Exercises 1

1. With homogeneous point and line coordinates in the plane, $x = (x_1, x_2, x_3)^T$ and $l = (l_1, l_2, l_3)^T$, the equation of a straight line can be written $l^T x = 0$. Show using the same notation that

   (a) the intersection x of two straight lines l and l' is given by $x = l \times l'$,

   (b) the line l between two points x and x' can be written as $l = x \times x'$,

   where "$\times$" denotes the cross product.

2. Projective transformations between two planar surfaces (which includes the perspective projection between a plane in the world and the image plane) can be expressed as transformations of the type

$$y = Ax$$

   where $x = (x_1, x_2, x_3)^T$ and $y = (y_1, y_2, y_3)^T$ are homogeneous coordinates in each respective plane and A is a non-singular 3×3-matrix. Determine the following geometric objects transformed using this projection:

   (a) a line $l^T x = 0$, where l is a vector of length 3,

   (b) an ellipse $x^T C x = 0$, where C is a positive definite 3×3-matrix.

3. Assume you have four points in a plane with coordinates $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(1, 1)$ respectively. Illustrate the mappings of these points, computed through:

   (i) the similarity transformation

$$\begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

   (ii) the affine transformation

$$\begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

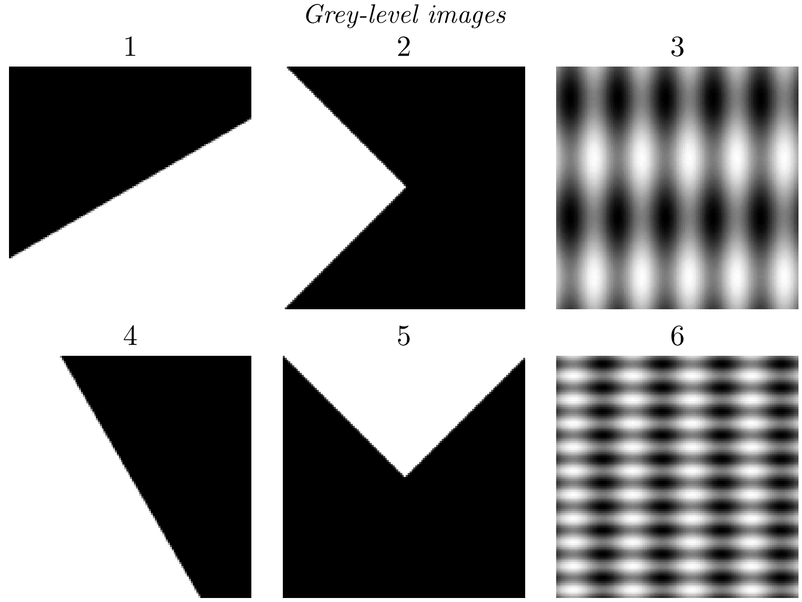
(iii) den projective transformation

$$\begin{pmatrix} 2 & 2 & 1 \\ -2 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

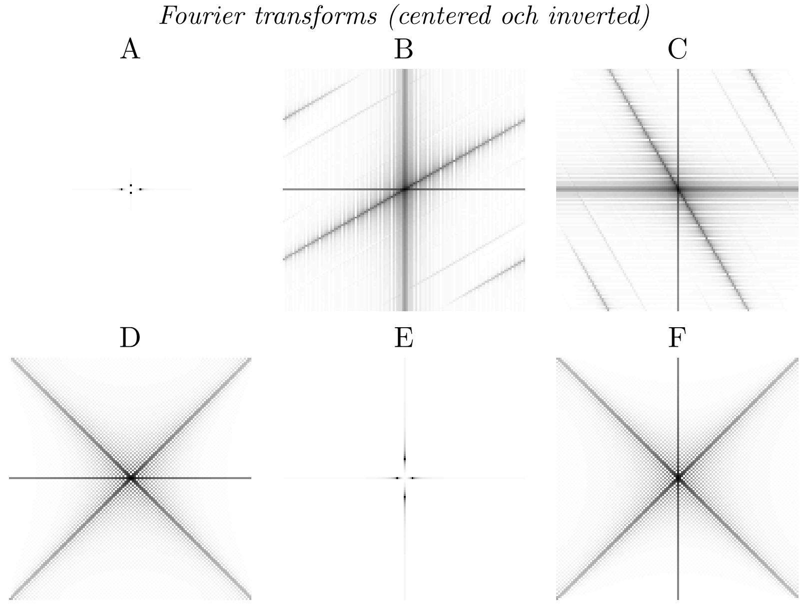
Interpret the results geometrically.

4. The figures below shows six images and their centered and inverted Fourier transforms (The dark areas in the Fourier transforms correspond to high values). Match the correct images to the correct Fourier transforms. Motivate your answers well.

Grey-level images



Fourier transforms (centered och inverted)



5. A grey-level image $f\colon \Omega \to [0, z_{max}]$ has a histogram

$$p(z) = \frac{\pi}{2z_{max}} \sin\left(\frac{\pi}{2}\frac{z}{z_{max}}\right).$$

Determine the grey-level transformation $T\colon [0, z_{max}] \to [-\frac{z_{max}}{2}, \frac{z_{max}}{2}]$ so that the grey-levels in the transformed image $g(x, y) = T(f(x, y))$ become uniformly distributed in the interval $[-\frac{z_{max}}{2}, \frac{z_{max}}{2}]$. For which $z \in [0, z_{max}]$ does this transformation result in a stretching of grey-levels?

6. An image f normalized to the interval $[0, 1]$ has a normalized distribution function $p_f(z) = 2(1 - z)$. Compute the monotonically increasing grey-level transformation $z' = T(z)$ that transforms this image into an image g with distribution function $p_g(z') = 2z'$.

7. A filter consists of the following coefficients

$$[-1, -2, \underset{\cdot}{0}, 2, 1]$$

   (a) Compute and draw the transfer function in the Fourier domain.
   (b) Determine the differential expression that this filter corresponds to.
   (c) Can you divide this filter into a collection of simpler components?