HWK1: 1. (3.1-8.)

$\Omega(g(n,m)) = \{ f(n,m) : \exists c, n_0, m_0 > 0, \text{ s.t. } \forall n \geq n_0, m \geq m_0, \ 0 \leq c g(n,m) \leq f(n,m) \}$

$\Theta(g(n,m)) = \{ f(n,m) : \exists c_1, c_2, n_0, m_0 > 0, \text{ s.t. } \forall n \geq n_0 \ m \geq m_0 \ 0 \leq c_1 g(n,m) \leq f(n,m) \leq c_2 g(n,m) \}$

2. (3-3)

a.

$2^{2^{n+1}}$ $\quad 2^{2^n}$

$(n+1)!$ $\quad n!$

e^n

$n \cdot 2^n$

2^n

$(\frac{3}{2})^n$

$n^{\lg\lg n} = (\lg n)^{\lg n}$

$(\lg n)!$

n^3

$n^2 = 4^{\lg n}$

$n\lg n, \lg(n!)$

$n = 2^{\lg n}$

$\sqrt{n}$

$2^{\sqrt{2\lg n}}$

$\lg^2 n$

$\ln n$

$\sqrt{\lg n}$

b. $y = |\tan x|$

$\ln \ln n$

$2^{\lg^* n}$

$\lg^* n \quad \lg^*(\lg n)$

$\lg(\lg^*) n$

$n^{\frac{1}{\lg n}} = 1$

3.(3-4)

b. 错 令 $f(x)=x \quad g(x)=x^2$

$\lim\limits_{x\to\infty} \frac{f(x)+g(x)}{\min(f(x),g(x))} = \lim\limits_{x\to\infty} 1+x = \infty$

与题设矛盾

∴原题不成立.

c. 对 $f(n)=O(g(n)) \Rightarrow \exists c, n_0>0$. s.t 对 $\forall n>n_0$, $f(n)\le c\,g(n)$

又 $\lg(g(n))\ge 1$

$f(n)\ge 1$

∴此时 $\lg f(n) \le \lg(g(n))+\lg c$

令 $d = \frac{\lg c+\lg g(n)}{\lg g(n)} = \frac{\lg c}{\lg g(n)}+1 \le \lg c+1$

∴ $\exists d, n_0>0$ s.t 对 $\forall n>n_0$, $\lg(f(n)) \le d\lg(g(n))$

∴ $\lg(f(n)) = O(\lg(g(n)))$

d. 错 令 $f(n)=n \quad g(n)=2n$

$2^{f(n)}=2^n \quad 2^{g(n)}=4^n$

$\lim\limits_{n\to\infty} \frac{2^{g(n)}}{2^{f(n)}} = \infty$

∴原题设错误.

g. 错　令 $f(n)=4^n$　则 $f(\frac{n}{2})=2^n$

显然 $4^n \neq \Theta(2^n)$

h. 正确　令 $o(f(n)) = g(n)$

则 $\exists c, n_0$, 对 $\forall n \geq n_0$, $cf(n) \geq g(n)$

$\therefore (1+c)f(n) \geq f(n) + o(f(n)) \geq f(n)$

$\therefore f(n) + o(f(n)) = \Theta(f(n))$