Homework 1.

- The file name of your homework (in PDF) should be in the format: "學號-作業編號.pdf". For example: 00967999-hw1.pdf
- Please submit your homework to Tronclass **before 23:59, October 18 (Friday), 2024.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (5%) Which of these sentences are propositions? What are the truth values of those that are propositions?
   (a) The moon is made of green cheese.
   (Sol.) Yes, F
   (b) $2^n \geq 100$.
   (Sol.) No
   (c) 2+3=5.
   (Sol.) Yes, T
   (d) x+2=11.
   (Sol.) No
   (e) Answer this question.
   (Sol.) No
2. (5%) Determine whether each of these conditional statements is true or false.
   (a) If 1+1=2, then dogs can fly.
   (Sol.) F
   (b) If 1+1=2, then 2+2=5.
   (Sol.) F
   (c) If 1+1=3, then 2+2=5.
   (Sol.) T
   (d) If 2+2=4, then 1+2=3.
   (Sol.) T
   (e) If 1+1=3, then 2+2=4.
   (Sol.) T
3. (10%) (1) Let p and q be the propositions
   p: You drive over 65 miles per hour.
   q: You get a speeding ticket.
   Write these propositions using p and q and logical connectives (including negations).
   (a) You will get a speeding ticket if you drive over 65 miles per hour.
   (Sol.) $p \rightarrow q$

(b) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

(Sol.) $p \rightarrow q$

(c) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

(Sol.) $q \rightarrow p$

(d) You do not drive over 65 miles per hour.

(Sol.) $\neg p$

(e) You drive over 65 miles per hour, but you do not get a speeding tick.

(Sol.) $p \wedge \neg q$

(2) Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

(a) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(Sol.) $p \wedge q \wedge r$

(b) To get an A in this class, it is necessary for you to get an A on the final.

(Sol.) $r \rightarrow p$

(c) You get an A in this class, but you do not do every exercise in this book.

(Sol.) $r \wedge \neg q$

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(Sol.) $p \wedge \neg q \wedge r$

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(Sol.) $(p \wedge q) \rightarrow r$

4. (10%) Show that each of these conditional statements is a tautology by **<u>using truth tables</u>**.

(a) $(p \wedge q) \rightarrow p$

(Sol.)

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T

T	F	F	T
F	T	F	T
F	F	F	T

(b) $\neg p \to (p \to q)$

(Sol.)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(c) $(p \wedge q) \to (p \to q)$

(Sol.)

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(d) $[\neg p \wedge (p \vee q)] \to q$

(Sol.)

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

(e) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

(Sol.)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

5. (10%) Show that each conditional statement is a tautology by **applying a chain of logical identities**. (Do not use truth tables.)

(a) $(p \wedge q) \to p$

(Sol.) $(p \wedge q) \to p \equiv \neg(p \wedge q) \vee p \equiv \neg p \vee \neg q \vee p \equiv (p \vee \neg p) \vee \neg q \equiv \mathbf{T} \vee \neg q \equiv \mathbf{T}$

(b) $\neg p \to (p \to q)$

(Sol.) $\neg p \to (p \to q) \equiv p \vee (p \to q) \equiv p \vee (\neg p \vee q) \equiv (p \vee \neg p) \vee q \equiv \mathbf{T} \vee q \equiv \mathbf{T}$

(c) $(p \wedge q) \to (p \to q)$

(Sol.) $(p \wedge q) \to (p \to q) \equiv \neg(p \wedge q) \vee (\neg p \vee q) \equiv \neg p \vee \neg q \vee \neg p \vee q \equiv (\neg p \vee \neg p) \vee (\neg q \vee q) \equiv \neg p \vee \mathbf{T} \equiv \mathbf{T}$

(d) $[\neg p \wedge (p \vee q)] \to q$

(Sol.) $[\neg p \wedge (p \vee q)] \to q \equiv \neg[\neg p \wedge (p \vee q)] \vee q \equiv p \vee \neg(p \vee q)] \vee q \equiv (p \vee q) \vee \neg(p \vee q) \equiv \mathbf{T}$

(e) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

(Sol.) $[(p \to q) \wedge (q \to r)] \to (p \to r) \equiv \neg[(p \to q) \wedge (q \to r)] \vee (p \to r) \equiv \neg(p \to q) \vee \neg(q \to r) \vee (p \to r) \equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r) \equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r) \equiv [\neg p \vee (p \wedge \neg q)] \vee [r \vee (q \wedge \neg r)] \equiv [(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee [(r \vee q) \wedge (r \vee \neg r)] \equiv [\mathbf{T} \wedge (\neg p \vee \neg q)] \vee [(r \vee q) \wedge \mathbf{T}] \equiv (\neg p \vee \neg q) \vee (r \vee q) \equiv (\neg p \vee r) \vee (\neg q \vee q) \equiv (\neg p \vee r) \vee \mathbf{T} \equiv \mathbf{T}$

6. (10%) (1) Let P(x) be the statement “$x = x^2$.” If the domain consists of the integers, what are these truth values?

(a) P(−1)

(Sol.) **F**

(b) $\exists x$ P(x)

(Sol.) **T**

(c) $\forall x$ P(x)

(Sol.) **F**

(2) Let C(x) be the statement “x has a cat,” let D(x) be the statement “x has a dog,” and let F(x) be the statement “x has a ferret.” Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

(a) Some student in your class has a cat and a ferret, but not a dog.

(Sol.) $\exists x\ (C(x) \wedge F(x) \wedge \neg D(x))$

(b) No student in your class has a cat, a dog, and a ferret.

(Sol.) ¬ ∃x (C(x) ∧ D(x) ∧ F(x))

7. (10%) (1) Use rules of inference to show that if ∀x (P(x) ∨ Q(x)), ∀x (¬Q(x) ∨ S(x)), ∀x (R(x)→¬S(x)), and ∃x ¬P(x) are true, then ∃x ¬R(x) is true.

(Sol.)

Step	Reason
1. $\exists x \neg P(x)$	Premise
2. $\neg P(c)$	Existential instantiation from (1)
3. $\forall x(P(x) \vee Q(x))$	Premise
4. $P(c) \vee Q(c)$	Universal instantiation from (3)
5. $Q(c)$	Disjunctive syllogism from (4) and (2)
6. $\forall x(\neg Q(x) \vee S(x))$	Premise
7. $\neg Q(c) \vee S(c)$	Universal instantiation from (6)
8. $S(c)$	Disjunctive syllogism from (5) and (7)
9. $\forall x(R(x) \rightarrow \neg S(x))$	Premise
10. $R(c) \rightarrow \neg S(c)$	Universal instantiation from (9)
11. $\neg R(c)$	Modus tollens from (8) and (10)
12. $\exists x \neg R(x)$	Existential generalization from (11)

(2) Use resolution to show the hypotheses “Allen is a bad boy or Hillary is a good girl” and “Allen is a good boy or David is happy” imply the conclusion “Hillary is a good girl or David is happy.

(Sol.) Let p be “Allen is a good boy," let q be “Hillary is a good girl," and let r be “David is happy."

Step	Reason
1. $\neg p \vee q$	Premise
2. $p \vee r$	Premise
3. $q \vee r$	Resolution from (1) and (2)

8. (30%) Let $P(x)$ be “x is perfect”; let $F(x)$ be “x is your friend”; and let the domain be all people. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) No one is perfect.

(Sol.) $\forall x \ \neg P(x)$

(b) Not everyone is perfect.

(Sol.) $\neg \forall x \, P(x)$

(c) All your friends are perfect.

(Sol.) $\forall x(F(x) \rightarrow P(x))$

(d) At least one of your friends is perfect.

(Sol.) $\exists x(F(x) \wedge P(x))$

(e) Everyone is your friend and is perfect.

(Sol.) $\forall x(F(x) \wedge P(x))$ (or $(\forall x F(x)) \wedge (\forall x P(x))$)

(f) Not everybody is your friend or someone is not perfect.

(Sol.) $(\neg \forall x F(x)) \vee (\exists x \ \neg P(x))$

9. (10%) Show that if n is an integer and n^3+5 is odd, then n is even using
   (a) a proof by contraposition.
   (Sol.) Assume that n is odd, by definition, $n = 2k + 1$ for some integer k. Then $n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$. By definition, $n^3 + 5$ is two times some integer $(4k^3 + 6k^2 + 3k + 3)$, it is even.
   (b) a proof by contradiction.
   (Sol.) Suppose "$n^3 + 5$ is odd then n is even" is false. Which means "$n^3 + 5$ is odd and n is odd." Because n is odd and the product of two odd numbers is odd, it follows that n^2 is odd and then n^3 is odd, too. This means that $n^3 + 5$ is even. So, the supposition that "$n^3 + 5$ is odd and n is odd." is false. Therefore, "$n^3 + 5$ is odd then n is even" is true.