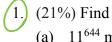
Homework 3.

- The file name of your homework (in PDF) should be in the format: "學號-作業編 號.pdf". For example: 00957999-hw3.pdf
- Please submit your homework to Tronclass before 23:59, December 8 (Sunday), 2024.

(可以用 word 檔寫完後轉成 pdf 檔上傳,或是手寫後拍照後存成 pdf 檔上傳)



- (a) $11^{644} \mod 645$
- (b) $3^{2003} \mod 99$
- (c) 123¹⁰⁰¹ mod 101
- $7^{121} \mod 13$.
- (e) $23^{1002} \mod 41$
- **(f)** gcd(1529, 14039)
- (g) gcd(1111, 0)

- (21%) Expansion conversion
- (a) Convert 97644 to a binary expansion.
- (b) Convert $(10\ 1011\ 0101)_2$ to a decimal expansion.
- (c) Convert (423)₈ to a binary expansion.
- (d) Convert (1010 1010 1010)₂ to an octal expansion.
- (e) Convert (135AB)₁₆ to an octal expansion.
- (f) Convert (BADFACED)₁₆ to an octal expansion.
- (g) Convert $(1011\ 0111\ 1011)_2$ to an octal expansion.

- (12%) Find the sum and the product of each of these pairs of numbers. Express your answers as the same base.
- to it (a) (100 0111)₂, (111 0111)₂
- (b) (112)₃, (210)₃
- (c) (763)₈, (147)₈
- (d) (1AE)₁₆, (BBC)₁₆

- (6%) Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find Hard the integer c with $0 \le c \le 12$ such that
 - (a) $c \equiv 11b \pmod{13}$.
 - (b) $c \equiv 2a + 3b \pmod{13}$.
 - (c) $c \equiv a^3 b^3 \pmod{13}$.

- 5. (8%) Find each of these values.
- (a) (177 mod 31 · 270 mod 31) mod 31
- (b) $(-133 \mod 23 + 261 \mod 23) \mod 23$
- (c) $(32^3 \mod 13)^2 \mod 11$ (d) $(99^2 \text{ mod } 32)^3 \text{ mod } 15$
- - 6. (10%) Express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.
 - (a) 117, 213
 - (b) 124, 323
 - - 7<mark>)</mark> (12%) Find <u>all</u> solutions (寫出通式):



- (a) $4x \equiv 5 \pmod{9}$
- (b) $34x \equiv 77 \pmod{89}$

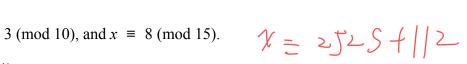
(c) $15x^2 + 19x \equiv 5 \pmod{11}$ (Hint: Show the congruence is equivalent to the congruence $15x^2 + 19x + 6$

 $\equiv 0 \pmod{11}$.



(d) Find all solutions, if any, to the system of congruences $x \equiv 5 \pmod{6}$, $x \equiv$





- (10%)
 - (a) Show that for every positive integer n,



$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$
.

Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.

Basis Step: $a^0 = 1$ is true by the definition of a^0 .

Inductive Step: Assume that $a^{j} = 1$ for all nonnegative integers j with $j \le k$. Then we can get

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$$



Homework 4.

- The file name of your homework (in PDF) should be in the format: "學號-作業編號.pdf". For example: 00957999-hw4.pdf
- Please submit your homework to Tronclass before 23:59, December 22 (Sunday),
 2024.

(可以用 word 檔寫完後轉成 pdf 檔上傳,或是手寫後拍照後存成 pdf 檔上傳)



(3%) How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$

2

to guarantee that at least one pair of these numbers add up to 16 (其中有兩個數加

起來大於等於 16)?



2. (4%) A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

3,

- (a) How many socks must he take out to be sure that he has at least two socks of the same color?
- (b) How many socks must be take out to be sure that he has at least two black socks?

(3,

(4%) (a) How many subsets with an odd number of elements does a set with 10 elements have?

(b) How many subsets with more than two elements does a set with 100 elements have?

4.)

- (16%) How many positive integers between 100 and 999 inclusive
 - (a) are divisible by 7?
 - (b) are odd?
 - (c) have the same three decimal digits?
 - (d) are not divisible by 4?
 - (e) are divisible by 3 or 4?
 - (f) are not divisible by either 3 or 4?
 - (g) are divisible by 3 but not by 4?
 - (h) are divisible by 3 and 4?
 - (4%) (a) What is the coefficient of x^9 in $(2-x)^{19}$?
- (b) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

6. (4%) The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \le k$ ≤ 10 , is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's

triangle. (直接利用 Pascal's identity 產生下一列的答案)

12%) (a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time.

- (b) What are the initial conditions?
- (c) In how many ways can this person climb a flight of eight stairs?
- 8.) (25%) Solve these recurrence relations together with the initial conditions given.

(a)
$$a_n = 2a_{n-1}$$
 for $n \ge 1$, $a_0 = 3$

(b)
$$a_n = 5a_{n-1} - 6a_{n-2}$$
 for $n \ge 2$, $a_0 = 1$, $a_1 = 0$ $a_1 \ge 2a_1 + 3$

(c)
$$a_n = a_{n-2} / 4$$
 for $n \ge 2$, $a_0 = 1$, $a_1 = 0$

(d)
$$a_n = 6a_{n-1} - 8a_{n-2}$$
 for $n \ge 2$, $a_0 = 4$, $a_1 = 10$

(e)
$$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$
 with $a_0 = 5$, $a_1 = -9$, and $a_2 = 15$ (10%) (a) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$.

- b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 4$.
- 0/8%) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 24$$

where x_i , i = 1, 2, 3, 4, 5, is a nonnegative integer such that

(a)
$$x_1 \ge 1$$
?

(b)
$$x_i \ge 2$$
 for $i = 1, 2, 3, 4, 5$?

(c)
$$0 \le x_1 \le 10$$
?

(d)
$$0 \le x_1 \le 3$$
, $1 \le x_2 < 4$, and $x_3 \ge 15$?

- (11) (10%) (a) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n$.
 - (b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 5$.

三个小子玄分六个不同的玩具, 共有幾種分法?