ECO 420Y — Homework 6

Joe White

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1. Main Idea of the Fama–French Three-Factor Model

The Fama–French Three-Factor Model extends the CAPM by adding two additional systematic risk factors that help explain cross-sectional differences in stock returns. The three factors are:

- **Market excess return (MKT):** $R_M - r_f$.
- **SMB (Small Minus Big):** return on small firms minus big firms.
- **HML (High Minus Low):** return on value stocks minus growth stocks.

The model for stock i states:

$$R_i - r_f = \alpha_i + \beta_{M,i}(R_M - r_f) + \beta_{SMB,i} \cdot SMB + \beta_{HML,i} \cdot HML + \varepsilon_i.$$

Where:

- $\beta_{M,i}$: sensitivity to market risk, similar to beta in CAPM.
- $\beta_{SMB,i}$: exposure to the size effect; positive values behave like small firms.
- $\beta_{HML,i}$: exposure to the value effect; positive values behave like value stocks.

2. Estimating the Factor Loadings

To estimate $(\beta_M, \beta_{SMB}, \beta_{HML})$ for a stock:

1. Collect historical time series of:
   - stock returns R_i,
   - risk-free rate r_f,
   - Fama–French factors: MKT, SMB, HML.
2. Compute the excess return:
$$R_i^e = R_i - r_f.$$
3. Run the time-series regression:
$$R_i^e = \alpha_i + \beta_{M,i} MKT + \beta_{SMB,i} SMB + \beta_{HML,i} HML + \varepsilon_i.$$
4. The fitted coefficients are the estimated factor loadings.

3. Expected Return Under the FFTFM

Given:

$$r_f = 0.03, \qquad E[R_M] = 0.09, \qquad E[SMB] = 0.02, \qquad E[HML] = 0.01,$$
$$\beta_M = 1.1, \qquad \beta_{SMB} = 0.5, \qquad \beta_{HML} = -0.3.$$

The Fama–French expected return is:

$$E[R_i] = r_f + \beta_M(E[R_M] - r_f) + \beta_{SMB}\,E[SMB] + \beta_{HML}\,E[HML]$$
$$= 0.03 + 1.1(0.09 - 0.03) + 0.5(0.02) - 0.3(0.01).$$

Computuing each term:

$$\begin{aligned} \text{Market term:} \quad & 1.1 \times 0.06 = 0.066, \\ \text{SMB term:} \quad & 0.5 \times 0.02 = 0.010, \\ \text{HML term:} \quad & -0.3 \times 0.01 = -0.003. \end{aligned}$$

Then plugging in:

$$E[R_i] = 0.03 + 0.066 + 0.010 - 0.003 = 0.103.$$

$$\boxed{E[R_i] = 10.3\%}$$

Final Answer

The expected return predicted by the Fama–French Three-Factor Model is 10.3%.