Homework 3.

- The file name of your homework (in PDF) should be in the format: “學號-作業編號.pdf”. For example: 00957999-hw3.pdf
- Please submit your homework to Tronclass **before 23:59, December 8 (Sunday), 2024.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (21%) Find
   5
   (a) 11^{644} mod 645
   (b) 3^{2003} mod 99
   (c) 123^{1001} mod 101
   (d) 7^{121} mod 13.
   (e) 23^{1002} mod 41
   (f) gcd(1529, 14039)
   (g) gcd(1111, 0)
2. (21%) Expansion conversion
   3
   (a) Convert 97644 to a binary expansion.
   (b) Convert $(10\ 1011\ 0101)_2$ to a decimal expansion.
   (c) Convert $(423)_8$ to a binary expansion.
   (d) Convert $(1010\ 1010\ 1010)_2$ to an octal expansion.
   (e) Convert $(135AB)_{16}$ to an octal expansion.
   (f) Convert $(BADFACED)_{16}$ to an octal expansion.
   (g) Convert $(1011\ 0111\ 1011)_2$ to an octal expansion.
3. (12%) Find the sum and the product of each of these pairs of numbers. Express your answers as the same base.
   4
   (a) $(100\ 0111)_2$, $(111\ 0111)_2$ 加法
   (b) $(112)_3$, $(210)_3$
   (c) $(763)_8$, $(147)_8$
   (d) $(1AE)_{16}$, $(BBC)_{16}$
4. (6%) Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \le c \le 12$ such that Hard
   1
   (a) $c \equiv 11b \pmod{13}$.
   (b) $c \equiv 2a + 3b \pmod{13}$.
   (c) $c \equiv a^3 - b^3 \pmod{13}$.

5. (8%) Find each of these values.

2

(a) (177 mod 31 · 270 mod 31) mod 31

(b) (−133 mod 23 + 261 mod 23) mod 23

(c) $(32^3 \bmod 13)^2 \bmod 11$

(d) $(99^2 \bmod 32)^3 \bmod 15$

6. (10%) Express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.

6

gcd

(a) 117, 213

(b) 124, 323

7. (12%) Find **all** solutions (寫出通式):

7.

(a) $4x \equiv 5 \pmod 9$

(b) $34x \equiv 77 \pmod{89}$

(c) $15x^2 + 19x \equiv 5 \pmod{11}$

same

(Hint: Show the congruence is equivalent to the congruence $15x^2 + 19x + 6 \equiv 0 \pmod{11}$.)

(d) Find all solutions, if any, to the system of congruences $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, and $x \equiv 8 \pmod{15}$.

數列似化

x ≡ 2525 + 112

8. (10%)

8.

(a) Show that for every positive integer n,

$$1 \cdot 2 + 2 \cdot 3 + \cdots \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

(b) Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.

Basis Step: $a^0 = 1$ is true by the definition of a^0.

Inductive Step: Assume that $a^j = 1$ for all nonnegative integers j with $j \le k$. Then we can get

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$$

n×n! 類加 證明

Homework 4.

- The file name of your homework (in PDF) should be in the format: “學號-作業編號.pdf”. For example: 00957999-hw4.pdf
- Please submit your homework to Tronclass **before 23:59, December 22 (Sunday), 2024.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (3%) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15}
2、 to guarantee that at least one pair of these numbers add up to 16 (其中有兩個數加起來大於等於 16)?

2. (4%) A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
3、 (a) How many socks must he take out to be sure that he has at least two socks of the same color?
   (b) How many socks must he take out to be sure that he has at least two black socks?

3. (4%) (a) How many subsets with an odd number of elements does a set with 10 elements have?
4、 (b) How many subsets with more than two elements does a set with 100 elements have?

4. (16%) How many positive integers between 100 and 999 inclusive
1、 (a) are divisible by 7?
   (b) are odd?
   (c) have the same three decimal digits?
   (d) are not divisible by 4?
   (e) are divisible by 3 or 4?
   (f) are not divisible by either 3 or 4?
   (g) are divisible by 3 but not by 4?
   (h) are divisible by 3 and 4?

5. (4%) (a) What is the coefficient of x^9 in $(2 - x)^{19}$?
5、 (b) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

6. (4%) The row of Pascal’s triangle containing the binomial coefficients $\binom{10}{k}$, $0 \le k$
6、 ≤ 10, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle. (直接利用 Pascal's identity 產生下一列的答案)

7. (12%) (a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time.

8,

假設爬的格1、2、3

(b) What are the initial conditions?
(c) In how many ways can this person climb a flight of eight stairs?

8. (25%) Solve these recurrence relations together with the initial conditions given.

9,

(a) $a_n = 2a_{n-1}$ for $n \geq 1, a_0 = 3$
(b) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2, a_0 = 1, a_1 = 0$ $a_n = 2a_{n-1} + 3^n$
(c) $a_n = a_{n-2}$ / 4 for $n \geq 2, a_0 = 1, a_1 = 0$
(d) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 10$
(e) $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5, a_1 = -9$, and $a_2 = 15$

10, 9. (10%) (a) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$.
b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 4$.

10. (8%) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = \cancel{21}, 8$$

7,

where x_i, $i = 1, 2, 3, 4, 5$, is a nonnegative integer such that
(a) $x_1 \geq 1$?
(b) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?
(c) $0 \leq x_1 \leq 10$?
(d) $0 \leq x_1 \leq 3$, $1 \leq x_2 < 4$, and $x_3 \geq 15$?

11. (10%) (a) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n$.
11, (b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 5$.

三个小孩分六个不同的玩具，
共有幾種分法？