Exercises 2

1. A one-dimensional classification problem for three classes, A, B and C and pdf-s $f_A(x)$, $f_B(x)$, $f_C(x)$ is defined as:

   $p_A = 1/2, f_A(x) = 1/8$ for $x \in [-4, 4]$

   $p_B = 1/3, f_B(x) = 3(1 - x^2)/4$ for $x \in [-1, 1]$

   $p_C = 1/6, f_C(x) = x/8$ for $x \in [0, 4]$

   Estimate optimal classification boundaries and decision rules for the system. Explain the general estimation steps and draw a figure that describes the estimation process and the results.

2. For a classification problem with two classes C_A and C_B, are the a priori probabilities $p_A = 3/4$ and $p_B = 1/4$. Assume the following pdf-s

   $$p(\bar{z}|C_k) = \frac{1}{2\pi|det\Sigma_k|^{1/2}} e^{-(\bar{z}-m_k)^T\Sigma_k^{-1}(\bar{z}-m_k)/2}$$

   with

   $$m_A = m_B = 0,$$

   and

   $$\Sigma_A = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, \Sigma_B = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}.$$

   Estimate decision boudaries for the problem!

3. Describe how to estimate the equation of a line that minimizes the sum of squared orthogonal distances for a set of points. Apply it on a following set of points:

   $$p_1 = (-6, -2), p_2 = (-3, -1), p_3 = (0, 0), p_4 = (1, 1), p_5 = (3, 2)$$

4. An image has been smoothed with the following kernel:

$$h = k \cdot [1, 5, 10, 10, 5, 1]$$

Can repeated convolutions of an image with the kernel

$$g = \frac{1}{2}[1, 1]$$

be used to obtain the same result as with the first kernel? If yes, how many convolutions are needed? If no, explain the reasons why.

What should the constant k be so that the filter gain is equal to 1?

5. (a) Derive a mask that approximates the first partial derivative in the x-direction when convolved with an image.

(b) Derive a mask, d_{xxx} for generating the third order derivative using the masks $d_x = 1/2(1, 0, -1)$, and $d_{xx} = (1, -2, 1)$ corresponding to the first and second order derivatives.