INVERTING BAND-PASS FILTER WITH REAL OPERATIONAL AMPLIFIER

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INVERTUJÍCÍ PÁSMOVÁ PROPUST S REÁLNÝM OPERAČNÍM ZESILOVAČEM

Abstrakt

Práce řeší vliv prvního pólu (ω_1) operačního zesilovače na vlastosti invertující pásmové propusti. Skutečnou charakteristickou frekvenci f_r pásmové propusti lze určit pomocí jednoduchého vztahu, který již v sobě zahrnuje i vliv prvního pólu operačního zesilovače. Je-li požadován příliš velký součin $\omega_0 Q$, musí být velký i tranzitní extrapolovaný úhlový kmitočet $\omega_T = A_0\omega_1$ operačního zesilovače; ω_0 je ideálně požadovaný charakteristický kmitočet; Q je požadovaný činitel jakosti invertující pásmové propusti; A_0 je stejnosměrné zesílení operačního zesilovače.

Teoretické úvahy jsou doloženy experimentálními měřeními a tabulkou, která definuje požadavky kladené na reálný operační zesilovač, požadujeme-li realizaci ω_0 (f_0) s předepsanou tolerancí. Požadavky mohou být až překvapivě náročné například již pro nevelkou hodnotu Q = 2 a požadovanou toleranci $1 > f_r / f_0 > 0{,}99$ (chyba ve stanovení f_0 pod 1%).

One of the most practical configuration of the band-pass filter is in Fig.1. In the first-order analysis of linear circuits it is most convenient to assume the operational amplfier to be ideal. The ideal operational amplifier is characterized by an infinite open loop gain (A_0), infinite input impedance (R_i), zero output impedance (R_o). These ideal conditions greatly simplify the analysis of active filters in undemanding applications at very low frequencies.

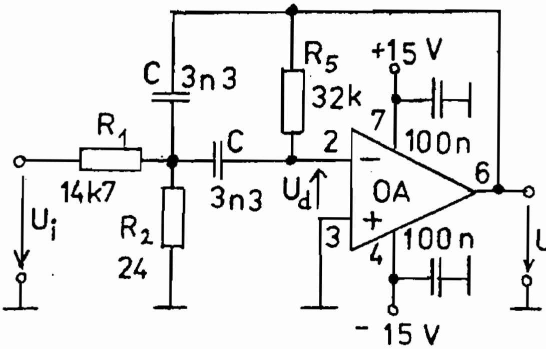

Fig.1. Inverting band-pass filter

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In exacting applications, the design must be based on a more accurate op amp model. One of the most important of the op amp characteristics is the frequency response ($A(p)$). In general, it is determined by many poles and zeros; however, in order to assure stability in closed-loop feedback configuration, most op amps are designed to have only one dominant negative real pole at $p = -\omega_l$ so that a suitably accurate op amp model is*

$$A(p) = \frac{U_o(p)}{U_d(p)} = \frac{A_0\omega_l}{p + \omega_l} \cong \frac{\omega_T}{p} \tag{1}$$

where ω_T is unity gain-bandwidth frequency (gain-bandwidth-product, unity gain-bandwidth) defined as

$$\omega_T = A_0 \cdot \omega_l \tag{2}$$

and ω_l is, of course, the 3 dB frequency of the op amp gain.

If we apply the fundamental rules from circuit theory**, we can determine (Fig.1) the transfer function (voltage gain) as follows:

$$\frac{U_o}{U_i} = -\frac{p / [R_1C(1 + 1 / A(p))]}{p^2 + p\dfrac{2 + (2 + R_5 / R_1 + R_5 / R_2) / A(p)}{C R_5(1 + 1 / A(p))} + \dfrac{1 / R_1 + 1 / R_2}{C^2R_5}} \tag{3}$$

We are now able to study properties of the band-pass filter.

If the op amp is ideal, it means $A(p) \to \infty$, the transfer function is given by Eq.(4)

$$\left(\frac{U_o}{U_i}\right)_{id} = -\frac{p / (R_1C)}{p^2 + p2 / (CR_5) + (1 / R_1 + 1 / R_2) / (C^2R_5)} \tag{4}$$

We can now easy derive an expression for the *ideal*:

characteristic frequency ω_0

$$\omega_0^2 = (1 / R_1 + 1 / R_2) / (C^2R_5) \tag{5}$$

Q-factor

$$Q = \sqrt{R_5 / R_1 + R_5 / R_2} / 2 \tag{6}$$

passband gain H

$$H = - R_5 / (2R_1) \tag{7}$$

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* NOTE 1: $p = j\omega$ for steady-state solution

** NOTE 2: Assume the op amp output impedance to be zero and the op amp input impedance to be infinite.

If the op amp is not ideal, it's parameters can have a drastic effect on filter response. Combining Eq.(1) and Eq.(3) gives Eq.(8)

$$\frac{U_o}{U_i} = -\frac{p(CR_5 / R_1) / a}{p^2\left(1 + \frac{p}{\omega_T} \cdot \frac{C^2R_5}{a}\right) + p\frac{2C + (1/R_1 + 1/R_2)/\omega_T}{a} + \frac{1/R_1 + 1/R_2}{a}} \tag{8}$$

where a is given by Eq.(9)

$$a = C^2R_5 + C(2 + R_5/R_1 + R_5/R_2)/\omega_T \tag{9}$$

It is evident that $C_2 R_5 / a \leq 1$ and Eq.(8) may be simplified for $\omega / \omega_T < 0{,}1$:

$$\frac{U_o}{U_i}(\omega / \omega_T \leq 0{,}1) \cong -\frac{p(CR_5 / R_1) / a}{p^2 + p\frac{2C + (1/R_1 + 1/R_2)/\omega_T}{a} + \frac{1/R_1 + 1/R_2}{a}} \tag{10}$$

Substituting the value a in Eq.(10), we get an expression for the *real characteristic frequency* ω_r:

$$\omega_r^2 = \frac{1/R_1 + 1/R_2}{a} = \frac{\omega_0^2}{1 + \omega_V / \omega_T} \tag{11}$$

where [Eq.(5), Eq.(6)]

$$\omega_V = \frac{2/R_5 + 1/R_1 + 1/R_2}{C} = \omega_0 (1/Q + 2Q) \tag{12}$$

is the *"own" characteristic frequency* of the operational circuit.

Using Eqn's (5), (6), (7), (12) we can calculate (refer to Fig.1):

f_0 = 55 080 Hz; Q = 18,3; H = - 1,088;

$f_V = \omega_V / (2\pi) = f_0(1/Q+2Q) = 2{,}019.10^6$ Hz

Table 1 summarizes properties of the band-pass filter with non-ideal op amps (experimentally measured). The quantity f_r / f_0 (the fifth column in Tab.1) can be determined from Eq.(11) as follows:

$$\omega_r / \omega_0 = f_r / f_0 = 1 / \sqrt{1 + f_V / f_T} = 1 / \sqrt{1 + (1/Q + 2Q) f_0 / f_T} \tag{11a}$$

Table 1. Properties of the band-pass filter in Fig.1 with non-ideal op amps [* - external compensation capacitor 30 pF]

OP AMP	f_T MHz	f_r kHz	f_r / f_0 -	f_r / f_0 -	f_0 / f_T -
MAC156 (LF156)	≅ 5	48.1	0,874	0,844	0,011
MAC155 (LF155)	≅ 3	42.0	0,763	0,773	0,018
MAA748 (μA748)	≅ 0,8 *	28.7	0,521	0,533	0,069
MAA741 (μA741)	≅ 0,6	25.4	0,461	0,479	0,092
	MEASURED			Eq.(11a)	
Eq.(5); f_0 = 55 080 Hz					

We are now able derive an expression for the needed f_T if the required magnitude of a frequency deviation is $\Delta f_0 = f_0 - f_r$, consequently

$$f_r / f_0 = 1 - \Delta f_0 / f_0 \tag{13}$$

If Eq.(13) is substituted into Eq.(11a), we obtain

$$f_r / f_0 = 1 - \Delta f_0 / f_0 = 1 / \sqrt{1 + f_V / f_T} \tag{14}$$

Solving Eq.(14) for f_T we can obtain an expression

$$f_T \geq f_V \Big/ \left[1 / (1 - \Delta f_0 / f_0)^2 - 1\right] = f_V \Big/ \left[(f_0 / f_r)^2 - 1\right] \tag{15}$$

consequently

$$f_T / f_0 \geq (1 / Q + 2Q) \Big/ \left[(f_0 / f_r)^2 - 1\right] \tag{16}$$

If the required deviation $\Delta f_0 / f_0$ is 0,05 (or 0,01) we can calcule the needed unity gain-frequency $f_T(0{,}05) > 18{,}7$ MHz [or $f_T(0{,}01) > 99{,}3$ MHz].

CONCLUSION

It is evident, that the experimentally measured characteristic frequency f_r can be calculated from the simple equation (11a). Data shown in Table 2 are obtained from this equation, by substituting various values from 1 to 50 for Q and from 0,0001 to 0,1 for f_0 / f_T ($f_T / f_0 = 10 - 10^4$).

Table 2. Data obtained from the equation (11a)

f_T / f_0	f_0 / f_T	f_r / f_0						ERROR
10 000	0,0001	**0,9999**	**0,9998**	**0,9995**	**0,9990**	**0,9980**	**0,9950**	< 1 %
5 000	0,0002	**0,9997**	**0,9996**	**0,9990**	**0,9980**	**0,9960**	**0,9901**	
2 000	0,0005	**0,9993**	**0,9989**	**0,9975**	**0,9950**	**0,9901**	0,9759	
1 000	0,001	**0,9985**	**0,9978**	**0,9949**	**0,9901**	0,9806	0,9535	< 10 %
500	0,002	**0,9970**	**0,9955**	**0,9900**	0,9805	0,9622	0,9129	
200	0,005	**0,9926**	**0,9889**	0,9754	0,9532	0,9128	**0,8165**	
100	0,01	0,9853	0,9782	0,9526	0,9125	**0,8450**	**0,7071**	
50	0,02	0,9713	0,9578	0,9114	**0,8446**	**0,7451**	**0,5773**	> 10 %
20	0,05	0,9325	0,9035	**0,8138**	**0,7062**	**0,5771**	**0,4082**	
10	0,1	**0,8771**	**0,8305**	**0,7036**	**0,5764**	**0,4470**	**0,3015**	
Q		1	2	5	10	20	50	
1/Q + 2Q		3	4,5	10,2	20,1	40,05	100,02	

If the product $2Q\omega_0$ is large [Eqn's (12), (16)], then the *own characteristic frequency* ω_V of the operational circuit becomes large and the *needed op amp unity gain-frequency* (f_T) *becomes very large.*

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