Solution of the linear circuits by Signal Flow Graphs

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Josef Punčochář
VSB - Technical University of Ostrava
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Solution of the linear circuits by Signal Flow Graphs

Josef Punčochář

We will describe the algorithm for solution of linear circuits by means of signal flow graphs - a process in which it is not necessary division by some members of the equations before merging the graphs of the sub-parts of the circuit. Such a procedure will allow establishing an algorithm for the direct compilation of a signal flow graph for the circuit to be analyzed - without the equations describing the circuit being assembled.

1. GRAPH ACCORDING TO THE FINAL SYSTEM OF LINEAR EQUATIONS – THE *A TYPE* OF ARRANGEMENT

Basic considerations will be demonstrated on the interconnection of a passive four-terminal circuit and an active three-terminal circuit. It follows from the considerations that the algorithms can be extended to the interconnection of any (linear) structures. In [1, 2, 3, 9 and 19] is to the circuit in Fig. 1 assigned the resulting admittance model:

	1	2,(a)	3,(b)			
1	Y_{11p}	$-Y_{12p}$	$-Y_{13p}$	U_1		I_1
2,(a)	$-Y_{12p}$	$Y_{22p}+Y_{aa}$	$-Y_{23p}+Y_{ab}$	U_2	=	I_2
3,(b)	$-Y_{13p}$	$-Y_{23p}+Y_{ba}$	$Y_{33p}+Y_{bb}$	U_3		I_3

(1)

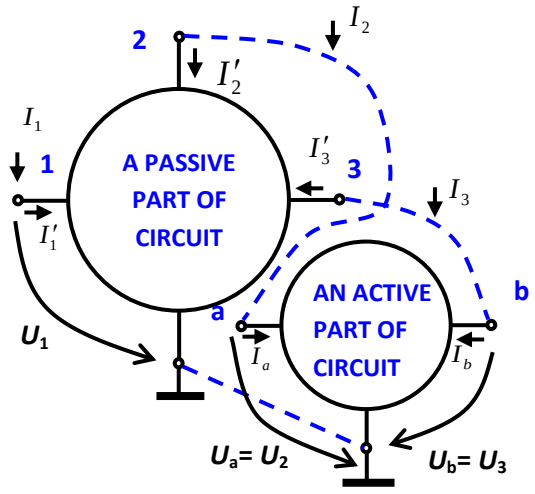


Fig.1. Parallel interconnection of the “passive four-terminal” and the “active three-terminal

The PASSIVE PART OF CIRCUIT is described as follows:

	1	2	3
1	Y_{11p}	$-Y_{12p}$	$-Y_{13p}$
2	$-Y_{12p}$	Y_{22p}	$-Y_{23p}$
3	$-Y_{13p}$	$-Y_{23p}$	Y_{33p}

The ACTIVE PART OF CIRCUIT is described below:

	a	b
a	Y_{aa}	Y_{ab}
b	Y_{ba}	Y_{bb}

I_i are exciting currents, U_i are nodal voltages, Y_{11p}, ... ,Y_{mmp} are elements of the admittance matrix; Y_{kkp} are sums of admittances of elements connected to the k-th node - they are always positive; Y_{rsp} are sums of admittances of elements connected between the r-th and s-th nodes - all these elements are negative ($r \neq s$).

The matrix description of the active part of circuit must be done for the “given arrow convention”.

The **parallel interconnection** is described (for our example) by equations:

$U_2 = U_a$; $U_3 = U_b$.

The following equations are derived from the **incidences** (interconnections):

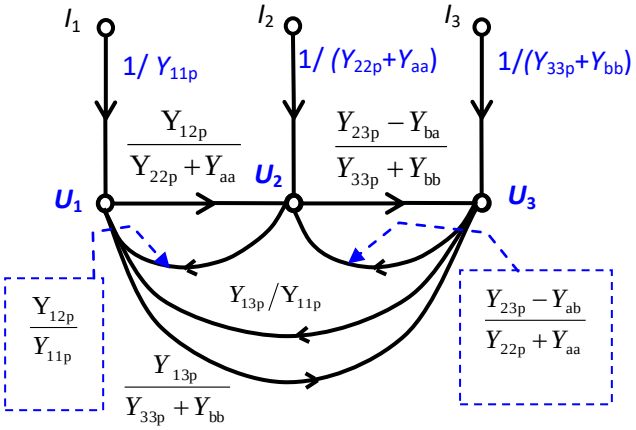
$$I_1 = I_1'; I_2 = I_2' + I_a; I_3 = I_3' + I_b$$

Let us now translate the system of equations (1) into the „primary equation" form:

$$\begin{aligned} &U_1 Y_{11p} - U_2 Y_{12p} - U_3 Y_{13p} = I_1 \\ &-U_1 Y_{12p} + U_2 (Y_{22p} + Y_{aa}) - U_3 (Y_{23p} - Y_{ab}) = I_2 \\ &-U_1 Y_{13p} - U_2 (Y_{23p} - Y_{ba}) + U_3 (Y_{33p} + Y_{bb}) = I_3 \end{aligned} \qquad (2)$$

From each equation, all members outside the "diagonal" are transferred to the right side of the equation, follows a division by the nodal admittance - the **A type arrangement** - and the oriented graph construction according to commonly known rules - Figure 2.

$$\begin{aligned} U_1 &= \frac{I_1}{Y_{11p}} + U_2 \frac{Y_{12p}}{Y_{11p}} + U_3 \frac{Y_{13p}}{Y_{11p}} \\ U_2 &= \frac{I_2}{Y_{22p} + Y_{aa}} + U_1 \frac{Y_{12p}}{Y_{22p} + Y_{aa}} + U_3 \frac{Y_{23p} - Y_{ab}}{Y_{22p} + Y_{aa}} \\ U_3 &= \frac{I_3}{Y_{33p} + Y_{bb}} + U_1 \frac{Y_{13p}}{Y_{33p} + Y_{bb}} + U_2 \frac{Y_{23p} - Y_{ba}}{Y_{33p} + Y_{bb}} \end{aligned} \qquad (3)$$



Node	Nodal admittance
U_1	Y_{11p}
U_2	$Y_{22p} + Y_{aa}$
U_3	$Y_{33p} + Y_{bb}$

Fig.2. Signal flow graph of the corresponding equation system (1) - after adjusting the system of equations - "**A type**"

Obviously, it is not easy to determine the algorithm for direct compilation of graph based on this "procedure". It has been shown that this procedure is useful only for active elements with the infinite (or zero) output admittance (zero output resistance) and (preferably) with the zero input admittance (infinite input resistance) where we can use a simple algorithm for a direct graph construction.

If the ***output admittance is infinite*** we use the fact that all the branch transmissions (gains of the passive part of the graph) into the node with infinite "node admittance" (output of the amplifier – voltage controlled voltage source) are divided by this infinite node admittance - the ***passive part of transfers*** to that node (output of amplifier) are simply ***canceled***.

If the ***output admittance is zero*** all the branch ***transmissions*** (gains of the passive part of the graph) into this output node (output of the amplifier – voltage controlled current source) ***stay the same***. The node admittance of this node does not change now.

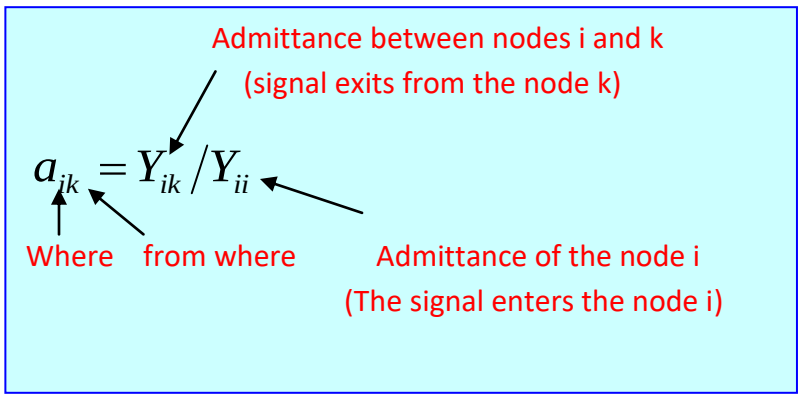
1.1 A direct algorithm of the signal flow graph compilation for ideal voltage controlled voltage source [1]

Based on relationships (3), the ***following algorithm*** can be defined:

1) We put on the input admittances of the active element to the circuit diagram (if are different from zero) - these are now considered as an element of the passive part of the circuit.

2) For each topological node, current source, and voltage source, we assign a graph node.

3) For the passive part of the circuit we determine the transfer (***gain, transmission, transmittance)*** a_{ik} (for each node **i**) from all other nodes **k** according to the relation:



Based on the above considerations, it is obvious that the transmissions to the ***nodes*** representing the ***ideal voltage source*** are zero (infinite node admittance), ie the ***branches*** of the passive part of the circuit entering such nodes ***are simply disconnected***.

4) The current source I_i entering into node **i** has a + 1 / Y_{ii} transmission. Transfer from any node to the source does not exist (the ideal source we can not affect).

5) We add the branches that define the active element between the corresponding node graphs.

6) We solve the required gains (transmissions).

1.1.1 *Some models of active elements* [1, 7] (intuitively defined models)

Some models (graphs) of active elements are shown in Figures 3 to 6. In the simplest case, infinite input impedance (zero admittance) is assumed which does not affect the node admittance of those nodes into which the inputs are connected. Furthermore, **zero output impedance (infinite admittance) is considered**, resulting in zero transmissions of branches representing the passive part of the circuit to the corresponding node (this node admittance is infinite now). Input and output admittances are no longer displayed mostly.

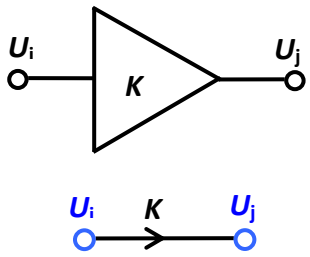
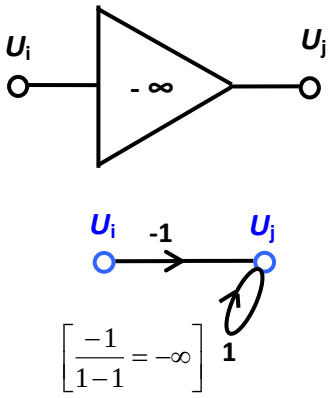
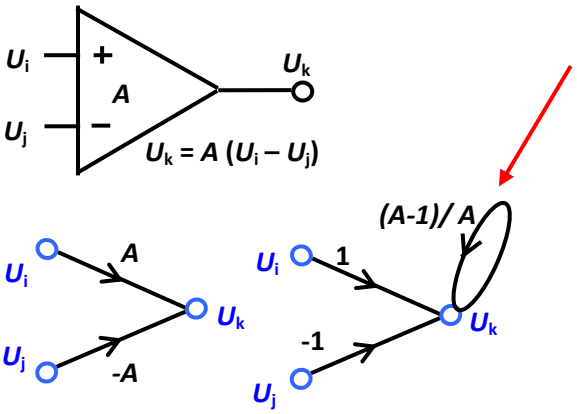


Fig.3. Basic model of amplifier with a gain K



$$U_j = -1 \cdot U_i + 1 \cdot U_j$$
$$U_j \cdot (1-1) = -1 \cdot U_i$$
$$U_j = \frac{-1}{1-1} \cdot U_i$$

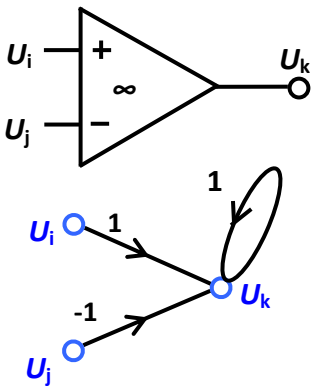
Fig.4. An inverting amplifier model with infinite gain



$$U_k = U_k \cdot \frac{A-1}{A} + 1 \cdot U_i - 1 \cdot U_j$$
$$U_k - U_k \cdot \frac{A-1}{A} = +1 \cdot U_i - 1 \cdot U_j$$
$$U_k \cdot \left(1 - \frac{A-1}{A}\right) = +1 \cdot U_i - 1 \cdot U_j$$

$$U_k = U_i \cdot \frac{1}{1-(A-1)/A} - U_j \cdot \frac{-1}{1-(A-1)/A} =$$
$$= A(U_i - U_j)$$

Fig.5. Differential amplifier model with gain A



$$U_k = U_k \cdot 1 + 1 \cdot U_i - 1 \cdot U_j$$
$$U_k - U_k \cdot 1 = +1 \cdot U_i - 1 \cdot U_j$$
$$U_k \to \infty \cdot U_i - \infty \cdot U_j$$

Fig.6. Model of an ideal OPA

1.1.2 *Example* 1 [1]

Let us investigate the structure in Fig. 7 for exciting from an ideal voltage source U_1 to the node number 1.

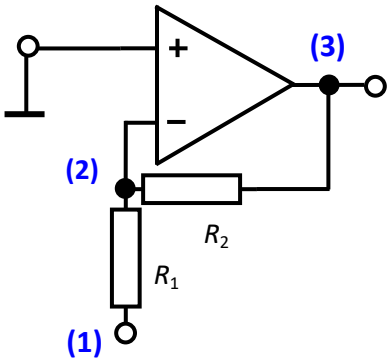


Fig.7. Inverting amplifier with the no ideal OPA – its gain is A

The admittance of node 1 is infinite when exciting from an ideal voltage source (zero output resistance), all transmissions of the circuit to node 1 are so zero (we "cut" them out).

The admittance of the node 2 is

$$Y_{22} = 1/R_1 + 1/R_2 = G_1 + G_2$$

The admittance of the node 3 is infinite due to OZ output zero resistance, so transmissions to this node are zero too.

Transmission of passive part of the circuit *from the node 1 into the node 2* (the admittance between nodes is G_1) is

$$a_{21} = \frac{G_1}{G_1+G_2}$$

Transmission of the circuit *from the node 2 into the node 1* a_{12} is zero.

Transmission of the circuit *from the node 2 into the node 3* a_{32} is zero.

Transmission of passive part of the circuit *from the node 3 into the node 2* (the admittance between nodes is G_2) is

$$a_{23} = \frac{G_2}{G_1+G_2}$$

Transmission (gain) of amplifier from the node 2 to the node 3 is -*A*. The non-inverting input is connected to a "zero signal" - so it is not applied gain +*A* here.

The resulting signal flow graph is in Fig. 8.

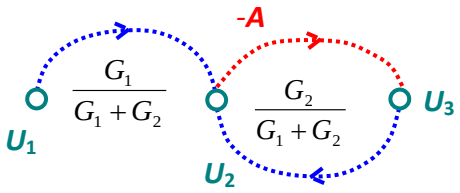


Fig.8. The resulting signal flow graph of the inverting amplifier – Fig.7; ideal voltage signal source U_1

We can easily determine (Mason's rule or simplifications see BOX1 or BOX2) that

$$\frac{U_3}{U_1} = -\frac{G_1}{G_2}\cdot\frac{1}{1+\frac{G_1+G_2}{AG_2}} = -\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{1+R_2/R_1}{A}}$$

BOX1 - Mason's Gain Rule (general form) – from node x_i to the node x_j

We remove all incoming branches to node x_i. From a physical point of view, this is logical. Assigning a "fixed quantity" to the node x_i - voltage for example - nothing can affect this quantity any more. If x_i has only outgoing branches then graph is unchanged. Now gain is

$$G_{ji} = \frac{x_j}{x_i} = \frac{(\sum_1^n P_k \Delta_k)}{\Delta}$$

P_k is the gain of the k–th forward path (we have n different forward paths)
Δ is the "graph determinant"

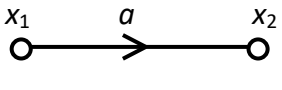
$$\Delta = 1 - \sum(\text{all loop gains}) + \sum(\text{products of non} - \text{touching} - \text{loop gain pairs}) - \cdots$$

$$\ldots - \sum(\text{products of non} - \text{touching} - \text{loop gain triplets}) + \cdots$$

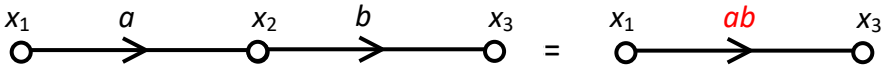
$\Delta_k = \Delta$ after removal of the k-th forward path (subgraf determinant)

Path: Collection of branches linked together in same direction.
Forward path: Path from input node (x_i) to output node (x_j) - node is not visited more than once.
Gain of forward path: Product of all gains of branches in the forward path.
Loop: Path that originates and terminates at the same node. No other node is visited more than once.
Loop gain: Product of branch gains in a loop.
Non-touching: Two parts of a SFG are non-touching if they do not share at least one node.

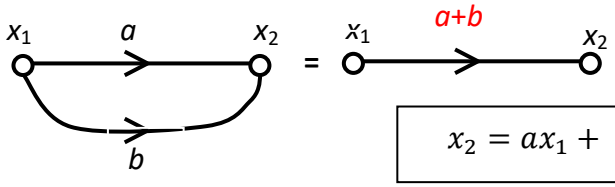
BOX2 – signal flow graph algebra, basic simplifications



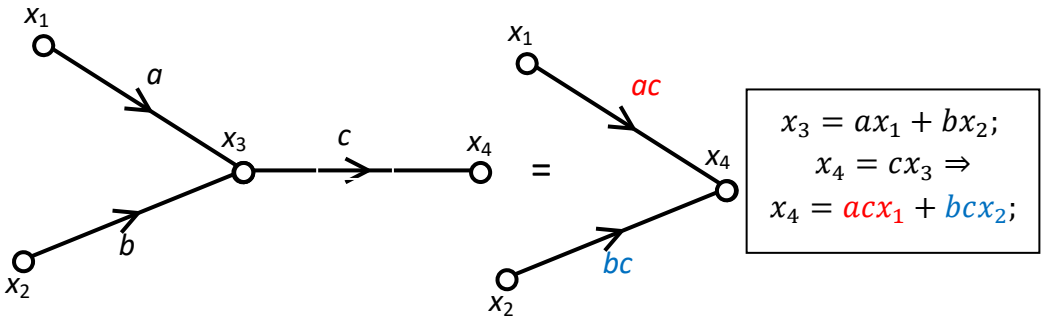
$$x_2 = ax_1$$



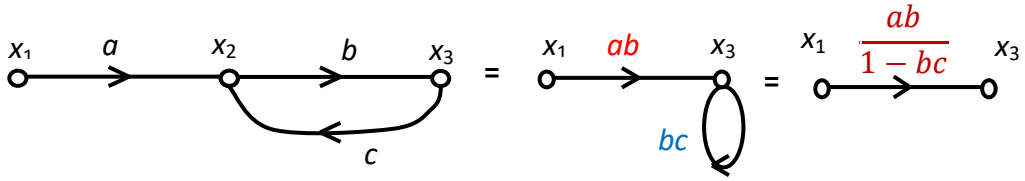
$$x_2 = ax_1;\ x_3 = bx_2 \ \Rightarrow\ x_3 = abx_1$$



$$x_2 = ax_1 + bx_1 = (a + b)x_1$$



$$x_3 = ax_1 + bx_2;$$
$$x_4 = cx_3 \Rightarrow$$
$$x_4 = acx_1 + bcx_2;$$



$$x_2 = ax_1 + cx_3;\ x_3 = bx_2;\ \Rightarrow\ x_3 = abx_1 + bcx_3 \Rightarrow x_3 = \frac{ab}{1 - bc}$$

1.2 A direct algorithm of the signal flow graph compilation for non-ideal voltage controlled voltage source [1] – with output admittance

If we have a non-ideal voltage source with non-infinite G_o the problem comes. In [7] was this problem solved as in Fig. 9. It turned out, however, that this approach does not lead to good results [20] - this is suitable for infinite or zero G_o only - as above. The right solution we can find in Fig. 10.

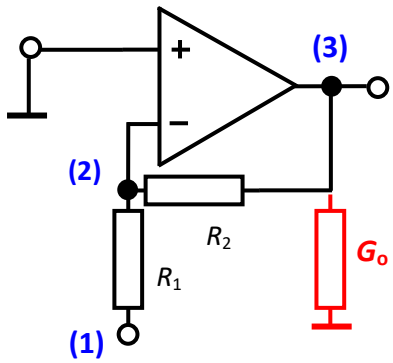


Fig.9. A non-correct model of the amplifier output admittance (red)

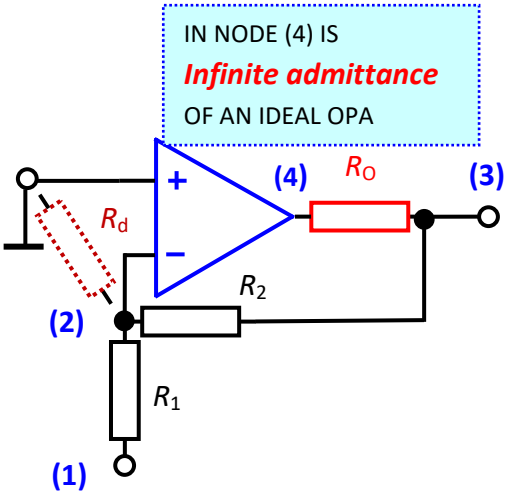


Fig. 10 A more realistic OPA model – output resistance R_O is depicted by means of an "external element R_o" as well as a differential input resistance R_d

Now we can use the above algorithm again. We easily get graph in Fig.11.

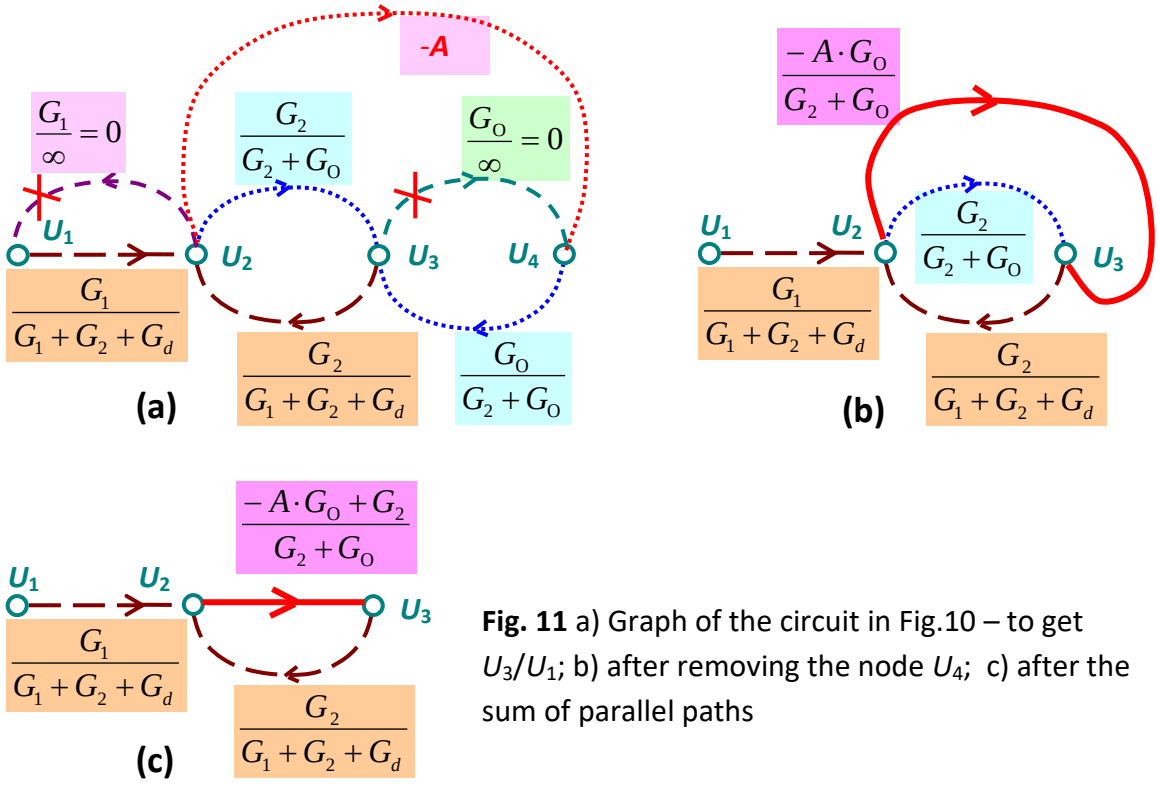


Fig. 11 a) Graph of the circuit in Fig.10 – to get U_3/U_1; b) after removing the node U_4; c) after the sum of parallel paths

Now we are able easily derive relationship below

$$\frac{U_3}{U_1}=-\frac{R_2}{R_1}\cdot\frac{1-R_o/(AR_2)}{1+\frac{1+(R_2+R_o)/R_1+(R_2+R_o)/R_d}{A}}$$

If $R_o \to 0$ then

$$\frac{U_3}{U_1}=-\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{1+R_2/R_1+R_2/R_d}{A}}$$

If $R_o \to 0$ and $R_d \to \infty$ then

$$\frac{U_3}{U_1}=-\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{1+R_2/R_1}{A}}$$

And right now it's time to look at the previous results - and there is a consensus.

1.3 An input impedance of electronic structures

For demonstration, use the structure in Fig. 10. From the graph in Fig. 11c, we can easily determine transmission

$$\frac{U_2}{U_1} = \frac{G_1}{G_1 + G_2 + G_d} \cdot \frac{1}{1 - \frac{-AG_o + G_2}{G_o + G_2} \cdot \frac{G_2}{G_1 + G_2 + G_d}} = |G_o \gg G_2| \cong \cdots$$

$$\cdots \cong \frac{G_1}{G_1 + G_2 + G_d + AG_2}$$

Now the current I_1 flowing through the resistor R_1 is

$$I_1 = \frac{U_1 - U_2}{R_1} = \frac{U_1 - U_1 \cdot \frac{G_1}{G_1 + G_2 + G_d + AG_2}}{R_1}$$

Thus the ***input impedance*** is

$$\frac{U_1}{I_1} = \ldots = \frac{1}{G_1} + \frac{1}{G_2 + G_d + AG_2} = R_1 + \frac{R_2}{1 + A + R_2 / R_d}$$

However, we can use another procedure - generally applicable to each node of the circuit. Exciting signal is current I_1 into node 1 now. Corresponding graph we have in Fig. 12. If the source of the signal is the current source (I_1), the transfer from the node 2 to the node 1 remains in the graph too!

Now we can use Mason's rule.
We have just one forward path (from current source into U_1) – its gain is $P_1 = 1/G_1$.

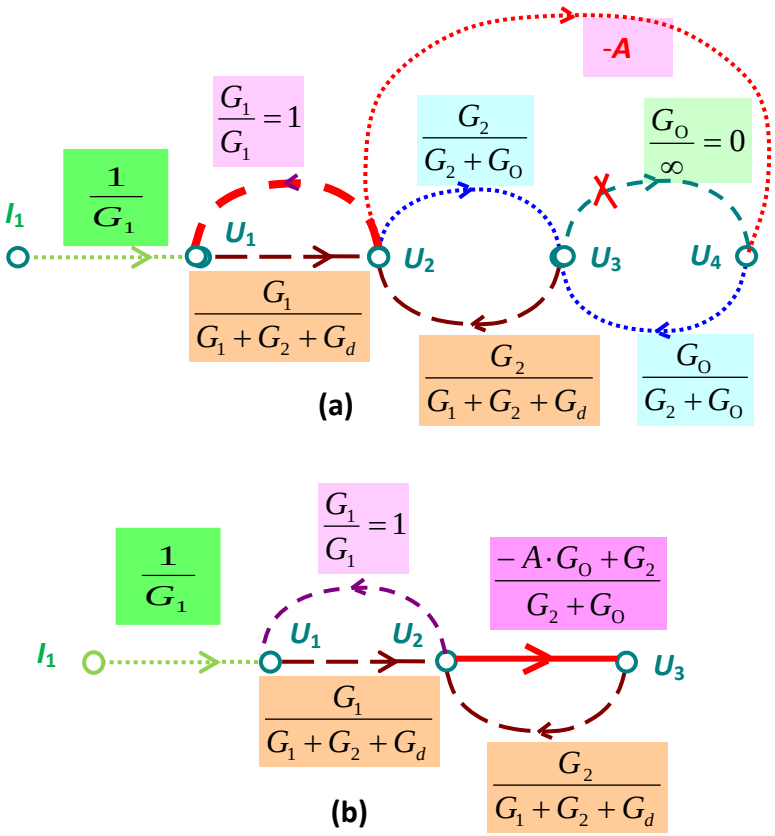


Fig. 12 a) Graph of the circuit in Fig.10 – to get U_1/I_1; b) after removing the node U_4

The graph determinant (two loops – in touch)

$$\Delta = 1 - \left[\frac{G_1}{G_1+G_2+G_d}\cdot 1 + \frac{-AG_o+G_2}{G_o+G_2}\cdot\frac{G_2}{G_1+G_2+G_d}\right] = |G_o \gg G_2| = \frac{G_2+G_d+AG_2}{G_1+G_2+G_d}$$

The subdeterminant (P_1 disconnect "loop $U_1 - U_2$"- node U_1 is removed; "loop $U_2 - U_3$"is maintained)

$$\Delta_1 = 1 - \left[\frac{-AG_o+G_2}{G_o+G_2}\cdot\frac{G_2}{G_1+G_2+G_d}\right] = |G_o \gg G_2| = \frac{G_1+G_2+G_d+AG_2}{G_1+G_2+G_d}$$

Now

$$\frac{U_1}{I_1} = \frac{(1/G_1)\cdot\frac{G_1+G_2+G_d+AG_2}{G_1+G_2+G_d}}{\frac{G_2+G_d+AG_2}{G_1+G_2+G_d}} = (1/G_1)\cdot\frac{G_1+G_2+G_d+AG_2}{G_2+G_d+AG_2} =$$

$$= (1/G_1)\cdot\left(1+\frac{G_1}{G_2+G_d+AG_2}\right) = \frac{1}{G_1}+\frac{1}{G_2+G_d+AG_2}$$

But it is already a known result from above.

If we need to know ***output resistance*** we excite node 3 with current I_3 and the node voltage U_1 is zero, thus transfers from node 1 are zero too. This fact will be reflected in the graph of Fig. 12b as illustrated in Fig.13. The nodal admittance of node 3 is known, so transmission from I_3 to U_3 is clear.

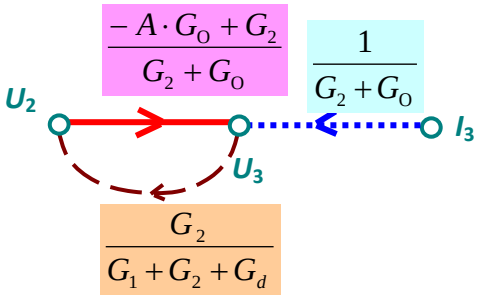


Fig.13 Graph for derivation of the output node impedance (output excited by current I_3)

I is evident that ***output resistance (impedance)*** is

$$\frac{U_3}{I_3}=\frac{1}{G_2+G_O}\cdot\frac{1}{1-\frac{-A\cdot G_O+G_2}{G_2+G_O}\cdot\frac{G_2}{G_1+G_2+G_d}}=\ldots=\left|G_d\to 0\right|=\frac{1}{G_O+\frac{G_1G_2+G_2AG_O}{G_1+G_2}}$$

$$\frac{U_3}{I_3}=\frac{R_o}{\frac{R_o}{R_1+R_2}+1+A\cdot\frac{R_1}{R_1+R_2}}$$

1.3 A direct algorithm of the signal flow graph compilation for voltage controlled current source [20]

It can be relatively easily to determine an algorithm for voltage controlled current source (OTA). Basic “circuit situation” is seen in Fig.14. OTA transmissions between inputs (voltage - ideally infinite input resistance) and its output (current - ideally infinite output resistance) are described in the usual manner by means of transconductance G_m.

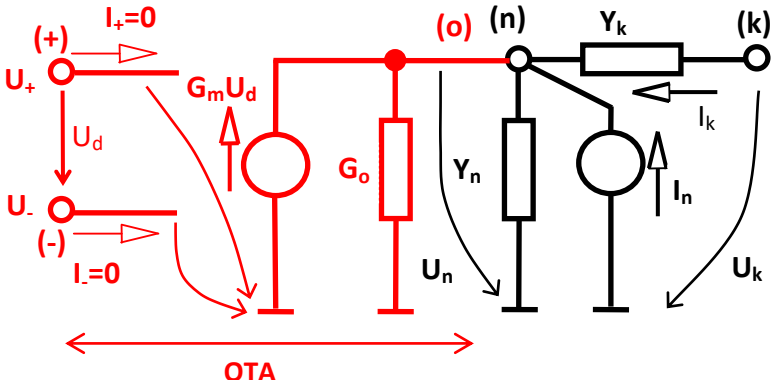


Fig. 14 Into node (n) is connected the OTA output **(o),** the current source I_n (signal current), an admittance Y_k toward the node (k) and an admittance Y_n toward the reference

From the situation shown in Fig.14 is to see that

$$U_n = \frac{G_m \cdot (U_+ - U_-) + I_n + Y_k \cdot (U_k - U_n)}{G_0 + Y_n}$$

After adjusting the relationship we get

$$U_n = \frac{G_m}{G_0 + Y_n + Y_k} \cdot (U_+ - U_-) + \frac{I_n}{G_0 + Y_n + Y_k} + \frac{Y_k \cdot U_k}{G_0 + Y_n + Y_k}$$

Now the situation is clear. Transconductance G_m must be divided by nodal admittance on its output. In fact, this is in line with the previous considerations - transmission from the current node is divided by the nodal admittance of the node to which the current flows.

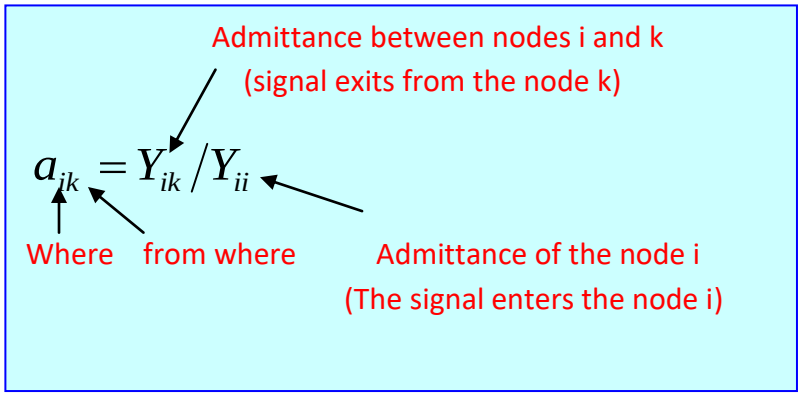
Fig. 15 Signal flow graph (model) of the OTA;

Y_{OE} = the total admittance of the node to which it is connected OTA output (Including its output conductance G_o if it is non - zero)

Based on our considerations, the ***following algorithm*** can be defined:

1) We draw the input and output OTA admittances into the circuit diagram - next they are part of the circuit (now the output admittance is connected correctly to the reference node).

2) For each topological node, current source, and voltage source, we assign a graph node.
3) Now we determine the transfer (***gain, transmission, transmittance)*** a_{ik} (for each node **i**) from all other nodes **k** according to the relation:



4) The current source I_i entering into node **i** has a + 1 / Y_{ii} transmission. Transfer from any node to the source does not exist (the ideal source we can not affect).
5) Between the corresponding nodes of the graph we add the branches with the transmission ± Gm and these transmissions are divided by the total admittance of the node to which the OTA output is connected - see Fig.14 (the resulting transfer between the voltage nodes is so dimensionless, which is physically correct).
6) We solve the required gains (transmissions) as before.

1.2.1 ***Example*** **2** [14, 20]

Fig.16 shows the low-pass filter with OTA and a voltage buffer.

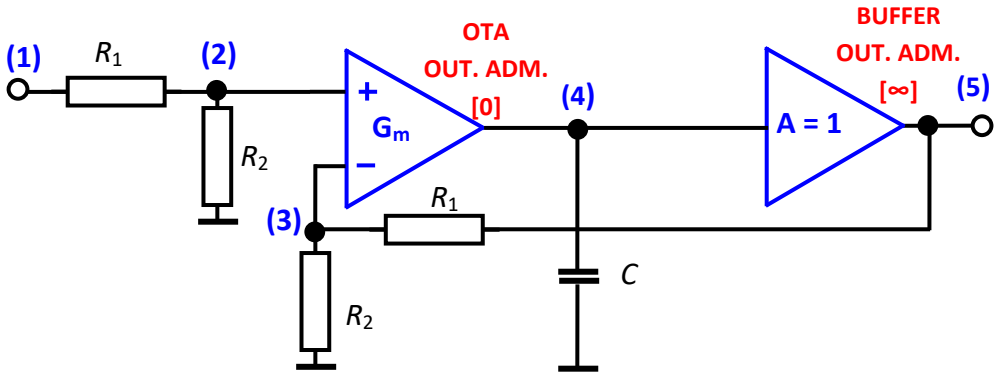


Fig. 16 The 1-st order Low Pass filter

Fig.17 shows the transmissions of the passive part of the circuit of Fig.16, we consider the excitation from the voltage source U_1, so the transfers to the node 1 are "canceled" - it is not necessary to mark them. For the same reason, we will not mark transmissions of the passive part of the circuit to node 5 because here is connected the output of the follower with a

voltage transfer A = 1 and an infinite output admittance. Resistances R2 connected into nodes 2 and 3 will be depicted as the node admittances of nodes 2 and 3 only (here in the same way G1 + G2), because the second terminal is connected to the reference node. The transfer from the reference node is zero (reference ≡ zero signal) and transmission to the reference node is simply "not" because the node admittance of the reference node is infinite (the node reference has a zero impedance). For the same reason, the capacity C will be included only to own admittance of the node 4 - as shown in Fig.17 - see **p.C** in square brackets (p = jω - steady harmonic state).

Now we can add the admittance (transadmittance) ± Gm of the active element (OTA) from nodes 2 and 3 to node 4 and divide it by admittance of this node (here is the OTA output) – by the admittance pC (resulting transfer between nodes representing the voltage is thus without dimension, which is physically right). Next, we add a voltage transfer A = 1 from node 4 to node 5 - Fig.18.

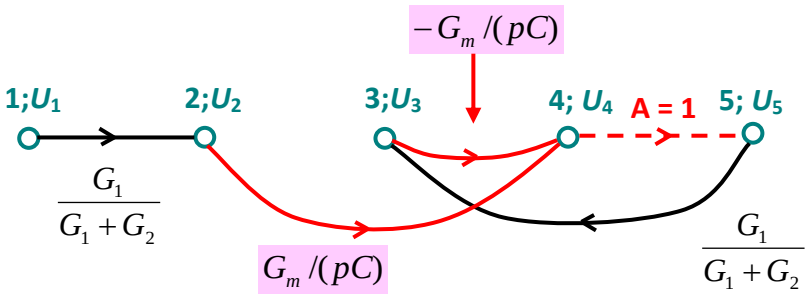


Fig. 18 Graph of the circuit on Fig.16

The graph of Fig.18 is redrawn to a more suitable form - Fig.19.

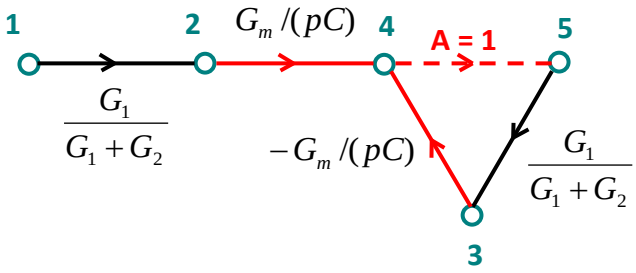


Fig. 19 Redrawn graph from Fig.18

Now it is evident that

$$\frac{U_5}{U_1}=\frac{\frac{G_1}{G_1+G_2}\cdot\frac{G_m}{pC}\cdot 1}{1-\left[1\cdot\frac{G_1}{G_1+G_2}\cdot\frac{-G_m}{pC}\right]}=\frac{\frac{G_mG_1}{G_1+G_2}\cdot\frac{1}{C}}{p+\frac{G_mG_1}{G_1+G_2}\cdot\frac{1}{C}}$$

The result deserves discussion. Transduction G_m of industrially produced OTA can usually be controlled by current (voltage). Transmission describes the 1st order low pass filter with

$$\omega_0 = \frac{G_m G_1}{G_1 + G_2} \cdot \frac{1}{C}$$

Therefore, the characteristic frequency can be controlled by current (voltage).

2. GRAPH ACCORDING TO THE FINAL SYSTEM OF LINEAR EQUATIONS – THE *B TYPE* OF ARRANGEMENT (MB signal flow graphs)

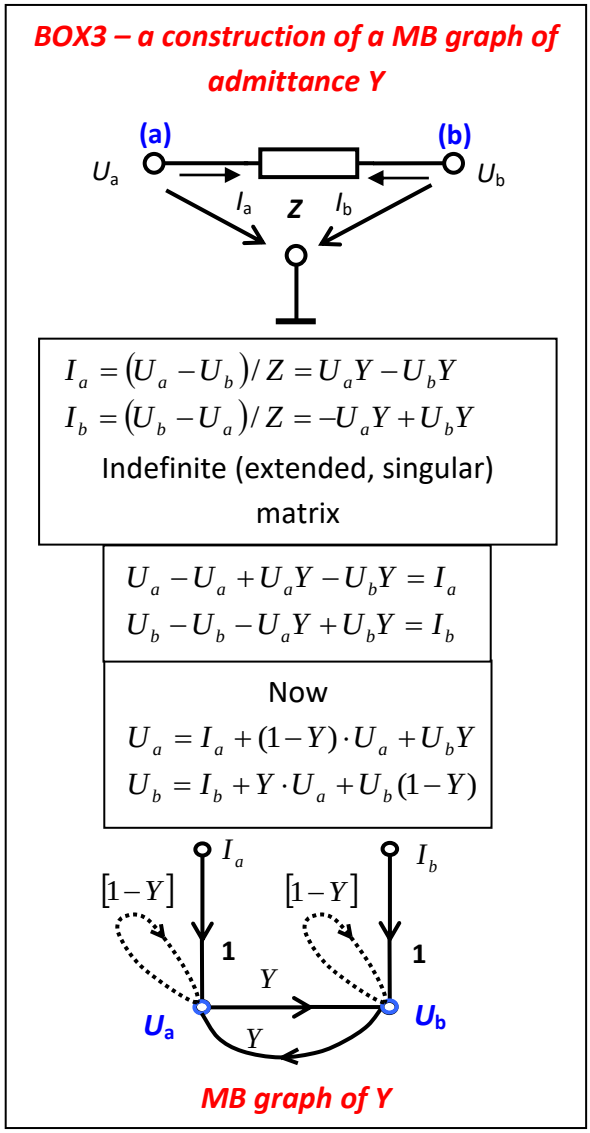
It is evident that creating models of active circuits we have some difficulties. We can largely eliminate these by modifying the initial equations (2) by way "of B"[21] - by "appropriate addition of zero" as (U_k - U_k) to equations (equivalent rearrangement of system equations):

$$\begin{aligned}
&U_1 - U_1 + U_1 Y_{11p} - U_2 Y_{12p} - U_3 Y_{13p} = I_1 \\
&U_2 - U_2 - U_1 Y_{12p} + U_2 (Y_{22p} + Y_{aa}) - U_3 (Y_{23p} - Y_{ab}) = I_2 \\
&U_3 - U_3 - U_1 Y_{13p} - U_2 (Y_{23p} - Y_{ba}) + U_3 (Y_{33p} + Y_{ab}) = I_3
\end{aligned}$$

From each equation we now can to separate the "diagonal" element - but without dividing it – this way:

$$\begin{aligned}
U_1 &= I_1 + (1 - Y_{11p}) \cdot U_1 + Y_{12p} \cdot U_2 + Y_{13p} \cdot U_3 \\
U_2 &= I_2 + Y_{12p} \cdot U_1 + \left(1 - Y_{22p} - Y_{aa}\right) \cdot U_2 + (Y_{23p} - Y_{ab}) \cdot U_3 \\
U_3 &= I_3 + Y_{13p} \cdot U_1 + (Y_{23p} - Y_{ba}) \cdot U_2 + \left(1 - Y_{33p} - Y_{ab}\right) \cdot U_3
\end{aligned}$$

Now we are able construct signal flow graph (where there is yet "no division") if we know the matrix description of circuits (elements of circuit). Let us construct first a signal flow graph (MB) of general admittance – Box3.



If $U_b = 0$ (node b connected to reference point) then

$$I_a = (U_a - 0)/Z = U_aY \quad \Rightarrow$$
$$U_a = I_a + (1-Y)\cdot U_a$$

We get MB graph in Fig. 20. Current graph nodes ($I_{a,b}$) we use only if current signals (sources) inputs topological nodes.

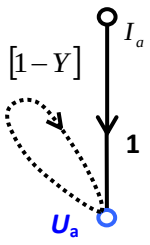


Fig. 20 MB graph of a resistor which connected to reference node

"Statement" in square brackets describes the property of a graph node - but it is "separate" now.

Let us see on a simple divider in Fig. 21a.

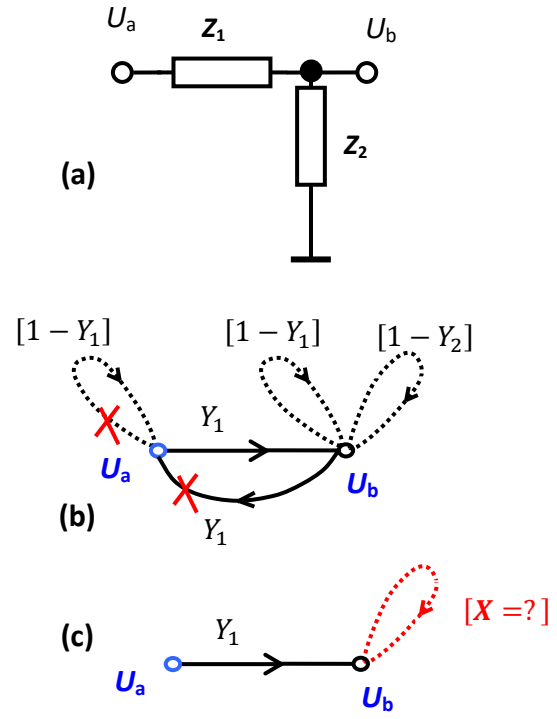


Fig. 21 (a) – a voltage divider; (b) – its "default" MB graph; (c) – its final MB graph

We suppose a voltage "excitation U_a" – the ***passive part of transfers*** into this node are ***canceled*** – Fig.21b. We can see that there are still two loops to the node U_b (of the graph). What can we do now? What is X in Fig.21c?

But we know that it is valid

$$U_b = U_a \cdot \frac{Z_2}{Z_1 + Z_2} = U_a \cdot \frac{Y_1}{Y_1 + Y_2}$$

For the graph in Fig.21c we can write

$$U_b = Y_1 U_a + \boldsymbol{X} U_b$$

thus

$$U_b = \frac{Y_1}{1 - \boldsymbol{X}} U_a$$

And now it must be

$$U_a \cdot \frac{Y_1}{Y_1 + Y_2} = \frac{Y_1}{1 - \boldsymbol{X}} U_a$$

It is evident that now

$$Y_1 + Y_2 = 1 - \boldsymbol{X} \implies \boldsymbol{X = 1 - Y_1 - Y_2}$$

So algebra is intended to "sum square brackets" (for MB graphs only):

$$\boldsymbol{[1 - Y_1] + [1 - Y_2] + \cdots + [1 - Y_n] = [1 - Y_1 - Y_2 - \cdots - Y_n]}$$

Every resulting node loop can have "only once number 1". It is the basic property "of MB" – more details is in [20, 21].

Direct construction of MB graph is now able:

1) For each topological node, current source, and voltage source, we assign a graph node.
2) The current source I_i into node (i) has transfer +1. Transfer from any node to the source does not exist (the ideal sources cannot be affected).
3) For each element R, L, C, we assign a graph according to BOX3 or Fig. 20 (without current nodes unless a current source signal is connected to the node).
4) We will replace the active elements (amplifiers) between the corresponding nodes with their graphs obtained from their admittance models by way of arrangement B.
5) After drawing the graphs of all circuit elements we sum at each node all loop in square brackets:

> Transfer of the resulting node loop = the sum of "all loops" connected to the node, but (see above)
> $$\boldsymbol{[1 - Y_1] + [1 - Y_2] + \cdots + [1 - Y_n] = [1 - Y_1 - Y_2 - \cdots - Y_n]}$$
> $\boldsymbol{Y_u = Y_1 + Y_2 + \cdots + Y_n}$ **– It is the node admittance**

6) Now it is possible to solve the graph in all common ways - it is a "full-fledged" graph according to Mr. Mason [16]. It is possible to remove the node admittance loops, that is, divide all the branch transfers entering into the node by the member (in accordance with the rules for editing the graph) $1-[1-Y_u]=Y_u$ - again by admittance of the node, but now defined fully exactly.

2.1 MB graph of OPA

Let us derive the MB graph of OPA – Fig.22 – for example.

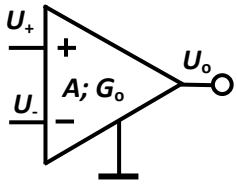


Fig. 22 Idealized OPA - defined only gain ***A*** and the output conductance $\boldsymbol{G_o}$ - input currents are zero - idealized

The admittance matrix [2, 3, and 9] is

	(+)	(-)	(o)			
(+)	0	0	0	U_+	=	I_+
(-)	0	0	0	U_-		I_-
(o)	$-A.G_o$	$A.G_o$	G_o	U_o		I_o

From this we easily get:

$$I_+ = 0.U_+ + 0.U_- + 0.U_o$$
$$I_- = 0.U_+ + 0.U_- + 0.U_o$$
$$I_o = -A.G_o.U_+ + A.G_o.U_- + G_o.U_o$$

Now

$$U_+ = I_+ + [1\text{-}0].U_+ + 0.U_- + 0.U_o$$
$$U_- = I_- + 0.U_+ + [1\text{-}0].U_- + 0.U_o$$
$$U_o = I_o + A.G_o.U_+ - A.G_o.U_- + [1\text{-}G_o].U_o$$

The assignment of the signal flow graph to the equation system is already evident - Fig.23.

If we want to include the OPA input differential resistance, it is sufficient to apply the algorithm for the differential resistance R_d connected between the non-inverting (+) and inverting (-) inputs - Fig. 24, for the R_d we use the graph of BOX3.

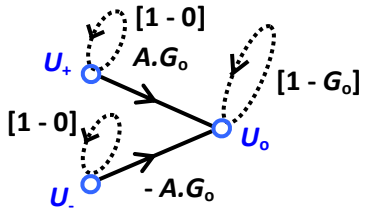


Fig. 23 The model (graph) of idealized OPA - only ***A*** and G_o defined - current inputs are not plotted

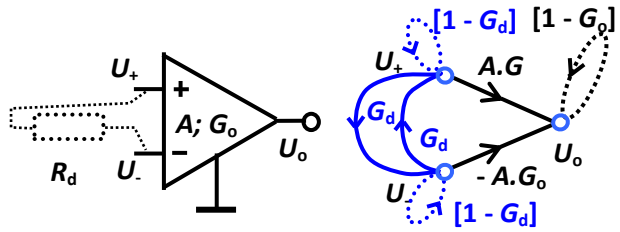


Fig. 24 The model (graph) of the OPA – with differential resistance R_d

However, simpler is to include the input impedances of the amplifier structure directly into the passive part of the circuit - the final result is the same.

We do not modify graph before adding it to the "overall structure" - it is necessary to keep loops of nodal admittances. The same problems would arise as in the methodology used in [7, 1] - with the correct determination of the influence of total admittance of nodes on transmissions into this graph nodes. This statement applies to models of all n - terminals.

Let's repeat solution of the structure in Fig. 7 - by means of the MB graphs. For each node (of graph; nodes of the investigated circuit and graph nodes are the same for the method used), we will draw only one node admittance loop for better clarity, to which we will gradually add the square brackets that we add according to the defined rule - Fig. 25.

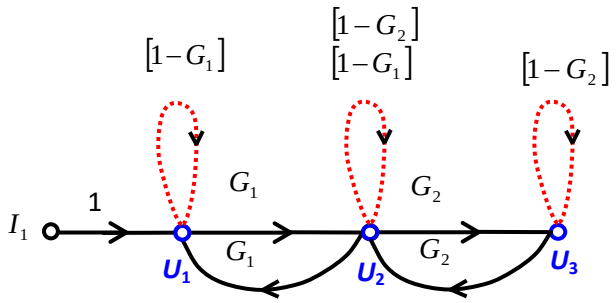


Fig. 25 MB graph of the passive part of the structure in Fig.7

Fig. 26 shows the graph "with the operational amplifier" - the transmissions "from zero" and "into zero" are meaningless, therefore only the path between inverting input and output of the OPA is used.

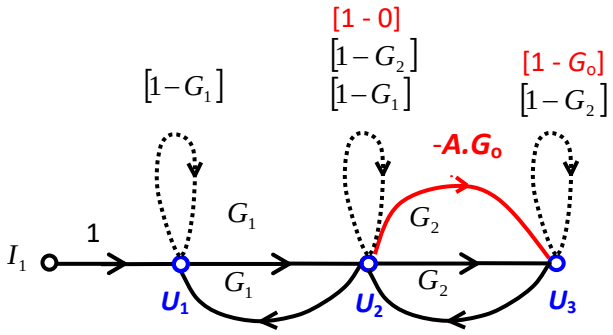


Fig. 26 Supplemented graph with OPA – before the sum of own loop transmissions

In the graph are all components, we can add the data in square brackets according to the rule for the MB graph – see Fig.27.

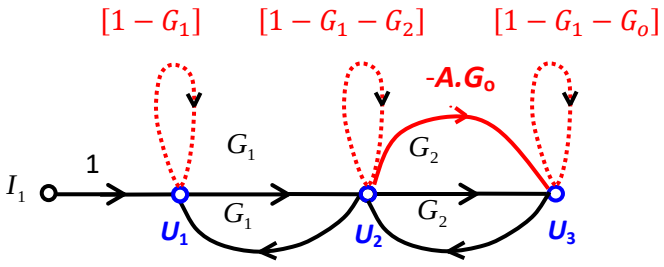


Fig. 27 The resulting MB graph of the structure of Fig.7

This is already a "full-fledged" graph, according to Mr. Mason. We can solve it directly using Mason's rule or by editing. For greater clarity of the graph, the basic removal of nodal loops according to point 6 of the algorithm leads - Fig.28.

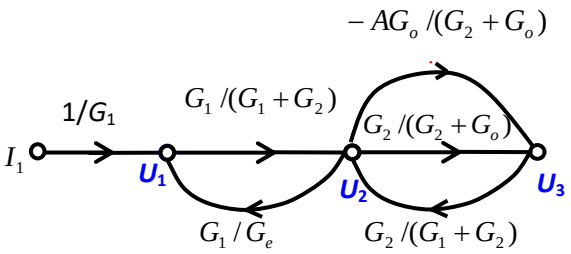


Fig. 28 Edited graph from Fig.27

If we determine graph transmission from node 1 to node 3, we "cut" all the branches entering node 1 (this corresponds to the connection of the ideal voltage source to node 1) - Fig. 29. For the infinite value G_o, our graph will be the same as the graph in Fig. 8.

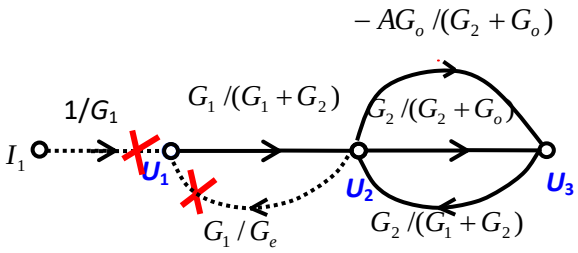


Fig. 29 Edited graph from Fig.28 – for determining the gain U_3/U_1

By editing the graph or using a Mason rule, it can be determined that graph transmission (in this case, the circuit voltage transfer) is

$$\frac{U_3}{U_1} = \ldots = \frac{-A\cdot G_o\cdot G_1 + G_1\cdot G_2}{G_1\cdot G_2 + G_1\cdot G_o + G_2\cdot G_o + A\cdot G_2\cdot G_o}$$

Upon editing the previous relationship, we get

$$\frac{U_3}{U_1} = -\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{1+(R_2+R_o)/R_1}{A}} + \frac{R_o}{R_o+R_2+R_1\cdot(1+A)}$$

The first member corresponds to the commonly-referenced relationship for amplifying the inverting structure; the second member defines the less frequently described forward transmission to the non-zero output resistance of the non-ideal operational amplifier.

3. MASON – COATES GRAPHS (MC graphs) [5, 6]

The starting point for the MC graphs methodology can again be a set of relations (2):

$$U_1Y_{11p} - U_2Y_{12p} - U_3Y_{13p} = I_1$$
$$-U_1Y_{12p} + U_2(Y_{22p}+Y_{aa}) - U_3(Y_{23p}-Y_{ab}) = I_2$$
$$-U_1Y_{13p} - U_2(Y_{23p}-Y_{ba}) + U_3(Y_{33p}+Y_{bb}) = I_3$$

The following modification ("**Modification C**") was used:

$$U_1 Y_{11p} = I_1 + U_2 Y_{12p} + U_3 Y_{13p}$$
$$U_2 (Y_{22p} + Y_{aa}) = I_2 + U_1 Y_{12p} + U_3 (Y_{23p} - Y_{ab})$$
$$U_3 (Y_{33p} + Y_{bb}) = I_3 + U_1 Y_{13p} + U_2 (Y_{23p} - Y_{ba})$$

The important thing is that we "do not divide" by the nodal admittance now, but to the node of the graph we "write this nodal admittance" - for information" - to the non-oriented loop of the node. This non-oriented loop is not actually a transmission; it only allows us to properly construct the resulting node admittance.

For MB graphs where equivalent equations were made for each equation, the same nodal admittance appears with the negative sign - subtracted from number 1.

If we make considerations comparable to the considerations in article 2 of this thesis, it is obvious that the graph MC is obtained from the MB graph so that we simply remove the orientation arrow of loop and its transmission

[1-Y_a] we simply replace by Y_a

and vice versa we can do it too.

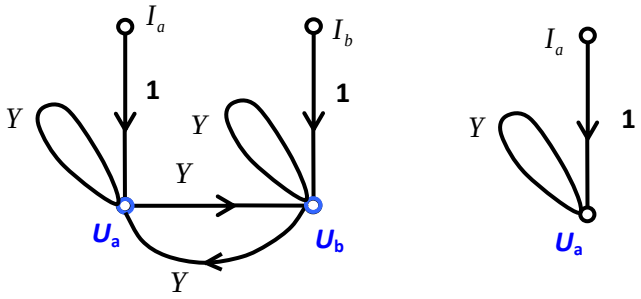


Fig. 30 MC graph of admittance *Y* and "grounded" *Y* – see BOX3 and Fig.20 too

Direct construction of MC graph

1) For each topological node, current source, and voltage source, we assign a graph node.
2) The current source I_i into node (i) has transfer +1. Transfer from any node to the source does not exist (the ideal sources cannot be affected).

3) For each element R, L, C, we assign a graph according to Fig. 30 (without current nodes unless a current source signal is connected to the node).
4) We will replace the active elements (amplifiers) between the corresponding nodes with their graphs obtained from their admittance models by way of arrangement C (MC graphs).
5) After drawing the graphs of all circuit elements, we "sum" for each node all gains of non-oriented loops:

The resulting admittance of the non-oriented node loop = the sum of admittances of the all non-oriented loops which are connected with this node.

6) *At this point we have a MC signal flow graph of the analyzed circuit. If we now want to get a "full-fledged" graph by Mr. Mason, we remove the non-oriented loops (carrying the total node admittance information) by dividing all branch transfers into the node directly by admittance Y_u which is attributed to the non-oriented node loop. This is how we get a graph where each branch actually defines transmission, and we can solve it by "modifying" or by means of Mason's rule.*
7) Another option is to solve the directly acquired MC graph using the modified Mason rule, see [5, 6]. Modification is necessary because non-oriented loops are not actually transmissions.

Let's repeat solution of the structure in Fig. 7 - by means of the MC graphs. For the clarity, we will now draw only one non-oriented loop to the each node, to which we will be gradually attribute admittances that we then "sum together" on the end. In the Fig.31 is a MC graph of the passive part of the circuit. The MC graph of the operational amplifier is obtained easily from the MB graph in Fig.23 – see Fig.32

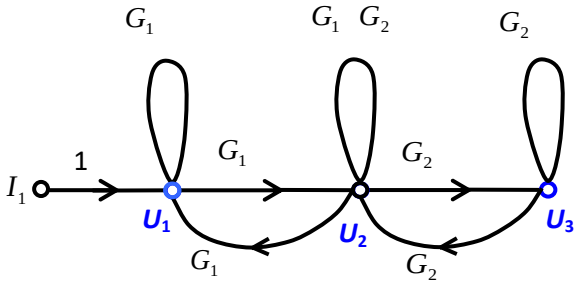


Fig. 31 MC graph of passive part of the circuit from Fig.7

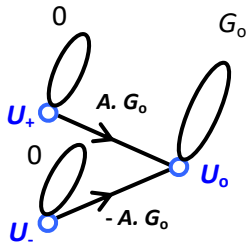


Fig. 32 MC graph of the OPA – no current inputs

The resulting MC graph is in Fig.33, transfer of the non-inverting input "from zero" does not make sense to draw. Admittances of non-oriented loops are already "summed" (square brackets). If we now divide gains of the branches entering the nodes with the appropriate nodal admittance - that is by an expression in the corresponding non-oriented loop (and

then we remove it) - we get a graph that is identical to the situation in Fig. 28. This graph is already solved by the previously mentioned conventional procedures. And another discussion will be the same.

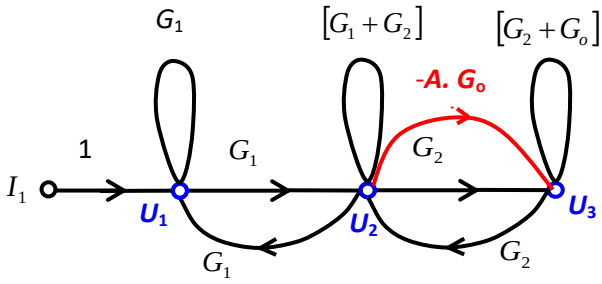


Fig. 33 The resulting MC graph of the inverting amplifier

4. ANOTHER EXAMPLE

Another example to be solved is in Fig.34. This circuit was solved in [22] by some different methods. And now we use signal flow graph with an ideal OPA – see Fig.6; the algorithm from chapter 1.1.

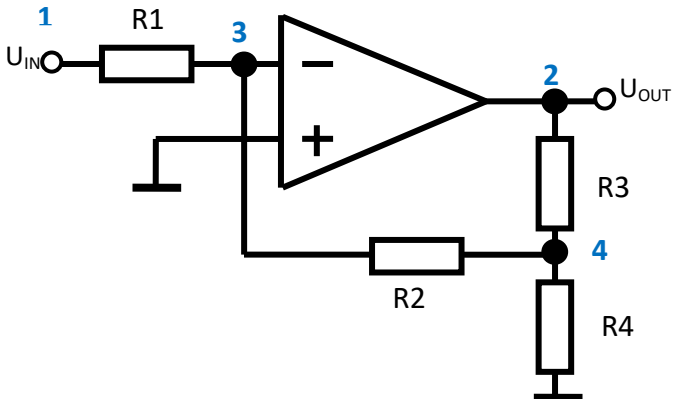


Fig.34 The inverting structure with OPA – "a T in feedback"

The signal flow graph is in Fig.35 – we suppose voltage signal U_1 and infinite output admittance of the OPA – thus in the node 2.

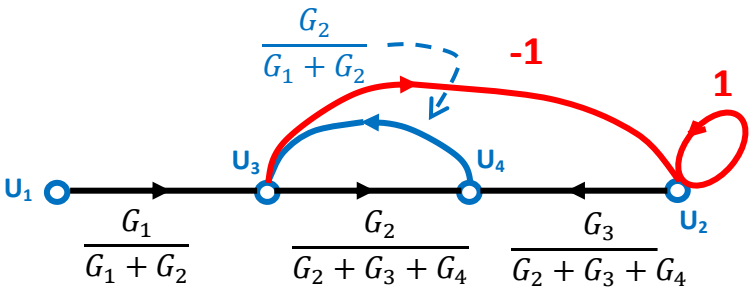


Fig.35 Signal flow graph of the circuit in Fig.34

We use Mason's rule. There are in the graph:

Three loops: 1 and $\frac{G_2}{G_2+G_3+G_4} \cdot \frac{G_2}{G_1+G_2}$ and $\frac{G_2}{G_1+G_2} \cdot (-1) \cdot \frac{G_3}{G_2+G_3+G_4}$.

Non-touching loops: 1 and $\frac{G_2}{G_2+G_3+G_4} \cdot \frac{G_2}{G_1+G_2}$

Thus the graph determinant is

$$\Delta = 1 - \left[1 + \frac{G_2}{G_2+G_3+G_4} \cdot \frac{G_2}{G_1+G_2} - \frac{G_2}{G_1+G_2} \cdot \frac{G_3}{G_2+G_3+G_4}\right] + \frac{G_2}{G_2+G_3+G_4} \cdot \frac{G_2}{G_1+G_2}$$

$$\Delta = \frac{G_2}{G_1+G_2} \cdot \frac{G_3}{G_2+G_3+G_4}$$

One Forward path (from U_1 into node U_2):

$$P_1 = \frac{G_1}{G_1+G_2} \cdot (-1)$$

This path disconnect all loops, so subdeterminant is

$$\Delta_1 = 1$$

Now we easily determine that

$$\frac{U_2}{U_1} = \frac{P_1 \cdot \Delta_1}{\Delta} = -\frac{\frac{G_1}{G_1+G_2}}{\frac{G_2}{G_1+G_2} \cdot \frac{G_3}{G_2+G_3+G_4}} = -\frac{\boldsymbol{R2}}{\boldsymbol{R1}} \cdot \left(\boldsymbol{1} + \frac{\boldsymbol{R3}}{\boldsymbol{R4}} + \frac{\boldsymbol{R3}}{\boldsymbol{R2}}\right)$$

It is the same result as in [22].

5. CONCLUSION

This article assigns in a new way models based on admittance models (type B modification) - MB signal flow graphs - and describes the methodology for use of them. There are described another active elements in [21 and 20].

An unambiguous relationship can be found between the MB models (and methodology) obtained by the described procedure and the MC graphs of the signal graphs described, for example, in [5, 6] - the Mason-Coates graphs. What is important is that, in the procedure described here, the signal graph in each step is a "true" graph - this is no longer the case for the Mason-Coates graphs. At the same time, it is clearly demonstrated how you can easily get the MC graph from the MB graph and vice versa.

It is legitimate to ask whether it is preferable to solve the circuit by means of matrixes or graphs. It is very easy to build a graph of the passive part of the circuit. But if the graph

contains too many loops, it may be difficult to solve it. In this case, matrix methods may be more advantageous. But for simpler circuits, the graphs give us an elegant methodology - we get the result after drawing one signal flow graph - and its solution, of course.

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