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No.
Date

1.

(a), true.

~~(b)~~

~~(c)~~

(d), false.

~~(e)~~

2.

(a) false

(b) true.

(c) true.

(d) false.

(e) true.

3.

(1) (a) $\neg p$.

(b) $p \land \neg q$.

(c) $p \to q$

(d) $p \to q$

(e) $p \to q$.

(2)

(a) $r \land \neg q$.

(b) $p \land q \land r$

(c) $p \to r$.

(d) $p \land \neg q \land r$

(e) $p \land q \to r$.

4.

(a)

p	q	$p \land q$	$p \land q \to p$.
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(b)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(c)

p	q	$p \land q$	$p \to q$	$(p \land q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(d)

p	q	$\neg p$	$p \lor q$	$\neg p \land (p \lor q)$	$[\neg p \land (p \lor q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	F	F	T

4-(e):

p	q	r	$p\rightarrow q$	$q\rightarrow r$	$(p\rightarrow q)\wedge(q\rightarrow r)$	$p\rightarrow r$	$[(p\rightarrow q)\wedge(q\rightarrow r)]\rightarrow(p\rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

5、

(a)

$(p \wedge q) \rightarrow p \equiv \neg(p \wedge q) \vee p$

$\equiv (\neg p \vee \neg q) \vee p$

$\equiv \neg p \vee \neg q \vee p$

$\equiv T \vee \neg q$

$\equiv T$

(b)

$\neg p \rightarrow (p \rightarrow q) \equiv \neg(\neg p) \vee (p \rightarrow q)$

$\equiv p \vee (p \rightarrow q)$

$\equiv p \vee (\neg p \vee q)$

$\equiv p \vee \neg p \vee q$

$\equiv T \vee q$

$\equiv T$

(c) $(p \wedge q) \rightarrow (p \rightarrow q)$

$\equiv \neg(p \wedge q) \vee (p \rightarrow q)$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\equiv \neg p \vee \neg q \vee \neg p \vee q$

$\equiv \neg p \vee \neg q \vee q$

$\equiv \neg p \vee T$

$\equiv T$

(d) $[\neg p \wedge (p \vee q)] \rightarrow q$

$\equiv \neg[\neg p \wedge (p \vee q)] \vee q$

$\equiv [p \vee \neg(p \vee q)] \vee q$

$\equiv p \vee (\neg p \wedge \neg q) \vee q$

$\equiv p \vee q \vee (\neg p \wedge \neg q)$

$\equiv (p \vee q \vee \neg p) \wedge (p \vee q \vee \neg q)$

$\equiv (T \vee q) \wedge (p \vee T)$

$\equiv T \wedge T \equiv T$

5-(e):

$[(p\to q)\land(q\to r)]\to(p\to r)$

$\equiv\neg[(p\to q)\land(q\to r)]\lor(p\to r)$

$\equiv\neg(p\to q)\lor\neg(q\to r)\lor(p\to r)$

$\equiv\neg(\neg p\lor q)\lor\neg(\neg q\lor r)\lor(\neg p\lor r)$

$\equiv(p\land\neg q)\lor(q\land\neg r)\lor(\neg p\lor r)$

$\equiv\neg p\lor(p\land\neg q)\lor r\lor(q\land\neg r)$

$\equiv(\neg p\lor p)\land(\neg p\lor\neg q)\lor(r\lor q)\land(r\lor\neg r)$

$\equiv T\land(\neg p\lor\neg q)\lor(r\lor q)\land T$

$\equiv(\neg p\lor\neg q)\lor(r\lor q)$

$\equiv\neg p\lor r\lor T\equiv T$

6.

(1) (a) false

(b) true

(c) false.

(2)

$C(x)=$ "x has a cat"
$D(x)=$ "x has a dog"
$F(x)=$ "x has a ferret"

(a)

$\exists x(C(x)\land F(x)\land\neg D(x))$

(b) $\neg\exists x(C(x)\land D(x)\land F(x))$

7. $\exists x \neg R(x)$.

(1)

1. $\forall x(P(x) \vee Q(x))$ premise
2. $\forall x(\neg Q(x) \vee S(x))$ premise.
3. $\forall x(R(x) \to \neg S(x))$ premise.
4. $\exists x \neg P(x)$ premise.
5. $\neg P(c)$ by EI from (4)
6. $P(c) \vee Q(c)$ by UI from (1)
7. $Q(c)$ by Disjunctive syllogism from (5),(6)
8. $\neg Q(c) \vee S(c)$ by UI from (2).
9. $S(c)$ by Disjunctive syllogism from (7)(8)
10. $R(c) \to \neg S(c)$ by UI from (3)
11. $\neg R(c) \vee \neg S(c)$ by Logical equivalence from (10)
12. $\neg(\neg S(c))$ by Double negation law from (9)
13. $\neg R(c)$ by Disjunctive syllogism (11),(12)
14. $\exists x \neg R(x)$ by EG from (13)

(2)

p = "Allen is a bad boy.
q = "Hillary is a good girl"
r = "David is happy"

Resolution:
$(p \vee q) \wedge (\neg p \vee r)$
$\equiv q \wedge R$

1. $p \vee q$ premise
2. $\neg p \vee r$ premise.
3. $q \vee r$ by Resolution from (1), (2)

8. $\qquad\qquad\qquad\qquad p \qquad\qquad\qquad\qquad\qquad q.$

n is a intege and n^3+5 is odd, then n is even.

(a) by contraposition:

⇒ $\neg q \rightarrow \neg p$.

⇒ Assume that n is odd: $\neg q$.

⇒ $n = 2k+1$

contraposition:

($\neg q \rightarrow \neg p$, then $p \rightarrow q$.).

$$(2k+1)^3+5 = (8k^3+12k^2+6k+1)+5$$
$$= 8k^3+12k^2+6k+6$$
$$= 2(4k^3+6k^2+3k+3)$$

⇒ $\exists\, L \in \mathbb{Z} = 4k^3+6k^2+3k+3$ s.t.

$n^3+5 = 2L$.

⇒ n^3+5 is even $\neg p$.

since the contraposition is true, the original statement is also true.

(b) by contradiction:

Assume that n^3+5 is odd and n is not even,

⇒ $\exists\, k \in \mathbb{Z}$ s.t.

$n = 2k+1$

$$n^3+5 = (2k+1)^3+5 = (8k^3+12k^2+6k+1)+5$$
$$= 2(4k^3+6k^2+3k+3).$$

$\exists\, L \in \mathbb{Z} = 4k^3+6k^2+3k+3$ s.t.

$n^3+5 = 2L$. ⇒ n^3+5 is even.

⇒ $\ast$

No.
Date : :

9.

$$A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B).$$ $p \rightarrow q \equiv \neg q \rightarrow \neg p.$

$$\equiv \forall x(\neg(x \in B) \rightarrow \neg(x \in A))$$

$$\equiv \forall x(x \in \overline{B} \rightarrow x \in \overline{A})$$

$$\equiv \overline{B} \subseteq \overline{A}$$

$$A \subseteq B \leftrightarrow \overline{B} \subseteq \overline{A}$$

10.

$$(A-B) \cap (B-C) \cap (A-C)$$

$$= (A \cap \overline{B}) \cap (B \cap \overline{C}) \cap (A \cap \overline{C})$$

$$= A \cap (\overline{B} \cap B) \cap \overline{C}) \cap (A \cap \overline{C})$$

$$= A \cap (\phi \cap \overline{C}) \cap (A \cap \overline{C})$$

$$= A \cap (\phi) \cap (A \cap \overline{C})$$

$$= A \cap A \cap \overline{C} \cap \phi = \phi$$

11.

(1)

(a) $f(n) = n^2+1$

Assume $f(a) = f(b)$

$$a^2+1 = b^2+1$$

$$a^2 = b^2$$

$$a^2 - b^2 = 0$$

$$(a+b)(a-b) = 0.$$, ex. $a=1, b=-1$.

(b) $f(n) = n^3$.

Assume $f(a) = f(b)$

$$a^3 = b^3$$

$$a^3 - b^3 = 0.$$

$$(a-b)(a^2+ab+b^2) = 0.$$

$\Rightarrow a-b=0$ or $a^2+ab+b^2=0. \Rightarrow a=b \Rightarrow$ one-to-one.

No.

Date

11-(2).

(a) $f(m,n) = m^2 - n^2 \Rightarrow$ No, ex: $f(m,n) \neq 2$.

(b) $f(m,n) = m + n \Rightarrow$ Yes.

(c) $f(m,n) = m^2 + n^2 \Rightarrow$ No; ex: $f(m,n) \neq 3$.

x^2 : 1, 4, 9, 16, 25 ...