: ed_ini = -40 -36 -24	Test on picture: ed_ini = -425964 (p11) -177664 (p12)
---	---

- Follow how the energy changes from iteration to iteration when you use the sequential update rule to approach an attractor.

Test on T5.1:

e_out = -40 -68 -68 -36 -56 -68 -24 -72 -72	When stuck in a not expected point: e_out = -40 -68 -68 -68 -36 -56 -68 -68 -24 -56 -68 -68
--	---

Using sequential update: randomly update 10% units(100)

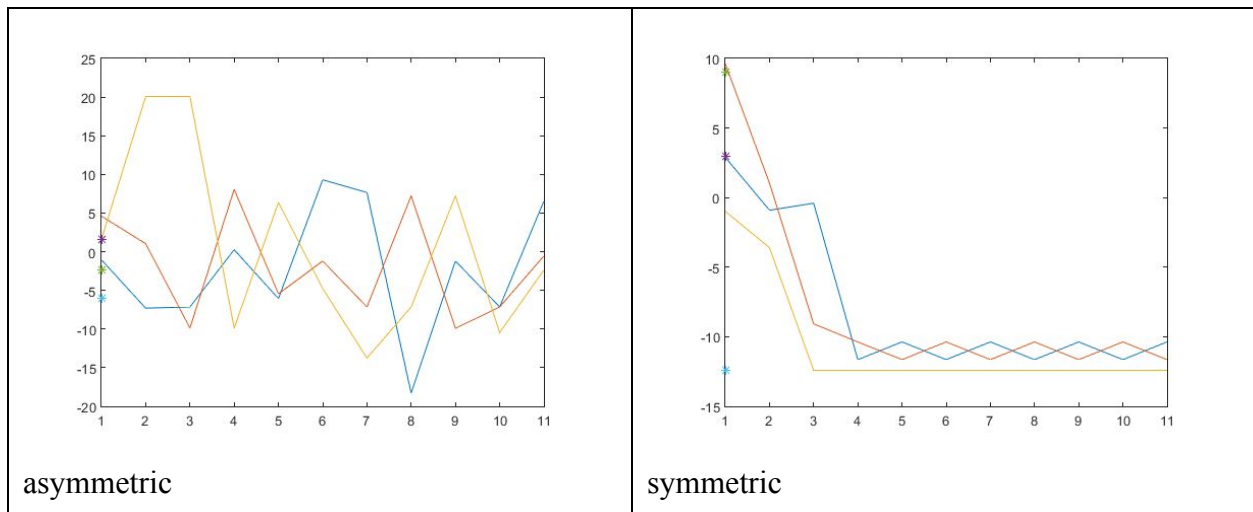


$$E1(\text{end}) = -1473936 = p1$$

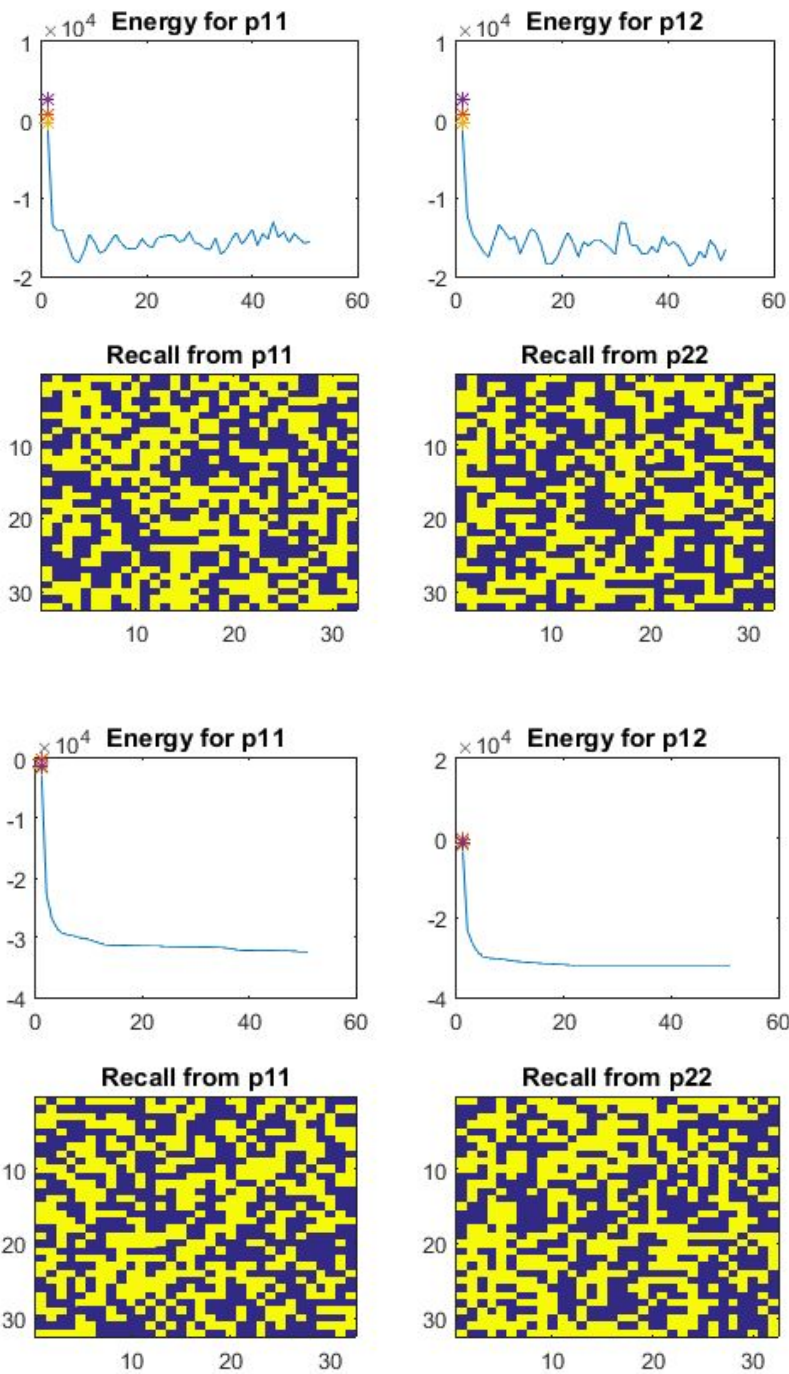
$$E2(\text{end}) = -1398416 = p2$$

- Generate a weight matrix by setting the weights to normally distributed random numbers, and try iterating an arbitrary starting state. What happens?
- Make the weight matrix symmetric (e.g. by setting $w = 0.5 \cdot (w + w')$). What happens now? Why?

Tset on T5.1

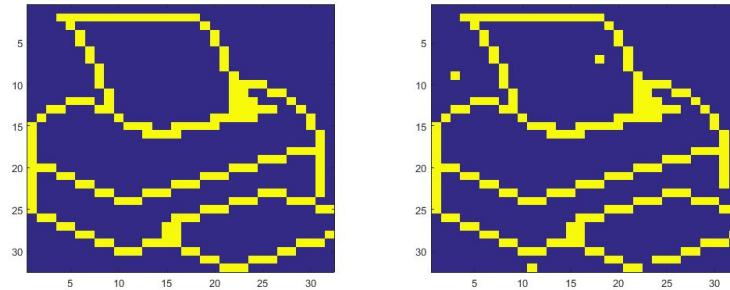


Test on picture:



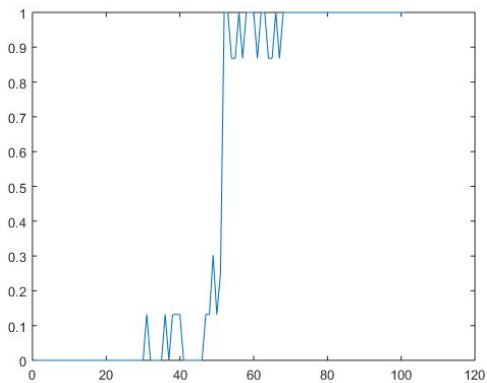
5.4 Distortion Resistance

function of flip: flip(p1,5): randomly choose 5 pixels in the picture and turn it to the opposite value, i.e. turn 1 to 0 and 0 to 1.

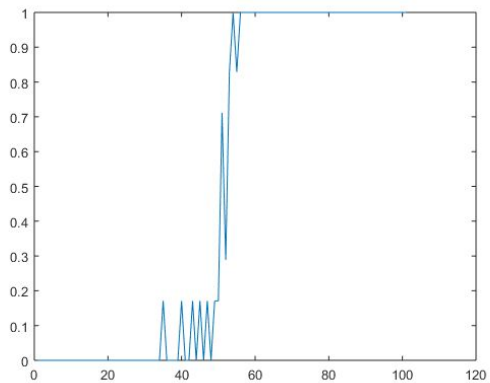


Noise test:

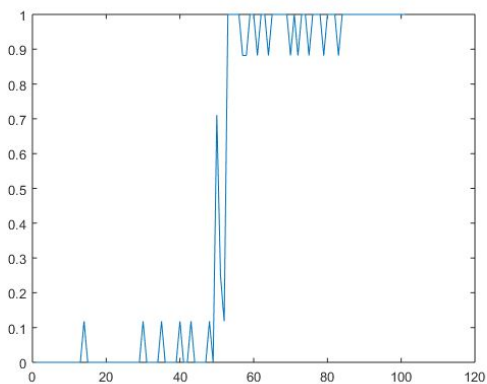
Trained with p1, p2, p3, and also noise is added to p1, p2 and p3



picture 1



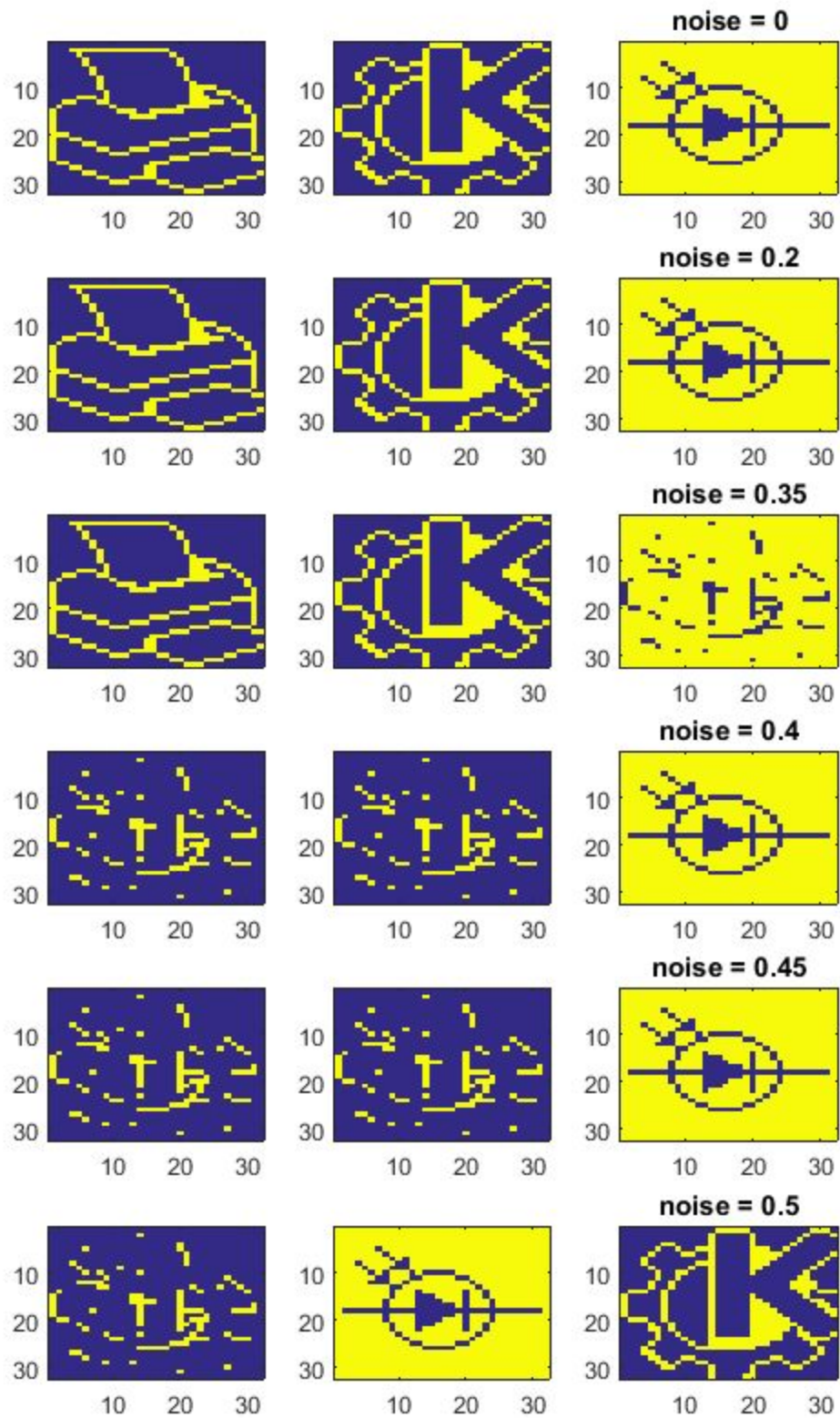
picture 2

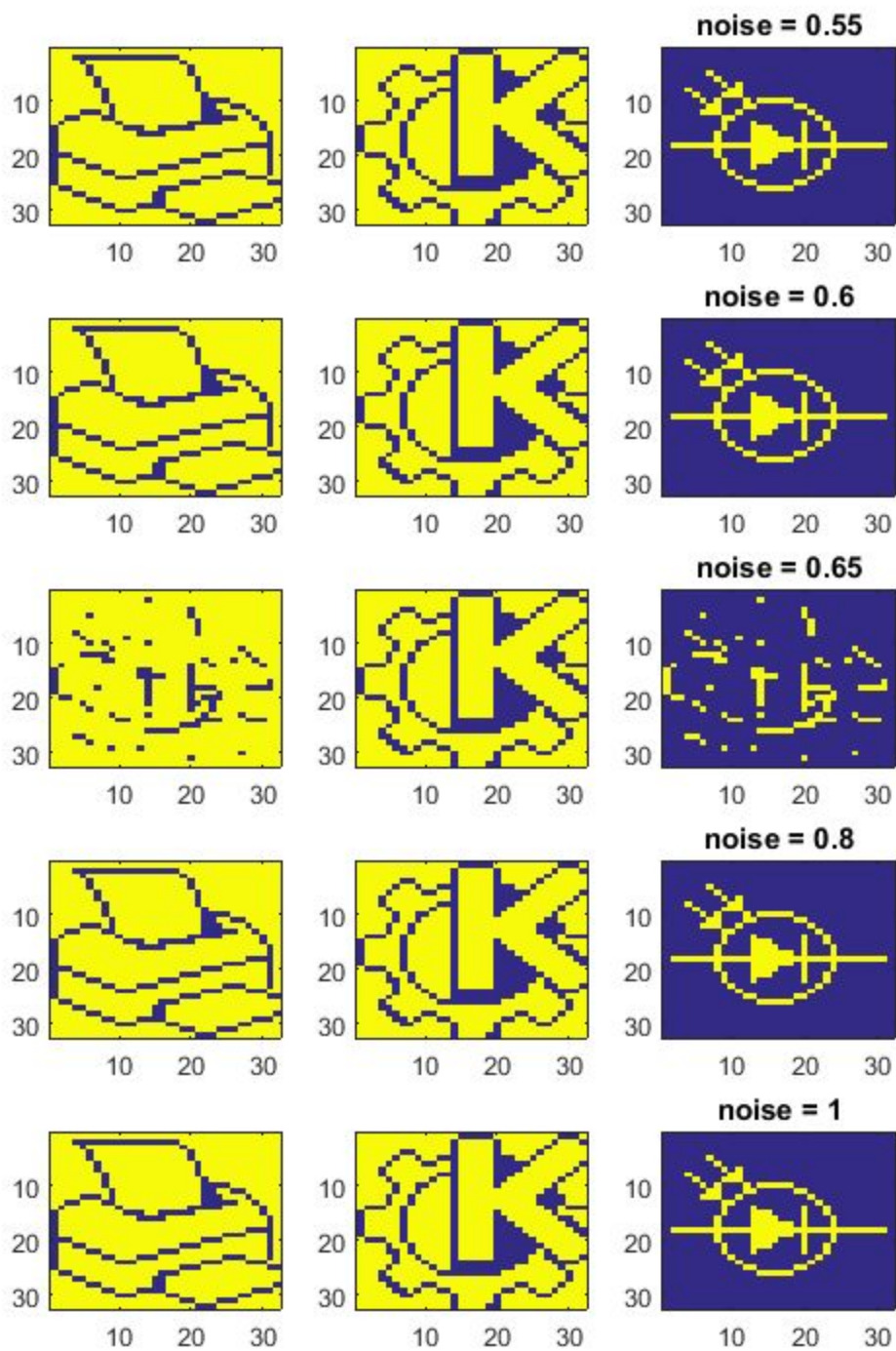


picture 3

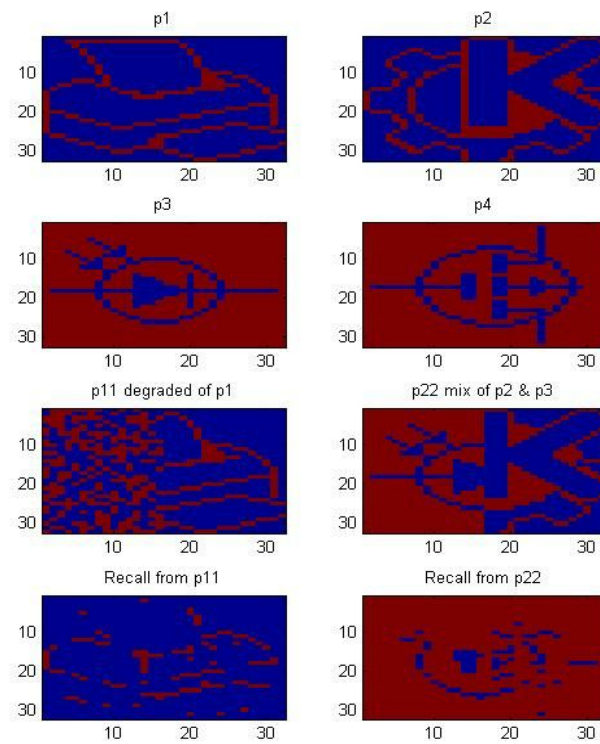
Conclusion: a good restoration when noise < 0.4 , and when noise > 0.5 , the picture tends to restore to an inverse of colour.

noise = [0, 0.2, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.8, 1];

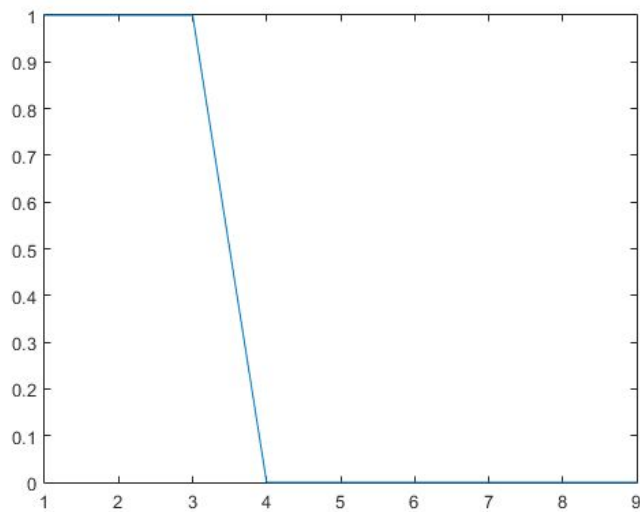


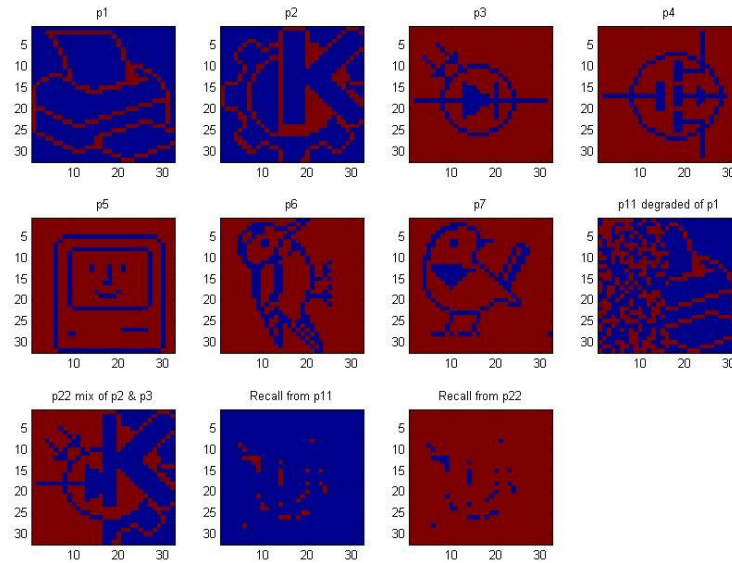


5.5 capacity



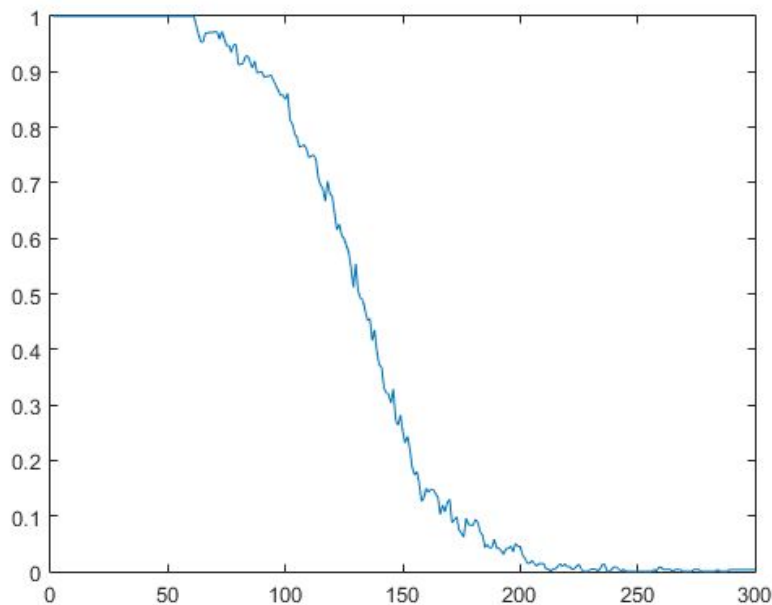
Result: by adding p4 into the weight matrix, no patterns could be safely stored after 10 iterations. The drop in performance is abrupt.





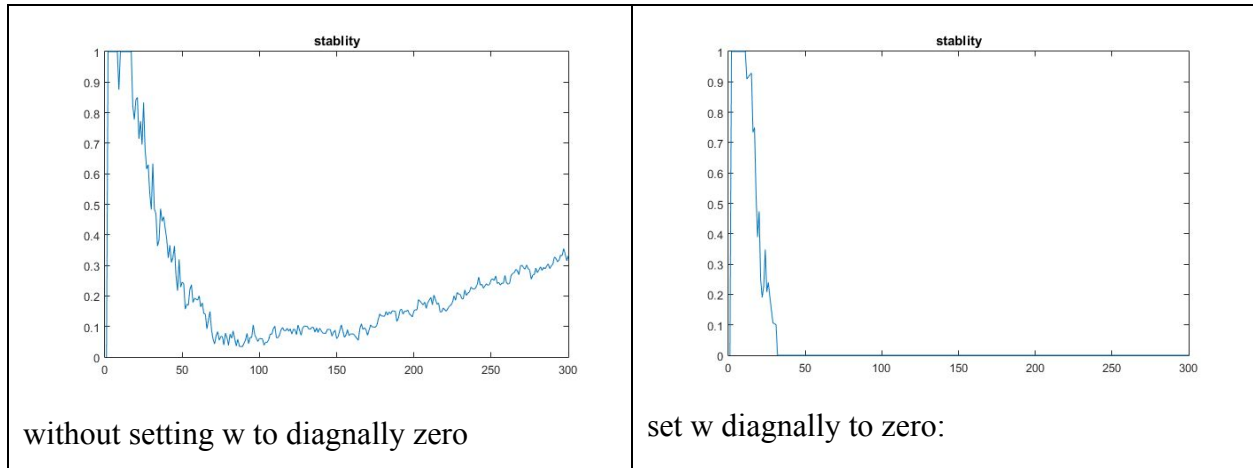
Result: by adding some random patterns the memory is partly stored, but not all.

Difference between random patterns and the pictures: random pictures have more entropy (more energy in general), hence increases the capacity.



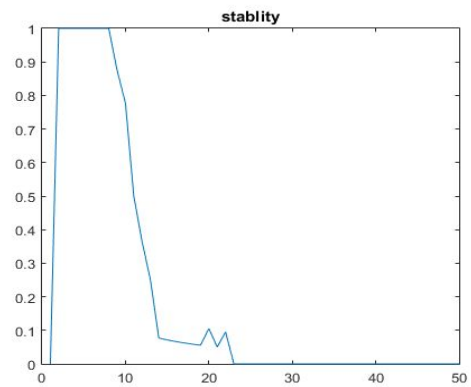
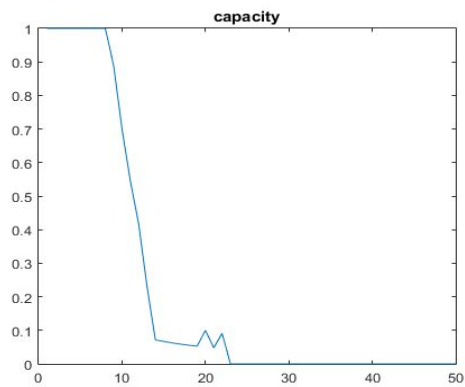
Increasing randomly created training patterns from 1 to 300, the capacity drop as the figure shows. $0.138 \times 1024 = 141.312$.

Stability:

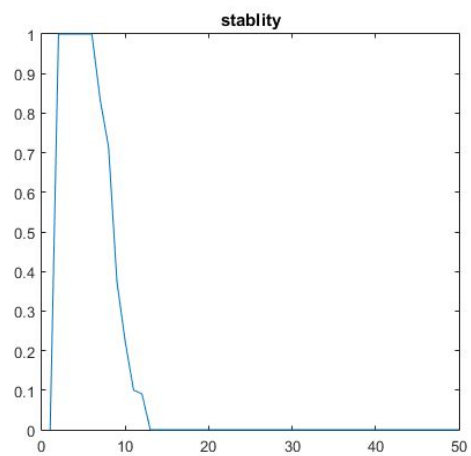
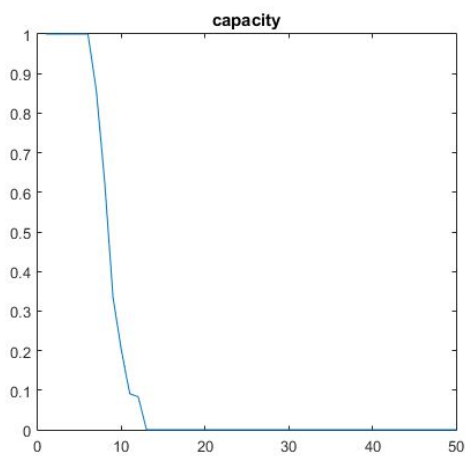


Bias:

without setting w to diagonally zero



set w diagonally to zero:



5.6 sparse patterns

Set $\rho = 0.1$ and $\theta = 0.1$, the capacity is increased to around 50.

Set $\rho = 0.05$ and $\theta = 0.1$, the capacity is increased to around 110.

