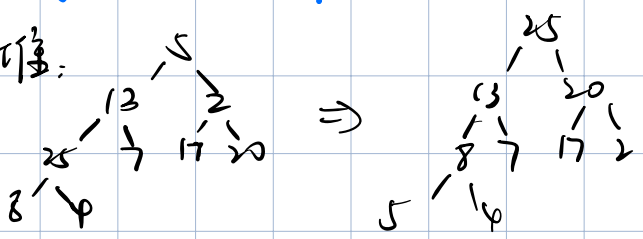
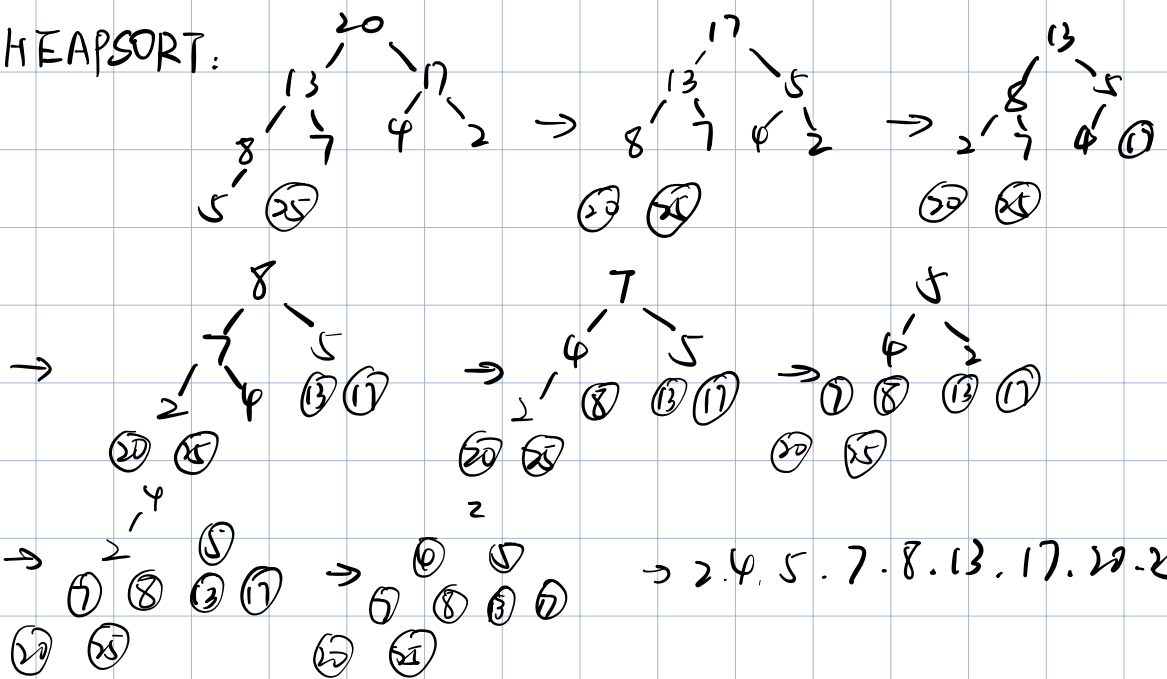
HWK4. 卢宏宇 2023202269

6.4-1：首先建堆：



HEAPSORT：



→ 2.4.5.7.8.13.17.20.25.

6.5-9. 这里heap中元素为链表指针，根据链表指针指向的value排序

```
Extern-Sort (Linkhead[k])

    A[k].
    BuildLinkList(head)
     current = head.

    for i=1 to k.
        A[i] = Linkhead[i]
    BUILD-Min-HEAP(A).
    While A.heapsize ≠ 0:
        pointer = HEAP-EXTRACT-MIN(A)
```

```
current -> value = pointer -> value.
current = current -> next
pointer = pointer -> next
if pointer ≠ nullptr
    HEAP-INSERT(A, pointer)
return head
```

7.2-5 设最小深度为 d_1

则 $n\cdot(\alpha)^{d_1}=1 \Rightarrow d_1=\log_{\alpha}\frac{1}{n}=\frac{-\lg n}{\lg\alpha}$

最大深度为 d_2

则 $n\cdot(1-\alpha)^{d_2}=1 \Rightarrow d_2=\log_{1-\alpha}\frac{1}{n}=\frac{-\lg n}{\lg(1-\alpha)}$

7.5 a. 共有 C_n^3 种取法，对于 $A[i]$ 为主元时共有 $(i-1)(n-i)$ 种取法.

$$p_i=\frac{(i-1)(n-i)}{C_n^3}$$

b. 平凡：$\frac{1}{n}$

$$\lim_{n\to\infty}\frac{\frac{6\cdot(\frac{n+1}{2}-1)(n-\frac{n+1}{2})}{n(n-1)(n-2)}}{\frac{1}{n}}=\frac{3}{2}$$ 增加到 $\frac{3}{2}$ 倍.

c. 平凡：$\frac{1}{3}$

$$\lim_{n\to\infty}\frac{\int_{\frac{n}{3}}^{\frac{2n}{3}}\frac{(x-1)(n-x)}{C_n^3}dx}{\frac{1}{3}}=\lim_{n\to\infty}3\int_{\frac{n}{3}}^{\frac{2n}{3}}\frac{-x^2+(n+1)x-n}{C_n^3}dx$$

$$=\lim_{n\to\infty}\frac{3}{C_n^3}\left[-\frac{7}{81}n^3+\frac{(n+1)}{6}\cdot n^2-\frac{n^2}{3}\right]$$

$$=\frac{13}{9}$$

增加到 $\frac{13}{9}$ 倍.

d. 根据书中计算 RANDOMIZED-QUICKSORT 期望时间的计算 $\frac{(i-1)(n-i)}{C_n^3}$ $\frac{(i+1)(n-i)}{C_n^3}$

同理可得 $E(X) \approx \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{C_n^3 + C_n^3}{\int_i^j \frac{(x-1)(n-x)}{C_n^3} dx}$

$$\approx \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{jn - n - i^2 + i + jn - n - j^2 + j}{-\frac{1}{3}(j^3 - i^3) + \frac{1}{2}(n+1)(j^2 - i^2) + j - i}$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Omega\left(\frac{1}{j-i+1}\right)$$

$$= \Omega(n \lg n)$$

∴ 只改变期望常数项因子.