資工系1B 01357101 黃翊宏

1、

$\{1,3,5,7,9,11,13,15\}$

$\{1,15\}$、$\{3,13\}$、$\{5,11\}$、$\{7,9\}$

N個鴿子，4個籠子。(廣義)

$\lceil \frac{x}{4} \rceil = 2$, $1 \times 4 + 1 = 5$ #

2、

(a) $\lceil \frac{x}{2} \rceil = 2$, $1 \times 2 + 1 = 3$ #

(b) in worst case, a dozen brown & 2 black , $12+2=14$ #

3、

(a) $C^{10}_1 + C^{10}_3 + C^{10}_5 + C^{10}_7 + C^{10}_9$

$= 10 + 120 + 252 + 120 + 10 = 512$ #

(b) $2^{100} - C^{100}_1 - C^{100}_2 - 1$

$= 2^{100} - 100 - 4950 - 1$

$= 2^{100} - 5051$

4、

(a)

100~999，7的倍數

min：105

max：994

$a_1 = 105$

$a_n = 105 + (n-1)7$

$= 98 + 7n$

$994 = 98 + 7n$, $n = 128$ #

(b)

$\frac{(999-100)+1}{2} = 450$ #

(c)

111、222……999

$1 \times 9 = 9$ #

(d)

100~999，4的倍數

min：100

max：996

$996 = 100 + (n-1)4 = 96 + 4n$, $n = 225$

$900 - 225 = 675$ #

(e)

100~999，3的倍數

min：102 , max：999

$999 = 102 + (n-1)3 = 99 + 3n$, $n = 300$ #

100~999，12的倍數

min：108 , max = 996

$996 = 108 + (n-1)12 = 96 + 12n$, $n = 75$

$675 - 300 + 75 = 450$ #

4、

(f) $900-450=\underline{450}$ #

(g) $300-75=\underline{225}$ #

(h) $\underline{75}$ #

5、

(a) $C^{19}_{10}\times 2^{10}\times(-1)^9=92378\times1024\times-1=\underline{-94595072}$ #

(b) $C^{17}_{8}\times 3^8\times 2^9=24310\times6561\times512=\underline{81662929920}$ #

6、

$1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1$

$C^{10}_0 \quad C^{10}_1 \quad C^{10}_2 \quad C^{10}_3 \quad C^{10}_4 \quad C^{10}_5 \quad C^{10}_6 \quad C^{10}_7 \quad C^{10}_8 \quad C^{10}_9 \quad C^{10}_{10}$

$\underline{1} \quad \underline{11} \quad \underline{55} \quad \underline{165} \quad \underline{330} \quad \underline{462} \quad \underline{462} \quad \underline{330} \quad \underline{165} \quad \underline{55} \quad \underline{11} \quad \underline{1}$

$C^{11}_0 \quad C^{11}_1 \quad C^{11}_2 \quad C^{11}_3 \quad C^{11}_4 \quad C^{11}_5 \quad C^{11}_6 \quad C^{11}_7 \quad C^{11}_8 \quad C^{11}_9 \quad C^{11}_{10} \quad C^{11}_{11}$

7、

(a)

let a_n represent the ways to n climb stairs,

$a_1=1 \rightarrow$ 1 way.

$a_2=2 \rightarrow$ 2 ways, take 1 stair 2 time or take 2 stair at a time.

$a_3=3=1+2=a_1+a_2 \rightarrow$ divide problem to climb 1 floor & 2 floor
Add way numbers.

$a_4=5=2+3=a_2+a_3$

$\vdots$

$a_n=a_{n-1}+a_{n-2}\ ,\quad n\geq 3$

(b)

$a_1 = 1$ 、 $a_2 = 2$

(c)

$a_1 = 1$ $\quad a_5 = 8$

$a_2 = 2$ $\quad a_6 = 13$

$a_3 = 3$ $\quad a_7 = 21$

$a_4 = 5$ $\quad a_8 = 34$ ✱

8、

(a) $a_n = 2a_{n-1}$, $n \geq 1$, $a_0 = 3$

let $a_n = r^n$ $\qquad a_1 = 2 \times 3 = 6$

$r^n = 2r^{n-1}$, $r = 2$ $\qquad 3 = \alpha_1 \times 2^0 + 0$, $\alpha_1 = 3$

$a_n = \alpha_1 2^n + \alpha_2 n 2^n$ $\qquad 6 = 3 \times 2 + 2\alpha_2$, $\alpha_2 = 0$

$a_n = \underline{3 \times 2^n}$ ✱

(b) $a_n = 5a_{n-1} - 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 0$

let $r^n = a_n$

$r^n = 5r^{n-1} - 6r^{n-2}$

$r^2 = 5r - 6$

$r^2 - 5r + 6 = (r-3)(r-2)$, $r = 3$ or 2

$a_n = \alpha_1 3^n + \alpha_2 2^n$

$1 = \alpha_1 + \alpha_2$

$0 = 3\alpha_1 + 2\alpha_2$

$$\begin{array}{r} \begin{cases} 3\alpha_1 + 3\alpha_2 = 3 \\ 3\alpha_1 + 2\alpha_2 = 0 \end{cases} \\ - \qquad\qquad \\ \hline \alpha_2 = 3 \end{array}$$

$1 = \alpha_1 + 3$, $\alpha_1 = -2$

$\Rightarrow \underline{a_n = -2 \times 3^n + 3 \times 2^n}$ ✱

8、

(c) $a_n = \frac{1}{4}a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 0$

let $r^n = a_n$

$r^n = \frac{1}{4}r^{n-2}$, $r^2 = \frac{1}{4}$, $r = \pm\frac{1}{2}$

$a_n = \alpha_1(\frac{1}{2})^n + \alpha_2(-\frac{1}{2})^n$

$1 = \alpha_1 + \alpha_2$

$0 = \frac{1}{2}\alpha_1 - \frac{1}{2}\alpha_2$

$\Rightarrow 0 = \alpha_1 - \alpha_2$

$$+\begin{cases} \alpha_1 + \alpha_2 = 1 \\ \alpha_1 - \alpha_2 = 0 \end{cases}$$

$2\alpha_1 = 1$, $\alpha_1 = \frac{1}{2}$

$1 = \frac{1}{2} + \alpha_2$, $\alpha_2 = \frac{1}{2}$

$a_n = \frac{1}{2} \times (\frac{1}{2})^n + \frac{1}{2}(-\frac{1}{2})^n$

$= (\frac{1}{2})^{n+1} - (-\frac{1}{2})^{n+1}$ #

(d) $a_n = 6a_{n-1} - 8a_{n-2}$, $n \geq 2$, $a_0 = 4$, $a_1 = 10$

let $r^n = a_n$

$r^n = 6r^{n-1} - 8r^{n-2}$

$r^2 = 6r - 8$, $r^2 - 6r + 8 = (r-4)(r-2) = 0$, $r = 4$ or 2

$a_n = \alpha_1 4^n + \alpha_2 2^n$

$4 = \alpha_1 + \alpha_2$

$10 = 4\alpha_1 + 2\alpha_2$

$$-\begin{cases} 4\alpha_1 + 2\alpha_2 = 10 \\ 2\alpha_1 + 2\alpha_2 = 8 \end{cases}$$

$2\alpha_1 = 2$, $\alpha_1 = 1$

$4 = 1 + \alpha_2$, $\alpha_2 = 3$

$\Rightarrow a_n = 4^n + 3 \times 2^n$ #

8、

(e) $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$, $a_0 = 5$, $a_1 = -9$, $a_2 = 15$

let $a_n = r^n$

$r^n = -3r^{n-1} - 3r^{n-2} - r^{n-3}$

$r^3 = -3r^2 - 3r - 1$, $r^3 + 3r^2 + 3r + 1 = (r+1)^3$, $r = -1$

$a_n = \alpha_1(-1)^n + \alpha_2 n(-1)^n + \alpha_3 n^2(-1)^n$

$5 = \alpha_1$, $-9 = -5 - \alpha_2 - \alpha_3$

$4 = \alpha_2 + \alpha_3$

$15 = 5 + 2\alpha_2 + 4\alpha_3$

$10 = 2\alpha_2 + 4\alpha_3$

$$\begin{cases} 2\alpha_2 + 4\alpha_3 = 10 \\ -\ 2\alpha_2 + 2\alpha_3 = 8 \end{cases}$$

$2\alpha_3 = 2$, $\alpha_3 = 1$

$10 = 2\alpha_2 + 4$

$6 = 2\alpha_2$, $\alpha_2 = 3$

$\Rightarrow a_n = \underline{5(-1)^n + 3n(-1)^n + n^2(-1)^n}$ \#

✓

9、

(a) $a_n = 2a_{n-1} + 2n^2$, find all solation.

let $a_n = r^n$, $r^n = 2r^{n-1}$, $r = 2 \Rightarrow a_n^{(h)} = \alpha 2^n$

Suppose $P_n = cn^2 + dn + e$

$cn^2 + dn + e = 2c(n-1)^2 + 2d(n-1) + 2e + 2n^2$

$= 2c(n^2 - 2n + 1) + 2dn - 2d + 2e + 2n^2$

$= \cancel{2cn^2} \cancel{-4cn} + 2c \cancel{+2dn} - 2d + 2e \cancel{+2n^2}$

$= (2c+2)n^2 + (-4c+2d)n + 2c - 2d + 2e$

$2c + 2 = c$, $c = -2$

$-4c + 2d = d$, $8 + 2d = d$, $d = -8$

$(2\times -2) - (2\times -8) + 2e = e$, $12 + 2e = e$, $e = -12$

$a_n^{(P)} = -2n^2 - 8n - 12$, $a_n = -2n^2 - 8n - 12 + \alpha 2^n$ ✓

9.

(b) $a_n = -2n^2 - 8n - 12 + \alpha 2^n$, $a_1 = 4$

$a_1 = -2 - 8 - 12 + 2\alpha$

$= -22 + 2\alpha$

$-22 + 2\alpha = 4$

$2\alpha = 26$, $\alpha = 13$

$\Rightarrow a_n = \underline{-2n^2 - 8n - 12 + 13 \times 2^n}$

10. $x_1 + x_2 + x_3 + x_4 + x_5 = 21$

where x_i, $i = 1, 2, 3, 4, 5$ is a nonnegative integer.

(a) $x_1 \geq 1$

let $y_1 = x_1 - 1$

$y_1 + x_2 + x_3 + x_4 + x_5 = 20$

$H^5_{20} = C^{24}_{20} = \dfrac{\cancel{24} \times 23 \times 22 \times \cancel{21}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= 21 \times 22 \times 23$

$= \underline{10626}$ #

(b) $x_i \geq 2$, $i = 1, 2, 3, 4, 5$

let $y_i = x_i - 2$

$y_1 + y_2 + y_3 + y_4 + y_5 = 11$

$H^5_{11} = C^{15}_{11} = \dfrac{15 \times \cancel{14} \times 13 \times \cancel{12}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= \underline{1365}$ #

(c) $0 \leq x_1 \leq 10$

全: $H^5_{21} = C^{25}_{21}$

$x_1 \geq 10$:

let $y_1 = x_1 - 10$

$y_1 + x_2 + x_3 + x_4 + x_5 = 10$

$H^5_{10} = C^{14}_{10}$

$C^{25}_{21} - C^{14}_{10} = \dfrac{25 \times \cancel{24} \times 23 \times \cancel{22}}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4}} - \dfrac{\cancel{14} \times 13 \times \cancel{12} \times 11}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= 50 \times 23 \times 11 - 77 \times 13$

$= 12650 - 1001 = \underline{11649}$ #

10. $x_1+x_2+x_3+x_4+x_5=21$

where x_i, $i=1,2,3,4,5$ is a nonnegative integer.

(d) $0 \le x_1 \le 3$, $1 \le x_2 < 4$, $x_3 \ge 15$

1.) $x_1 \ge 1$, $x_3 \ge 15$

let $y_1 = x_1 - 1$, $y_3 = x_3 - 15$

$y_1 + x_2 + y_3 + x_4 + x_5 = 5$

$H^5_5 = C^9_5 = \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6}^{2}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 126$

2.) $x_1 \ge 4$, $x_2 \ge 1$, $x_3 \ge 15$

let $y_1 = x_1 - 4$, $y_2 = x_2 - 1$, $y_3 = x_3 - 15$

$y_1 + y_2 + y_3 + x_4 + x_5 = 1$

$H^5_1 = C^5_1 = 5$

3.) $x_2 \ge 4$, $x_3 \ge 15$

let $y_2 = x_2 - 4$, $y_3 = x_3 - 15$

$x_1 + y_2 + y_3 + x_4 + x_5 = 2$

$H^5_2 = C^6_2 = 15$,

4.) $x_1 \ge 4$, $x_2 \ge 4$, $x_3 \ge 15$

$4+4+15 > 21$, no solutions.

$(1)-(2)-(3) = 126-5-15 = 106$ #

11.

(a) $a_n = 2a_{n-1} + 3^n$, find all solution.

let $a_n = r^n$, $r^n = 2r^{n-1}$, $r=2 \Rightarrow a_n^{(h)} = \alpha 2^n$

Suppose $P_n = c3^n$

$c3^n = 2c3^{n-1} + 3^n$

$c = 2c3^{-1} + 1$

$c = \frac{2c+3}{3}$

$3c = 2c+3$

$c = 3$

$\Rightarrow a_n^{(P)} = 3 \times 3^n = 3^{n+1}$

$\Rightarrow a_n = \alpha 2^n + 3^{n+1}$ #

(b) $a_n = \alpha 2^n + 3^{n+1}$, $a_1 = 5$

$5 = 2\alpha + 9$

$-4 = 2\alpha$, $\alpha = -2$ #

$a_n = -2 \times 2^n + 3^{n+1}$

$= -2^{n+1} + 3^{n+1}$ #