At the beginning of mathematical objects

JF Durand, M1, université Lille 1

Introduction

- Do you have a problem with 27 ? (three times three)
- No ? Well you are able to conceptualize (abstraction).

Plan

- The Birth of equations
- Formalization
- Groups

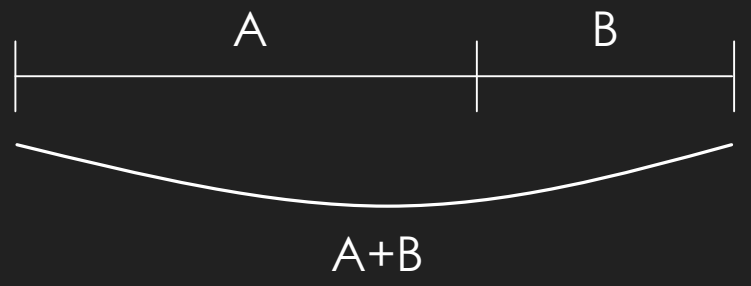
The birth of Equations

- This object called : unknown in mathematics has appeared for the first time in the Antiquity.
- The problem was : « Division in Extrem and Mean Ratio »

The birth of Equations

The Goal is to divide a segment in two.

The little section divide by the tall section equals the tall section divide by the entiere segment.



$$\frac{B}{A} = \frac{A}{A+B}$$

The birth of Equations

- Others problems talking about area.
- Egyptian : A number and its sevenths equals nineteen. What is this number ?
- Greek : Thales, Euclide, Pythagore

Solving methods are geometrical or numerical and always about lenght, area or volume.

Formalization

- In the ninth century, Al Khwarizmy invent two object :
  - Unknown
  - Equation
- He still uses sentences but gave up the thing behind the number (length, area, volumes… etc)
- Now we can study equations for themselves.

Formalization

- In 1830, Galois created the concept of « group »
  - For solving equations of higher degree
  - Open the way to a lot of derived object : circle, division ring

Groups

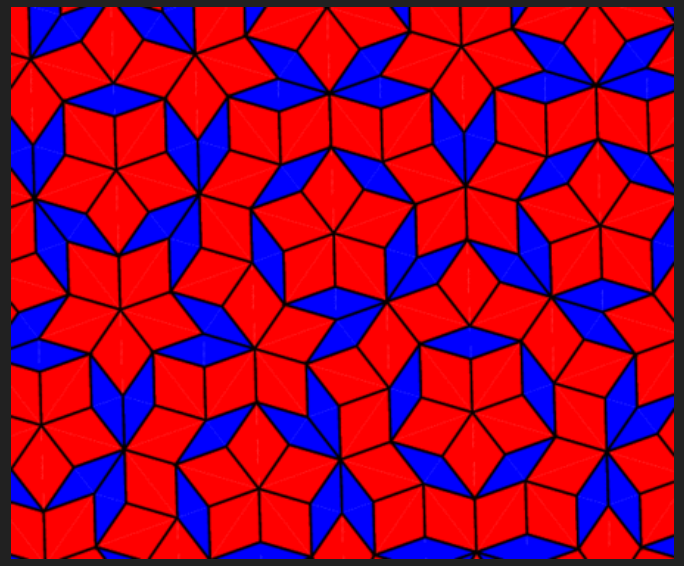
- A group is a couple (G,S)
  - G a set : [a,b]
    - (number,variable,..etc)
  - S an operation : •
    - (+,-,*,/,..etc)

Groups

- A group has to respect four rules :
  - Closure
    - a,b in G; • in G
  - Associativity
    - a,b,c in G: (a•b)•c = a•(b•c)
  - Identity Element (e)
    - e,a in G; e•a = a•e = a
  - Inverse Element
    - e,a,b in G; a•b = b•a = e

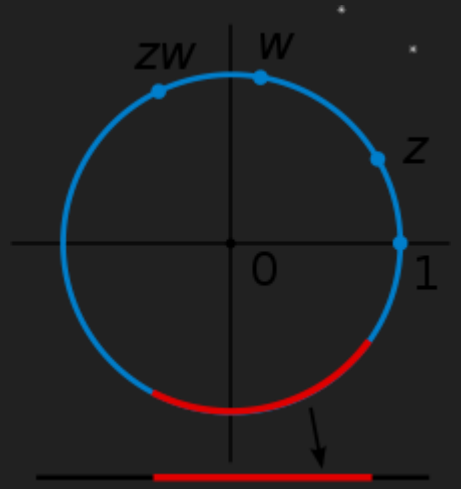
Groups

- In practice



17 regular paving like this one in the plane

Ammonia, group of order 6



Trigonometric circle with multiplication is an usual group

Conclusion

- Mathematics are universal ?
- « Mathematics should be an obviousness for everyone, because it's only a logical concatenation, which is in theroy a formality of the 'common sense' shared by everybody » Poincaré

Bibliography

- Podcast Science
- R. Herz-Fischler, A Mathematical History of Division in Extreme and Mean Ratio, Wilfrid Laurier.
- https://fr.wikipedia.org/wiki/Al-Khw%C3%A2rizm%C3%AE
- https://fr.wikipedia.org/wiki/Groupe_(math%C3%A9matiques)
- http://images.math.cnrs.fr/Un-concept-mathematique-trois.html

Complex Number

- Tartaglia find a way to solve 3 degrees of equations
- Sometimes in the middle of the calculation he falls of a monster.
- A monster is a square root of a negative number.
- These numbers are called imaginary numbers.

Complexe Number

- Later D'Alembert found that these monster is a multiple of a number that mulitply -1.
- Written a+ib
- Complexe is not the meaning of difficulty here, but the fact of their composition in two member.