Homework 2.

- The file name of your homework (in PDF) should be in the format: "學號-作業編號.pdf". For example: 00957999-hw2.pdf
- Please submit your homework to Tronclass **before 23:59, October 28 (Saturday), 2023.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (5%) Determine whether *f* is a function from **Z** to **R** if
   (a) $f(n) = \pm n$
   (b) $f(n) = \sqrt{n^2 + 1}$
   (c) $f(n) = 1/(n^2 - 4)$

2. (10%) (a) If $f$ and $f \circ g$ are one-to-one, does it follow that $g$ is one-to-one?
   (b) If $f$ and $f \circ g$ are onto, does it follow that $g$ is onto?

3. (10%) Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.
   (a) $a_n = 6a_{n-1}, a_0 = 2$
   (b) $a_n = na_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1$
   (c) $a_n = a_{n-1} + a_{n-3}, a_0 = 1, a_1 = 2, a_2 = 0$
   (d) $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$
   (e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$

4. (15%) Find the solution to each of these recurrence relations and initial condition. (請寫出計算過程)
   (a) $a_n = 2a_{n-1} - 3, a_0 = -1$
   (b) $a_n = a_{n-1} + 2n + 3, a_0 = 4$
   (c) $a_n = 2na_{n-1}, a_0 = 1$

5. (10%) Show that the sequence$\{a_n\}$is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if
   (a) $a_n = -n + 2$
   (b) $a_n = 3(-1)^n + 2^n - n + 2$

6. (10%) (a) Show that the union of a countable number of countable sets is countable.
   (b) Show that the set $\boldsymbol{Z}^+ \times \boldsymbol{Z}^+$ is countable.

7. (10%) (a) Find $\boldsymbol{AB}$ if $\boldsymbol{A} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$.

   (b) Let $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find $\boldsymbol{A} \vee \boldsymbol{B}$ and $\boldsymbol{A} \wedge \boldsymbol{B}$.

8. (10%) Use the definition of "$f(x)$ is $O(g(x))$" to show that $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$.

9. (10%) Use the definition of "$f(x)$ is $O(g(x))$" to show that $(x^2 + 1)/(x + 1)$ is $O(x)$.

10. (10%) Use the definition of "$f(x)$ is $O(g(x))$" to show that $(x^3 + 2x)/(2x + 1)$ is $O(x^2)$.