MATH 441 FINAL REPORT: AIRPLANE BOARDING

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Abstract. Existing studies have not yet produced a one-size-fits-all optimal strategy for boarding an airplane. Different airlines continue to utilize different strategies, despite having similarly designed airplanes. The objective of any boarding strategy is to minimize the delays in order to make the boarding as efficient as possible as well as lessen the discomfort of loading passengers. Building on Kuo (2015), we implement a simplification of the model the author established, followed by examining how different combinations of the model's variables (including number of rows and seats) impact the optimal strategy for boarding. Starting with a toy model plane, we further develop our model by adding seat interferences caused by passengers having to get up to allow another passenger to enter their seat. While we found that boarding strategies remain similar, there are changes to the previous seating strategies for rows assigned with multiple groups. We also vary other parameters such as the number of groups allowed, and the fraction of passengers from the previous group that is not yet seated. We found that increasing the number of groups reduces the objective value significantly, but exponentially increases the computation time. Lastly, the optimal boarding strategy varies as we vary the fraction from the previous group not seated. Throughout our research, we found that the Reverse Pyramid scheme, a back to front boarding strategy, emerged the most universally-efficient boarding strategy across a variety of airplane models.

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1. Introduction

1.1. **Overview.** The goal of this project is to examine different boarding processes used by various airlines and for various airplane models, and determine the conditions under which each is optimal. We expand on the results of Kuo (2015). We begin by implementing the model the author used and then discuss and validate our findings. We answer some of the open questions posed by the original paper, and explore additional questions they did not consider.

1.2. **Motivation.** It is well-known that airplane boarding is often a tedious and painful process. Our motivation is to find ways to expedite boarding, to make the process as painless (and perhaps even pleasant) and efficient as possible.

1.3. **Specific Questions in this Line of Research.** First, we will explore the optimal boarding strategy for a toy example, with only 8 rows and 6 seats per row, with 3 seats on each side of a central aisle. We will attempt to verify the previous results on this toy example.

The second step involves applying the model to a realistic setup. We investigate how the optimal solution changes when moving from the toy example to a realistic airplane layout. To begin, we apply the model to the Airbus A320 layout, with 23 rows of seats and 3 seats on each side of a central aisle (note that this includes the economy seating only).

The third sort of questions we answer are related to the effects of tweaking the model's parameters. We explore how the optimal strategy for boarding a plane changes as the size and layout of the plane change, and what types of boarding group arrangements are most suitable. We consider whether a different strategy should be utilized if the number of aisles increases. We also investigate how the optimal strategy changes as the amount of overlap between groups decreases (i.e. fewer people from previous group remain when next group boards). Furthermore, we vary the number of groups allowed and investigate its impact on the objective.

2. Models

2.1. **Basic Model.** The simplest model we start from is a simplification of the model used by Kuo. It will consider only delays caused by interference among passengers in the aisle of a single-aisle plane with N rows of seats and S seats per row, and not seat interferences caused by passengers within the same row. The passengers will be split into G boarding groups that are then boarded one at a time. To cut down on computation time, we assume the boarding groups are equally sized (or differ by at most 1, if necessary).

The decision variables will be defined as follows, for i in $1, \dots, N$, j in $1, \dots, S$, and k in $1, \dots, G$:

$$x_{i,j,k} = \begin{cases} 1 & \text{if a passenger is assigned to row } i \text{, seat } j \text{, and group } k \\ 0 & \text{otherwise} \end{cases}$$

We assume that all seats on the plane are occupied. Since each passenger will be assigned to only one group, we can add the following constraint:

$$\sum_{k=1}^{G} x_{i,j,k} = 1$$

Also, we assume that the group sizes do not have differences larger than 1:

$$\left| \sum_{i=1}^{N} \sum_{j=1}^{S} x_{i,j,k_1} - \sum_{i=1}^{N} \sum_{j=1}^{S} x_{i,j,k_2} \right| \leq 1$$

We will now define aisle interferences. We consider two different kinds of aisle interference: (1) **between-group interference** for interference occurring between passengers in different boarding groups and (2) **within-group interference** for interference occurring between passengers in the same group.

Our goal is to minimize the time delay caused by aisle interferences. We let t_a denote the time delay caused by an aisle interference, $AB_{i,j,k}$ denote the number of between-group aisle interferences faced by the passenger assigned to row i, seat j, and group k, and $AW_{i,j,k}$ denote the number of within-group aisle interferences faced by the passenger assigned to row i, seat j, and group k. The objective function is therefore defined as

$$\text{minimize } t_a \sum_{i=1}^{N} \sum_{j=1}^{S} \sum_{k=1}^{G} (AB_{i,j,k} + AW_{i,j,k})$$

We assume that a certain portion of the previous group will not yet be seated by the time the next group begins boarding and denote this fraction by α, with $0 \leq \alpha \leq 1$.

If a person is assigned to row i, seat j, and group k, they will experience between-group aisle interference with any person who is assigned to rows 1 through i and group $k-1$. Since a fraction, α, of the previous group are assumed to have not yet been seated, the expected number of between-group interferences this person will experience is

$$\alpha \sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k-1}$$

However, the person in row i and seat j may not be assigned to group k, in which case $AB_{i,j,k} = 0$, so we define the number of between-group interferences as follows:

$$AB_{i,j,k} = \begin{cases} \alpha \sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k-1} & \text{if } x_{i,j,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

To calculate the expected number of within-group interferences a person assigned to row i, seat j, and group k will face, we assume that the passengers within each group line up randomly at the gate, so this person is equally likely to be first, second, or so on, to board in group k. Again, an interference can only occur with another passenger in rows 1 through i. The total number of passengers in group k seated in rows 1 through i is

$$\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k}$$

so the person assigned to row i and seat j has probability

$$\frac{1}{\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k}}$$

of being first, second, or so on, to board within this subset of group k. Hence, the expected number of intra-group interferences this person will face is

$$\frac{1}{\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k}} \left(0 + 1 + 2 + \cdots + \left[\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k} \right] - 1 \right) = \frac{1}{2} \left(\left[\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k} \right] - 1 \right)$$

Again, we need to consider the possibility that the passenger in row i and seat j is not assigned to group k, so we define the number of within-group seat interferences as

$$AW_{i,j,k} = \begin{cases} \frac{1}{2} \left(\left[\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k} \right] - 1 \right) & \text{if } x_{i,j,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

To capture these variables in the linear program, we introduce the following constraints, where M is some large constant (such as 10^6):

$$AB_{i,j,k} \geq -M(1 - x_{i,j,k}) + \alpha \sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k-1}$$

$$AW_{i,j,k} \geq -M(1 - x_{i,j,k}) + \frac{1}{2} \left(\left[\sum_{u=1}^{i} \sum_{j=1}^{S} x_{u,j,k} \right] - 1 \right)$$

Note that these constraints allow us to require that $AB_{i,j,k} = AW_{i,j,k} = 0$ when $x_{i,j,k} = 0$, as desired.

2.2. **Include Seat Interferences.** In a more refined model, we add seat interference to the model. As with aisle interference, we consider two types of seat interference: (1) **between-group seat interference** as interference occurring between passengers from different boarding groups, and (2) **within-group seat interference** as interference occurring between passengers from the same boarding group.

Let t_s denote the time of each seat interference, $SB_{i,j,k}$ denote the number of between-group seat interferences faced by the passenger assigned to row i, seat j, and group k, and $SW_{i,j,k}$ denote the number of within-group seat interferences faced by the passenger assigned to row i, seat j, and group k. Therefore, The objective function we are trying to minimize for this refined model is

$$\text{minimize } t_a \sum_{i=1}^{N}\sum_{j=1}^{S}\sum_{k=1}^{G}(AB_{i,j,k}+AW_{i,j,k}) + t_s \sum_{i=1}^{N}\sum_{j=1}^{S}\sum_{k=1}^{G}(SB_{i,j,k}+SW_{i,j,k})$$

For both types of seat interference, we consider interferences only between each passenger and other passengers seated in the same row, on the same side of the aisle, and closer to the aisle. For example, on a plane like the Airbus A320 with a single aisle and 3 seats on either side of the aisle, if a passenger is in seat 1 (a window seat), then the passenger might incur interferences with passengers in seats 2 and 3 (middle and aisle seats, respectively); if a passenger is in seat 5, then they will only experience interference will a passenger in seat 4. The following model describes the seat interferences present in a plane with this layout.

Between-group seat interferences will occur between this passenger and passengers in groups 1 through $k-1$. As described above, we need to discuss different situations for different seat numbers j since the number of interferences a person will experience depends on the seat number j.

When $j = 1$, the expected number of between-group seat interference this person will experience is

$$\sum_{w=1}^{k-1} x_{i,2,k} + \sum_{w=1}^{k-1} x_{i,3,k}$$

Similar to aisle interference, we need to consider the case when the person in row i and seat j is not assigned to group k, so we define the number of between-group seat interference as follows for $j = 1$:

$$SB_{i,1,k} = \begin{cases} \sum_{w=1}^{k-1} x_{i,2,w} + \sum_{w=1}^{k-1} x_{i,3,w} & \text{if } x_{i,1,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Similarly, for $j = 2$, 5 and 6:

$$SB_{i,2,k} = \begin{cases} \sum_{w=1}^{k-1} x_{i,3,w} & \text{if } x_{i,2,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$SB_{i,5,k} = \begin{cases} \sum_{w=1}^{k-1} x_{i,4,w} & \text{if } x_{i,5,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$SB_{i,6,k} = \begin{cases} \sum_{w=1}^{k-1} x_{i,4,w} + \sum_{w=1}^{k-1} x_{i,5,w} & \text{if } x_{i,6,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

When $j = 3$ or 4, there is no seat interference, so

$$SB_{i,3,k} = SB_{i,4,k} = 0$$

.

These expressions can be combined into a single expression as:

$$SB_{i,j,k} = \begin{cases} \sum_{w=1}^{k-1} \sum_{v=j+1}^{3} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \le 3 \\ \sum_{w=1}^{k-1} \sum_{v=4}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \ge 4 \\ 0 & \text{otherwise} \end{cases}$$

Within-group seat interferences will occur only between this passenger and passengers in the same group k. Again, the number of interferences a passenger will experience depends on their seat number. It is clear that for passengers sitting in seats 3 and 4, who are next to the aisle, will not experience any seat interference when they are seated, so

$$SW_{i,3,k} = SW_{i,4,k} = 0$$

.

For passengers in seat $j = 1$, there are 4 cases in which interference would happen:

(1) When the passengers in row i and seats 2 and 3 are both not assigned to group k, then $x_{i,2,k} = x_{i,3,k} = 0$ and the number of within-group seat interferences is 0.
(2) When the passenger in row i and seat 2 is assigned to group k, but not seat 3, then $x_{i,2,k} = 1$ and $x_{i,3,k} = 0$. Assuming random line up within each group, the probability that this passenger stands in front of (and hence gets seated before) the passenger in seat 1 is 0.5. Therefore, the expected interference in this case is $0.5 \times 1 = 0.5(x_{i,2,k} + x_{i,3,k}) = 0.5$.
(3) When the passengers in row i and seat 3 is assigned to group k, but not seat 2, the expected interference is $0.5 \times 1 = 0.5(x_{i,2,k} + x_{i,3,k}) = 0.5$, similar to (2).
(4) When the passengers in row i and seats 2 and 3 are both assigned to group k, which means that $x_{i,2,k} = 1$ and $x_{i,3,k} = 1$, there are 6 possible situations (the order of the passengers in seat $j = 1$, $j = 2$ and $j = 3$ get seated):
1-2-3 (0 interferences), 1-3-2 (0 interferences),
2-1-3 (1 interference), 2-3-1 (2 interferences),
3-1-2 (1 interference), 3-2-1 (2 interferences),
and each has the same probability $\frac{1}{6}$. Therefore, the expected number of within-group inferences is $\frac{1}{6}(0 + 0 + 1 + 2 + 1 + 2) = 1 = 0.5(x_{i,2,k} + x_{i,3,k})$.

Considering the situation when the passenger in row i and seat $j = 1$ is not assigned to group k ($x_{i,1,k} = 0$), we have

$$SW_{i,1,k} = \begin{cases} 0.5(x_{i,2,k} + x_{i,3,k}) & \text{if } x_{i,1,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Similarly, for passengers in row i and seat $j = 6$, we have

$$SW_{i,6,k} = \begin{cases} 0.5(x_{i,4,k} + x_{i,5,k}) & \text{if } x_{i,6,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

For passengers in seat $j = 2$, there are 2 cases:

(1) When the passenger in row i and seat 3 is not assigned to group k, then $x_{i,3,k} = 0$ and the number of within-group seat interferences is 0.
(2) When the passenger in row i and seat 3 is assigned to group k, $x_{i,3,k} = 1$. Assuming random line up within each group, the probability that this passenger stands in front of (and get seated before) the passenger in seat $j = 2$ is 0.5. Therefore, the expected number of interferences in this case is $0.5 = 0.5x_{i,3,k}$.

Considering the situation when the passenger in row i and seat $j = 1$ is not assigned to group k ($x_{i,2,k} = 0$), we have

$$SW_{i,2,k} = \begin{cases} 0.5x_{i,3,k} & \text{if } x_{i,2,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Similarly, for passengers in row i seat $j = 5$, we have

$$SW_{i,5,k} = \begin{cases} 0.5x_{i,4,k} & \text{if } x_{i,5,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

As before, we can capture the SB and SW variables in the linear program by introducing one constraint per variable, where each constraint contains a term $-M(1 - x_{i,j,k})$ to require that $SB_{i,j,k} = SW_{i,j,k} = 0$ when $x_{i,j,k} = 0$, as desired. The full model is in appendix.

2.3. **Allow Multiple Aisles and Different Seat Arrangements.** A refinement of the above model is to allow different interior layouts, such as having two aisles with three sections of seats or having two seats on either side of a single aisle. Both of these arrangements are commonplace, so it would be useful to see how the optimal boarding strategy changes.

To model planes with a single aisle where the number of seats on either side of the aisle is not equal to 3, only the constraints controlling the seat interference variables need to change. Though a generic model can be built based on arbitrary S, the most common aircraft model with only one aisle, other than three seats per side of the aisle, has two seats per side of the aisle. For this case where $S = 4$, the seat interference expressions need to modified as follows:

$$SB_{i,j,k} = \begin{cases} \sum_{w=1}^{k-1} \sum_{v=j+1}^{2} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 2 \\ \sum_{w=1}^{k-1} \sum_{v=3}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$SW_{i,j,k} = \begin{cases} 0.5 \sum_{v=j+1}^{2} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 2 \\ 0.5 \sum_{v=3}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

To model planes with multiple aisles, it is necessary to modify the constraints for all types of interference. Consider the Airbus A380, which has two aisles, 3 seats per row on the outside of each aisle, and 4 seats per row between the two aisles. Enumerate the seats left-to-right, as before, so seats 1-3 are on the left side of the first aisle, seats 4-7 are between the aisles, and seats 8-10 are on the right side of the aisle. We assume that passengers in seats 1-5 will board via the left aisle and passengers in seats 6-10 will board via the right aisle.

Having two aisles means that a passenger's seat number impacts the number of aisle interferences they will face, because they will now only experience aisle interferences with the passengers who board via the same aisle as them:

$$AB_{i,j,k} = \begin{cases} \alpha \sum_{u=1}^{i} \sum_{j=1}^{5} x_{u,j,k-1} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 5 \\ \alpha \sum_{u=1}^{i} \sum_{j=6}^{10} x_{u,j,k-1} & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 6 \\ 0 & \text{otherwise} \end{cases}$$

$$AW_{i,j,k} = \begin{cases} \frac{1}{2} \left(\sum_{u=1}^{i} \sum_{j=1}^{5} (x_{u,j,k}) - 1 \right) & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 5 \\ \frac{1}{2} \left(\sum_{u=1}^{i} \sum_{j=6}^{10} (x_{u,j,k}) - 1 \right) & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 6 \\ 0 & \text{otherwise} \end{cases}$$

The seat interferences a passenger will face will change similarly:

$$SB_{i,j,k} = \begin{cases} \sum\limits_{w=1}^{k-1} \sum\limits_{v=j+1}^{3} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 3 \\ \sum\limits_{w=1}^{k-1} \sum\limits_{v=4}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 5 \\ \sum\limits_{w=1}^{k-1} \sum\limits_{v=j+1}^{7} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 7 \\ \sum\limits_{w=1}^{k-1} \sum\limits_{v=8}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 8 \\ 0 & \text{otherwise} \end{cases}$$

$$SW_{i,j,k} = \begin{cases} 0.5 \sum\limits_{v=j+1}^{3} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 3 \\ 0.5 \sum\limits_{v=4}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 5 \\ 0.5 \sum\limits_{v=j+1}^{7} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \leq 7 \\ 0.5 \sum\limits_{v=8}^{j-1} x_{i,v,w} & \text{if } x_{i,j,k} = 1 \text{ and } j \geq 8 \\ 0 & \text{otherwise} \end{cases}$$

It is worth noting that the layout changes only affect the interference expressions, and have no effect on the objective function nor the decision variable definitions.

3. Data

As discussed above, we consider a few different airplane layouts. Preliminary results were produced using the toy model with 8 rows of 6 seats each to validate the models. We also use this model to explore the effects of varying certain parameters, because its relatively expedient runtime makes such testing feasible. Furthermore, we examine optimal boarding strategies for two real airplane models, the Airbus A320, which has 23 rows, six seats per row and one aisle, and the Airbus A380, which has two aisles and 33 rows.

In Kuo's paper, the constants t_a and t_s are assumed, based on previous literature, to be 2.4 and 3.6, respectively.

4. Algorithms

We use the Gurobi libraries in Python to solve the ILP we described above; the Gurobi Optimization Solver uses branch-and-bound techniques. The scripts used to generate our results can be found in the appendix.

5. Results

5.1. **Basic Model.** We first implemented a basic model, taking into account only delays caused by aisle interferences, to generate preliminary results using the toy example with 8 rows and 6 seats per row. Passengers are split among 3 boarding groups. We assume that half of the passengers in the previous group are not yet seated when the next group begins boarding (i.e. $\alpha = 0.5$) and that the delay caused by each aisle interference is 2.4 seconds (i.e. $t_a = 2.4$).

The boarding group assignment of each passenger is illustrated in Figure 1. The total delay caused by passenger interferences is 576 seconds, or 9.6 minutes. Note that this quantity is not equal to the total time required to board the plane, because we allow interferences to happen simultaneously. However, it is still a meaningful measure because it measures the total time of interference all passengers experienced, so it is positively correlate to the total passenger satisfaction. The optimal boarding strategy using the basic model is back-to-front, where the back rows of the plane are boarded first and the front rows boarded last. Using this optimal strategy, all boarding groups are the same size. Because we do not include any delays based on seat interferences, the seat order within the rows which contain passengers are boarded in multiple zones is randomly arranged.

5.2. **Include Seat Interferences.** The next step was adding seat interferences to the basic model. As before, we assume $\alpha = 0.5$, and assign penalties $t_a = 2.4$ and $t_s = 3.6$ to interferences. We again split the passengers into 3 boarding groups.

Figure 2 shows the optimal boarding group assignment when seat interferences are included. The total delay caused by passenger interference is 632.8 seconds, or 10.6 minutes. The plane is again boarded from back to front; seat order within each row, however, is no longer random and we begin to observe a boarding pattern referred to as Reverse Pyramid (RP)[1] in the literature.

5.3. **Changes in Inputs and Coefficients.** In order to push beyond the results of Kuo's paper we needed to experiment with different inputs to the model. What we found quickly was that this is computationally difficult especially when dealing with larger/realistic plane sizes. To that end we experimented primarily over small planes such as the single aisle 8 row by 6 seat layout seen earlier.

We found that changes in α led to compelling results when $G \leq 3$ as we attempted to match the results from Kuo with a variable N and $0 \leq \alpha \leq 1$. We confirmed that at low α values the model would initially optimize to an Outside-In (OI)[2] loading scheme while at a high α the Reverse Pyramid scheme would be adopted and furthermore total interference grows proportionally to α.

What we found particularly interesting while investigating this is that we discovered that, as α approaches 1 from 0, this transition in loading scheme isn't gradual- there is a "breaking point" afterwards the model's optimal layout remains fixed as RP.

For example Figure 3 shows (from left to right) the seating layouts for $N = 4$, $S = 6$, $G = 3$ and $0 \leq \alpha \leq 1$ over step size 0.1, we see the breaking point occurs around $\alpha = 0.4$ (the fifth seating diagram) however in Figure 4 with $N = 8$ and the same alpha step size we see the breaking point occurs somewhere around 0.5.

We concluded from these tests that there is most likely a unique minimal breaking point α_b given constant N, S, and small G afterwhich an airplane should be loaded in the RP method. Given the computation time and the number of variables it is difficult to derive a formula to find α_b nor is it truly realistic for an airline to compute α prior to loading as it is by definition a measurement of disorganization.

We also discovered exciting results when adjusting group size G. Although the graph on the right is not true optimal solution (it was only run to a 10% GAP), Figure 5 illustrates how chaotic the loading strategy becomes as G grows. The juiciest part of adjusting the group size however is how reliably the total interferences decreases as G increases. As the following table displays, the savings in interference are significantly impacted:

G	1	2	3	4	6	9
obj. val:	914.39	519.2	383.2	313.6	203.258	135.62

Unfortunately the computation time for greater group sizes grows at an unprecedented rate compared to the other inputs and coefficients we've seen- referring again to Figure 5, it took only 2.5 minutes to calculate the solution on the left whereas it took over 4 hours!

5.4. **Airbus A320 and A380.** We lastly look to the results running the model over the larger, realistic input of an Airbus A320 and A380 and the layouts produced by these two planes can be found in Figure 6 and Figure 7 respectively. While the output layout of the LP with these inputs were unsurprising and the length of the planes making them hard to experiment upon, we began to re-investigate the significance of the objective value.

6. Discussion

As discussed in Section 5.2, the total delay incurred by passengers on a plane with 8 rows, 6 seats per row, and one central aisle is 632.8 seconds. We were interested in investigating how adding a second aisle would reduce or increase the overall delay that passengers face. We tested this idea by running our model on another plane with 8 rows and 6 seats per row, with 2 seats in between the two aisles and 2 seats on either

[1]The plane is loaded rear windows first, followed by rear middle, front window, front middle, rear aisle, and finally front aisle

[2]Window seat passengers are loaded first, followed by middle seat then lastly aisle seat passengers.

side of the aisles. Under this group assignment, the total delay experienced by passengers is 586.8 seconds, 46 seconds less than the total delay for the single-aisled plane. This suggests that airlines will experience more expedient boarding for multi-aisled planes than for single-aisled planes of the same size. As noted above, the same Reverse Pyramid boarding strategy is utilized for the multi-aisled plane.

Moving the door from the front of the plane to the middle, so passengers can walk to both directions at the same time, will surely reduce the boarding time. However, the change of door location will not affect group assignments. It can be seen as adding another lane and shorter the plane length to $\frac{N}{2}$.

The biggest challenge we've had is solving the ILP proved time-consuming; while running the toy example with only 8 rows took around 3 minutes to complete, running the realistic 23 row plane took over 30 minutes. Furthermore, as discussed in Section 5.3 attempting the model with varying inputs took hours to compute to just a decent approximation. This prevented us from exploring the solution space as much as we'd have liked.

In order to combat the combinatorial burden, we implemented Gurobi's `TimeLimit` and `MIPGap` parameters which stopped the optimization process early upon exceeding a given time limit[3] or meeting a specified gap percentage respectively [4]. The results are rather reliable as an estimation and are usually very close to the optimal solution at a complete run. The minor variation is at the rows when two different groups contact which usually have only a few group numbers assigned differently from the fully run optimal.

7. Questions for Future Research

Our next steps are to run our current model over larger problem sizes with bounds on computation time and to extend our model to include empty or pre-occupied layover seats.

Further questions we may or may not have the time to research are (in no specific order):

(1) Empty seat modeling: Given n unfilled seats, how significant is the impact of having these seats sporadically distributed vs. strategically (ie. pseduo-randomly arranged vs. all in one row at the back of the plane)? Do different groupings of empty seats produce fewer interferences?
(2) Straggler modeling: given G groups, occasionally within each group are passengers who either refuse to board with their group (or are just in the washroom and miss the boarding call). Given a fraction of absentees per group, comparatively how much interference is created/saved by all the stragglers loading up as one disorganized group after the rest of the passengers have boarded (not counting for time wasted on the runway).
(3) Wrong seat modeling: given some subset with size ≥ 1 of currently boarded passengers who board into the wrong seats (say $x_{i,j,1}$ boards with group 1 but sits in $x_{i,j,3}$'s seat who boards with group 3) how much additional interference is caused considering that this situation creates additional aisle and seating interference when $x_{i,j,1}$ and $x_{i,j,3}$ swap and then have $x_{i,j,1}$ find their correct seat. -be difficult to model, maybe with a variable
(4) Optimal actual boarding time: instead of using the total delay experienced by all passengers on the plane, define the objective function to calculate the actual time taken from the first passenger getting in the plane to the last one being seated.

8. Conclusion

In this paper, binary linear programming is used to identify the optimal boarding strategy for different parameter sets. Based on the Airbus A320 and A380, two commonly used aircraft, we developed our linear programming model such that the objective function is the total delay across the plane that passengers experience during boarding. It is interesting to see that as α increases, the optimal boarding group assignment shifts from an Outside-In to Reverse Pyramid loading scheme. This implies the RP method is resilient to high amounts of human disorganization which is a reassuring result as the RP scheme is a popular strategy in aircraft loading around the world. We found that increasing the total number of groups made significant improvements to the objective value but without any discerning pattern thus making it very difficult to derive a modular strategy for any given input. In general, we can conclude that the Reverse Pyramid boarding strategy is the safest one to apply as it is easy to implement and compute and it works well with small group counts; we also admit that, due to the number of constraints and variables required by the model

[3] depending on the experiment this was anywhere from 60 seconds to 8 hours.

[4] usually between 10% and 40%

and the current computational limitations, we are unable to definitively say that no better strategy could be obtained through further analysis, such as finding an optimal G value for a plane prior to running the optimization.

References

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[2] TripAdvisor LLC. (2015). Delta Seat Map: Airbus A320 (320/32R) V1. Retrieved from `https://www.seatguru.com/airlines/Delta_Airlines/Delta_Airlines_Airbus_A320.php#seatMapKeyLegend`

[3] TripAdvisor LLC. (2015). Singapore Airlines Seat Maps: Airbus A380-800(388) V1. Retrieved from `https://www.seatguru.com/airlines/Singapore_Air/Singapore_Air_Airbus_A380.php`

Appendix A. Parameters

$x_{i,j,k}$	The decision variables, 1 for passenger in row i seat j assigned to group k, 0 for not.
N	Number of rows of seats.
S	Number of seats per row.
G	Number of boarding groups.
α	The portion of the previous group will not yet be seated by the time the next group begins boarding.
t_a	Time required to clear an aisle interference.
t_s	Time required to clear a seat interference.
$AB_{i,j,k}$	Expected number of between-group aisle interference the passenger in row i seat j group k will experience.
$AW_{i,j,k}$	Expected number of within-group aisle interference the passenger in row i seat j group k will experience.
$SB_{i,j,k}$	Expected number of between-group seat interference the passenger in row i seat j group k will experience.
$SW_{i,j,k}$	Expected number of within-group seat interference the passenger in row i seat j group k will experience.
M	A big number, 10^6.

Figures

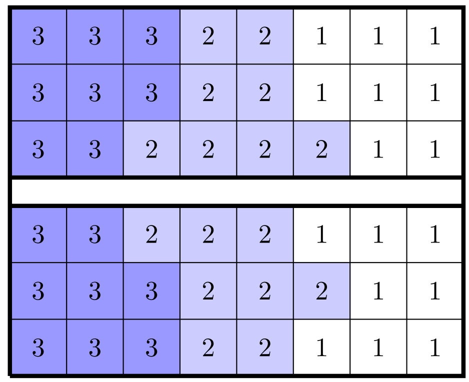

Figure 1. Optimal boarding strategy for plane with 8 rows and 6 seats per row, using basic model. The numbers represent the boarding group to which each passenger is assigned. In this figure and all following plane layout diagrams we have the front of the plane on the left, the back on the right and the negative space between nodes to be aisles.

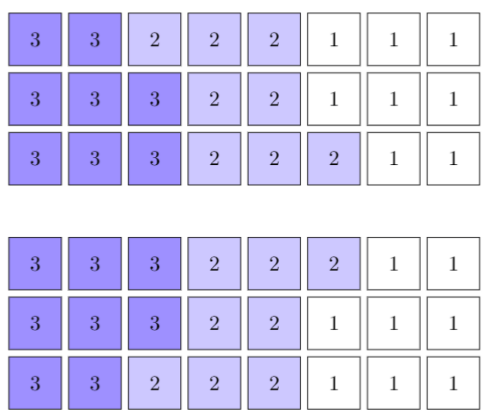

Figure 2. Optimal boarding strategy for plane with 8 rows and 6 seats per row, using model including seat interferences. The numbers and colors represent the boarding group to which each passenger is assigned.

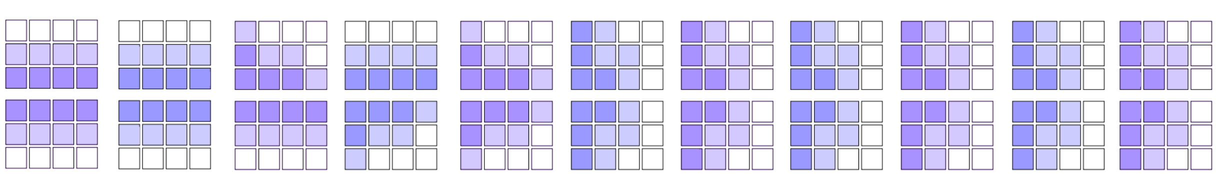

Figure 3. Optimal boarding strategies for plane with 4 rows and 6 seats per row, with $0 \leq \alpha \leq 1$ with step size $= 0.1$ (read left to right).

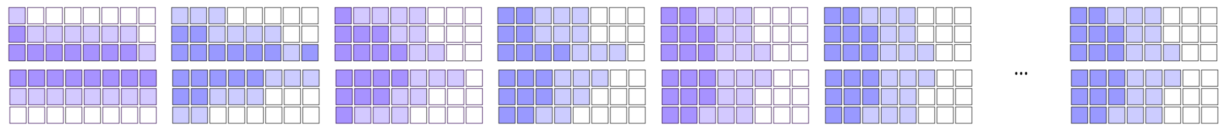

Figure 4. Optimal boarding strategies for plane with 8 rows and 6 seats per row, with $0.1 \leq \alpha \leq 1$ with step size $= 0.1$, $\alpha = 0.6, \ldots, 0.9$ omitted as they are consistent with the 5^{th} and 6^{th} diagram where $\alpha = 0.5, \alpha = 1$ (read left to right)

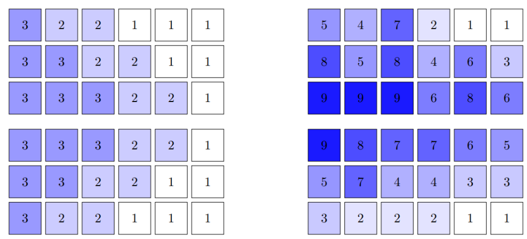

Figure 5. Optimal boarding strategy for plane with 8 rows and 6 seats per row, using model including seat interferences. The numbers and colors represent the boarding group to which each passenger is assigned.

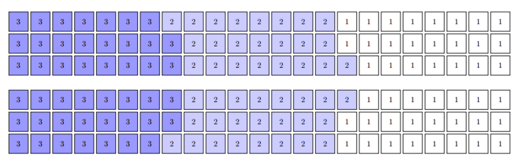

Figure 6. Optimal boarding strategy for Airbus A320 which has 6 seats by 23 rows and $G = 3$.

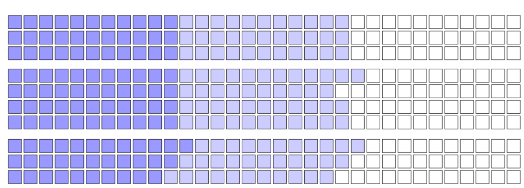

Figure 7. Optimal boarding strategy for Airbus A380 which has 10 seats by 33 rows and $G = 3$.