Eric numbers

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1 Eric numbers

1.1 Basics:

One day on the bus I started thinking about integers of the form $(xy)_{10}$ st. $xy = x(y) + y^2$. ie) $45 = 4(5) + 5^2$ and decided to formulate a method for finding them.
Known relatives: Kaprekar ($45^2 = 2025$, $45 = 20 + 25$), Hashand ($4 + 5 = 9$, $9|45$) and Triangular ($0, 1, 3, 6, 10, 15 \dots$) numbers. Products of two squares ($45 = 6^2 + 3^2$).

1.2 Findings so far:

Defined and found $f((xi)_{10}) := f(x, i) = xi + i^2 = (x+1)i + 2a_i$ where a_i denotes the i^{th} triangular number.
ie. $f(4, 5) = 5(5) + 2a_5 = 30 + 2(10)$
Notes on a_i:
$a_0 = 0$
$a_i = a_{i-1} + (i - 1)$
Quick proof by induction:
let $f(x, y) = x(y) + y^2 = (x + 1)y + 2a_y$ for $y \leq i$.
then:

$$\begin{aligned}
x(i+1) + (i+1)^2 &= xi + x + i^2 + 2i + 1 \\
&= (xi + i^2) + x + 2i + 1 \\
&= ((x+1)i + 2a_i) + x + 2i + 1 \ \texttt{(by hyp.)} \\
&= (x+1)i + 2(a_i + i) + x + 1 \\
&= (x+1)(i+1) + 2(a_i + i)
\end{aligned}$$

Follows that Eric numbers of two digits are roots of the function $F(x, i) = f(x, i) - 10x - i$ and generalized to $F(x, i) = f_s(x, i) - 10^s x - i$ for numbers with greater than 2 digits where s denotes the split point from the right ie. $f_2((1025)_1 0) = f(10, 25)$, $f_1((1025)_1 0) = f(102, 5)$.
Rearranging the original formula to solve for x however we find a very simple function for finding all possible Eric numbers:

$$x = -\frac{i^2 - i}{i - 10^{(\lfloor log(i) \rfloor + 1)}}$$

Iterating over i yields the first 5 Eric numbers:

i:	4	5	8	9	10	...
$(xi)_{10}$:	24	45	248	729	110	...