Homework 3.

1.

(d) $7^{121} \bmod 13$

By Fermat's little theorem,

$(7^{12})^k \equiv 1 \pmod{13},\ k \in \mathbb{Z}^+$

$7^{121} = 7^{12 \times 10 + 1}$

$7^{121} \bmod 13$

$= (7^{12})^{10}\, 7 \bmod 13$

$= [((7^{12})^{10} \bmod 13) \times (7 \bmod 13)] \bmod 13$

$= 1 \times 7 \bmod 13 = 7$ ※

(f) $\gcd(1529, 14039)$

$$\begin{array}{r|r|r|l}
5 & 1529 & 14039 & 9 \\
 & 1390 & 13761 & \\
\hline
 & 139 & 278 & 2 \\
 & & 278 & \\
\hline
 & & 0 &
\end{array}$$

$\gcd(1529, 14039) = 139$ ※

(g) $\gcd(1111, 0) = 1111$ ※

2.

(a) $97644_{10} \rightarrow$ BIN

$$\begin{array}{r|l}
2 & 97644 - 0 \\
2 & 48822 - 0 \\
2 & 24411 - 1 \\
2 & 12205 - 1 \\
2 & 6102 - 0 \\
2 & 3051 - 1 \\
2 & 1525 - 1 \\
2 & 762 - 0 \\
2 & 381 - 1 \\
 & 190
\end{array}
\qquad
\begin{array}{r|l}
2 & 190 - 0 \\
2 & 95 - 1 \\
2 & 47 - 1 \\
2 & 23 - 1 \\
2 & 11 - 1 \\
2 & 5 - 1 \\
2 & 2 - 0 \\
 & 1
\end{array}$$

97644_{10}
$= 10111110101101100_2$

(b) $1010110101_2 \rightarrow$ DEC

$$\begin{array}{cccccccccc}
1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
2^9 & & 2^7 & & 2^5 & 2^4 & & 2^2 & & 2^0
\end{array}$$

$= 512 + 128 + 32 + 16 + 4 + 1 = 693_{10}$ ※

(c) $423_8 \rightarrow$ BIN

$$\begin{array}{ccc}
4 & 2 & 3 \\
8^2 & 8^1 & 8^0 \\
100 & 010 & 011 \\
421 & 421 & 421
\end{array} \Rightarrow 100010011_2$$

(d) $101010101010_2 \rightarrow$ OCT

$$\begin{array}{cccc}
101 & 010 & 101 & 010_2 \\
5 & 2 & 5 & 2
\end{array} \Rightarrow 5252_8$$

(e) $135AB_{16} \rightarrow$ OCT

A	10
B	11
C	12
D	13
E	14
F	15

$$\begin{array}{ccccc}
1 & 3 & 5 & A & B \\
0001 & 0011 & 0101 & 1010 & 1011
\end{array}$$

$$\begin{array}{cccccc}
000 & 010 & 011 & 010 & 110 & 101 \; 011 \\
0 & 2 & 3 & 2 & 6 & 5 \quad 3
\end{array}$$

$135AB_{16} = 232653_8$

(f) $BADFACED_{16} \rightarrow$ OCT

$$\begin{array}{cccccccc}
B_{11} & A_{10} & D_{13} & F_{15} & A_{10} & C_{12} & E_{14} & D_{13} \\
1011 & 1010 & 1101 & 1111 & 1010 & 1100 & 1110 & 1101
\end{array}$$

$$\begin{array}{ccccccccccc}
010 & 111 & 010 & 110 & 111 & 111 & 010 & 110 & 011 & 101 & 101 \\
2 & 7 & 2 & 6 & 7 & 7 & 2 & 6 & 3 & 5 & 5
\end{array}$$

$BADFACED_{16} = 27267726355_8$

(g) $1011\ 0111\ 1011_2 \to$ OCT

101 101 111 011

5 5 7 3

$101101111011_2 = 5573_8$

3、

(a) $100\ 0111_2$ 、 $111\ 0111_2$

$$\begin{array}{r} 100\ 0111 \\ +111\ 0111 \\ \hline 1011\ 1110 \end{array}$$

$100\ 0111 = 71$ (6, 2 1 0)

$111\ 0111 = 119$ (6 5 4, 2 1 0)

sum = 1011 1110

$$\begin{array}{r} 71 \\ \times 119 \\ \hline 639 \\ 71 \\ 71 \\ \hline 8449 \end{array}$$

$8449_{10} = 10000100000001$

4、 $a \equiv 4 \pmod{13}$,

$b \equiv 9 \pmod{13}$,

(a) $C \equiv 11b \pmod{13}$

$C \equiv 99 \pmod{13}$

$\equiv 8 \pmod{13}$, $C = 8$ #

(b) $C \equiv 2a + 3b \pmod{13}$

$C \equiv 35 \pmod{13}$

$\equiv 9 \pmod{13}$, $C = 9$ #

(c) $C \equiv a^3 - b^3 \pmod{13}$

$\equiv -665 \pmod{13}$

$\equiv -2 \pmod{13}$

$\equiv 11 \pmod{13}$, $C = 11$ #

5、

(a)

$(177 \bmod 31 \times 270 \bmod 31) \bmod 31$

$= (22 \times 22) \bmod 31$

$= 19$ #

(b)

$(-133 \bmod 23 + 261 \bmod 23) \bmod 23$

$= (5 + 8) \bmod 23$

$= 13$ #

(c)

$(32^3 \bmod 13)^2 \bmod 11$

$= ((32 \bmod 13)^3 \bmod 13)^2 \bmod 11$

$= 64 \bmod 11$

$= 9$ #

(d)

$(99^2 \bmod 32)^3 \bmod 15$

$= ((99 \bmod 32)^2 \bmod 32)^3 \bmod 15$

$= 729 \bmod 15 = 9$ #

6、

(a) 117, 213

$213 = 1 \times 117 + 96$

$117 = 1 \times 96 + 21$

$96 = 4 \times 21 + 12$

$21 = 1 \times 12 + 9$

$12 = 1 \times 9 + 3$

$9 = 3 \times 3 + 0$

$\gcd(117, 213) = 3$

$3 = 12 - 9$

$= 1 \times 12 + (-1) \times (21 - 12)$

$= 2 \times 12 - 21$

$= 2 \times (96 - 4 \times 21) - 21$

$= 2 \times 96 - 9 \times 21$

$= 2 \times 96 - 9 \times (117 - 96)$

$= 11 \times 96 - 9 \times 117$

$= 11 \times (213 - 117) - 9 \times 117$

$= 11 \times 213 - 20 \times 117$ #

(b) 124, 323

$323 = 2\times124+75$
$124 = 1\times75+49$
$75 = 1\times49+26$
$49 = 1\times26+23$
$26 = 1\times23+3$
$23 = 7\times3+2$
$3 = 1\times2+1$
$2 = 2\times\underset{\sim}{1}+0$
$\gcd(124,323) = 1$

$1 = 3-2$
$= 3-(23-7\times3)$
$= 8\times3-23$
$= 8\times(26-23)-23$
$= 8\times26-9\times23$
$= 8\times26-9\times(49-26)$
$= 17\times26-9\times49$
$= 17\times(75-49)-9\times49$
$= 17\times75-26\times49$
$= 17\times75-26\times(124-75)$
$= 43\times75-26\times124$
$= 43\times(323-2\times124)-26\times124$
$= 43\times323-112\times124$ #

7.

(a) $4x \equiv 5 \pmod 9$

$9 = 2\times4+1$
$4 = 4\times1+0$
$\gcd(4,9) = 1$

$1 = 9-2\times4$
$= 1\times9+\underset{\sim}{(-2)}\times4$

$-2+9 = 7$

$7\times4x \equiv 7\times5 \pmod 9$
$x \equiv 35 \pmod 9$
$\equiv 8 \pmod 9$

$x = 8+9k,\ k\in\mathbb{Z}$

(b) $34x \equiv 77 \pmod{89}$

$89 = 2\times34+21$
$34 = 1\times21+13$
$21 = 1\times13+8$
$13 = 1\times8+5$
$8 = 1\times5+3$
$5 = 1\times3+2$
$3 = 1\times2+1$
$2 = 2\times1+0$

$1 = 3-2$
$= 3-(5-3)$
$= 2\times3-1\times5$
$= 2\times(8-5)-1\times5$
$= 2\times8-3\times5$
$= 2\times8-3\times(13-8)$
$= 5\times8-3\times13$
$\vdots$
$= 13\times89+\underset{\sim}{(-34)}\times34$

$-34+89 = 55$

$55\times34x \equiv 55\times77 \pmod{89}$

$x \equiv 52 \pmod{89}$

$52+89k,\ k\in\mathbb{Z}$

(c)

Ans: $3+11k, k\in\mathbb{Z}$ or
$6+11k, k\in\mathbb{Z}$

(C) $15x^2+19x \equiv 5 \pmod{11}$ $\quad 15x^2+19x+6=0$

< Hint: $15x^2+19x+6 \equiv 0 \pmod{11}$ >

$$\begin{array}{ccc} 5x & \times & 3 \\ 3x & & 2 \end{array}$$

$(5x+3)(3x+2)=0$

$\underbrace{(5x+3)}_{①}\underbrace{(3x+2)}_{②} \equiv 0 \pmod{11}$

①

$5x+3 \equiv 0 \pmod{11}$
$5x\cancel{+3+8} \equiv 0+8 \pmod{11}$
$5x \equiv 8 \pmod{11}$

$11=2\times5+1$ $\quad 1=11-2\times5$
$5=5\times1+0$ $\quad =11+(-2)\times5$
$\gcd(5,11)=1$ $\quad$ inverse: $-2+k$

$-2+11=9$
$9\times5x \equiv 9\times8 \pmod{11}$
$x \equiv 72 \pmod{11}$
$x \equiv 6 \pmod{11}$

②

$3x+2 \equiv 0 \pmod{11}$
$3x\cancel{+2+9} \equiv 0+9 \pmod{11}$
$3x \equiv 9 \pmod{11}$

$11=3\times3+2$ $\quad 1=3-2$
$3=1\times2+1$ $\quad =3-(11-3\times3)$
$2=2\times1+0$ $\quad =4\times3-11$
$\gcd(3,11)=1$ $\quad$ inverse: $4+k$

$4+11=15$
$15\times3x \equiv 15\times9 \pmod{11}$
$x \equiv 135 \pmod{11}$
$\equiv 3 \pmod{11}$

Ans: $x=3$ or 6

Ans: $3+11k, k\in\mathbb{Z}$ or
$6+11k, k\in\mathbb{Z}$

(d) $x \equiv 5 \pmod 6$

$x \equiv 3 \pmod{10}$, find all solution.

$x \equiv 8 \pmod{15}$

$x = 6t+5,\ t \in \mathbb{Z}$

$6t+5 \equiv 8 \pmod{15}$

$6t \equiv 3 \pmod{15} \rightarrow 2t \equiv 1 \pmod 5$

$\gcd(6, 15) = 3$

$$\begin{array}{r|r|r|l} 2 & 6 & 15 & 2 \\ & 6 & 12 & \\ \hline & 0 & 3 & \end{array}$$

$2t \equiv 1 \pmod 5$

$5 = 2 \times 2 + 1$ $\quad 1 = 5 - 2 \times 2$

$2 = 2 \times 1 + 0$ $\quad = 5 + \underbrace{(-2)}\times 2$

inverse: $-2+k$

$-2+5=3$

$3 \times 2t \equiv 3 \times 1 \pmod 5$

$t \equiv 3 \pmod 5,\ 5k+3 = t,\ k \in \mathbb{Z}$

兩兩一組處理!

$x = 6(5k+3)+5$

$= 30k+23$

$x \equiv 23 \pmod{30}$

$x \equiv 3 \pmod{10}$

$x = 10u+3,\ u \in \mathbb{Z}$

$10u+3 \equiv 23 \pmod{30}$

$10u \equiv 20 \pmod{30} \rightarrow u \equiv 2 \pmod 3$

$\gcd(10, 30) = 10$ $\qquad u = 3n+2,\ n \in \mathbb{Z}$

$x = 10(3n+2)+3$

$= 30n+23$

$x \equiv 23 \pmod{30}$

Ans: $x = 30n+23,\ n \in \mathbb{Z}$ ✓

Homework . 4

5、

(a) $C^{19}_{10} \times 2^{10} \times (-1)^{9} = 92378 \times 1024 \times -1 = -94595072$ #

(b) $C^{17}_{8} \times 3^{8} \times 2^{9} = 24310 \times 6561 \times 512 = 81662929920$ #

三个小孩分六个不同的玩具，
共有幾種分法?

$3^{6} = 729$

10、 $x_1+x_2+x_3+x_4+x_5=21$

where x_i, $i=1、2、3、4、5$ is a nonnegative integer、

(a) $x_1 \ge 1$

let $y_1 = x_1 - 1$

$y_1+x_2+x_3+x_4+x_5=20$

$H^5_{20} = C^{24}_{20} = \frac{\cancel{24}^{63}\times 23\times 22\times \cancel{21}^{7}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$

$= 21\times 22\times 23$

$= 10626$ #

(b) $x_i \ge 2$, $i=1、2、3、4、5$

let $y_i = x_i - 2$

$y_1+y_2+y_3+y_4+y_5=11$

$H^5_{11} = C^{15}_{11} = \frac{15\times\cancel{14}^{7}\times 13\times\cancel{12}^{3}}{\cancel{4}\times 3\times\cancel{2}\times 1}$

$=1365$ #

(c) $0\le x_1\le 10$

全: $H^5_{21} = C^{25}_{21}$

$x_1 \ge 10$:

let $y_1 = x_1 - 10$

$y_1+x_2+x_3+x_4+x_5=10$

$H^5_{10} = C^{14}_{10}$

$C^{25}_{21} - C^{14}_{10} = \frac{25\times\cancel{24}^{2}\times 23\times\cancel{22}^{11}}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}_{1}} - \frac{\cancel{14}^{7}\times 13\times\cancel{12}^{3}\times 11}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$

$= 50\times 23\times 11 - 77\times 13$

$= 12650 - 1001 = 11649$ #

(d) $0\le x_1\le 3$, $1\le x_2 < 4$, $x_3\ge 15$

1.) $x_1 \ge 1$, $x_3\ge 15$

let $y_1 = x_1 - 1$, $y_3 = x_3 - 15$

$y_1+x_2+y_3+x_4+x_5=5$

$H^5_5 = C^9_5 = \frac{9\times\cancel{8}^{2}\times 7\times\cancel{6}^{2}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} = 126$

2.) $x_1\ge 4$, $x_2\ge 1$, $x_3 \ge 15$

let $y_1 = x_1 - 4$、$y_2 = x_2 - 1$、$y_3 = x_3 - 15$

$y_1+y_2+y_3+x_4+x_5=1$

$H^5_1 = C^5_1 = 5$

3.) $x_2\ge 4$, $x_3\ge 15$

let $y_2 = x_2 - 4$, $y_3 = x_3 - 15$

$x_1+y_2+y_3+x_4+x_5=2$

$H^5_2 = C^6_2 = 15$、

4.) $x_1\ge 4$, $x_2\ge 4$, $x_3\ge 15$

$4+4+15>21$, no solutions.

$(1)-(2)-(3) = 126-5-15 = 106$ #

11、

(a) $a_n = 2a_{n-1} + 3^n$, find all solution、

let $a_n = r^n$, $r^n = 2r^{n-1}$, $r = 2 \Rightarrow a_n^{(h)} = \alpha 2^n$

Suppose $P_n = c3^n$

$c3^n = 2c3^{n-1} + 3^n$

$c = 2c3^{-1} + 1$

$c = \frac{2c+3}{3}$

$3c = 2c + 3$

$c = 3$

$\Rightarrow a_n^{(p)} = 3 \times 3^n = 3^{n+1}$

$\Rightarrow a_n = \alpha 2^n + 3^{n+1}$ #

(b) $a_n = \alpha 2^n + 3^{n+1}$, $a_1 = 5$

$5 = 2\alpha + 9$

$-4 = 2\alpha$, $\alpha = -2$ #

$a_n = -2 \times 2^n + 3^{n+1}$

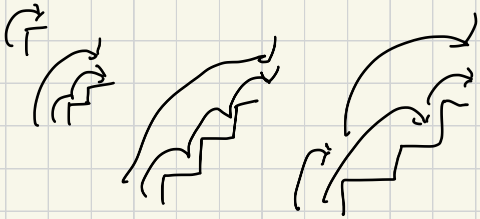
$= -2^{n+1} + 3^{n+1}$ #

7、 限定爬的梯子1、2、3

$a_1 = 1$

$a_2 = 2$

$a_3 = 4$



$a_n = a_{n-1} + a_{n-2}$, $n \geq 4$

$a_4 = 6$, $a_5 = 10$, $a_6 = 16$, $a_7 = 26$, $a_8 = 42$