Exercise 8 - (E11.3)

From the picture of the given network we can extract its parameters:

$$\mathbf{W}_1 = \begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}, \ \mathbf{B}_1 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} \tag{1}$$

$$\mathbf{W}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ b_2 = 0.5 \tag{2}$$

Since we are using *purelin* as activation functions we'll have $a_i^m = n_i^m$, where m is the index layer, and i is the neuron's index in the layer. The activations in the first layer will then be:

$$a_1^1 = n_1^1 = -2p_1 - p_2 - 0.5 \tag{3}$$
$$a_2^1 = n_2^1 = p_1 + 3p_2 - 0.5 \tag{4}$$

While in the second layer:

$$a_1^2 = n_1^2 = a_1^1 + a_2^1 + 0.5 \tag{5}$$

Replacing the values for a_1^1 and a_2^1:

$$a_1^2 = n_1^2 = (-2p_1 - p_2 - 0.5) + (p_1 + 3p_2 - 0.5) + 0.5 \tag{6}$$
$$\tag{7}$$
$$a_1^2 = n_1^2 = -p_1 + 2p_2 - 0.5 \tag{8}$$

Therefore, the given 2-layer neural network will produce the same output as the 1-layer network with the following parameters:

$$\mathbf{W} = \begin{bmatrix} -1 & 2 \end{bmatrix}, \ b = -0.5 \tag{9}$$