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Distributed Coordination of Heterogeneous Multi-Agent Systems with Output Feedback Control

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***Abstract*—This paper investigates a heterogeneous multi-agent system consensus control problem under the leader-follower formation. The state dimensions of multi-agent systems (MASs) are diverse, which makes it difficult to analyze and design the controller for the multi-agent formation consistency problem. Therefore, an output feedback control scheme is designed for MAS consensus to ensure the output convergence for each follower. First, a discrete-time model of the leader-follower system is established. Then, an output feedback controller is provided with formation topology to ensure the formation consensus of the heterogeneous MAS. Finally, simulation results are shown to verify the designed control strategies.**

***Index Terms*—Formation consistency, output feedback, heterogeneous multi-agent systems.**

I. Introduction

The distributed control problem and leader-follower scenario of heterogeneous multi-agent systems (MASs) have been widely studied in many research fields, such as unmanned aircraft vehicle (UAV) and unmanned underground vehicle (UGV) formation coordination [1]. The leader-follower formation approach is a common strategy for the homogeneous MAS formation. Increasing engineering applications resort to leader-follower formation. However, the different state dimensions of heterogeneous MASs make the controller design more complex [2]. Therefore, the study of the heterogeneous MAS formation consensus is necessary, which can provide important reference for effective practical applications.

During the past few years, the MAS formation control problem has been widely investigated from various perspectives, such as agent dynamics and distributed control problems [3] - [4]. In [5], a static network-based distributed control protocol under an undirected graph was designed to handle with the MAS distributed control problem. Combined with the known Lyapunov asymptotical stability theory, a state feedback control scheme was presented for homogeneous MAS in a practical experiment [6]. The above results of MAS are mainly applied in continuous-time dynamic systems. For a discrete-time MAS, in order to maintain the formation, a leader-follower configuration was studied in [7]. In [8], a leader-follower discrete-time linear MAS with unknown disturbance was investigated for formation consensus problem. However, in most of practical engineering, the multi-agent systems often consist of different structure agents, such as the UAV and UGV air-ground formation, and the conventional methods cannot be used to analyze the formation consensus problems, directly. Therefore, the problem of heterogeneous MASs formation consensus needs to be considered further.

In the heterogeneous MAS, the dimensions and structures of each agent are different, which constrain the system controller design [9]. To solve this problem, a heterogeneous MAS such as the UAV and UGV formation should be considered, and the system output can be introduced for controller design [10]. In [11], a dynamic output distributed control strategy using the internal model principle was presented for the MAS with output containment. In [12], a discrete-time output feedback distributed controller was presented for MAS with external disturbance, which was solved by a linear matrix inequality (LMI) method. However, there are few researches investigating the consensus control problem for heterogeneous MAS with topology analysis of the discrete-time dynamic which needs to be further studied.

Inspired by the facts stated above, a distributed control scheme is designed for the heterogeneous MAS formation consensus problem. This paper is organized as follows: In section II, some preliminaries of the heterogeneous MAS are introduced. Section III provides the distributed output feedback control strategy based on linear matrix inequality method. Section IV provides a new numerical simulate example. The conclusions of this paper are drawn in Section V.

Notation: The superscript "T" represents the transpose of matrix. $\lambda(\bullet)$ denotes the eigenvalue of matrix. The two-norm of a vector is described by $\|\bullet\|$. R^n represents the dimensional Euclidean space. The symbol $\otimes$ means the Kronecker product, and $*$ means the symmetry term of matrix.

II. Problem Formulation

The MAS includes a tracked target, a formation-leader and n formation-followers. An undirected graph $\mathbb{G}$ is described by the network of n agents, and $\bar{\mathbb{G}}$ denotes the graph that consists of $\mathbb{G}$ with the leader via directed edges. Let $a_{ij} = a_{ji}(i = 1, ..., n; j = 1, ..., n)$ with $a_{ii} = 0$ denote the nonzero

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connection weights between agents i and j. $g_i > 0$ means that between the leader and the follower i has a directed link. O is the in-degree matrix that $O = diag(d_i)$, where $d_i = \sum_{j=1}^{n} a_{ij}$. Then the Laplacian matrix of the graph $\mathbb{G}$ is denoted by L [13].

The dynamics of each formation-follower is described by [11]

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + Bu_i(k) \\ y_i(k) &= Cx_i(k) \end{aligned} \tag{1}$$

where $x_i(k) \in R^q$, $y_i(k) \in R^m$ and $u_i(k) \in R^d$ are represented the system state, measurement output and control input, respectively.

The system of the formation-leader l is described by [11]

$$\begin{aligned} x_l(k+1) &= Ax_l(k) + Bu_l(k) \\ y_l(k) &= Cx_l(k) \end{aligned} \tag{2}$$

where $x_l(k) \in R^q$ is the system state of the leader l, $u_l(k) \in R^d$ is the control input, and $y_l(k) \in R^m$ is the output.

Assuming that the system structure of the tracked target is different with other agents, and the tracked target can transfer the system dynamics to the formation-leader, then the systems of the tracked target, the formation-leader and the formation-followers consist of the heterogeneous MAS. The dynamic discrete-time system of the tracked target is described as follows:

$$\begin{aligned} x_0(k+1) &= A_0x_0(k) + r(k) \\ y_0(k) &= C_0x_0(k) \end{aligned} \tag{3}$$

where $x_0(k) \in R^p$ and $y_0(k) \in R^m$ are the system state and measurement output, respectively. $r(k)$ is the external bounded signal. A_0 and C_0 are the matrices with appropriate dimensions, and A_0 is Hurwitz stable.

The expected formation configuration vector for the followers is specified by $\xi(k) = \left[\xi_1^T(k), \xi_2^T(k), ..., \xi_n^T(k)\right]^T$, where $\xi_i(k) \in R^m$ and $\xi_i(k)$ is bounded.

Remark 1: For the multi-agent formation, the formation vectors are pre-given in order to maintain pre-designed formations for the MAS, which guaranteed that $\xi_i(k) \neq \xi_j(k)$, more details could see in Refs. [13]- [14].

In this paper, a distributed control scheme is designed to ensure that $\lim_{k\to\infty} \|(y_l(k) - \xi_l(k)) - y_0(k)\| = 0$ and $\lim_{k\to\infty} \|y_l(k) - (y_i(k) - \xi_i(k))\| = 0$. To make sure the tracking error convergence, the following assumptions are adopted.

Assumption 1 [15]: With the tracked target as a root node, there exists a spanning tree in the graph $\bar{\mathbb{G}}$.

Assumption 2 [16]: There exist solutions X and Φ for the linear matrix equations as follows:

$$\begin{aligned} CX &= C_0 \\ XA_0 &= AX \end{aligned} \tag{4}$$

Assumption 3 [17]: For a given formation vector $\xi_i(k)$, there exist a state vector $\vartheta_i(k)$ and a compensation input $\chi_i(k)$ which satisfies the following condition:

$$\begin{aligned} &\lim_{k\to\infty} (A\vartheta_i(k) - \vartheta_i(k+1) + B\chi_i(k)) = 0 \\ &\xi_i(k) = C\vartheta_i(k), i = 1, 2, ..., n \end{aligned} \tag{5}$$

Remark 2: As shown in Assumption 3, the formation vector all can use a constant gain matrix K_1 to obtain the feasible formations. In (5), $\lim_{k\to\infty} ((A + BK_1)\vartheta_i(k) - \vartheta_i(k+1) + B\hat{\chi}_i(k)) = 0$, which can be regarded as a special situation of (5) by choosing $\chi_i(k) = K_1\vartheta_i(k) + \hat{\chi}_i(k)$. If $(A+BK_1)$ is Hurwitz stable, the proposed feasible constraint (5) is available to obtain [17]. Besides, $\hat{\chi}_i(k)$ is designed as $\hat{\chi}_i(k) \neq 0$ and $\hat{\chi}_i(k) \neq \hat{\chi}_j(k)$, which means $\xi_i(k) \neq 0$ and $\xi_i(k) \neq \xi_j(k)$ to avoid the formation collusion.

Then, to guarantee the leader tracks the target, the tracking error between the leader and the target is denoted as follows:

$$e(k) = x_l(k) - \vartheta_l(k) - Xx_0(k) \tag{6}$$

The output tracking error is defined by

$$\eta_i(k) = Cx_l(k) - (Cx_i(k) - \xi_i(k)) \tag{7}$$

Let the external bounded signal $r(k) = \Theta\hat{r}(k)$, where Θ is an adjustment constant dimension matrix which satisfies $BF = X\Theta$. For leader l, the state feedback controller parameters Φ, G and F are designed as

$$u_l(k) = Ge(k) + F\hat{r}(k) + \chi_l(k) \tag{8}$$

Invoking (2), (3), (6) and (8), it yields

$$\begin{aligned} e(k+1) &= x_l(k+1) - \vartheta_l(k+1) - Xx_0(k+1) \\ &= Ax_l(k) + BGe(k) + BF\hat{r}(k) \\ &\quad + B\chi_l(k) - AXx_0(k) \\ &\quad - X\Theta\hat{r}(k) - \vartheta_l(k+1) \\ &= A(x_l(k) - \vartheta_l(k) + \vartheta_l(k)) + BGe(k) \\ &\quad + BF\hat{r}(k) - X\Theta\hat{r}(k) + B\chi_l(k) - \vartheta_l(k+1) \\ &= (A + BG)e(k) + A\vartheta_l(k) + B\chi_l(k) \\ &\quad - \vartheta_l(k+1) \end{aligned} \tag{9}$$

Let $e_i(k) = (y_j(k) - \xi_j(k)) - (y_i(k) - \xi_i(k))$, the neighbor relative output error information $e_{y_i}(k)$ of each follower is described as

$$\begin{aligned} e_{y^i}(k) &= \sum_{j=1}^{n} a_{ij}e_i(k) + g_i\eta_i(k) \\ &= \sum_{j=1}^{n} a_{ij}\left((y_j(k) - \xi_j(k)) - (y_i(k) - \xi_i(k))\right) \\ &\quad + g_i\left(y_l(k) - (y_i(k) - \xi_i(k))\right) \end{aligned} \tag{10}$$

In the MAS network, if each agent transmits the status information for other agents, the MAS network may generate congestion due to the large data communication. Therefore, the output feedback control method is introduced for heterogeneous MAS formation. The output feedback distributed controller N of each follower is designed by

$$u_i(k) = Ne_{y^i}(k) + F\hat{r}(k) + \chi_i(k) + \chi_l(k) \tag{11}$$

Combined with link weights a_{ij} and g_i, the communication topology of the graph is normalized as

$$\sum_{j=1}^{n} a_{ij} + g_i = d_i + g_i = 1 \tag{12}$$

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Define that $\tilde{y}_i(k) = y_i(k) - \xi_i(k)$ and $\tilde{y}_j(k) = y_j(k) - \xi_j(k)$. Then combined with (1), (2), (7) and (11), we have

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + BNe_{y^i}(k) + BF\hat{r}(k) \\ &\quad + B\Phi x_0(k) + B\chi_i(k) + B\chi_l(k) \\ &= Ax_i(k) + BNe_{y^i}(k) + BF\hat{r}(k) + B\Phi x_0(k) \\ &\quad + B\chi_i(k) + B\chi_l(k) \end{aligned} \tag{13}$$

Letting $\tilde{x}_i(k) = x_i(k) - \vartheta_i(k)$ invoking with Assumption 3, it yields that $\tilde{y}_i(k) = C\tilde{x}_i(k)$ and $\tilde{y}_j(k) = C\tilde{x}_j(k)$, respectively. Invoking (7), (10) and (12), we have

$$\begin{aligned} e_{y^i}(k) &= \sum_{j=1}^{n} a_{ij}(\tilde{y}_j(k) - \tilde{y}_i(k)) + g_i(y_l(k) - \tilde{y}_i(k)) \\ &= \sum_{j=1}^{n} a_{ij}\tilde{y}_j(k) + g_i y_l(k) \\ &\quad - \left(\sum_{j=1}^{n} a_{ij}\tilde{y}_i(k) + g_i\tilde{y}_i(k) \right) \\ &= \sum_{j=1}^{n} a_{ij}\tilde{y}_j(k) + g_i y_l(k) - \tilde{y}_i(k) \end{aligned} \tag{14}$$

Combined with (13) and (14), we have

$$\begin{aligned} \tilde{x}_i(k+1) &= x_i(k+1) - \vartheta_i(k+1) \\ &= Ax_i(k) + BNe_{y^i}(k) + B\Phi x_0(k) \\ &\quad + BF\hat{r}(k) + B\chi_i(k) + B\chi_l(k) - \vartheta_i(k+1) \\ &= A\left(x_i(k) - \vartheta_i(k) + \vartheta_i(k)\right) + BNe_{y^i}(k) \\ &\quad + BF\hat{r}(k) + B\chi_i(k) \\ &\quad + B\chi_l(k) - \vartheta_i(k+1) \\ &= A\tilde{x}_i(k) + BNe_{y^i}(k) + BF\hat{r}(k) \\ &\quad + B\chi_l(k) + A\vartheta_i(k) + B\chi_i(k) - \vartheta_i(k+1) \end{aligned} \tag{15}$$

The uniformity output tracking error (7) is denoted as follows:

$$\varepsilon_i(k) = \tilde{x}_i(k) - x_l(k) \tag{16}$$

According to Assumption 3, χ_i is a known signal, which means that $\vartheta_i(k+1)$ can be obtained at moment k. Then, let $\nabla_i(k) = A\vartheta_i(k) + B\chi_i(k) - \vartheta_i(k+1)$ and invoking (15) and (16), we obtain

$$\begin{aligned} \varepsilon_i(k+1) &= \tilde{x}_i(k+1) - x_l(k+1) \\ &= A\tilde{x}_i(k) + BN\sum_{j=1}^{n} a_{ij}C\tilde{x}_j(k) \\ &\quad + BNg_iCx_l(k) - BNC\tilde{x}_i(k) \\ &\quad + BF\hat{r}(k) + B\chi_l(k) + \nabla_i(k) \\ &\quad - Ax_l(k) - BGe(k) - BF\hat{r}(k) - B\chi_l(k) \end{aligned} \tag{17}$$

The equation (17) can be written as

$$\begin{aligned} \varepsilon_i(k+1) &= A\tilde{x}_i(k) + BN\sum_{j=1}^{n} a_{ij}C\tilde{x}_j(k) \\ &\quad + BNg_iCx_l(k) - BNC\tilde{x}_i(k) + \nabla_i(k) \\ &\quad - Ax_l(k) - BGe(k) \end{aligned} \tag{18}$$

According to (12), we have $g_i - 1 = -\sum_{j=1}^{n} a_{ij}$, then combined with (16) and (18), it yields that

$$\begin{aligned} \varepsilon_i(k+1) &= A\tilde{x}_i(k) - Ax_l(k) + BN\sum_{j=1}^{n} a_{ij}C\tilde{x}_j(k) \\ &\quad + BN(g_i - 1 + 1)Cx_l(k) \\ &\quad - BNC\tilde{x}_i(k) + \nabla_i(k) - BGe(k) \\ &= A\varepsilon_i(k) + BN\sum_{j=1}^{n} a_{ij}C\tilde{x}_j(k) \\ &\quad - BN\sum_{j=1}^{n} a_{ij}Cx_l(k) - BNC\tilde{x}_i(k) \\ &\quad + BNCx_l(k) + \nabla_i(k) - BGe(k) \\ &= A\varepsilon_i(k) + BN\sum_{j=1}^{n} a_{ij}C\varepsilon_j(k) \\ &\quad - BNC\varepsilon_i(k) + \nabla_i(k) - BGe(k) \\ &= \hat{A}\varepsilon_i(k) + BN\sum_{j=1}^{n} a_{ij}C\varepsilon_j(k) \\ &\quad + \nabla_i(k) - BGe(k) \end{aligned} \tag{19}$$

where $\hat{A} = A - BNC$.

By defining the augment vectors as $\hat{\varepsilon} = col(\varepsilon_1, ..., \varepsilon_n)$, $\bar{\eta} = col(\eta_1, ..., \eta_n)$, $\hat{\nabla} = col(\nabla_1, \nabla_2, ..., \nabla_n)$, $\hat{e} = col(\overbrace{e, e, ..., e}^{n})$, $\hat{\nabla}_l = col(\overbrace{\nabla_l, \nabla_l, ..., \nabla_l}^{n})$ and the augment matrix as $\hat{C} = col(\overbrace{C, C, ..., C}^{n})$. Based on the weighted graph, we have $H = L + O$, then the error systems (9) and (19) can be expressed as

$$\begin{aligned} \hat{\varepsilon}(k+1) &= \left(I_N \otimes \hat{A} + H \otimes BNC \right) \hat{\varepsilon}(k) \\ &\quad - (I_N \otimes BG)\hat{e}(k) + \hat{\nabla}(k) \\ \hat{e}(k+1) &= I_N \otimes (A + BG)\hat{e}(k) + \hat{\nabla}_l(k) \end{aligned} \tag{20}$$

Let $E(k) = \left[\begin{array}{c} \hat{e}^T(k), \hat{\varepsilon}^T(k) \end{array} \right]^T$ and $\bar{\nabla}(k) = \left[\hat{\nabla}_l^T(k), \hat{\nabla}^T(k) \right]^T$, equation (20) can be written as

$$\begin{aligned} E(k+1) &= \bar{A}E(k) + \bar{\nabla}(k) \\ \bar{\eta} &= \bar{C}E(k) \end{aligned} \tag{21}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} \bar{A}_1 & 0 \\ \bar{A}_2 & \bar{A}_3 \end{bmatrix} \\ \bar{A}_1 &= I_N \otimes (A + BG) \\ \bar{A}_2 &= -I_N \otimes BG \\ \bar{A}_3 &= I_N \otimes \hat{A} + H \otimes BNC \\ \bar{C} &= \left[\hat{C}, -\hat{C} \right] \end{aligned} \tag{22}$$

The objective of this paper is to achieve that the follower system (1) and leader system (2) could track the target system (3) within a given formation. For the controller designed in (8) and (11), the tracking control condition is given in the next section. To solve the control gains, the following definition and Lemma are adopted.

Definition 1 [18]: For any given initial time k_0, if the system (21) satisfying

$$\|E(k)\| < \delta\rho^{(k-k_0)} \|E(k_0)\|, \forall k \geq k_0 \tag{23}$$

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then the system (21) is exponentially stable, where $\delta > 0$ and $0 < \rho < 1$ are constant, and ρ is the decay rate.

Lemma 1 [19]: For $S = S^T > 0$ and $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, the following conditions are equivalent:

$$\begin{aligned} &(1) S < 0 \\ &(2) S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0 \\ &(3) S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0 \end{aligned}$$

III. Main Results

The solution conditions of the output consensus formation-tracking problem are proposed in this section.

Theorem 1: For multi-agent follower system (1), leader system (2) and target system (3), there exist gain-scheduled controllers given in equations (8) and (11), if there exist positive definite symmetric matrices P and Q satisfy

$$\Omega < 0 \tag{24}$$

where

$$\begin{aligned} &\Omega = \begin{bmatrix} -\rho P & \bar{A}^T \\ * & -Q \end{bmatrix} \\ &Q = P^{-1} \end{aligned}$$

then, the closed-loop system (21) is exponentially stable. Here, $\bar{A}$ is given in (22). The closed-loop system exponentially decay rate is ρ.

Proof: The Lyapunov function candidate is considered by

$$V(k) = E^T(k) P E(k) \tag{25}$$

The dynamics of $V(k)$ with exponential forward difference is defined as

$$\begin{aligned} &V(k+1) - \rho V(k) \\ &= E^T(k+1) P E(k+1) - \rho E^T(k) P E(k) \\ &= E^T(k) \bar{A}^T P \bar{A} E(k) + 2E^T(k) \bar{A}^T P \bar{\nabla}(k) \\ &\quad + \bar{\nabla}^T(k) P \bar{\nabla}(k) - \rho E^T(k) P E(k) \end{aligned} \tag{26}$$

Combined with Assumption 3, it yields $\lim\limits_{k\to\infty} \nabla_i(k) = 0$, $\lim\limits_{k\to\infty} \nabla_l(k) = 0$ and $\lim\limits_{k\to\infty} \bar{\nabla}(k) = 0$. Then equation (26) can be deduced as

$$\begin{aligned} &\lim_{k\to\infty} V(k+1) - \rho V(k) \\ &= E^T(k) \bar{A}^T P \bar{A} E(k) - \rho E^T P E(k) \end{aligned} \tag{27}$$

According to (24), (27) with Lemma 1, it deduces that $V(k+1) - \rho V(k) < 0$, which means that

$$V(k) < \rho V(k-1) < \ldots < \rho^{k-k_0} V(k_0) \tag{28}$$

Due to $P > 0$, let $c_1 = \max \lambda(P)$ and $c_2 = \min \lambda(P)$, combined with (25), it yields

$$\begin{aligned} &V(k) = E^T(k) P E(k) \geq c_2 \left\| E(k) \right\| \\ &V(k_0) = E^T(k_0) P E(k_0) \leq c_1 \left\| E(k_0) \right\| \end{aligned} \tag{29}$$

Let $\delta = c_1/c_2$, together with (28), further implies

$$\left\| E(k) \right\| \leq c_1 \rho^{k-k_0} \left\| E(k_0) \right\| \tag{30}$$

Then invoking with Definition 1, the closed-loop system (21) is exponential stability. This completes the proof. The controller solution will be given in the following.

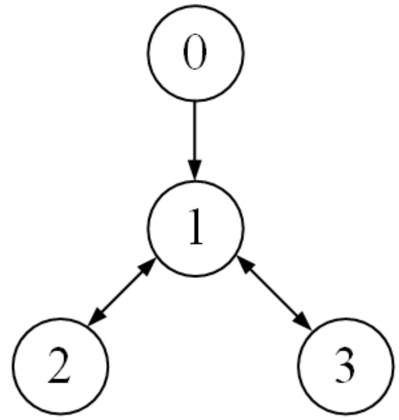

Fig. 1. Communication graph

Theorem 2. For multi-agent follower system (1), leader system (2) and target system (3), there exist gain-scheduled controllers given in equations (8) and (11), if there exist positive definite symmetric matrices P_1, P_2, Q_1, Q_2, and control gains N, K, Φ, G and F satisfy

$$\hat{\Omega} < 0 \tag{31}$$

where

$$\begin{aligned} &\hat{\Omega} = \begin{bmatrix} -P_1 & 0 & \bar{A}_1^T & \bar{A}_2^T \\ * & -P_2 & 0 & \bar{A}_3^T \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} \\ &Q_1 = -P_1^{-1}, Q_2 = -P_2^{-1} \end{aligned} \tag{32}$$

then the closed-loop system (21) is exponentially stable. Here, $\bar{A}_1, \bar{A}_2$ and $\bar{A}_3$ are given in (22). The closed-loop system exponentially decay rate is ρ.

Proof: According to Theorem 1, let $P_1 = diag\{\overbrace{P, P, \ldots, P}^{n}\}$ and $P_2 = diag\{\overbrace{P, P, \ldots, P}^{n}\}$, then invoking $\bar{A}_1$, $\bar{A}_2$ and $\bar{A}_3$ in (22), equation (24) can be written as

$$\bar{\Omega} = \begin{bmatrix} -P_1 & 0 & \bar{A}_1^T & \bar{A}_2^T \\ * & -P_2 & 0 & \bar{A}_3^T \\ * & * & -P_1^{-1} & 0 \\ * & * & * & -P_2^{-1} \end{bmatrix} \tag{33}$$

Let $P_1^{-1} = Q_1, P_2^{-1} = Q_2$, then $P_1 = P_1^T > 0, P_2 = P_2^T > 0$. Now by utilizing Lemma 1 and (33), one can obtain that

$$\hat{\Omega} < 0 \tag{34}$$

where $\hat{\Omega}$ is given in (32). Therefore, the controller gains of the closed-loop system (21) are solved. The proof is completed.

IV. Simulation

The simulate results are shown to illustrate the validity of the proposed distributed control scheme in this section.

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A. Simulation Setup

The MAS in the simulation is comprised by four agents, the topology digraph of the heterogeneous multi-agent system is depicted in Fig. 1. In Fig. 1, the system structure of the agent 0 is different with agents 1 - 3, which is considered as the tracked-target, the agent 1 is considered as the leader meanwhile agent 2 and agent 3 are considered as the followers. The dynamic of the agents 1 - 3 is described as follows [20]:

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + Bu_i(k) \\ y_i(k) &= Cx_i(k) \end{aligned} \tag{35}$$

the system parameters are given by [20]

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0.002 & 0.002 & 0 \\ 0 & 1 & 0.005 & 0 \\ 0 & 0 & 0.9704 & 0 \\ 0.002 & 0 & 0.002 & 1 \end{bmatrix} \\ B &= \begin{bmatrix} 1 & 1 & 1.0305 & 1 \end{bmatrix}^T \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

The dynamic equation of the agent 0 is given below [21]

$$\begin{aligned} x_0(k+1) &= A_0x_0(k) + r(k) \\ y_0(k) &= C_0x_0(k) \end{aligned} \tag{36}$$

The system parameters of the agent 0 are given by [21]

$$\begin{aligned} A_0 &= \begin{bmatrix} 1 & 0 & 0.001 \\ 0 & 1 & 0.001 \\ -0.022 & -0.022 & -1 \end{bmatrix} \\ C_0 &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \end{aligned}$$

According to the above analysis, let $r(k) = \Theta\hat{r}(k)$, Θ is chosen as $\Theta = \begin{bmatrix} 4.497 & 2.485 & -0.882 \end{bmatrix}^T$, and $\hat{r}(k)$ is chosen as $\hat{r}(k) = 2$. From Theorem 2, the controller gains are obtained as

$$\begin{aligned} G &= \begin{bmatrix} -0.2598 & -0.2086 & -0.2469 \end{bmatrix}^T \\ F &= 4.497, K = -1.701, N = -0.202 \end{aligned}$$

B. Simulation Results

Let $\hat{\chi}_1 = 0.8$, $\hat{\chi}_2 = -0.47$ and $\hat{\chi}_3 = -0.8$, under the proposed distributed control scheme, the simulation results are depicted in Figs. 2 - 3, respectively. Fig. 2 shows the MAS's tracking trajectory. Fig. 3 shows the relative trajectory error of the formation. From these simulation results, it illustrates that the designed distributed tracking control scheme is effective for the heterogeneous multi-agent formations.

Conclusion

The formation consensus control problem was proposed for the discrete-time heterogeneous MAS in this paper. Firstly, a distributed control scheme had been proposed for the heterogeneous MAS consensus control problem. The controller parameters were calculated by the derived sufficient conditions. Finally, the valid of the designed control scheme is shown by the simulations.

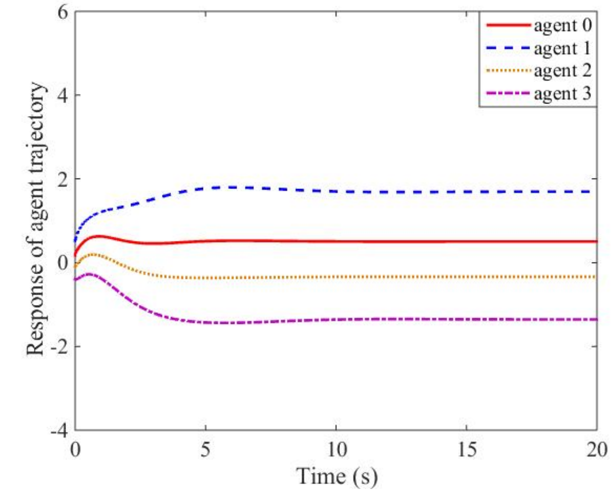

Fig. 2. Response of follower trajectory

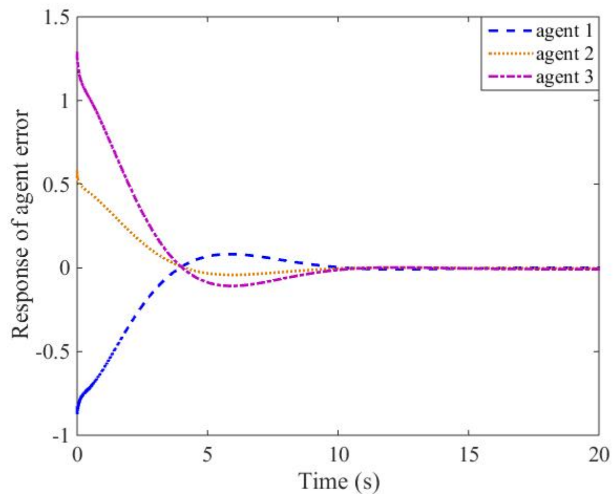

Fig. 3. Response of follower error

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