# Homework #2
# CSCI 5521
# Professor Kuang

Christopher White

March 14, 2019

## Question 1

### 1(a)

$$p(\mathbf{x}|\Sigma_i) = \frac{1}{(2\pi)^{d/2}|\Sigma_i|^{1/2}} \exp(-\frac{1}{2}(\mathbf{x}-\mu_i)^T\Sigma_i^{-1}(\mathbf{x}-\mu_i))$$

$$\log p(\mathbf{x}|\Sigma_i) = -\frac{1}{2}(\mathbf{x}-\mu_i)^T\Sigma_i^{-1}(\mathbf{x}-\mu_i) - \frac{d}{2}\log 2\pi - \frac{1}{2}\log|\Sigma_i|$$

$$L(\Sigma_i|\mathbf{x}^t) = \sum_{t=1}^{N} \log p(\mathbf{x}^t|\Sigma_i)$$

$$= \sum_{t=1}^{N} -\frac{1}{2}(\mathbf{x}-\mu_i)^T\Sigma_i^{-1}(\mathbf{x}-\mu_i) - \frac{d}{2}\log 2\pi - \frac{1}{2}\log|\Sigma_i|$$

$$= -\frac{Nd}{2}\log 2\pi - \frac{N}{2}\log|\Sigma_i| - \frac{1}{2}\sum_{t=1}^{N}(\mathbf{x}-\mu_i)^T\Sigma_i^{-1}(\mathbf{x}-\mu_i)$$

**For Model 1:**

$$S_i = \frac{\sum_t r_i^t (x^t - m_i)(x^t - m_i)^T}{\sum_t r_i^t}$$

**For Model 2:**

$$L(\Sigma_i|\mathbf{x}^t) = -\frac{Nd}{2}\log 2\pi - \frac{N}{2}\log|\Sigma_i| - \frac{1}{2}\sum_{t=1}^{N}(\mathbf{x}^t-\mu)^T\Sigma_i^{-1}(\mathbf{x}^t-\mu)$$

$$\frac{d}{d\Sigma_i} L(\Sigma_i|\mathbf{x}^t) = \frac{d}{d\Sigma_i}\left(-\frac{N}{2}\log|\Sigma_i| - \frac{1}{2}\sum_{t=1}^{N}(\mathbf{x}^t-\mu)^T\Sigma_i^{-1}(\mathbf{x}^t-\mu)\right)$$

$$= -\frac{N}{2}\frac{1}{|\Sigma_i|}|\Sigma_i|(\Sigma_i^{-T}) - \frac{1}{2}\sum_{t=1}^{N}\left(-\Sigma_i^{-T}(\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\Sigma_i^{-T}\right) = 0$$

$$N\Sigma_i^{-T} = \sum_{t=1}^{N}\left(\Sigma_i^{-T}(\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\Sigma_i^{-T}\right)$$

$$\Sigma_i^T N = \sum_{t=1}^{N}\left((\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\right)$$

$$\Sigma_i^T = \frac{\sum_{t=1}^{N}\left((\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\right)}{N}$$

However, $\Sigma_i^T = \Sigma_i$ because it is symmetric.

$$S_i = \frac{\sum_{t=1}^{N}\left((\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\right)}{N}$$

$$S_1 = S_2$$

$$S_1 + S_2 = 2S = \frac{\sum_{t=1}^{N_1}\left((\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\right)}{N_1} + \frac{\sum_{t=1}^{N_2}\left((\mathbf{x}^t-\mu)(\mathbf{x}^t-\mu)^T\right)}{N_2}$$

$$S = \frac{1}{2}(S_1 + S_2)$$

**For Model 3:**

$$S_i = \alpha_i I$$

$$L(\Sigma_i|\mathbf{x}^t) = \sum_{t=1}^{N}\log p(\mathbf{x}^t|\Sigma_i)$$

$$= -\frac{Nd}{2}\log 2\pi - \frac{N}{2}\log|\alpha_i I| - \frac{1}{2}\sum_{t=1}^{N}(\mathbf{x}^t-\mu_i)^T\alpha_i^{-1}I(\mathbf{x}^t-\mu_i)$$

$$\frac{d}{d\alpha_i}L(\alpha_i|\mathbf{x}^t) = -\frac{N}{2}\frac{d}{d\alpha_i}\log\alpha_i^d - \frac{1}{2}\sum_{t=1}^{N}\frac{d}{d\alpha_i}(\mathbf{x}^t-\mu_i)^T\alpha_i^{-1}(\mathbf{x}^t-\mu_i)$$

$$= -\frac{N}{2}\frac{\mathrm{d}}{\alpha_i} + \frac{1}{2\alpha_i^2}\sum_{t=1}^{N}(\mathbf{x}^t-\mu_i)^T(\mathbf{x}^t-\mu_i) = 0$$

$$N\mathrm{d}\alpha_i = \sum_{t=1}^{N}(\mathbf{x}^t-\mu_i)^T(\mathbf{x}^t-\mu_i)$$

$$\alpha_i = \frac{\sum_{t=1}^{N}(\mathbf{x}^t-\mu_i)^T(\mathbf{x}^t-\mu_i)}{N\mathrm{d}}$$

DISCRIMINANT:

$$g_i(\mathbf{x}^t) = \log\hat{P}(C_i) - \frac{d}{2}\log 2\pi - \frac{d}{2}\log\alpha_i - \frac{1}{2\alpha_i}(\mathbf{x}^t-m_i)^T(\mathbf{x}^t-m_i)$$

## 1(b)

See results in MATLAB window of `MultiGaussian()` by running `script_1b.m`.

```
TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.2
mu1:    0.43062     2.0235     3.1758    -2.4272    -2.5234     3.2378    -5.5208    -6.6921
mu2:     4.5841     6.4933     6.4265     1.6891     2.2943     8.3626   -0.16579    -1.8048
-----------------------------------------
S1:     1.9283    0.23454    0.77191     1.0321    0.43221     1.2648     1.1728    -1.232
S1:    0.23454      3.659    0.31235   -0.13452     1.5821     1.0301   -0.19427     3.2756
S1:    0.77191    0.31235     8.1131     1.3347   -0.42859     1.7792    0.35497    0.23201
S1:     1.0321   -0.13452     1.3347     4.2296    0.94821     0.7471     1.0682      1.981
S1:    0.43221     1.5821   -0.42859    0.94821     4.1354     1.0028   -0.54523     3.4385
S1:     1.2648     1.0301     1.7792     0.7471     1.0028     4.0696   -0.19595     2.3004
S1:     1.1728   -0.19427    0.35497     1.0682   -0.54523   -0.19595     4.2163    -1.7099
S1:     -1.232     3.2756    0.23201      1.981     3.4385     2.3004    -1.7099      17.095
-----------------------------------------
S2:     3.4733     2.0992      2.608     2.6506     1.7989     1.8568     2.7181     2.9765
S2:     2.0992     5.8673     2.2109     2.7575     3.2032     2.9295     2.7963     5.9386
S2:      2.608     2.2109     8.8389     3.4241     2.8666     2.2617     2.7934     5.2586
S2:     2.6506     2.7575     3.4241     8.2867     3.6293     2.6992     2.0496     8.5185
S2:     1.7989     3.2032     2.8666     3.6293     5.6484     2.9483     3.2971     4.8906
S2:     1.8568     2.9295     2.2617     2.6992     2.9483     3.7835     2.2673     4.5415
```

```
S2:     2.7181     2.7963     2.7934     2.0496     3.2971     2.2673     8.3339     4.3679
S2:     2.9765     5.9386     5.2586     8.5185     4.8906     4.5415     4.3679     20.135

PC11 =

    0.3000


PC21 =

    0.7000


mu11 =

    0.4306    2.0235    3.1758   -2.4272   -2.5234    3.2378   -5.5208   -6.6921


mu21 =

    4.5841    6.4933    6.4265    1.6891    2.2943    8.3626   -0.1658   -1.8048


S11 =

    1.9283    0.2345    0.7719    1.0321    0.4322    1.2648    1.1728   -1.2320
    0.2345    3.6590    0.3123   -0.1345    1.5821    1.0301   -0.1943    3.2756
    0.7719    0.3123    8.1131    1.3347   -0.4286    1.7792    0.3550    0.2320
    1.0321   -0.1345    1.3347    4.2296    0.9482    0.7471    1.0682    1.9810
    0.4322    1.5821   -0.4286    0.9482    4.1354    1.0028   -0.5452    3.4385
    1.2648    1.0301    1.7792    0.7471    1.0028    4.0696   -0.1960    2.3004
    1.1728   -0.1943    0.3550    1.0682   -0.5452   -0.1960    4.2163   -1.7099
-1.2320    3.2756    0.2320    1.9810    3.4385    2.3004   -1.7099   17.0954


S21 =

    3.4733    2.0992    2.6080    2.6506    1.7989    1.8568    2.7181    2.9765
    2.0992    5.8673    2.2109    2.7575    3.2032    2.9295    2.7963    5.9386
    2.6080    2.2109    8.8389    3.4241    2.8666    2.2617    2.7934    5.2586
    2.6506    2.7575    3.4241    8.2867    3.6293    2.6992    2.0496    8.5185
    1.7989    3.2032    2.8666    3.6293    5.6484    2.9483    3.2971    4.8906
    1.8568    2.9295    2.2617    2.6992    2.9483    3.7835    2.2673    4.5415
    2.7181    2.7963    2.7934    2.0496    3.2971    2.2673    8.3339    4.3679
    2.9765    5.9386    5.2586    8.5185    4.8906    4.5415    4.3679   20.1353
```

```
ErrTest1 =

    0.2000
         0
         0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.23
mu1:     1.0658     2.6548     3.2977    -1.6793    -1.4987     4.3959    -4.2138    -4.9679
mu2:     2.8221     4.4669     4.8537    0.51923    0.37638     6.2585    -2.6611    -3.8175
-----------------------------------------
S1:     1.3816   -0.27405    0.68356    0.28188     1.0317    0.47828   -0.29784    0.30388
S1:   -0.27405      2.633     2.1369    0.95217     1.1237   -0.46126  -0.052341     2.7848
S1:    0.68356     2.1369     7.1517     1.5667    0.84272    0.19932     2.7759     2.8105
S1:    0.28188    0.95217     1.5667     3.1578    0.68105   -0.47999    0.24738     2.3803
S1:     1.0317     1.1237    0.84272    0.68105     2.2748 -0.0038062   -0.58546    0.77435
S1:    0.47828   -0.46126    0.19932   -0.47999 -0.0038062     1.2469    0.68031   -0.20541
S1:   -0.29784  -0.052341     2.7759    0.24738   -0.58546    0.68031     4.1857   -0.75495
S1:    0.30388     2.7848     2.8105     2.3803    0.77435   -0.20541   -0.75495     10.834
-----------------------------------------
S2:     2.9707     1.0855     2.9514    0.57866  -0.092368   0.067599    0.88675     1.7887
S2:     1.0855     5.0624     2.4893    0.17835     2.7711   -0.32819    0.50148     0.0514
S2:     2.9514     2.4893      11.826   0.089452    0.83721    0.12939   0.022258   -0.84529
S2:    0.57866    0.17835   0.089452     5.4082   0.054843     1.0249     4.3627     2.3291
S2:  -0.092368     2.7711    0.83721   0.054843     4.3167     1.0396     0.5454     1.6923
S2:   0.067599   -0.32819    0.12939     1.0249     1.0396     2.7282     1.7313     2.1908
S2:    0.88675    0.50148   0.022258     4.3627     0.5454     1.7313     11.481     2.7512
S2:     1.7887     0.0514   -0.84529     2.3291     1.6923     2.1908     2.7512     13.119

PC12 =

    0.3000


PC22 =

    0.7000


mu12 =

    1.0658    2.6548    3.2977   -1.6793   -1.4987    4.3959   -4.2138   -4.9679
```

```
mu22 =

    2.8221    4.4669    4.8537    0.5192    0.3764    6.2585   -2.6611   -3.8175


S12 =

    1.3816   -0.2741    0.6836    0.2819    1.0317    0.4783   -0.2978    0.3039
-0.2741    2.6330    2.1369    0.9522    1.1237   -0.4613   -0.0523    2.7848
    0.6836    2.1369    7.1517    1.5667    0.8427    0.1993    2.7759    2.8105
    0.2819    0.9522    1.5667    3.1578    0.6811   -0.4800    0.2474    2.3803
    1.0317    1.1237    0.8427    0.6811    2.2748   -0.0038   -0.5855    0.7744
    0.4783   -0.4613    0.1993   -0.4800   -0.0038    1.2469    0.6803   -0.2054
-0.2978   -0.0523    2.7759    0.2474   -0.5855    0.6803    4.1857   -0.7550
    0.3039    2.7848    2.8105    2.3803    0.7744   -0.2054   -0.7550   10.8345


S22 =

    2.9707    1.0855    2.9514    0.5787   -0.0924    0.0676    0.8867    1.7887
    1.0855    5.0624    2.4893    0.1783    2.7711   -0.3282    0.5015    0.0514
    2.9514    2.4893   11.8262    0.0895    0.8372    0.1294    0.0223   -0.8453
    0.5787    0.1783    0.0895    5.4082    0.0548    1.0249    4.3627    2.3291
-0.0924    2.7711    0.8372    0.0548    4.3167    1.0396    0.5454    1.6923
    0.0676   -0.3282    0.1294    1.0249    1.0396    2.7282    1.7313    2.1908
    0.8867    0.5015    0.0223    4.3627    0.5454    1.7313   11.4810    2.7512
    1.7887    0.0514   -0.8453    2.3291    1.6923    2.1908    2.7512   13.1191


ErrTest2 =

    0.2300
         0
         0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.12
mu1:    0.97473     2.6233      3.177    -1.4652    -1.3053      4.516    -4.3197    -5.5215
mu2:     1.4916     3.1655     3.6504   -0.81618   -0.35148     5.1345     -3.277    -4.7293
-----------------------------------------
S1:    0.26426   0.084393   0.061156  -0.085321 -0.0060191  -0.076364  -0.016044   0.080466
```

```
S1:   0.084393    0.42581  -0.084981   -0.10514  -0.012187  -0.089623    0.11651   -0.07582
S1:   0.061156  -0.084981    0.60629  -0.037661   -0.09252  -0.033265    0.08249   0.013285
S1:  -0.085321   -0.10514  -0.037661    0.44002  -0.035577   -0.14464   -0.10444   0.074589
S1: -0.0060191  -0.012187   -0.09252  -0.035577    0.43269   0.053367  -0.099753   0.014154
S1:  -0.076364  -0.089623  -0.033265   -0.14464   0.053367    0.59442   0.070731  -0.024553
S1:  -0.016044    0.11651    0.08249   -0.10444  -0.099753   0.070731    0.54428  -0.030454
S1:   0.080466   -0.07582   0.013285   0.074589   0.014154  -0.024553  -0.030454    0.39816
----------------------------------------
S2:     2.7945    0.41095   -0.38458    0.59849   -0.41287    0.16596    0.35736   -0.48835
S2:    0.41095     2.5371   -0.34972  -0.036702   -0.29616    0.05865    0.17826   0.088163
S2:   -0.38458   -0.34972     2.1605   -0.25595  -0.049037  -0.030287   0.035702    -0.2838
S2:    0.59849  -0.036702   -0.25595     2.9685   -0.87774    0.10262   -0.34245   -0.32702
S2:   -0.41287   -0.29616  -0.049037   -0.87774     3.5095   0.074612   -0.76268    0.19421
S2:    0.16596    0.05865  -0.030287    0.10262   0.074612     2.9193    0.64239    0.35185
S2:    0.35736    0.17826   0.035702   -0.34245   -0.76268    0.64239     2.9478    0.27836
S2:   -0.48835   0.088163    -0.2838   -0.32702    0.19421    0.35185    0.27836     2.6123

PC13 =

    0.3000


PC23 =

    0.7000


mu13 =

    0.9747    2.6233    3.1770   -1.4652   -1.3053    4.5160   -4.3197   -5.5215


mu23 =

    1.4916    3.1655    3.6504   -0.8162   -0.3515    5.1345   -3.2770   -4.7293


alpha1 =

    0.2643    0.0844    0.0612   -0.0853   -0.0060   -0.0764   -0.0160    0.0805
    0.0844    0.4258   -0.0850   -0.1051   -0.0122   -0.0896    0.1165   -0.0758
    0.0612   -0.0850    0.6063   -0.0377   -0.0925   -0.0333    0.0825    0.0133
-0.0853   -0.1051   -0.0377    0.4400   -0.0356   -0.1446   -0.1044    0.0746
-0.0060   -0.0122   -0.0925   -0.0356    0.4327    0.0534   -0.0998    0.0142
-0.0764   -0.0896   -0.0333   -0.1446    0.0534    0.5944    0.0707   -0.0246
-0.0160    0.1165    0.0825   -0.1044   -0.0998    0.0707    0.5443   -0.0305
```

```
    0.0805   -0.0758    0.0133    0.0746    0.0142   -0.0246   -0.0305    0.3982


alpha2 =

    2.7945    0.4110   -0.3846    0.5985   -0.4129    0.1660    0.3574   -0.4884
    0.4110    2.5371   -0.3497   -0.0367   -0.2962    0.0586    0.1783    0.0882
-0.3846   -0.3497    2.1605   -0.2560   -0.0490   -0.0303    0.0357   -0.2838
    0.5985   -0.0367   -0.2560    2.9685   -0.8777    0.1026   -0.3425   -0.3270
-0.4129   -0.2962   -0.0490   -0.8777    3.5095    0.0746   -0.7627    0.1942
    0.1660    0.0586   -0.0303    0.1026    0.0746    2.9193    0.6424    0.3518
    0.3574    0.1783    0.0357   -0.3425   -0.7627    0.6424    2.9478    0.2784
-0.4884    0.0882   -0.2838   -0.3270    0.1942    0.3518    0.2784    2.6123


ErrTest3 =

    0.1200
         0
         0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.17
mu1:    0.43062     2.0235     3.1758    -2.4272    -2.5234     3.2378    -5.5208    -6.6921
mu2:     4.5841     6.4933     6.4265     1.6891     2.2943     8.3626   -0.16579    -1.8048
-----------------------------------------
S1:     2.7008     1.1668     1.6899     1.8414     1.1156     1.5608     1.9454    0.87225
S1:     1.1668     4.7632     1.2616     1.3115     2.3927     1.9798      1.301     4.6071
S1:     1.6899     1.2616      8.476     2.3794      1.219     2.0204     1.5742     2.7453
S1:     1.8414     1.3115     2.3794     6.2581     2.2887     1.7232     1.5589     5.2498
S1:     1.1156     2.3927      1.219     2.2887     4.8919     1.9755     1.3759     4.1646
S1:     1.5608     1.9798     2.0204     1.7232     1.9755     3.9265     1.0357      3.421
S1:     1.9454      1.301     1.5742     1.5589     1.3759     1.0357     6.2751      1.329
S1:    0.87225     4.6071     2.7453     5.2498     4.1646      3.421      1.329     18.615
-----------------------------------------
S2:     2.7008     1.1668     1.6899     1.8414     1.1156     1.5608     1.9454    0.87225
S2:     1.1668     4.7632     1.2616     1.3115     2.3927     1.9798      1.301     4.6071
S2:     1.6899     1.2616      8.476     2.3794      1.219     2.0204     1.5742     2.7453
S2:     1.8414     1.3115     2.3794     6.2581     2.2887     1.7232     1.5589     5.2498
S2:     1.1156     2.3927      1.219     2.2887     4.8919     1.9755     1.3759     4.1646
S2:     1.5608     1.9798     2.0204     1.7232     1.9755     3.9265     1.0357      3.421
S2:     1.9454      1.301     1.5742     1.5589     1.3759     1.0357     6.2751      1.329
S2:    0.87225     4.6071     2.7453     5.2498     4.1646      3.421      1.329     18.615
```

```
PC11 =

    0.3000


PC21 =

    0.7000


mu11 =

    0.4306    2.0235    3.1758   -2.4272   -2.5234    3.2378   -5.5208   -6.6921


mu21 =

    4.5841    6.4933    6.4265    1.6891    2.2943    8.3626   -0.1658   -1.8048


S11 =

    2.7008    1.1668    1.6899    1.8414    1.1156    1.5608    1.9454    0.8722
    1.1668    4.7632    1.2616    1.3115    2.3927    1.9798    1.3010    4.6071
    1.6899    1.2616    8.4760    2.3794    1.2190    2.0204    1.5742    2.7453
    1.8414    1.3115    2.3794    6.2581    2.2887    1.7232    1.5589    5.2498
    1.1156    2.3927    1.2190    2.2887    4.8919    1.9755    1.3759    4.1646
    1.5608    1.9798    2.0204    1.7232    1.9755    3.9265    1.0357    3.4210
    1.9454    1.3010    1.5742    1.5589    1.3759    1.0357    6.2751    1.3290
    0.8722    4.6071    2.7453    5.2498    4.1646    3.4210    1.3290   18.6154


S21 =

    2.7008    1.1668    1.6899    1.8414    1.1156    1.5608    1.9454    0.8722
    1.1668    4.7632    1.2616    1.3115    2.3927    1.9798    1.3010    4.6071
    1.6899    1.2616    8.4760    2.3794    1.2190    2.0204    1.5742    2.7453
    1.8414    1.3115    2.3794    6.2581    2.2887    1.7232    1.5589    5.2498
    1.1156    2.3927    1.2190    2.2887    4.8919    1.9755    1.3759    4.1646
    1.5608    1.9798    2.0204    1.7232    1.9755    3.9265    1.0357    3.4210
    1.9454    1.3010    1.5742    1.5589    1.3759    1.0357    6.2751    1.3290
    0.8722    4.6071    2.7453    5.2498    4.1646    3.4210    1.3290   18.6154


ErrTest1 =
```

```
    0.2000
    0.1700
        0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.55
mu1:     1.0658     2.6548     3.2977    -1.6793    -1.4987     4.3959    -4.2138    -4.9679
mu2:     2.8221     4.4669     4.8537    0.51923    0.37638     6.2585    -2.6611    -3.8175
-----------------------------------------
S1:     2.1762    0.40574     1.8175    0.43027    0.46967    0.27294    0.29445     1.0463
S1:    0.40574     3.8477     2.3131    0.56526     1.9474   -0.39472    0.22457     1.4181
S1:     1.8175     2.3131     9.4889    0.82808    0.83997    0.16435     1.3991    0.98259
S1:    0.43027    0.56526    0.82808      4.283    0.36795    0.27246     2.3051     2.3547
S1:    0.46967     1.9474    0.83997    0.36795     3.2958    0.51791  -0.020032     1.2333
S1:    0.27294   -0.39472    0.16435    0.27246    0.51791     1.9875     1.2058    0.99271
S1:    0.29445    0.22457     1.3991     2.3051  -0.020032     1.2058     7.8334    0.99815
S1:     1.0463     1.4181    0.98259     2.3547     1.2333    0.99271    0.99815     11.977
-----------------------------------------
S2:     2.1762    0.40574     1.8175    0.43027    0.46967    0.27294    0.29445     1.0463
S2:    0.40574     3.8477     2.3131    0.56526     1.9474   -0.39472    0.22457     1.4181
S2:     1.8175     2.3131     9.4889    0.82808    0.83997    0.16435     1.3991    0.98259
S2:    0.43027    0.56526    0.82808      4.283    0.36795    0.27246     2.3051     2.3547
S2:    0.46967     1.9474    0.83997    0.36795     3.2958    0.51791  -0.020032     1.2333
S2:    0.27294   -0.39472    0.16435    0.27246    0.51791     1.9875     1.2058    0.99271
S2:    0.29445    0.22457     1.3991     2.3051  -0.020032     1.2058     7.8334    0.99815
S2:     1.0463     1.4181    0.98259     2.3547     1.2333    0.99271    0.99815     11.977

PC12 =

    0.3000


PC22 =

    0.7000


mu12 =

    1.0658    2.6548    3.2977   -1.6793   -1.4987    4.3959   -4.2138   -4.9679
```

```
mu22 =

    2.8221    4.4669    4.8537    0.5192    0.3764    6.2585   -2.6611   -3.8175


S12 =

    2.1762    0.4057    1.8175    0.4303    0.4697    0.2729    0.2945    1.0463
    0.4057    3.8477    2.3131    0.5653    1.9474   -0.3947    0.2246    1.4181
    1.8175    2.3131    9.4889    0.8281    0.8400    0.1644    1.3991    0.9826
    0.4303    0.5653    0.8281    4.2830    0.3679    0.2725    2.3051    2.3547
    0.4697    1.9474    0.8400    0.3679    3.2958    0.5179   -0.0200    1.2333
    0.2729   -0.3947    0.1644    0.2725    0.5179    1.9875    1.2058    0.9927
    0.2945    0.2246    1.3991    2.3051   -0.0200    1.2058    7.8334    0.9981
    1.0463    1.4181    0.9826    2.3547    1.2333    0.9927    0.9981   11.9768


S22 =

    2.1762    0.4057    1.8175    0.4303    0.4697    0.2729    0.2945    1.0463
    0.4057    3.8477    2.3131    0.5653    1.9474   -0.3947    0.2246    1.4181
    1.8175    2.3131    9.4889    0.8281    0.8400    0.1644    1.3991    0.9826
    0.4303    0.5653    0.8281    4.2830    0.3679    0.2725    2.3051    2.3547
    0.4697    1.9474    0.8400    0.3679    3.2958    0.5179   -0.0200    1.2333
    0.2729   -0.3947    0.1644    0.2725    0.5179    1.9875    1.2058    0.9927
    0.2945    0.2246    1.3991    2.3051   -0.0200    1.2058    7.8334    0.9981
    1.0463    1.4181    0.9826    2.3547    1.2333    0.9927    0.9981   11.9768


ErrTest2 =

    0.2300
    0.5500
         0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.47
mu1:    0.97473     2.6233      3.177    -1.4652    -1.3053      4.516    -4.3197    -5.5215
mu2:     1.4916     3.1655     3.6504   -0.81618   -0.35148     5.1345     -3.277    -4.7293
----------------------------------------
S1:     1.5294    0.24767   -0.16171    0.25658   -0.20944   0.044799    0.17066   -0.20394
S1:    0.24767     1.4815   -0.21735  -0.070922   -0.15417  -0.015487    0.14738  0.0061713
S1:   -0.16171   -0.21735     1.3834   -0.14681  -0.070779  -0.031776   0.059096   -0.13526
```

```
S1:    0.25658  -0.070922   -0.14681     1.7042   -0.45666  -0.021009   -0.22345   -0.12622
S1:   -0.20944   -0.15417  -0.070779   -0.45666     1.9711    0.06399   -0.43122    0.10418
S1:   0.044799  -0.015487  -0.031776  -0.021009    0.06399     1.7568    0.35656    0.16365
S1:    0.17066    0.14738   0.059096   -0.22345   -0.43122    0.35656     1.7461    0.12396
S1:   -0.20394  0.0061713   -0.13526   -0.12622    0.10418    0.16365    0.12396     1.5052
----------------------------------------
S2:     1.5294    0.24767   -0.16171    0.25658   -0.20944   0.044799    0.17066   -0.20394
S2:    0.24767     1.4815   -0.21735  -0.070922   -0.15417  -0.015487    0.14738  0.0061713
S2:   -0.16171   -0.21735     1.3834   -0.14681  -0.070779  -0.031776   0.059096   -0.13526
S2:    0.25658  -0.070922   -0.14681     1.7042   -0.45666  -0.021009   -0.22345   -0.12622
S2:   -0.20944   -0.15417  -0.070779   -0.45666     1.9711    0.06399   -0.43122    0.10418
S2:   0.044799  -0.015487  -0.031776  -0.021009    0.06399     1.7568    0.35656    0.16365
S2:    0.17066    0.14738   0.059096   -0.22345   -0.43122    0.35656     1.7461    0.12396
S2:   -0.20394  0.0061713   -0.13526   -0.12622    0.10418    0.16365    0.12396     1.5052

PC13 =

    0.3000


PC23 =

    0.7000


mu13 =

    0.9747    2.6233    3.1770   -1.4652   -1.3053    4.5160   -4.3197   -5.5215


mu23 =

    1.4916    3.1655    3.6504   -0.8162   -0.3515    5.1345   -3.2770   -4.7293


alpha1 =

    1.5294    0.2477   -0.1617    0.2566   -0.2094    0.0448    0.1707   -0.2039
    0.2477    1.4815   -0.2174   -0.0709   -0.1542   -0.0155    0.1474    0.0062
-0.1617   -0.2174    1.3834   -0.1468   -0.0708   -0.0318    0.0591   -0.1353
    0.2566   -0.0709   -0.1468    1.7042   -0.4567   -0.0210   -0.2234   -0.1262
-0.2094   -0.1542   -0.0708   -0.4567    1.9711    0.0640   -0.4312    0.1042
    0.0448   -0.0155   -0.0318   -0.0210    0.0640    1.7568    0.3566    0.1636
    0.1707    0.1474    0.0591   -0.2234   -0.4312    0.3566    1.7461    0.1240
-0.2039    0.0062   -0.1353   -0.1262    0.1042    0.1636    0.1240    1.5052
```

```
alpha2 =

    1.5294    0.2477   -0.1617    0.2566   -0.2094    0.0448    0.1707   -0.2039
    0.2477    1.4815   -0.2174   -0.0709   -0.1542   -0.0155    0.1474    0.0062
-0.1617   -0.2174    1.3834   -0.1468   -0.0708   -0.0318    0.0591   -0.1353
    0.2566   -0.0709   -0.1468    1.7042   -0.4567   -0.0210   -0.2234   -0.1262
-0.2094   -0.1542   -0.0708   -0.4567    1.9711    0.0640   -0.4312    0.1042
    0.0448   -0.0155   -0.0318   -0.0210    0.0640    1.7568    0.3566    0.1636
    0.1707    0.1474    0.0591   -0.2234   -0.4312    0.3566    1.7461    0.1240
-0.2039    0.0062   -0.1353   -0.1262    0.1042    0.1636    0.1240    1.5052


ErrTest3 =

    0.1200
    0.4700
         0

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.24
mu1:    0.43062     2.0235     3.1758    -2.4272    -2.5234     3.2378    -5.5208    -6.6921
mu2:     4.5841     6.4933     6.4265     1.6891     2.2943     8.3626   -0.16579    -1.8048
-----------------------------------------
S1:     5.7331          0          0          0          0          0          0          0
S1:          0     5.7331          0          0          0          0          0          0
S1:          0          0     5.7331          0          0          0          0          0
S1:          0          0          0     5.7331          0          0          0          0
S1:          0          0          0          0     5.7331          0          0          0
S1:          0          0          0          0          0     5.7331          0          0
S1:          0          0          0          0          0          0     5.7331          0
S1:          0          0          0          0          0          0          0     5.7331
-----------------------------------------
S2:      7.931          0          0          0          0          0          0          0
S2:          0      7.931          0          0          0          0          0          0
S2:          0          0      7.931          0          0          0          0          0
S2:          0          0          0      7.931          0          0          0          0
S2:          0          0          0          0      7.931          0          0          0
S2:          0          0          0          0          0      7.931          0          0
S2:          0          0          0          0          0          0      7.931          0
S2:          0          0          0          0          0          0          0      7.931

PC11 =
```

```
    0.3000


PC21 =

    0.7000


mu11 =

    0.4306    2.0235    3.1758   -2.4272   -2.5234    3.2378   -5.5208   -6.6921


mu21 =

    4.5841    6.4933    6.4265    1.6891    2.2943    8.3626   -0.1658   -1.8048


S11 =

    5.7331


S21 =

    7.9310


ErrTest1 =

    0.2000
    0.1700
    0.2400

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.55
mu1:     1.0658     2.6548     3.2977    -1.6793    -1.4987     4.3959    -4.2138    -4.9679
mu2:     2.8221     4.4669     4.8537    0.51923    0.37638     6.2585    -2.6611    -3.8175
-----------------------------------------
S1:     3.9713          0          0          0          0          0          0          0
S1:          0     3.9713          0          0          0          0          0          0
S1:          0          0     3.9713          0          0          0          0          0
```

```
S1:          0          0          0     3.9713          0          0          0          0
S1:          0          0          0          0     3.9713          0          0          0
S1:          0          0          0          0          0     3.9713          0          0
S1:          0          0          0          0          0          0     3.9713          0
S1:          0          0          0          0          0          0          0     3.9713
----------------------------------------
S2:     7.0124          0          0          0          0          0          0          0
S2:          0     7.0124          0          0          0          0          0          0
S2:          0          0     7.0124          0          0          0          0          0
S2:          0          0          0     7.0124          0          0          0          0
S2:          0          0          0          0     7.0124          0          0          0
S2:          0          0          0          0          0     7.0124          0          0
S2:          0          0          0          0          0          0     7.0124          0
S2:          0          0          0          0          0          0          0     7.0124

PC12 =

    0.3000


PC22 =

    0.7000


mu12 =

    1.0658    2.6548    3.2977   -1.6793   -1.4987    4.3959   -4.2138   -4.9679


mu22 =

    2.8221    4.4669    4.8537    0.5192    0.3764    6.2585   -2.6611   -3.8175


S12 =

    3.9713


S22 =

    7.0124


ErrTest2 =
```

```
    0.2300
    0.5500
    0.5500

TABLE OF TEST RESULTS

P(C1): 0.3
P(C2): 0.7
Error Rate: 0.05
mu1:    0.97473     2.6233      3.177    -1.4652    -1.3053      4.516    -4.3197    -5.5215
mu2:     1.4916     3.1655     3.6504   -0.81618   -0.35148     5.1345     -3.277    -4.7293
-----------------------------------------
S1:     0.4478          0          0          0          0          0          0          0
S1:          0     0.4478          0          0          0          0          0          0
S1:          0          0     0.4478          0          0          0          0          0
S1:          0          0          0     0.4478          0          0          0          0
S1:          0          0          0          0     0.4478          0          0          0
S1:          0          0          0          0          0     0.4478          0          0
S1:          0          0          0          0          0          0     0.4478          0
S1:          0          0          0          0          0          0          0     0.4478
-----------------------------------------
S2:     2.7661          0          0          0          0          0          0          0
S2:          0     2.7661          0          0          0          0          0          0
S2:          0          0     2.7661          0          0          0          0          0
S2:          0          0          0     2.7661          0          0          0          0
S2:          0          0          0          0     2.7661          0          0          0
S2:          0          0          0          0          0     2.7661          0          0
S2:          0          0          0          0          0          0     2.7661          0
S2:          0          0          0          0          0          0          0     2.7661

PC13 =

    0.3000


PC23 =

    0.7000


mu13 =

    0.9747    2.6233    3.1770   -1.4652   -1.3053    4.5160   -4.3197   -5.5215
```

```
mu23 =

    1.4916    3.1655    3.6504   -0.8162   -0.3515    5.1345   -3.2770   -4.7293


alpha1 =

    0.4478


alpha2 =

    2.7661


ErrTest3 =

    0.1200
    0.4700
    0.0500
```

## 1(c)

Table 1: Error Results for Test Data on 3 Models

| Model | Error Test #1 | Error Test #2 | ErrorTest#3 |
|---|---|---|---|
| 1 | 0.20 | 0.23 | 0.12 |
| 2 | 0.17 | 0.55 | 0.47 |
| 3 | 0.24 | 0.55 | 0.05 |

From the table of error rates, the models each work best on a different data set. Model 2 has the lowest error for test set #1. Model 1 has the lowest error on test set #2. And model 3 has the lowest error on test set #3.

The data is test set #1 is best fit by model 2. This implies that the covariance for class 1 is close to class 2 for this data.

Test set #2 is best modeled by model 1. Model 1 makes the fewest assumptions about the covariance of the data. Since this model worked best, the data may be dependent in $\mathbf{x}$ and has unique variances for each dimension. And the covariance is different for class 1 and class 2.

The model 3 represents data that is independent in $\mathbf{x}$ and has the same variance for all dimensions for a given class. Since test set #3 is best fit by model 3, this indicates that the data in #3 is independent in $\mathbf{x}$ and that a single class variance is a good approximation across the dimensions.

# Question 2

## 2(a)

```
ERROR RESULTS FOR 2(A)
------------------
The error rate for k=1 is: 0.053872
The error rate for k=3 is: 0.040404
The error rate for k=5 is: 0.043771
The error rate for k=7 is: 0.053872
```

## 2(b)

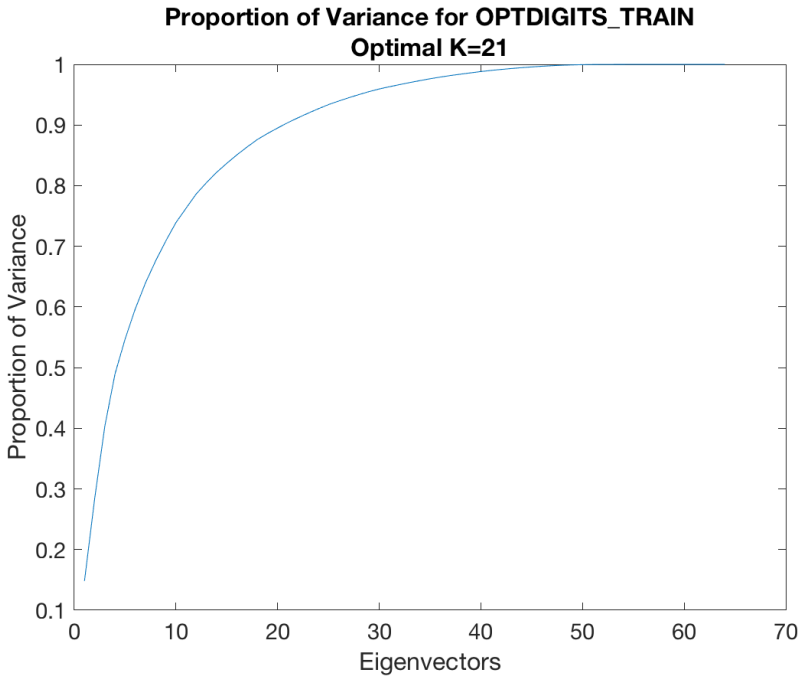


Figure 1: 2(b) Proportion of Variance: Optimal K is 21

```
ERROR RESULTS FOR 2(B)
------------------
Error rate for k=1: 0.047138
Error rate for k=3: 0.047138
Error rate for k=5: 0.0538721
Error rate for k=7: 0.0538721
```

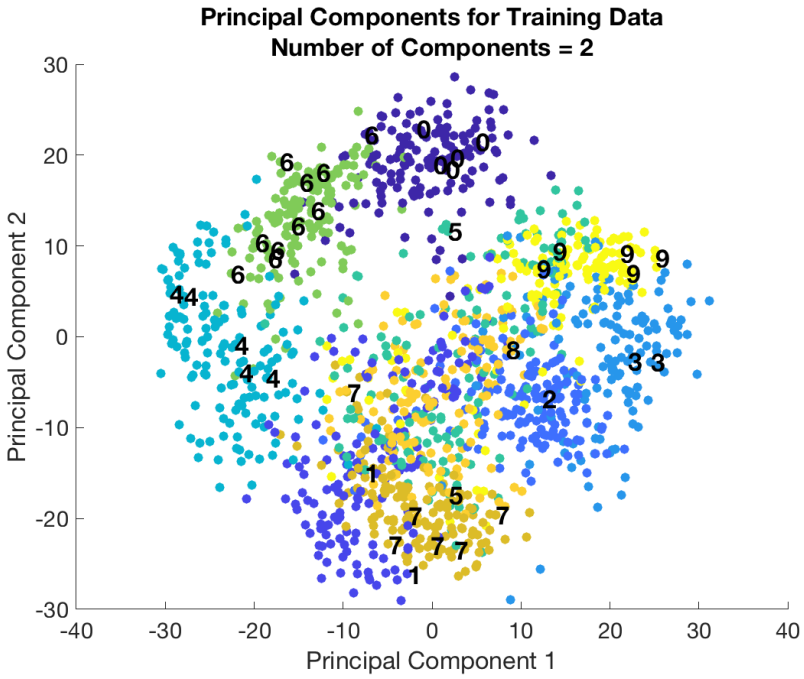


Figure 2: 2(c) PCA Plot for First Two Components on Training Data

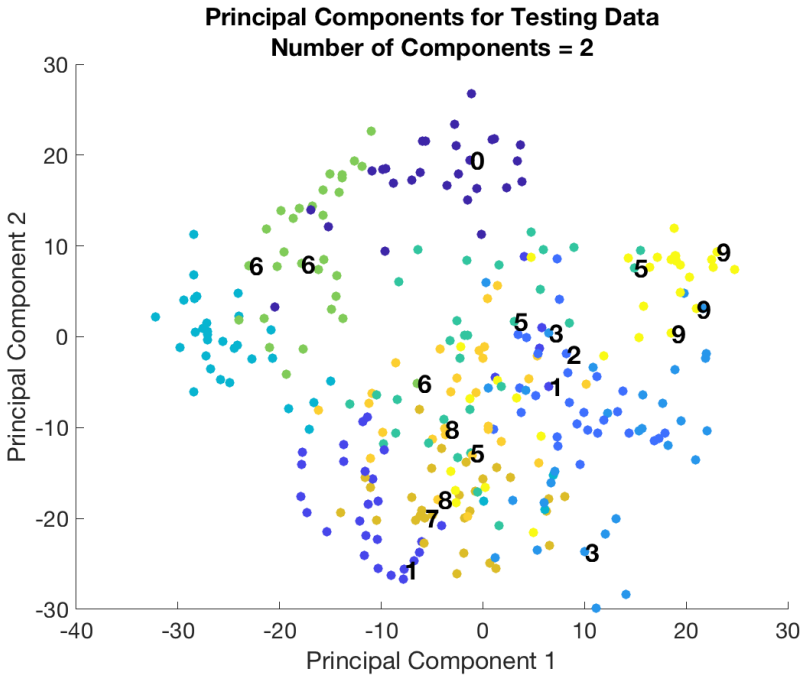


Figure 3: 2(c) PCA Plot for First Two Components on Test Data

## 2(c)

## 2(d)

```
RESULTS TABLE FOR 2(D)
--------------
L   k   Error
--------------
2   1   0.750842
2   3   0.771044
2   5   0.744108
4   1   0.707071
4   3   0.717172
4   5   0.676768
9   1   0.47138
9   3   0.444444
9   5   0.424242
```

## 2(e)

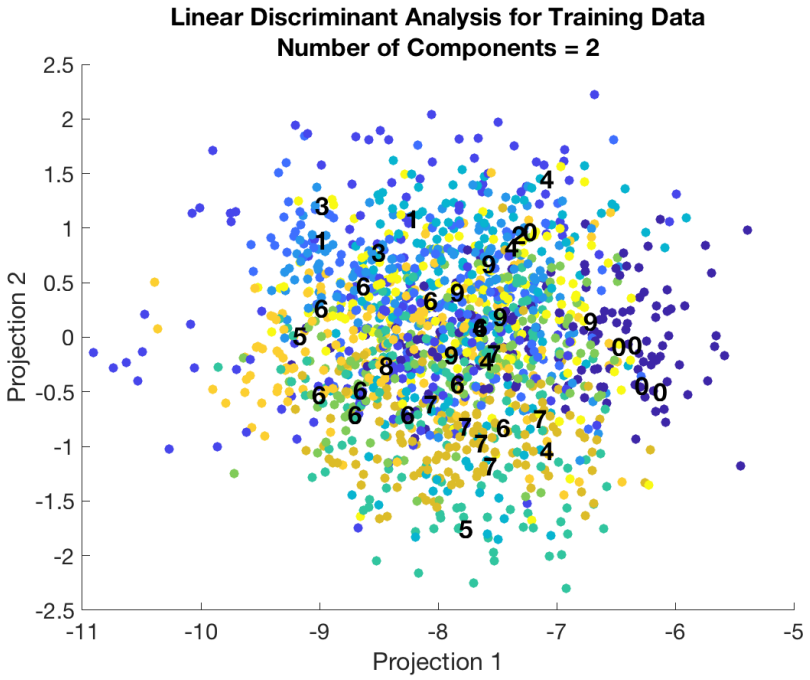


Figure 4: 2(e) LDA Plot for First Two Components on Training Data

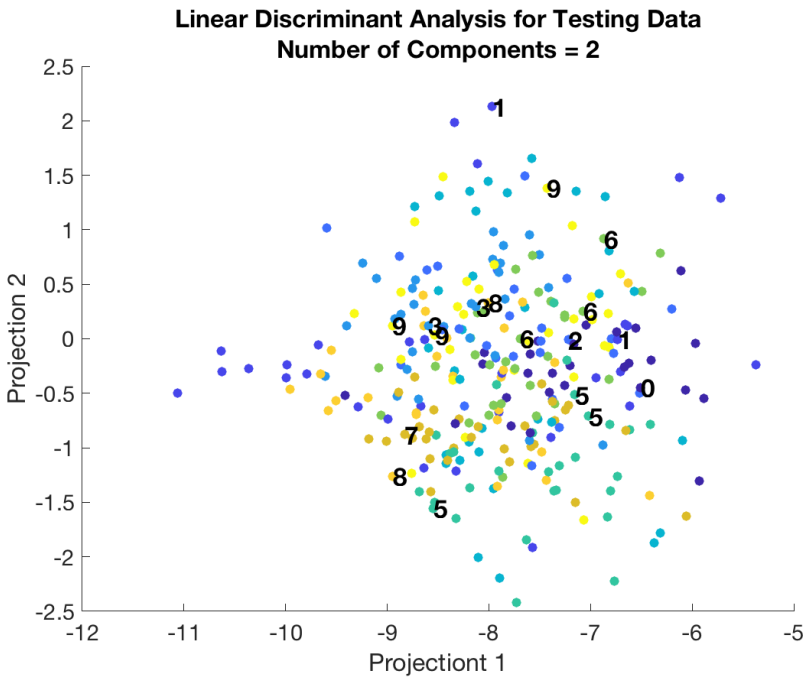


Figure 5: 2(e) LDA Plot for First Two Components on Test Data

# Question 3

## 3(a)

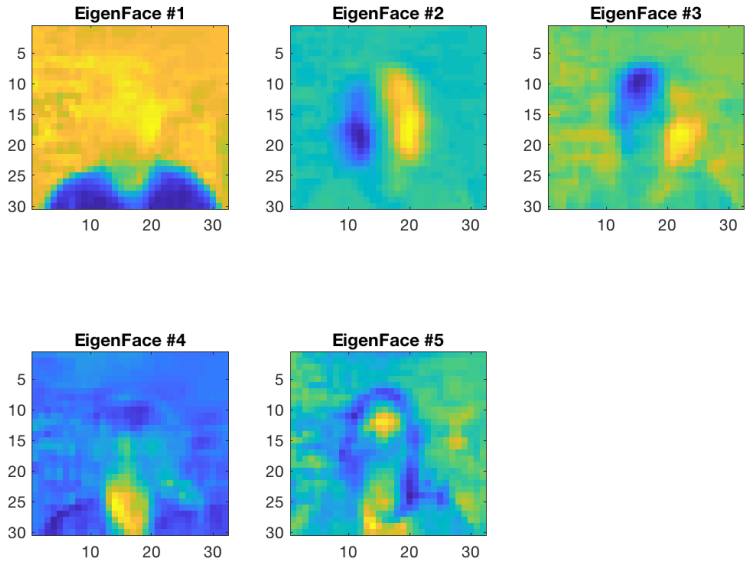


Figure 6: 3(a) First Five Eigen-Faces

## 3(b)

```
ERROR RESULTS FOR 3(B)
------------------
Error rate for k=1: 0.112903
Error rate for k=3: 0.233871
Error rate for k=5: 0.41129
Error rate for k=7: 0.435484
```

## 3(c)

The results in the following three pictures show the back projected images compared to the originals. In the first graphic, the images are blurry and only vaguely look like the originals. In the second graphic, some facial features are visible as are the sunglasses. In the final graphic for K of 100, the features are more visible than K of 10 or 50. In all of the graphics there is error as the images are not as clear as the originals.

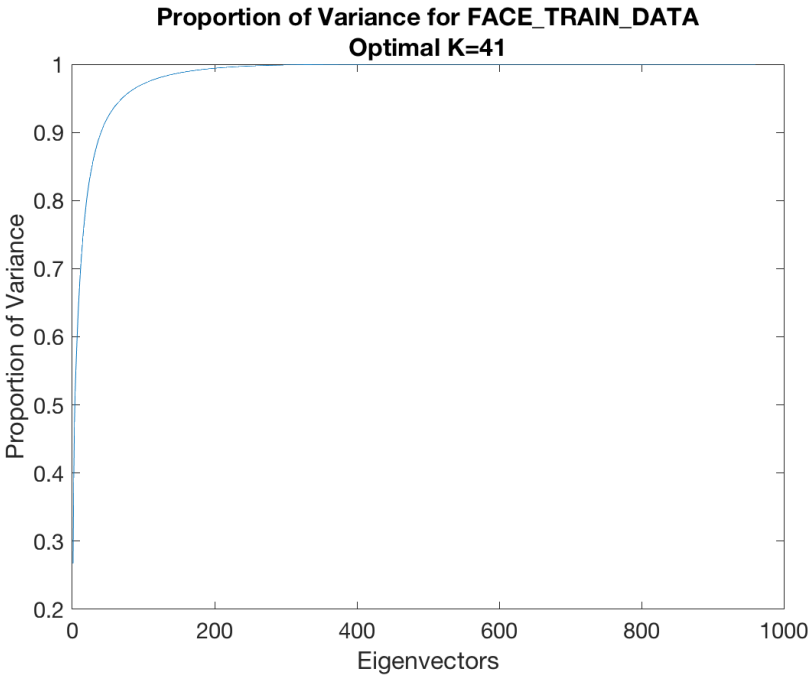


Figure 7: 3(b) Proportion of Variance: Optimal K is 41

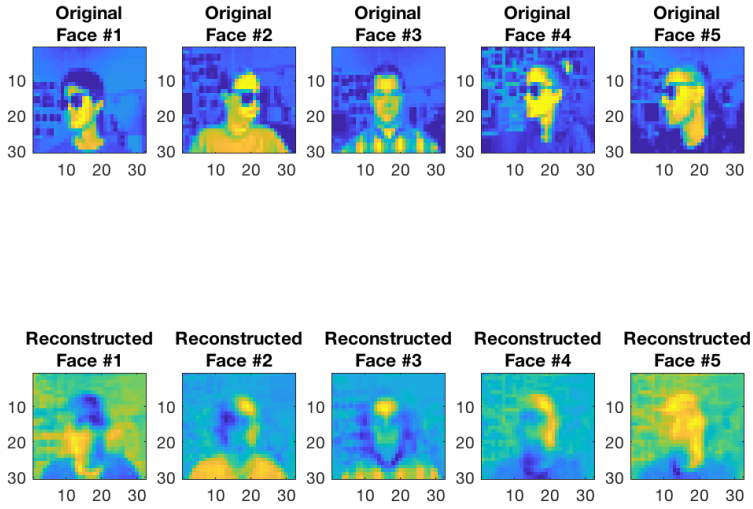


Figure 8: 3(c) Back Projected Results for First 10 Components

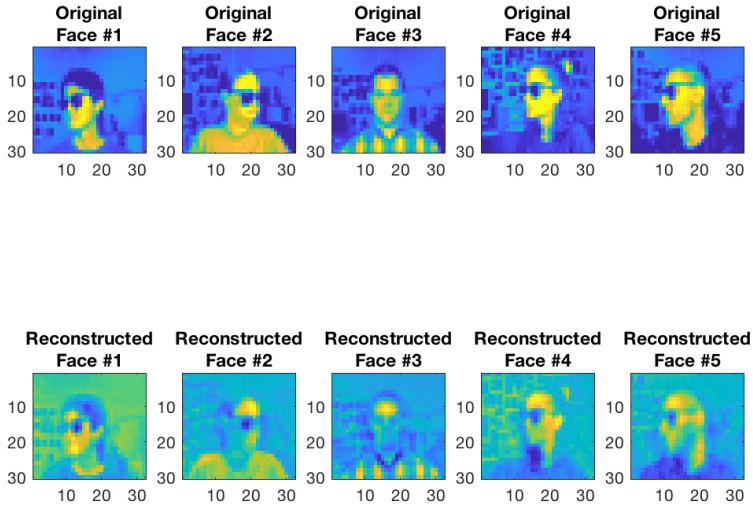


Figure 9: 3(c) Back Projected Results for First 50 Components

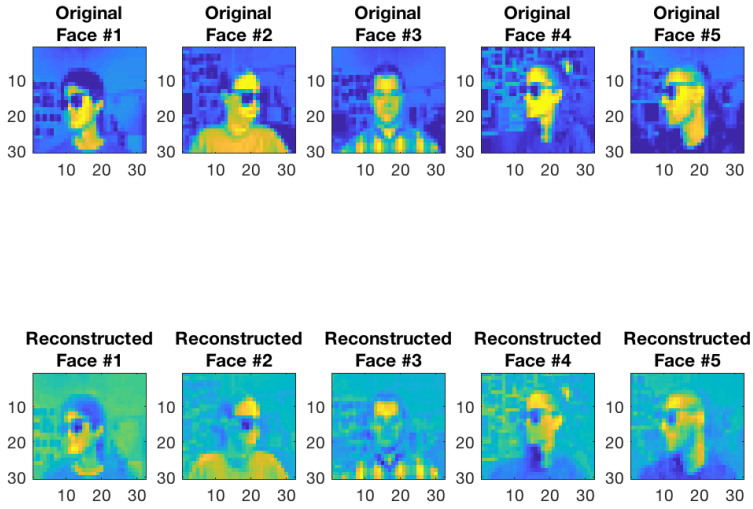


Figure 10: 3(c) Back Projected Results for First 100 Components