Multi-period Portfolio optimization using Dynamic Programming

M.Tech Term Project-1

in

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by

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Subhadeep Paul

Abstract

This report explores the application of dynamic programming in multi-period portfolio optimization, addressing the dynamic nature of financial markets. Utilizing a recursive approach, the study formulates an optimization framework that adapts asset allocations across sequential time periods. The methodology accounts for evolving risk-return profiles, enabling investors to make informed decisions in a changing market environment. Results demonstrate the effectiveness of dynamic programming in enhancing long-term portfolio performance and resilience to market uncertainties, providing valuable insights for strategic investment planning.

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Introduction

Multi-period portfolio optimization is a strategic financial approach involving the sequential selection of portfolio weights over a defined time horizon. This methodology offers substantial benefits, enabling investors to achieve financial goals, enhance risk management, and tailor investment strategies to individual needs. By navigating the evolving market landscape, investors can make informed decisions, maximizing expected returns while minimizing risk. This proactive approach fosters adaptability, empowering individuals to navigate dynamic financial environments and optimize their portfolios over both finite and infinite time frames. Ultimately, multi-period portfolio optimization is a powerful tool for investors seeking sustained success and alignment with their unique financial objectives.

This report delves into the realm of multi-period portfolio optimization, employing a comprehensive approach that integrates Mean-Variance Optimization with Rebalancing, Dynamic Programming, Monte Carlo Simulation, and Multi-Period Risk Measures. The study recognizes the dynamic nature of financial markets and the evolving risk landscape, offering a nuanced analysis of portfolio strategies over extended time horizons.

Mean-Variance Optimization with Rebalancing serves as the foundation for constructing efficient portfolios, while Dynamic Programming enhances decision-making by accounting for sequential adjustments over time. Monte Carlo Simulation provides a probabilistic framework for assessing a spectrum of future market scenarios, enriching the analysis with a diverse range of potential outcomes. Additionally, Multi-Period Risk Measures are employed to gauge portfolio resilience and downside protection across the extended investment horizon. The synthesis of these methodologies enables a holistic evaluation of portfolio performance, considering both risk and return dynamics. Through empirical validation and scenario-based analyses, the report contributes valuable insights for investors navigating the complexities of multi-period investment planning, facilitating more informed decision-making in dynamic financial landscapes.

Research Objective

With the exception of the riskless asset, the returns of assets are subject to randomness. While the precise distributions of these returns remain unknown, their means, variances, and interrelationships are assumed to be understood. These factors, encompassing means, variances, and covariances, undergo random fluctuations periodically, collectively forming a stochastic market. This configuration represents the states of a Markov chain, introducing serial correlation among returns across different periods. The dynamic nature of the market state prompts corresponding changes in returns over time. In essence, our model articulates a scenario wherein asset returns are influenced and modulated by the inherent stochasticity of the market.

We let Y_n denote the state of the market at period n so that $Y = Y_n; n = 0, 1, 2, ...$ is a Markov chain with a discrete state space E and transition matrix Q.

Let $R(i)$ denote the random vector of asset returns in any period given that the stochastic market is in state i. The excess return of the kth asset in state i is $R^e_k(i) = R_k(i) - r_f(i)$, where r_f is the risk-free asset. It follows that:

$$R^e_k(i) = E[R^e_k(i)] = r_k(i) - r_f(i),$$

$$\sigma_{kj}(i) = Cov(R_k(i) - r_f(i), R_j(i) - r_f(i))$$

where $r_f(i)$ and $r(i)$ are column vectors.

Let X_n represent the investor's wealth at period n, with X_T denoting the final wealth at the investment horizon's conclusion. The vector $u = (u_1, u_2, ..., u_m)$ specifies the allocations invested in risky assets $(1, 2, ..., m)$ during a specific period. Irrespective of the chosen investment policy, the stochastic progression of the investor's wealth adheres to the wealth dynamics equation.

$$X_{n+1}(u) = R(Y_n)'u + (X_n - 1'u)r_f(Y_n)$$

The model assumptions can be succinctly outlined as follows:

- Unlimited borrowing and lending are permissible at the current return of the riskless asset in each period,
- Short selling is permitted for all assets throughout the investment horizon,
- Capital additions or withdrawals are not permitted during the investment horizon, and
- Transaction costs and fees are considered negligible.

$$P_1(\sigma) : \max E_1[X_T]$$
$$\text{s.t.} \quad \mathrm{Var}(X_T) < \sigma,$$
$$X_{n+1}(u) = r_f(Y_n)X_n + R_e(Y_n)'u,$$

The objective function and constraint for a time series portfolio optimization problem. The objective function maximizes the expected return of the portfolio, subject to a constraint on the risk of the portfolio. The risk of the portfolio is measured by its variance. The variables in the equation are:-

- $X_T$is the value of the portfolio at time T
- $E[X_T]$ is the expected return of the portfolio at time T
- $Var[X_T]$ is the variance of the portfolio at time T
- σ is the maximum allowable risk of the portfolio

The constraint ensures that the portfolio is not too risky, while the objective function maximizes the expected return of the portfolio.
The constraint on the risk of the portfolio is important because investors do not want to lose too much money. However, investors also want to maximize their returns. The time series portfolio optimization problem solves this trade-off by finding the portfolio with the highest expected return that still meets the investor's risk constraint.

Previous Work

3.1 Multiperiod portfolio optimization models in stochastic markets(By U. C Elikyurt, S. O Zekici, 2007)

This study delves into multiperiod portfolio optimization incorporating both riskless and risky assets. It introduces a dynamic approach where returns are contingent on economic conditions, modeled through a Markov chain. Various optimization models, such as safety-first, coefficient of variation, and quadratic utility functions, are examined. The study employs dynamic programming to identify optimal portfolio management policies over the defined time horizon. Illustrative cases exemplify the application of these models, providing insights into the solution process and revealing optimal policies. This research contributes to a nuanced understanding of strategic portfolio management in the context of evolving economic conditions and diverse utility functions.

3.2 Portfolio optimization with multiple risky assets (By Xiaoling Mei, Victor DeMiguel, Francisco Nogales, 2016)

This research delves into the complexities of multiperiod portfolio optimization by incorporating multiple risky assets and transaction costs. Specifically addressing proportional costs, the study introduces a no-trade region, providing a framework for efficient computation. Additionally, for market impact costs, the research proposes a state-dependent rebalancing region boundary, offering a strategic approach to trading. Empirical findings underscore the crucial importance of accounting for transaction costs in portfolio optimization, emphasizing the potential for significant losses when overlooked. By delineating no-trade regions and incorporating state-dependent boundaries, the study contributes practical methods to navigate the impact of transaction costs on portfolio performance. This nuanced approach enhances the understanding of multiperiod optimization, guiding investors toward more realistic and effective strategies in the presence of multiple risky assets and transaction complexities.

3.3 Convolutional reinforcement learning portfolio management(By Farzan Soleymani, Eric Paquet, 2021)

"DeepPocket" emerges as an innovative deep graph convolutional reinforcement learning framework designed for financial portfolio management. This cutting-edge approach capitalizes on the dynamic interrelationships among financial instruments, portraying them as a graph. By leveraging deep graph convolutional networks and reinforcement learning, DeepPocket surpasses traditional market indexes when tested on real-life datasets across diverse investment periods, demonstrating robust performance even during challenging periods like the Covid-19 crisis. The model's ability to capture time-varying complexities in financial markets enhances its adaptability and predictive power. DeepPocket's outperformance not only highlights its efficacy in optimizing portfolio management strategies but also underscores the potential for advanced machine learning techniques to navigate volatile market conditions, providing valuable insights for investors seeking resilient and adaptive solutions in the ever-evolving landscape of financial

Methods

Below are the methods I have used in my report for the construction of the portfolio.

4.1 Markowitz portfolio optimization: - Problems

Markowitz portfolio optimization, while foundational, encounters several limitations. It lacks consideration for time dynamics, as it is designed for single-period decisions and doesn't adapt well to the dynamic nature of multi-period financial markets. Additionally, the model ignores the crucial aspect of dynamic portfolio rebalancing, offering no guidance on handling periodic adjustments and associated transaction costs. Another drawback lies in its assumption of stationarity, assuming constant expected returns and covariances over time. This assumption doesn't align with the evolving nature of real-world asset returns and risks. These limitations highlight the need for more sophisticated approaches, such as dynamic programming or Monte Carlo simulation, in multi-period portfolio optimization.

4.2 Mean Variance Optimisation with periodic rebalancing

Mean-variance optimization (MVO) is a prevalent method in portfolio optimization, seeking to maximize expected return for a specified risk level, measured by portfolio return variance. In the context of multi-period portfolio optimization, the focus extends beyond a single period. The goal is to identify optimal asset allocations over multiple periods, acknowledging the dynamic nature of the investment horizon. This approach addresses the evolving risk-return landscape, allowing investors to make informed decisions across changing market conditions and timeframes, enhancing the adaptability and effectiveness of portfolio strategies.

4.3 Dynamic Programming

Dynamic Programming is a powerful mathematical technique employed to tackle intricate optimization problems by breaking them into more manageable subproblems, optimizing each part individually, and combining the solutions to derive an overall optimal solution. In the context of portfolio optimization, Dynamic Programming proves invaluable for decision-making across multiple periods.

In this study, the method is applied to a simulated economy featuring one risk-free asset and two risky assets. The simulation aims to validate the dynamic programming approach by running the program on a representative dataset. Through this analysis, the effectiveness of dynamic programming in optimizing asset allocation over time and adapting to evolving market dynamics can be assessed, offering insights into its practical applicability in addressing the complexities of multi-period portfolio optimization.

4.4 Monte Carlo simulation

Monte Carlo simulation is employed for comprehensive portfolio optimization and decision support in a dynamic, multi-period investment framework.

- **Modeling Returns:** Implement Monte Carlo simulation to effectively model future return distributions for the portfolio assets, accommodating the inherent uncertainties in financial markets.

- **Scenario Generation:** Utilize the simulation to generate diverse random market scenarios, enabling the simulation of various paths for asset prices over multiple periods, capturing a wide range of potential market dynamics.

- **Portfolio Construction:** Optimize portfolios for each generated scenario, incorporating dynamic adjustments over time to enhance adaptability to evolving market conditions.

- **Performance Evaluation:** Evaluate portfolio performance across the generated simulations, leveraging risk metrics to provide a comprehensive assessment of potential outcomes and associated risks.

- **Decision Support:** Offer valuable insights derived from simulation results to support informed investment decisions, enabling stakeholders to navigate the complexities of a dynamic, multi-period investment landscape effectively.

4.5 Stochastic optimization

Stochastic optimization serves as a powerful tool for navigating market uncertainties and optimizing portfolios in dynamic financial landscapes.

- **Modeling Asset Returns:** Embrace stochastic optimization to portray asset returns as random variables, effectively capturing the inherent uncertainty prevalent in financial markets.
- **Scenario-Based Modeling:** Employ scenario-based modeling to generate a spectrum of future scenarios, reflecting diverse potential market outcomes and enhancing the model's adaptability to dynamic conditions.
- **Risk Measures Integration:** Integrate risk measures such as Value at Risk (VaR) and Conditional Value at Risk (CVaR) to quantitatively assess downside risk, providing a comprehensive understanding of potential portfolio vulnerabilities.
- **Dynamic Decision-Making:** Facilitate dynamic decision-making by adjusting portfolios over time, responding to evolving risks and market conditions to optimize performance.
- **Consideration of Transaction Costs and Constraints:** Account for transaction costs and practical constraints, ensuring the feasibility of implementing optimized portfolios in real-world scenarios.
- **Monte Carlo Simulation for Performance Assessment:** Utilize Monte Carlo simulation to evaluate portfolio performance under diverse stochastic scenarios, offering valuable insights into the robustness and adaptability of the optimization strategy.

4.6 Adam's algorithm

Adam's algorithm is a gradient-based optimization technique that synergizes the strengths of RMSprop (Root Mean Square Propagation) and momentum. Tailored for training deep neural networks, such as convolutional neural networks (CNNs) and recurrent neural networks (RNNs), Adam efficiently balances adaptive learning rates and momentum. This amalgamation enhances convergence speed and model performance, making it a favored optimization algorithm in the realm of deep learning, where intricate architectures and large datasets are prevalent.

Simulation Results

5.1 Monte Carlo Simulation results

The Monte Carlo Simulation results are presented herein, focusing on a market scenario with two assets. Figure 5.1 illustrates the dynamic changes in returns as the portfolio composition of the two assets is altered. Delving deeper, Figures 5.2 and 5.3 provide insights into the nuanced relationship between returns and the contributions of risky and riskless assets. These visualizations offer a comprehensive understanding of the portfolio's performance under varying conditions, shedding light on the impact of asset allocation on returns. The Monte Carlo Simulation serves as a robust analytical tool, allowing for the exploration of a range of market scenarios and providing investors with valuable insights to inform decision-making. The figures presented in this report contribute to a nuanced comprehension of the interplay between asset composition and returns in a dynamic financial landscape, aiding in the formulation of informed and adaptive investment strategies.

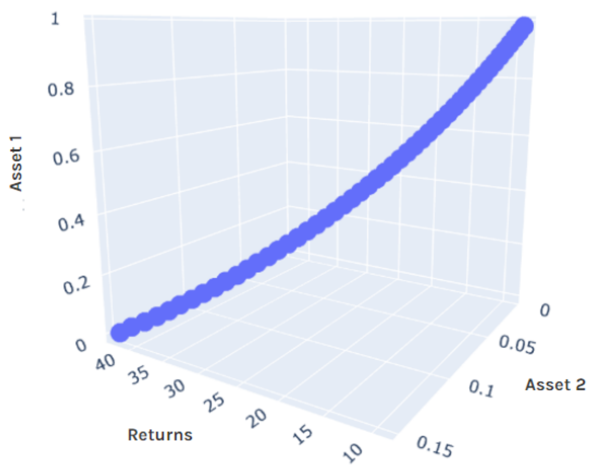

Figure 5.1: The dynamic changes in returns as the portfolio composition of the two assets is altered

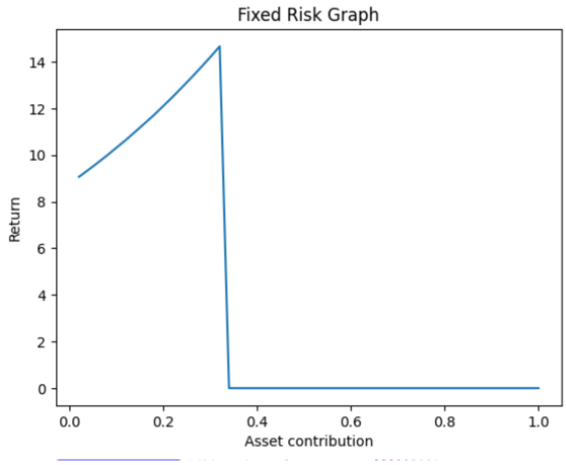

Figure 5.2: The nuanced relationship between returns and the contributions of risky asset

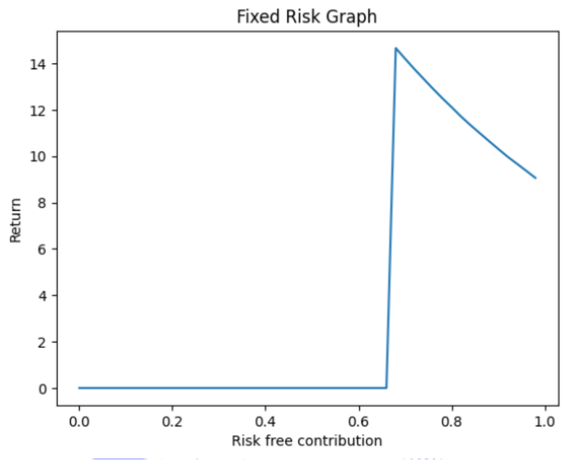

Figure 5.3: The nuanced relationship between returns and the contributions of riskless asset

5.2 Dynamic Programming Results

In the context of dynamic programming, Figure 5.4 depicts the fluctuation in risk for a 2-asset portfolio relative to returns. This visualization provides a comprehensive perspective on the risk-return trade-offs inherent in the portfolio optimization process. Furthermore, Figure 5.5 delves into the impact of the risky asset's contribution on returns, specifically for fixed return levels. These figures offer valuable insights into the intricate relationship between risk and returns within a dynamic programming framework. By analyzing the trade-offs and sensitivities, investors gain a deeper understanding of how strategic adjustments in asset composition can influence both risk exposure and expected returns. These visualizations contribute to the overall narrative of multi-period portfolio optimization, guiding investors in the formulation of robust and adaptive investment strategies aligned with their risk preferences and return objectives.

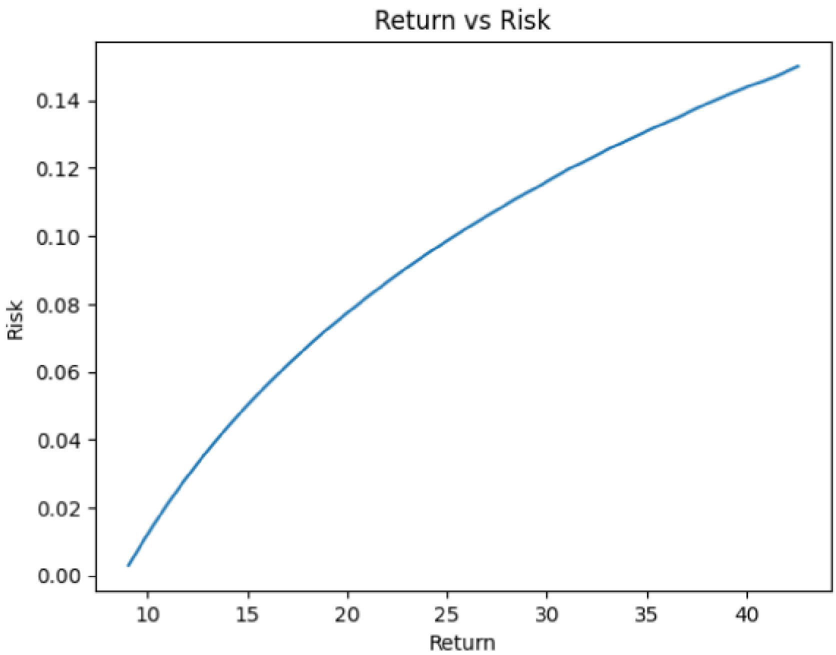

Figure 5.4: Fluctuation in risk for a 2-asset portfolio relative to returns

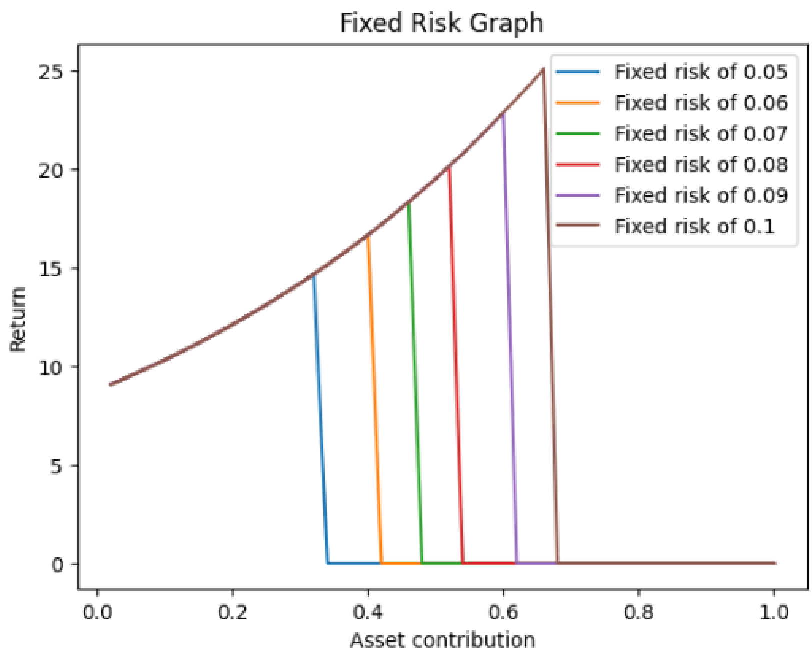

Figure 5.5: Impact of the risky asset's contribution on returns for fixed return levels

Managerial Implications

- **Wealth Accumulation:** Multi-period portfolio optimization facilitates wealth accumulation by dynamically adjusting asset allocations over time, maximizing returns and compounding wealth through strategic investment decisions.
- **Risk Management:** It enhances risk management by continuously assessing and adapting portfolio allocations to changing market conditions, aiming to minimize downside risk and preserve capital over the extended investment horizon.
- **Diversification:** Multi-period optimization emphasizes diversification, spreading investments across various assets and adjusting allocations based on evolving market dynamics, reducing the impact of individual asset volatility on the overall portfolio.
- **Adaptation to Market Conditions:** This approach ensures adaptation to market conditions by utilizing historical data and forecasting techniques, enabling the portfolio to respond proactively to shifts in economic environments, thus enhancing resilience.
- **Incorporating Constraints:** Multi-period optimization considers constraints such as liquidity, transaction costs, and regulatory requirements, ensuring that the portfolio adheres to specified limitations while striving to achieve optimal returns over time.
- **Monitoring and Rebalancing:** Regular monitoring and rebalancing are integral, as multi-period optimization continuously evaluates the portfolio's performance, making adjustments to maintain alignment with financial goals, risk tolerance, and changing market conditions.

Conclusion

In conclusion, the integration of Monte Carlo simulations and dynamic programming in multi-period portfolio optimization has provided a robust foundation for strategic decision-making. The simulations have allowed for a comprehensive exploration of potential future scenarios, while dynamic programming has enabled the identification of optimal policies over the specified time horizon. As we move forward, the forthcoming implementation of stochastic optimization and the application of Adam's algorithm represent exciting prospects for further refining our approach. These advanced techniques promise to enhance the model's adaptability to stochastic market conditions, fostering a more nuanced understanding of risk and return dynamics. This ongoing evolution in methodology underscores the commitment to continuous improvement and innovation in multi-period portfolio optimization, ensuring a resilient and adaptive framework for investors in an ever-changing financial landscape.

Bibliography

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[3] Convolutional reinforcement learning portfolio management(By Farzan Soleymani, Eric Paquet, 2021)