Appendix B

Lemma 1 *(One degree of transitivity).* If there is a node a under the view V which belongs to SP point and $in = 1$, then if $(b, a) \in E_d$, b is also belongs to SP point. Similarly, $out = 1$, then if $(a, b) \in E_d$, b is also belongs to SP point. We formalize it as follows

$$For\ a\ view\ V \bullet \exists a \in SP \wedge a_{in} = 1 \to (b, a) \in E_d \wedge b \in SP \tag{1}$$

$$For\ a\ view\ V \bullet \exists a \in SP \wedge a_{out} = 1 \to (a, c) \in E_d \wedge c \in SP \tag{2}$$

Proof . Since the proofs of formula (2) and formula (1) are similar, we only give the proof of formula (1) below. Because a belongs to SP and the defination 14 of SP, we can divide the L into two subsets, the subset L_1 is composed of nodes x_i which has path from x_i to a and subset L_2 is composed of nodes y_j which has path from a to y_j.

$$L_1 = \{\bigcup_{i=1}^{M} x_i | \forall x_i \in L_1 \bullet \exists path = (n_i = v_0, e_1, v_1, ..., e_k, v_k = n_j) \wedge n_i = x_i, n_j = a\}$$

$$L_2 = \{\bigcup_{i=1}^{N} y_j | \forall y_j \in L_2 \bullet \exists path = (n_i = v_0, e_1, v_1, ..., e_k, v_k = n_j) \wedge n_i = a, n_j = y_j\}$$

Since $in_a = 1$, if and only if there exists (b, a) belonging to E_d and b is unique, so we can construct the new subsets L'_1 and L'_2 which can support b to be an element of set SP.

$$L'_1 = L_1 - \{b\}$$

$$L'_2 = L_2 + \{a\}$$

it is obviously that for all paths from L'_1 to b exist as long as we delete the sequence (b, a) from L_1 to a, because we know that $in_a = 1$ and can infer that sequence $\langle b, a\rangle$ exists in every path from x_i to a according to the set L_1. Similarly, all paths from b to L'_2 also exist, as long as the sequence (b, a) is added to the path a to L_2.

$$\forall x_i \in L_1 \wedge x_i \neq b \wedge path(x_i, a) = (x_i = v_0, e_1, v_1, ..., b, e_k, v_k = a)$$

$$\Rightarrow \exists x_i \in L'_1 \wedge path(x_i, b) = (x_i = v_0, e_1, v_1, ...b = b)$$

$$\forall y_j \in L_2 \wedge y_j \neq a \wedge path(a, y_j) = (a = v_0, e_1, v_1, ..., e_k, v_k = y_j)$$

$$\Rightarrow \exists y_j \in L'_2 \wedge path(b, y_j) = (b = b, e_{(b,a)}, v_0, e_1, v_1, ...e_k, v_k = y_j)$$

Lemma 2 *(SP structure).* A set SP in any view constitutes continuous sequence(no direction).

$$For\ view\ V \bullet\ \forall a_i \in SP \wedge SP = \bigcup_{i=1}^{n} a_i \to \exists path = (a_1, ...a_n) \tag{3}$$

Proof . Fist if we only have one element in set SP, the equation clearly holds. We have n nodes $a_1...a_n$ belong to SP and form $path = (a_1...a_n)$, assuming that node x belongs to SP but there is no path to construct by nodes $\{a_1...a_n\} \cup x$. Then we have relation $x \in SP \wedge \nexists path[\{a_1...a_n\} \cup x] \Rightarrow \exists a_i \in SP \wedge ((x, a_i), (a_{i-1}, a_i) \in E_d) \vee ((a_i, x), (a_i, a_{i+1}) \in E_d))$ By definition of SP, the nodes in SP must be interconnected, and because there are no rings in

L, there are no rings in SP. We can infer that

$$\exists a_i \in SP \wedge (((x, a_i), (a_{i-1}, a_i) \in E_d) \vee (a_i, x), (a_i, a_{i+1}) \in E_d))) \Rightarrow x \notin SP$$

We consider the case where (x, a_i) and (a_{i-1}, a_i) belong to E_d, we can infer that there is no path from node x to a_{i-1} or from node a_{i-1} to x, this contradicts that x belongs to SP, so the hypothesis is not tenable and the proof is done.

Lemma 3 *(split invariance).* The set SP in the original view is still the subset of the sum of the subviews SP collection. We assume b is the point the divide and the subviews are V_i.

$$For\ all\ view\ V \bullet\ \forall d_i \in SP \wedge b \in SP \rightarrow \exists V_i \bullet\ d_i \in SP_i \tag{4}$$

Proof . In the following, we only give the proof under operator $\frown$, and it obviously holds for parallel operation. As for the operations < and >, it can be simply analyzed and obtained that all elements in SP are all in L_1. Therefore, we only need to give the proof under operator $\frown$ to deduce the establishment of all operators. As in **Lemma 1**, first construct sets L_1 and L_2 with b as the split point.

(1) Consider first the case where the set SP contains only one element, which clearly holds.

(2) Consider the case where the set SP contains two elements named a and b. When b is the split point, both a and b are elements of the set SP in L_1. When a is the split point, a is the element of the set SP inL_1, and b is the element of the set SP in L_2.

(3) If the number of elements in the set SP is n, we can further obtain the following property through **Lemma 2**.

$$path = (a_1, ...a_n) \Rightarrow out_{a_1} = 1 \wedge in_{a_n} = 1 \wedge in_{a_i} = out_{a_i} = 1, i = 2, ..n - 1$$

we assum that $d_i = a_j, b = a_k, j \leq k$ and construct $L_{12} = \{a_{j+1}, ..., a_k\}$ and $L_{11} = L_1 - L_{12}$, where L_{12} can form $path = (a_j, a_{j+1}, ..., a_k)$ through **Lemma 2**, we can infer that

$$\forall x_i \in L_{12} \Rightarrow \exists t \in \{a_{j+1}, ..., a_k\} \wedge t = x_i \wedge path(d_i, x_i) = (d_i = a_j, ..., t = x_i) \tag{5}$$

$$\begin{aligned}
&\forall x_i \in L_1 \wedge path(x_i, a) = (x_i = v_0, e_1, v_1, ..., v_t, a_1, ..., a_k = a) \\
&\Rightarrow \forall x_i \in L_1 - \{a_k\} \wedge path(x_i, a_{k-1}) = (x_i = v_0, e_1, v_1, ...v_t, a_1, ..., a_{k-1} = a_{k-1}) \\
&\Rightarrow ... \\
&\Rightarrow \forall x_i \in L_1 - \{a_{j+1}, ..., a_k\} \wedge path(x_i, a_j) = (x_i = v_0, e_1, v_1, ...v_t, a_1, ..., a_j = a_j) \\
&\Leftrightarrow \forall x_i \in L_{11} \wedge path(x_i, d_i) = (x_i = v_0, e_1, v_1, ...v_t, a_1, ..., a_j = d_i)
\end{aligned} \tag{6}$$

By equations 5 and 6, we can get that if $d_i = a_j, a_k = a, j \leq k$, d_i is the element of the set SP in L_1, where L_{11} is the set which can reach d_i, L_{12} is the set which can be reached by d_i. Similarly, we can get that when $j > k$, d_i is the element of set SP in L_2 and the proof is done.

Proof 1 for Theorem *(Elimination of unit view p).* First we proof the equation 1 which shows below. Notice all the formulas φ_i are atomic formulas.

$$\left\langle \varphi_1 <^{p}_{\varphi_3} \right\rangle \Leftrightarrow \varphi_1 \frown \varphi_3$$

The model M satisfy the formula on the left side of the equation, if and only if the view V can be divied into three part, and satisfy the definition 14 of operations $\lessdot$. And when we apply abbreviation notation $\langle\rangle$ to the formula, the unit view p should be omitted, so the relation of V_i can be represented as follow, where we use z to represent the element of L_p.

$$V_i = (L_i, X_i, SP_i, E), i = 1, 2, 3 \wedge T_{com}$$
$$\wedge \exists a \in SP, z, c \in L, a \neq z \neq c, in_z = in_c = 1, out_a = 2 \wedge (a, z), (a, c) \in E_d$$

and after we eliminate the p, unfold T_{com} and simplify the conditions, note that we mentioned in Definition 16 that when the out degree is equal to 2, we no longer to assign p to subview, so the conditions are equivalently transformed as

$$V_i = (L_i, X_i, SP_i, E), i = 1, 2 \wedge \exists a \in SP, b \in L, a \neq b, in_b = 1, out_a = 1 \wedge (a, b) \in E_d$$
$$\wedge\, a \in L_1 \wedge b \in L_2 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cup L_2 = L \wedge SP_i = FindSp(L_i)$$

we unfold T_{chop} on the right side of the equation and obtain that

$$V_i = (L_i, X_i, SP_i, E), i = 1, 2 \wedge \exists m \in SP, n \in L, out_m = in_n = 1 \wedge m \neq n \wedge (m, n) \in E_d$$
$$m \in L_1 \wedge n \in L_2 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cup L_2 = L \wedge SP_i = FindSp(L_i)$$

let m replace a, n replace b, so the conditions on the left side are equal to the right side, and the proof 1 is done.

Proof 2 for Theorem *(Elimination of unit view p).* We give equation 2 and the proof of the equation as follows

$$\left\langle \varphi_1 <^{\varphi_2}_{p <^{\varphi_3}_{\varphi_4}} \right\rangle \Leftrightarrow \varphi_1 <_3 \begin{matrix} \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{matrix}$$

Before giving an rough proof, we give the definition of $<_3$ which is similar to $<$, but the notation $<_3$ represent that it has three branches along the road.

$$V_i = (L_i, X_i, SP_i, E), i = 1, 2, 3, 4 \wedge \exists a \in SP, b, c, d \in L, a \neq b \neq c \neq d, in_{b,c,d} = 1, out_a = 3$$
$$\wedge (a, b), (a, c), (a, d) \in E_d \wedge a \in L_1 \wedge b \in L_2 \wedge c \in L_3 \wedge d \in L_4 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cap L_3 = \emptyset$$
$$\wedge L_1 \cap L_4 = \emptyset \wedge L_2 \cap L_3 = \emptyset \wedge L_3 \cap L_4 = \emptyset \wedge L_1 \cup L_2 \cup L_3 \cup L_4 = L \wedge SP_i = FindSp(L_i)$$

first we consider $p <^{\varphi_3}_{\varphi_4}$ as φ_3' and $L_3' = L_3 \cup L_4 \cup p$. Therefore, the conditions on the left side with fold φ_3' can be equivalently expressed as

$$V_i = (L_i, X_i, SP_i, E), i = 1, 2, 3 \wedge T_{com}$$
$$\wedge \exists a \in SP, b, c \in L, a \neq b \neq c, in_b = in_c = 1, out_a \geq 2 \wedge (a, b), (a, c) \in E_d$$

because c belongs to L_3' and don't equal to a by **Lemma 3**, we can infer that φ_3 or φ_4 exactly has lane c, and we also know that z is the same lane with unit 0 assigned from a. Then we unfold the φ_3' with unit view p by the defination of $<$, and exactly know $\exists\, d, e \in L_3', (a,d), (a,e) \in E_d$

$$V_i = (L_i, X_i, SP_i, E), i = 1,2,3 \wedge T_{com}$$

$$\wedge\ \exists z \in SP, c, d \in L, z \neq c \neq d, in_c = in_d = 1, out_z = 2 \wedge (z,c), (z,d) \in E_d$$

then we merge the above conditions and unfold T_{com}, which is equivalent to

$V_i = (L_i, X_i, SP_i, E), p = (L_p, X_p, SP_p, E), i = 1,2,3,4$

$\exists a \in SP, b, c, d \in L, a \neq b \neq c \neq d, in_b = 1, (a,b), (a,c), (a,d) \in E_d, out_a = 3$

$\wedge\ a \in L_1, b, p \in L_2, c \in L_3, d \in L_4 \wedge L_1 \cap L_2 = p \wedge L_1 \cap \{L_3 \cup L_4 \cup p\} = p \wedge p = a, x_p = 0$

$\wedge\ L_2 \cap L_3 = \emptyset \wedge L_3 \cap L_4 = \emptyset \wedge L_1 \cup L_2 \cup L_3 \cup L_4 = L \wedge SP_i = FindSp(L_i)$

Further, by applying notation $\langle\rangle$ to omit the unit view p and use distributive law of $\cup$, the conditions can be equivalently transformed into

$V_i = (L_i, X_i, SP_i, E), i = 1,2,3,4 \wedge \exists a \in SP, b, c, d \in L, a \neq b \neq c \neq d, in_{b,c,d} = 1, out_a = 3$

$\wedge\ (a,b), (a,c), (a,d) \in E_d \wedge a \in L_1, b \in L_2 \wedge c \in L_3 \wedge d \in L_4 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cap L_3 = \emptyset$

$\wedge\ L_1 \cap L_4 = \emptyset \wedge L_2 \cap L_3 = \emptyset \wedge L_3 \cap L_4 = \emptyset \wedge L_1 \cup L_2 \cup L_3 \cup L_4 = L \wedge SP_i = FindSp(L_i)$

compared conditions with $<_3$, the conditions on the left side are equal to the right side, and the proof 2 is done.

Proof 3 for Theorem *(Elimination of unit view p).* We give equation 3 and the proof of the equation as follows

$$\left\langle \varphi_1 <_3 \begin{matrix} p \\ \varphi_2 \\ \varphi_3 \end{matrix} \right\rangle \Leftrightarrow \varphi_1 <^{\varphi_2}_{\varphi_3}$$

according to the defination of $<_3$ which is given in proof 2, the conditions on the left side can be expressed as

$V_i = (L_i, X_i, SP_i, E), i = 1,2,3,4 \wedge \exists a \in SP, b, c, z \in L, a \neq b \neq c \neq z, in_{b,c,z} = 1, out_a = 3$

$\wedge\ (a,b), (a,c), (a,z) \in E_d \wedge a \in L_1 \wedge b \in L_2 \wedge c \in L_3 \wedge z \in L_4 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cap L_3 = \emptyset$

$\wedge\ L_1 \cap L_4 = \emptyset \wedge L_2 \cap L_3 = \emptyset \wedge L_3 \cap L_4 = \emptyset \wedge L_1 \cup L_2 \cup L_3 \cup L_4 = L \wedge SP_i = FindSp(L_i)$

where V_4 is equal to the unit view p, so the conditions transform equivalently to

$$V_i = (L_i, X_i, SP_i, E), i = 1,2,3 \wedge \exists a \in SP, b, c \in L, a \neq b \neq c, in_b = in_c = 1, out_a = 2$$

$$\wedge\ (a,b), (a,c) \in E_d \wedge a \in L_1 \wedge b \in L_2 \wedge c \in L_3 \wedge L_1 \cap L_2 = \emptyset \wedge L_1 \cap L_3 = \emptyset$$

$$\wedge\ L_2 \cap L_3 = \emptyset \wedge L_1 \cup L_2 \cup L_3 = L \wedge SP_i = FindSp(L_i)$$

compared conditions with $<$, the conditions on the left side are equal to the right side, and the proof 3 is done.