Maths Ext 2 2023 trial questions

broken into syllabus dot points

yr 12 Vectors

Questions with answers and reference

Vext2_1 BASIC ALGEBRA AND PROJECTIONS

MC QUESTIONS

Q1 WESTERN Q4

The magnitude of the vector $\underset{\sim}{u} = a\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}$ is 7.

What is the value of a ?

A. $2\sqrt{6}$

B. $6\sqrt{2}$

C. 19

D. 35

Q2 ACE YR Q3

If two vectors are $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} + 12\underset{\sim}{j} - 3\underset{\sim}{k}$, what is their scalar product ?

(A) -5

(B) 19

(C) $4\underset{\sim}{i} - 12\underset{\sim}{j} + 3\underset{\sim}{k}$

(D) $5\underset{\sim}{i} + 11\underset{\sim}{j} - 4\underset{\sim}{k}$

Q3 WESTERN Q6

The vectors $\underset{\sim}{u} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + p\underset{\sim}{j} - 3\underset{\sim}{k}$ are perpendicular.

What is the value of p ?

A. -1

B. $-\frac{3}{4}$

C. $\frac{3}{4}$

D. 1

Q4 ACE YR Q5

What is the magnitude of the vector $\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$?

(A) $\sqrt{17}$

(B) $\sqrt{35}$

(C) 17

(D) 35

Q5 SYD TECH Q1

The magnitude of the vector $\underset{\sim}{u} = a\underset{\sim}{i} - 6\underset{\sim}{j} + 8\underset{\sim}{k}$ is 12.

What is a possible value of *a*?

A. $2\sqrt{11}$

B. $11\sqrt{2}$

C. 44

D. 2

Q6 SYD TECH Q4

The vectors $\underset{\sim}{u} = 6\underset{\sim}{i} - 2\underset{\sim}{j} + p\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + p\underset{\sim}{j} + 3\underset{\sim}{k}$ are perpendicular.
What is the value of *p*?

A. -12

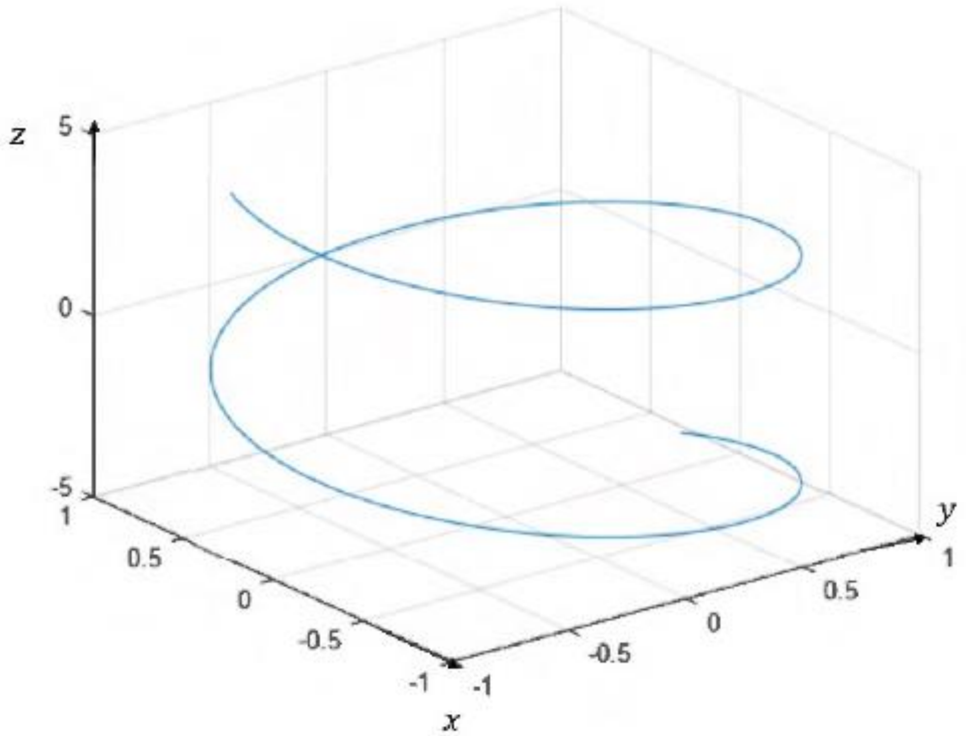
B. $-\frac{12}{5}$

C. $\frac{12}{5}$

D. 12

Q7 SYD TECH Q9

The parametric equation of the curve shown is:



A. $x = t^2, y = t, \ z = 1$

B. $x = sint, y = cos2t, \ z = t$

C. $x = sint, y = cost, \ z = 1$

D. $x = sint, y = cost, \ z = t$

Q8 Syd g q1

The points A, B and C are collinear where $\overrightarrow{OA} = \underset{\sim}{i} + \underset{\sim}{j}$, $\overrightarrow{OB} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, and $\overrightarrow{OC} = 3\underset{\sim}{i} + a\underset{\sim}{j} + b\underset{\sim}{k}$.

What are the values of a and b?

A. $a = -3,\ b = -2$

B. $a = 3,\ b = -2$

C. $a = -3,\ b = 2$

D. $a = 3,\ b = 2$

Q9 NSG Q1

Given $A(1,-2,3)$ and $B(5,0,-1)$, which of the following is a unit vector in the direction of $\overrightarrow{BA}$?

A. $-4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

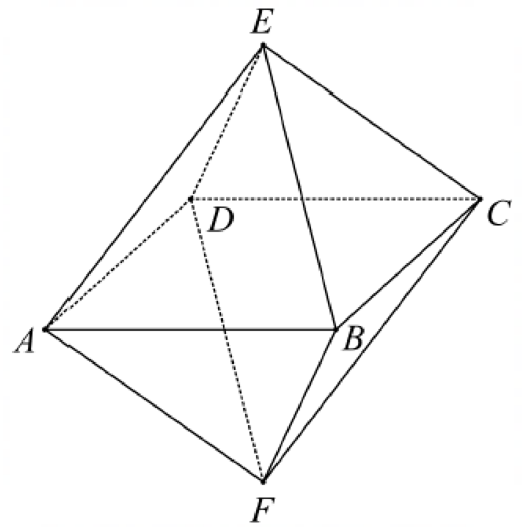
B. $4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

C. $-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

D. $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

Q10 MATHS BANK Q2

A regular octahedron is shown in the diagram below. All edges are of equal length.



Which of the following vectors is equal to $\overrightarrow{EC} + \overrightarrow{FC}$?

A. $\overrightarrow{AC}$

B. $\overrightarrow{CA}$

C. $\overrightarrow{EF}$

D. $\overrightarrow{FE}$

Q11 CSSA Q4

The amount of apples, bananas and oranges sold by a fruit seller over a year is shown in the table below.

Fruit	Amount Sold (tonnes)	Profit ($/tonne)
Apples	25	530
Bananas	55	380
Oranges	10	410

Let $\underset{\sim}{a} = \begin{pmatrix} 25 \\ 55 \\ 10 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 530 \\ 380 \\ 410 \end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Which of the following expressions calculates the average profit in dollars per tonne of fruit sold over the year?

A. $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{c}}{\underset{\sim}{b} \cdot \underset{\sim}{c}}$

B. $\dfrac{\underset{\sim}{b} \cdot \underset{\sim}{c}}{\underset{\sim}{a} \cdot \underset{\sim}{c}}$

C. $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{c}}$

D. $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{a} \cdot \underset{\sim}{c}}$

Q12 ASCHAM Q4

Two vectors are such that $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$.

What is the correct evaluation of $|\underset{\sim}{u}| \times \underset{\sim}{v} \bullet \underset{\sim}{v}$?

A. $\begin{pmatrix} \frac{2}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \end{pmatrix}$

B. $\begin{pmatrix} 8\sqrt{14} \\ 4\sqrt{14} \\ 8\sqrt{14} \end{pmatrix}$

C. $36\sqrt{14}$

D. 84

Q13 GIRRAWEEN Q4

Given that $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are non-zero vectors, the contrapositive of:

if $\underset{\sim}{p}.\underset{\sim}{q} = 0$ *then* $\underset{\sim}{p} \perp \underset{\sim}{q}$ is

(A) If $\underset{\sim}{p} \perp \underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} = 0$

(B) If $\underset{\sim}{p}$ is NOT $\perp$ $\underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} \neq 0$

(C) If $\underset{\sim}{p} \perp \underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} \neq 0$

(D) If $\underset{\sim}{p}$ is NOT $\perp$ $\underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} = 0$

Q14 GIRRAWEEN Q5

if $\underset{\sim}{p}.\underset{\sim}{q} = 0$ *then* $\underset{\sim}{p} \perp \underset{\sim}{q}$ is

(A) If $\underset{\sim}{p} \perp \underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} = 0$

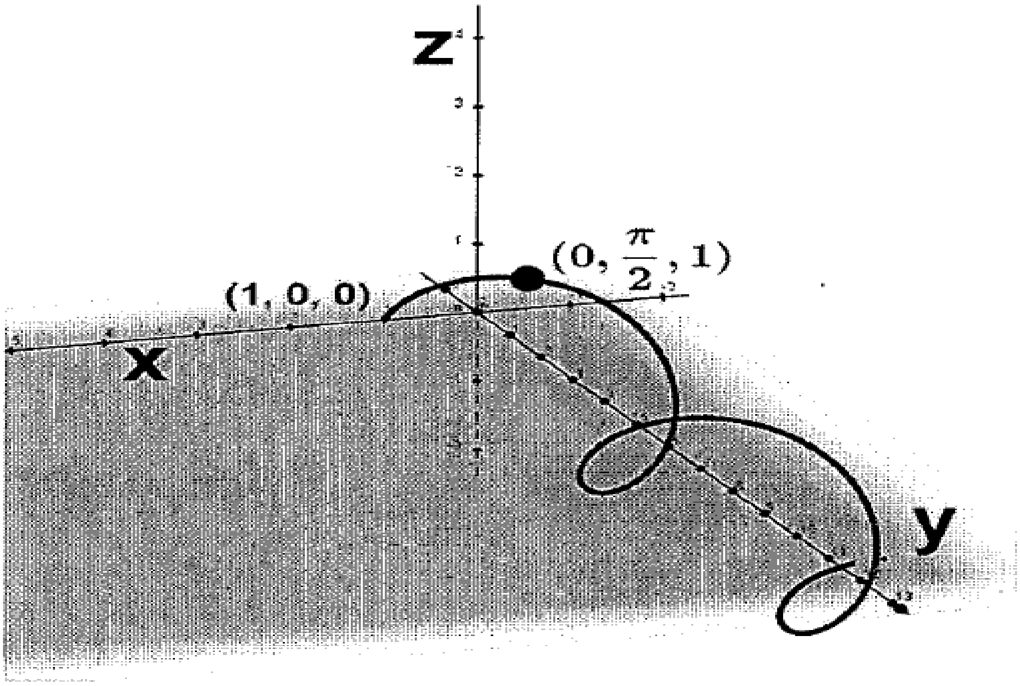
(B) If $\underset{\sim}{p}$ is NOT $\perp$ $\underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} \neq 0$

(C) If $\underset{\sim}{p} \perp \underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} \neq 0$

(D) If $\underset{\sim}{p}$ is NOT $\perp$ $\underset{\sim}{q}$ then $\underset{\sim}{p}.\underset{\sim}{q} = 0$

Q15 GIRRAWEEN Q6

A curve in three dimensional space is pictured below.



The equation for this curve is

(A) $(\cos t, \sin t, t)$ **(B)** $(\cos t, t, \sin t)$

(C) $(t, \cos t, \sin t)$ **(D)** $(\sin t, t, \cos t)$

Q16 HAH Q5

$OABC$ is a rectangle with $\overrightarrow{OA} = 3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ and $\overrightarrow{OC} = 6\boldsymbol{i} + 4\boldsymbol{j} + a\boldsymbol{k}$ for some constant a.

What is the value of a?

A. -2 B. -3

C. -5 D. -6

Q17 INDEP Q3

What is the size of the acute angle θ between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{k}$?

(A) $\theta = \frac{\pi}{6}$

(B) $\theta = \frac{\pi}{5}$

(C) $\theta = \frac{\pi}{4}$

(D) $\theta = \frac{\pi}{3}$

Q18 NSB Q3

Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} 0.6 \\ 0 \\ t \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ -5 \end{pmatrix}$ respectively.

The following two statements are made about $\mathbf{u}$ and $\mathbf{v}$:

(1) when $t = -1$, $\mathbf{u}$ and $\mathbf{v}$ are parallel

(2) when $t = -0.8$, $\mathbf{u}$ is a unit vector

Which of the following is true?

A Neither statement is correct.
B Only statement (1) is correct.
C Only statement (2) is correct.
D Both statements are correct.

Q19 SYD GRAMMAR Q2

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ satisfy $(\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = 0$, which of the following must be true?

(A) $\underset{\sim}{a} = \pm \underset{\sim}{b}$

(B) $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

(C) $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$

(D) $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$

Q20 SYD GRAMMAR Q7

What is the angle made between the line $\underset{\sim}{r} = \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, where $\lambda \in \mathbb{R}$, and the yz-plane?

You may assume $a, b, c > 0$.

(A) $\sin^{-1}\left(\dfrac{a}{\sqrt{a^2+b^2+c^2}}\right)$

(B) $\sin^{-1}\left(\dfrac{b}{\sqrt{a^2+b^2+c^2}}\right)$

(C) $\tan^{-1}\left(\dfrac{b}{\sqrt{a^2+c^2}}\right)$

(D) $\tan^{-1}\left(\dfrac{c}{\sqrt{a^2+b^2}}\right)$

Q21 ABBOTS Q6

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then $\left|\hat{a}+\hat{b}+\hat{c}\right|$ is:

A. 1

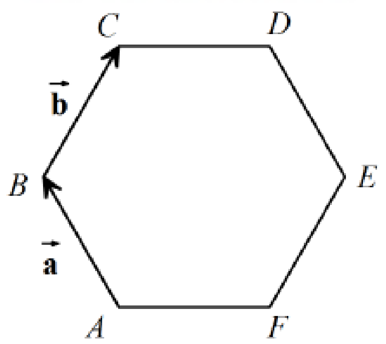
B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

Q22 ABBOTS Q8

If $\vec{a}$ and $\vec{b}$ are the vectors forming consecutive sides of a regular hexagon $ABCDEF$, as shown, then the vector representing the side CD is:



A. $\vec{a}+\vec{b}$

B. $\vec{a}-\vec{b}$

C. $\vec{b}-\vec{a}$

D. $-(\vec{a}+\vec{b})$

Q23 PEM Q4

Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} 0.6 \\ 0 \\ t \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ -5 \end{pmatrix}$ respectively.

The following two statements are made about $\mathbf{u}$ and $\mathbf{v}$:

(1) when $t=-1$, $\mathbf{u}$ and $\mathbf{v}$ are parallel

(2) when $t=-0.8$, $\mathbf{u}$ is a unit vector

Which of the following is true?

(A) Neither statement is correct.

(B) Only statement (1) is correct.

(C) Only statement (2) is correct.

(D) Both statements are correct.

Q24 PEM Q6

A vector perpendicular to the plane that contains the vectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix}$ is

(A) $6\mathbf{i}+15\mathbf{j}-9\mathbf{k}$

(B) $6\mathbf{i}+15\mathbf{j}+9\mathbf{k}$

(C) $6\mathbf{i}-15\mathbf{j}+9\mathbf{k}$

(D) $-6\mathbf{i}+15\mathbf{j}+9\mathbf{k}$

Vext2_1 BASIC ALGEBRA AND PROJECTIONS

SHORT ANSWER QUESTIONS

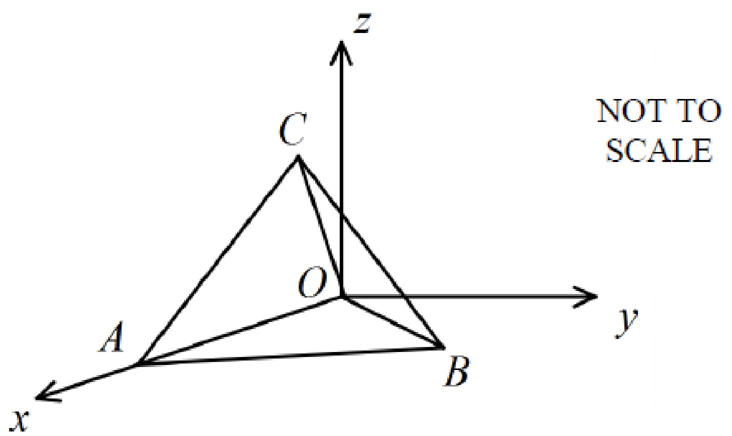
Q25 WESTERN Q13a 2MKS

Find the angle between the vectors $\vec{a} = 3i - 2j + 2k$ and $\vec{b} = 2i + 2j + 4k$, correct to the nearest minute.

Q26 ABBOTS Q16a 4MKS

The faces of the tetrahedron $OABC$ comprise equilateral triangles of side length one unit. Its' base, OAB lies on the xy-plane. Two of the vertices are O and $A(1,0,0)$. The vertex C is above the xy-plane.

Show that the coordinates of C are $\left(\frac{1}{2}, \frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{3}\right)$.



Q27 PEM Q11b 2MKS

) It is given that the point R is $(2,1,-1)$, $\overrightarrow{RS}=\begin{pmatrix}-4\\-1\\2\end{pmatrix}$ and $\overrightarrow{RT}=3\overrightarrow{RS}$.

Find the coordinates of T.

Q28 ACE YR Q15c 5MKS

Position vectors of the points A, B and C, relative to an origin O, are $-\underset{\sim}{i}-\underset{\sim}{j}$, $\underset{\sim}{j}+2\underset{\sim}{k}$, and $4\underset{\sim}{i}+\underset{\sim}{k}$ respectively.

(i) Find $\overrightarrow{AB}$. **1**

(ii) Find $|\overrightarrow{AB}|$. **1**

(iii) Prove that $\angle ABC$ is a right angle. **3**

Q29 ACE YR Q16d 6MKS

The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has the equation:

l_1: $\underset{\sim}{r}=6\underset{\sim}{i}+19\underset{\sim}{j}-\underset{\sim}{k}+\lambda(\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k})$

(i) Find the values of a and b. **2**

(ii) The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin. Find the position vector of point P. **4**

Q30 SYD TECH Q12b 3MKS

A triangle is formed in three – dimensional space with vertices A(1, -2, 3) B(2, 3, 1) and C(-1, 3, 2)

Find the size of $\angle ABC$, correct to the nearest degree. **3**

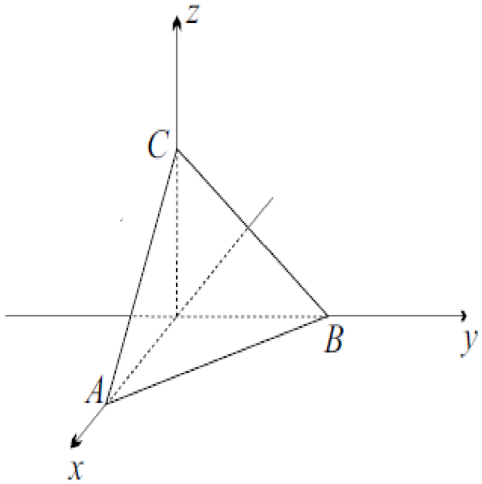
Q31 THERE IS NO Q31

Q32 ACE Q11b 2MKS

The point A has position vector $\overrightarrow{O\text{A}} = 2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k}$ relative to an origin O.
Find a unit vector parallel to $\overrightarrow{OA}$.

Q33 SYD GRAMMAR Q12a 3MKS

Let ABC be a triangle with vertices $A(1,0,0)$, $B(0,1,0)$ and $C(0,0,k)$ for some constant $k > 0$. This is shown below:



(i) Express $\overrightarrow{AC}$ and $\overrightarrow{BC}$ as column vectors. $\boxed{1}$

(ii) Find the value of k that will make $\angle ACB = 45^\circ$. Write your answer correct to 2 decimal places. $\boxed{2}$

Q34 SYD GRAMMAR Q13a 4MKS

Consider the points $A(5,0,2)$, $B(2,3,-4)$ and $C(8,-3,5)$.

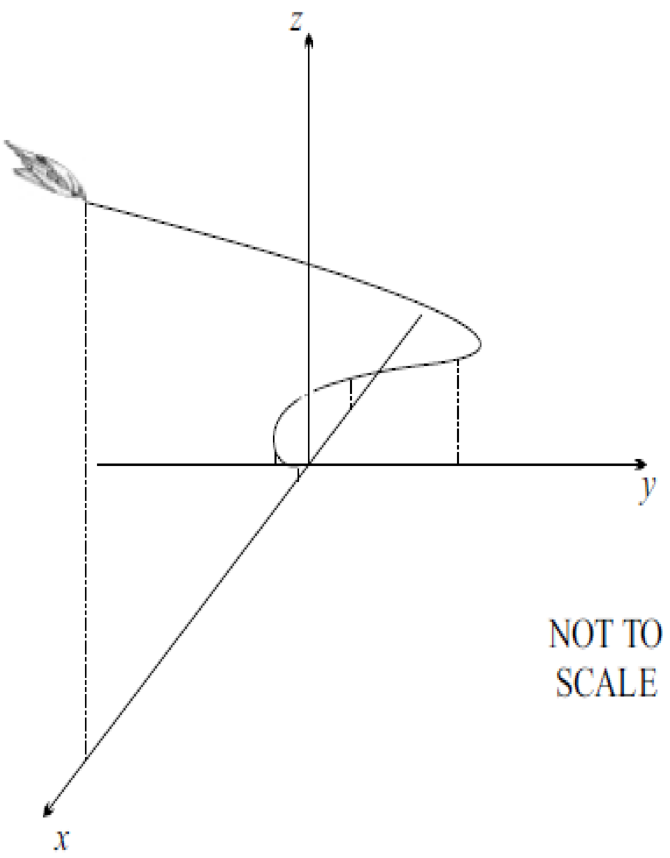
(i) Find an equation for the line AB. $\boxed{1}$

(ii) Show that the point C is not on the line AB. $\boxed{1}$

(iii) Find the point on the line AB which is closest to point C. $\boxed{2}$

Q35 SYD GRAMMAR Q14d 5MKS

A trained falcon is soaring above a crowd before gliding down to land on its trainer. In order to maintain constant eye contact without having to move its eyes or head, it glides along a path known as a logarithmic spiral, as shown below:



The position of the falcon, t seconds after beginning its descent, is given by

$$\underset{\sim}{r} = \begin{pmatrix} 20e^{-t}\cos t \\ 20e^{-t}\sin t \\ 10e^{-t} \end{pmatrix},$$

where the units of the components are in metres.

(i) Find an expression for the velocity $\underset{\sim}{v}$ of the falcon at any time t, and hence calculate its speed as it begins its descent. $\boxed{2}$

(ii) Show that the angle between $\underset{\sim}{r}$ and $\underset{\sim}{v}$ is constant. $\boxed{3}$

Q36 SYD G Q11b 3MKS

Consider the vectors $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + p\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} + 4\underset{\sim}{j} + 5\underset{\sim}{k}$.

For what values of p are $\underset{\sim}{b} + \underset{\sim}{a}$ and $\underset{\sim}{b} - \underset{\sim}{c}$ perpendicular?

Q37 SYD G Q12c 3MKS

Consider the lines $\underset{\sim}{r_1} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$ and $\underset{\sim}{r_2} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$, where $\lambda, \mu \in \mathbb{R}$.

Assuming these lines are neither parallel nor perpendicular, determine whether the lines intersect or are skew.

Q38 NSB Q15b 6MKS

b) Raymond and Jordan have model airplanes, which take off from level ground. Jordan's airplane takes off after Raymond's. Distances are measured in metres.

The position of Raymond's airplane t seconds after it takes off is given by

$$\underset{\sim}{r_1} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$$

i Find the speed of Raymond's airplane **1**

ii Find the height of Raymond's airplane after two seconds **1**

iii The position of Jordan's airplane s seconds are it takes off is given by **1**

$$\underset{\sim}{r_2} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$$

Show that the paths of the airplanes are perpendicular

iv The two airplanes collide at the point $(-23, 20, 28)$. **3**

How long after Raymond's airplane takes does Jordan's airplane take off?

Q39 INDEP Q12c 4MKS

With respect to a fixed origin O, the point P has position vector $2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}$ and the line L that passes through P has vector equation $\underset{\sim}{r}=\left(2\underset{\sim}{i}+\underset{\sim}{j}+3\underset{\sim}{k}\right)+\lambda\left(\underset{\sim}{i}-2\underset{\sim}{j}-\underset{\sim}{k}\right)$ for some scalar parameter λ. The line L is perpendicular to the plane $x-2y-z=3$.

(i) Show that the point P does not lie in the plane $x-2y-z=3$ and find the position vector of the point Q where the line L meets the plane. **3**

(ii) Hence find in simplest exact form the shortest distance from the point P to the plane $x-2y-z=3$. **1**

Q40 CSSA Q11b 4MKS

Consider the two points $A(2,2,2)$ and $B(2,-2,2)$.

(i) Find $\overrightarrow{AB}$. **1**

(ii) Find $|\overrightarrow{AB}|$. **1**

(iii) Find $\angle AOB$. Give your answer correct to the nearest degree. **2**

Q41 ASCHAM Q11a 2MKS

Find the acute angle (to the nearest degree) between the vectors

$$\underset{\sim}{u}=\begin{pmatrix}2\\-1\\3\end{pmatrix} \text{ and } \underset{\sim}{v}=\begin{pmatrix}3\\2\\5\end{pmatrix}.$$

Q42 INDEP Q13a 2MKS

Find the projection of the vector $\underset{\sim}{a}=2\underset{\sim}{i}+4\underset{\sim}{j}$ on the vector $\underset{\sim}{b}=4\underset{\sim}{i}+3\underset{\sim}{j}$.

Q43 INDEP Q13d 6MKS

With respect to a fixed origin O, the lines L_1 and L_2 have vector equations $\underset{\sim}{r}_1 = (-9+2\lambda)\underset{\sim}{i} + \lambda\underset{\sim}{j} + (10-\lambda)\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = (3+3\mu)\underset{\sim}{i} + (1-\mu)\underset{\sim}{j} + (17+5\mu)\underset{\sim}{k}$ respectively where λ and μ are scalar parameters. The point A with position vector $5\underset{\sim}{i} + 7\underset{\sim}{j} + 3\underset{\sim}{k}$ lies on L_1. The point B is the reflection of the point A in the line L_2.

(i) Find the position vector of the point of intersection P of the lines L_1 and L_2. **2**

(ii) Show that L_1 and L_2 are perpendicular to each other. **2**

(iii) Find the position vector of the point B. **2**

Q44 ASCHAM Q11c 5MKS

Relative to a fixed origin O, the point A has position vector $2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$, the point B has position vector $4\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and the point C has position vector $a\underset{\sim}{i} + 5\underset{\sim}{j} - 2\underset{\sim}{k}$, where a is a constant and $a > 0$. D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(i) Find the position vector of D. **2**

(ii) If $\left|\overrightarrow{AC}\right| = 4$, find the value of a. **3**

Q45 CSSA Q14b 4 MKS

The position of an object after t seconds is given by the vector equation

$$\underset{\sim}{r} = \cos\frac{\pi t}{4}\underset{\sim}{i} + \left(\cos\frac{\pi t}{4} + \sin\frac{\pi t}{4}\right)\underset{\sim}{j} + \sin\frac{\pi t}{4}\underset{\sim}{k}.$$

(i) What is the position of the object after 3 seconds? **1**

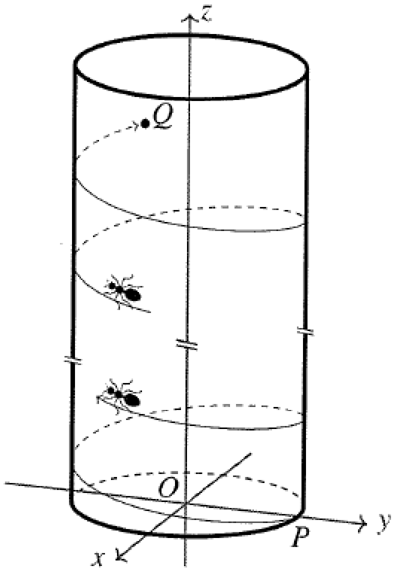
(ii) Find the vector equation of the tangent to the path taken by the object after 3 seconds. **3**

Q46 NSB Q14b 2MKS

The tangent from the point $P(1, \sqrt{6}, 3)$ to the sphere $x^2 + y^2 + z^2 = 1$ intersects the sphere at the point C. Find $|\overrightarrow{PC}|$

Q47 CSSA Q16b 7MKS

An ant follows a spiral path up a cylindrical column from $P(0,7,0)$ to the point Q as shown in the diagram below.



The ant's position $\underset{\sim}{r}$ in centimetres after t seconds is given by the vector equation below.

$$\underset{\sim}{r}(t) = 7\sin\frac{\pi t}{32}\underset{\sim}{i} + 7\cos\frac{\pi t}{32}\underset{\sim}{j} + \frac{t}{15}\underset{\sim}{k}$$

(i) Find the coordinates of the point Q if $\left|\overrightarrow{OQ}\right| = 25$. **3**

(ii) How many times has the ant crossed the line $x = 7$ on its journey to Q? **2**

(iii) The ant crawls back from Q to P along the shortest path possible. How far did it crawl on this leg of its journey? Give your answer correct to one decimal place. **2**

Vext2_2 GEOMETRY PROOFS

SHORT ANSWER QUESTIONS

Q48 PEM Q16b 5MKS

Given three vectors $\vec{a}, \vec{b},$ and $\vec{c}$, define

$$\vec{u} = \left(\vec{b}\cdot\vec{c}\right)\vec{a} - \left(\vec{c}\cdot\vec{a}\right)\vec{b}$$
$$\vec{v} = \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$$
$$\vec{w} = \left(\vec{b}\cdot\vec{a}\right)\vec{c} - \left(\vec{c}\cdot\vec{b}\right)\vec{a}$$

The vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle.

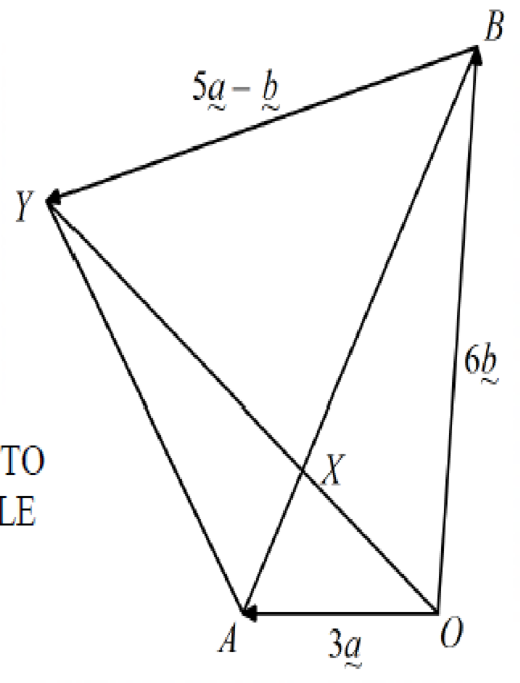
(i) Prove $\vec{u}, \vec{v}, \vec{w}$ also form a triangle. **2**

(ii) Calculate $\vec{u}\cdot\vec{c}$. **1**

(iii) Prove that the two triangles formed by $\vec{a}, \vec{b}, \vec{c}$ and $\vec{u}, \vec{v}, \vec{w}$ are similar. **2**

Q49 ABBOTS Q12c 4MKS

The diagram below shows quadrilateral $OAYB$ with $\overrightarrow{OA} = 3\underset{\sim}{a}$ and $\overrightarrow{OB} = 6\underset{\sim}{b}$.



NOT TO SCALE

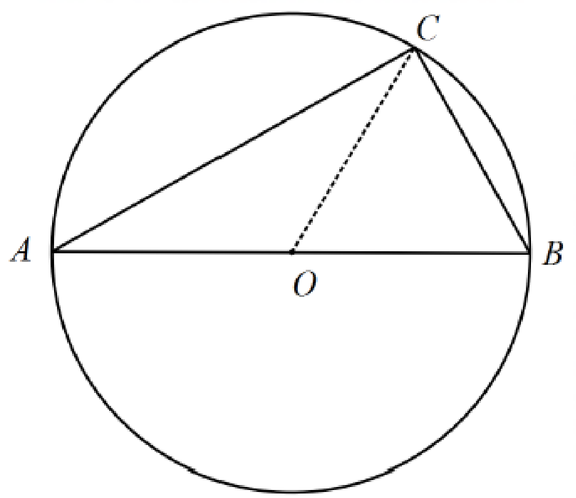
(i) Express $\overrightarrow{AB}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **1**

(ii) X is the point on AB such that $AX : XB = 1:2$ and $\overrightarrow{BY} = 5\underset{\sim}{a} - \underset{\sim}{b}$. **3**

Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$.

Q50 ABBOTS Q13c 3MKS

Using vectors, prove that the angle at the circumference of a semi-circle from the ends of the diameter is a right angle.

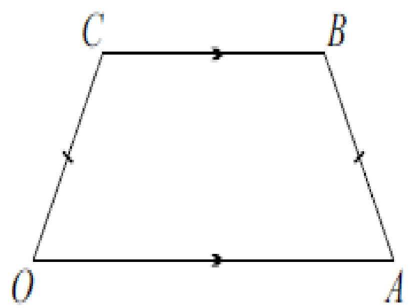


NOT TO SCALE

Q51 SYD GRAMMAR Q16c 3MKS

An isosceles trapezium $OABC$ is a quadrilateral with sides OA and BC parallel but NOT equal in length, and sides OC and AB equal in length but not parallel.

This is shown below:



Let $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{c} = \overrightarrow{OC}$.

Show, using vector methods, that the diagonals OB and AC are equal in length.

Q52 SYD B Q16b 5MKS

Given three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, define

$$\vec{u} = (\vec{b} \cdot \vec{c})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}$$
$$\vec{v} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$
$$\vec{w} = (\vec{b} \cdot \vec{a})\vec{c} - (\vec{c} \cdot \vec{b})\vec{a}$$

The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ form a triangle.

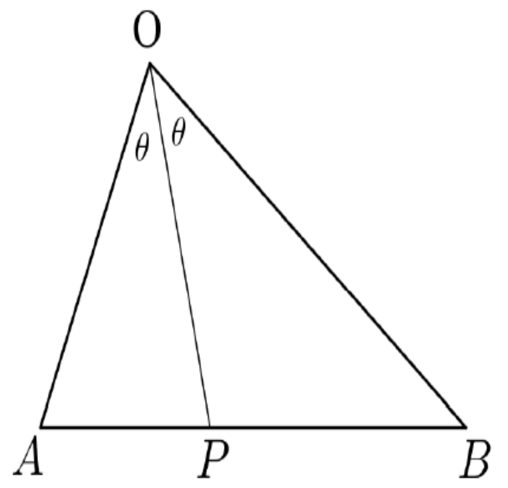
(i) Prove that $\vec{u}$, $\vec{v}$, and $\vec{w}$ also form a triangle. **2**

(ii) Calculate $\vec{u} \cdot \vec{c}$. **1**

(iii) Prove that the two triangles formed by $\vec{a}$, $\vec{b}$, & $\vec{c}$ and $\vec{u}$, $\vec{v}$, & $\vec{w}$ are similar. **2**

Q53 NSB Q14d 3MKS

d)



In ΔOAB the point $P(a, b, c)$ lies on AB such that OP bisects the angle at O and P divides AB in the ratio $m:n$

OP is extended to C such that $AO \parallel CB$.

If $O = (0,0,0), A\ (1,3,2), B(4,2,-6)$

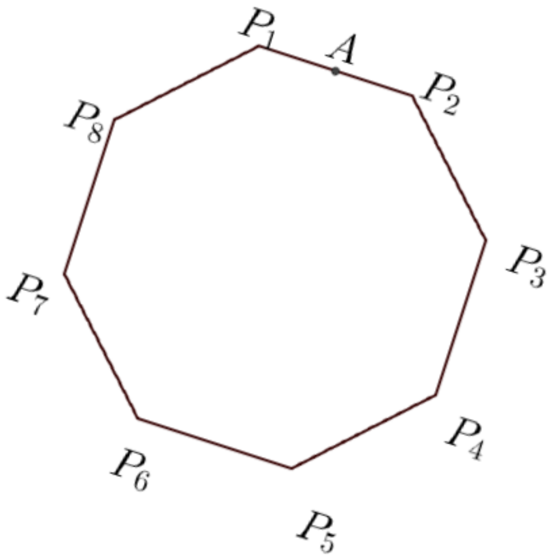
i) Find the value of $\frac{m}{n}$ **2**

ii) Hence or otherwise, find the coordinates of P **1**

Q54 NSB Q16b 5MKS

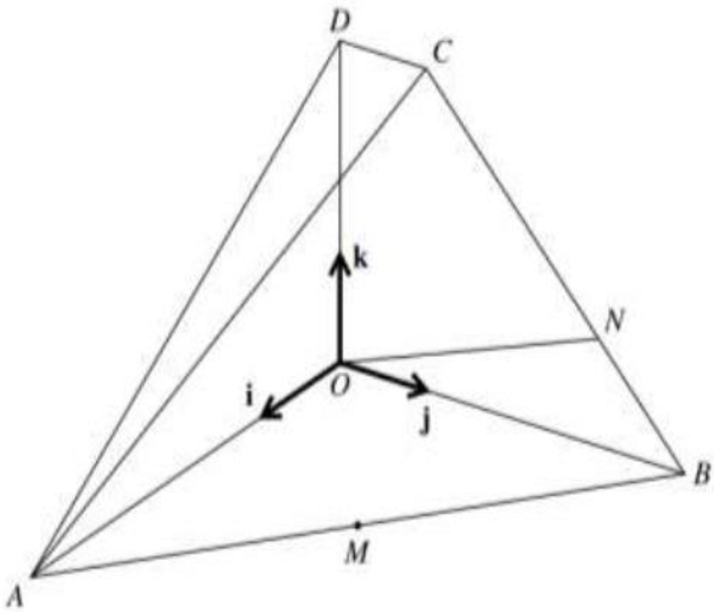
In the diagram, $P_1P_2P_3P_4P_5P_6P_7P_8$ is regular octagon with side length 4. A is the midpoint of P_1P_2.

Find the value of $|\overrightarrow{AP_1} + \overrightarrow{AP_2} + \overrightarrow{AP_3} + \overrightarrow{AP_4} + \overrightarrow{AP_5} + \overrightarrow{AP_6} + \overrightarrow{AP_7} + \overrightarrow{AP_8}|$



Q55 JAMES R Q11e 7MKS

In the diagram below, $OABCD$ is a solid figure where $|\overrightarrow{OA}| = |\overrightarrow{OB}| = 4$ units and $|\overrightarrow{OD}| = 3$ units. The edge $\overrightarrow{OD}$ is vertical, $\overrightarrow{DC}$ is parallel to $\overrightarrow{OB}$ and $|\overrightarrow{DC}| = 1$ unit. The base, OAB, is horizontal and $\angle AOB = 90°$. Unit vectors $\underset{\sim}{i}, \underset{\sim}{j}$ and $\underset{\sim}{k}$ are parallel to $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OD}$ respectively. The midpoint of $\overrightarrow{AB}$ is M and the point N on $\overrightarrow{BC}$ is such that $\overrightarrow{NC} = 2\overrightarrow{BN}$.



(i) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of $\underset{\sim}{i}, \underset{\sim}{j}$ and $\underset{\sim}{k}$. **2**

(ii) Calculate the angle between $\overrightarrow{MD}$ and $\overrightarrow{ON}$. **2**

(iii) Using vector methods, show that the length of the perpendicular from M to $\overrightarrow{ON}$ is $\sqrt{\frac{22}{5}}$ units. **3**

Q56 JAMES R Q13d 6MKS

Consider the line ℓ_1 joining $\begin{pmatrix}-3\\-1\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\0\\3\end{pmatrix}$.

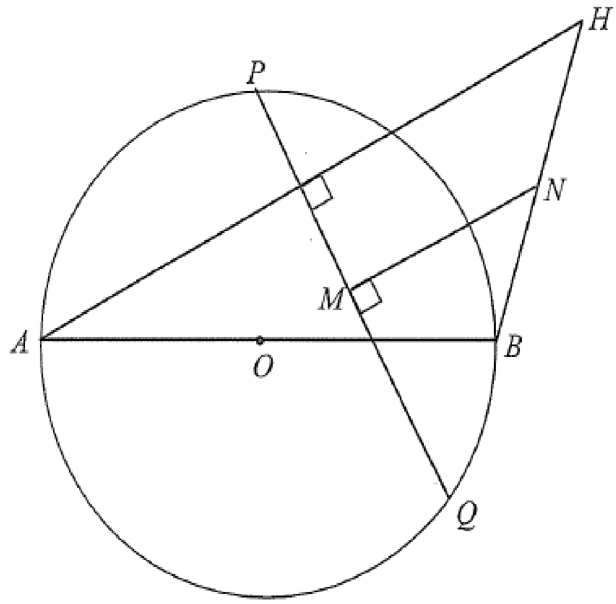
(i) Determine the vector equation of ℓ_1. **2**

Another line, ℓ_2, is defined by the vector equation $\underset{\sim}{r} = \begin{pmatrix}0\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}a\\1\\-1\end{pmatrix}$, where $\lambda, a \in \mathbb{R}$.

(ii) Find the possible values of a when the angle between ℓ_1 and ℓ_2 is $\frac{\pi}{4}$. **2**

(iii) ℓ_1 and ℓ_2 have a unique point of intersection when $a \neq 2$. Find the point of intersection in terms of a. **2**

Q57 INDEP Q14c 6MKS



In the diagram, AB is a diameter of a circle with centre O and PQ is a chord of the circle that is not perpendicular to AB. The perpendicular from A to PQ is produced to the point H outside the circle. M is the midpoint of PQ and N is the point on BH such that $MN \perp PQ$. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OP} = \underset{\sim}{p}$, $\overrightarrow{OQ} = \underset{\sim}{q}$ and $\overrightarrow{AH} = \underset{\sim}{h}$.

i) By writing $\overrightarrow{OM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$, show that the points O, M and N are collinear. **2**

ii) If $\overrightarrow{BN} = \lambda \overrightarrow{BH}$ for some scalar λ, express $\overrightarrow{ON}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{h}$ and λ. **1**

iii) Hence show that N is the midpoint of BH. **3**

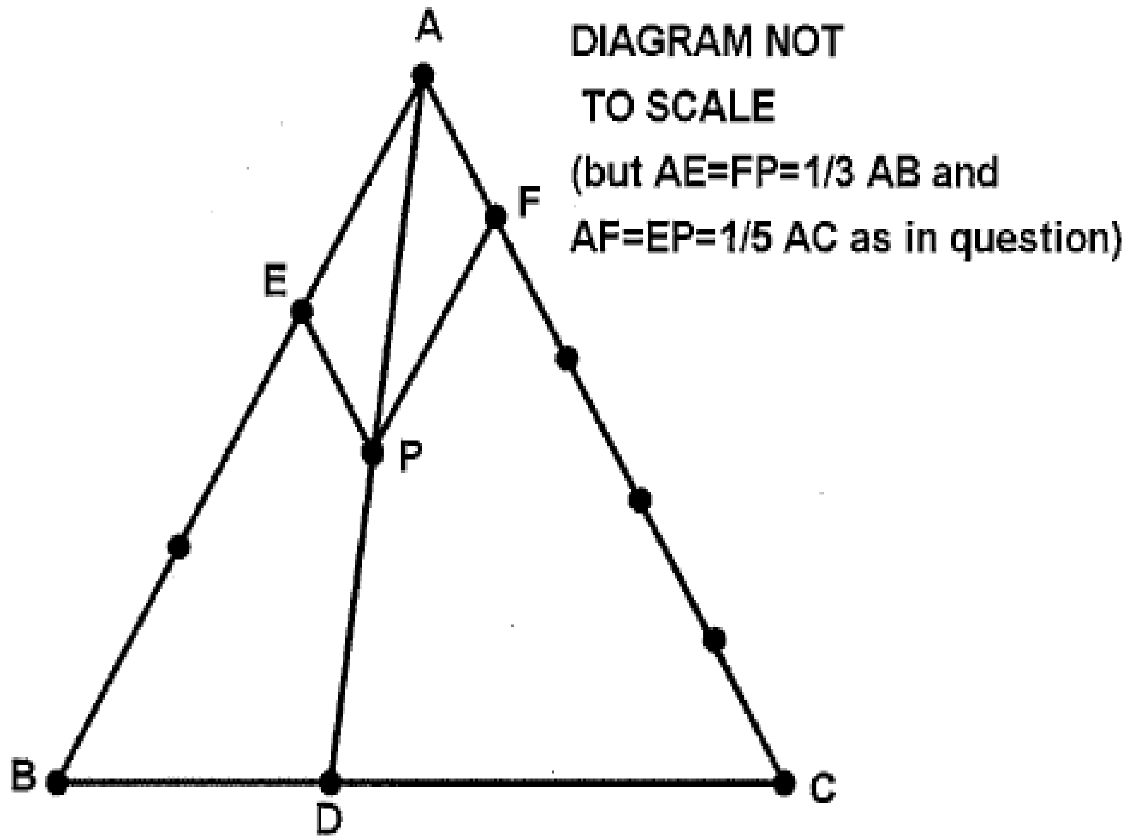
Q58 GIRRAWEEN Q15c 3MKS

ABC is an equilateral triangle. $\overrightarrow{AB} = \underset{\sim}{p}$ and $and\ \overrightarrow{AC} = \underset{\sim}{q}$.

$\overrightarrow{AE} = \overrightarrow{FP} = \frac{1}{3}\underset{\sim}{p}$ and $\overrightarrow{AF} = \overrightarrow{EP} = \frac{1}{5}\underset{\sim}{q}$. D is on BC so that

$\overrightarrow{BD} = \lambda\,\overrightarrow{BC}$. *(So note that D is NOT the midpoint of BC).*

(See diagram).

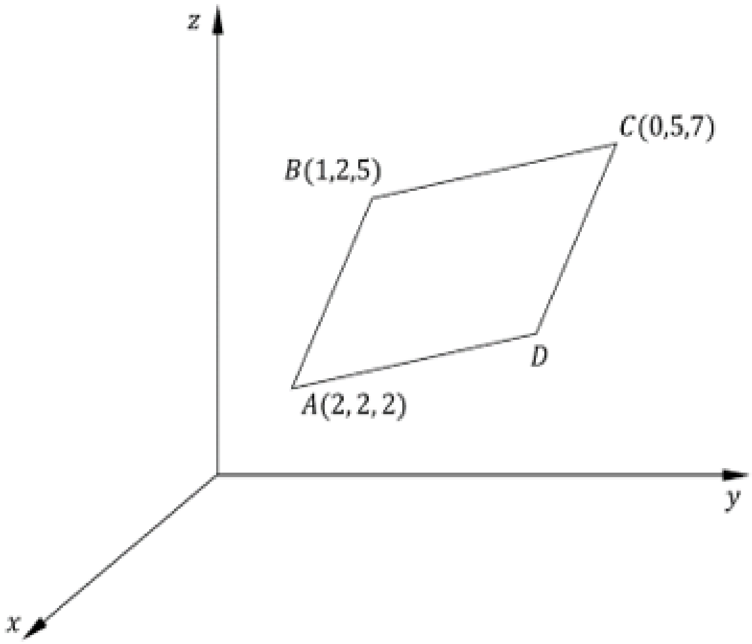


(i) Show that $\overrightarrow{BD} = \lambda(\underset{\sim}{q} - \underset{\sim}{p})$ and $AD = \underset{\sim}{p} + \lambda(\underset{\sim}{q} - \underset{\sim}{p})$ **1**

(ii) Find the value of λ. **2**

Q59 TTA Q11c 3MKS

The parallelogram $ABCD$ is shown, with $A(2,2,2)$, $B(1,2,5)$ and $C(0,5,7)$.



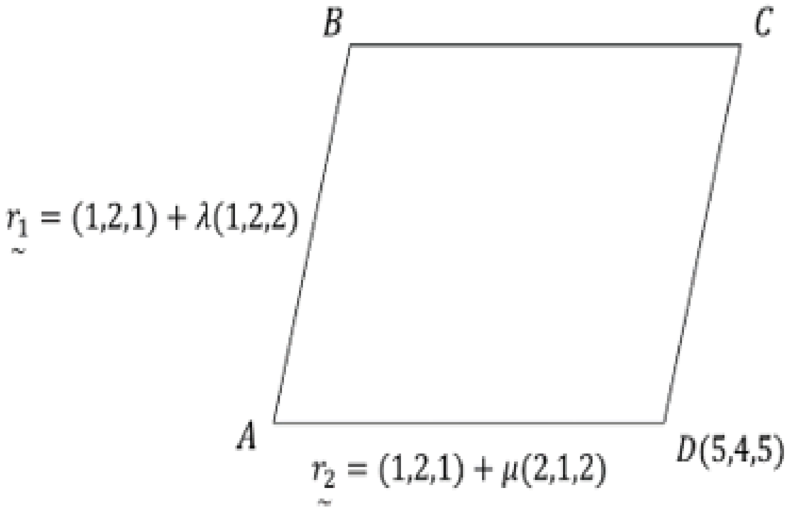
(i) Show that the coordinates of D are $(1,5,4)$. **2**

(ii) Show that the diagonals of the parallelogram bisect each other. **1**

Q60 TTA15b 6MKS

Two sides of the rhombus $ABCD$ are formed by $\underset{\sim}{r_1} = (1,2,1) + \lambda(1,2,2)$ passing through A and B, and $\underset{\sim}{r_2} = (1,2,1) + \mu(2,1,2)$ passing through A and D.

The coordinates of D are $(5,4,5)$.



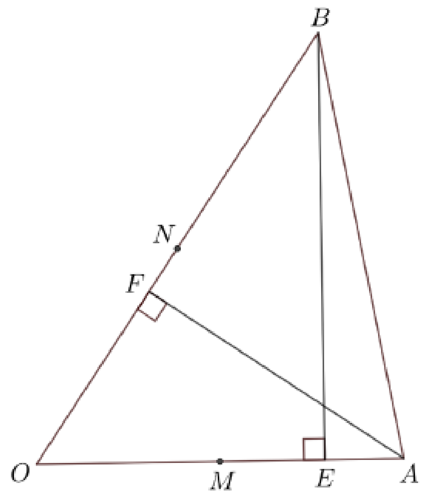
(i) Find the coordinates of B. **3**

(ii) Find the area of the rhombus, giving your answer correct to 2 decimal places. **3**

Q61 HAH Q13d 3MKS

In ΔOAB below, BE is the altitude from B to OA, and AF is the altitude from A to OB.

$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.



Given that M, N are the midpoints of OA, OB respectively,

use vector methods to show that $|\overrightarrow{OM}| \times |\overrightarrow{OE}| = |\overrightarrow{ON}| \times |\overrightarrow{OF}|$.

Vext2_3 LINES and SPHERES

MC QUESTIONS

Q62 NSG Q5

The vector equation of straight line l_1 is given by $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

A second line l_2 passes through the points $A(4,0,1)$ and $B(3,-1,1)$.

What is the value of the parameter λ at the point of intersection of the two lines?

A. $\frac{1}{2}$

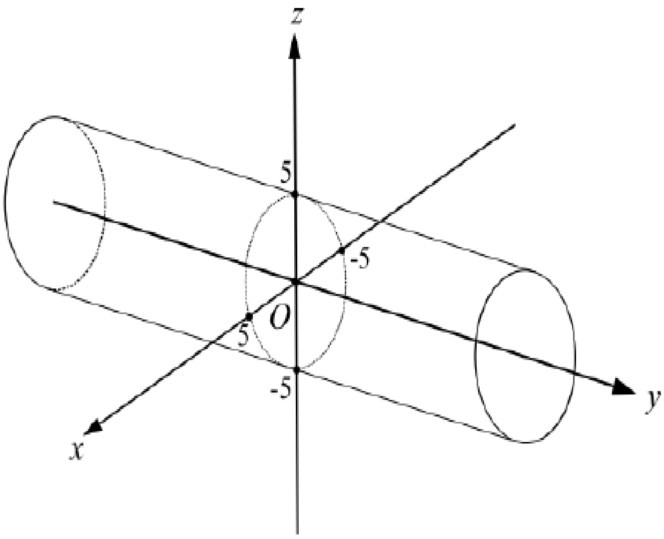
B. 1

C. 2

D. 3

Q63 MATHS BANK Q4

The diagram below is of a hollow cylinder of infinite height in three dimensions, centred about the y-axis.



What is the Cartesian equation that describes this cylinder?

A. $y^2 + z^2 = 25$

B. $x^2 + z^2 = 25$

C. $x^2 + z^2 = (5 - y)^2$

D. $y^2 + z^2 = (5 - x)^2$

Q64 ASCHAM Q3

Which of the following is a vector equation of the line joining the points $A(1,3)$ and $B(-2,4)$?

A. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

B. $\underset{\sim}{r} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

C. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

D. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Q65 PEM Q7

A Cartesian equation for a sphere with centre $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and radius 3 is

(A) $x^2 + x + y^2 + 2y + z^2 - 2z = 0$

(B) $x^2 + 2x + y^2 + 4y + z^2 - 4z = 0$

(C) $x^2 - x + y^2 - 2y + z^2 + 2z = 0$

(D) $x^2 - 2x + y^2 - 4y + z^2 + 4z = 0$

Q66 TTA Q5

Each pair of lines given below intersects at $\begin{pmatrix}1\\2\\3\end{pmatrix}$.

Which pair of lines are perpendicular?

A. $\ell_1\colon\ \underset{\sim}{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\1\end{pmatrix}$ and $\ell_2\colon\ \underset{\sim}{r} = \begin{pmatrix}3\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\0\end{pmatrix}$

B. $\ell_1\colon\ \underset{\sim}{r} = \begin{pmatrix}2\\4\\6\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\ell_2\colon\ \underset{\sim}{r} = \begin{pmatrix}-2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\3\end{pmatrix}$

C. $\ell_1\colon\ \underset{\sim}{r} = \begin{pmatrix}0\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\-3\end{pmatrix}$ and $\ell_2\colon\ \underset{\sim}{r} = \begin{pmatrix}-1\\-2\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$

D. $\ell_1\colon\ \underset{\sim}{r} = \begin{pmatrix}3\\-2\\8\end{pmatrix} + \lambda\begin{pmatrix}-2\\4\\-5\end{pmatrix}$ and $\ell_2\colon\ \underset{\sim}{r} = \begin{pmatrix}3\\4\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\1\end{pmatrix}$

Q67 HAH Q6

The vector equation of a sphere is given by $\left|\underset{\sim}{v} - \begin{pmatrix}3\\0\\-2\end{pmatrix}\right| = 52$.

Where does the point with position vector $\begin{pmatrix}-3\\0\\2\end{pmatrix}$ lie with respect to the sphere?

A. At the centre of the sphere.

B. Within the sphere, but not at the centre.

C. On the surface of the sphere.

D. Outside of the sphere.

Q68 JAMES R Q3

A line has equation $\underset{\sim}{r}(t) = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$, $\lambda \in \mathbb{R}$. Which of the following is parallel to this line?

(A) $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -15 \end{pmatrix}$

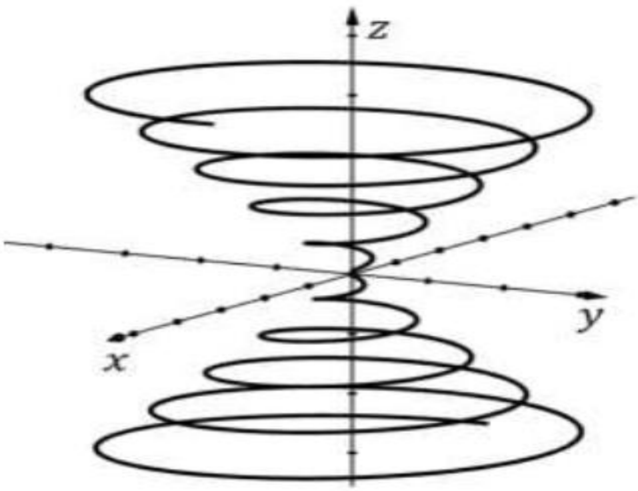
(B) $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -6 \\ -15 \end{pmatrix}$

(C) $\underset{\sim}{r} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

(D) $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$

Q69 JAMES R Q9

Which of the equations best represent the curve below?



(A) $\underset{\sim}{r}(t) = (\cos t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j} + (t)\underset{\sim}{k}$

(B) $\underset{\sim}{r}(t) = (t\cos t)\underset{\sim}{i} + (t\sin t)\underset{\sim}{j} + (t)\underset{\sim}{k}$

(C) $\underset{\sim}{r}(t) = (t\cos t)\underset{\sim}{i} + (t\sin t)\underset{\sim}{j} + \left(\frac{1}{t}\right)\underset{\sim}{k}$

(D) $\underset{\sim}{r}(t) = (\cos t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j} + \left(\frac{1}{t}\right)\underset{\sim}{k}$

Vext2_3 LINES and SPHERES

SHORT ANSWER QUESTIONS

Q70 WESTERN Q11a 2MKS

Find the vector equation of the line passing through $(3, 6, 4)$ and $(-3, 4, -2)$.

Q71 WESTERN Q12b 3MKS

Find the point of intersection of the lines:

$$r = \binom{2}{3} + s\binom{1}{2} \text{ and } r = \binom{3}{2} + t\binom{1}{4}.$$

Q72 PEM Q12b 5MKS

The line l_1 has equation

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where λ is a parameter.

The line l_2 has the equation

$$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

(i) Show that l_1 and l_2 lie in the same plane. **2**

(ii) Write down a vector equation for the plane containing l_1 and l_2. **1**

(iii) To the nearest minute, find the acute angle between l_1 and l_2. **2**

Q73 PEM Q12d 3MKS

Find the shortest distance from the point $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ to the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

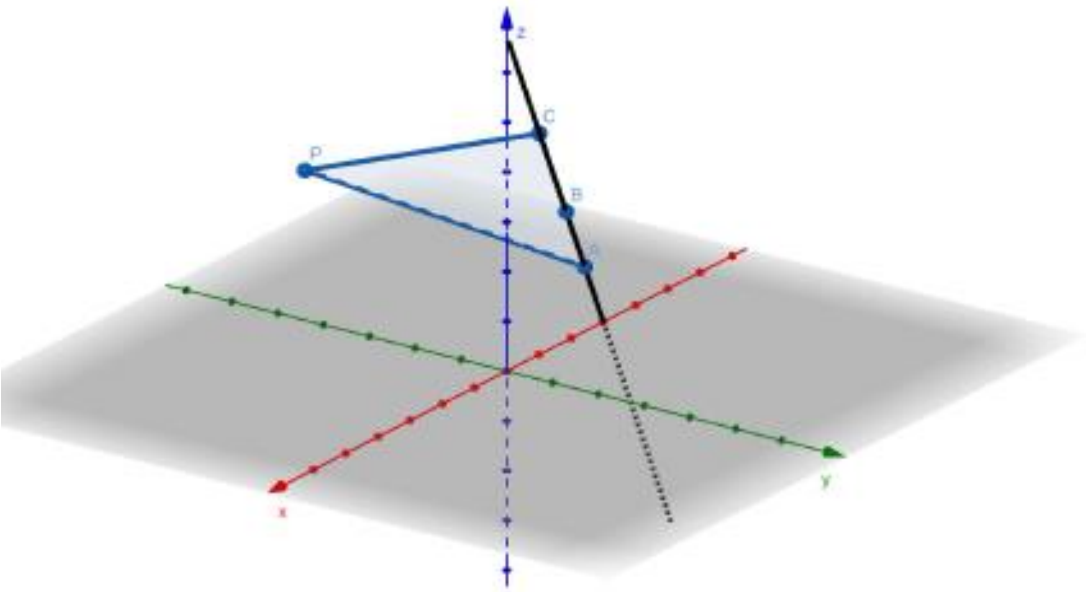
Q74 ABBOTS Q15a 5MKS

) Consider the line, l, with vector equation $\underset{\sim}{r}(\lambda) = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ and the points

$A = (-1, 1, 2)$ and $B = (1, 2, 4)$ lie on the line. Let $\underset{\sim}{b} = \overrightarrow{AB}$, so

$$\underset{\sim}{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \text{ (DO NOT prove this).}$$

Let P be the point $(2, -3, 4)$.



(i) Find the projection of $\overrightarrow{AP}$ onto the line l, and hence the perpendicular distance from P to the line, l. **3**

(ii) Hence find the coordinates of the point, C on the line l such that the area of ΔAPC is 15 square units. **2**

Q75 ACE YR Q13b 4MKS

Lines l_1 and l_2 are given below, relative to a fixed origin O.

$$l_1:\ \underset{\sim}{r} = \left(11\underset{\sim}{i} + 2\underset{\sim}{j} + 17\underset{\sim}{k}\right) + \lambda\left(-2\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}\right)$$

$$l_2:\ \underset{\sim}{r} = \left(-5\underset{\sim}{i} + 11\underset{\sim}{j} + p\underset{\sim}{k}\right) + \mu\left(-3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$$

where λ and μ are scalar parameters.

(i) Given that line l_1 and l_2 intersect, find the value of p. **3**

(ii) Hence find the point of intersection of line l_1 and l_2. **1**

Q76 SYD TECH Q12d 3MKS

Let l_1 be the line with equation $\underset{\sim}{r} = (-\underset{\sim}{i} + 2\underset{\sim}{j}) + \lambda(2\underset{\sim}{i} + 5\underset{\sim}{j}),\ \lambda \in \mathbb{R}$.

The line l_2 passes through the point A(1, -2) and is parallel to l_1.

Find the equations of l_2 in the form $y = mx + c$.

Q77 Syd g q14b 5mks

Consider a sphere S, centred at point $C(2, -1, 0)$ with radius $\sqrt{29}$.

Consider also the line ℓ with parametric equations

$$x = \lambda + 1,\ y = \lambda,\ z = 2\lambda + 3.$$

(i) Find the vector equation of line ℓ, writing your answer in the form $\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{d}$, where $\underset{\sim}{a}$ and $\underset{\sim}{d}$ are expressed as column vectors. **1**

It is known that ℓ intersects the surface of S at points P and Q.

(ii) Find the coordinates of P and Q. **3**

(iii) Hence, or otherwise, determine whether PQ is a diameter of S, showing all necessary working. **1**

Q78 SYD G Q15c 7MKS

Suppose that line ℓ_1 has vector equation

$$\underset{\sim}{r} = \lambda \begin{pmatrix} \cos\phi + \sqrt{3} \\ \sqrt{2}\sin\phi \\ \cos\phi - \sqrt{3} \end{pmatrix}$$

and that line ℓ_2 has vector equation

$$\underset{\sim}{r} = \mu \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

where $\lambda,\ \mu \in \mathbb{R}$.

(i) Show that the acute angle θ between ℓ_1 and ℓ_2 is independent of ϕ. **3**

(ii) A plane has equation $x - z = 4\sqrt{3}$. The line ℓ_2 meets this plane at C. **1**

Find the coordinates of C.

(iii) The line ℓ_1 intersects the plane $x - z = 4\sqrt{3}$ at the point P. **3**

Show that as ϕ varies, P describes a circle of centre C and radius $2\sqrt{2}$.

Q79 NSG Q12d 5MKS

The two points $A(-1,2,3)$ and $B(-1,3,5)$ lie in a three-dimensional coordinate system.

(i) Find the vector equation of the sphere centred at A and passing through B. **2**

(ii) Determine if the line l_1 defined by $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ is a tangent to the sphere. **3**

Q80 MATHS BANK Q11c 3MKS

Two lines, L_1 and L_2, are defined by the Cartesian equations:

$$L_1 : \ y = 7, \ \frac{x-8}{2} = \frac{z+3}{4}$$

$$L_2 : \quad \frac{x-1}{k} = \frac{y-5}{-8} = \frac{z}{3}$$

Find the value of k, where $k \in \mathbb{Z}$, for which L_1 and L_2 would be perpendicular.

Q81 MATHS BANK Q12c 5MKS

(i) Find the point of intersection, $P(x, y, z)$, between the lines: **3**

$$\underset{\sim}{r_1} = \begin{pmatrix} -11 \\ 17 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{r_2} = \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

where $\lambda, \mu \in \mathbb{R}$.

(ii) Find the vector equation of the line that is concurrent with the two lines above, and passes through the point $Q(2, 6, 8)$. **2**

Q82 MATHS BANK Q13a 2MKS

A, B, and C are three collinear points where $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$, and O is the origin.

B lies between A and C such that $\left|\overrightarrow{AB}\right| = 5\left|\overrightarrow{BC}\right|$.

Find vector $\underset{\sim}{c}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Q83 MATHS BANK Q14b 4MKS

Two lines have vector equations for $\lambda,\ \mu \in \mathbb{R}$:

$$\underset{\sim}{r_1} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{r_2} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

Find the shortest distance between the two lines.

Q84 JAMES R Q11d 3MKS

Find the equations of a sphere whose centre is at $(1,0,1)$ and touches the line $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ where $\lambda \in \mathbb{R}$.

Q85 HAH Q13 abc 12 MKS

(a) Let $\underset{\sim}{u} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} a \\ 2 \\ -1 \end{pmatrix}$.

Given that the vector projection of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is $\begin{pmatrix} \frac{4}{9} \\ -\frac{2}{9} \\ -\frac{4}{9} \end{pmatrix}$, find the value of a. **2**

(b) The points A, B, C are collinear where $\overrightarrow{OA} = i - j$, $\overrightarrow{OB} = -3i - k$ and $\overrightarrow{OC} = 2i + aj + bk$ for some constants a and b.

What are the values of a and b? **3**

(c) A sphere S_1 with centre $C(-3, -5, 10)$ passes through the point with coordinates $A(3, -3, 6)$.

(i) Show that the vector equation of S_1 is $\left| \underset{\sim}{u} - \begin{pmatrix} -3 \\ -5 \\ 10 \end{pmatrix} \right| = 2\sqrt{14}$. **1**

(ii) Write down the Cartesian equation of S_1. **1**

(iii) The vector equation of another sphere of S_2 is $\left| \underset{\sim}{r} - \begin{pmatrix} -9 \\ 4 \\ 7 \end{pmatrix} \right| = \sqrt{14}$

Prove that the two spheres S_1 and S_2 touch each other at a single point. **2**

(iv) The vector equation of the line m is given as

$$\underset{\sim}{r} = \begin{pmatrix} -6 \\ -3 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \text{where } \lambda \in \mathbb{R}.$$

Find the value(s) of λ where the line m intersects the sphere S_1. **3**

Q86 GIRRAWEEN Q13a 9MKS

(i) Show BY FINDING IT (NOT BY SUBSTITUTION) that the **4**

point of intersection of $\begin{pmatrix} -9 \\ -14 \\ -3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 23 \\ -4 \\ -27 \end{pmatrix} + \lambda_2 \begin{pmatrix} 10 \\ 1 \\ -16 \end{pmatrix}$

is $\begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix}$.

(ii) Show that $\begin{pmatrix} -9 \\ -14 \\ -3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ is a tangent to the sphere **2**

$(x-1)^2 + (y+2)^2 + (z-4)^2 = 21$ and show that the point

of intersection of this line with the sphere is also $\begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix}$.

(iii) Find the equation of the straight line perpendicular to both **3**

$\begin{pmatrix} -9 \\ -14 \\ -3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 23 \\ -4 \\ -27 \end{pmatrix} + \lambda_2 \begin{pmatrix} 10 \\ 1 \\ -16 \end{pmatrix}$ passing through $\begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix}$.

Hence or otherwise show that $\begin{pmatrix} 23 \\ -4 \\ -27 \end{pmatrix} + \lambda_2 \begin{pmatrix} 10 \\ 1 \\ -16 \end{pmatrix}$ is also a tangent

to the sphere $(x-1)^2 + (y+2)^2 + (z-4)^2 = 21$.

Q87 ASCHAM Q15d 5MKS

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1 : \underset{\sim}{r} = \left(10\underset{\sim}{i} - 9\underset{\sim}{k}\right) + \lambda\left(-\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right)$$

$$l_2 : \underset{\sim}{r} = \left(17\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}\right) + \mu\left(5\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right),$$

where λ and μ are scalar parameters.

Show that l_1 and l_2 meet and find the position vector of the point of intersection.

Q88 CSSA Q12a 5MKS

Consider the two lines

$$L_1: \begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \quad \text{and} \quad L_2: \begin{pmatrix} -1 \\ 2 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}.$$

(i) Find the value of k given $B(9, k, 24)$ lies on L_2. **2**

(ii) Find the point of intersection of L_1 and L_2. **3**

Q89 CSSA Q13c 3MKS

Consider the sphere with vector equation $\left| \underset{\sim}{r} - \begin{pmatrix} 3 \\ -12 \\ 4 \end{pmatrix} \right| = 3.$

(i) Show the point $(5, -10, 3)$ lies on the sphere. **1**

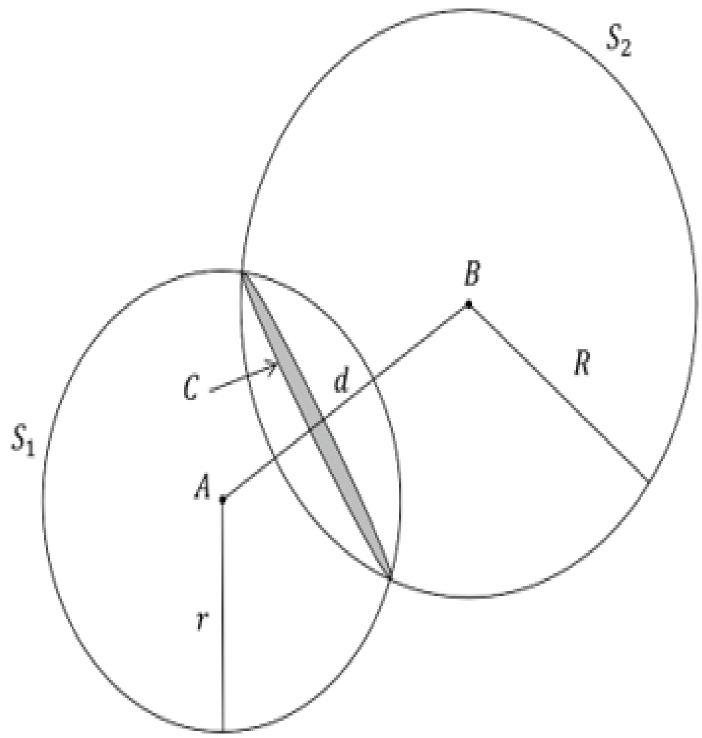
(ii) Find the point on the sphere farthest from the origin. **2**

Q90 TTA Q16d 5MKS

Two spheres, S_1 and S_2, intersect in the circle C.

The sphere S_1 is centred at $A(a, b, c)$ with radius r and the sphere S_2 is centred at $B(a+m, b+n, c+p)$ with radius R, where $a,\ b,\ c,\ m,\ n,\ p,\ r$ and R are real, with $r,\ R > 0$.

The distance between the centres of the spheres is d.



(i) Prove that the circle C lies on the sphere S_3, which has AB as a diameter, only if $m^2 + n^2 + p^2 = r^2 + R^2$. **4**

(ii) Hence, or otherwise, show that the intersection of the spheres $x^2 + y^2 + z^2 = 25$ and $(x-3)^2 + (y-4)^2 + (z-12)^2 = 144$ is a circle lying on a third sphere, which has the interval joining the centres of the first two spheres as a diameter. **1**

Q91 ASCHAM Q13d 4MKS

A sphere has a centre at $(3,-3,4)$ and its radius is 6 units.

A line has equation $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$.

(i) Write down the vector equation of the sphere. **1**

(ii) Determine whether the line is a tangent to the sphere, clearly justifying your conclusion. **3**

ANSWERS

Q1 ANS A

$\lvert \underset{\sim}{u} \rvert = \sqrt{a^2 + (-3)^2 + (-4)^2}$ $7 = \sqrt{a^2 + 9 + 16}$ $7 = \sqrt{a^2 + 25}$ $49 = a^2 + 25$ $a^2 = 49 - 25 = 24$ $a = \sqrt{24} = 2\sqrt{6}$	**A**

Q2 ANS A

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 1 \times 4 + (-1) \times 12 + (-1) \times (-3)$ $= -5$	1 Mark: A

Q3 ANSWER C

If Perpendicular, then $u \cdot v = 0$ $(3 \times 2) - 4p + (1 \times -3) = 0$ $6 - 4p - 3 = 0$ $-4p = -3$ $p = \frac{3}{4}$	**C**

Q4 ANS B

$\sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + (-3)^2 + 5^2}$ $= \sqrt{35}$	1 Mark: B

Q5 ANS A

1. $\sqrt{a^2 + 36 + 64} = 12$

$a^2 + 100 = 144$

$a = 2\sqrt{11}$ (A)

Q6 ANS A

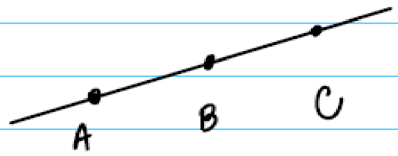
4. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0 \quad 12 - 2p + 3p = 0$

$p = -12$ Ⓐ

Q7 ANS D

Q8 Ans C

5)



$\overrightarrow{AB} = \lambda \overrightarrow{BC}$

$2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k} - \underset{\sim}{i} - \underset{\sim}{j} = \lambda\left(3\underset{\sim}{i} + a\underset{\sim}{j} + b\underset{\sim}{k} - 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)$

$\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k} = \lambda\left(\underset{\sim}{i} + (a+1)\underset{\sim}{j} + (b-1)\underset{\sim}{k}\right)$

$$\begin{cases} \lambda = 1 \\ \lambda(a+1) = -2 \\ \lambda(b-1) = 1 \end{cases}$$

$\therefore \begin{cases} a+1 = -2 \\ b-1 = 1 \end{cases} \qquad \therefore \begin{cases} a = -3 \\ b = 2 \end{cases} \qquad \therefore$ Ⓒ

Q9 ANS C

$\overrightarrow{BA} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{(-4)^2 + 2^2 + 4^2} = \sqrt{36} = 6$

unit vector $= \frac{1}{6}\begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix} = -\frac{2}{3}\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j} + \frac{2}{3}\underset{\sim}{k}$ $\boxed{C}$

Q10 ANS A

$$\overrightarrow{FC} = \overrightarrow{AE}$$
$$\therefore \overrightarrow{FC} + \overrightarrow{EC} = \overrightarrow{AE} + \overrightarrow{EC}$$
$$= \overrightarrow{AC}$$

Answer: A

Q11 ANS D

$$\frac{\$\text{profit}}{\text{tonnes sold}} = \frac{25 \times 530 + 55 \times 380 + 10 \times 410}{25 + 55 + 10}$$
$$= \frac{\begin{pmatrix}25\\55\\10\end{pmatrix} \cdot \begin{pmatrix}530\\380\\410\end{pmatrix}}{\begin{pmatrix}25\\55\\10\end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix}} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{a} \cdot \underset{\sim}{c}}$$

Hence D

Q12 ANS C

4. $|\underset{\sim}{u}| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$

$\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2 = 4^2 + 2^2 + 4^2 = 36$

$\therefore |\underset{\sim}{u}| \times \underset{\sim}{v} \cdot \underset{\sim}{v} = 36\sqrt{14}$ (C)

Q13 ANS B

Q14 ANS A

Q15 ANS B

(6) $t = y = 0 \quad x = \cos 0 = 1 \quad z = \sin 0 = 0$
$(1, 0, 0)$.

$t = y = \frac{\pi}{2}. \quad x = \cos\frac{\pi}{2} = 0. \quad y = \sin\frac{\pi}{2} = 1.$
$(0, \frac{\pi}{2}, 1)$.

Parametric equation is
$(\cos t, t, \sin t)$ (B)

Q16 ANS C

$\overrightarrow{OA} \perp \overrightarrow{OC}$, so $\overrightarrow{OA} \cdot \overrightarrow{OC} = 0$

$3 \times 6 - 2 \times 4 + 2a = 0$

$a = -5$ $\therefore C$

Q17 ANS A

A	$\cos\theta = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert} = \dfrac{4+0+2}{\sqrt{6}\sqrt{8}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ $\therefore \theta = \frac{\pi}{6}$

Q18 ANS D

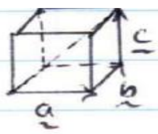
Q19 ANS B

B $(a-b)\cdot(a+b) = |a|^2 - |b|^2 = 0$

Q20 ANS A

Q21 ANS C

G. $\hat{\underset{\sim}{a}} + \hat{\underset{\sim}{b}} + \hat{\underset{\sim}{c}}$ will be the diagonal of a 1 unit cube.



$|\hat{\underset{\sim}{a}} + \hat{\underset{\sim}{b}}| = \sqrt{1^2+1^2}$

$= \sqrt{2}$

$|\hat{\underset{\sim}{a}} + \hat{\underset{\sim}{b}} + \hat{\underset{\sim}{c}}| = \sqrt{\sqrt{2}^2+1^2}$

$= \sqrt{3}$.

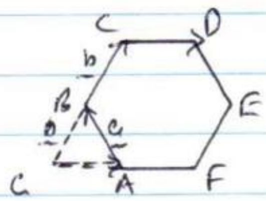
C

Q22 ANS C

$\vec{CD} = \vec{GA}$

now $\vec{GA} = \underset{\sim}{b} - \underset{\sim}{a}$

$\therefore \vec{CD} = \underset{\sim}{b} - \underset{\sim}{a}$



C

Q23 ANS D

Q24 ANS A

Q25 ANS

$$\vec{a}\cdot\vec{b} = (3\times 2)+(-2\times 2)+(2\times 4) = 6-4+8 = 10$$

$$|\vec{a}| = \sqrt{3^2+(-2)^2+2^2} = \sqrt{17}$$
$$|\vec{b}| = \sqrt{2^2+2^2+4^2} = \sqrt{24} = 2\sqrt{6}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{10}{\sqrt{17}\times 2\sqrt{6}} = 60^\circ\,20' \quad \text{(nearest minute)}$$

Q26 ANS

a) Let C have coordinates (x, y, z)

Given $|\vec{OA}| = |\vec{OC}| = 1$

& ΔAOC is equilateral $\Rightarrow \angle AOC = \pi/3$.

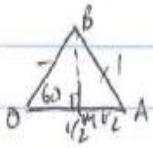
$\therefore \vec{OA}\cdot\vec{OC} = |\vec{OA}||\vec{OC}|\cos\pi/3$

ie $\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = 1\times 1\times\frac{1}{2}$.

$x+0+0 = \frac{1}{2}$

$x = 1/2$ $\boxed{\checkmark}$

Now B is the altitude of ΔOAB.



$MB = \sin 60^\circ$

$= \sqrt{3}/2$.

$\therefore B = \left(\frac{1}{2}, \sqrt{3}/2, 0\right)$ $\boxed{\checkmark}$

So $\vec{OB}\cdot\vec{OC} = |\vec{OB}||\vec{OC}|\cos\pi/3$

$\begin{pmatrix}\frac{1}{2}\\ \sqrt{3}/2\\ 0\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{2}\\ y\\ z\end{pmatrix} = 1\times 1\times\frac{1}{2}$.

$\frac{1}{4}+\frac{\sqrt{3}}{2}y+0 = \frac{1}{2}$.

$\frac{\sqrt{3}}{2}y = \frac{1}{4}$

$y = \frac{1}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{6}$ $\boxed{\checkmark}$

Finally $|\vec{OC}| = 1$

$\therefore \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{6}\right)^2+z^2} = 1$

$\frac{1}{4}+\frac{3}{36}+z^2 = 1$

$z^2 = 1-\frac{1}{4}-\frac{1}{12}$

$= \frac{2}{3}$

$z = \sqrt{2/3}\times\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}}{3}$ $\boxed{\checkmark}$

$\therefore C = \left(\frac{1}{2}, \frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{3}\right)$

Q27 ANS

-

$$\overrightarrow{OT} = \overrightarrow{OR} + \overrightarrow{RT}$$
$$= \overrightarrow{OR} + 3\overrightarrow{RS}$$
$$= \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + 3\begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} -10 \\ -2 \\ 5 \end{pmatrix}$$

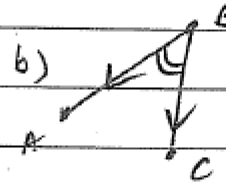
Q28 ANS

15(c) (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (\underset{\sim}{j} + 2\underset{\sim}{k}) - (-\underset{\sim}{i} - \underset{\sim}{j})$ $= \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$
15(c) (ii)	$\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2 + z^2}$ $= \sqrt{1^2 + 2^2 + 2^2}$ $= 3$
15(c) (iii)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ $= (4\underset{\sim}{i} + \underset{\sim}{k}) - (\underset{\sim}{j} + 2\underset{\sim}{k})$ $= 4\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ $\lvert\overrightarrow{BC}\rvert = \sqrt{x^2 + y^2 + z^2}$ $= \sqrt{4^2 + (-1)^2 + (-1)^2}$ $= 3\sqrt{2}$ $\cos\angle CAB = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert}$ $= \dfrac{1 \times 4 + 2 \times (-1) + 2 \times (-1)}{3 \times 3\sqrt{2}}$ $= \dfrac{0}{9\sqrt{2}} = 0$ $\angle CAB = 90^\circ$

Q29 ANS

16(d) (i)	$\underset{\sim}{i}$ component $6+\lambda=0$ $\lambda=-6$ $\underset{\sim}{j}$ component $19-6\times 4=a$ $a=-5$ $\underset{\sim}{k}$ component $-1-6\times(-2)=b$ $\therefore$ $a=-5$ and $b=11$.
16(d) (ii)	$\overrightarrow{OP}=(6+\lambda)\underset{\sim}{i}+(19+4\lambda)\underset{\sim}{j}+(-1-2\lambda)\underset{\sim}{k}$ Direction vector of l_1: $\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k}$ $\overrightarrow{OP}$ and l_1 are perpendicular $\left[(6+\lambda)\underset{\sim}{i}+(19+4\lambda)\underset{\sim}{j}+(-1-2\lambda)\underset{\sim}{k}\right]\cdot(\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k})=0$ Hence $6+\lambda+(19+4\lambda)4+(-1-2\lambda)-2=0$ $6+\lambda+76+16\lambda+2+4\lambda=0$ $21\lambda+84=0$ $\lambda=-4$ Therefore $\overrightarrow{OP}=(6-4)\underset{\sim}{i}+(19+4\times(-4))\underset{\sim}{j}+(-1-2\times(-4))\underset{\sim}{k}$ $=2\underset{\sim}{i}+3\underset{\sim}{j}+7\underset{\sim}{k}$

Q30 ANS

b) 

$\vec{BA} = \begin{pmatrix} -1 \\ -5 \\ 2 \end{pmatrix}$ $\quad \vec{BC} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$\vec{BA} \cdot \vec{BC} = \sqrt{1+25+4} \times \sqrt{9+0+1} \cos\theta$

$3 + 0 + 2 = \sqrt{30} \cdot \sqrt{10} \cos\theta$

$5 = 10\sqrt{3} \cos\theta$

$\cos\theta = 1/2\sqrt{3}$

$\theta \doteqdot 73 \cdot 22' \ldots \simeq 73^\circ$

Q32 ANS

11(b)	$\lvert\overrightarrow{OA}\rvert = \sqrt{2^2 + 6^2 + (-3)^2}$ $= \sqrt{49} = 7$ $\widehat{\overrightarrow{OA}} = \dfrac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert}$ $= \dfrac{1}{7}(2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k})$

Q33 ANS

12a) i) $\vec{AC} = \begin{pmatrix} -1 \\ 0 \\ k \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 0 \\ -1 \\ k \end{pmatrix}$ ✓

ii) If $\angle ACB = 45^\circ$, then

$$\frac{\vec{AC} \cdot \vec{BC}}{|\vec{AC}|\,|\vec{BC}|} = \cos 45^\circ$$

so $$\frac{k^2}{\sqrt{k^2+1} \cdot \sqrt{k^2+1}} = \frac{1}{\sqrt{2}}$$ ✓

$\Rightarrow \quad \sqrt{2}\,k^2 = k^2 + 1$

$(\sqrt{2} - 1)k^2 = 1$

$k^2 = \dfrac{1}{\sqrt{2} - 1}$

$k \doteqdot 1 \cdot 55$ ✓

Q34 ANS

13a) $A(5,0,2)$, $B(2,3,-4)$, $C(8,-3,5)$

i) $\underset{\sim}{r} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$

$$= \begin{pmatrix}5\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}-3\\3\\-6\end{pmatrix}$$ ✓

ii) Suppose $\begin{pmatrix}8\\-3\\5\end{pmatrix} = \begin{pmatrix}5\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}-3\\3\\-6\end{pmatrix}$

then $8 = 5 - 3\lambda \Rightarrow \lambda = -1$

but equating z coefficients gives

$$5 = 2 - 6\lambda$$
$$\Rightarrow 6\lambda = -3$$
$$\Rightarrow \lambda = -\tfrac{1}{2}$$ ✓

since these do not agree, C is not on AB.

iii) $\overrightarrow{AC} = \begin{pmatrix}3\\-3\\3\end{pmatrix}$

$$\text{proj}_{\overrightarrow{AB}}\overrightarrow{AC} = \frac{\begin{pmatrix}3\\-3\\3\end{pmatrix}\cdot\begin{pmatrix}-3\\3\\-6\end{pmatrix}}{\left|\begin{pmatrix}-3\\3\\-6\end{pmatrix}\right|^2}\cdot\begin{pmatrix}-3\\3\\-6\end{pmatrix}$$

$$= \frac{-36}{54}\begin{pmatrix}-3\\3\\-6\end{pmatrix}$$

$$= \begin{pmatrix}2\\-2\\4\end{pmatrix}$$ ✓

Q35 ANS

14 d)

$$\underset{\sim}{r} = \begin{pmatrix} 20e^{-t} \cos t \\ 20e^{-t} \sin t \\ 10e^{-t} \end{pmatrix}$$

$$\underset{\sim}{v} = \begin{pmatrix} -20e^{-t} \cos t - 20e^{-t} \sin t \\ -20e^{-t} \sin t + 20e^{-t} \cos t \\ -10e^{-t} \end{pmatrix}$$

when $t = 0$, $\underset{\sim}{v} = \begin{pmatrix} -20 \\ 20 \\ -10 \end{pmatrix}$

$$|\underset{\sim}{v}| = 30 \text{ m/s}$$

ii)

$$\underset{\sim}{r} \cdot \underset{\sim}{v} = \left(-400e^{-2t} \cos^2 t - 400e^{-2t} \sin t \cos t\right) + \left(-400e^{2t} \sin^2 t + 400e^{-2t} \sin t \cos t\right) + \left(-100e^{-2t}\right)$$

$$= -500e^{-2t}$$

$$|\underset{\sim}{r}|^2 = 400e^{-2t} \cos^2 t + 400e^{-2t} \sin^2 t + 100e^{-2t}$$

$$= 500e^{-2t}$$

$$|\underset{\sim}{r}| = 10\sqrt{5}\, e^{-t}$$

$$|\underset{\sim}{v}|^2 = \left(400e^{-2t} \cos^2 t + 800e^{-2t} \cos t \sin t + 400e^{-2t} \sin^2 t\right) + \left(400e^{-2t} \sin^2 t - 800e^{-2t} \cos t \sin t + 400e^{-2t} \cos^2 t\right) + 100e^{-2t}$$

14d) ii) (cont.)

$$|\underset{\sim}{v}|^2 = 900e^{-2t}$$

$$|\underset{\sim}{v}| = 30e^{-t}$$ ✓

Let θ be the angle between $\underset{\sim}{r}$ & $\underset{\sim}{v}$

then $$\cos\theta = \frac{\underset{\sim}{r} \cdot \underset{\sim}{v}}{|\underset{\sim}{r}|\,|\underset{\sim}{v}|}$$

$$= \frac{-500e^{-2t}}{(10\sqrt{5}e^{-t})(30e^{-t})}$$

$$\cos\theta = \frac{-\sqrt{5}}{3}$$ ✓

$$\doteqdot 138^\circ$$

(The falcon looks at a constant angle of 42°).

Q36 ANS

(b) $\underset{\sim}{b} + \underset{\sim}{a} = 3\underset{\sim}{i} + (p+2)\underset{\sim}{j} + 7\underset{\sim}{k}$

$\underset{\sim}{b} - \underset{\sim}{c} = 4\underset{\sim}{i} + (p-4)\underset{\sim}{j} - \underset{\sim}{k}$ ✓

$(\underset{\sim}{b} + \underset{\sim}{a}) \cdot (\underset{\sim}{b} - \underset{\sim}{c}) = 0$

$12 + (p+2)(p-4) - 7 = 0$ ✓

$p^2 - 2p - 3 = 0$

$(p-3)(p+1) = 0$

$\therefore p = 3, -1$ ✓

Q37 ANS

Lines intersect if :

$1 + \lambda = 1 - \mu$

$\lambda = -\mu \quad (1)$

$3 - 4\lambda = 2 + 3\mu$

$1 - 4\lambda = 3\mu \quad (2)$

$-2 + 7\lambda = -1 + \mu$

$-1 + 7\lambda = \mu \quad (3)$

Sub (1) into (2):

$1 + 4\mu = 3\mu$

$\mu = -1$ ✓

$\therefore \lambda = 1$

Sub these into (3):

$-1 + 7(1) = -1$

This is false.

$\therefore$ No points of intersection.

(i.e. the lines are skew)

Q38 ANS

i) $\sqrt{4^2+2^2+4^2} = 6\text{ m/s}$

ii) $t = 2, \quad z = 0 + 2 \times 4$
$= 8\text{ m}$

iii) $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix} = -4 \times 4 + 2 \times (-6) + 4 \times 7$
$= 0$

iv) $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 0 \end{pmatrix}$

$5 - 4t = -39 + 4s$
$6 + 2t = 44 - 6s$
$4t = 7s$

$t = 7, \; s = 4$

Jordan airplane takes off 3 seconds after Raymond's

Q39 ANS

At $P(2,1,3)$, $x-2y-z=2-2-3=-3\neq 3$. Hence P does not lie in the plane.

At $Q(2+\lambda,1-2\lambda,3-\lambda)$, $(2+\lambda)-2(1-2\lambda)-(3-\lambda)=3$. $\therefore \lambda=1$, $Q(3,-1,2)$

$6\lambda-3=3$ $\qquad$ $\overrightarrow{OQ}=3\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k}$

$PQ^2=1^2+2^2+1^2$ Hence required distance is $\sqrt{6}$ units.

Q40 ANS

$$\overrightarrow{AB}=0\underset{\sim}{i}-4\underset{\sim}{j}+0\underset{\sim}{k}$$

$$|\overrightarrow{AB}|=\sqrt{4^2}=4$$

$$\cos\angle AOB=\frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{|\overrightarrow{OA}|\times|\overrightarrow{OB}|}=\frac{4-4+4}{\left(\sqrt{2^2+2^2+2^2}\right)^2}=\frac{1}{3}.$$ Hence $\angle AOB=\cos^{-1}\left(\frac{1}{3}\right)\approx 71°$.

Q41 ANS

11 a) $\cos\theta=\frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}=\frac{2\times3+-1\times2+3\times5}{\sqrt{2^2+(-1)^2+3^2}\times\sqrt{3^2+2^2+5^2}}$

$=\frac{19}{\sqrt{14}\times\sqrt{38}}$ ✓ $=0.82375\ldots$

$\theta=34.537$

$=35°$ (n. degree) ✓

Q42 ANS

$$proj_{\underset{\sim}{b}}\,\underset{\sim}{a}=\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\underset{\sim}{b}=\frac{8+12}{4^2+3^2}\left(4\underset{\sim}{i}+3\underset{\sim}{j}\right)=\tfrac{16}{5}\underset{\sim}{i}+\tfrac{12}{5}\underset{\sim}{j}$$

Q43 ANS

At intersection point P,

$-9+2\lambda = 3+3\mu$ (1)

$\lambda = 1-\mu$ (2)

$10-\lambda = 17+5\mu$ (3)

$(1)+3\times(2) \Rightarrow -9+5\lambda = 6 \qquad \lambda = 3,\ \mu = -2$

Testing (3): $LHS = 10-3 = 7,\ RHS = 17-10 = 7$

Hence L_1, L_2 intersect at $(-9+6, 3, 10-3)$ $\therefore$ at $P(-3,3,7)$

$$\overrightarrow{OP} = -3\underset{\sim}{i} + 3\underset{\sim}{j} + 7\underset{\sim}{k}$$

L_1, L_2 have direction vectors $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, $\underset{\sim}{v} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ respectively. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 6-1-5 = 0 \quad \therefore L_1 \perp L_2$

B lies on L_1 such that $\overrightarrow{PB} = -\overrightarrow{PA}$. Then $\overrightarrow{OB} = \overrightarrow{OP} + \overrightarrow{PB} = \overrightarrow{OP} - \overrightarrow{PA} = (-3\underset{\sim}{i} + 3\underset{\sim}{j} + 7\underset{\sim}{k}) - (8\underset{\sim}{i} + 4\underset{\sim}{j} - 4\underset{\sim}{k})$

$$\therefore \overrightarrow{OB} = -11\underset{\sim}{i} - \underset{\sim}{j} + 11\underset{\sim}{k}$$

Q44 ANS

c) i) $\overrightarrow{AB} = \begin{pmatrix} 4-2 \\ -2-3 \\ 3-{-4} \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix}$ ✓ let $D = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} d_1-4 \\ d_2-{-2} \\ d_3-3 \end{pmatrix} = \begin{pmatrix} d_1-4 \\ d_2+2 \\ d_3-3 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{BD} \Rightarrow \begin{pmatrix} d_1-4 = 2 \\ d_2+2 = -5 \\ d_3-3 = 7 \end{pmatrix}$

$\therefore d_1 = 6,\ d_2 = -7,\ d_3 = 10$

and position vector D is: $6\underset{\sim}{i} - 7\underset{\sim}{j} + 10\underset{\sim}{k}$. ✓

11 c(ii) $|\vec{AC}| = 4$

$\vec{AC} = \begin{pmatrix} a-2 \\ 5-3 \\ -2--4 \end{pmatrix} = \begin{pmatrix} a-2 \\ 2 \\ 2 \end{pmatrix}$ ✓

$|\vec{AC}|^2 = (a-2)^2 + 2^2 + 2^2 = 4^2$

$\therefore a^2 - 4a + 4 + 4 + 4 = 4 \times 4$

$a^2 - 4a = 4$

$a^2 - 4a - 4 = 0$ ✓

$a^2 - 4a + 4 = 8$

$(a-2)^2 = 4 \times 2$

$a - 2 = \pm 2\sqrt{2}$

$a = 2 \pm 2\sqrt{2}$

but $a > 0$ and so $a = 2 + 2\sqrt{2}$ ✓

Q45 ANS

When $t = 3$, $\underset{\sim}{r} = -\frac{1}{\sqrt{2}}\underset{\sim}{i} + 0\underset{\sim}{j} + \frac{1}{\sqrt{2}}\underset{\sim}{k}$

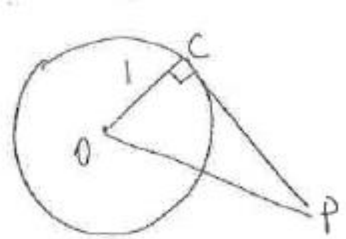
$$\underset{\sim}{v}(t) = \frac{\pi}{4}\left[-\sin\left(\frac{\pi}{4}t\right)\underset{\sim}{i} + \left(-\sin\left(\frac{\pi}{4}t\right) + \cos\left(\frac{\pi}{4}t\right)\right)\underset{\sim}{j} + \cos\left(\frac{\pi}{4}t\right)\underset{\sim}{k}\right]$$

$$\underset{\sim}{v}(3) = \frac{\pi}{4}\left[-\frac{1}{\sqrt{2}}\underset{\sim}{i} + \left(-\frac{1}{\sqrt{2}} + -\frac{1}{\sqrt{2}}\right)\underset{\sim}{j} + \frac{1}{\sqrt{2}}\underset{\sim}{k}\right] = \frac{\pi}{4\sqrt{2}}\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

Therefore the tangent will be given by: $\underset{\sim}{r} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$, for $\lambda \in \mathbb{R}$.

Q46 ANS

14b



$|\vec{OP}| = \sqrt{1 + 6 + 9} = 4$

$|\vec{PC}| = \sqrt{4^2 - 1^2} = \sqrt{15}$

Q47 ANS

$$|OQ|^2 = 25^2 = 49\sin^2\left(\frac{\pi t}{32}\right) + 49\cos^2\left(\frac{\pi t}{32}\right) + \left(\frac{t}{15}\right)^2$$
$$625 = 49 + \frac{t^2}{15^2}$$
$$t^2 = 576 \times 15^2$$
$$t = 24 \times 15 = 360 \text{ seconds}$$

So, if $t = 360$, then $\frac{\pi t}{32} = \frac{45\pi}{4}$

$$\underset{\sim}{r}(t) = 7\sin\frac{45\pi}{4}\,\underset{\sim}{i} + 7\cos\frac{45\pi}{4}\,\underset{\sim}{j} + \frac{360}{15}\,\underset{\sim}{k}$$
$$= 7\sin\frac{-3\pi}{4}\,\underset{\sim}{i} + 7\cos\frac{-3\pi}{4}\,\underset{\sim}{j} + 24\,\underset{\sim}{k}$$

Therefore $Q = \left(\frac{-7}{\sqrt{2}}, \frac{-7}{\sqrt{2}}, 24\right)$

The line $x = 7$ is crossed when the $\underset{\sim}{i}$ component is equal to 7.

$$7\sin\frac{\pi t}{32} = 7$$
$$\frac{\pi t}{32} = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$$
$$t = 16, 80, 144, 208, 272, 336, 400, \cdots$$

Hence line $x = 7$ is crossed 6 times in the 360 seconds it takes the ant to crawl to Q.

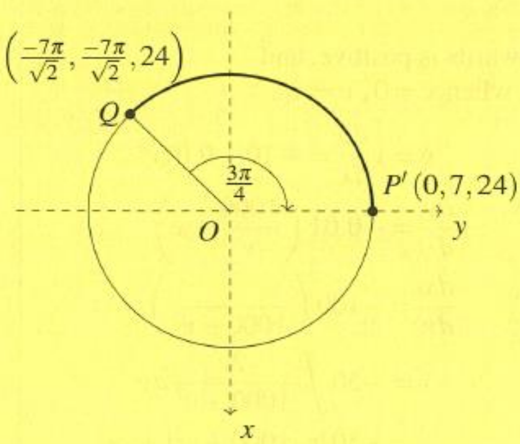
As illustrated in the diagrams on the right, the most direct path from Q to P can be found by considering the cylindrical column as a rectangle with Q and P as opposite vertices.

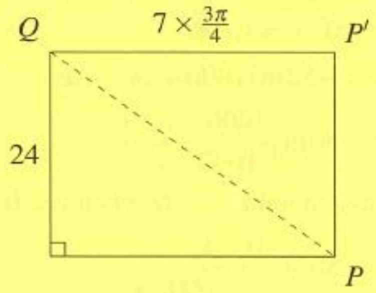
Let the point P' be at the same height of Q and directly above P. Hence $P' = (0, 7, 24)$, and the points Q and P' lie on the plane $z = 24$, as in the first diagram on the right.

The arc length from Q to P' will be $7 \times \frac{3\pi}{4}$. This is also represented in the second diagram, and this gives:

$$\text{The ant's path QP} = \sqrt{24^2 + \left(\frac{21\pi}{4}\right)^2}$$
$$\approx 29.1\,\text{cm}$$

ABOVE VIEW





Q48 ANS

$\vec{a},\vec{b},\vec{c}$ form a triangle $\Rightarrow \vec{a}+\vec{b}+\vec{c}=\vec{0}$

$$\begin{aligned}\vec{u}+\vec{v}+\vec{w} &= \left(\vec{b}\cdot\vec{c}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}+\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}+\left(\vec{b}\cdot\vec{a}\right)\vec{c}-\left(\vec{c}\cdot\vec{b}\right)\vec{a}\\ &= \left(\vec{b}\cdot\vec{c}\right)\vec{a}-\left(\vec{c}\cdot\vec{b}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}+\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}+\left(\vec{b}\cdot\vec{a}\right)\vec{c}\\ &= \vec{0} \quad \text{since } \vec{p}\cdot\vec{q}=\vec{q}\cdot\vec{p}\end{aligned}$$

Hence $\vec{u},\vec{v},\vec{w}$ also form a triangle

$$\begin{aligned}\vec{u}\cdot\vec{c} &= \left(\left(\vec{b}\cdot\vec{c}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}\right)\cdot\vec{c}\\ &= \left(\vec{b}\cdot\vec{c}\right)\vec{a}\cdot\vec{c}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}\cdot\vec{c}\\ &= 0\end{aligned}$$

Question 16(b) (iii)

Criteria	Marks
• Provides a correct solution	2
• Extends part (ii) to the other two corresponding sides, or equivalent merit	1

Sample answer:

From (ii),
$\vec{u}$ and $\vec{c}$ are perpendicular. Similarly, $\vec{v}$ and $\vec{a}$, and $\vec{w}$ and $\vec{b}$ are respectively perpendicular.

The sides of the triangle formed with $\vec{u},\vec{v},\vec{w}$ are perpendicular to the sides of the triangle formed with $\vec{a},\vec{b},\vec{c}$, hence the two triangles are equiangular, that is are similar.

Q49 ANS

(i)	$\vec{AB} = \vec{OB} - \vec{OA}$ $= 6\underset{\sim}{b} - 3\underset{\sim}{a}$ $= 3(2\underset{\sim}{b} - \underset{\sim}{a})$ ☑	1
(ii)	$\vec{OY} = \vec{OB} + \vec{BY}$ $= 6\underset{\sim}{b} + 5\underset{\sim}{a} - \underset{\sim}{b}$ $= 5(\underset{\sim}{a} + \underset{\sim}{b})$ ☑ Since $AX : XB = 1:2$ $\vec{AX} = \frac{1}{3}\vec{AB}$ Now $\vec{OX} = \vec{OA} + \vec{AX}$ $= 3\underset{\sim}{a} + \frac{1}{3}(6\underset{\sim}{b} - 3\underset{\sim}{a})$ $= 3\underset{\sim}{a} + 2\underset{\sim}{b} - \underset{\sim}{a}$ $= 2(\underset{\sim}{a} + \underset{\sim}{b})$ ☑ $\therefore \vec{OX} : \vec{OY} = 2(\underset{\sim}{a} + \underset{\sim}{b}) : 5(\underset{\sim}{a} + \underset{\sim}{b})$ $= 2:5$ So $\vec{OX} = \frac{2}{5}\vec{OY}$ ☑	3

Q50 ANS

Since O is the centre of the circle

$|\vec{OA}| = |\vec{OB}| = |\vec{OC}|$ radii of circle.

also $\vec{OA} = -\vec{OB}$

Now $\vec{AC} = \vec{OC} - \vec{OA}$

$= \vec{OC} + \vec{OB}$ $(\vec{OA} = -\vec{OB})$

& $\vec{CB} = \vec{OB} - \vec{OC}$ ☑

If $\angle ACB = 90°$ then

$\vec{AC} \cdot \vec{CB} = 0$ ☑

$\vec{AC} \cdot \vec{CB} = (\vec{OC} + \vec{OB})(\vec{OB} - \vec{OC})$

$= \vec{OC} \cdot \vec{OB} - \vec{OC} \cdot \vec{OC}$

$+ \vec{OB} \cdot \vec{OB} - \vec{OC} \cdot \vec{OB}$

$= -|\vec{OC}|^2 + |\vec{OB}|^2$

$= 0$

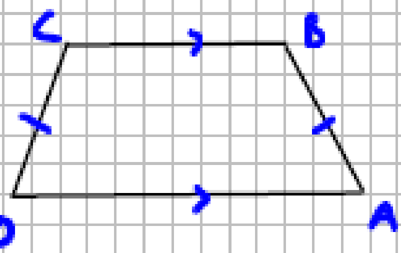
as $\underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$

& $|\vec{OB}| = |\vec{OC}|$ ☑

$\therefore \angle ACB = 90°$

Q51 ANS

16c) Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{c} = \overrightarrow{OC}$



$\overrightarrow{CB} \parallel \overrightarrow{OA}$ so

$\overrightarrow{CB} = k\underset{\sim}{a}$ for some $k \neq 1$.

Then $\overrightarrow{AB} = -\underset{\sim}{a} + \underset{\sim}{c} + k\underset{\sim}{a}$

$= (k-1)\underset{\sim}{a} + \underset{\sim}{c}$

$|\overrightarrow{OC}| = |\overrightarrow{AB}|$ so $|\overrightarrow{OC}|^2 = |\overrightarrow{AB}|^2$

$|\underset{\sim}{c}|^2 = \left((k-1)\underset{\sim}{a} + \underset{\sim}{c}\right) \cdot \left((k-1)\underset{\sim}{a} + \underset{\sim}{c}\right)$

$= (k-1)^2 |\underset{\sim}{a}|^2 + 2(k-1)\underset{\sim}{a} \cdot \underset{\sim}{c} + |\underset{\sim}{c}|^2$

$\Rightarrow \quad 0 = (k-1)^2 |\underset{\sim}{a}|^2 + 2(k-1)\underset{\sim}{a} \cdot \underset{\sim}{c}$

$(k-1) \neq 0$

so $\quad (k-1)|\underset{\sim}{a}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$

$\Rightarrow \quad 2\underset{\sim}{a} \cdot \underset{\sim}{c} = (1-k)|\underset{\sim}{a}|^2$

Now $\overrightarrow{OB} = \underset{\sim}{c} + k\underset{\sim}{a}$

and $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$|\overrightarrow{OB}|^2 = |\underset{\sim}{c}|^2 + 2k\,\underset{\sim}{a} \cdot \underset{\sim}{c} + k^2 |\underset{\sim}{a}|^2$

$= |\underset{\sim}{c}|^2 + k(1-k)|\underset{\sim}{a}|^2 + k^2 |\underset{\sim}{a}|^2$

$= |\underset{\sim}{c}|^2 + k|\underset{\sim}{a}|^2$

$|\overrightarrow{AC}|^2 = |\underset{\sim}{c}|^2 - 2\underset{\sim}{a} \cdot \underset{\sim}{c} + |\underset{\sim}{a}|^2$

$= |\underset{\sim}{c}|^2 - (1-k)|\underset{\sim}{a}|^2 + |\underset{\sim}{a}|^2$

$= |\underset{\sim}{c}|^2 + k|\underset{\sim}{a}|^2$

$= |\overrightarrow{OB}|^2$

so $|\overrightarrow{AC}| = |\overrightarrow{OB}|$ as required.

Q52 ANS

b) i) $\vec{a}, \vec{b}, \vec{c}$ form a triangle

$\therefore \vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}, \vec{b}, \vec{c} \neq \vec{0}$

$\vec{u} + \vec{v} + \vec{w} = \cancel{(\vec{b}\cdot\vec{c})\vec{a}} - \cancel{(\vec{c}\cdot\vec{a})\vec{b}} + \cancel{(\vec{a}\cdot\vec{c})\vec{b}} - \cancel{(\vec{a}\cdot\vec{b})\vec{c}} + \cancel{(\vec{b}\cdot\vec{a})\vec{c}} - \cancel{(\vec{c}\cdot\vec{b})\vec{a}}$

$= \vec{0}$

$\therefore \vec{u}, \vec{v}, \vec{w}$ also forms a triangle

ii) $\vec{u}\cdot\vec{c} = \left[(\vec{b}\cdot\vec{c})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}\right]\cdot\vec{c}$

$= (\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{c}) - (\vec{c}\cdot\vec{a})(\vec{b}\cdot\vec{c})$

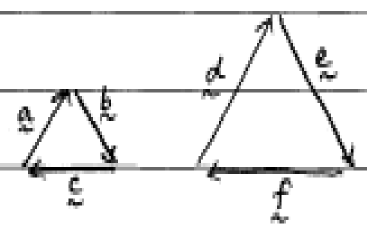
$= 0$

iii) $\vec{u}$ and $\vec{c}$ are perpendicular

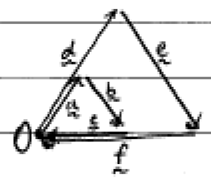
similarly $\vec{v}$ and $\vec{a}$ are perpendicular

$\vec{w}$ and $\vec{b}$ are perpendicular

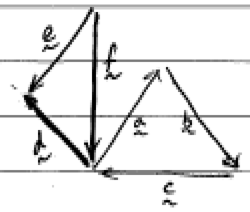
consider $\vec{d}, \vec{e}, \vec{f}$ which are in the same direction (respectively) as $\vec{a}, \vec{b}, \vec{c}$ and also form a triangle



If we make $\underset{\sim}{a}$ & $\underset{\sim}{d}$ position vectors
its easy to see that those two triangles will be similar



Now, consider $\vec{d}, \vec{e}, \vec{f}$ which are a $90°$ rotation anticlockwise of $\vec{a}, \vec{b}, \vec{c}$ respectively.



rotations preserve congruency and so $90°$ anticlockwise for each would also be fine.

What if $\vec{a}, \vec{b}$ were anticlockwise while $\vec{c}$ was clockwise

we would have vectors $\vec{d}, \vec{e}, -\vec{f}$

Assume $\vec{d}, \vec{e}, -\vec{f}$ also forms a triangle

ie $\vec{d} + \vec{e} + -\vec{f} = \vec{0}$ ___ ①

$\vec{d} + \vec{e} + \vec{f} = \vec{0}$ ___ ②

② − ①

$\vec{d} + \vec{e} + \vec{f} - (\vec{d} + \vec{e} - \vec{f}) = \vec{0}$

$2\vec{f} = \vec{0}$

$\vec{f} = \vec{0}$

A contradiction ∴ can't form a triangle

$\vec{u}, \vec{v}, \vec{w}$ are perpendicular to $\vec{c}, \vec{a}, \vec{b}$ respectively
Since $\vec{u}, \vec{v}, \vec{w}$ form a triangle
they must be a rotation of $90°$ in the same direction and thus the triangles formed by $\vec{a}, \vec{b}, \vec{c}$ and $\vec{u}, \vec{v}, \vec{w}$ will be similar.

Q53 ANS

ii) $\vec{OP} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 3 \\ -1 \\ -8 \end{pmatrix}$ from (i)

$= \frac{1}{3}\begin{pmatrix} 6 \\ 8 \\ -2 \end{pmatrix}$

14d

$|\vec{OA}| = \sqrt{1+3^2+2^2} = \sqrt{14}$

$|\vec{OB}| = \sqrt{4^2+2^2+6^2} = 2\sqrt{14}$

$\vec{OP} = \vec{OA} + \lambda\vec{AB}$

$= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -1 \\ -8 \end{pmatrix}$

$= \begin{pmatrix} 1+3\lambda \\ 3-\lambda \\ 2-8\lambda \end{pmatrix}$

In $\triangle OAP$, $\cos\theta = \dfrac{\vec{OA}\cdot\vec{OP}}{|\vec{OA}||\vec{OP}|}$

In $\triangle OBP$, $\cos\theta = \dfrac{\vec{OB}\cdot\vec{OP}}{|\vec{OB}||\vec{OP}|}$

$\therefore \dfrac{\vec{OA}\cdot\vec{OP}}{|\vec{OA}||\vec{OP}|} = \dfrac{\vec{OB}\cdot\vec{OP}}{|\vec{OB}||\vec{OP}|}$

$\vec{OA}\cdot\vec{OP} = \dfrac{\vec{OB}\cdot\vec{OP}}{2}$

$2\left[1+3\lambda+3(3-\lambda)+2(2-8\lambda)\right] = \left[4(1+3\lambda)+2(3-\lambda)+6(2-8\lambda)\right]$

$\lambda = \frac{1}{3}$

$\lambda = \dfrac{m}{m+n}$

$\dfrac{1}{3} = \dfrac{m}{m+n}$

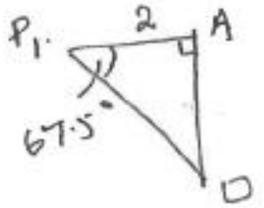
$m+n = 3m$

$n = 2m$

$\boxed{\dfrac{m}{n} = \dfrac{1}{2}}$

Q54 ANS

16 (b) Let O be the centre of the regular octahedron $P_1P_2P_3P_4P_5P_6P_7P_8$.



$|\vec{OA}| = 2\tan 67.5° = 2t$

$\tan 135° = \frac{2t}{1-t^2}$

$-1 = \frac{2t}{1-t^2}$

$t^2 - 2t - 1 = 0$

$t = 1 \pm \sqrt{2}$

as $t > 0$

$t = 1 + \sqrt{2}$

$\therefore |\vec{OA}| = 2(1+\sqrt{2})$ units

$\vec{OP_i}$ $(i = 1, 2, \dots 8)$ are the 8th roots of

$z^8 = |\vec{OP_i}|$

$\Rightarrow$ sum of the roots $\sum_{i=1}^{8} \vec{OP_i} = 0$

$\Rightarrow \sum_{i=1}^{8} (\vec{OA} + \vec{AP_i}) = 0$

$\Rightarrow \sum_{i=1}^{8} \vec{AP_i} = -8\,\vec{OB}$

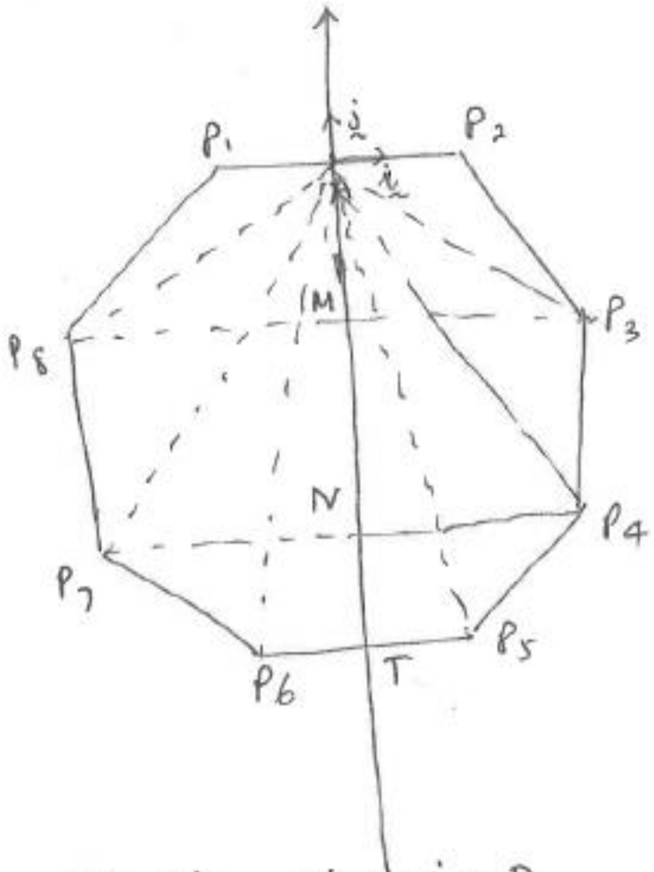
$\Rightarrow \left|\sum_{i=1}^{8} \vec{AP_i}\right| = 8|\vec{OA}|$

$= 8[2(1+\sqrt{2})]$

$= 16(1+\sqrt{2})$ units

ie $|\vec{AP_1} + \vec{AP_2} + \vec{AP_3} + \vec{AP_4} + \vec{AP_5} + \vec{AP_6} + \vec{AP_7} + \vec{AP_8}| = 16(1+\sqrt{2})$ units

16(b) Alternative solution 1



$$\overrightarrow{AP_1} + \overrightarrow{AP_2} = -2\underset{\sim}{i} + 2\underset{\sim}{i} = 0$$

$$\overrightarrow{AP_3} + \overrightarrow{AP_8} = 2\overrightarrow{AM} = -4\sqrt{2}\,\underset{\sim}{j}$$

$$\overrightarrow{AP_4} + \overrightarrow{AP_7} = 2\overrightarrow{AN} = -2(2\sqrt{2}+4)\underset{\sim}{j}$$

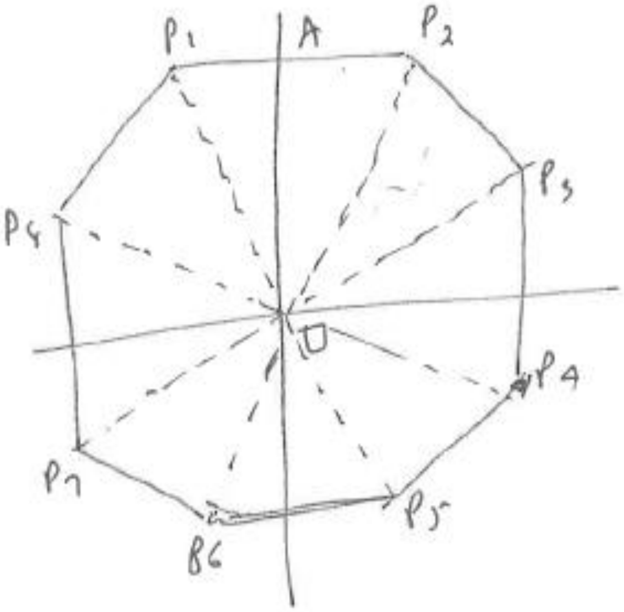
$$\overrightarrow{AP_5} + \overrightarrow{AP_6} = 2\overrightarrow{AT} = -2(2\sqrt{2}+4+2\sqrt{2})\underset{\sim}{j}$$

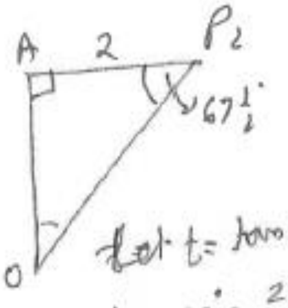
$$\therefore \left|\overrightarrow{AP_1} + \overrightarrow{AP_2} + \overrightarrow{AP_3} + \overrightarrow{AP_4} + \overrightarrow{AP_5} + \overrightarrow{AP_6} + \overrightarrow{AP_7} + \overrightarrow{AP_8}\right| = \left|\left(\left[-4\sqrt{2} - 2(2\sqrt{2}+4) - 2(2\sqrt{2}+4+2\sqrt{2})\right]\underset{\sim}{j}\right)\right|$$

$$= \left|(-16-16\sqrt{2})\underset{\sim}{j}\right|$$

$$= \underline{\underline{16(1+\sqrt{2})}} \text{ units}$$

16 (b) Alternative solution 2





$\overrightarrow{AP_1} = \overrightarrow{AO} + \overrightarrow{OP_1}$

$\overrightarrow{AP_2} = \overrightarrow{AO} + \overrightarrow{OP_2}$

$\overrightarrow{AP_3} = \overrightarrow{AO} + \overrightarrow{OP_3}$

$\overrightarrow{AP_4} = \overrightarrow{AO} + \overrightarrow{OP_4}$

$\overrightarrow{AP_5} = \overrightarrow{AO} + \overrightarrow{OP_5} = \overrightarrow{AO} - \overrightarrow{OP_1}$

$\overrightarrow{AP_6} = \overrightarrow{AO} + \overrightarrow{OP_6} = \overrightarrow{AO} - \overrightarrow{OP_2}$

$\overrightarrow{AP_7} = \overrightarrow{AO} + \overrightarrow{OP_7} = \overrightarrow{AO} - \overrightarrow{OP_3}$

$\overrightarrow{AP_8} = \overrightarrow{AO} + \overrightarrow{OP_8} = \overrightarrow{AO} - \overrightarrow{OP_4}$

Let $t = \tan 67.5^\circ$

$\Rightarrow \tan 135^\circ = \frac{2t}{1-t^2}$

$-1 = \frac{2t}{1-t^2}$

$t^2 - 2t - 1 = 0$

$t = 1 \pm \sqrt{2}$

$t > 0$

$\therefore t = 1+\sqrt{2}$

$|\overrightarrow{AO}| = 2t$

$= 2 + 2\sqrt{2}$

Adding: $\left|\overrightarrow{AP_1} + \overrightarrow{AP_2} + \overrightarrow{AP_3} + \overrightarrow{AP_4} + \overrightarrow{AP_5} + \overrightarrow{AP_6} + \overrightarrow{AP_7} + \overrightarrow{AP_8}\right| =$

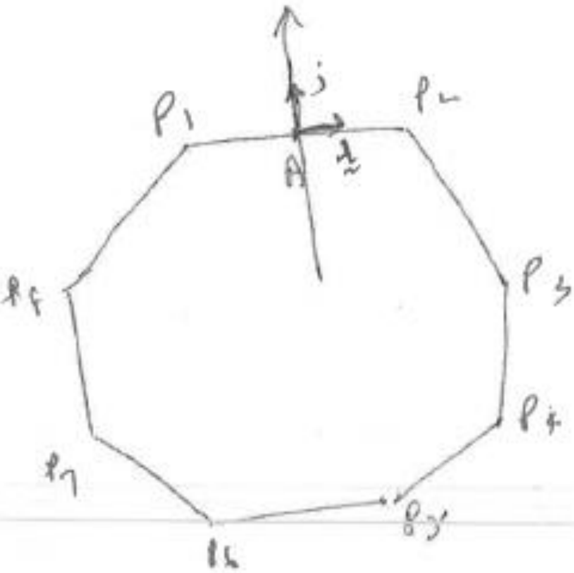
$|8\overrightarrow{AO}|$

$= 8|\overrightarrow{AO}|$

$= 8(2+2\sqrt{2})$

$= 16 + 16\sqrt{2}$ units

16(b) Alternative Solution 3



$$\overrightarrow{AP_2} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad \cancel{\overrightarrow{AP_1} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}}$$

$$\overrightarrow{AP_3} = \begin{pmatrix} 2+2\sqrt{2} \\ -2\sqrt{2} \end{pmatrix} \qquad \cancel{\overrightarrow{AP_8} = \begin{pmatrix} -2-2\sqrt{2} \\ -2\sqrt{2} \end{pmatrix}}$$

$$\overrightarrow{AP_4} = \begin{pmatrix} 2+2\sqrt{2} \\ -2-2\sqrt{2} \end{pmatrix}$$

$$\overrightarrow{AP_5} = \begin{pmatrix} 2 \\ -4-4\sqrt{2} \end{pmatrix}$$

$$\therefore \overrightarrow{AP_2} + \overrightarrow{AP_3} + \overrightarrow{AP_4} + \overrightarrow{AP_5} = \begin{pmatrix} 8+6\sqrt{2} \\ -8-8\sqrt{2} \end{pmatrix}$$

By symmetry $\overrightarrow{AP_1} + \overrightarrow{AP_8} + \overrightarrow{AP_7} + \overrightarrow{AP_6} = \begin{pmatrix} -8-6\sqrt{2} \\ -8-8\sqrt{2} \end{pmatrix}$

$$\therefore \left| \overrightarrow{AP_1} + \overrightarrow{AP_2} + \overrightarrow{AP_3} + \overrightarrow{AP_4} + \overrightarrow{AP_5} + \overrightarrow{AP_6} + \overrightarrow{AP_7} + \overrightarrow{AP_8} \right| = \left| \begin{pmatrix} 0 \\ -16-16\sqrt{2} \end{pmatrix} \right|$$

$$= (16+16\sqrt{2}) \text{ units}$$

Q55 ANS

$$\therefore \begin{pmatrix} 2+2\lambda \\ 1+\lambda \\ 1+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0$$

① recognising radius $\perp$ tangent

$$4+4\lambda+1+\lambda+2+4\lambda=0$$
$$9\lambda+7=0$$
$$\lambda=\frac{-7}{9}$$

① valid progress

$\therefore$ Equation of sphere is

$$\left| \underset{\sim}{r} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right| = |\overrightarrow{CP}|$$

$$= \left| \begin{pmatrix} 4/9 \\ 2/9 \\ -5/9 \end{pmatrix} \right|$$

$$\left| \underset{\sim}{r} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right| = \frac{\sqrt{5}}{3}$$

① final

or

$$(x-1)^2+y^2+(z-1)^2=\frac{5}{9}$$

11e) i) Given $\overrightarrow{OA} = 4\underset{\sim}{i}$, $\overrightarrow{OB} = 4\underset{\sim}{j}$, $\overrightarrow{OC} = 3\underset{\sim}{k}$, $\overrightarrow{DC} = \underset{\sim}{j}$

$$\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BO} + \overrightarrow{OD}$$
$$= \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BO} + \overrightarrow{OD}$$
$$= \frac{1}{2}(4\underset{\sim}{j} - 4\underset{\sim}{i}) - 4\underset{\sim}{j} + 3\underset{\sim}{k}$$
$$= 2\underset{\sim}{j} - 2\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$$
$$= -2\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$$

Better written in this order.

$$\overrightarrow{ON} = \overrightarrow{OB} + \overrightarrow{BN}$$
$$= \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$$
$$= 4\underset{\sim}{j} + \frac{1}{3}\left((3\underset{\sim}{k} + \underset{\sim}{j}) - 4\underset{\sim}{j}\right)$$
$$= 4\underset{\sim}{j} + \underset{\sim}{k} - \underset{\sim}{j}$$
$$= 3\underset{\sim}{j} + \underset{\sim}{k}$$

ii) Let the required angle be θ.
Then
$$\overrightarrow{MD} \cdot \overrightarrow{ON} = |MD||ON|\cos\theta$$

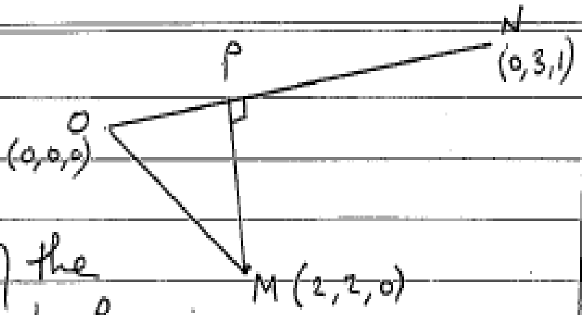
$$\cos\theta = \left|\frac{\overrightarrow{MD} \cdot \overrightarrow{ON}}{|\overrightarrow{MD}||\overrightarrow{ON}|}\right|$$

Abs. value to make angle acute,

$$= \left|\frac{0 - 6 + 3}{\sqrt{17}\,\sqrt{10}}\right|$$
$$= \frac{3}{\sqrt{170}}$$

$\Rightarrow$ $\theta = 76°42'$ to nearest minute

$76{\cdot}7°$, $103°18'$, $103{\cdot}3°$ also accepted as question was not very discerning iii)



Let point of intersection of the perpendicular be P.

<u>Method 1</u> Find OP, the projection of OM on to ON. Then use Pythagoras on ΔOPM.

$OP = \text{Proj}_{\vec{ON}} \vec{OM} = \dfrac{\vec{OM} \cdot \vec{ON}}{|\vec{ON}|^2} \vec{ON}$ — 1 for correct projection formula

$= \dfrac{0+6+0}{3^2+1^2} \vec{ON}$

$= \dfrac{6}{10} \vec{ON}$

$\therefore |\vec{OP}| = \dfrac{3}{5} |\vec{ON}|$

$= \dfrac{3\sqrt{10}}{5}$ — 1 for a correct formulation for answer

Now $|PM|^2 = OM^2 - OP^2$ (Pythagoras)

$= 8 - 90/25$

$= 110/25$

$= 22/5$

$\therefore |PM| = \sqrt{\dfrac{22}{5}}$ units. — 1 for correct answer, suitably presented (ie. enough working)

(N.B. The diagram was useful to see what was required.

iii) Method 2 Find the co-ordinates of P then calculate the length of the resulting vector MP.

$\overrightarrow{OP} = \lambda\overrightarrow{ON}$ for some $\lambda \in \mathbb{R}$

$= 3\lambda \underset{\sim}{j} + \lambda \underset{\sim}{k}$

Now $\overrightarrow{OP} \cdot \overrightarrow{PM} = 0$ (Perpendicular)

i.e. $(3\lambda \underset{\sim}{j} + \lambda \underset{\sim}{k}) \cdot (2\underset{\sim}{i} + (2-3\lambda)\underset{\sim}{j} - \lambda \underset{\sim}{k}) = 0$

$\therefore 3\lambda(2-3\lambda) - \lambda^2 = 0$

$6\lambda - 9\lambda^2 - \lambda^2 = 0$

$10\lambda^2 = 6\lambda$ | 1 for $\lambda = 3/5$ or

$\lambda = 3/5 \quad (\lambda \neq 0)$ | equivalent

$\therefore \overrightarrow{OP} = \frac{9}{5}\underset{\sim}{j} + \frac{3}{5}\underset{\sim}{k}$

$\therefore \overrightarrow{PM} = 2\underset{\sim}{i} + \frac{1}{5}\underset{\sim}{j} - \frac{3}{5}\underset{\sim}{k}$

$\therefore |PM| = \sqrt{4 + \frac{1}{25} + \frac{9}{25}}$ A | 1 for a form of $|PM|$

$= \sqrt{\frac{100+1+9}{25}}$ B

$= \sqrt{\frac{110}{25}}$ C

$= \sqrt{\frac{22}{5}}$ units. D | 1 for correctly deduced answer (It is NOT satisfactory to jump from A to D.)

Q56 ANS

d) i) $\begin{pmatrix}-1\\0\\3\end{pmatrix} - \begin{pmatrix}-3\\-1\\4\end{pmatrix} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$

$\therefore l_1 : \underset{\sim}{r} = \begin{pmatrix}-1\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}, \quad \lambda \in \mathbb{R}$

1 mark for correctly finding the direction vector

OR

1 mark for correct solution, defining lambda

$r = \begin{pmatrix}-3\\-1\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}, \quad \lambda \in \mathbb{R}$

ii) $l_2: \underset{\sim}{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}, \quad \lambda, a \in \mathbb{R}$

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\right| \left|\begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}\right| \cos\theta$$

1 mark for setting this statement

1 mark for the correct solution

Case I $\theta = \frac{\pi}{4}$

$$2a+1+1 = \sqrt{6} \cdot \sqrt{a^2+2} \cdot \frac{1}{\sqrt{2}}$$

$$2a+2 = \sqrt{3(a^2+2)}$$

$$(2a+2)^2 = 3(a^2+2)$$

$$a^2+8a-2 = 0$$

$$\therefore a = -4+3\sqrt{2}$$

(reject negative)

Case II $\theta = \frac{3\pi}{4}$

$$2a+1+1 = \sqrt{6} \cdot \sqrt{a^2+2} \cdot \left(-\frac{1}{\sqrt{2}}\right)$$

$$2a+2 = -\sqrt{3(a^2+2)}$$

$$(2a+2)^2 = 3(a^2+2)$$

$$a^2+8a-2 = 0$$

$$\therefore a = -4-3\sqrt{2}$$

(reject positive)

$$\therefore a = -4 \pm 3\sqrt{2}$$

iii) $\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}$

$$\begin{cases} -1+2\lambda = a\mu & (1) \\ \lambda = 1+\mu & (2) \\ 3-\lambda = 2-\mu & (3) \end{cases}$$

1 mark for setting equations 1 and 2 OR equations 1 and 3

1 mark for the correct solution

Sub (2) into (1): $-1+2(1+\mu) = a\mu$

$$1+2\mu = a\mu$$

$$\therefore \mu = \frac{1}{a-2} \quad (a \neq 2)$$

$\therefore$ The point of intersection $= \left(\frac{a}{a-2}, -1+\frac{1}{a-2}, 2-\frac{1}{a-2}\right)$

$$= \left(\frac{a}{a-2}, \frac{a-1}{a-2}, \frac{2a-5}{a-2}\right)$$ ✓

Q57 ANS

$\overrightarrow{OM}=\frac{1}{2}\left(\underset{\sim}{p}+\underset{\sim}{q}\right)$ Then $\overrightarrow{OM}\cdot\overrightarrow{PQ}=\frac{1}{2}\left(\underset{\sim}{p}+\underset{\sim}{q}\right)\cdot\left(\underset{\sim}{q}-\underset{\sim}{p}\right)=\frac{1}{2}\left(\underset{\sim}{q}\cdot\underset{\sim}{q}-\underset{\sim}{p}\cdot\underset{\sim}{p}\right)=0$ since $OP=OQ$ (radii)

Then $\angle OMN=\angle OMQ+\angle QMN=90^\circ+90^\circ=180^\circ$. Hence O, M, N are collinear.

$$\overrightarrow{ON}=\overrightarrow{OB}+\overrightarrow{BN}=-\underset{\sim}{a}+\lambda\,\overrightarrow{BH}=-\underset{\sim}{a}+\lambda\left(\overrightarrow{BA}+\overrightarrow{AH}\right)=-\underset{\sim}{a}+\lambda\left(2\underset{\sim}{a}+\underset{\sim}{h}\right)$$

$$\therefore\ \overrightarrow{ON}=\left(2\lambda-1\right)\underset{\sim}{a}+\lambda\underset{\sim}{h}$$

$AH\perp PQ\Rightarrow\underset{\sim}{h}\cdot\left(\underset{\sim}{q}-\underset{\sim}{p}\right)=0$ and O, M, N collinear $\Rightarrow\overrightarrow{ON}\cdot\overrightarrow{PQ}=0$ since $MN\perp PQ$.

$$\therefore\left\{\left(2\lambda-1\right)\underset{\sim}{a}+\lambda\underset{\sim}{h}\right\}\cdot\left(\underset{\sim}{q}-\underset{\sim}{p}\right)=0$$

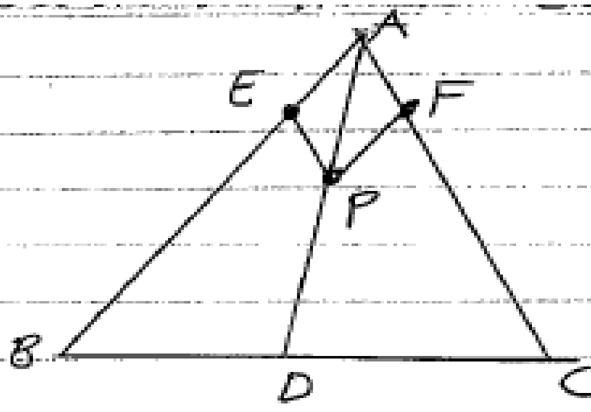
$$\therefore\left(2\lambda-1\right)\underset{\sim}{a}\cdot\left(\underset{\sim}{q}-\underset{\sim}{p}\right)=0$$

But AB, PQ are not perpendicular $\Rightarrow\underset{\sim}{a}\cdot\left(\underset{\sim}{q}-\underset{\sim}{p}\right)\neq0$. Hence $2\lambda-1=0$. $\therefore\lambda=\frac{1}{2}$ and $BN=\frac{1}{2}BH$.

Hence N is the midpoint of BH.

Q58 ANS

Q(15)(c)



(i) $\overrightarrow{BD} = \lambda \overrightarrow{BC}$

$= \lambda(\overrightarrow{BA} + \overrightarrow{AC})$

$= \lambda(-\underset{\sim}{p} + \underset{\sim}{q})$

$= \lambda(\underset{\sim}{q} - \underset{\sim}{p})$.

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= \underset{\sim}{p} + \lambda(\underset{\sim}{q} - \underset{\sim}{p})$

(ii) Letting $\overrightarrow{AP} = k\overrightarrow{AD}$

$\overrightarrow{AP} = k[\underset{\sim}{p} + \lambda\underset{\sim}{q} - \lambda\underset{\sim}{p}]$

$\overrightarrow{AP} = (k - k\lambda)\underset{\sim}{p} + k\lambda\underset{\sim}{q}$ (1)

As $\overrightarrow{AP} = \overrightarrow{AE} + \overrightarrow{EP}$

$\overrightarrow{AP} = \frac{1}{3}\underset{\sim}{p} + \frac{1}{5}\underset{\sim}{q}$. (2)

$\therefore$ Equating (1) & (2):

$\frac{1}{3}\underset{\sim}{p} + \frac{1}{5}\underset{\sim}{q} = (k - k\lambda)\underset{\sim}{p} + k\lambda\underset{\sim}{q}$

$\therefore k - k\lambda = \frac{1}{3}$ & $k\lambda = \frac{1}{5}$.

$\therefore k - \frac{1}{5} = \frac{1}{3} \Rightarrow k = \frac{8}{15}$.

As $k\lambda = \frac{1}{5}$, $\frac{8}{15}\lambda = \frac{1}{5}$

$\lambda = \frac{1}{5} \times \frac{15}{8}$

$= \frac{3}{8}$.

$\therefore \lambda = \frac{3}{8}$ & $\overrightarrow{BD} = \frac{3}{8}\overrightarrow{BC}$.

Q59 ANS

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \qquad \boxed{A}\\ &= \overrightarrow{OA} + \overrightarrow{BC}\\ &= \begin{pmatrix}2\\2\\2\end{pmatrix} + \begin{pmatrix}0-1\\5-2\\7-5\end{pmatrix}\\ &= \begin{pmatrix}2\\2\\2\end{pmatrix} + \begin{pmatrix}-1\\3\\2\end{pmatrix}\\ &= \begin{pmatrix}1\\5\\4\end{pmatrix} \qquad \boxed{B}\end{aligned}$$

Alternative solution:

Let D be $\begin{pmatrix}x\\y\\z\end{pmatrix}$

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{BA}\\ \begin{pmatrix}x-0\\y-5\\z-7\end{pmatrix} &= \begin{pmatrix}2-1\\2-2\\2-5\end{pmatrix} \qquad \boxed{A}\\ \begin{pmatrix}x-0\\y-5\\z-7\end{pmatrix} &= \begin{pmatrix}1\\0\\-3\end{pmatrix}\\ \begin{pmatrix}x\\y\\z\end{pmatrix} &= \begin{pmatrix}1\\0\\-3\end{pmatrix} + \begin{pmatrix}0\\5\\7\end{pmatrix}\\ &= \begin{pmatrix}1\\5\\4\end{pmatrix} \qquad \boxed{B}\end{aligned}$$

Question 11 (c) (ii)

Criteria	M
• Provides correct solution	

Sample answer:

The midpoint of $\overrightarrow{AC}$ is $\frac{1}{2}\left[\begin{pmatrix}2\\2\\2\end{pmatrix} + \begin{pmatrix}0\\5\\7\end{pmatrix}\right] = \begin{pmatrix}1\\3.5\\4.5\end{pmatrix}$

The midpoint of $\overrightarrow{BD}$ is $\frac{1}{2}\left[\begin{pmatrix}1\\2\\5\end{pmatrix} + \begin{pmatrix}1\\5\\4\end{pmatrix}\right] = \begin{pmatrix}1\\3.5\\4.5\end{pmatrix}$ $\boxed{A}$ (midpoints coincide)

Since the midpoints of the two diagonals coincide the diagonals bisect each other.

Q60 ANS

Both lines pass through $\begin{pmatrix}1\\2\\1\end{pmatrix}$, which must be A. $\boxed{A}$

$$|\overrightarrow{AD}| = \sqrt{(5-1)^2+(4-2)^2+(5-1)^2}$$
$$= 6$$

$|\overrightarrow{AB}| = |\overrightarrow{AD}| = 6$ since $ABCD$ is a rhombus.

$$\therefore \left|\lambda\begin{pmatrix}1\\2\\2\end{pmatrix}\right| = 6 \qquad \boxed{B}$$

$$\lambda \times \sqrt{1^2+2^2+2^2} = 6$$
$$\lambda \times 3 = 6$$
$$\lambda = 2$$

$$\therefore B \text{ is } \begin{pmatrix}1\\2\\1\end{pmatrix} + 2\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}3\\6\\5\end{pmatrix} \qquad \boxed{C}$$

$$\cos\angle BAD = \frac{\begin{pmatrix}2\\4\\4\end{pmatrix}\cdot\begin{pmatrix}4\\2\\4\end{pmatrix}}{6\times 6} \qquad \boxed{A}$$
$$= \frac{8+8+16}{36}$$
$$= \frac{32}{36}$$
$$\angle BAD = \cos^{-1}\left(\frac{8}{9}\right) \qquad \boxed{B}$$
$$\text{Area} = 2\times\left(\frac{1}{2}\times 6\times 6\times\sin\left(\cos^{-1}\frac{8}{9}\right)\right) \qquad \text{area comprises two congruent triangles}$$
$$= 16.4924225\ldots$$
$$= 16.49 \text{ units}^2 \text{ (2 dp)} \qquad \boxed{C}$$

Alternative solution:

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \begin{pmatrix}3\\6\\5\end{pmatrix} + \begin{pmatrix}5-1\\4-2\\5-1\end{pmatrix} = \begin{pmatrix}7\\8\\9\end{pmatrix} \qquad \boxed{A}$$
$$|AC| = \sqrt{(7-1)^2+(8-2)^2+(9-1)^2} = \sqrt{136}$$
$$|BD| = \sqrt{(5-3)^2+(4-6)^2+(5-5)^2} = \sqrt{8} \qquad \boxed{B}$$
$$\text{Area} = \frac{1}{2}\times|AC|\times|BD|$$
$$= \frac{1}{2}\times\sqrt{136}\times\sqrt{8}$$
$$= 16.49 \text{ units}^2 \text{ (2 dp)} \qquad \boxed{C}$$

Q61 ANS

$$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} = \frac{1}{2}\underset{\sim}{a} \quad \text{and} \quad \overrightarrow{OE} = \lambda\overrightarrow{OA} = \lambda\underset{\sim}{a}$$

$$\overrightarrow{ON} = \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}\underset{\sim}{b} \quad \text{and} \quad \overrightarrow{OF} = \mu\overrightarrow{OB} = \mu\underset{\sim}{b}$$

$$\left|\overrightarrow{OM}\right|\left|\overrightarrow{OE}\right|\cos\theta = \overrightarrow{OM}\cdot\overrightarrow{OE}$$

$$\left|\overrightarrow{OM}\right|\left|\overrightarrow{OE}\right|\cos 0 = \frac{1}{2}\underset{\sim}{a}\cdot\lambda\underset{\sim}{a}$$

$$\therefore \left|\overrightarrow{OM}\right|\left|\overrightarrow{OE}\right| = \frac{1}{2}\lambda\underset{\sim}{a}\cdot\underset{\sim}{a} \quad ...[1]$$

$$\text{similarly } \left|\overrightarrow{ON}\right|\left|\overrightarrow{OB}\right| = \frac{1}{2}\mu\underset{\sim}{b}\cdot\underset{\sim}{b} \quad ...[2]$$

now, $\overrightarrow{OA} \perp \overrightarrow{BE}$

so, $\overrightarrow{OA}\cdot\overrightarrow{BE} = 0$

$$\overrightarrow{OA}\cdot\left(\overrightarrow{OE} - \overrightarrow{OB}\right) = 0$$

$$\underset{\sim}{a}\cdot\left(\lambda\underset{\sim}{a} - \underset{\sim}{b}\right) = 0$$

$$\lambda\underset{\sim}{a}\cdot\underset{\sim}{a} = \underset{\sim}{a}\cdot\underset{\sim}{b} \quad ...[3]$$

similarly, $\overrightarrow{OB} \perp \overrightarrow{AF}$

$$\mu\underset{\sim}{b}\cdot\underset{\sim}{b} = \underset{\sim}{a}\cdot\underset{\sim}{b} \quad ...[4]$$

$$\therefore \quad \lambda\underset{\sim}{a}\cdot\underset{\sim}{a} = \mu\underset{\sim}{b}\cdot\underset{\sim}{b} \qquad \left(\textit{from } [3] \,\&\, [4]\right)$$

$$\frac{1}{2}\lambda\underset{\sim}{a}\cdot\underset{\sim}{a} = \frac{1}{2}\mu\underset{\sim}{b}\cdot\underset{\sim}{b}$$

$$\therefore \left|\overrightarrow{OM}\right|\left|\overrightarrow{OE}\right| = \left|\overrightarrow{ON}\right|\left|\overrightarrow{OF}\right| \qquad \left(\textit{from } [1] \,\&\, [2]\right)$$

Q62 ANS B

5. $l_2: \begin{pmatrix} 4+\mu \\ 2+\mu \\ 1 \end{pmatrix}$ $l_1: \begin{pmatrix} 1+2\lambda \\ -\lambda \\ 1 \end{pmatrix}$

① $4+\mu = 1+2\lambda$

② $2+\mu = -\lambda \Rightarrow \mu = -\lambda - 2$ Sub into ①

③ $1 = 1$

$4-\lambda-2 = 1+2\lambda$

$3\lambda = 1$

$\lambda = \frac{1}{3}$

$\boxed{B}$

Q63 ANS B

The cylinder is parallel to the y-axis, and the cross-sectional circle is a circle centred at the origin, passing through $x = -5$, $x = 5$, $z = -5$ and $z = 5$.

The equation of the cylinder is $x^2 + z^2 = 25$.

Answer: B

Q64 ANS D

3. direction of $\overrightarrow{AB}$ is $\begin{pmatrix} -2-1 \\ 4-3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$\therefore$ equation can only be D. or $-\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

D

Q65 ANS A

Q66 ANS A

5 A

Looking for the dot product of the direction vectors being zero.

$$\text{A: } \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = -2 + 2 + 0 = 0, \text{ so A.}$$

Q67 ANS B

Q6. distance $= \sqrt{(3+3)^2 + 0^2 + (-2-2)^2}$

$= \sqrt{52} < 52$

$\therefore$ point is inside sphere (but not at centre) $\therefore$ B

Q68 ANS A

(A)

$\begin{pmatrix} 3 \\ -6 \\ -15 \end{pmatrix} = -3\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$

Q69 ANS B

Q70 ANS

The required line has direction $\begin{pmatrix} -3-3 \\ 4-6 \\ -2-4 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \\ -6 \end{pmatrix}$

Using $(3, 6, 4)$, the equation becomes

$$r = (3i + 6j + 4k) + \lambda(-6i - 2j - 6k)$$

Using $(-3, 4, -2)$

$$r = (-3i + 4j - 2k) + \lambda(-6i - 2j - 6k)$$

Can use $\begin{pmatrix} 3--3 \\ 6-4 \\ 4--2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix}$ as the direction vector.

Using $(3, 6, 4)$, the equation becomes

$$r = (3i + 6j + 4k) + \lambda(6i + 2j + 6k)$$

Using $(-3, 4, -2)$

$$r = (-3i + 4j - 2k) + \lambda(6i + 2j + 6k)$$

Q71 ANS

$$r = \begin{pmatrix}2\\3\end{pmatrix} + s\begin{pmatrix}1\\2\end{pmatrix} \text{ and } r = \begin{pmatrix}3\\2\end{pmatrix} + t\begin{pmatrix}1\\4\end{pmatrix}$$

$$r = \begin{pmatrix}2+s\\3+2s\end{pmatrix} \qquad r = \begin{pmatrix}3+t\\2+4t\end{pmatrix}$$

$$2 + s = 3 + t \;---(1)$$

$$3 + 2s = 2 + 4t \;---(2)$$

From (1)

$$s = t + 1$$

$$\therefore 3 + 2(t+1) = 2 + 4t$$

$$3 + 2t + 2 = 2 + 4t$$

$$3 = 2t$$

$$t = 1\frac{1}{2}$$

$$\therefore \text{Point of intersection} = \begin{pmatrix}3 + 1\frac{1}{2}\\2+6\end{pmatrix} = \begin{pmatrix}4\frac{1}{2}\\8\end{pmatrix}$$

Q72 ANS

In vector form, we have

$$\begin{pmatrix}1\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\3\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1+\lambda\\-\lambda\\3+2\lambda\end{pmatrix}=\begin{pmatrix}1+2\mu\\-1-\mu\\4+3\mu\end{pmatrix}$$

From the first component, $\lambda = 2\mu$.

In the second component, $\lambda = 1+\mu$.

Solving $2\mu = 1+\mu$ gives $\mu = 1 \Rightarrow \lambda = 2$

Checking the third component, $3+2\lambda = 7$ and $4+3\mu = 7$

Since both vectors agree for all components when $\mu = 1, \lambda = 2$

the lines intersect at a point $\begin{pmatrix}3\\-2\\7\end{pmatrix}$. Hence the lines lie in the same plane.

-

$$\text{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}1\\-1\\2\end{pmatrix}+q\begin{pmatrix}2\\-1\\3\end{pmatrix}$$

Note: equivalent answers can be had with $\begin{pmatrix}3\\-2\\7\end{pmatrix}$ replaced with $\begin{pmatrix}1\\0\\3\end{pmatrix}$ or $\begin{pmatrix}1\\-1\\4\end{pmatrix}$

$$\begin{aligned}\cos\theta &= \frac{\begin{pmatrix}2\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\2\end{pmatrix}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}\\ &= \frac{2+1+6}{\sqrt{14}\sqrt{6}}\\ &= \frac{9}{\sqrt{84}}\\ \therefore \theta &= 10^\circ 53' 36.22'' \approx 10^\circ 54' \text{ to nearest minute}\end{aligned}$$

Q73 ANS

Let P be a point on $\mathbf{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$, say $P=\begin{pmatrix}1+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}$ and let $A=\begin{pmatrix}3\\1\\2\end{pmatrix}$.

Then $\overrightarrow{DP}=\begin{pmatrix}1+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}-\begin{pmatrix}3\\1\\2\end{pmatrix}=\begin{pmatrix}2\lambda-2\\-1-\lambda\\\lambda-1\end{pmatrix}$ and $\left|\overrightarrow{DP}\right|$ is minimised when $\overrightarrow{DP}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$

i.e. when $2(2\lambda-2)-(-1-\lambda)+(\lambda-1)=0$

$$6\lambda-4=0 \quad \therefore \lambda=\frac{2}{3}$$

Then $P=\begin{pmatrix}7/3\\-2/3\\5/3\end{pmatrix}$ and the minimum distance is

$$\left|\begin{pmatrix}7/3\\-2/3\\5/3\end{pmatrix}-\begin{pmatrix}3\\1\\2\end{pmatrix}\right|=\left|\begin{pmatrix}-2/3\\-5/3\\-1/3\end{pmatrix}\right|=\sqrt{\left(\frac{-2}{3}\right)^2+\left(\frac{-5}{3}\right)^2+\left(\frac{-1}{3}\right)^2}=\frac{\sqrt{30}}{3}$$

Q74 ANS

(i)

$r(\lambda) = \begin{bmatrix}-1\\1\\2\end{bmatrix} + \lambda\begin{bmatrix}2\\1\\2\end{bmatrix}$

$A = (-1, 1, 2) \qquad B = (1, 2, 4)$

$\vec{AB} = \begin{bmatrix}2\\1\\2\end{bmatrix} \qquad P = (2, -3, 4)$

(i) $\vec{AP} = \begin{bmatrix}2--1\\-3-1\\4-2\end{bmatrix}$

$= \begin{bmatrix}3\\-4\\2\end{bmatrix}$ ✓

$\text{Proj}_{\underset{\sim}{b}}\vec{AP} = \dfrac{\underset{\sim}{b}\cdot\vec{AP}}{|\underset{\sim}{b}|^2}\underset{\sim}{b}$

$= \dfrac{2\times3 - 1\times4 + 2\times2}{2^2+1^2+2^2}\begin{bmatrix}2\\1\\2\end{bmatrix}$

$= \dfrac{6}{9}\begin{bmatrix}2\\1\\2\end{bmatrix}$

$= \dfrac{2}{3}\begin{bmatrix}2\\1\\2\end{bmatrix}$ ✓

let d be perpendicular distance from P to l

$\therefore d = |\text{Proj}_{\underset{\sim}{b}}\vec{AP} - \vec{AP}|$

$= \left|\begin{bmatrix}\frac{4}{3}-3\\\frac{2}{3}+4\\\frac{4}{3}-2\end{bmatrix}\right|$

$= \sqrt{(-5/3)^2 + (14/3)^2 + (-2/3)^2}$

$= \sqrt{\dfrac{225}{9}}$

$=$ 5 units ✓

(ii)

Let C be some point on the line l:

$C = \begin{bmatrix}-1\\1\\2\end{bmatrix} + \mu\begin{bmatrix}2\\1\\2\end{bmatrix}$ for some μ.

ie. $C = (-1+2\mu,\ 1+\mu,\ 2+2\mu)$

$\therefore \vec{AC} = \begin{bmatrix}2\mu\\\mu\\2\mu\end{bmatrix}$

if the area $\Delta ACP = 15\,u^2$ we have

$\frac{1}{2}\times|\vec{AC}|\,d = 15.$

$\therefore \quad \frac{1}{2}\times|\vec{AC}|\times5 = 15$

$|\vec{AC}| = 6$ ✓

So $\sqrt{(2\mu)^2 + (\mu)^2 + (2\mu)^2} = 6$

$9\mu^2 = 36$

$\mu^2 = 4$

$\therefore \mu = \pm2$

So $C = (-1+4,\ 1+2,\ 2+4)$ if $\mu = 2$

$= (3, 3, 6)$

or $C = (-1-4,\ 1-2,\ 2-4)$ if $\mu = -2$ ✓

$= (-5, -1, -2)$

Q75 ANS

13(b) (i)	Line l_1 intersects the line l_2 then: $(11\underset{\sim}{i} + 2\underset{\sim}{j} + 17\underset{\sim}{k}) + \lambda(-2\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k})$ $= (-5\underset{\sim}{i} + 11\underset{\sim}{j} + p\underset{\sim}{k}) + \mu(-3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$ $11 - 2\lambda = -5 - 3\mu$ ① $2 + \lambda = 11 + 2\mu$ ② $17 - 4\lambda = p + 2\mu$ ③ Equation ① + 2 × ② $15 = 17 + \mu$ then $\mu = -2$ Equation ② $2 + \lambda = 11 - 4$ then $\lambda = 5$ Equation ③ $17 - 4 \times 5 = p + 2 \times -2$ $p = 17 - 20 + 4$ $= 1$ $\therefore$ The value of p is 1.
13(b) (ii)	From 13(b)(i) $\lambda = 5,\ \mu = -2$ and $p = 1$ l_1: $(11\underset{\sim}{i} + 2\underset{\sim}{j} + 17\underset{\sim}{k}) + 5(-2\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}) = \underset{\sim}{i} + 7\underset{\sim}{j} + -3\underset{\sim}{k}$ or l_2: $(-5\underset{\sim}{i} + 11\underset{\sim}{j} + \underset{\sim}{k}) + -2(-3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}) = \underset{\sim}{i} + 7\underset{\sim}{j} + -3\underset{\sim}{k}$ $\therefore$ Point of intersection is $\underset{\sim}{i} + 7\underset{\sim}{j} + -3\underset{\sim}{k}$.

Q76 ANS

i) parallel $\therefore$ same direction vector

$$l_2 = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$x = 1 + 2\mu \qquad \therefore \mu = \frac{x-1}{2}$$

$$y = -2 + 5\mu$$

$$\text{and } y = -2 + \left(\frac{x-1}{2}\right) \cdot 5$$

$$y = -2 + \frac{5x}{2} - \frac{5}{2}$$

$$y = \frac{5x}{2} - \frac{9}{4}$$

Q77 ANS

i) $\underset{\sim}{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda+1 \\ \lambda \\ 2\lambda+3 \end{pmatrix}$

$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ ✓

ii) Equation of sphere:

$\left| \underset{\sim}{r} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right| = \sqrt{29}$

$\left| \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right| = \sqrt{29}$

$\left| \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \right| = \sqrt{29}$

$\therefore (\lambda-1)^2 + (\lambda+1)^2 + (2\lambda+3)^2 = 29$ ✓

$\lambda^2 - 2\lambda + 1 + \lambda^2 + 2\lambda + 1 + 4\lambda^2 + 12\lambda + 9 = 29$

$6\lambda^2 + 12\lambda - 18 = 0$

$\lambda^2 + 2\lambda - 3 = 0$

$(\lambda+3)(\lambda-1) = 0$

$\therefore \lambda = -3$ or $\lambda = 1$ ✓

when $\lambda = -3$: $P = (1-3, 0-3, 3-6) = (-2, -3, -3)$

When $\lambda = 1$: $Q = (1+1, 0+1, 3+2) = (2, 1, 5)$ ✓

* Points should not be written as column vectors.

iii) $PQ = \sqrt{(-2-2)^2 + (-3-1)^2 + (-3-5)^2}$

$= \sqrt{16+16+64}$

$= \sqrt{96}$

$= 2 \times \sqrt{24}$

$\neq 2 \times \sqrt{29}$ ✓ (where $\sqrt{29}$ is the radius of sphere S).

$\therefore$ PQ is <u>NOT</u> a diameter of the sphere.

Q78 ANS

$\boxed{Q15}$ c/i) $\underset{\sim}{r_1} = \lambda \begin{bmatrix} \cos\phi + \sqrt{3} \\ \sqrt{2}\sin\phi \\ \cos\phi - \sqrt{3} \end{bmatrix} \quad : L_1$

$\underset{\sim}{r_2} = \mu \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad : L_2$

$\underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = |\underset{\sim}{r_1}| \cdot |\underset{\sim}{r_2}| \cdot \cos\theta$

$\cos\theta = \dfrac{\underset{\sim}{r_1} \cdot \underset{\sim}{r_2}}{|\underset{\sim}{r_1}| \cdot |\underset{\sim}{r_2}|} = \dfrac{\cos\phi + \sqrt{3} + 0 + \sqrt{3} - \cos\phi}{|\underset{\sim}{r_1}| \cdot |\underset{\sim}{r_2}|}$ ✓

$\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{(\cos\phi + \sqrt{3})^2 + (\sqrt{2}\sin\phi)^2 + (\cos\phi - \sqrt{3})^2} \times \sqrt{1^2 + 0^2 + (-1)^2}}$

$\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{\cos^2\phi + 2\sqrt{3}\cos\phi + 3 + 2\sin^2\phi + \cos^2\phi - 2\sqrt{3}\cos\phi + 3} \times \sqrt{2}}$

$\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{2(\sin^2\phi + \cos^2\phi) + 6} \times \sqrt{2}}$ ✓

$\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{8} \cdot \sqrt{2}} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

$\cos\theta$ is independent of ϕ. ✓

OR $\theta = \dfrac{\pi}{6}$. The acute angle between L_1 and L_2 is independent of ϕ.

$\boxed{Q15}$

c(ii) $l_2 : \underset{\sim}{r} = \mu\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}\mu\\0\\-\mu\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$

subs into the plane

$$x - z = 4\sqrt{3}$$
$$\mu - -\mu = 4\sqrt{3}$$
$$2\mu = 4\sqrt{3}$$
$$\mu = 2\sqrt{3}$$

$\therefore$ The coordinate of C : $C(x, y, z)$

$C(2\sqrt{3}, 0, -2\sqrt{3})$ ✓

c(iii) $l_1 : x = \lambda(\cos\phi + \sqrt{3}),\ y = \sqrt{2}\lambda\sin\phi$

$z = \lambda(\cos\phi - \sqrt{3})$

subs into the plane: $x - z = 4\sqrt{3}$

$$\lambda(\cos\phi + \sqrt{3}) - \lambda(\cos\phi - \sqrt{3}) = 4\sqrt{3}\ ✓$$
$$\lambda . 2\sqrt{3} = 4\sqrt{3}$$
$$\lambda = 2\ ✓$$

$P\left(2(\cos\phi + \sqrt{3}),\ 2\sqrt{2}\sin\phi,\ 2(\cos\phi - \sqrt{3})\right)$

$PC = \sqrt{(2\cos\phi)^2 + (2\sqrt{2}\sin\phi)^2 + (2\cos\phi)^2} \overset{✓}{=} \sqrt{8}$

$PC = 2\sqrt{2}$ $\therefore$ as ϕ varies, P traces out a circle of centre C and radius $2\sqrt{2}$.

AW 3 Marks for Clearly Showing the radius $= 2\sqrt{2}$ units.

Q79 ANS

(b) (i)

$\vec{AB} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$|\vec{AB}| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}$

Vector eqn of sphere is $\left| \underset{\sim}{s} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right| = \sqrt{5}$

b)(ii)

$l_1: \quad \underset{\sim}{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 + \mu \\ 4 - 3\mu \\ 2 + 2\mu \end{pmatrix}$

Sub $\underset{\sim}{r}$ into equation of sphere

$$\left| \begin{pmatrix} -1+\mu \\ 4-3\mu \\ 2+2\mu \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right| = \sqrt{5}$$

$$\left| \begin{matrix} \mu \\ 2-3\mu \\ -1+2\mu \end{matrix} \right| = \sqrt{5}$$

$$\mu^2 + (2-3\mu)^2 + (-1+2\mu)^2 = 5$$

$$\mu^2 + 4 - 12\mu + 9\mu^2 + 1 - 4\mu + 4\mu^2 = 5$$

$$14\mu^2 - 16\mu + 5 = 5$$

$$14\mu^2 - 16\mu = 0$$

$$2\mu(7\mu - 8) = 0$$

$$\mu = 0, \ \frac{8}{7}$$

$\mu = 0$: point is $(-1, 4, 2)$

$\mu = \frac{8}{7}$ point is $\left(\frac{1}{7}, \frac{4}{7}, \frac{30}{7}\right)$

Q80 ANS

For L_1, let $\lambda \in \mathbb{R}$ such that:

$$\lambda = \frac{x-8}{2}, \lambda = \frac{z+3}{4} \text{ and } y = 7$$

i.e. $x = 2\lambda + 8$, $z = 4\lambda - 3$ and $y = 0\lambda + 7$

$$\therefore L_1 = \begin{pmatrix} 8 \\ 7 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

For L_2, let $\mu \in \mathbb{R}$ such that:

$$\mu = \frac{x-1}{k}, \mu = \frac{y-5}{-8} \text{ and } \mu = \frac{z}{3}$$

i.e. $x = k\mu + 1$, $y = 5 - 8\mu$ and $z = 3\mu$

$$\therefore L_2 = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} k \\ -8 \\ 3 \end{pmatrix}$$

If $L_1 \perp L_2$, then $\begin{pmatrix} k \\ -8 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} = 0$

i.e $2k + 12 = 0 \quad \therefore k = -6$

Q81 ANS

Let $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$

i.e. $x: -11 + 3\lambda = -1 + \mu \quad (1)$

$y: \quad 17 - 4\lambda = -5 + 3\mu \quad (2)$

$z: \quad -7 + 2\lambda = -3 + 2\mu \quad (3)$

$2 \times (1) - (3) \Rightarrow -15 + 4\lambda = 1$

$\therefore \lambda = 4$

substituting $\lambda = 4$ into (1) gives $\mu = 2$.

Test by substituting $\lambda = 4$ and $\mu = 2$ into (2)
i.e. $17 - 4 \times 4 = -5 + 3 \times 2$ which is true.

$\therefore$ the lines intersect at $P(1, 1, 1)$

$$\overrightarrow{PQ} = \begin{pmatrix} 2-1 \\ 6-1 \\ 8-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}$$

Therefore the line required has equation $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}$ where $\gamma \in \mathbb{R}$.

Q82 ANS

$$
\begin{aligned}
\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} \\
&= \overrightarrow{OA} + \overrightarrow{AB} + \frac{1}{5}\overrightarrow{AB} \\
&= \overrightarrow{OA} + \frac{6}{5}\overrightarrow{AB} \\
\therefore \underset{\sim}{c} &= \underset{\sim}{a} + \frac{6}{5}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \\
&= \frac{1}{5}\left(6\underset{\sim}{b} - \underset{\sim}{a}\right)
\end{aligned}
$$

Q83 ANS

Let $P \in \underset{\sim}{r_1}$ and $Q \in \underset{\sim}{r_2}$ such that

P has coordinates $(1-\lambda, -1, 3+2\lambda)$,
Q has coordinates $(-1+2\mu, 1+2\mu, 4+\mu)$

$$\overrightarrow{PQ} = \begin{pmatrix} -2+\lambda+2\mu \\ 2+2\mu \\ 1-2\lambda+\mu \end{pmatrix}$$

$\overrightarrow{PQ}$ is shortest when it is perpendicular to both $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$.

i.e. $\begin{pmatrix} -2+\lambda+2\mu \\ 2+2\mu \\ 1-2\lambda+\mu \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = 0 \qquad (1)$

and $\begin{pmatrix} -2+\lambda+2\mu \\ 2+2\mu \\ 1-2\lambda+\mu \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 0 \qquad (2)$

From (1), $2-\lambda-2\mu+2-4\lambda+2\mu = 0 \Rightarrow \lambda = \frac{4}{5}$

From (2), $-4+2\lambda+4\mu+4+4\mu+1-2\lambda+\mu = 0 \Rightarrow \mu = -\frac{1}{9}$

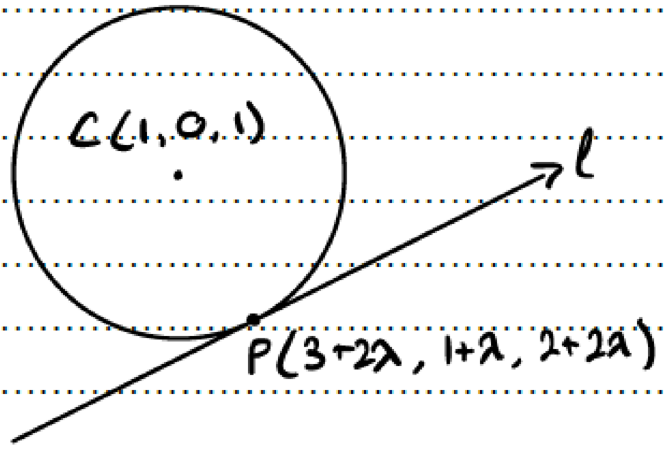
$$\text{Hence, } \overrightarrow{PQ} = \begin{pmatrix} -2+\frac{4}{5}+2\times-\frac{1}{9} \\ 2+2\times-\frac{1}{9} \\ 1-2\times\frac{4}{5}+-\frac{1}{9} \end{pmatrix} = \frac{1}{9}\begin{pmatrix} -\frac{64}{5} \\ 16 \\ -\frac{32}{5} \end{pmatrix}$$

$$\left|\overrightarrow{PQ}\right| = \frac{1}{9}\sqrt{\left(-\frac{64}{5}\right)^2 + 16^2 + \left(-\frac{32}{5}\right)^2} = \frac{16}{15}\sqrt{5}$$

Therefore the shortest distance between $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$ is $\frac{16}{15}\sqrt{5}$ units.

Q84 ANS

d,



Let $C = (1, 0, 1)$ be the centre of the sphere and P the point of tangency between the sphere and line.

Then since P lies on the line, it has coordinates $(3+2\lambda, 1+\lambda, 2+2\lambda)$ for some λ.

Now, we want $\overrightarrow{CP} \cdot \underset{\sim}{v} = 0$, where $\underset{\sim}{v}$ is the direction of l

Q85 ANS

(a)	$\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}=\dfrac{\underset{\sim}{v}\cdot\underset{\sim}{u}}{\underset{\sim}{u}\cdot\underset{\sim}{u}}\times\underset{\sim}{u}$ $=\dfrac{2a-2+2}{4+1+4}\times\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ $\begin{pmatrix}\frac{4}{9}\\-\frac{2}{9}\\-\frac{4}{9}\end{pmatrix}=\dfrac{2a}{9}\times\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ $\dfrac{9}{2}\begin{pmatrix}\frac{4}{9}\\-\frac{2}{9}\\-\frac{4}{9}\end{pmatrix}=a\times\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ $\begin{pmatrix}2\\-1\\-2\end{pmatrix}=a\times\begin{pmatrix}2\\-1\\-2\end{pmatrix}\qquad\therefore a=1$
(b)	$\overrightarrow{AB}=-3\underset{\sim}{i}-\underset{\sim}{k}-(\underset{\sim}{i}-\underset{\sim}{j})\qquad=-4\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$ $\overrightarrow{BC}=2\underset{\sim}{i}+a\underset{\sim}{j}+b\underset{\sim}{k}-(-3\underset{\sim}{i}-\underset{\sim}{k})\;=5\underset{\sim}{i}+a\underset{\sim}{j}+(b+1)\underset{\sim}{k}$ $\overrightarrow{BC}=\lambda\overrightarrow{AB}$ $5\underset{\sim}{i}+a\underset{\sim}{j}+(b+1)\underset{\sim}{k}=\lambda(-4\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k})$ $-4\lambda=5\qquad a=\lambda\qquad b+1=-\lambda$ $\lambda=-\dfrac{5}{4}\;\rightarrow\qquad =-\dfrac{5}{4}\qquad b=\dfrac{5}{4}-1=\dfrac{1}{4}$

(c)(i)

$$\text{radius} = \sqrt{(-3-3)^2 + (-5+3)^2 + (10-6)^2}$$
$$= \sqrt{56} = 2\sqrt{14}$$

centre is $(-3,-5,10)$

$$\therefore S_1 \text{ is } \left| \underset{\sim}{u} - \begin{pmatrix} -3 \\ -5 \\ 10 \end{pmatrix} \right| = 2\sqrt{14}$$

(c)(ii)

cartesian equation of S_1 is

$$(x+3)^2 + (y+5)^2 + (z-10)^2 = 56$$

(c)(iii)

Distance between centres is

$$\sqrt{(-9+3)^2 + (4+5)^2 + (7-10)^2}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$
$$= \sqrt{14} + 2\sqrt{14} \quad (\underline{\text{sum of the two radii}})$$

$\therefore$ S_1 & S_2 meet at a single point

(c)(iv)

Equate $\underset{\sim}{v}$ and $\underset{\sim}{u}$ (ie sub $\underset{\sim}{v}$ into S_1)

$$\left| \begin{pmatrix} -6+2\lambda \\ -3+\lambda \\ 11+\lambda \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \\ 10 \end{pmatrix} \right| = 2\sqrt{14}$$

$$\left| \begin{pmatrix} -3+2\lambda \\ 2+\lambda \\ 1+\lambda \end{pmatrix} \right| = 2\sqrt{14}$$

$$(-3+2\lambda)^2 + (2+\lambda)^2 + (1+\lambda)^2 = \left(2\sqrt{14}\right)^2$$
$$9 - 12\lambda + 4\lambda^2 + 4 + 4\lambda + \lambda^2 + 1 + 2\lambda + \lambda^2 = 56$$
$$6\lambda^2 - 6\lambda - 42 = 0$$
$$\lambda^2 - \lambda - 7 = 0$$
$$\lambda = \frac{1 \pm \sqrt{29}}{2}$$

Q86 ANS

Solutions : GHS 4U 2023 Trial p.6

(13)(a)(i) $\begin{pmatrix}-9\\-14\\-3\end{pmatrix}+\lambda_1\begin{pmatrix}3\\2\\2\end{pmatrix}=\begin{pmatrix}23\\-4\\-27\end{pmatrix}+\lambda_2\begin{pmatrix}10\\1\\-16\end{pmatrix}$

Equating x: $-9+3\lambda_1=23+10\lambda_2$
$3\lambda_1-10\lambda_2=32$ (1) ✓

Equating y: $-14+2\lambda_1=-4+\lambda_2$
$2\lambda-\lambda_2=10$ (2) ✓

Equating z: $-3+2\lambda_1=-27-16\lambda_2$
$2\lambda_1+16\lambda_2=-24$ (3)

(3)−(2) $2\lambda_1+16\lambda_2=-24$ (3)
$2\lambda_1-\lambda_2=10$ (2)
$17\lambda_2=-34$
$\lambda_2=-2.$

Sub. $\lambda_2=-2$ in (2)
$2\lambda_1+2=10$ ✓
$\lambda_1=4.$

Be careful. students must sub. into an equation already used here.

Sub. $\lambda_1=4, \lambda_2=-2$ in (1) to check skewness: $3\times4-10\times-2=32.$ ✓ → MUST do this!

So point of intersection
$=\begin{pmatrix}-9\\-14\\-3\end{pmatrix}+4\begin{pmatrix}3\\2\\2\end{pmatrix}$
$=\begin{pmatrix}3\\-6\\5\end{pmatrix}.$

(ii) For 1st line
$x=-9+3\lambda_1, y=-14+2\lambda_1, z=-3+2\lambda_1.$
Sub. in to sphere,
$(x-1)^2+(y+2)^2+(z-4)^2=21.$
$(3\lambda_1-10)^2+(2\lambda_1-12)^2+(2\lambda_1-7)^2=21.$
$17\lambda_1^2-136\lambda_1+293=21.$
$\lambda^2-8\lambda+16=0$ $\xrightarrow{-21,\ \div17}$

Could also show tangent using $\Delta=(-8)^2-4\times1\times16=0.$

$(\lambda_1-4)^2=0 \Rightarrow \lambda_1=4.$ Only 1 point of intersection → tangent.

Point of intersection $=\begin{pmatrix}-9\\-14\\-3\end{pmatrix}+4\begin{pmatrix}3\\2\\2\end{pmatrix}=\begin{pmatrix}3\\-6\\5\end{pmatrix}.$

Q.(13)(a)(iii) Let direction vector of line $\perp$ to both

$$= \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = 0 \quad \& \begin{pmatrix} 10 \\ 1 \\ -16 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = 0$$

$3x_1 + 2y_1 + 2z_1 = 0 \ (1)$

$10x_1 + y_1 - 16z_1 = 0 \ (2) \times 2 = (3)$

$3x_1 + 2y_1 + 2z_1 = 0 \ (1) \ -$

$20x_1 + 2y_1 - 32z_1 = 0 \ (3)$

$-17x_1 + 34z_1 = 0 \ \div -17$

$x_1 \quad 2z_1 = 0$

$x_1 = \cdot 2z_1.$

Sub. in (1): $\cdot 6z_1 + 2y_1 + 2z_1 = 0$

$y_1 = -4z_1.$

Letting $z_1 = 1, x_1 = -2, y_1 = 2.$

Line $\perp$ to both

$$= \begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix} + \lambda_3 \begin{pmatrix} \cdot 2 \\ -4 \\ 1 \end{pmatrix}.$$

If Line 3 [just found] is radius, then Line 2 is also a tangent as it is $\perp$ to radius.

Finding if $\begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix} + \lambda_3 \begin{pmatrix} \cdot 2 \\ -4 \\ 1 \end{pmatrix}$ passes through $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$

$x: 3 + 2\lambda_3 = 1. \ \lambda_3 = -1.$

Sub. $\lambda_3 = -1$ in Line 3:

$$\begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix} - 1 \begin{pmatrix} \cdot 2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} = \text{sphere centre.}$$

As Line 2 is $\perp$ to radius at point of contact on surface of sphere, line $2 = \begin{pmatrix} 23 \\ -4 \\ -27 \end{pmatrix} + \lambda_2 \begin{pmatrix} 10 \\ 1 \\ -16 \end{pmatrix}$ is ALSO tangent to sphere.

NOTE: Could also use "or otherwise" & show only 1 point of intersection of Line 2 & Sphere.

Q87 ANS

15 (d) $\ell_1: \underset{\sim}{r} = \begin{pmatrix} 10-\lambda \\ \lambda \\ -9+2\lambda \end{pmatrix}$ 2 $\ell_2: \underset{\sim}{r} = \begin{pmatrix} 17+5\mu \\ 1-\mu \\ 3+3\mu \end{pmatrix}$

for ℓ_1 and ℓ_2 to cross, solve for λ and μ.

$10-\lambda = 17+5\mu$ (1) and $\lambda = 1-\mu$ (2)

Sub (2) into (1):

$10-(1-\mu) = 17+5\mu$

$9-17 = 4\mu$

$-8 = 4\mu$ so $\mu = -2$ ✓

and $\lambda = 1-(-2)$ (from (2))

$= 3$ ✓

Check: $-9+2\lambda = 3+3\mu$ ✓

when $\lambda = 3$, LHS $= -9+2\times 3 = -3$

$\mu = -2$, RHS $= 3+3\times -2 = -3 =$ LHS

$\therefore$ ℓ_1 and ℓ_2 cross at: $\begin{pmatrix} 10-3 \\ 3 \\ -9+2\times 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix}$ ✓

$7\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$

Q88 ANS

$$\begin{pmatrix} 9 \\ k \\ 24 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$$

So, $9 = -1 + 2\mu \quad \Rightarrow \quad \mu = 5.$
Further, $k = 2 - 3\mu \quad \Rightarrow \quad k = -13.$

From $\underset{\sim}{i}$, $7 + \lambda = -1 + 2\mu \quad \Rightarrow \quad \lambda = -8 + 2\mu$, and from $\underset{\sim}{j}$, $-1 + 3\lambda = 2 - 3\mu$

Substituting the first into the second gives: $-1 + 3(-8 + 2\mu) = 2 - 3\mu$

$9\mu = 2 + 24 + 1$

Which gives $\mu = 3$ and $\lambda = -2$. Substituting in L_1 or L_2 gives the point $(5, -7, 12)$.

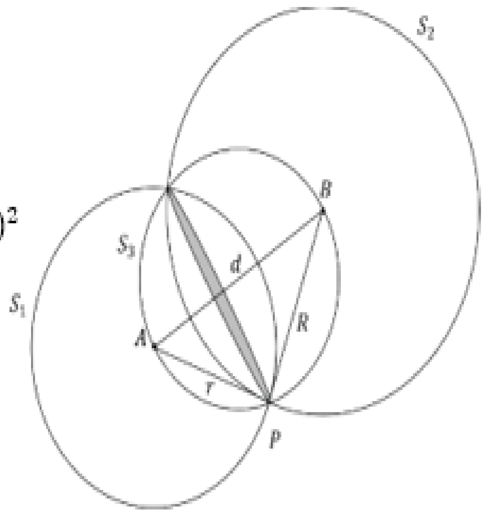
Q89 ANS

$$\text{LHS}=\left|\begin{pmatrix}5\\-10\\3\end{pmatrix}-\begin{pmatrix}3\\-12\\4\end{pmatrix}\right|=\sqrt{2^2+2^2+(-1)^2}=3=\text{RHS}$$

Centre of sphere is $(3,-12,4)$, and the point on the sphere farthest from the origin would be 3 units (one radius) farther along the vector $\overrightarrow{OC}$.
Now, $|\overrightarrow{OC}|=\sqrt{3^2+(-12)^2+4^2}=13$, so the point P on the surface of the sphere farthest from O will be given by: $\overrightarrow{OP}=\dfrac{13+3}{13}\begin{pmatrix}3\\-12\\4\end{pmatrix}$. Hence the point $P=\left(\dfrac{48}{13},-\dfrac{192}{13},\dfrac{64}{13}\right)$.

Q90 ANS

Sample answer:



$$d^2=\left|\overrightarrow{AB}\right|^2=(a+m-a)^2+(b+n-b)^2+(c-p-c)^2$$
$$=m^2+n^2+p^2 \qquad (1) \quad \boxed{A}$$

Case 1: If $m^2+n^2+p^2=r^2+R^2$

From (1): $\qquad d^2=r^2+R^2 \qquad (2)$

Let P be any point on C.

From (2): $\left|\overrightarrow{AB}\right|^2=\left|\overrightarrow{AP}\right|^2+\left|\overrightarrow{PB}\right|^2$ so ΔAPB is right angled at P $\quad \boxed{B}$

$\therefore \overrightarrow{AP}\cdot\overrightarrow{PB}=0$

$\therefore P$ lies on the sphere S_3 with diameter AB

Since P is any point on C then C lies on S_3 if $m^2+n^2+p^2=r^2+R^2$. $\quad \boxed{C}$

Case 2: If $m^2+n^2+p^2\neq r^2+R^2$

From (1): $\qquad d^2\neq r^2+R^2 \qquad (3)$

$\therefore \left|\overrightarrow{AB}\right|^2\neq\left|\overrightarrow{AP}\right|^2+\left|\overrightarrow{PB}\right|^2$ so ΔAPB is not right angled at P

$\therefore \overrightarrow{AP}\cdot\overrightarrow{PB}\neq 0$

$\therefore P$ does not lie on the sphere S_3.

Since P is any point on C then C does not lie on S_3 if $m^2+n^2+p^2\neq r^2+R^2$. $\quad \boxed{D}$

$\therefore$ the circle C lies on the sphere S_3 whose diameter is AB
only if $m^2+n^2+p^2=r^2+R^2$

$x^2 + y^2 + z^2 = 25$ is a sphere with centre $(0,0,0)$ and radius 5, so $r = 5$.
$(x-3)^2 + (y-4)^2 + (z-12)^2 = 144$ is a sphere with centre $(3, 4, 12)$ and radius 12,
so $m = 3, n = 4, p = 12$ and $R = 12$.

$$m^2 + n^2 + p^2 = 3^2 + 4^2 + 12^2 = 169$$
$$r^2 + R^2 = 5^2 + 12^2 = 169$$
$$\therefore\ m^2 + n^2 + p^2 = r^2 + R^2 \qquad \boxed{A}$$

So from (i) we can see that the two spheres intersect in a circle that lies on a sphere with a diameter whose ends are at the centres of the spheres.

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13 (d)(i) $(x-3)^2 + (y+3)^2 + (z-4)^2 = 6^2 = 36$ ✓ ①

(ii) $\underset{\sim}{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+2\lambda \\ 5-\lambda \\ 4-\lambda \end{pmatrix}$

sub $x = 1+2\lambda$, $y = 5-\lambda$ and $z = 4-\lambda$ into ①:

$(1+2\lambda-3)^2 + (5-\lambda+3)^2 + (4-\lambda-4)^2 = 36$

$(2(\lambda-1))^2 + (8-\lambda)^2 + (-\lambda)^2 = 36$

$4(\lambda^2-2\lambda+1) + 64 - 16\lambda + \lambda^2 + \lambda^2 = 36$

$4\lambda^2 + 2\lambda^2 - 8\lambda - 16\lambda + 4 + 64 = 36$

$6\lambda^2 - 24\lambda + 68 - 36 = 0$

$6\lambda^2 - 24\lambda + 32 = 0$ ✓

$3\lambda^2 - 12\lambda + 16 = 0$

for λ, check $\Delta = (-12)^2 - 4 \times 3 \times 16$

$= 144 - 192$

$= -48 < 0$

$\therefore$ there is no real solution for λ ✓

and so the line and sphere do not touch $\therefore$ the line is not a tangent to the sphere. ✓