Filters studies

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Tasks of type to design low-pass filter (Butterworth, Bessel, and Chebyshev) are somewhat easy actually. In today's difficult times, it is necessary to offer students a slightly different view of the problem. No "how only" but also "why this solution". Where is the starting point?

Description of the possible situation

There is an insulated metallic (wired) system at the risky workplace, which enables participants (from critical places) to communicate with the dispatcher - completely independent of other devices. On the same line, there is a signal still present with a frequency f_v = 10 kHz and a level of 100 mV, which indicates, for example, the presence of a burner flame. There are also 50 Hz and 100 Hz disturbances on the line (common industrial disturbances) with a level of approx. 50 mV. The speech signal level is about 700 mV. The line impedance is 300 Ω.

Task:

Design filters for the acoustic path!

Situation analysis:

Assume that the level of each interfering component must not exceed 2% of the level of the acoustic signal - ie the value of 14 mV. Therefore, interference at 50 Hz and 100 Hz must be suppressed at least 50/14 = 3.57 times (by 11.06 dB) and interference at 10 kHz must be suppressed at least 100/14 = 7.14 times (by 17.1 dB). For an acoustic call, a frequency band of 300 Hz to 3500 Hz is sufficient; at the cut-off frequencies, the transmission drop may be 3 dB (this is not a measuring system - there the requirements would be more demanding). From this point of view, the Butterworth approximation of the filter satisfies - it is sufficient.

We therefore need to design a filter with an input impedance significantly greater than 300 Ω, a low frequency of 300 Hz with suppression (attenuation; against the pass band) 11 dB at 100 Hz (**point A** in Fig. 1; 50 Hz is a smaller frequency value, more will be suppressed - see below Fig. 1); with an upper frequency of 3500 Hz and with a suppression of 17 dB (against the pass band) at a frequency of 10 kHz (**point B** in Fig. 1). The situation is captured in the picture - Fig. 1.

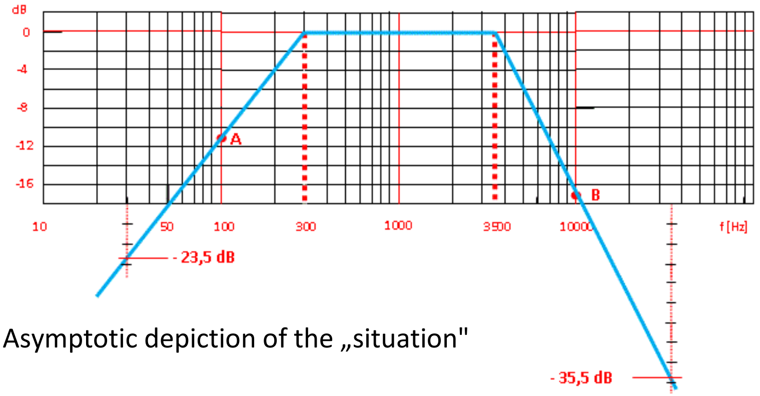


Fig. 1 Asymptotic depiction of the „situation"

It is clear from the requirements shown that an asymptote with a slope of +23.5 dB / dec (change of transmission between 30 and 300 Hz) passes through point A. The 1st-order high-pass filter has a slope of only +20 dB / dec, so **we must use a 2nd-order high-pass filter** with a slope of +40 dB / dec and a characteristic frequency of 300 Hz.

It is also clear from the requirements shown that an asymptote with a slope of -35.5 dB / dec (change in transmission between 3500 and 35000 Hz) passes through point B. The 1st-order low-pass filter has a slope of only -20 dB blanks, so **we must use a 2nd-order low-pass filter** with a slope of -40 dB / dec and a characteristic frequency of 3500 Hz.

After all, if we have chosen the Butterworth approximation, we can also determine the necessary filter orders by calculation [Theory of electronic circuits, Chapter7; Operační zesilovače v elektronice _Operational amplifiers in electronics, p. 337 and p. 339]. The required order *n of the low pass filter* for a given approximation is

$$n = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(\omega_1/\omega_0)} = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(f_1/f_0)}$$

α is the required attenuation in dB at the frequency $\boldsymbol{\omega_1 > \omega_0}$

ω_0 is the characteristic frequency of the filter.

It is therefore for our conditions $f_0 = 3500\ Hz;\ \alpha = 17\ dB$ at 10 kHz; therefore, the required low-pass order *n* is

$$n = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(f_1/f_0)} = \frac{log\left(10^{17/10} - 1\right)}{2 \cdot log(10^4/3500)} = \frac{log(50{,}12 - 1)}{2 \cdot log(2{,}86)} = \frac{1{,}69}{0{,}91} = 1{,}85$$

Because we are able "do" only whole values of *n*, we must use the next larger number, ie 2! This corresponds to the simple previous "graphical solution of the problem" ☺

For high pass filter it is true formula for determining of the filter order *n* in the reciprocal form:

$$n = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(\boldsymbol{\omega_0/\omega_1})} = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(\boldsymbol{f_0/f_1})}$$

α is the required attenuation in dB at the frequency $\boldsymbol{\omega_1 < \omega_0}$ *(log in the denominator must always "give" a positive number so that n is a positive number!)*

ω_0 is the characteristic frequency of the filter.

It is therefore for our conditions $f_0 = 300\ Hz;\ \alpha = 11\ dB$ at 100 Hz; therefore, the required high-pass order *n* is

$$n = \frac{log(10^{1{,}1} - 1)}{2 \cdot log(3)} = \frac{1{,}06}{0{,}95} = 1{,}11$$

So we have to choose the high pass of the 2nd order. It is clear that at 50 Hz, the suppression will be even greater.

By cascading HP2 and LP2 (Butterworth approximation) we achieve the required properties.

Punčochář, J.: **Filters - study of one simple case (case study);** 22.10.2020

At this point, we have actually solved the so-called **approximation problem** for the initial situation. The required filters can be solved in various ways, but HP2 and LP2 - Butterworth approximations must always be implemented.

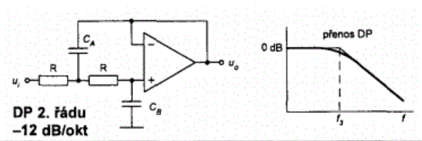
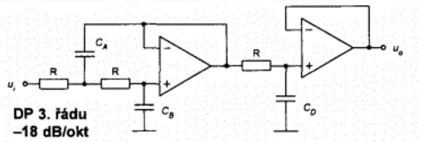
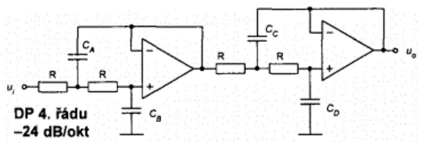
Implementation 1

Very convenient solutions are some tables of filters. See examples below:

Implementation of LP2 to LP4; related to f_3 (also for Bessel approximation) –Tab 27

Tabulka 27

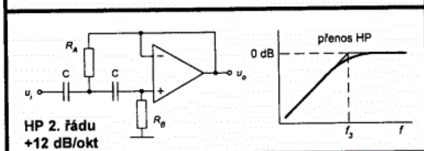
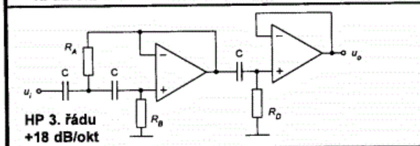
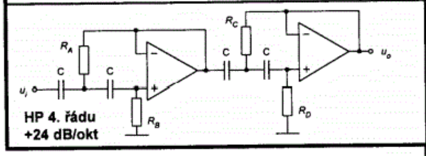
Realizace Besselových a Butterworthových DP 2. až 4. řádu; vztaženo k f_3 (frekvence poklesu přenosu o 3 dB; f_0 - charakteristická frekvence v aproximačních vztazích pro požadovaný filtr - přenos na f_0 dle aproximace); filtry vhodné pro reproduktorové výhybky.

Zapojení	Bessel	Butterworth	Poznámky
	$C_A = 0{,}9076 / [R \cdot 2\pi f_3]$ $C_B = 0{,}6809 / [R \cdot 2\pi f_3]$	$C_A = 1{,}414 / [R \cdot 2\pi f_0]$ $C_B = 0{,}7071 / [R \cdot 2\pi f_0]$	$C_1 \rightarrow C_B$ $C_2 \rightarrow C_A$ *obr. 254*
	$C_A = 0{,}9548 / [R \cdot 2\pi f_3]$ $C_B = 0{,}4998 / [R \cdot 2\pi f_3]$ $C_D = 0{,}7560 / [R \cdot 2\pi f_3]$	$C_A = 2 / [R \cdot 2\pi f_0]$ $C_B = 0{,}5 / [R \cdot 2\pi f_0]$ $C_D = 1 / [R \cdot 2\pi f_0]$	$C_1 \rightarrow C_B$ $C_2 \rightarrow C_A$ *obr. 254* ---------- $C_1 \rightarrow C_D$ *obr. 253*
	$C_A = 0{,}7298 / [R \cdot 2\pi f_3]$ $C_B = 0{,}6699 / [R \cdot 2\pi f_3]$ $C_C = 1{,}0046 / [R \cdot 2\pi f_3]$ $C_D = 0{,}3872 / [R \cdot 2\pi f_3]$	$C_A = 1{,}0824 / [R \cdot 2\pi f_0]$ $C_B = 0{,}9239 / [R \cdot 2\pi f_0]$ $C_C = 2{,}6130 / [R \cdot 2\pi f_0]$ $C_D = 0{,}3827 / [R \cdot 2\pi f_0]$	$C_1 \rightarrow C_B, C_D$ $C_2 \rightarrow C_A, C_C$ *obr. 254*
	$f_3 = f_0 \cdot k_n$	$f_3 = f_0$	R = 4k7 až 10k

Implementation of HP2 to HP4; related to f_3 (also for Bessel approximation) –Tab 30

Tabulka 30

Realizace Besselových a Butterworthových HP 2. až 4. řádu (vhodné pro reproduktorové výhybky), f_3 - pokles přenosu o 3 dB.

Zapojení	Bessel	Butterworth	Poznámky
	$R_a = 1{,}1017 / [C \cdot 2\pi f_3]$ $R_B = 1{,}4688 / [C \cdot 2\pi f_3]$	$R_A = 0{,}7071 / [C \cdot 2\pi f_0]$ $R_B = 1{,}414 / [C \cdot 2\pi f_0]$	$R_2 \rightarrow R_A$ $R_1 \rightarrow R_B$ *obr. 262*
	$R_A = 1{,}0474 / [C \cdot 2\pi f_3]$ $R_B = 2{,}0008 / [C \cdot 2\pi f_3]$ $R_D = 1{,}3228 / [C \cdot 2\pi f_3]$	$R_A = 0{,}5 / [C \cdot 2\pi f_0]$ $R_B = 2 / [C \cdot 2\pi f_0]$ $R_D = 1 / [C \cdot 2\pi f_0]$	$R_2 \rightarrow R_A$ $R_1 \rightarrow R_B$ *obr. 262* ---------- $R_1 \rightarrow R_D$ *obr. 261*
	$R_A = 1{,}3701 / [C \cdot 2\pi f_3]$ $R_B = 1{,}4929 / [C \cdot 2\pi f_3]$ $R_C = 0{,}9952 / [C \cdot 2\pi f_3]$ $R_D = 2{,}5830 / [C \cdot 2\pi f_3]$	$R_A = 0{,}9239 / [C \cdot 2\pi f_0]$ $R_B = 1{,}0824 / [C \cdot 2\pi f_0]$ $R_C = 0{,}3827 / [C \cdot 2\pi f_0]$ $R_D = 2{,}6130 / [C \cdot 2\pi f_0]$	$R_2 \rightarrow R_A, R_C$ $R_1 \rightarrow R_B, R_D$ *obr. 262*
	$f_3 = f_0 / k_n$	$f_3 = f_0$	C = 4n7 až 10n

Note to the tables: the slope of n x 6 dB / oct corresponds to the slope of n x 20 dB / dec; octave - twice the frequency. Bessel filters here already calculate with the 3 dB frequency of the transmission, their characteristic frequency can be calculated using k_n - see the literature.

In the given tables, the transmission of filters in the pass band is equal to one (0 dB), the required properties (quality factor Q) are set only by the ratio of passive components - see the literature.

The filtering structure may then look as shown in Fig. 2:

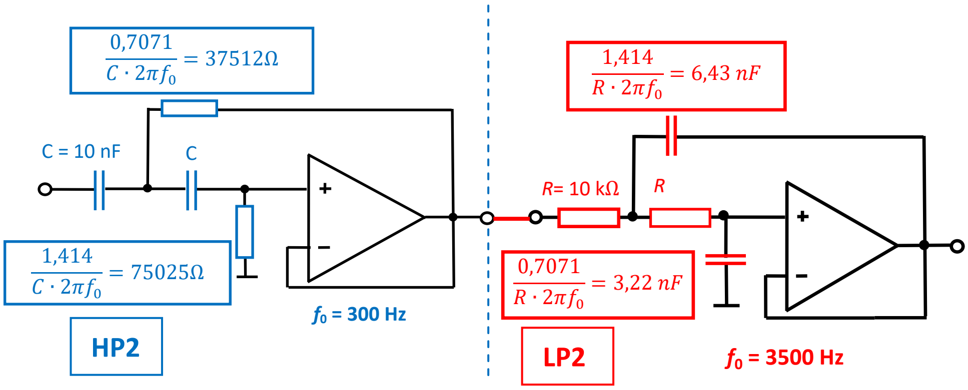


Fig. 2 Cascade connection of HP2 and LP2 required properties; transmission in the bandwidth is 1 (0 dB)

We still have to solve the problem of line impedance (300 Ω). This resistor would be in series with the "first" capacitance C. This would change the properties of the high pass filter. The output resistance of the signal source should ideally be zero. Therefore, we will connect, for example, the non-inverting configuration of the operational amplifier in front of the high-pass filter, this way we can easily ensure a considerable input resistance - at the same time we can also set the additional gain of the entire chain.

The situation for low-pass filter is good, because the output resistance of high-pass filter is very small compared to 10 k Ω - so the properties of low-pass filter do not change. The resulting connection with an input resistance of, for example, 100 kΩ and an adjustable transmission in the pass band is shown in Fig. 3.

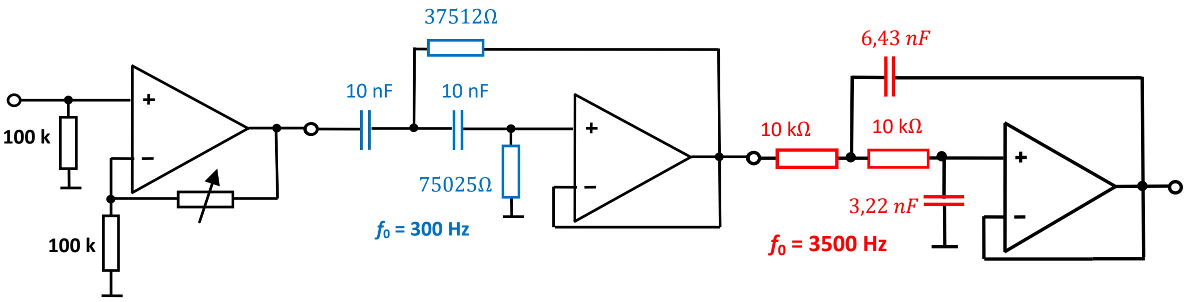


Fig. 3 Cascade connection of HP2 and LP2 of required properties; transmission in the bandwidth adjustable by an input buffer amplifier

Implementation 2

If we do not have suitable tables, we must delve a little deeper into theory. The normalized transfer for 2nd order Butterworth filters is [Theory of electronic circuits, Chapter7]:

$$\frac{1}{s^2 + 1{,}414s + 1}$$

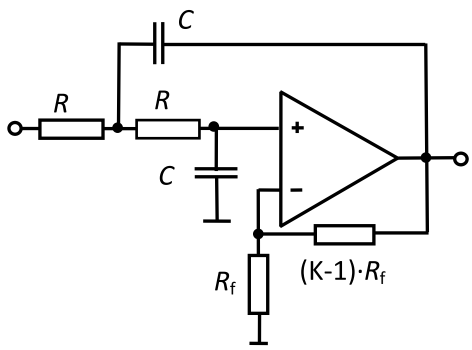
For the <u>low-pass filter</u>, we transform the normalized frequency by the relation $\boldsymbol{s \rightarrow p/\omega_0}$

By elementary modifications we obtain the transmission of the Butterworth low-pass filter of the 2nd order:

$$\frac{\omega_0^2}{p^2 + 1{,}414p\omega_0 + \omega_0^2} = \frac{\omega_0^2}{p^2 + p\dfrac{\omega_0}{0{,}707} + \omega_0^2} = \frac{\omega_0^2}{p^2 + p\dfrac{\omega_0}{Q} + \omega_0^2}$$

We must therefore implement a 2nd order low pass filter with a quality factor Q = 0.707.

This time we will opt for the frequently used structure, which we are able to set very well, because the setting of the quality factor does not affect the characteristic frequency - this is a significant advantage in practice. We can't choose any gain arbitrarily, but that's not too much of a problem - Fig. 4.



It applies here: $\boldsymbol{\omega_0^2 = 1/(RC)^2; Q = 1/(3-K)}$;
K is the gain of the non-inverting amplifier.
Filter design:
1) We choose the value C; (or R)
2) Then we calculate $R = 1/(\omega_0 C)$; or ($C = 1/(\omega_0 R)$)
3) Determine the value of K =3 - 1/Q. We choose a suitable value of R_f and calculate the value (K-1).R_f.

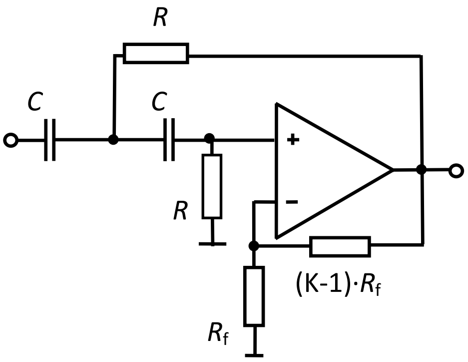
Fig. 4 The 2nd order low pass variant - Sallen-Key filter

We require a low pass filter with $\omega_0 = 2\pi \cdot 3500$; we choose the value of R = 10 k Ω; then we calculate $C = 1/(\omega_0 R) = 1/(2\pi \cdot 3500 \cdot 10^4)$; C = 4,55 nF. The needed gain is K =3 - 1/Q = 3 – 1/0,707=3 -1,414 = 1,586. We choose R_f = 10 k Ω; then the resistor (K-1).R_f = 5, 86 k Ω will provide the needed gain to ensure the required quality factor.

For the <u>high pass filter</u>, we transform the normalized frequency by a reciprocal relation $\boldsymbol{s \rightarrow \omega_0/p}$.
By elementary modifications we obtain the transmission:

$$\frac{p^2}{p^2 + 1{,}414p\omega_0 + \omega_0^2} = \frac{p^2}{p^2 + p\dfrac{\omega_0}{0{,}707} + \omega_0^2}$$

For the Butterworth approximation, we must therefore implement a second-order high-pass filter with a quality factor $Q = 0{,}707$. We will use a reciprocal circuit to the circuit of Fig. 4 – see Fig. 5.



It applies here too: $\boldsymbol{\omega_0^2 = 1/(RC)^2; Q = 1/(3-K)}$;
K is the gain of the non-inverting amplifier.
Filter design:
1) We choose the value C; (or R)
2) Then we calculate $R = 1/(\omega_0 C)$; or ($C = 1/(\omega_0 R)$)
3) Determine the value of K =3 - 1/Q. We choose a suitable value of R_f and calculate the value (K-1).R_f.

Fig. 5 The 2nd order high pass variant - Sallen-Key filter

We require a high pass filter with $\omega_0 = 2\pi \cdot 300$; we choose $R = 10\ \text{k}\,\Omega$; then we calculate the value $C = 1/(\omega_0 R) = 1/(2\pi \cdot 300 \cdot 10^4)$; $C = 53{,}1$ nF. The needed gain is K =3 - 1/Q = 3 – 1/0,707=3 -1,414 = 1,586. We choose R_f = 10 k Ω; then the resistor (K-1).R_f = 5, 86 k Ω will provide the needed gain to ensure the required quality factor.

It is obvious that the same quality factor for HP2 and LP2 requires the same gain K = 1.586 for this type (solution) of filters. This means that the transmission in the bandwidth will now be 1,586 x 1,586 = 2,515 (+8 dB). All attenuations against transmission in the pass band are maintained - the whole "blue" waveform in Fig. 1 simply moves up 8 dB.

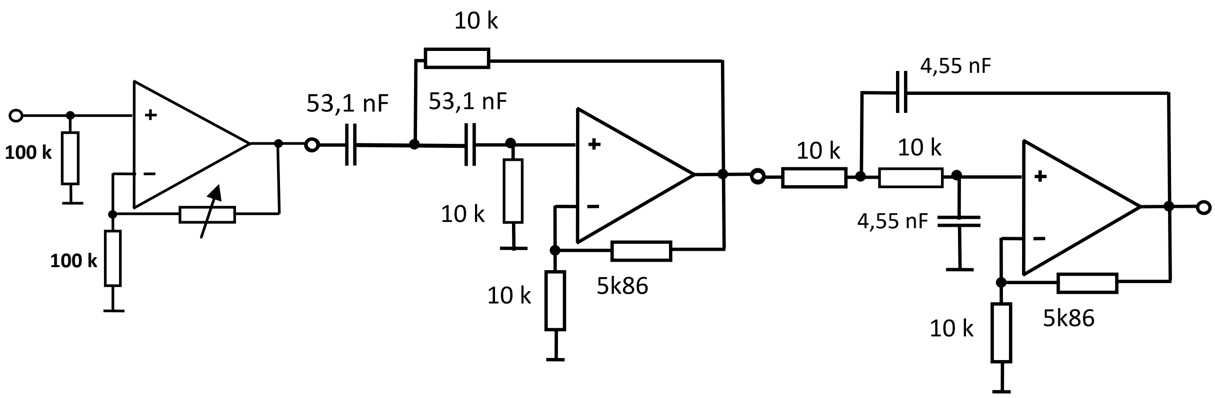


Fig. 6 Cascade connection of HP2 and LP2 of required properties, the transmission in the bandwidth is +8 dB and is further adjustable by an input buffer amplifier.

Other realizations

Of course, other "hardware" implementations of transfer functions are also possible. For the 2nd order Butterworth approximation, we must always ensure a quality factor of 0.707 and an appropriate frequency. The problem could even be solved by passive circuits and "isolating amplifiers" - Fig. 7. Just realize the basic features of a series resonant circuit. However, the realization of inductances, especially at low frequencies, is a thorny issue. They are bulky and heavy, expensive and always non-linear!

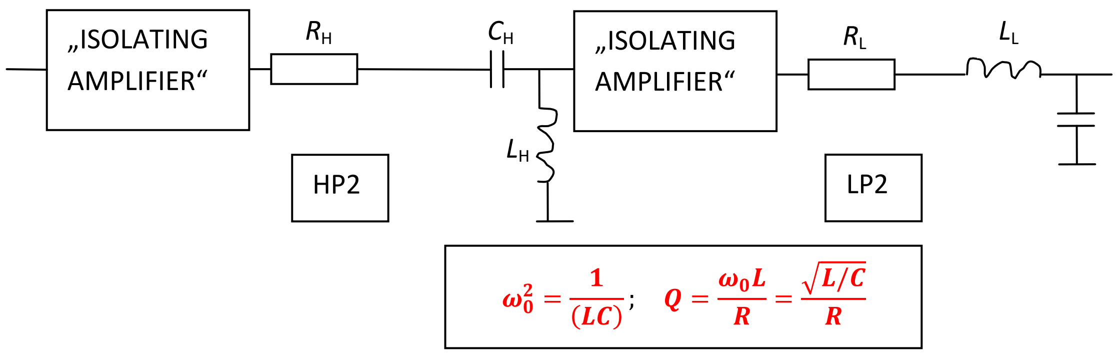


Fig. 7 Cascading of passive filters and isolating amplifiers

In all the previous considerations, we have quietly assumed that the operational amplifiers used are ideal. However, this is never true. Analysis of connections with real amplifiers is possible, but usually very demanding – see texts at the end. For an indicative assessment of the requirements for the operational amplifiers used, I recommend thinking in the first step of the "graphic criterion" below:

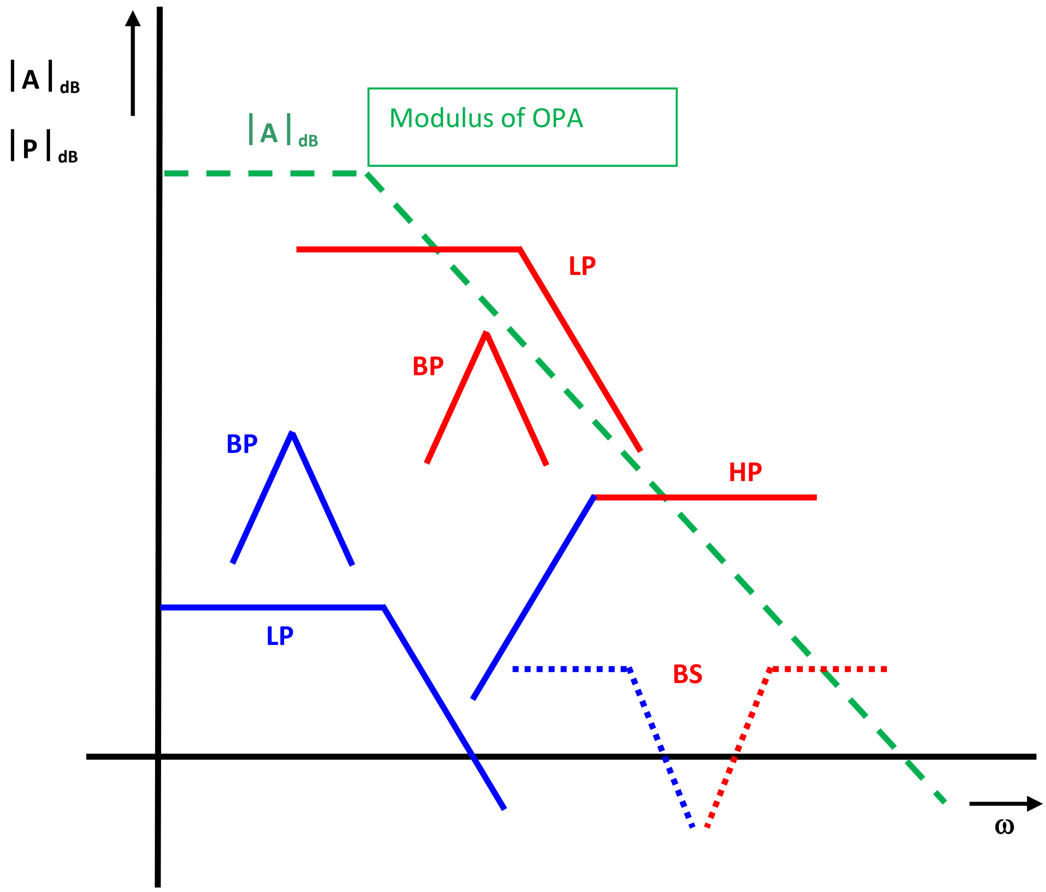


Fig. 8 A „Graphical criterion of operational amplifier usability“: **modulus of needed transfer function is much less than modulus of OPA gain – blue lines – satisfactory functions; red lines - not satisfactory functions**

After absorbing this material, you should perhaps be able to work "reasonably" with at least the Butterworth approximations. For this, all the necessary data are in the study materials. Further expansion is beyond the scope of this course - for time and "volume" reasons. Rather, the point is that we can analyze filters using the method of generalized nodal voltage analysis also. One dedicated lecture - as an application of a certain method of circuit analysis - cannot replace the systematic teaching (usually several-semester) of filter theory.

Mohylová, J. – Punčochář, J.: Theory of electronic circuits. www.researchgate.net/publication/

Punčochář, J.: Operační zesilovače v elektronice. www.researchgate.net/publication/

Punčochář, J.: Low-Pass Filters SALLEN and KEY With Real Operational Amplifiers. www.researchgate.net/publication/

Punčochář, J. - Mohylová, J. – Zajaczek, S.: Band Stop Filter with a Synthetic Inductor with Series Resistance and a Real Operational Amplifier. www.researchgate.net/publication/

Kolář, J. – Punčochář, J.: Band stop filters with real operational amplifier. www.researchgate.net/publication/

Punčochář, J.: The fifth journey – a short look in unreliable country of "R filters". www.researchgate.net/publication/