Subject : Dynamic Model of a small glider   Date:

For zero thrust:

For flat earth: $\dot{h} = V\sin\gamma$

$\dot{x} = V\cos\chi\cos\gamma$

$\dot{y} = V\sin\chi\cos\gamma$

$\dot{V} = -\frac{D}{m} - g\sin\gamma$

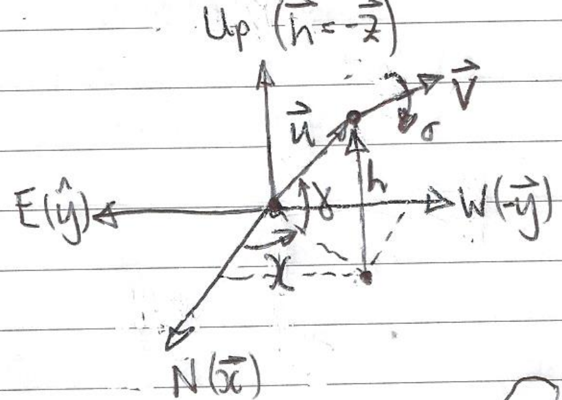
$\dot{\gamma} = \frac{1}{mV}(L\cos\sigma + C\sin\sigma) - \frac{g}{V}\cos\gamma$

$\dot{\chi} = \frac{1}{mV\cos\gamma}(L\sin\sigma - C\cos\sigma)$

$$\begin{cases} V = \|\vec{V}\| \\ h = -z \\ \gamma = \text{elevation angle} \\ \chi = \text{azimuth angle} \\ \sigma = \text{bank angle} \end{cases}$$

D, C, L is the aerodynamic forces

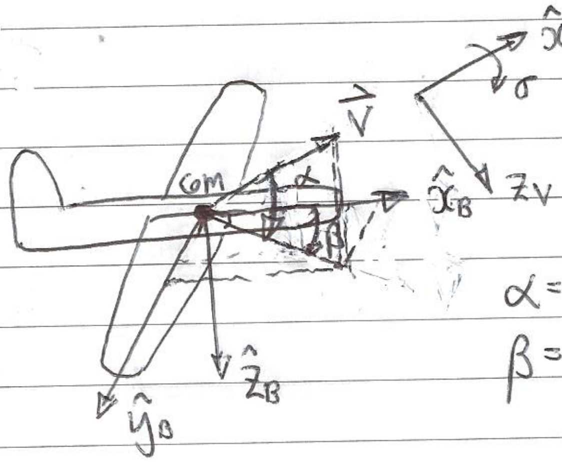
Velocity $\vec{V} = [\dot{x} \ \dot{y} \ \dot{z}]^T$, Position $\vec{u} = \begin{cases} \hat{x} = \text{North direction} \\ \hat{y} = \text{East direction} \\ \hat{z} = \text{Down direction} \end{cases}$



Euler 3 2 1

$\hat{u}_V = R_{VI}(\chi, \gamma, \sigma)$ Velocity frame axis

$= [\hat{x}_V, \hat{y}_V \ \hat{z}_V]^T$



α = angle of attack

β = sideslip angle

Aerodynamic force vector:

$$F = -D\hat{x}_V - C\hat{y}_V - L\hat{z}_V$$

$$\hat{u}_B = [\hat{x}_b \ \hat{y}_b \ \hat{z}_b]^T = R_{BV}(-\beta, \alpha, 0)\,\hat{u}_V$$

$(0.0254\,m = 1'')$

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$D = \bar{q}\, S\, C_D(\alpha, \beta)$ (Drag force)
$C = \bar{q}\, S\, C_c(\beta)$ (Side force)
$L = \bar{q}\, S\, C_L(\alpha)$ (Lift force)

$\bar{q} = \frac{1}{2} \rho V^2$

Estimation of aerodynamic coeff :

l = wingspan = $102.4''$ (2.6m)
$AR = \frac{l^2}{S} = 11.08$ $\Rightarrow$ $S = 946.45$ inch^2 ($61.06\ dm^2$)
W = 5 lb (2.254 kg)

$\therefore$

Measured
$l_h = 0.86$ m
$l_v = 0.93$ m

S_F = Fuselage area $\simeq 221$ inch^2 (for $102.4''$ wingspan)
l_t = Fuselage moment arm $= 0.28\, l = 28.7''$ (0.73 m)
V_H = ~~0.4~~ 0.56 Horizontal tail volume ratio
V_V = ~~0.02~~ 0.037 Vertical tail volume ratio
$\bar{c} = 1.02\, l / AR$ (inch) $= 9.43''$ (0.24 m) = mean chord
e = Oswald efficiency factor = 0.95

$0.56 = \frac{S_H l_h}{S \bar{c}} = \frac{0.096 \times 0.86}{0.61 \times 0.24}$
$0.037 = \frac{S_V l_v}{S\, l} = \frac{0.063 \times 0.93}{0.61 \times 2.6}$

Infinite-span wing with aerodynamic characteristics (Reynolds No 150,000) :
$a_0 = 0.1 \frac{180}{\pi} = 5.73\ rad^{-1}$ (lift curve slope)
$\alpha_0 = -2.5 \frac{\pi}{180} = -0.0436$ rad (zero point)

C_d = wing profile drag $= 0.01 + 0.05 (C_L - 0.4)^2$
$C_{DF} = 0.008$ = fuselage drag coeff
$C_{DT} = 0.01$ = tail drag coeff
$C_{DE} = 0.002$ = misc extra drag

$S = l^2 / AR$ = Wing surface area $= 946.45$ inch^2 ($0.6105\ m^2$)
$0.096\, m^2 = S_T = V_H \bar{c} S / l_t = 124.4$ inch^2 (~~0.0803~~ m^2) Horizontal tail surface area
$0.063\, m^2 = S_V = V_V\, l\, S / l_t = 67.5$ inch^2 (~~0.0435~~ m^2) Vertical tail surface area

$C_{L\alpha} = \frac{a_0}{1 + a_0 / (\pi e AR)}$ (Finite wing slope) $= 4.883$
$\Rightarrow C_L(\alpha) = C_{L\alpha} (\alpha - \alpha_0)$ (Glider lift coeff)

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Drag coeff $C_D(\alpha)$:

$$C_{D_0} = C_{DF}\frac{S_F}{S} + C_{DT}\frac{S_T+S_V}{S} + C_{DE} + C_{d_0}$$

$$= 0.008\,\frac{221}{946.45} + 0.01\,\frac{\cancel{124.4+67.5}}{\cancel{946.45}}\left(\frac{0.096+0.063}{0.61}\right) + 0.002 + 0.01 = 0.016$$

Fuselage   tail wings   extra   wing

$$\Rightarrow C_D(\alpha) = C_{D_0} + C_{dL}\left(C_L(\alpha) - C_{L_{min}}\right)^2 + C_L^2(\alpha)/(\pi e AR)$$

$$= 0.016 + 0.05\left(C_L(\alpha) - 0.4\right)^2 + C_L^2(\alpha)/33.05$$

Drag due to sideslip β :   (Assume $AR_V = 0.5AR = 5.54$)

$$C_{C\beta} = \frac{a_0}{1 + a_0/(\pi e AR_V)}\left(\frac{S_V}{S}\right) = \cancel{0.303}\ (0.44)$$

$$\Rightarrow C_c(\beta) = C_{c\beta}\cdot\beta = \cancel{0.303}\beta\ \ 0.44\beta$$

$$\therefore C_D(\beta) = C_c^2(\beta)\,\frac{1}{\pi e AR_V}\left(\frac{S}{S_V}\right) = C_c^2(\beta)\,\cancel{0.8485}\,(0.59)$$

$$\Rightarrow \therefore C_D(\alpha,\beta) = 0.016 + 0.05\left(C_L(\alpha) - 0.4\right)^2 + C_L^2(\alpha)/33.05 + C_c^2(\beta)\,\cancel{0.8485}\ 0.59$$

($m = 2.5$ kg)

Alt(m)	Vtrim	αtrim
1000	11.25 m/s	4.2°
2000	11.8 m/s	4.2°
3000	12.5 m/s	4.2°
4000	13.4 m/s	4.2°
5000	14.5 m/s	4.2°
6000	16.0 m/s	4.2°
7000	18.0 m/s	4.2°
8000	21.0 m/s	4.2°
9000	26.2 m/s	4.2°
10000	38.2 m/s	4.2°
500	10.97 m/s	4.2°

~~Vtrim = 2e-14.x^4 - 3e-10.x^3 + 1e-6 x^2 - 0.0023x + 12.046~~

$\Rightarrow$ Lookup table

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Input to simulation:

m	glider mass
α, β, σ	angles (assume constant)
$\vec{u}(0)$	initial position
$\chi(0), \gamma(0)$	initial heading and elevation
$\vec{V}(0)$	initial velocity vector

t = 0 to end:

Calculate from $\alpha, \beta \Rightarrow L, D, C$

Integrate $\dot{V}, \dot{\gamma}, \dot{\chi}$

Integrate $\dot{x}, \dot{y}, \dot{z}$

Output $\vec{u}(t), V(t), \gamma(t), \chi(t)$

Positional Dynamics of Glider

Subject : Date:

XYZ = Forces in glider body axis coordinates
UVW = Linear velocity vector coordinates in body axis
PQR = Angular velocity coordinates in body axis

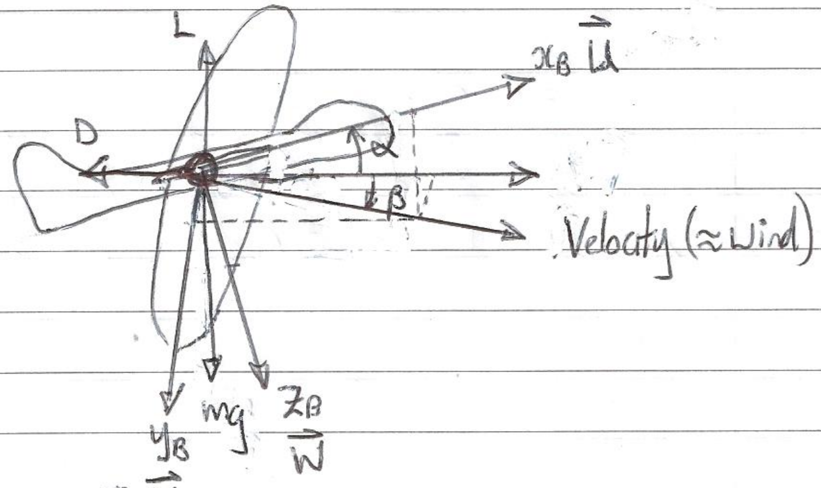
$$X = m(\dot{U} - VR + WQ)$$
$$Y = m(\dot{V} + UR - WP)$$
$$Z = m(\dot{W} - UQ + VP)$$

$$\begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \omega_{xi} \\ \omega_{yi} \\ \omega_{zi} \end{bmatrix}$$

$$\begin{bmatrix} \dot{N} \\ \dot{E} \\ \dot{D} \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = \underset{B/I}{A}(\psi, \theta, \phi) \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad \begin{cases} \psi = \text{Yaw} \\ \theta = \text{Pitch} \\ \phi = \text{Roll} \end{cases}$$

$$\alpha = \tan^{-1}\left(\frac{W}{U}\right), \quad \beta = \tan^{-1}\left(\frac{V}{U}\right), \quad \bar{V} = \sqrt{U^2 + V^2 + W^2}$$

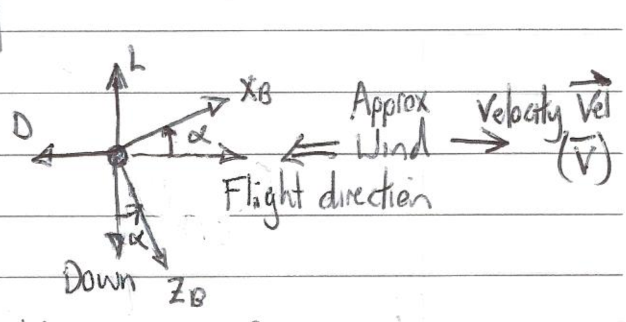
$$\begin{bmatrix} X^G \\ Y^G \\ Z^G \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} mg$$



$$\begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix} = qS \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} = \underbrace{\underset{W/B}{A}(-\beta, \alpha, 0)}_{\approx \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ +\sin\alpha & 0 & \cos\alpha \end{bmatrix}} \begin{bmatrix} -C_D \\ -C_C \\ -C_L \end{bmatrix} qS \quad \left\{ q = \tfrac{1}{2}\rho\bar{V}^2 \right\}$$

Appox small angles:

$$\Rightarrow C_X = -C_D\cos\alpha + C_L\sin\alpha$$
$$C_Y = -C_C$$
$$C_Z = -C_L\cos\alpha - C_D\sin\alpha$$



$\Rightarrow$ Calculate C_L, C_D, C_C according to glider model in NASA Paper
$\{C_L(\alpha), C_D(\alpha,\beta), C_C(\beta)\}$

Rotational Dynamics of Glider

Subject : Date:

LMN = Moment vector coordinates in body axis (RPY)
UVW = Linear Velocity vector coordinates in body axis ($X_B Y_B Z_B$)
PQR = Angular velocity coordinates in body axis
δ_E, δ_R = Elevator and rudder deflections

$$\bar{V} = \sqrt{u^2 + v^2 + w^2} \qquad \begin{cases} u = \bar{V}\cos\alpha\cos\beta \\ v = \bar{V}\sin\beta \\ w = \bar{V}\sin\alpha\cos\beta \end{cases} \qquad \begin{aligned} \alpha &= \text{AoA} \\ \beta &= \text{Sideslip} \end{aligned}$$

$$\begin{cases} L = \dot{P} I_{xx} + QR(I_{zz} - I_{yy}) \\ M = \dot{Q} I_{yy} + PR(I_{xx} - I_{zz}) \\ N = \dot{R} I_{zz} + PQ(I_{yy} - I_{xx}) \end{cases} \qquad \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \omega_{xi} \\ \omega_{yi} \\ \omega_{zi} \end{bmatrix} \qquad \begin{aligned} I_{xx} &= 0.2\ \text{kgm}^2 \\ I_{yy} &= 0.36\ \text{kgm}^2 \\ I_{zz} &= 0.525\ \text{kgm}^2 \end{aligned}$$

Aerodynamic Moments: $q = \frac{1}{2}\rho\bar{V}^2$

$$\begin{aligned} L^A &= qSbC_l \\ M^A &= qS\bar{c}C_m \\ N^A &= qSbC_n \end{aligned} \qquad \begin{cases} C_l = C_{l\beta}\beta + \frac{b}{2\bar{V}}C_{lP}P + \frac{b}{2\bar{V}}C_{lR}R + C_{l\delta R}\delta_R \\ C_m = C_{m0} + C_{m\alpha}\alpha + \frac{\bar{c}}{2\bar{V}}C_{mQ}Q + C_{m\delta E}\delta_E \\ C_n = C_{n\beta}\beta + \frac{b}{2\bar{V}}C_{nP}P + \frac{b}{2\bar{V}}C_{nR}R + C_{n\delta R}\delta_R \end{cases}$$

Typically

C_{m0} = Static pitch moment coeff = 0.0
$C_{m\alpha}$ = -0.2954 = Pitch stiffness coeff
C_{mQ} = -10.281 = Pitch damping coeff
$C_{m\delta E}$ = -1.5852 = Elevator moment coeff on pitch

$C_{l\beta}$ = -0.0331 = Roll lateral coeff
C_{lP} = -0.4248 = Roll damping coeff
C_{lR} = 0.0450 = Roll moment coeff
$C_{l\delta R}$ = 0.0080 = Rudder moment coeff on roll

$C_{n\beta}$ = 0.086 = Yaw stiffness coeff
C_{nP} = -0.0251 = Yaw moment due to roll
C_{nR} = -0.125 = Yaw damping coeff
$C_{n\delta R}$ = -0.1129 = ~~Yaw~~ Rudder moment coeff on yaw