Filters studies 2

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Tasks of type to design low-pass filter (Butterworth, Bessel, and Chebyshev) are somewhat easy actually. In today's difficult times, it is necessary to offer students a slightly different view of the problem. No "how only" but also "why this solution". Where is the starting point?

Description of the somewhat more difficult situation

The text "Filters studies" stated the following:

There is an insulated metallic (wired) system at the risky workplace, which enables participants (from critical places) to communicate with the dispatcher - completely independent of other devices. On the same line, there is a signal still present with a frequency f_v = 10 kHz and a level of 100 mV, which indicates, for example, the presence of a burner flame. There are also 50 Hz and 100 Hz disturbances on the line (common industrial disturbances) with a level of approx. 50 mV. The speech signal level is about 700 mV. The line impedance is 300 Ω.

During the debate with Lutz von Wangenheim, I discovered that suitable solution with cascade connection LP2 and HP2 is a bit simple (:-)) and this could lead to simplified considerations about filters design. Therefore, I do a small change in the input conditions.

The whole assignment remains, only the following applies:

… There is a signal still present with a frequency f_v = **8 kHz** and a level of 100 mV …

Task is the same as before:

Design filters for the acoustic path!

Situation analysis:

Assume that the level of each interfering component must not exceed 2% of the level of the acoustic signal - ie the value of 14 mV. Therefore, interference at 50 Hz and 100 Hz must be suppressed at least 50/14 = 3.57 times (by 11.06 dB) and interference at **8 kHz** (*new value*) must be suppressed at least 100/14 = 7.14 times (by 17.1 dB). For an acoustic call, a frequency band of 300 Hz to 3500 Hz is sufficient; at the cut-off frequencies, the transmission drop may be 3 dB (this is not a measuring system - there the requirements would be more demanding). From this point of view, the Butterworth approximation of the filter satisfies - it is sufficient.

We therefore need to design a filter with an input impedance significantly greater than 300 Ω, a low frequency of 300 Hz with suppression (attenuation; against the pass band) 11 dB at 100 Hz (**point A** in Fig. 1; 50 Hz is a smaller frequency value, more will be suppressed - see below Fig. 1); with an upper frequency of 3500 Hz and with a suppression of 17 dB (against the pass band) at a frequency of **8 kHz** (**point B** in Fig. 1). The situation is captured in the picture - Fig. 1.

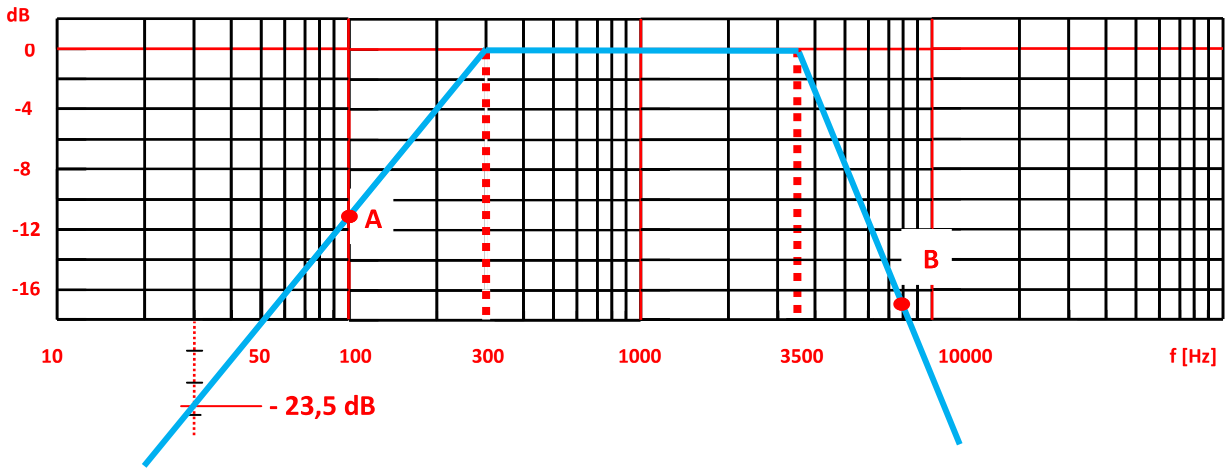


Fig. 1 Asymptotic depiction of the „new situation"

It is clear from the requirements shown that an asymptote with a slope of +23.5 dB / dec (change of transmission between 30 and 300 Hz) still passes through point A. The 1st-order high-pass filter has a slope of only +20 dB / dec, so **we must use a 2nd-order high-pass filter** with a slope of +40 dB / dec and a characteristic frequency of 300 Hz. So we can use the same solution as before – in *"Filters studies".*

It is also clear from the new requirements shown that an asymptote with a slope of -15 dB / oct (change in transmission between 3500 and 7000 Hz) passes through point B. The 2nd-order low-pass filter has a slope of only -12 dB/oct (= -40 dB/dec), so **we must use a 3rd-order low-pass filter** with a slope of -18 dB /oct (=60 dB/dec) and a characteristic frequency of 3500 Hz.

For the Butterworth approximation, we can also determine the necessary filter orders by calculation. The required order <u>n of the low pass filter</u> for a given approximation is

$$n = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(\omega_1/\omega_0)} = \frac{log\left(10^{\alpha/10} - 1\right)}{2 \cdot log(f_1/f_0)} = \frac{log\left(10^{17/10} - 1\right)}{2 \cdot log(8000/3500)} =$$

$$= \frac{log(50{,}12 - 1)}{2 \cdot log(2{,}286)} = \frac{1{,}69}{0{,}718} = 2{,}35$$

α is the required attenuation in dB at the frequency $\boldsymbol{\omega_1 > \omega_0}$

ω_0 is the characteristic frequency of the filter.

Because we are able "do" only whole values of *n*, we must use the next larger number, ie **3**! This corresponds to the simple previous "graphical solution" of the problem" ☺

By cascading HP2 (old) and new **LP3** (Butterworth approximation) we achieve the new required properties.

At this point, we have actually solved the so-called **approximation problem** for the initial situation. The required filters can be solved in various ways, but HP2 and LP3 - Butterworth approximations must always be implemented.

Possible implementation

If we do not have suitable tables, we must delve a little deeper into theory. The normalized transfer for the 3rd order Butterworth filters is [Theory of electronic circuits, Chapter7]:

$$\frac{1}{s^2 + 1.s + 1} \cdot \frac{1}{s + 1}$$

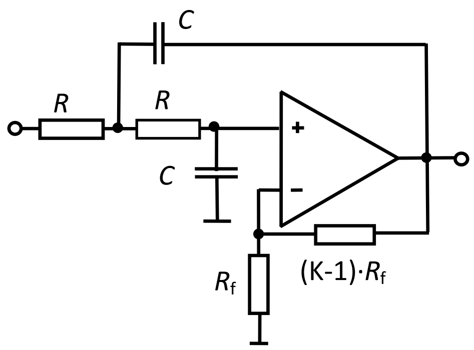
For the low-pass filter, we transform the normalized frequency by the relation $\boldsymbol{s \rightarrow p/\omega_0}$

By elementary modifications we obtain the transmission of the Butterworth low-pass filter of the 3rd order:

$$\frac{\omega_0^2}{p^2 + 1 \cdot p \cdot \omega_0 + \omega_0^2} \cdot \frac{\omega_0}{p + \omega_0}$$

We must therefore implement just one 2nd order low pass filter with a quality factor **Q = 1**, and one 1st order filter and we connect these filters in cascade – ω_0 of all filters is the same as ω_0 for the "all cascade"– for "Butterworth" only!

This time we will opt for the frequently used structure, which we are able to set very well, because the setting of the quality factor does not affect the characteristic frequency - this is a significant advantage in practice. We can't choose any gain arbitrarily, but that's not too much of a problem - Fig. 2.



It applies here: $\boldsymbol{\omega_p^2 = 1/(RC)^2; Q = 1/(3-K)}$;
K is the gain of the non-inverting amplifier.
Filter design:
1) We choose the value C; (or R)
2) Then we calculate $R = 1/(\omega_p C)$; or $(C = 1/(\omega_p R))$
3) Determine the value of K =3 - 1/Q. We choose a suitable value of R_f and calculate the value $(K-1).R_f$.

Fig. 2 The 2nd order low pass variant - Sallen-Key filter

Now we require a low pass filter (for "Butterworth") with $\omega_p = \omega_0 = 2\pi \cdot 3500$; we choose the value of $R = 10\ \mathrm{k}\Omega$; then we calculate $C = 1/(\omega_0 R) = 1/(2\pi \cdot 3500 \cdot 10^4)$; $C = 4{,}55\ \mathrm{nF}$. The needed gain is now **K** =3 - 1/Q = 3 – 1/1=**2 (new value)**. We choose $R_f = 10\ \mathrm{k}\ \Omega$; then the resistor $(K-1).R_f = 10\ \mathrm{k}\ \Omega$ (new value) will provide the needed gain **2** to ensure the required quality factor.

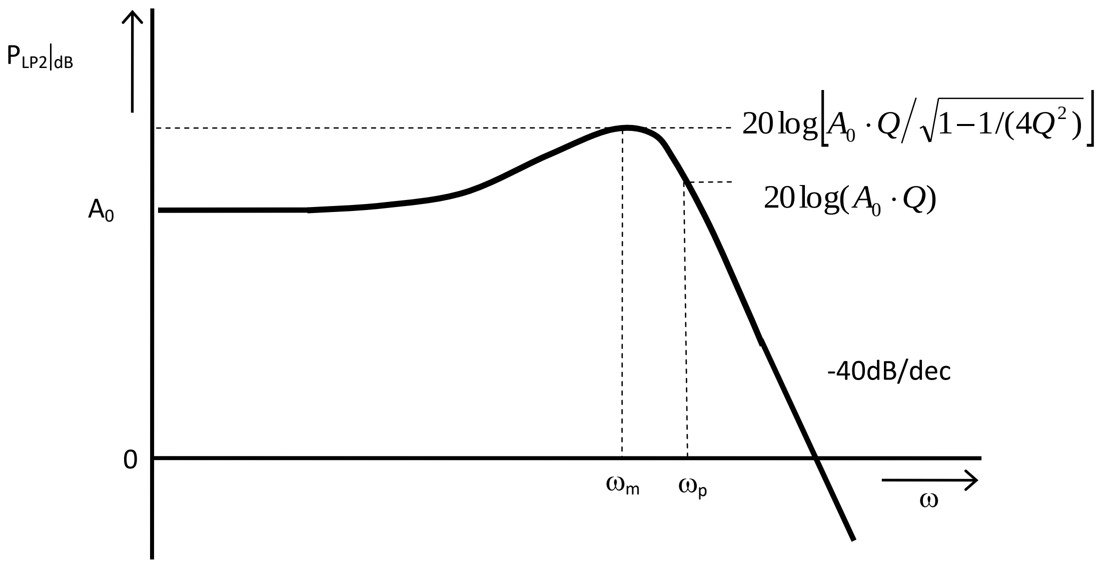


Fig. 3 Magnitude of the LP2 in dB: here **A_0 = K; ω_p= ω_0**

Fig.3 provides us with an important aid for the practical settings of the needed second order block properties. We see the maximum (at ω_m) transmission during the measurement, not so ω_p. So we set the maximum

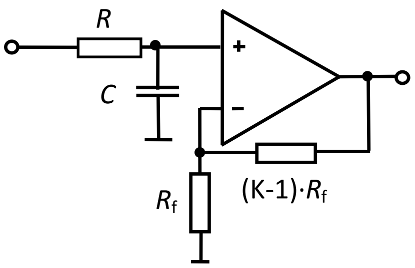
$$\frac{KQ}{\sqrt{1-1/(4Q^2)}} = \frac{2 \cdot 1}{\sqrt{1-1/4}} = 2{,}31; (7{,}27\ dB)$$

at the appropriate ω_m

$$\omega_m = \omega_p \cdot \sqrt{1-1/(2Q^2)} = \omega_p \cdot \sqrt{1-1/2} = \omega_p \cdot 0{,}707 = 2\pi \cdot 2474$$

<u>The first order filter realization</u>

One simple realization of the first order filter is in Fig. 4.



$$\frac{U_2}{U_1} = \frac{K \cdot \omega_p}{p + \omega_p}$$

$$\omega_p = 1/(RC)$$

Fig. 4 Noninverting 1st order LP; an operational amplifier creates noninverting structure with gain K; a characteristic frequency is $\omega_p = \frac{1}{RC} = |Butterworh| = \omega_0$

The design process is trivial. We choose the value of R = 10 k Ω; then we calculate $C = 1/(\omega_0 R)$ $=1/(2\pi \cdot 3500 \cdot 10^4)$; C = 4,55 nF. We set K as needed; it does not affect the frequency ratios here.

The whole LP3 filter is shown in Fig. 5. Only by cascading the filters of Fig. 2 and Fig. 4 do we get the transmission according to "Mr. Butterworth". HP2 and input (buffer) amplifier are the same as in “Filters studies”.

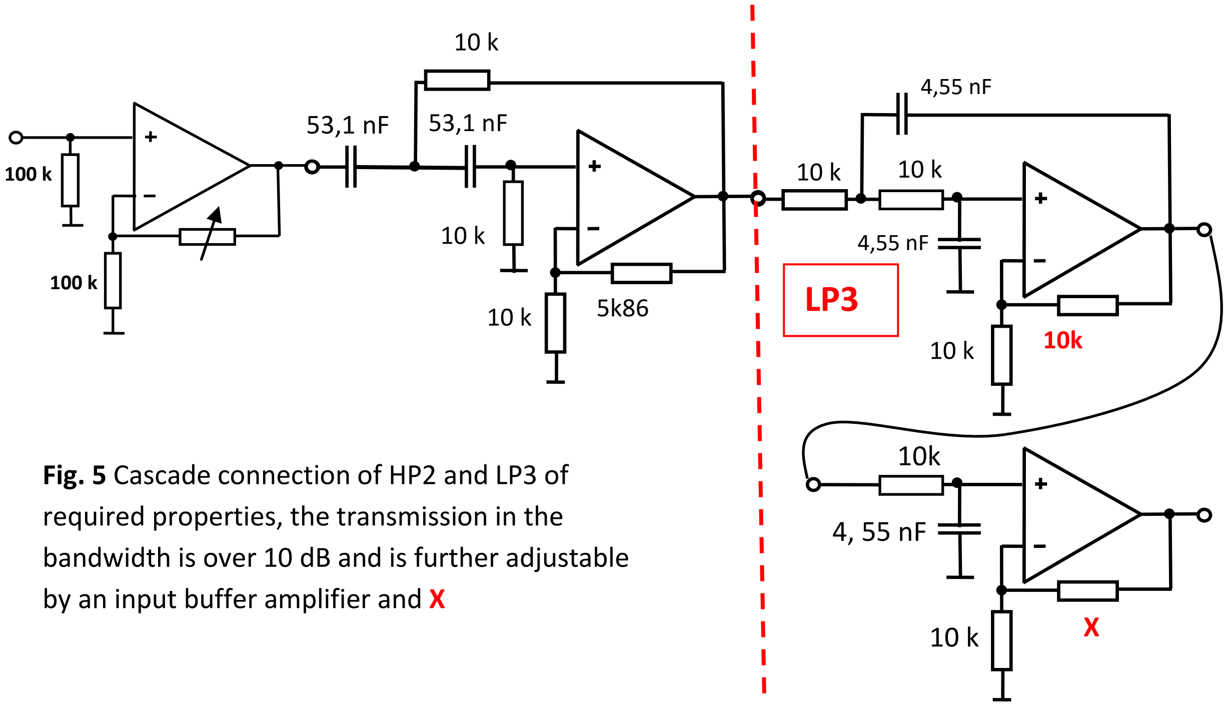


Fig. 5 Cascade connection of HP2 and LP3 of required properties, the transmission in the bandwidth is over 10 dB and is further adjustable by an input buffer amplifier and **X**

The gain of HP2 is 1.586; the gain of LP3 is 2 x gain of its first order part. This means that the transmission of the all filter (without input amplifier) is: 1,586 x (2 x gain of its first order part).

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