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1.

(a) False

(b) Not a propositions.

(c) True

(d) Not a propositions.

(e) Not a propositions.

2.

(a) "1+1=2" → "Dogs can fly"

$T \to F$, False#

(b) "1+1=2" → "2+2=5"

$T \to F$, False#

(c) "1+1=3" → "2+2=5"

$F \to F$, True#

(d) "2+2=4" → "1+2=3"

$T \to T$, True#

(e) "1+1=3" → "2+2=4"

$F \to T$, True#

3.

(1)

(a) $p \to q$

(b) $p \to q$

(c) $p \to q$ $q \to p$

(d) $\neg p$

(e) $p \to \neg q$

(2)

(a) $p \wedge q \wedge r$

(b) $p \to r$ $r \to p$

(c) $r \wedge \neg q$

(d) $p \wedge \neg q \wedge r$

(e) $p \wedge q \to r$

4.

(a) $(p \wedge q) \to p$

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(b) $\neg p \to (p \to q)$

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(c) $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(d) $[\neg p \wedge (p \vee q)] \to q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	~~F~~ T	~~F~~ T	T
F	F	T	F	F	T

(e) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p\to q)\wedge(q\to r)$	$p \to r$	$[(p\to q)\wedge(q\to r)]\to(p\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

5.

(a) $(p \wedge q) \to p \equiv \neg(p \wedge q) \vee p$ because $p \to q \equiv (\neg p) \vee q$

$\equiv (\neg p \vee \neg q) \vee p$ by the 1st De Morgan's law

$\equiv \neg p \vee p \vee \neg q$

$\equiv T \vee \neg q$ by Negation law

$\equiv T$ by Domination

(b) $\neg p \to (p \to q) \equiv p \vee (p \to q)$ because $p \to q \equiv (\neg p) \vee q$

$\equiv p \vee (\neg p) \vee q$ because $p \to q \equiv (\neg p) \vee q$

$\equiv T \vee q$ by Negation law

$\equiv T$ by Domination

(c) $(p\wedge q)\to(p\to q) \equiv (p\wedge q)\to((\neg p)\vee q)$ because $p\to q\equiv(\neg p)\vee q$

$\equiv \neg(p\wedge q)\vee(\neg p)\vee q$ because $p\to q\equiv(\neg p)\vee q$

$\equiv (\neg p)\vee(\neg q)\vee(\neg p)\vee q$ by 1st De Morgan's law

$\equiv (\neg p)\vee(\neg p)\vee(\neg q)\vee q$

$\equiv T\vee(\neg p)$ by Indemptent law & Negation law

$\equiv T$ ※

(d) $[\neg p\wedge(p\vee q)]\to q \equiv \neg(\neg p\wedge(p\vee q))\vee q$ because $p\to q\equiv(\neg p)\vee q$

$\equiv (p\vee\neg(p\vee q))\vee q$ by 1st De Morgan's law

$\equiv p\vee\neg p\wedge\neg q\vee q$ by 2nd De Morgan's law

$= T\wedge T$ by Negation law

$= T$ ※

(e) $((p\to q)\wedge(q\to r))\to(p\to r)$

$\equiv \neg((p\to q)\wedge(q\to r))\vee(p\to r)$ because $p\to q\equiv(\neg p)\vee q$

$\equiv \neg(p\to q)\vee\neg(q\to r)\vee(p\to r)$ by 1st De Morgan's law

$\equiv \neg((\neg p)\vee q)\vee\neg((\neg q)\vee r)\vee((\neg p)\vee r)$ because $p\to q\equiv(\neg p)\vee q$

$\equiv (p\wedge\neg q)\vee(q\wedge\neg r)\vee(\neg p\vee r)$ by 2nd De Morgan's law

$\equiv \neg p\vee(p\wedge\neg q)\vee r\vee(q\wedge\neg r)$

$\equiv T\wedge(\neg p\vee\neg q)\vee(r\vee q)\wedge T$ by Negation law

$\equiv (\neg p\vee\neg q)\vee(r\vee q)$ by Identity law

$\equiv \neg p\vee r\vee T \equiv T$ ※ by Negation law.

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(1)

(a) $P(-1)$

$x=-1$, $x^2=1$

$x \neq x^2$, false ※

(b) $\exists x\, P(x)$

let $x=1$, $x^2=1$

$x=x^2$, true ※

(c) $\forall x\, P(x)$

let $x_1=1$, $x_1^2=1$

$x_1=x_1^2$

let $x_2=2$, $x_2^2=4$

$x_2 \neq x_2^2$

false ※

(2)

$C(x)$ = "x has a cat."

$D(x)$ = "x has a dog."

$F(x)$ = "x has ferret."

(a) $\exists x\,(C(x) \wedge F(x) \wedge \neg D(x))$

(b) $\forall x\,(\neg C(x) \wedge \neg D(x) \wedge \neg F(x))$

$(\neg \exists x\,(C(x) \wedge D(x) \wedge F(x)))$

7、

(1)

Step	Reason
1. $\exists x \neg P(x)$	Premise
2. $\neg P(c)$	Existential instantiation from (1)
3. $\forall x(P(x) \vee Q(x))$	Premise
4. $P(c) \vee Q(c)$	Universal instantiation from (3)
5. $Q(c)$	Disjunctive syllogism from (4) and (2)
6. $\forall x(\neg Q(x) \vee S(x))$	Premise
7. $\neg Q(c) \vee S(c)$	Universal instantiation from (6)
8. $S(c)$	Disjunctive syllogism from (5) and (7)
9. $\forall x(R(x) \rightarrow \neg S(x))$	Premise
10. $R(c) \rightarrow \neg S(c)$	Universal instantiation from (9)
11. $\neg R(c)$	Modus tollens from (8) and (10)
12. $\exists x \neg R(x)$	Existential generalization from (11)

1、$\forall x(P(x) \vee Q(x))$	premise.
2、$\forall x(\neg Q(x) \vee S(x))$	premise.
3、$\forall x(R(x) \rightarrow \neg S(x))$	premise、
4、$\exists x \neg P(x)$	premise、
5、$\neg P(c)$	EI from 4.
6、$P(c) \vee Q(c)$	UI from 1.
7、$(\neg P(c) \wedge (P(c) \vee Q(c))) \rightarrow Q(c)$	Disjunctive Syllogism from 5.,6、
8、$\neg Q(c) \vee S(c)$	UI from 2、
9、$(Q(c) \wedge (\neg Q(c) \vee S(c))) \rightarrow S(c)$	Disjunctive Syllogism from 7.,8、
10、$R(c) \rightarrow \neg S(c)$	UI from 3、
11、$S(c) \wedge (R(c) \rightarrow \neg S(c)) \rightarrow \neg R(c)$	Modus Tollens from 9、10.
12、$\exists x \neg R(x)$	EG from 11、

(2)

<key: Use Resolution $((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$>

p: "Allen is a bad boy"

q: "Hillary is a good girl"

r: "David is happy"

1、$p \vee q$	premise
2、$\neg p \vee r$	premise
3. $((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$	Resolution from 1.,2、

8.

$P(x)$: "x is prefect"

$F(x)$: "x is your friend"

(a) $\forall x(\neg P(x))$

(b) $\exists x(\neg P(x))$ $\neg\forall x P(x)$

(c) $\exists x(F(x) \wedge \forall x(P(x)))$ $\forall x(F(x) \rightarrow P(x))$

(d) $\exists x(F(x) \wedge \exists x(P(x))$

(e) $\forall x(F(x) \wedge P(x))$

(f) $\neg\forall x(F(x)) \vee \neg\exists x(P(x))$

$(\exists x(\neg P(x)))$

9. p : "n is an integer and n^3+5 is odd."

q : "n is even"

(a)

Assume n is odd $(\neg q)$

$\Rightarrow n = 2k+1$

$(2k+1)^3+5 = (8k^3+12k^2+12k+1)+5$

$= 8k^3+12k^2+12k+6 = 2(4k^3+6k^2+6k+3) = 2j$

$n^3+5 = 2j \Rightarrow n^3+5$ is even. $(\neg p)$

Since the contraposition is true $(\neg q \rightarrow \neg p)$

The original statement is also true. $(p \rightarrow q)$

(b)

Assume that $\underbrace{n^3+5 \text{ is odd}}_{(p)}$, and n is not even $(\neg q)$

$\Rightarrow \exists k \in \mathbb{Z}$ s.t. $n = 2k+1$

$$n^3+5 = (2k+1)^2+5 = 8k^3+12k^2+12k+6$$
$$= 2(4k^3+6k+6k+3)$$
$$= 2j$$

$\exists j \in \mathbb{Z}$ s.t. $n^3+5 = 2j$

$\hookrightarrow$ n^3+5 is even (q), not odd.

Assume that n^3+5 is odd, but consequences shows $\exists j \in \mathbb{Z}$ s.t. $n^3+5 = 2j$,
This contradicts the assumption that n^3+5 is odd.

(a) a proof by contraposition.

(Sol.) Assume that *n* is odd, by definition, $n = 2k + 1$ for some integer k. Then $n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$. By definition, $n^3 + 5$ is two times some integer $(4k^3 + 6k^2 + 3k + 3)$, it is even.

(b) a proof by contradiction.

(Sol.) Suppose "$n^3 + 5$ is odd then n is even" is false. Which means "$n^3 + 5$ is odd and n is odd." Because n is odd and the product of two odd numbers is odd, it follows that n^2 is odd and then n^3 is odd, too. This means that $n^3 + 5$ is even. So, the supposition that "$n^3 + 5$ is odd and n is odd." is false. Therefore, "$n^3 + 5$ is odd then n is even" is true.