Homework 3.

- The file name of your homework (in PDF) should be in the format: “學號-作業編號.pdf”. For example: 00957999-hw3.pdf
- Please submit your homework to Tronclass **before 23:59, December 11 (Monday), 2023.**

(可以用 word 檔寫完後轉成 pdf 檔上傳，或是手寫後拍照後存成 pdf 檔上傳)

1. (6%) Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \le c \le 12$ such that
   (a) $c \equiv 11b \pmod{13}$.
   (b) $c \equiv 2a + 3b \pmod{13}$.
   (c) $c \equiv a^3 - b^3 \pmod{13}$.
2. (8%) Find each of these values.
   (a) $(177 \bmod 31 \cdot 270 \bmod 31) \bmod 31$
   (b) $(-133 \bmod 23 + 261 \bmod 23) \bmod 23$
   (c) $(32^3 \bmod 13)^2 \bmod 11$
   (d) $(99^2 \bmod 32)^3 \bmod 15$
3. (21%) Expansion conversion
   (a) Convert 97644 to a binary expansion.
   (b) Convert $(10\ 1011\ 0101)_2$ to a decimal expansion.
   (c) Convert $(423)_8$ to a binary expansion.
   (d) Convert $(1010\ 1010\ 1010)_2$ to an octal expansion.
   (e) Convert $(135AB)_{16}$ to an octal expansion.
   (f) Convert $(BADFACED)_{16}$ to an octal expansion.
   (g) Convert $(1011\ 0111\ 1011)_2$ to an octal expansion.
4. (12%) Find the sum and the product of each of these pairs of numbers. Express your answers as the same base.
   (a) $(100\ 0111)_2$, $(111\ 0111)_2$
   (b) $(112)_3$, $(210)_3$
   (c) $(763)_8$, $(147)_8$
   (d) $(1AE)_{16}$, $(BBC)_{16}$
5. (21%) Find
   (a) $11^{644} \bmod 645$

(b) $3^{2003} \bmod 99$
(c) $123^{1001} \bmod 101$
(d) $7^{121} \bmod 13$.
(e) $23^{1002} \bmod 41$
(f) gcd(1529, 14039)
(g) gcd(1111, 0)

6. (10%) Express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.
   (a) 117, 213
   (b) 124, 323
7. (12%) Find all solutions:
   (a) $4x \equiv 5 \pmod 9$
   (b) $34x \equiv 77 \pmod{89}$
   (c) $15x^2 + 19x \equiv 5 \pmod{11}$
   (Hint: Show the congruence is equivalent to the congruence $15x^2 + 19x + 6 \equiv 0 \pmod{11}$.)
   (d) Find all solutions, if any, to the system of congruences $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, and $x \equiv 8 \pmod{15}$.
8. (10%)
   (a) Show that for every positive integer n,
   $$1 \cdot 2 + 2 \cdot 3 + \cdots \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$
   (b) Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
   Basis Step: $a^0 = 1$ is true by the definition of a^0.
   Inductive Step: Assume that $a^j = 1$ for all nonnegative integers j with $j \le k$. Then note that
   $$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$$