Low-Pass Filters SALLEN and KEY With Real Operational Amplifiers

1 author:

Josef Punčochář
VSB - Technical University of Ostrava

Low-Pass Filters SALLEN and KEY With Real Operational Amplifiers

Doc. Dr. Ing. Josef Punčochář
Department of Theoretical Electrical Engineering
Faculty of Electrical Engineering and Computer Science
VŠB - Technical University Ostrava
e-mail: josef.puncochar@vsb.cz

In [13] is a good comparison of properties of second order low-pass filter with real voltage feedback op amps (VFA) and with real current feedback op amps (CFA). But there are some inaccuracies in the [13]. This paper deals with an analysis of ***low-pass Sallen and Key filter.*** *It describes the influence of* ***gain-bandwidth product*** *and* ***output resistance of real operational amplifier*** *over frequency response of the circuit and possibly* ***circuit modification*** *- too overcome (partly) the influence of the output resistance.*

1. Introduction

A denomination "current feedback amplifier - CFA" by itself is not very good name. From the point of view of "feedback" - the feedback in the Sallen and Key filters, for example, is always "voltage" - dependent on the output voltage U_o (and not on the output current !). But there are different properties of inverting inputs, there: VFA - the inverting input impedance is infinite; CFA - the inverting input impedance is zero. The inverting input voltage is equal to noninverting input voltage - for CFA and VFA, too (VFA - the inverting input voltage is created by feedback circuit - an error signal is difference of voltages; CFA - the inverting input voltage is created by input follower - sometimes named OTA - an error signal is inverting input current - converted on the output voltage by means of transimpedance). *If a CFA transimpedance is large enough (ideally infinite), a resulting inverting input current is the same for CFA and VFA - ideally zero.* The low inverting input impedance (and simple CFA construction) assures (for the same technology) better frequency response of CFA compared to VFA (but worse DC properties of CFA). But the CFA is so commonly used name today that we can hardly change it.

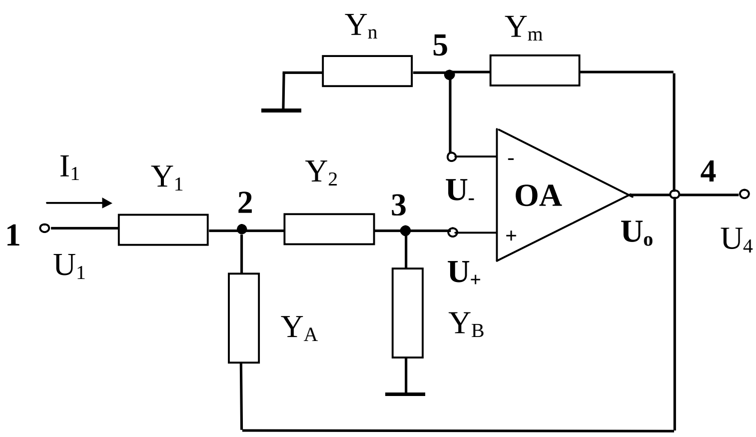

Fig.1. Filters Sallen and Key

A structure of the circuit is still the same - for CFA and VFA - see Fig.1. If we use CFA, we can not omit appropriate resistive (no capacitance)Y_m - never ! Both amplifiers create ***noninverting voltage amplifier*** - with certain impedance and frequency properties. These properties are formally (from the point of view of circuit model) the same.

2. Generalized nodal voltage analysis

2.1 A matrix description of VFA [14]

The assignment and sign convention for input and output voltages and currents of operational amplifiers is shown in Fig.2.

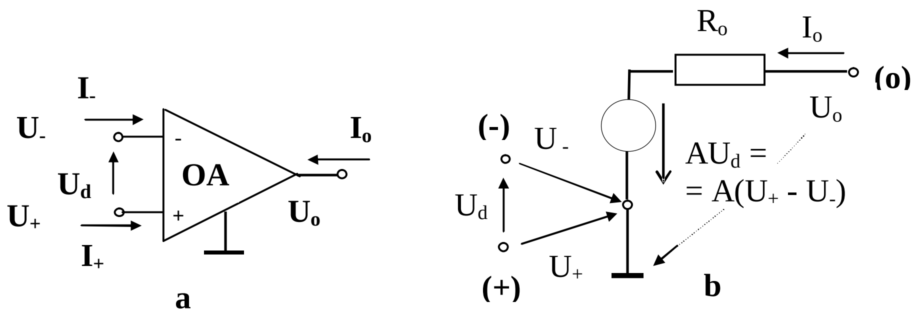

Fig.2. a) Symbol of VFA ; b) signal model of VFA

The ground (reference) terminal provides a reference point for the three others (Fig. 2b):

- noninverting input (+)
- inverting input (-)
- output (o)

The simplified signal model (for $I_+ = I_- = 0$) is shown in Fig.2b. We can easy determine equations

$I_+ = 0.U_+ + 0.U_- + 0.U_o$
$I_- = 0.U_+ + 0.U_- + 0.U_o$
$I_o = [U_o - A(U_+ - U_-)]/R_o = -AG_oU_+ + AG_oU_- + G_oU_o$

A matrix form (model) of the equations is

$$\begin{array}{c|ccc|} & (+) & (-) & (o) \\ (+) & 0 & 0 & 0 \\ (-) & 0 & 0 & 0 \\ (o) & -AG_o & AG_o & G_o \end{array} \begin{bmatrix} U_+ \\ U_- \\ U_o \end{bmatrix} = \begin{bmatrix} I_+ \\ I_- \\ I_o \end{bmatrix} \qquad (1)$$

The eq.(1) describes the ***matrix admittance model of the operational amplifier*** with the ***output resistance*** R_o (= $1/G_o$) and ***voltage gain*** A.

2.2 Results for real and ideal op amps

Now we can to analyse the linear electronic circuit in Fig.1. First we number nodes. There is only one signal current source I_1 in the circuit. We use the admittance matrix from the eq. (1) and basic rules of generalised nodal voltage analysis - we get:

	1	2	3 (+)	4 (o)	5 (-)			
1	Y_1	$-Y_1$	0	0	0	U_1		I_1
2	$-Y_1$	$Y_1+Y_A+Y_2$	$-Y_2$	$-Y_A$	0	U_2		0
3 (+)	0	$-Y_2$	$Y_2+Y_B+(0)$	$0+(0)$	$0+(0)$	U_3	=	0
4 (o)	0	$-Y_A$	$0+(-AG_o)$	$Y_A+Y_m+(G_o)$	$-Y_m+(AG_o)$	U_4		0
5 (-)	0	0	$0+(0)$	$-Y_m+(0)$	$Y_m+Y_n+(0)$	U_5		0

ROW COL.

( ) ↔ ( ) - „coincidences"; we add respective matrix elements from the eq.(1);

e.g.: (o) ↔ (-) give us ... $+(AG_o)$, (-) ↔ (o) give us ... +(0)

This set of equations defines the electronic circuit in the Fig.1 with the real operational amplifier (we suppose zero input currents only).

In general, the frequency response of the op amp is determined by many poles and zeros; however, in order to assure stability in closed-loop feedback configurations, most modern op amps are designed to have a dominant real pole at $p = -\omega_1$, so the suitable model of an op amp voltage gain [1, 2, 8] is ($p = j\omega$ for steady state solution)

$$A(p) = \omega_T/(p + \omega_1) \cong \omega_T/p \qquad (2)$$

consequently

$$A(s) = (\omega_T/\omega_0)/ (p/\omega_0) = \gamma/s \qquad (3)$$

where ω_T is gain-bandwidth product defined as $\omega_T = A_0\omega_1$; A_0 is the op amp´s dc gain; ω_1 is the 3-dB frequency; $\gamma = \omega_T/\omega_0$ is the normalised gain-bandwidth product; $s = p/\omega_0$ is the normalised frequency ($j\omega/\omega_0$ for steady state solution).

In reality an output impedance has an effect on filters response, too. The op amp open - circuit impedance is considered to be ohmic, $R_0 = 1/G_o$ [1].

Solving matrix set of equations [3] for

$Y_1 = G_1 = 1/R_1$; $Y_2 = G_2 = 1/R_2$; $Y_A = pC_A$; $Y_B = pC_B$;
$Y_m = G_m = 1/R_m$; $Y_n = G_n = 1/R_n$

give us ***voltage transfer function*** $P(s) = U_4/U_1$ of the analysed circuit (A - see eq.(3)):

$$P(s) = \frac{R_o/R_1}{D + R_o/R_{12}} \cdot \frac{s^3 + s^2\,\omega_0/\omega_A + \gamma R_1/R_o}{s^3 + s^2\,\dfrac{\gamma/H + D/Q + H\omega_0/\omega'_A}{D + R_o/R_{12}} + s\,\dfrac{\gamma/(HQ) + D}{D + R_o/R_{12}} + \dfrac{\gamma/H}{D + R_o/R_{12}}} \qquad (4)$$

where

$\omega_o = 1/\sqrt{R_1 R_2 C_A C_B}$ is the ideal characteristic frequency

$H = 1+R_m/R_n$ is the ideal „dc gain“

$1/Q = \sqrt{\frac{R_1 C_B}{R_2 C_A}} + \sqrt{\frac{R_2 C_B}{R_1 C_A}} + \sqrt{\frac{R_1 C_A}{R_2 C_B}}(1-H)$ defines the „ideal Q“

$D = (R_m + R_n + R_o)/(R_m + R_n) = |R_m + R_n \rangle\rangle R_o| \approx 1$

$\omega_A = 1/(R_1 C_A)$; $R_{12} = R_1 R_2/(R_1 + R_2)$; $1/\omega'_A = (R_1 + R_o/(DH))C_A$

For an ideal op amp ($\gamma \to \infty$) we can derive the known „ideal normalised low-pass“ transfer function

$$P_{id} = U_o/U_i = H/(s^2 + s/Q + 1) \tag{4a}$$

3. Discussion of results

3.1 Zeros - feedforward transmission [3, 4, 5]

The ***feedforward transmission*** through the Sallen and Key feedback impedances („into R_o“) takes place at high frequencies as a result of the decrease in the open-loop gain A(s). The transfer function P(s) has a cubic equation in the numerator. It can be determined that for $\sqrt[3]{\gamma R_1/R_o} >> \omega_o/\omega_A$ we can write

$$s^3 + s^2\,\omega_0/\omega_A + \gamma R_1/R_o \cong s^3 + \gamma R_1/R_o \tag{5}$$

This has three solutions for *s* which make the numerator equal to zero:

$$s_{Z1} = -\sqrt[3]{\gamma R_1/R_o} \quad ; \; s_{Z2,3} = |s_{Z1}| \cdot \exp(\pm j\pi/3)$$

It means that

$$|s_{Z1}| = |s_{Z2}| = |s_{Z3}| = \sqrt[3]{\gamma R_1/R_o}$$

consequently the zero frequency is ($|s_Z| = w_Z/w_0$)

$$\omega_Z = \omega_{Z1} = \omega_{Z2} = \omega_{Z3} = \omega_0 \cdot \sqrt[3]{\gamma R_1/R_o} = \sqrt[3]{\omega_0^2 \omega_T R_1/R_o} \tag{6}$$

In the frequency domain (Fig.3), those three zeros will manifest themselves by stopping the decrease in the closed-loop gain. If $w_Z >> w_0$ we can derive

$$P_Z \cong P_{id}(s = s_Z) = \frac{H}{s_Z^2 + s_Z/Q + 1} \cong \frac{H}{s_Z^2} = \frac{H\omega_0^2}{\omega_Z^2} \tag{7}$$

and (refer to Fig.3)

$$P_{ZdB} = 20\log P_Z = 20\log H - 20\log(\omega_Z/\omega_0)^2 = 20\log H - \frac{40}{3}\log\left(\frac{\omega_T R_1}{\omega_0 R_o}\right) \tag{8}$$

As the value of *s* becomes larger, the transfer function increases and approaches value

$$P(\infty) = \frac{R_o/R_1}{D + R_o/R_{12}} = |D \approx 1| = 20\log\left(\frac{R_o // R_2}{R_1 + R_o // R_2}\right) \tag{9}$$

where $P(\infty)$ is the ***feedforward transmission for*** $|s| > w_T/\omega_0$ and $R_2 // R_o$ represents a parallel combination of resistors. If R_1, $R_2 >> R_o$ we easy determine from the eq.(9) that $P(\infty) \cong 20\log(R_o / R_1)$.

3.2 Poles - real characteristic frequency [3, 12]

The loci of ***poles*** are obtained by factoring the denominator of eq.(4) for different values of γ, R_o/R_1 and Q. The equation

$$s^3 + bs^2 + cs + d = 0 \qquad (10)$$

has one real root [b, c, d - see denominator of eq.(4)], and if this is found, say $s = s_1$ (***high frequency pole***), the equation can be factorised into the suitable form

$$(s - s_1)\cdot(s^2 + sk_r / Q_r + k_r^2) = 0 \qquad (11)$$

or

$$(s - s_1)\cdot(s - s_2)\cdot(s - s_3) = 0 \qquad (12)$$

where $s_{2,3} = -\alpha \pm j\beta$ are two complex-conjugate roots - ***dominant poles***.

Comparing eq.(11) and eq.(12), k_r, Q_r, α and β must satisfy:

$$k_r / Q_r = 2\alpha \qquad \text{or} \qquad \alpha = k_r / (2Q_r) \qquad (13)$$

$$k_r^2 = \alpha^2 + \beta^2 \qquad \text{or} \qquad \beta^2 = k_r^2 \left[1 - 1/(4Q_r^2)\right] \qquad (14)$$

$$Q_r = \sqrt{\alpha^2 + \beta^2} / (2\alpha) \qquad (15)$$

where $k_r = \omega_r/\omega_o$ is the ***normalised characteristic frequency „with real operational amplifier“***

Q_r is the ***„real Q“.***

Comparing eq.(10) and eq.(11) we get (exact equations)

$$s_1 = -b + k_r / Q_r \qquad (16)$$

$$k_r^2 = d / (-s_1) \qquad (17)$$

$$Q_r = \sqrt{d(-s_1)} / (c - k_r^2) \qquad (18)$$

We are able to solve eq.(10) completely if we know the real root s_1. We can use exact Cardan´s solution. But we can use subsequent solution, too.

3.3 Approximate determining of the root s_1 [3, 12]

It is evident that for the op amp with $\gamma >> 1$ is $b >> 1$, $k_r \to 1$, $Q_r \to Q$, consequently $s_1 \approx -b$. Combining this and eqs.(17), (18) and again (16) gives

$$s_1 \approx -b + c / b - d / b^2 \qquad (19)$$

Repeated combining eq.(19) and eqs.(17), (18) and (16) gives (for $b^2 >> c$)

$$s_1 \approx -b + bc / (b^2 - c) - d / b^2 \qquad (20)$$

For example, we assume H = 1 ($R_m = 0$, $R_n \to \infty$), $R_1 = R_2 = R$. The expressions for ω_o^2, 1/Q, R_{12}, $1/\omega'_A$ simplify to: $\omega_o^2 = 1/(R^2 C_A C_B)$; $1/Q = 2 \cdot \sqrt{C_B / C_A}$; $R_{12} = R/2$; $1/\omega'_A = (R + R_o)C_A$; consequently:
$\omega_o / \omega'_A = 2Q(1 + R_o / R)$;

$$b = \frac{\gamma + 1/Q + 2Q(1 + R_o / R)}{1 + 2R_o / R}; \qquad c = \frac{\gamma / Q + 1}{1 + 2R_o / R}\ ; \qquad d = \frac{\gamma}{1 + 2R_o / R}$$

The results are summarised in Table 1.

Table 1. Some results of analysis - poles
Notes: ECS - Exact Cardan´s Solution; HNE - Has Not Effect; NE - No Exist;
IOA - Ideal Op Amp; ROA - Real Op Amp; → the same result as on the right

ROVNICE →			(20)	(17)	(18)	ECS	(17)	(18)	POZN.
Q	γ	R_o/R	$-s_1$	k_r	Q_r	$-s_1$	k_r	Q_r	
Q	**¥**	**HNE**	**NE**	**1**	**Q**	**NE**	**1**	**Q**	**IOA**
		0	101,2	→	→	101,2	0,9949	0,5025	
	100	0,1	83,933	→	→	83,933	0,9964	0,5023	ROA
0,5		0,5	49,751	→	→	49,750	1,0025	0,5013	
		0	36,059	→	→	36,058	0,9852	0,5073	
C_A/C_B	35	0,1	29,799	0,9893	0,5067	29,797	0,9894	0,5066	ROA
= 1		0,5	17,258	1,0070	0,5039	17,251	1,0072	0,5038	
		0	16,142	0,9640	0,5175	16,136	0,9642	0,5174	
	15	0,1	13,206	0,9729	0,5163	13,198	0,9732	0,5161	ROA
		0,5	7,2913	1,0141	0,5110	7,2564	1,0166	0,5099	
0,6			30,042	→	→	30,042	0,9853	0,6107	
1	35	0,1	30,891	→	→	30,893	0,9717	1,0331	ROA
3			34,664	→	→	34,664	0,9173	3,2732	
10			47,517	→	→	47,517	0,7835	11,871	
3	200		172,12	→	→	172,12	0,9840	3,0561	
	100	0,1	88,799	→	→	88,800	0,9687	3,1086	ROA
C_A/C_B	15		18,038	0,8325	3,4864	18,039	0,8324	3,4864	
=36	8		12,237	0,7381	3,5975	12,240	0,7380	3,5977	
10	200		184,99	→	→	184,99	0,9492	10,578	
	100	0,1	101,67	→	→	101,67	0,9053	11,027	ROA
	15		30,921	0,6358	11,709	30,862	0,6364	11,703	
C_A/C_B	200	0	220,1	→	→	220,1	0,9532	10,443	ROA
=400	200	0,5	114,97	→	→	114,97	0,9326	11,134	

3.4 Bode plots - eq.(4)

If we know poles and zeros of transfer function, we can easy construct Bode plots of the analysed circuit - see Fig.3. We described: $S_Z = \left| s^3 + \gamma R_1 / R_o \right|$ - zeros, $S_D = \left| s^2 + s k_r / Q_r + k_r^2 \right|$ - dominant poles ($k_r^2 \to 1$), $S_R = \left| s - s_1 \right|$ - real pole (of the transfer function). A discussion of the frequency response is evident.

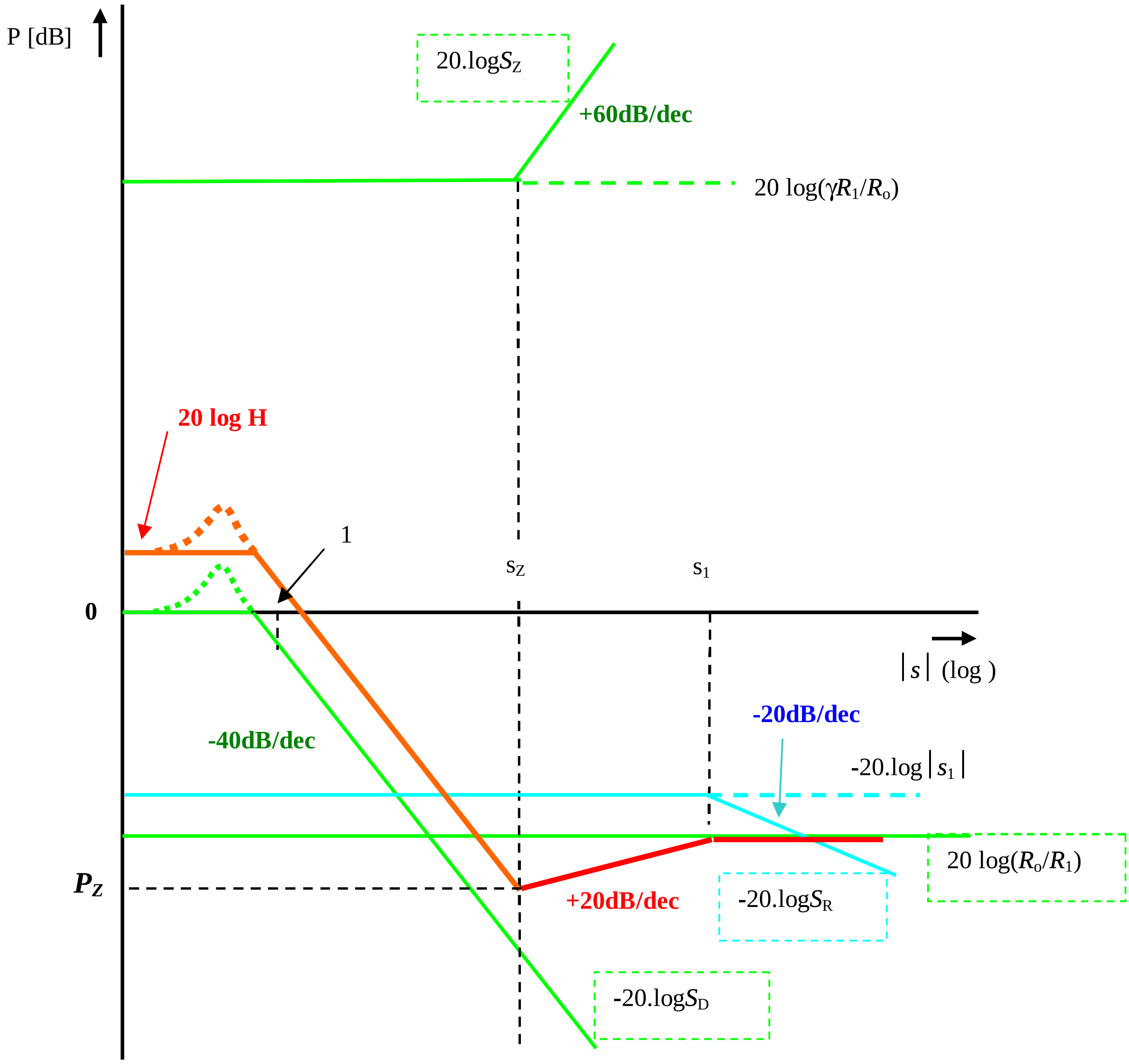

Fig.3. Bode plots of the transfer function (modules) - red line - complete result

3.5 Verification of the analysis - discussion of zeroes influence

Results of the analysis were verified already in [3]. Let us compare now zeroes magnitude with data from [13]. There were simulated transfer functions (H = 1, $C_1 \equiv C_A$ = 1 nF, $C_2 \equiv C_B$ = 100 pF, R_1 = 330 Ω, R_2 = 1084 Ω) for gain-bandwidth's f_T = 100 MHz and 1 GHz and output resistance's R_o = 1 mΩ; 1 Ω; 10 Ω; 100 Ω a 1 kΩ.

From the eq.(6) we determine

$$f_Z \cong f_0 \cdot \sqrt[3]{\gamma R_1 / R_o} = \sqrt[3]{f_0^2 f_T R_1 / R_o} = f_T \cdot \sqrt[3]{(f_o / f_T)^2 \cdot (R_1 / R_o)}$$

From relation $\omega_o = 1\big/\sqrt{R_1 R_2 C_A C_B}$ is evident that $f_o = 1\big/\left(2\pi\sqrt{R_1 R_2 C_A C_B}\right)$ = 841 489 Hz.

For f_T = 100 MHz we can determine from [13] that the zero frequencies f_Z are "in accordance with increasing R_o": ≅ 300; 28; 13; 6,3 and 2,8 MHz.
For given circumstances is

$$f_Z \cong 100 \cdot \sqrt[3]{(0{,}841489/100)^2 \cdot (330/R_o)} = 28{,}6 \cdot \sqrt[3]{1/R_o} \quad [\mathrm{MHz}]$$

thus we get stepwise: 286; 28; 13,27; 6,16 and 2,86 MHz.

For f_T = 1000 MHz we can determine from [13] that zero frequencies f_Z are "in accordance with R_o, too": ≅ 600; 60; 28; 13 and 6 MHz.

Thus $f_Z \cong 1000 \cdot \sqrt[3]{(0{,}841489/1000)^2 \cdot (330/R_o)} = 61{,}6 \cdot \sqrt[3]{1/R_o}$ [MHz]

and we get: 616; ; 61,6; 28,59; 13,27 and 6,16 MHz.

Discussion about determination of transfer function zeros - by means of eq.(6) - is needless, I think.

These transfer function zeros determine unambiguously properties of the analysed circuit as the low-pass filter. It is evident we wish an infinite value of these zeros. Unfortunately, the eq.(6) is valid - the magnitude of f_Z is proportional to cube root of f_T. The same conclusion is valid for R_1. The best solution is to choose an op amp with very small magnitude of the output resistance R_o.

4. Modification of the circuit

4.1 A correction transfer function pole - the third order low-pass filter

To overcome (partly) the effect of the op amp output resistance we can modify the RC network. What is required is a circuit having a falling response above frequency $f_z = \omega_z/(2\pi)$. This can be done by ***falling R_1 into two parts and adding a compensating capacitor*** C_k - see Fig.4. In the frequency domain, this new pole manifests itself by stopping the increase in the closed-loop gain (above f_z) and by decrease in the closed-loop gain for frequencies above f_T/H - see Fig.5 and Fig.6.

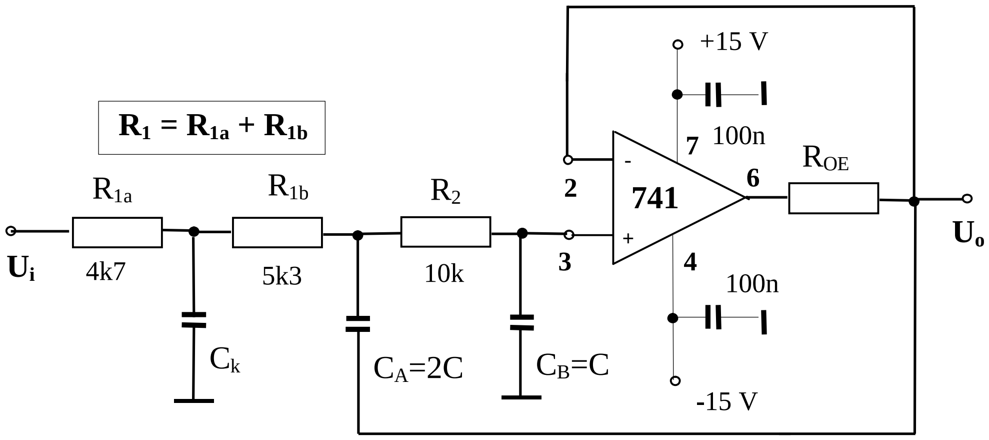

Fig.4. Practical realisation of Sallen - Key filter with opa „741“
[$R_o \cong 70\ \Omega$; $f_T \cong$ 0,6 MHz - f_T measured]

It is evident that H = 1+ 0/∞ = 1; $R_1 = R_2 = R = 10$ kΩ; $C_A = 2C_B = 2C$, thus
$f_0 = 1/(2\pi\sqrt{2}\,RC)$; $Q = 0{,}5\,(C_A/C_B)^{1/2} = 0{,}707$ - Butterworth approximation.
The output resistance of operational amplifier is changed by means of R_{OE} (see Fig.4), thus $R_o \cong 70\ \Omega + R_{OE}$, where 70 Ω is „the output resistance quality“ of 741.

The filter properties are described in the Table 2 and in the Fig. 5. The qualitative explanation of the s_K pole influence is depicted in the Fig.6 (C_{K1} - *deficient correction*, C_{K2} - *optimal correction* for second order filter). The correction pole is described by means of idealised term $s_K/(s - s_K)$, which module is

$$S_K = |s_K/(s - s_K)|.$$

Table 2. Some results of the circuit in Fig.4 analysis and measurement (C_{KOPT} = 440 pF determined experimentally)

f	$P(f) = U_o/U_i$ (U_i = 1 V)				
Hz	[- /dB]				
40	0,948/ -0,46	0,946/ -0,48	0,947/ -0,47	-	-
60	0,800/ -1,94	0,798/ -1,96	0,800/ -1,94	-	-
80	0,603/ -4,40	0,599/ -4,45	0,600/ -4,43	-	-
100	0,432/ -7,30	0,431/ -7,32	0,433/ -7,28	-	-
200	0,119/ -18,5	0,119/ -18,5	0,117/ -18,6	-	-
500	0,0193/ -34,3	0,0191/ -34,4	0,0195/-34,2	-	-
1 k	$4{,}62.10^{-3}$/-46,7	$4{,}52.10^{-3}$/-46,9	$4{,}57.10^{-3}$/ -46,8	0,990/ -0.09	0,987/ -0,11
2 k	$1{,}15.10^{-3}$/-58,8	$1{,}00.10^{-3}$/ -60,0	$9{,}77.10^{-4}$/ -60,2	0,979/ -0,18	0,971/ -0,26
3 k	$5{,}62.10^{-4}$/-65,0	$5{,}69.10^{-4}$/ -64,9	**$6{,}26.10^{-4}$/ -64,4**	0,882/ -1,09	0,869/ -1,21
4 k	$3{,}80.10^{-4}$/-68,4	**$3{,}55.10^{-4}$/ -69,0**	**$4{,}90.10^{-4}$/ -66,2**	0,750/ -2,50	0,748/ -2,52
5 k	$2{,}51.10^{-4}$/-72,0	**$2{,}60.10^{-4}$/-71,7**	**$5{,}31.10^{-4}$/ -65,5**	0,541/ -5,34	0,530/ -5,51
7 k	**$1{,}41.10^{-4}$/-77,0**	**$2{,}99.10^{-4}$/ -70,5**	$7{,}08.10^{-4}$/ -63,0	0,299/ -10,5	0,295/ -10,6
8 k	**$1{,}26.10^{-4}$/ -78**	$3{,}35.10^{-4}$/ -69,5	$7{,}94.10^{-4}$/ -62,0	0,211/ -13,5	0,209/ -13,6
10 k	**$1{,}58.10^{-4}$/ -76**	$4{,}12.10^{-4}$/ -67,7	$1{,}05.10^{-3}$/ -59,6	0,155/ -16,2	0,150/ -16,5
20k	$2{,}24.10^{-4}$/ -73	$7{,}50.10^{-4}$/ -62,5	$2{,}11.10^{-3}$/ -53,5	0,0376/ -28,5	0,0372/ -28,6
50k	$4{,}22.10^{-4}$/-67,5	$1{,}76.10^{-3}$/ -55,1	$5{,}01.10^{-3}$/ -46,0	$5{,}96.10^{-3}$/-44,5	$5{,}89.10^{-3}$/ -44,6
100 k	$7{,}76.10^{-4}$/-62,2	$3{,}80.10^{-3}$/ -48,4	$1{,}00.10^{-2}$/ -40,0	**$1{,}64.10^{-3}$/-55,7**	$1{,}60.10^{-3}$/ -55,9
200 k	$1{,}51.10^{-3}$/-56,4	$7{,}08.10^{-3}$/ -43,0	0,0197/ -34,1	**$1{,}30.10^{-3}$/ -57,7**	$1{,}26.10^{-3}$/ -58
300 k	$2{,}07.10^{-3}$/-53,7	$1{,}00.10^{-2}$/ -40,0	0,0316/ -30,0	**$1{,}80.10^{-3}$/ -54,9**	$1{,}26.10^{-3}$/ -58
500 k	$3{,}80.10^{-3}$/-48,4	0,0162/ -35,8	0,0468/ -26,6	$3{,}09.10^{-3}$/ -50,2	$1{,}17.10^{-3}$/ -58,6
700 k	$4{,}90.10^{-3}$/-46,2	0,0211/ -33,5	0,0442/ -27,1	$4{,}42.10^{-3}$/ -47,1	$7{,}24.10^{-4}$/ -62,8
1 M	$6{,}61.10^{-3}$/-43,6	0,0248/ -32,1	0,0432/ -27,3	$5{,}96.10^{-3}$/ -44,5	$5{,}01.10^{-4}$/ -66,0
1,5 M	$8{,}91.10^{-3}$/-41,0	0,0254/ -31,9	0,0422/ -27,5	$7{,}85.10^{-3}$/ -42,1	$3{,}98.10^{-4}$/ -68
2 M	$9{,}66.10^{-3}$/-40,3	0,0260/ -31,7	0,0412/ -27,7	$9{,}23.10^{-3}$/-40,7	$3{,}16.10^{-4}$/ -70
C [nF]	163			2,8	
R_o[Ω]	70	220	540	70	
C_K[pF]	no			no	440
f_0[Hz]	69			4019	
$\gamma = f_T/f_0$	8696			149,3	
$\gamma R_1/R_o$	$1{,}24.10^6$	$3{,}95.10^5$	$1{,}61.10^5$	$2{,}13.10^4$	
f_Z [kHz]- eq. (6)	7,412	5,063	3,754	111,407	
$P(\infty)$[dB]- eq. (9)	-43,1	-33,2	-25,4	-43,1	
P_Z [dB] - eq. (7)	-81	-74	-69	-57,7	

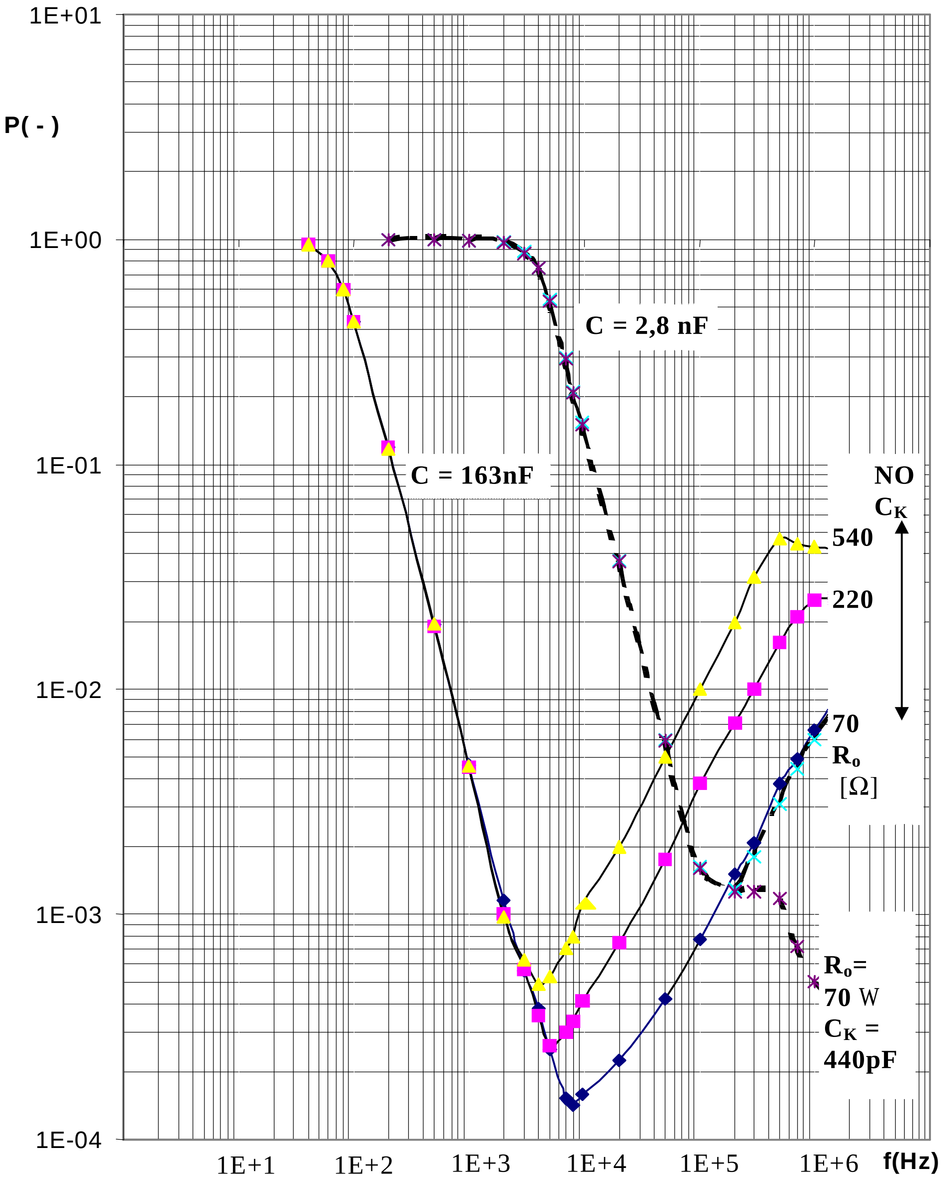

Fig.5. Measured frequency response of the circuit in the Fig.4

Just here we can see that for increasing module of C_k the correction pole will decrease - toward to the main transfer poles of the circuit. We get the familiar known ***third order low-pass filter***. Another asymptote (for this pole s_K) added in the Fig.3 enables us to judge (qualitative) properties of the third order low-pass filter with the real operational amplifier - see Fig.6, C_{K3}.

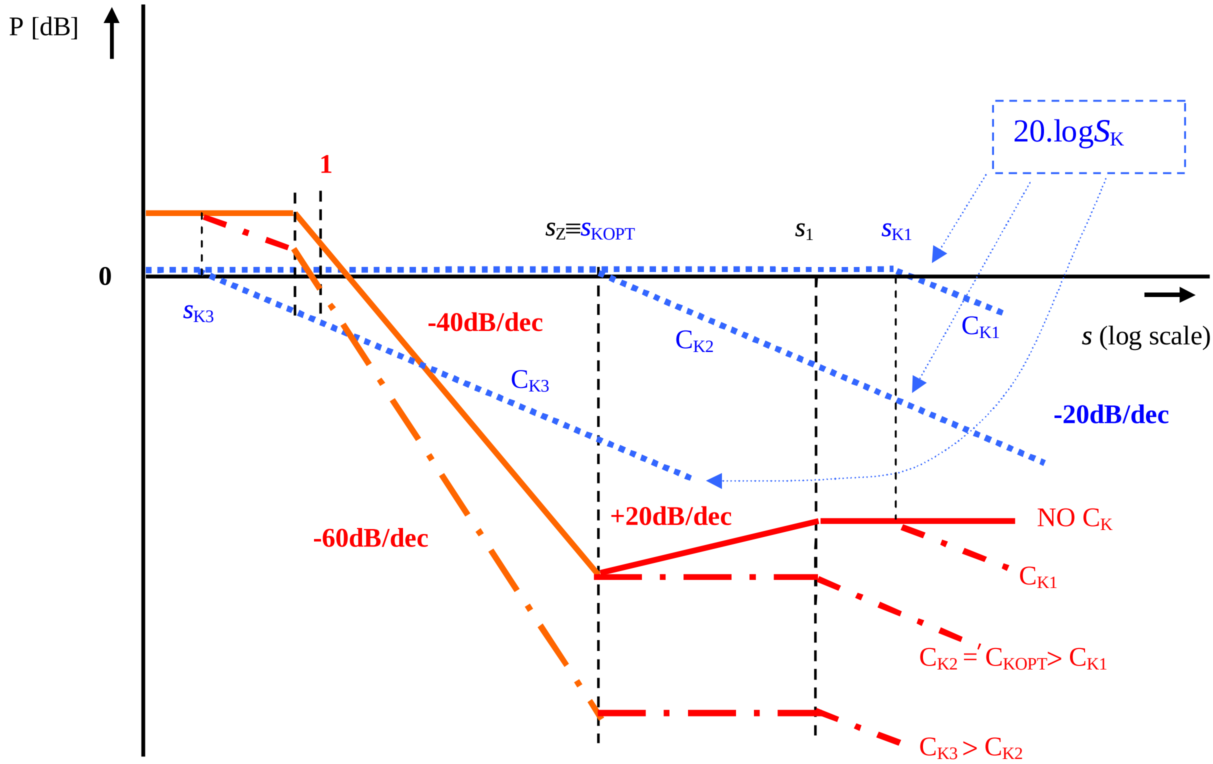

Fig.6. Qualitative depicting of correction pole influence(blue lines) on the overall circuit frequency response (red lines); $s_K \sim 1/C_K$; C_{K1} - deficient correction; $C_{K2} = C_{KOPT}$ - optimal correction; C_{K3} - the third order low-pass filter.

4.2 A buffer in the feedback path - nonreciprocal feedback

An interesting influence has a buffer (***follower - its ideally transfer is 1***) in the feedback path - Fig.7.

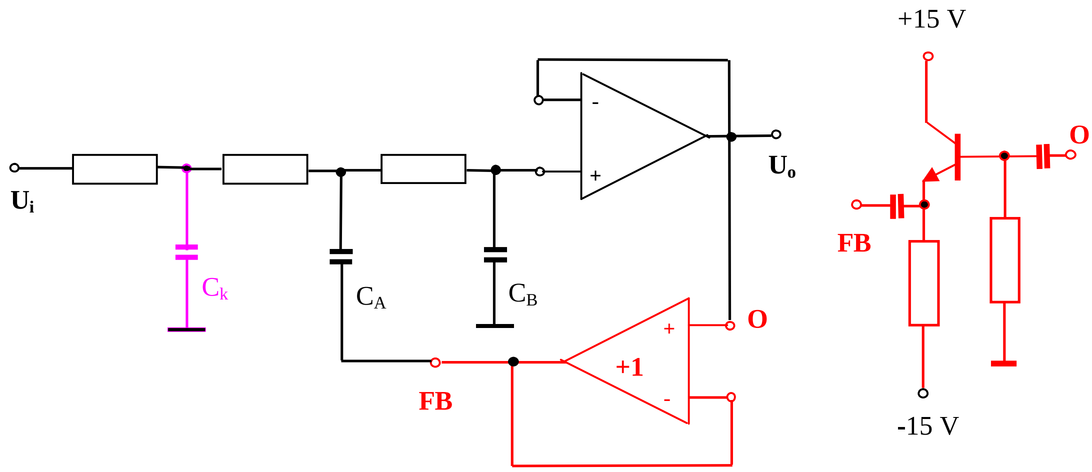

Fig.7. Second order low-pass filter (no C_K) - nonreciprocal feedback;
(the third order filter - large enough C_K).

The effect of the follower on the over-all filter performance has been simulated by a computer program (MCII). The feedforward transmission ideally dwindles - for ideal feedback follower. If we use **741** as the feedback follower, the low-frequency properties are unchanged, but high-frequency properties are rather more complicated (but better as before) - the follower is not ideal (phase response are more complicated). If we use a better (it means more costly) op amp as the feedback follower, a question presents: Is it not better to use this op amp direct as the "main op amp"? But due to alternating feedback "through C_A" it is possible to use (perhaps) a simple one transistor (ore more perfect transistor) follower - it is an interesting area to investigation (see red circuit in the Fig.7). Very interesting (similar) results has been got with the follower (in the feedback) and the C_K (large enough - the third order low-pass) - see Fig.7 and Fig.3 (C_{K3}).

4. Conclusion

There is demonstrated (just only) in the [13] that Sallen and Key filter properties depend on "main amplifier" properties. ***No matter what type of amplifier (VFA, CFA) is used if these "main amplifier" properties are the same***. But it is commonly known that CFA has better frequency (dynamic) responses compared to VFA (for the same technology). On the contrary the DC properties of VFA are more better than these of CFA.

The dominant poles are the pair of complex poles that correspond to the ideal poles but are shifted because of finite gain-bandwidth product and nonzero output resistance R_o. To see the effect graphically, we can use eqs.(13) and (14) and draw loci of the dominant poles. ***The results show that for $Q < 1$ the output resistance R_o rather compensates the negative influence of the finite g. For Q´s > 1 this is not valid; for increasing R_o the normalised characteristic frequency k_r invariably decreases from the ideal value 1.***

Results in the Table 1 show that for determining s_1 (to our purpose) we can use eq.(20), we need not use the exact Cardan´s solution. Also assumption in the eq.(5) is valid - for the second order low - pass filters, therefore the eq.(6) is valid, too.

But the ***feedforward transmission*** (due to R_o and finite γ, eq.(6)) ***is unpleasant*** problem, at all events. To overcome (partly) this effect, ***we can modify the RC network***. We fall ***R_1 into two parts and add compensating capacitor C_k***, see **Fig.4**. It is a good solution for the low Q´s second order Sallen - Key low-pass filters. An interesting solution of the problem is suggested in the **Fig. 7** - ***nonreciprocal feedback***.

It is interesting (too) that classical Bode plots description can give use still very useful qualitative results ("quick" and "low-cost") - see a third order filter properties estimation in the Fig. 6, for example.

LITERATURA

[1] Dostál, J.: *Operational Amplifiers*. ELSEVIER SCIENTIFIC PUBLISHING COMPANY. Amsterdam - Oxford - New-York, 1981, ISBN 0-444-99760-1

[2] Huelsman, L.P., and P.E. Allen, *Introduction to the Theory and Design of Active Filters*, Mc Graw-Hill Book Company, New-York, 1980

[3] Punčochář, J.: *Vliv vlastností reálného zesilovače na přenos některých zpětnova-zebních systémů*, Disertační doktorandská práce, VŠB-TU Ostrava, FEI, Katedra elektroniky, Ostrava, září 1995

[4] Punčochář, J.: *Sallen and Key Filters and Feedforward Transmission*. Nové smery v spracování signálov III, sborník II. časť, str. 219 až 222, Liptovský Mikuláš, 29. až 31. 5. 1996

[5] Punčochář, J.: *Sallen and Key Low-Pass Filters with Real Operational Amplifiers*. SPETO´97, TOM II, str. 325 až 328, Gliwice-Ustroň, 21. až 24. 5. 1997, ISBN 83-85940-19-7

[6] Punčochář, J.: *Nullor Nodal Analysis as a Result of a Nodal Analysis With Ideal OP Amps*. 21st SEMINAR on fundamentals of electrotechnics and circuit theory (SPETO), GLIWICE - USTROŇ, 20.-23. 5. 1998, p.p. 547 - 550, ISBN 83-85940-20-0

[7] Punčochář, J.: *Operační zesilovače nejen podle pana Soclofa*, Amatérské radio pro konstruktéry 52 (1993), č.4, str. 123-151

[8] Punčochář, J.: *Operační zesilovače v elektronice*. BEN-technická literatura, Praha 1996 - 2002, (1. až 5. vydání), ISBN 80-7300-059-8

[9] Punčochář, J.: *Dynamické vlastnosti operačních zesilovačů a jejich orientační určení z katalogových údajů*, Sdělovací technika 30 (1982), č.4, str. 123-127

[10] Punčochář, J.: *Analýza Bridgmanovy-Brennarovy struktury metodou uzlových napětí*. VI.sešit Katedry teoretické elektrotechniky, 14. listopadu 1997, p.p. 67 - 72

[11] Punčochář, J.: *Zobecněná metoda uzlových napětí*. Seminář teorie obvodů (STO-6), MODERNÍ SMĚRY VÝUKY ELEKTROTECHNIKY A ELEKTRONIKY, Sborník prací celostátní konference. (Katedra elektrotechniky a elektroniky VA Brno), Brno 24. - 25. 9. 1997, p.p. 160 - 163

[12] Punčochář, J.: *The loci of Sallen & Key low-pass filters*. Nové smery v spracování signálov IV, sborník , str. 246 až 249, Liptovský Mikuláš, 27. až 29. 5. 1998, ISBN 80-8040-071-7

[13] Vochyán, J.: *Porovnání vlastností aktivní dolní propusti s VFA a s CFA*. Elektronický časopis ELEKTROREVUE 2004/63 (www.elektrorevue.cz/clanky/04063)

[14] Punčochář, J.: *Operační zesilovače - historie a současnost*. BEN-technická literatura, Praha 2002, ISBN 80-7300-047-4.

[15] Biolek, D.: *Řešíme elektronické obvody (aneb kniha o jejich analýze)*. BEN-technická literatura, Praha 2004, ISBN 80-7300-125-X.

[16] Hájek, K.- Sedláček, J.: *Kmitočtové filtry*. BEN-technická literatura, Praha 2002, ISBN 80-7300-023-7.

[17] Punčochář, J.: *Analysis of Sallen and Key low-pass filters with real operational amplifiers*. TRANSACTIONS of the VŠB - Technical University Ostrava, NO. 1, VOL. V (1999), Electrical Engineering Series, pp.177 - 186, ISBN 80-7078-740-6