

# ECO 420Y — Homework 7

Joe White

2025-12-01

## Question 1

```
##### Question 1 #####
library(quantmod)
library(tseries)
library(zoo)

getSymbols("SP500", src = "FRED")
```

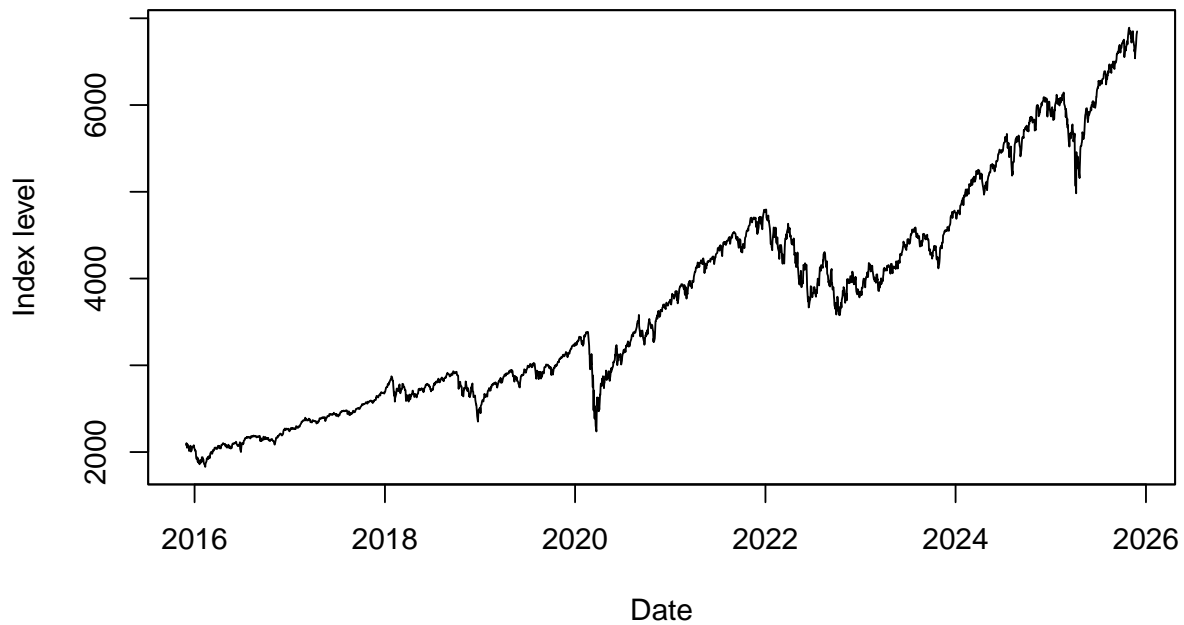
```
## [1] "SP500"
```

```
sp_zoo <- SP500
sp_raw <- coredata(sp_zoo)
sp_num <- as.numeric(sp_raw)
sp      <- na.omit(sp_num)

dates_clean <- index(sp_zoo)[!is.na(sp_raw)]
sp_ts <- zoo(sp, order.by = dates_clean)

plot(sp_ts,
      main = "SP500 Index from FRED",
      ylab = "Index level",
      xlab = "Date")
```

## SP500 Index from FRED



## Question 2

```
#===== Question 2 =====
```

```
adf_sp <- adf.test(sp, alternative = "stationary", k = 1)
adf_sp
```

```
##
## Augmented Dickey-Fuller Test
##
## data:  sp
## Dickey-Fuller = -1.9839, Lag order = 1, p-value = 0.5851
## alternative hypothesis: stationary
```

```
dsp_raw <- diff(sp)
dsp      <- na.omit(dsp_raw)

adf_dsp <- adf.test(dsp, alternative = "stationary", k = 1)
adf_dsp
```

```
##
```

```
## Augmented Dickey-Fuller Test
##
## data:  dsp
## Dickey-Fuller = -34.892, Lag order = 1, p-value = 0.01
## alternative hypothesis: stationary
```

From the ADF tests:

- For the SP500 level series, the p-value is 0.5851, so we do not reject the unit-root null. The level has a unit root (nonstationary).
- For the first difference, the p-value is 0.01, so we do reject the unit-root null. The differenced series is stationary.

### Question 3

```
===== Question 3 =====

fit_ar1 <- arima(dsp, order = c(1, 0, 0))
fit_ar1
```

```
##
## Call:
## arima(x = dsp, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##      -0.0861      1.8957
## s.e.   0.0199      0.7744
##
## sigma^2 estimated as 1778:  log likelihood = -12973.98,  aic = 25953.96
```

```
Box.test(fit_ar1$residuals, lag = 1, type = "Ljung")
```

```
##
## Box-Ljung test
##
## data:  fit_ar1$residuals
## X-squared = 0.05882, df = 1, p-value = 0.8084
```

For the AR(1) model, the Ljung-Box p-value is 0.8084.

Since this is greater than 0.05, we do not reject the null of no autocorrelation in the residuals, so the AR(1) model is adequate for the first difference of SP500.

## Question 4

```
#===== Question 4 =====
```

```
fc_ar1 <- predict(fit_ar1, n.ahead = 1)
```

```
fc_value <- fc_ar1$pred[1]
```

```
fc_se <- fc_ar1$se[1]
```

```
cat("One-step-ahead forecast of the first difference of SP500 is:",  
    round(fc_value, 4), "\n")
```

```
## One-step-ahead forecast of the first difference of SP500 is: -1.0818
```

```
cat("Standard error of the forecast is:",  
    round(fc_se, 4), "\n")
```

```
## Standard error of the forecast is: 42.1718
```