AGCD algorithm

The Approximate Greatest Common Divisor algorithm (AGCD algorithm) is an algorithm to find the tolerance of an arithmetic sequence, whose elements are expected to lie near elements of a significant arbitrary set, when each elements of the set is expected to lie near elements of the sequence with tolerance as its first term. In other words, AGCD algorithm estimates a certain constant, which indicates the greatest common divisor of the elements of the set, when the elements of the set can be considered to be approximately integer multiples of the constant.

Note that when the AGCD algorithm is used for a set, the greatest common divisor of the elements of the sequence of equalities expected in the set must be known as an overview. The AGCD algorithm can also be regarded as a rigorization of the greatest common divisor of the elements of the equi-difference sequence obtained as an overview.

The AGCD algorithm procedure is shown below.

[1]

$$E(A) := \left\{ b_n \;\middle|\; \begin{matrix} \forall n \in \mathbb{N}, \\ n \le |A| - 1, \end{matrix} \begin{cases} \Delta a_n < g_A \implies b_n = \dfrac{a_n + a_{n+1}}{2} \\ \Delta a_n \ge g_A \implies b_n = \Delta a_n \end{cases} \right\} \tag{1.1}$$

$$\left(a_n \le a_{n+1},\ \Delta a_n := a_{n+1} - a_n\right)$$

$$\forall n \in \mathbb{N}, \quad \begin{cases} E^1(A) := E(A) \\ E^{n+1}(A) := E\left(E^n(A)\right) \end{cases} \tag{1.2}$$

$$\{\alpha\} := E^{|A|-1}(A) \tag{1.3}$$

$$D : \left[\alpha - \Delta\varepsilon,\ \alpha + \Delta\varepsilon\right] \tag{1.4}$$

[2]

$$Z(\delta, A) := \sum_{k=1}^{|A|} |a_k - \delta| \qquad \left(a_k \in A\right) \tag{2.1}$$

$$G_A := \arg\min_{\delta \in D} Z(\delta, A) \tag{2.2}$$

In the above equation, the desired greatest common divisor is G_A. The AGCD algorithm consists of two stages [1], [2], where [1] limits the range within which the greatest common divisor is searched for and [2] is the actual approximation. In addition, g_A is the greatest common divisor that is obtained as an overview, and $\Delta\varepsilon$ is the number that determines the width of the range in which the greatest common divisor is searched, both of which are constants.

[1]

(1.1)

For any set A, let $a_n \leq a_{n+1}$ and $\Delta a_n = a_{n+1} - a_n$, and let b_n be the average of a_n and a_{n+1} for $\Delta a_n < g_A$, and let b_n be Δa_n for $\Delta a_n \geq g_A$, and determine the set $E(A)$ of b_n. In other words, the set A is regarded as a sequence of numbers, and if the difference for each a_n is less than the greatest common divisor g_A, which is determined as an overview, then b_n is the average of a_n and a_{n+1}, otherwise b_n is the difference Δa_n to obtain $E(A)$.

(1.2)

For any natural number n, let $E^1(A) = E(A)$, $E^{n+1}(A) = E^n\left(E(A)\right)$ be defined. That is, $E^n(A)$ is the set obtained by obtaining $E(A)$ from the set A for n times.

(1.3)

For a set A whose number of elements is $|A|$, the number of elements in $E^{|A|-1}(A)$ is 1. Let α be the only element of $E^{|A|-1}(A)$.

(1.4)

The search range D is defined as $\left[\alpha - \Delta\varepsilon,\ \alpha + \Delta\varepsilon\right]$. In other words, we assume that the greatest common divisor we seek exists in $\left[\alpha - \Delta\varepsilon,\ \alpha + \Delta\varepsilon\right]$. We move on to [2].

[2]

(2.1)

Determine the function Z with any real number δ and set A as arguments as the sum of the absolute values of the differences between the elements a_k and δ of all A.

(2.2)

For any real number δ in D and a function Z whose argument is a set A, let G_A be the greatest common divisor for finding δ such that $Z(\delta, A)$ takes the minimum value.

In the above manner, if for any set A, each element can be expected to lie near elements of a significant arithmetic sequence whose tolerance is the first term, then the approximate value of the greatest common divisor of the elements of the arithmetic sequence, G_A, can be obtained.